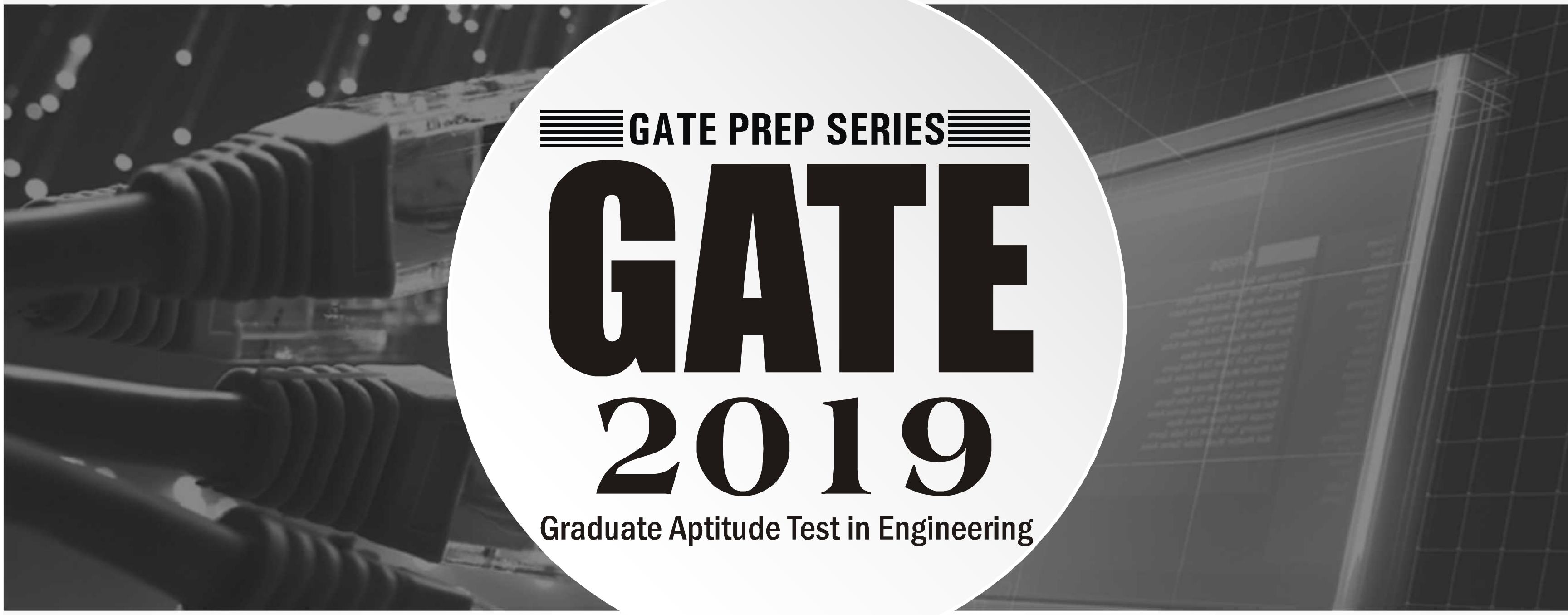

GATE PREP SERIES

GATE 2019

Graduate Aptitude Test in Engineering

Computer Science & Information Technology

G. K. PUBLICATIONS (P) LTD.

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GATE

Graduate Aptitude Test in Engineering

IIT Institutes

Indian Institute of Technology, Mumbai

Indian Institute of Technology, Kharagpur

Indian Institute of Technology, Delhi

GATE 2019 will be conducted by
Indian Institute of Technology, Chennai

GATE 2018 Conducted by
Indian Institute of Technology, Guwahati

Indian Institute of Technology, Roorkee

Indian Institute of Technology, Kanpur

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Preface

The Graduate Aptitude Test in Engineering (GATE) is an online exam conducted by the IITs for admissions to PG courses in IITs, IISc Bangalore, NITs and many state run universities as well as private universities. Also there are more than 37 PSUs that use GATE score for recruitments. A large number of corporates are also using GATE score as a tool to screen students for placements.

GK Publications is well known as the "publisher of choice" to students preparing for GATE and other technical test prep examinations in the country. We published first set of books in 1994 when GATE exam, both objective and conventional, was conducted in the paper and pencil environment, and used as a check point for entry to post graduate courses in IITs and IISCs. At that time, students had little access to technology and relied mainly on instructor led learning followed by practice with books available for these examinations.

A lot has changed since then!

Today, GATE is conducted in an online only mode with multiple choice and numerical based questions. The score is valid for three years and is used not only for post graduate courses but is also used by major PSUs for recruitment. Today's students have easy access to technology and the concept of a monologue within the classroom has changed to dialogue where students come prepared with concepts and then discuss topics. They learn a lot of things on the go with their mobile devices and practice for mock tests online.

We, as a leading publisher of GATE books, have also embraced change. Today, our books are no more guides only but come with a fully supported mobile app and a web portal. The mobile App provides access to video lectures, short tests and regular updates about the exam. The web portal in additional to what is available on the App provides full length mock tests to mimic the actual exam and help you gauge your level of preparedness. The combination of practice content in print, video lectures, and short and full length tests on mobile and web makes this product a complete courseware for GATE preparation.

We also know that improvement is a never ending process and hence we welcome your suggestions and feedback or spelling and technical errors if any. Please write to us at gkp@gkpublications.com

We hope that our small effort will help you prepare well for the examination.

We wish you all the best!

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About GATE

The Graduate Aptitude Test in Engineering (GATE) conducted by IISc and IITs has emerged as one of the bench mark tests for engineering and science aptitude in facilitating admissions for higher education (M.Tech./Ph.D.) in IITs, IISc and various other Institutes/Universities/ Laboratories in India. With the standard and high quality of the GATE examination in 23 disciplines of engineering and science subjects, it identifies the candidate's understanding of a subject and aptitude and eligibility for higher studies. During the last few years, GATE score is also being used as one of the criteria for recruitment in Government Organizations such as Cabinet Secretariat, and National/State Public Sector Undertakings in India. Because of the importance of the GATE examination, the number of candidates taking up GATE exams has increased tremendously. GATE exams are conducted by the IITs and IISc as a computer based test having multiple choice questions and numerical answer type questions. The questions are mostly fundamental, concept based and thought provoking. From 2017 onwards GATE Exam is being held in Bangladesh, Ethiopia, Nepal, Singapore, Sri Lanka and United Arab Emirates. An Institute with various nationalities in its campus widens the horizons of an academic environment. A foreign student brings with him/her a great diversity, culture and wisdom to share. Many GATE qualified candidates are paid scholarships/assistantship, especially funded by Ministry of Human Resources Development, Government of India and by other Ministries. Indian Institute of Technology Guwahati is the Organizing Institute for GATE 2018.

New in GATE 2017 Onward

- GATE examinations shall be conducted in Bangladesh, Ethiopia, Nepal, Singapore, Sri Lanka and United Arab Emirates. The pattern of examination remains same, exam timing/Center shall be notified in admit cards. The International candidates should have completed Bachelor's degree in Engineering or Master's degree in Science in relevant subject or are in the final year of the program.
- The session timings at GATE International Centers according to their local time shall be notified well in advance and will be displayed on GATE website (www.gate.iitr.ernet.in).
- The XE (Engineering Sciences) paper now has a new section on Atmospheric and Oceanic Sciences (section H). With this the number of sections in the XE paper has increased to eight (A-H).
- No restriction on second choice examination city to be in the same zone as first choice city. It can be anywhere in India. Thumb impression of candidates needs to be uploaded during the application procedure. Candidates who have completed the qualifying degree, need to upload scanned image of degree completion certificate or final year/semester mark sheet. However, candidates studying in the final year of their qualifying degree need to upload scanned image of pre-final year (or fifth/sixth semester) mark sheet, or provisional certificate letter from his/her Institute as proof of eligibility criterion. Similarly this applies to candidates of AMIE, AMICE and other similar programs.

Why GATE?

Admission to Post Graduate and Doctoral Programmes

Admission to postgraduate programmes with MHRD and some other government scholarships/ assistantships in engineering colleges/institutes is open to those who qualify through GATE. GATE qualified candidates with Bachelor's degree in Engineering/Technology/Architecture or Master's degree in any branch of Science/Mathematics/Statistics/Computer Applications are eligible for admission to Master/Doctoral programmes in Engineering/Technology/Architecture as well as for Doctoral programmes in relevant branches of Science with MHRD or other government scholarships/assistantships. Candidates with Master's degree in Engineering/ Technology/Architecture may seek admission to relevant PhD programmes with scholarship/ assistantship without appearing in the GATE examination.

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Financial Assistance

A valid GATE score is essential for obtaining financial assistance during Master's programs and direct Doctoral programs in Engineering/Technology/Architecture, and Doctoral programs in relevant branches of Science in Institutes supported by the MHRD or other Government agencies. As per the directives of the MHRD, the following procedure is to be adopted for admission to the post-graduate programs (Master's and Doctoral) with MHRD scholarship/ assistantship. Depending upon the norms adopted by a specific institute or department of the Institute, a candidate may be admitted directly into a course based on his/her performance in GATE only **or** based on his/her performance in GATE **and** an admission test/interview conducted by the department to which he/she has applied **and/or** the candidate's academic record. If the candidate is to be selected through test/interview for post-graduate programs, a minimum of 70% weightage will be given to the performance in GATE and the remaining 30% weightage will be given to the candidate's performance in test/interview and/or academic record, as per MHRD guidelines. The admitting institutes could however prescribe a minimum passing percentage of marks in the test/interview. Some colleges/institutes specify GATE qualification as the mandatory requirement even for admission without MHRD scholarship/assistantship.

To avail of the financial assistance (scholarship), the candidate must first secure admission to a program in these Institutes, by a procedure that could vary from institute to institute. Qualification in GATE is also a minimum requirement to apply for various fellowships awarded by many Government organizations. Candidates are advised to seek complete details of admission procedures and availability of MHRD scholarship/assistantship from the concerned admitting institution. The criteria for postgraduate admission with scholarship/assistantship could be different for different institutions. The management of the post-graduate scholarship/ assistantship is also the responsibility of the admitting institution. Similarly, reservation of seats under different categories is as per the policies and norms prevailing at the admitting institution and Government of India rules. *GATE offices will usually not entertain any enquiry about admission, reservation of seats and/or award of scholarship/assistantship*.

PSU Recruitments

As many as 37 PSUs are using GATE score for recruitment. It is likely that more number of PSUs may start doing so by next year. Below is the list of PSUs:

MDL, BPCL, GAIL, NLC LTD, CEL, Indian Oil, HPCL, NBPC, NECC, BHEL, WBSEDCL, NTPC, ONGC, Oil India, Power Grid, Cabinet Secretariat, Govt. of India, BAARC, NFL, IPR, PSPCL, PSTCL, DRDO, OPGC Ltd., THDC India Ltd., BBNL, RITES, IRCON, GHECL, NHAI, KRIBHCO, Mumbai Railway Vikas Corporation Ltd. (MRVC Ltd.), National Textile Corporation, Coal India Ltd., BNPM, AAI, NALCO, EdCIL India.

Important :

1. Admissions in IITs/IISc or other Institutes for M.Tech./Ph.D. through GATE scores shall be advertised separately by the Institutes and GATE does not take the responsibility of admissions.
2. Cabinet Secretariat has decided to recruit officers for the post of Senior Field Officer (Tele) (From GATE papers of EC, CS, PH), Senior Research Officer (Crypto) (From GATE papers of EC, CS, MA), Senior Research Officer (S&T) (From GATE papers EC, CS, CY, PH, AE, BT) in the Telecommunication Cadre, Cryptographic Cadre and Science & Technology Unit respectively of Cabinet Secretariat. The details of the scheme of recruitment shall be published in National Newspaper/Employment News by the concerned authority.
3. Some PSUs in India have expressed their interest to utilize GATE scores for their recruitment purpose. The Organizations who intend to utilize GATE scores shall make separate advertisement for this purpose in Newspapers etc.

Who Can Appear for GATE?

Eligibility for GATE

Before starting the application process, the candidate must ensure that he/she meets the eligibility criteria of GATE given in Table.

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Eligibility Criteria for GATE 2019

Qualifying Degree	Qualifying Degree/Examination (Descriptive)	Description of Eligible Candidates	Year of qualification cannot be later than
B.E./B.Tech./ B.Pharm.	Bachelor's degree holders in Engineering/ Technology (4 years after 10+2 or 3 years after B.Sc./Diploma in Engineering/ Technology)	Currently in the final year or already completed	2019
B. Arch.	Bachelor's degree holders of Architecture (Five years course)	Currently in the final year or already completed	2019
B.Sc. (Research)/ B.S.	Bachelor's degree in Science (Post-Diploma/4 years after 10+2)	Currently in the 4th year or already completed	2019
M. Sc./ M.A./MCA or equivalent	Master's degree in any branch of Science/Mathematics / Statistics / Computer Applications or equivalent	Currently in the final year or already completed	2019
Int. M.E. / M.Tech. (Post-B.Sc.)	Integrated Master's degree programs in Engineering / Technology (Four year program)	Currently in the 2nd/3rd/4th year or already completed	2021
Int. M.E./ M.Tech. or Dual Degree(after Diploma or 10+2)	Integrated Master's degree program or Dual Degree program in Engineering / Technology (Five year program)	Currently in the 4th/5th year or already completed	2020
Int. M.Sc./ Int. B.S.-M.S.	Integrated M.Sc. or Five year integrated B.S.-M.S. program	Currently in the final year or already completed	2019
Professional Society Examinations (equivalent to B.E./B.Tech./B.Arch.)	B.E./B.Tech./B.Arch. equivalent examinations, of Professional Societies, recognized by MHRD/UPSC/AICTE (e.g., AMIE by Institution of Engineers-India, AMICE by the Institute of Civil Engineers-India)	Completed section A or equivalent of such professional courses	NA

In case a candidate has passed one of the qualifying examinations as mentioned above in 2018 or earlier, the candidate has to submit the degree certificate / provisional certificate / course completion certificate / professional certificate / membership certificate issued by the society or institute. In case, the candidate is expected to complete one of the qualifying criteria in 2019 or later as mentioned above, he/she has to submit a certificate from Principal or a copy of marks card for section A of AMIE.

Certificate From Principal

Candidates who have to submit a certificate from their college Principal have to obtain one from his/her institution beforehand and upload the same during the online submission of the application form.

Candidates With Backlogs

Candidates, who have appeared in the final semester/year exam in 2019, but with a backlog (arrears/failed subjects) in any of the papers in their qualifying degree should upload a copy of any one of the mark sheets of the final year,

OR

obtain a declaration from their Principal along with the signature and seal beforehand and upload the same during the online submission of the application form.

GATE Structure

Structure of GATE

For the GATE examination, a candidate can apply for only one of the 23 papers listed in Tables below. The syllabus for each of the papers is given separately. Making a choice of the appropriate paper during GATE application is the responsibility of the candidate. Some guidelines in this respect are suggested below.

The candidate is expected to appear in a paper appropriate to the discipline of his/her qualifying degree. However, the candidate can choose any paper according to his/her admission plan, keeping in mind the eligibility criteria of the institutions in which he/she wishes to seek admission. For more details regarding the admission criteria in any particular institute, the candidate is advised to refer to the website of that institute.

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List of GATE Papers and Corresponding Codes

Paper	Code	Paper	Code
Aerospace Engineering	AE	Geology and Geophysics	GG
Agricultural Engineering	AG	Instrumentation Engineering	IN
Architecture and Planning	AR	Mathematics	MA
Biotechnology	BT	Mechanical Engineering	ME
Civil Engineering	CE	Mining Engineering	MN
Chemical Engineering	CH	Metallurgical Engineering	MT
Computer Science and Information Technology	CS	Petroleum Engineering	PE
		Physics	PH
Chemistry	CY	Production and Industrial Engineering	PI
Electronics and Communication Engineering	EC	Textile Engineering and Fibre Science	TF
Electrical Engineering	EE	Engineering Sciences	XE*
Ecology and Evolution	EY	Life Sciences	XL**

*XE Paper Sections	Code	**XL Paper Sections	Code
Engineering Mathematics (Compulsory)	A	Chemistry (Compulsory)	P
Fluid Mechanics	B	Biochemistry	Q
Materials Science	C	Botany	R
Solid Mechanics	D	Microbiology	S
Thermodynamics	E	Zoology	T
Polymer Science and Engineering	F	Food Technology	U
Food Technology	G		
Atmospheric and Oceanic Sciences	H		

*XE (Engineering Sciences) and **XL (Life Sciences) papers are of general nature and will comprise of Sections listed in the above table. More detailed explanation is given as following pages.

General Aptitude Questions

All the papers will have a few questions that test the General Aptitude (Language and Analytical Skills), apart from the core subject of the paper.

XE Paper

A candidate appearing in the XE paper has to answer the following

1. Section A – Engineering Mathematics

2. GA – General Aptitude

3. Any two of XE sections B to H

The choice of two sections from B to H can be made during the examination after viewing the questions. Only two optional sections can be answered at a time. A candidate wishing to change midway of the examination to another optional section must first choose to deselect one of the previously chosen optional sections (B to H).

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XL Paper

A candidate appearing in the XL paper has to answer the following

1. Section P – Chemistry

2. GA – General Aptitude

3. Any two of XL sections Q to U

The choice of two sections from Q to U can be made during the examination after viewing the questions. Only two optional sections can be answered at a time. A candidate wishing to change midway of the examination to another optional section must first choose to deselect one of the previously chosen optional sections (Q to U).

Duration and Examination Type

The GATE examination consists of a single paper of **3-hour** duration that contains **65** questions carrying a maximum of **100 marks**. The question paper will consist of both multiple choice questions (MCQ) and numerical answer type (NAT) questions. The pattern of question papers is discussed in following paragraphs.

The examination for all the papers will be carried out in an ONLINE Computer Based Test (CBT) mode where the candidates will be shown the questions in a random sequence on a computer screen. The candidates are required to either select the answer (for MCQ type) or enter the answer for numerical answer type question using a mouse on a virtual keyboard (keyboard of the computer will be disabled). Each candidate will be provided with a scribble pad for rough work. The scribble pad has to be returned after the examination. At the end of the 3-hour window, the computer will automatically close the screen from further actions.

Pattern of Question Papers

In all the papers, there will be a total of 65 questions carrying 100 marks, out of which 10 questions carrying a total of 15 marks will be on General Aptitude (GA).

In the papers bearing the codes AE, AG, BT, CE, CH, CS, EC, EE, IN, ME, MN, MT, PE, PI, TF and XE, the Engineering Mathematics will carry around **15% of the total marks**, the General Aptitude section will carry **15% of the total marks** and the **remaining 70% of the total marks** is devoted to the subject of the paper.

In the papers bearing the codes AR, CY, EY, GG, MA, PH and XL, the General Aptitude section will carry **15% of the total marks** and the **remaining 85% of the total marks** is devoted to the subject of the paper.

GATE 2018 would contain questions of two different types in various papers:

(*i*) **Multiple Choice Questions (MCQ)** carrying 1 or 2 marks each in all papers and sections. These questions are objective in nature, and each will have a choice of four answers, out of which the candidate has to mark the correct answer(s).

(*ii*) **Numerical Answer Questions** of 1 or 2 marks each in all papers and sections. For these questions the answer is a real number, to be entered by the candidate using the virtual keypad. No choices will be shown for this type of questions.

Marking Scheme

For **1-mark** multiple-choice questions, **1/3** mark will be deducted for a wrong answer. Likewise, for **2-mark** multiple-choice questions, **2/3** mark will be deducted for a wrong answer. **There is NO negative marking for numerical answer type questions**.

Consult syllabus before proceeding

General Aptitude (GA) Questions

In all papers, GA questions carry a total of **15 marks**. The GA section includes 5 questions carrying **1-mark** each (sub-total **5 marks**) and 5 questions carrying **2-marks** each (sub-total **10 marks**).

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Question Papers other than GG, XE and XL

These papers would contain **25** questions carrying **1-mark** each (sub-total **25 marks**) and **30** questions carrying **2-marks** each (sub-total **60 marks**). The question paper will consist of questions of multiple choice and numerical answer type. For numerical answer questions, choices will not be given. Candidates have to enter the answer (which will be a real number, signed or unsigned, e.g., 25.06, – 25.06, 25, – 25 etc.) using a virtual keypad. An appropriate range will be considered while evaluating the numerical answer type questions so that the candidate is not penalized due to the usual round-off errors.

GG (Geology and Geophysics) Paper

Apart from the General Aptitude (GA) section, the GG question paper consists of two parts: Part A and Part B. Part A is common for all candidates. Part B contains two sections: Section 1 (Geology) and Section 2 (Geo-physics). Candidates will have to attempt questions in Part A and either Section 1 or Section 2 in Part B.

Part A consists of **25** multiple-choice questions carrying **1-mark** each (sub-total **25 marks** and some of these may be numerical answer type questions). Each section in Part B (Section 1 and Section 2) consists of **30** multiple choice questions carrying **2-marks** each (sub-total **60 marks** and some of these may be numerical answer type questions).

XE Paper (Engineering Sciences)

In XE paper, Engineering Mathematics section (Section A) is **compulsory**. This section contains **11** questions carrying a total of **15 marks**: 7 questions carrying **1-mark** each (sub-total **7 marks**), and 4 questions carrying **2-marks** each (sub-total **8 marks**). Some questions may be of numerical answer type questions.

Each of the other sections of the XE paper (Sections B through H) contains 22 questions carrying a total of **35 marks**: 9 questions carrying **1-mark** each (sub-total **9 marks**) and 13 questions carrying **2-marks** each (sub-total **26 marks**). Some questions may be of numerical answer type.

XL Paper (Life Sciences)

In XL paper, Chemistry section (Section P) is **compulsory**. This section contains **15** questions carrying a total of **25 marks**: 5 questions carrying **1-mark** each (sub-total **5 marks**) and 10 questions carrying **2-marks** each (sub-total **20 marks**). Some questions may be of numerical answer type.

Each of the other sections of the XL paper (Sections Q through U) contains 20 questions carrying a total of **30 marks**: 10 questions carrying **1-mark** each (sub-total **10 marks**) and 10 questions carrying **2-marks** each (sub-total **20 marks**). Some questions may be of numerical answer type.

Note on Negative Marking for Wrong Answers

For a wrong answer chosen for the **multiple choice questions** (MCQs), there would be negative marking. For **1-mark** multiple choice questions, **1/3** mark will be deducted for a wrong answer. Likewise, for **2-mark** multiple choice questions, **2/3** mark will be deducted for a wrong answer. However, there is **NO negative marking for a wrong answer in numerical answer type questions**.

GATE Score

After the evaluation of the answers, the raw marks obtained by a candidate will be converted to a normalized GATE Score.

The GATE score will be computed using the formula given below.

Calculation of Normalized Marks for CE, CS, EC, EE and ME papers (multi-session papers)

In GATE, examination for some papers may be conducted in multi-sessions. Hence, for these papers, a suitable normalization is applied to take into account any variation in the difficulty levels of the question papers across different sessions. The normalization is done based on the fundamental assumption that "in all multi-session GATE papers, the distribution of abilities of candidates is the same across all the sessions". This assumption is justified since the number of

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candidates appearing in multi-session papers in GATE is large and the procedure of allocation of session to candidates is random. Further it is also ensured that for the same multi-session paper, the number of candidates allotted in each session is of the same order of magnitude.

Based on the above, and considering various normalization methods, the committee arrived at the following formula for calculating the normalized marks for the multi-session papers.

Normalization mark of j^{th} candidate in the i^{th} session $\widehat{M_{ij}}$ is given by

$$\widehat{M_{ij}} = \frac{\bar{M}_t^g - M_q^g}{\bar{M}_{ti} - M_{iq}} (M_{ij} - M_{iq}) + M_q^g$$

where

M_{ij} : is the actual marks obtained by the j^{th} candidate in i^{th} session

$\bar{M}_t^g$: is the average marks of the top 0.1% of the candidates considering all sessions

M_q^g : is the sum of mean and standard deviation marks of the candidates in the paper considering all sessions

$\bar{M}_{ti}$: is the average marks of the top 0.1% of the candidates in the i^{th} session

M_{iq} : is the sum of the mean marks and standard deviation of the i^{th} session

After evaluation of the answers, normalized marks based on the above formula will be calculated corresponding to the raw marks obtained by a candidate and the GATE Score will be calculated based on the normalized marks.

For all papers for which there is only one session, actual marks obtained will be used for calculating the GATE Score.

Calculation of GATE Score For All Papers

GATE 2018 score will be calculated using the formula

$$\text{GATE Score} = S_q + (S_t - S_q)\frac{(M - M_q)}{(\bar{M}_t - M_q)}$$

In the above formulae

M : marks obtained by the candidate (actual marks for single session papers and normalized marks for multi-session papers)

M_q : is the qualifying marks for general category candidate in the paper

$\bar{M}_t$: is the mean of marks of top 0.1% or top 10 (whichever is larger) of the candidates who appeared in the paper (in case of multi-session papers including all sessions)

S_q : 350, is the score assigned to M_q

S_t : 900, is the score assigned to $\bar{M}_t$

In the GATE 2017 score formula, M_q is usually 25 marks (out of 100) or + s, whichever is larger. Here μ is the mean and s is the standard deviation of marks of all the candidates who appeared in the paper.

After the declaration of results, GATE Scorecards can be downloaded by

(a) All SC/ST/PwD candidates whose marks are greater than or equal to the qualifying mark of SC/ST/PwD candidates in their respective papers, and

(b) All other candidates whose marks are greater than or equal to the qualifying mark of OBC (NCL) candidates in their respective papers.

There is no provision for the issue of hard copies of the GATE Scorecards

The GATE Committee has the authority to decide the qualifying mark/score for each GATE paper. In case any claim or dispute arises in respect of GATE, the Courts and Tribunals in Bangalore alone shall have the exclusive jurisdiction to entertain and settle any such dispute or claim.

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GATE Syllabus

Section 1: Engineering Mathematics

Discrete Mathematics

Propositional and first order logic. Sets, relations, functions, partial orders and lattices. Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations, generating functions.

Linear Algebra

Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

Calculus

Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.

Probability

Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

Section 2: Digital Logic

Boolean algebra. Combinational and sequential circuits. Minimization. Number representations and computer arithmetic (fixed and floating point).

Section 3: Computer Organization and Architecture

Machine instructions and addressing modes. ALU, data-path and control unit. Instruction pipelining. Memory hierarchy: cache, main memory and secondary storage; I/O interface (interrupt and DMA mode).

Section 4: Programming and Data Structures

Programming in C. Recursion. Arrays, stacks, queues, linked lists, trees, binary search trees, binary heaps, graphs.

Section 5: Algorithms

Searching, sorting, hashing. Asymptotic worst case time and space complexity. Algorithm design techniques: greedy, dynamic programming and divide - and - conquer. Graph search, minimum spanning trees, shortest paths.

Section 6: Theory of Computation

Regular expressions and finite automata. Context-free grammars and push-down automata. Regular and contex-free languages, pumping lemma. Turing machines and undecidability.

Section 7: Compiler Design

Lexical analysis, parsing, syntax-directed translation. Runtime environments. Intermediate code generation.

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Section 8: Operating System

Processes, threads, inter-process communication, concurrency and synchronization. Deadlock. CPU scheduling. Memory management and virtual memory. File systems.

Section 9: Databases

ER-model. Relational model: relational algebra, tuple calculus, SQL. Integrity constraints, normal forms. File organization, indexing (e.g., B and B+ trees). Transactions and concurrency control.

Section 10: Computer Networks

Concept of layering. LAN technologies (Ethernet). Flow and error control techniques, switching. IPv4/IPv6, routers and routing algorithms (distance vector, link state). TCP/UDP and sockets, congestion control. Application layer protocols (DNS, SMTP, POP, FTP, HTTP). Basics of Wi-Fi. Network security: authentication, basics of public key and private key cryptography, digital signatures and certificates, firewalls.

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Chapter-Wise Analysis

GATE PAPERS (Computer Science & Information Technology)

Subject	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Engineering Mathematics											
1 mark Questions	5	4	6	2	3	5	5	4	5	5	5
2 marks Questions	11	6	5	7	3	2	5	6	4	2	7
Total Marks	**27**	**16**	**16**	**16**	**9**	**9**	**15**	**16**	**13**	**9**	**19**
Theory of Computation											
1 mark Questions	3	4	1	3	4	1	5	1	3	3	1
2 marks Questions	6	3	3	3	1	2	6	3	3	4	2
Total Marks	**15**	**10**	**7**	**9**	**6**	**5**	**17**	**7**	**9**	11	5
Digital Logic											
1 mark Questions	4	2	3	3	2	3	3	1	2	1	2
2 marks Questions	1	0	2	3	2	1	5	2	1	3	1
Total Marks	**6**	**2**	**7**	**9**	**6**	**5**	**13**	**5**	**4**	**7**	4
Computer Organization & Architecture											
1 mark Questions	0	2	1	3	2	1	2	1	1	2	3
2 marks Questions	12	4	4	2	4	7	2	2	3	3	4
Total Marks	**24**	**10**	**9**	**7**	**10**	**15**	**6**	**5**	**7**	**8**	**11**
Programming & Data Structures											
1 mark Questions	1	1	3	4	2	2	0	5	5	4	4
2 marks Questions	3	3	5	7	6	5	2	3	6	7	4
Total Marks	**7**	**7**	**13**	**18**	**14**	**12**	**4**	**11**	**17**	**18**	**12**
Algorithm											
1 mark Questions	2	3	1	1	4	5	1	4	1	1	1
2 marks Questions	15	6	3	0	2	3	2	4	3	2	3
Total Marks	**32**	**15**	**7**	**1**	**8**	**11**	**5**	**12**	**7**	**5**	**7**
Compiler Design											
1 mark Questions	2	1	2	1	1	2	1	2	1	2	2
2 marks Questions	2	0	1	0	3	2	2	1	1	0	2
Total Marks	**6**	**1**	**4**	**1**	**7**	**6**	**5**	**4**	**3**	**2**	**6**
Operating System											
1 mark Questions	2	2	3	3	1	1	0	2	1	2	4
2 marks Questions	5	5	2	2	3	1	2	2	5	2	3
Total Marks	**12**	**12**	**7**	**7**	**7**	**3**	**4**	**6**	**11**	**6**	**10**
Data Bases											
1 mark Questions	1	0	3	0	3	1	3	1	3	2	0
2 marks Questions	5	5	2	3	3	4	2	2	0	2	2
Total Marks	**11**	**5**	**7**	**6**	**9**	**9**	**7**	**5**	**3**	**6**	**4**

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Subject	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Computer Networks											
1 mark Questions	1	0	2	5	3	4	4	2	2	3	3
2 marks Questions	4	5	3	2	3	2	2	3	4	2	2
Total Marks	**9**	**5**	**8**	**9**	**9**	**8**	**8**	**8**	**10**	**7**	**7**
***Software Engineering**											
1 mark Questions			1	0	1	0	0	1	1	0	0
2 marks Questions			0	0	0	0	1	0	1	0	0
Total Marks			**1**	**0**	**1**	**0**	**2**	**1**	**3**	**0**	**0**
***Web Technology**											
1 mark Questions			1	0	1	0	0	0	1	0	0
2 marks Questions			0	0	0	0	0	0	1	0	0
Total Marks			**1**	**0**	**1**	**0**	**0**	**0**	**3**	**0**	**0**
Reasoning & Aptitude											
1 mark Questions			5	5	5	5	5	5	5	5	5
2 marks Questions			5	5	5	5	5	5	5	5	5
Total Marks			**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**

**Topics not included in GATE 2017 Syllabus.*

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General Aptitude

Part I : Verbal Ability

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1 CHAPTER

English Grammar

ERRORS IN USE OF ARTICLES

ARTICLES

Indefinite (A, An)	Definite (The)

Use of '*An*'

1. Before words beginning with vowel sounds [a, e, i, o, u are called vowels, others are consonants].
   e.g. an apple, an egg, an owl.
2. Before words beginning with silent 'h' but sounds as vowel.
   e.g. an hour, an honourable man, an heir, an honest man.
3. F, H, L, M, N, R, S, X are letters that are not vowels but begin with vowel sound 'M' has the sound of 'em'. So, 'an' is used before abbreviations beginning with vowels of these letters.
   e.g. an M.L.A., an R.A.F., an N.C.C. officer, an F.I.R., an X-ray, an H.E. school, an S.P.

Use of '*A*'

1. In the sense of one.
   e.g. He couldn't speak a word to save himself.
   With 'one' (since 'one' begins with sound of 'w')
   e.g. a one-man show, a one-rupee note.
2. Before words beginning with consonant sound
   e.g. a boy, a box, a dog.
3. With Vowel letters having consonant value.
   e.g. a university, a unique article, a euphenism, a unit, a European language
4. With units and rate (per).
   e.g. He earns rupees five hundred a month.
5. In exclamatory expressions before singular countable nouns.
   *e.g. What **a** pretty girl !*
6. When two subjects or articles are thought of as a single unit.
   *e.g. He was ready with **a** cup and saucer.*
7. With certain expressions of quantity.
   *e.g. **a** lot of , **a** dozen, **a** great deal of, **a** couple.*
8. With a person's name to indicate that the person is perhaps unknown to the person addressed.
   *e.g. **A** Mr. roy is at the door.*
9. With a special meal (to celebrate something or in someone's honour).
   *e.g. I called my friends to **a** lunch to celebrate my success.*
10. To make a common noun of a proper noun.
   *e.g. This man is '**a** second Newton'.*
   (This phrase means 'a philosopher as great as Newton')

Use of '*The*'

1. When we speak of a particular person or thing already referred to.
   *e.g. I dislike **the** follow.*
2. When a singular noun represents a whole class.
   e.g. The mango is considered the king among fruits.
3. With name of
   (*i*) gulfs, rivers, oceans, islands and mountains e.g. the Himalayas, the Indian ocean, the Persian Gulf, the Red sea, the Andaman islands, the Brahmaputra river.
   (*ii*) Certain books
   e.g. the Vedas, the Puranas, the Bible, the Ramacharitmanas.
   (*iii*) Musical instruments
   e.g. the flute, the violin, the tabla, the trumpet.
   (*iv*) The inventions
   e.g. I hate the telephone for its constant ringing.
   (*v*) Parts of body
   *e.g. He was wounded in **the** leg.*
   (*vi*) Religious groups
   e.g. the Sikhs, the Hindus, the Parsees.
   (*vii*) Names enforcing law
   e.g. the Police, the Navy, the Air Force.
   (*viii*) Political parties
   e.g. the Congress, the Janata Pary, the B.J.P.
   (*ix*) Aeroplanes, ships, trains etc
   e.g. the Makalu (aeroplane), the Vikrant (Ship), the Rajdhani express (train).
   (*x*) Before names of an empire, dynasty or historical event
   e.g. the Gupta dynastry, the Old Stone Age, the First World War, the American Revolution.
   (*xi*) Clubs, foundations etc.
   e.g. the Lion's Club, the Ford Foundation.
   (*xii*) Before common nouns denoting unique things
   e.g. the sun, the sky, the earth, the worl.
   (*xiii*) With superlatives
   e.g.*He is **the** best boy in the class.*
   (*xiv*) With ordinals
   *e.g. He took **the** first taxi that came his way.*
   (*xv*) Before the comparative degree
   e.g.***The more** they get, **the** more they want.*
   (*xvi*) Before an adjective when the noun is understood
   *e.g. **The** poor would favour him.*

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ERRORS IN USE OF NOUNS

1. Nouns which are used in singular form

(*i*) Scenery, information, furniture, advice, machinery, stationery, news, poetry, business, mischief, fuel, issue, repair, bedding.

*e.g. The **scenery** of this place is worth seeing.*

(*ii*) Physics, Mathematics, Economics, Classics, Ethics, Athletics, innings, gallows.

*e.g. **Economics** is a very interesting subject.*

(*iii*) Brick, bread, fruit, word (as 'promise')

*e.g. Let me buy some **fruit**.*

(*iv*) Words like dozen, score, hundred, thousand, million when preceded by a numeral.

*e.g. He bought ten **dozen** oranges.*

(*v*) Expressions as a ten-rupee note, a two-hour journey, a four-mile walk, a Five-year plan, a six-man committee etc.

e.g. A ten-rupee note is lying there.

2. Nouns used only in plural form

(*i*) Cattle, police, poultry, people, gentry, peasantry, artillery.

*e.g. The **police have** caught the thief.*

(*ii*) Scissors, trousers, stockings, spectacles, shorts, alms, remains, riches, goods, measles.

e.g. My scissors are very sharp.

3. Nouns used both as singular and plural in the same form

(*i*) Dear, sheep, fish, apparatus, wages.

*e.g. The **wages** of sin **is** death.*

*The **wages** of the workers **have** been raised.*

*I saw **a sheep** grazing in the field.*

***Sheep are** sold cheaper than goat.*

(*ii*) Collective nouns as jury, public, team, audience, committee, government, audience, congregation, orchestra.

*e.g. The **team are** looking quite fit.*

*The **team has** not turned up yet.*

4. Use of Collective Nouns

- **crowd** of people
- **herd** of cattle/cows
- **team** of players
- **flight** of birds
- **bouquet** of flowers
- **Shoal/school** of fish
- **bundle** of sticks
- **army** of soldiers
- **flock** of sheep
- **crew** of sailors
- **swarm** of bees
- **garland** of beads
- **gang** of thieves
- **library** of books
- **mob** of angry people
- **fleet** of ships/cruisers
- **pack** of wolves
- **pack** of cards
- **loaf** of bread
- **heap** of corn
- **stock** of grain
- **class** of pupils
- **bunch** of grapes
- **herd** of lions
- **pile** of books
- **hoarde** of nomads
- **block** of flats
- **file** of papers

5. One of or **any of** is followed by plural words.

*e.g. I want **one of** the **books** kept on the table.*

***Any of** these **tools** may serve the purpose.*

6. Plural nouns are used with fractions and decimal over 1.

*e.g. It took us one and a half **hours.***

ERRORS IN USE OF PRONOUNS

Pronouns are words used to replace nouns or noun groups already mentioned.

Nominative/ Vocative case (comes before verb)	***Accurative/ Objective case (comes after verb)***	***Possessive case***	***Reflexive Pronoun***
I	Me	My	Myself
We	Us	Our	Ourselves
He	Him	His	Himself
She	Her	Her	Herself
They	Them	Their	Themselves
Who	Whom	Whose	—

1. The pronoun '*One*' must be followed by '*one's*'.

e.g. One must do one's duty to one's country.

2. When 'one' means 'one in number', the pronoun for it is third person singular pronoun (he, she, it).

e.g. *One of them has given up one's studies.* (×)

One of them has given up his studies. (✓)

3. 'Everyone' or 'Everybody' must be followed by 'his'.

e.g. Everyone should love his country.

4. Each, every, anyone, anybody must be followed by the singular pronoun of their person.

e.g. Anyone can do this if he tries.

5. 'Let' is followed by pronoun in the objective case.

e.g. Let him go.

6. 'But' and 'except' are followed by pronoun in the objective case.

e.g. Everyone attended the party except him.

'such as' is followed by pronoun in the subjective case.

e.g. I have no liking for such a man as he.

7. Verbs like enjoy, avail, pride, resign, apply, acquit, assert, absent are followed by reflexive pronouns.

e.g. He absented himself from the class.

We enjoyed ourselves at the party.

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8. Reflexive pronouns are never used with verbs keep, conceal, quality, spread, rest, stay.
   e.g. *I stayed away from my class.*
   He qualified in the test.
9. When first, second and third person singular pronouns (I, you and He) are used together, they are placed in the order : You, he and I.
   e.g. *You he and I are neighbours.*
   In case of plural pronouns, 'we' comes first, then 'you' and then 'they'.
   e.g. *We, you and they must work together.*
   But if we have only two persons including first, then first person pronoun is written first.
   e.g. *I and Sanjeev have done this job.*
10. 'Who' denotes subject and 'whom' denotes object.
   e.g. *Who do you think did the job ?*
11. 'Whose' is used for persons and 'which' for lifeless objects.
   e.g. *This is the table which I was talking about.*
12. 'Which' conveys additional information and 'that' explains a certain thing.
   e.g. *I will tell you the first thing which I remember.*
13. The following expressions usually take 'that' in place of 'who' or 'which'.
   Only, Any, It is, All, Superlatives.
   e.g. *He is the only man that can do it.*
   Any man that listens to you is a fool.
14. '*Each other*' is used for two; '*one another*' for more than two.
   e.g. *Rahul and Renu love each other.*
15. The complement of the verb *to be*, when it is expressed by a pronoun, should be in Nominative case.
   e.g. *It was he who did it.*
16. When the same person is the subject and object, it is necessary to use reflexive pronouns.
   e.g. *I cut me shaving this morning.* (×)
   I cut myself shaving this morning. (✓)
17. When a pronoun is the object of a verb or preposition it should be in objective case.
   e.g. *These books are for you and I.* (×)
   These books are for you and me. (✓)
   Between him and me there is an understanding.
18. The Relative Pronoun should be placed as near as possible to the antecedent.
   e.g. *I have read Shakespeare's works who was a great dramatist.* (×)
   I have read the works of Shakespeare who was a great dramatist.(✓)
19. The case of the pronoun following 'than' and 'as' is decided by mentally supplying the verb and completing the sentence.
   e.g. *She is taller than I (am)*
   I love more than (I love) him.

ERRORS IN USE OF PREPOSITIONS

A preposition is a word used with a noun or pronoun to show its relation to some other word in a sentence.

PREPOSITIONS OF TIME

1. **At** is used
   (a) with a definite point of time
   e.g. *I usually get up at 5 o'clock.*
   (b) with festivals
   e.g. *He will come at Holi.*
2. **In** is used :
   (a) with the parts of the day, (with **noon**, use **at**), months, seasons and years
   e.g. *He takes a walk in the afternoon.*
   (b) with the future tense referring to the period in which action may take place
   e.g. *You must be careful in future.*
3. **On** is used : with days and dates
   e.g. *My brother will arrive on Monday.*
4. **By** refers to the latest time at which an action will be over
   e.g. *The examination will be over by 5 p.m.*
5. **For** is used with perfect continuous tense showing the duration of action
   e.g. *I have been here for three years.*
6. **Since** is used with the point of time when action begins and continues :
   e.g. *He has been ill since last Tuesday.*
7. **From** refers to the starting point of action.
   e.g. *This water came from a spring.*

PREPOSITIONS OF POSITION

1. **At** refers to an exact point.
   e.g. *The tourist stayed at the Tourist Hotel.*
   He studied at Oxford.
2. **In** refers to larger areas.
   e.g. *He lives in Bombay.*
3. **Between** is used for two persons or two things.
   e.g. *Share these sweets between him and me.*
4. **Among** is used with more than two persons or things but before the word which starts with a consonant letter.
   e.g. *Divide the sweets among the three boys.*
5. **Amongst** is also used with more than two persons or things but before the word which starts with a vowel letter.
   e.g. *Divide the sweets amongst us.*
6. **Above** is used for **higher than.**
   e.g. *The sun rose above the horizon.*
7. **Under** is used for **vertically below.**
   e.g. *It is shady under the trees.*
8. **Below** is used for **lower than.**
   e.g. *When the sun sets it goes below the horizon.*
9. **Over** is used for **vertically above.**
   e.g. *There is an aircraft coming over.*
10. **Beneath** means a lower position.
   e.g. *the ground was slippery beneath her.*

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PREPOSITIONS OF DIRECTION

1. **To** is used to express motion from one place to another.
   e.g. We walked to the river and back.
2. **Towards** refers to direction.
   e.g. He saw me running towards him.
3. **Into** denotes motion towards the inside of something.
   e.g. She fell into a ditch.
4. **At** refers to aim.
   e.g. He aimed at the bird.
5. **For** denotes direction.
   e.g. I will leave for Pune today.
6. **Against** shows pressure.
   e.g. She cleaned the edge of her knife against the plate.
7. **Off** refers to separation.
   e.g. He was wiping sweat off his face.
8. **From** refers to the point of departure.
   e.g. The man parted from his friends.

OTHER USES OF PREPOSITION

1. **About** shows nearness.
   e.g. His father is about to retire.
2. **Along** stands for **in the same line.**
   e.g. She led them along the corridor.
3. **After** refers to sequence.
   e.g. She came after me.
4. **Across** means from one side.
   e.g. He blew a cloud of smoke across the table.
5. **Before** stands for **in front of.**
   e.g. A thief was brought before the judge.
6. **Behind** means **at the the back of.**
   e.g. She sat down behind the hedge.
7. **Beyond** means **on the farther side of.**
   e.g. This is beyond his power.
8. **Beside** means **by the side of.**
   e.g. *I sat down beside my wife.*
9. **Besides** means **in addition to.**
   e.g. He is guilty of five killings and more besides.

WORDS FOLLOWED BY PREPOSITIONS

A

- abound with
- abstain from
- according to
- accordance with
- accuse of
- accustomed to
- acquaint with
- acquit of
- adverse to
- affiliate to/with
- afflict with
- afraid of
- agree to/on
- aim at
- allegation
- about
- allude to
- alternate with
- alternative to
- angry with
- apply to (body)
- appreciation of
- apprise of
- approximate of
- arraign against
- arrest in
- arrive at/in
- ashamed of
- aspire to/after/at
- associate with
- assure of
- attribute to

B, C

- beset with
- beware of
- blink at
- boast of
- break off
- bungle over
- careful of
- cause of
- centre on
- charge with
- clamour for
- clamp on
- coerce into
- collide with
- comment on
- compatible with
- complain of
- comply with
- compliment on
- composed of
- concur in
- condole with
- conducive to
- confer on
- confirmation of
- conform to
- congratulate on
- conscious of
- contiguity with
- converge on
- convict of
- co-opt to
- cope with
- correspond with
- converge on
- convict of
- co-opt to
- cope with
- correspond with
- cost of
- credit with
- criticism of
- cure of

D

- decide on
- debar from
- delight in
- deliberate on
- denude of
- depart to
- departure for
- depend on/upon
- deprive of
- derive from
- derogate from
- derogatory to
- desist from
- despair of
- detrimental to
- devoid of
- difficulty in
- diffident of
- disabuse of
- disagree with
- disgusted with
- dispose of
- dissociate from
- divest of
- divide into
- dressed in

E

- effort in (—ing)
- election to
- embroil in
- emphasis on
- employ in
- enamoured of
- encroach on
- endow with
- enlarge on
- engage in
- essential to
- expostulate with
- extend to

F, G, I

- fed up with
- focus on
- forbid to
- fraught with
- fritter in
- full of
- give to
- grapple with
- good at
- identical with
- impose on
- incur on
- indict for
- indifferent to
- indulge in
- infatuate with

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- infect with
- infest with
- information on
- insensible to
- insist on
- interest in
- intimate to
- intoxicate with
- intrigue with
- intrude on
- investigation of
- involve in

J, K, L

- jealous of
- jeer at
- jump at
- just to
- knock at
- lash at
- lay siege to
- leave for
- live on (income)
- long for

O, P

- oblivious of
- operate on
- opinion on
- overcome with
- pay a call on
- persevere in
- persist in
- plan for
- plead for
- pleased with
- prefer to
- preference for
- present with
- prevent from
- priority to
- probe into
- prohibit from
- proud of

R, S

- recoil from
- reconcile to
- remand to
- remonstrate with
- repent of
- replete with
- reply to
- research on
- resolve on/to
- sail for
- satiate with
- satisfied with
- sentence to
- sick of
- side with
- sparing
- square with
- stickler for
- subversive of
- succeed in
- sue for
- superior to
- supply with
- sure of
- surprised at
- susceptible to
- suspect of
- sympathy for

T, U, V, W

- testify to
- thank for
- throw at
- tinker at
- tire of
- trace to
- trample on
- translate into
- try to (do)
- unequal to (the task)
- vie with
- vote for (a candidate)
- wait for (or await)
- ways of
- wish for

SOME SPECIAL CASES

- absolve from (blame) / of (sin)
- admit to (hospital, place, class)
- appear for (examination)/at
- begin on (not from Sunday)
- blow down (the roof)/off (steam)/over (pass off)
- buy in (market, store)/at (shop)
- come/go by (bus, train)
- come into (conflicit with)
- compare with (one man height with another's)/to (one man's height to a building)
- consist of (to denote the substance)/in (define the subject)
- deal with (somebody)/ in (goods)
- die of (disease)/from (other causes)
- differ from (to be different from)/with (to disagree)
- due to (used after a noun, not after a verb)
- entrust to (somebody a job)
- entrust (somebody) with a job)
- fill with (enthusiasm)/ in (a form)
- fire at (when not hit)/on (when injured to killed)
- glad of (the news)/at (having arrived safe)
- hostility to/in hand (being attended to)/ on hand (in possession of)
- ingratiate (oneself) with
- inquire into (matter) / of (person)/about. often (thing)
- invest with (power/in (blank)
- key of (the door)/to (a problem)
- level (charge) against.
- look at (consider evidence)/to (somebody for aid)
- meeting (called) for (Sunday)
- motive in (doing)/for (an act)
- neglect of (duty)/in (doing a thing)
- nominated to (board)
- noted for (good things)
- owing to (used after a verb, not after a noun)
- part from (people)/with (things)
- preventive for priority
- put out (light)/off (trip)
- Shoot at (when injured)/in (leg, head)
- take up (job)/to (a hobby)
- taste of (sensation)/for (liking)
- tear off (remove)/up (to pieces)
- vest with (The P.M. is vested with power to ...)/ in (Powers are vested in the P.M. to ...)

ERRORS IN USE OF CONJUNCTIONS

1. **Scarcely or hardly** is followed by **when**.
   e.g. *Hardly had I slept when the telephone rang.*
2. **Though** is followed by **yet**.
   e.g. *Though he worked hard yet he failed.*
3. **No sooner** is followed by **than.**
   e.g. *No sooner did we reach there than it began to rain.*
4. **Not only** is followed by **but also.**
   e.g. *Not only did he help her, but also dropped her home safely.*
5. **Lest** is negative and so should not be followed by **not**; it is followed by **should**.
   e.g. *Work hard lest you should fail.*
   If **would** or **may** is used in place of **should**, then **else** should be used in place of **lest.**
   e.g. *Give him water, else he may die.*
6. **Both** is complemented by **and**, not by **as well as.**
   e.g. *Both Amit and Satish are good at Science.*

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7. **So.....as** is used in negative sentences, whereas **as** .....**as** is used in affirmative sentences.
e.g. He is not so tall as his brother.

8. **Other** is followed by **than**.
e.g. He has no other claim than his wealth.

9. The word **reason** is not followed by **because,** but by **that.**
e.g. The reason why he didn't go was that his mother was ill.

10. **Because** denotes reason.
in order that denotes purpose.
e.g. He went to the doctor because he was ill.

11. Words such as regard, describe, define, treat, mention, depict, portray are followed by **as**.
e.g. I regard her as my sister.

12. **As** and **since** are also used to express reason.
e.g. As he was not there, I spoke to his brother.

13. **Neither** is followed by **nor** and both are followed by same auxiliary verb.
e.g. Neither Ravi nor Raju was seen.

14. **Either** is followed by **or**.
e.g. Either you are or he is to blame.

15. **Unless, until, if not, so that** should not be followed by **not.**
e.g. Wait here until I come.

16. **If** is used in conditional series
Whether is used in uncertainty.
e.g. I don't know whether he was present.

17. To express **time before** use **until** or **till** and to express **how long** use **as long as.**
*e.g. Work **as long as** you live.*

18. **Such** is followed by **as.**
e.g. We talked about such subjects as the weather.
Such is followed by **that** if we emphasize degree of something by mentioning the result.
e.g. The extent of the disaster was such that not a single man could survive.

19. **Like** is followed by pronoun; **as** is followed by a clause.
e.g. He runs like me.

ERRORS IN SUBJECT–VERB AGREEMENT

1. Singular subject must have singular verb.
e.g. He writes; I write.

2. Plural subject must have plural verb.
e.g. They write; We write.

3. Two subjects joined by **and** will always take a plural verb.
e.g. The doctor and nurse work together.
The doctor and nurses work together.
The doctors and nurse work together.

4. Two singular subjects joined by **or** or **nor** will take a singular verb.
e.g. A doctor or a nurse is working in the hospital.

5. A singular subject and a plural subject joined by **or** or **nor** will take a singular or plural verb depending on which subject is nearer the verb.
e.g. Neither Deepak nor his friends are joining the tour.
Neither his friends nor Deepak is joining the tour.

6. If the subject is singular and the predicate is plural, the verb must agree with its subject and not its predicate.
e.g. Physical conditioning and mental attitude are winning combination.
The winning combination is physical conditioning and mental attitude.

7. Indefinite pronouns such as
someone, somebody, each, nobody, anyone, anybody, one, no one, everyone, everybody, either, neither, etc. always take a singular verb.
e.g. Each of my friends calls me once a month.

8. Indefinite pronouns which indicate more than one (several, few, both, many) always take plural verbs
e.g. Both of the books required careful reading.

9. Collective nouns (fleet, army, committee, crowd) are singular when the group works together as a unit and hence take singular verbs.
e.g. The jury has reached its verdict.

10. Collective nouns are plural when the members of the group are acting individually and hence take plural verbs.
e.g. The jury have argued for five hours.

11. Some words (such as news, measles, mumps etc.) end in —*s* but represent a single thing. These words need singular verbs.
e.g. The 7 o'clock news is about to begin.

12. Some words (such as scissors, trousers, spectacles, shorts etc.) end in —*s* and seem to represent a single thing, but they are two parts to that single thing. These words take plural verbs.
e.g. The scissors are on the table.

13. Words (such as politics, ethics, athletics etc.) that end in —ies are usually singular and hence take singular verbs.
e.g. Mathematics is his favourite subject.
But these words are singular when they refer to a study, science or practice. If these words have modifiers with them, they become plural and hence take plural verbs.
e.g. His politics are somewhat divided.

14. Title of the books need singular verbs.
e.g. Great Expectations is a good book.

15. Some nouns in the plural form represent an amount, a fraction, or an element of time. These nouns are considered singular and hence take singular verbs.
e.g. Sixty minutes is enough to finish this work.

16. If two subjects are joined together by **as well as** the verb will act according to the first subject.
e.g. Students as well as the teacher are playing.

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17. The subject **Many a .....** is always followed by the singular verb.

e.g. *Many a man was drowned in the sea.*

18. If two subjects are joined together by **with** the verb will act according to the subject.

e.g. *The principal together with his students was seeing the final match.*

19. If subject is **The number of ...** use a singular verb.

e.g. *The number of books is very small.*

20. If the subject begins with **A number of ....** (= many), use a plural verb.

e.g. *A number of books are missing.*

21. If two subjects express one idea, use a singular verb.

e.g. *Bread and butter is wholesome food.*

22. When adjectives such as **much, less, little** and **more** are used as nouns, they must have a singular verb.

e.g. *A little of good habits makes our life happy.*

ERRORS IN THE USE OF TENSES

The changed forms of a verb that indicate time of the action are called **tenses** of the verb.

1. When the verb in the Principal Clause is in the Past tense, the verbs of the Subordinate Clauses should be in the Past tense.

e.g. *He said that he had finished his work.*

2. But a past tense in the Principal Clause may or may not be follwoed by the Past tense in the sub-ordinate clause if the latter expresses unviersal or habitual truth.

e.g. *The teacher said that the earth revolves round the sun.*

3. Any tense may be used in the sub-ordinate clause if it gives a comparison by using the word **than.**

e.g. *The teacher liked Anil better than he liked me.*

4. Any tense can be used when the subordinate clause is in a quotation.

e.g. *I said, "I am going to Delhi today".*

5. The Present Perfect Tense (subject + has/have + V_3) cannot be used when an expression of Past time (yesterday, last night, ago etc.) is used.

e.g. *We wrote to you yesterday about his mother.*

6. In conditional sentences the subordinate clause, beginning with if, is generally in Present or Past tense but not in future tense.

e.g. *If I go to Delhi, I shall see the Red Fort.*

7. With the phrases **as if** and **as though**, Past tense and plural form of the verb should be used.

e.g. *He behaves as if he were a millionaire.*

8. Past Perfect tense (subject + had + V_3) is used when a sentence refers to two past actions and one of them occurs earlier than the other.

e.g. *When my uncle came to India I had passed M.A.*

9. Words like **usually, generally, often, whenever** etc. are used in Present Indefinite tense.

e.g. *I usually go to play cricket.*

10. If the action began in the past and is still continuing in the present, use Present Perfect tense (Subject + has/have + been V_1 + ing).

e.g. *I have been reading in this class for two months.*

11. Do not use the Future or the Present tense after such expressions as **suppose that, it is high time, it is time, as if** etc.

e.g. *It is high time that you went home.*

GERUND & INFINITIVE

Use of Gerund (V_1 + ing)

1. When an action is being considered in general sense, gerund is used as subject.

e.g. *Swimming is his favourite pastime.*

2. Gerund is used as subject in short prohibitions

e.g. *Smoking is prohibited.*

3. Verbs such as help, stop, detest, avoid, finish, dread, mind, prevent, dislike, risk, deny, recollect, no good, no use, resent, delay, postpone, defer, enjoy, forgive, pardon, excuse, suggest etc. are followed by the gerund.

e.g. *It is no use saying.*

4. A gerund is placed after a preposition :

e.g. *She is tired of walking.*

But the prepositions **except** and **but** are followed by an infinitive.

5. A gerund, showing physical activity, is put after **come** or **go**

e.g. *He is to go shooting.*

6. Nouns, followed by a gerund, are put in the possessive case.

e.g. *He hates his brother's drinking everyday.*

7. If there is a sense of dislike, hesitation, risk etc. in a sentence, use gerund.

e.g. *I dislike reading cheap novels.*

8. Adjective before gerund are put in the possessive case.

e.g. *I do not mind your going away.*

9. **With a view to** is always followed by a gerund and not an adjective.

e.g. *We go to school with a view to studying.*

INFINITIVE (TO + V_1)

1. Verbs such as learn, remember, promise, swear, consent, neglect, refuse, purpose, try, endeavour, attempt, fail, care, hope, decide, hesitate, prepare, determine, undertake, manage, arrange, seem, forget, agree, regret etc. are followed by infinitive:

e.g. *We endeavour to invite such persons.*

We promise to do this work soon.

2. Verbs such as order, tell, invite, oblige, allow, permit, compel, teach, instruct, advise, tempt, encourage, urge, request, show, remind, forbid, warm etc. are followed by object and infinitive

e.g. *The teacher instructed the students to go.*

3. Verbs or expressions like—will, can, do, must, may, let are followed by infinitive without **to**

e.g. *Let him do this work.*

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4. Expressions like would rather, would sooner, rather than, sooner than, had better are followed by infinitive without **to**

e.g. You had better tell him.

5. **See** can be used with infinitive or without it

e.g. Go to see him.

6. The infinitive is used after adjectives like delight, angry, glad, astonished

e.g. I was astonished to find him.

7. The verb know is never directly followed by the infinitive. It is followed by a conjunction and then the infinitive

e.g. Do you know to play the harmonium ? (×)
Do you know how to play the harmonium ?(✓)

8. **...had better** is always followed by the infinitive without to

e.g. You had better to stop taking the medicine which has harmful side-effects. (×)
You had better stop taking the medicine which has harmful side-effects.(✓)

9. Infinitive is not used with **to** after verbs of perception like-to see, to hear, to feel, to watch etc.

e.g. I saw him cross the street.

10. Avoid using split-infinitive :

e.g. He planned to not go on a vacation this year.(×)
He planned not to go on a vacation this year.(✓)

AUXILIARY VERBS

These are helping verbs.

1. **May** implies permission, doubt or possibility.

e.g. It may rain tonight.

2. **Might** is the past form of **may** but it does not necessarily represent past time.

Often it implies more doubt than **may.**

e.g. If the clouds are salted, the rains might come.

3. **Can** is used to express ability.

e.g. He can do this work.

4. **Could** is the past form of **can** but it does not necessarily represent past time. Often it implies a more uncertain condition than **can.**

e.g. She could refuse, but she never does.

5. **Should** is the past form of **shall**. It is used in sub-ordinate clauses after **in case** and sometimes after **if.**

e.g. I shall get some money in case brother comes.

It is also used in past sentences with **so that** and **in order that :**

e.g. He turned the stereo down very low so that he should not disturb him.

- **Should** can also be used in sub-ordinate clauses when we are expressing the idea that something must be done or is important. The fact is that this happens after verbs like command, order, request, insist, suggest, advise etc. & after adjectives like important, vital, essential, necessary, eager, anxious, concerned etc.

  e.g. I am anxious that nobody should be hurt.

- **Should** is also used in sub-ordinate clauses in sentences where we express personal reactions to events. We express our reactions with words like—amazing, interesting, shocked, sorry, normal, natural, it's a shame etc.

  e.g. I am sorry you should think I did it on purpose.

6. **Must** is followed by the infinitive without **to.**

e.g. I must get up at five tomorrow.

Must is used to give strong advice or orders :

e.g. I really must stop drinking.

Must is used to say that we are sure about something :

e.g. I am in love—that must be nice.

Must is used after a past reporting verb.

e.g. I felt there must be something wrong.

7. **Should** and **Ought** have similar meanings, but **ought** is followed by **to. Ought to** has a more objective force and is used when we are talking about laws, duties and regulations.

e.g. We ought to see her tomorrow.

Should and **ought to** can also be used to talk about strong probability.

e.g. He has bought thirty pints of whisky—that ought to be enough.

To talk about things which did not happen, although they were supposed to, we use

should and **ought to** with the perfect infinitive :

e.g. The taxi should have arrived at 8.30.

8. **Do** is used to make question and negative forms of ordinary verbs.

e.g. Do you know Rajat ?

Do is used in question-tags and short answers

e.g. Does he know I am here ? —Yes, he does.

Do is used before an imperative to make the request more persuasive

e.g. Do accompany us.

Do is not used in questions which have **who, what or which** as their subject

e.g. Who said that ?
What happened ?
Which one's that ?

But if **who, what** or **which** is the object of the sentence, **do** is used

e.g. Who did you see ?
Which department do you want ?

Do is also used with an —**ing** form when we want to talk of activity that takes a certain time or that is repeated. In this case we find determiners like —the, my, some, much, etc.

e.g. I usually do most of my washing on Sundays.

ERRORS IN USE OF ADJECTIVES

Word qualifying a noun or pronoun is called an **adjective.**

1. The adjectives ending in —ior (prior, junior, senior, superior, inferior, posterior) take 'to' and not 'than' after them.

e.g. He is senior to me.

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2. Some adjectives like unique, ideal, perfect, extreme, complete, universal, infinite, perpetual, chief, entire, round, impossible are not compared.

e.g. It is the most unique book. (×)
It is a unique book (✓)

3. Comparative degree is used in comparing two things or persons.

e.g. It is the better of the two books.

Superlative degree is used in comparing more than two things or persons.

e.g. He *is the best* of the three boys.

4. Double comparatives and double superlatives must not be used.

e.g. He is more wiser than his brother. (×)
He is wiser than his brother. (✓)

5. When we compare two qualities in the same person or thing, the comparative ending —er is not used.

e.g. You are wiser than old. (×)
You are more wiser than old. (✓)

6. When two adjectives in superlative or comparative degree are used together, the one formed by adding 'more' or 'most' must follow the other adjective.

e.g. He is more intelligent and wiser than his brother. (×)
He is *wiser* and *more intelligent* than his brother. (✓)

7. When two adjectives with differing degrees of comparison are used they should be complete in themselves.

e.g. He is as wise, if not wiser than his brother. (×)
He is as wise as, if not wiser than his brother. (✓)

8. When two changes happen together, comparative degree is used in both.

e.g. The higher you go, the cooler you feel.

9. When comparative degee is used in superlative sense, it is followed by '*any other*'.

e.g. Kapil is better than any bowler. (×)
Kapil is better than any other bowler. (✓)

10. Compound adjective formed by adding 'worth' is placed after the noun it qualifies.

e.g. This is a worth seeing sight. (×)
This is a sight worth seeing. (✓)

11. When two or more comparatives are joined by **'and'**, they must be in the same degree.

e.g. Russel was one of the wisest and most learned men of the world.

12. When there are two objects of comparison, then to avoid repetition of noun, 'that' is used for singular noun and 'those' for plural noun.

e.g. The climate of Ranchi is better than Gaya. (×)
The climate of Ranchi is better than that of Gaya. (✓)

13. If comparison is made by using 'other', 'than' is used instead of 'but'.

e.g. He turned out to be no other than my old friend.

14. **Likely, certain** and **sure** are followed by '**to**'.

e.g. He is likely to win.

CONFUSED ADJECTIVES

1. **Beautiful** is used for woman; **handsome** for man.

e.g. He is a handsome youth.
She is a beautiful girl.

2. **Less** refers to quantity, **fewer** denotes number.

e.g. He takes no less than a litre of milk.
They have fewer books than I have.

3. **Last** is the final one; **Latest** is last upto the present.

e.g. Z is the last letter of the alphabet.
This is the latest edition of the book.

4. **Each** is used for one of two or more things; **every** is used for more than two things, taken as a group.

e.g. Each of the two boys was wrong.
He read every book I gave him.

5. **Older** refers to persons or things.

e.g. This tree is older than that.

Elder refers to persons only.

e.g. He is my elder brother.

6. **Little** means 'not much'.

A little means 'at least some'.

e.g. He slept little.
He slept a little.

7. **Farther** means 'more distant'.

Further means 'additional'.

e.g. Mumbai is farther than Delhi.
I shall get further information.

8. **Latter** means the second of two things.

e.g. *Keats and Byron are romantic poets, but I prefer the latter.*

Later refers to time.

e.g. She came to school later than I.

ERRORS IN USE OF ADVERBS

Adverbs are words that add information about the verb.

1. Adverbs of manner, place and time are usually placed after the verb or object.

e.g. He was running slowly.
Reena does her work carefully.

2. Adverbs of frequency (e.g. never, often, usually, always, rarely, etc.) and other adverbs (like already, almost, just, quite, nearly, hardly) are normally put between subject and verb. If there is more than one word in the verb, they are put after first word.

e.g. He often goes to Delhi.
I quite agree with you.

But if verb is 'am', 'is' and 'are', adverb is placed after the verb.

e.g. I am *never* late for school.

3. The adverb *enough* is placed after the adjective.

e.g. She is cunning enough to tackle him.

4. 'Ever' is sometimes incorrectly used for 'never'.

e.g. He seldom or ever tells a lie. (×)
He seldom or never tells a lie. (✓)

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5. Adverb 'not' shouldn't be used with words having negative meaning.
e.g. The teacher forbade me not to go. (×)
The teacher forbade me to go. (✓)

6. The word 'only' should be placed immediately before the word it modifies.
e.g. Hari answered only two questions.

7. An adverb should not be used before an infinitive.
e.g. He quickly did the job. (×)
He did the job quickly. (✓)

8. The auxiliaries *have to* and *used to* come after the adverb.
e.g. He often used to go to cinema.

SOME MORE TIPS OF USAGE

1. Avoid the use of **lots, a lot** and **a whole lot** in the sense of **much** or **a great deal.**
e.g. He expects to earn a lot of money on his sale of farm products. (×)
He expects to earn a great deal of money on his sale of farm products. (✓)

2. **Plenty** is a noun and is always followed by **of.**
e.g. He has plenty of room in his old house.

3. **Due to** always modifies a noun and not a verb. Hence, no sentence should begin with **due to,** it must be used only after some form of the verb **to be**.
e.g. His death was due to natural causes.

4. **Alright** is incorrect, use **all right.**
e.g. I think it is a quite alright if you stay. (×)
I think it is quite all right if you stay. (✓)

5. If the gender is not determined, use the pronoun of the masculine gender.
e.g. If anybody has got the book let her return it (×)
If anybody has got the book let him return it (✓)

6. When two nouns are closely connected, apostrophe —'s will be written after second only.
e.g. Ravi's and Shikha's mother is ill. (×)
Ravi and Shikha's mother is ill. (✓)

7. **Both** and **as well as** cannot be used together in the same sentence because both convey the same sense.
e.g. Both Pinki as well as Pooja are beautiful (×)
Both Pinki and Pooja are beautiful. (✓)

8. **Supposing** and **if** cannot be used together in the same sentence.
e.g. Supposing if he fails, what will he do ?(×)
If he fails, what will he do ? (✓)

9. **Endure** suggests **suffering,** usually in silence and **tolerate,** means to allow it with some degree of approval.
e.g. He endured the pain without complaint.
He will not tolerate laziness.

10. Avoid the use of **had** with **ought.**
e.g. You had not ougth to do it. (×)
You ought not do it. (✓)

11. **As** is used when one compares things or persons of equal or about equal size or quality, **so** is used when one compares things or persons which are unequal.
e.g. He weighs as much as his father. (×)
He does not weigh so much as his father. (✓)

12. Avoid using the double comparative and double superlative.
e.g. *This pen is the most costliest in this store.* (×)
This pen is the costliest in this store. (✓)

13. **Cent per cent** and **word by word** are wrong uses. The real terms are **hundred percent** and **word for word.**

14. When two comparatives are used in a sentence for proportion, **the** is used before both of them.
e.g. The higher we go, the cooler it is.

15. **Do the needful** is incorrect, write **do what is necessary.**

16. Never say **family members,** say **members of the family.**
e.g. His family members have gone to Mumbai. (×)
The members of his family have gone to Mumbai (✓)

17. **Else** is always followed by **but** and never by **than.**
e.g. It is nothing else but your pride which makes you say such a thing.

18. **All of** is used in conversation, not in standard written English.
e.g. She gave all of her antiques to the museum (×)
She gave all her antiques to the museum (✓)

19. **Talking terms** is wrong, use **speaking terms.**
e.g. He is not on talking terms with his brother (×)
He is not on speaking terms with his brother (✓)

20. Verbs like resemble, recommend, comprise, order, accompany, reach, join, shirk, attack, emphasise, discuss, fear, succeed, resist, request, assist, benefit, afford etc. are not followed by any preposition when used in the active voice.
e.g. He emphasised on the need for discipline. (×)
He emphasised the need for discipline. (✓)
She will accompany with you to Chennai. (×)
She will accompany you to Chennai. (✓)

21. The phrase **type of, sort of** and **kind of** do not follow **a** or **an.**
e.g. What kind of a friend are you ? (×)
What kind of friend are you ? (✓)

22. A full hour number follows **o'clock,** but in fraction it does not follow.
e.g. He left this place by 10.40 o'clock. (×)
He left this place by 10.40. (✓)

23. **Yet** means up to the present time something that has not happened.
e.g. His brothers are not married even yet.

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EXERCISE – I

MCQ TYPE QUESTIONS

Directions (Q. 1 – 20)

In this section you are required to spot errors in sentences. Read each sentence to find out whether there is any error in any of the parts. Errors, if any, are only in one of the parts. No sentence has more than one error. When you detect an error, in any of the parts of the sentence, choose the corresponding alphabet.

1. (*a*) The scenery here is not good
(*b*) I have lost my furnitures
(*c*) We have received no information
(*d*) He told his mother this news
(*e*) No error.

2. (*a*) Please excuse the trouble
(*b*) He took pains over his work
(*c*) In India there are many poor
(*d*) Scouts wear shorts
(*e*) No error.

3. (*a*) I spent the holidays with my family
(*b*) Gone him some blotting paper
(*c*) Write this in your note book
(*d*) Gone my kind regards to all
(*e*) No error.

4. (*a*) We had a good play of football
(*b*) We saw a play
(*c*) We like acting
(*d*) He is a tall man
(*e*) No error.

5. (*a*) The boy was wearing a new suit
(*b*) He took offence at this
(*c*) Please put your sign here
(*d*) Please put your signature here
(*e*) No error.

6. (*a*) He is my cousin brother
(*b*) We are all brothers
(*c*) I spent the middle of the day working
(*d*) He had a large amount of money
(*e*) No error.

7. (*a*) He got into bad company
(*b*) One of my servant tells me
(*c*) Some of my servants tell me
(*d*) The front of the house
(*e*) No error.

8. (*a*) I had a bathe in the sea
(*b*) I have hurt a toe
(*c*) Each of these days play games
(*d*) None of us went
(*e*) No error.

9. (*a*) A man should work hard
(*b*) A man should not waste his time
(*c*) A boy should not waste his time
(*d*) Here is my cup; please fill it
(*e*) No error.

10. (*a*) Have you a pencil ? I have not got one
(*b*) Is he coming ? Yes I think so
(*c*) He enjoyed during the holidays
(*d*) I asked for my pencil but he did not give it to me
(*e*) No error.

11. (*a*) The boy who does best he will get a prize
(*b*) Whoever does best will get a prize
(*c*) Who did this ? I
(*d*) He and I are brothers
(*e*) No error.

12. (*a*) I went for a walk with some friends
(*b*) He is wiser than I
(*c*) The master tested the boy if he could read English
(*d*) I shall see whether the brakes work well
(*e*) No error.

13. (*a*) Everyone is frightened when he sees a tiger
(*b*) None of us has seen him
(*c*) People strave when they have no money
(*d*) The size of the shoe should be the same as this shoe
(*e*) No error.

14. (*a*) Everyone knows this
(*b*) Every man knows this
(*c*) These all mangoes are ripe
(*d*) He hold the book in both hands
(*e*) No error.

15. (*a*) I have no any friends
(*b*) Neither man has come
(*c*) All idle man should do some work or other
(*d*) Shakespeare is greater than any other poet
(*e*) No error.

16. (*a*) Open your book at page six
(*b*) He is in class ninth
(*c*) This article costs ten rupees
(*d*) He came a second time
(*e*) No error.

17. (*a*) King George VI
(*b*) He is older than I
(*c*) He is better than I
(*d*) Raipur is Hotter thanSimla
(*e*) No error.

18. (*a*) He is worse than I
(*b*) A horse is more useful than a car
(*c*) He gets a small salary
(*d*) I our library the number of books is less
(*e*) No error.

19. (*a*) He is the more clever of the two
(*b*) He is the cleverer of the two
(*c*) From the three he is more clever
(*d*) Of the two plans this is the better
(*e*) No error.

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20. (*a*) He is becoming strong
(*b*) There is a very good teacher in that class
(*c*) He will spend the rest of his life here
(*d*) This is a sight worth seeing
(*e*) No error.

Directions (Q. 21 – 30)

Each question below has a sentence, from the choices provided, identify the one which best restates the given sentence and mark its number as the answer.

21. Uneasy lies the head that wears a crown.
(*a*) Uneasily lies the head that wears a crown.
(*b*) Uneasy is the head wearing a crown.
(*c*) The head which wears the crown lies uneasy.
(*d*) The head which is wearing the crown lies uneasy.

22. Had I been a little early, I would not have missed the train.
(*a*) Had I been a little early, I would not have missed the train.
(*b*) Had I been a little early than now, I would not have missed the train.
(*c*) Had I been a little earlier, I would not have missed the train.
(*d*) Had I only been a little early than now, I would not have missed the train.

23. She is as intelligent as her sister if not more.
(*a*) She is more intelligent than her sister.
(*b*) She is as intelligent as her sister if not more intelligent.
(*c*) She and her sister are equally intelligent.
(*d*) She is less intelligent than her sister.

24. You will not succeed unless you are optimistic.
(*a*) You will not be successfull unless you are optimistic.
(*b*) You will not succeed until you are an optimist.
(*c*) You will not succeed unless you don't be optimistic.
(*d*) You will not succeed unless you don't be an optimistic.

25. The students were advised to follow the instructions of the examiner.
(*a*) The instructions of the examiner were followed by the students.
(*b*) The students followed the examiner's instructions.
(*c*) The examiner's instructions were advised to be followed by the students..
(*d*) It has been advised to the students that they should follow the examiner's instructions.

26. Despite being ill, he attended the classes.
(*a*) Despite of being ill, he attended the classes.
(*b*) Despite his illness, he attended the classes.
(*c*) Inspite of his being ill, he attended classes.
(*d*) The classes were attended by him inspite of being ill.

27. That man is aggressive by nature is a hard fact of life and no one can deny it.
(*a*) That man is aggressive by nature is a hard fact of life which none can deny.
(*b*) That man is aggressive by nature is a hard fact of life and no one can deny it.
(*c*) That man is aggressive by nature is a hard fact of life and not one can deny.
(*d*) That man is aggressive by nature is hard for anyone to deny.

28. He is as old as I.
(*a*) He is as old as me.
(*b*) He and I are equally old.
(*c*) He and I are equally older.
(*d*) Both he and I are of the same age.

29. A man becomes wiser with age and experience.
(*a*) A man as he is aged and experienced becomes wiser.
(*b*) A man with age and experience becomes wiser.
(*c*) A man as he is aged and experienced becomes the wiser.
(*d*) With age and experience, a man becomes wiser.

30. Hazards of life cannot be negated but they can be quite effortlessly evaded.
(*a*) Hazards in life can be quite effortlessly evaded and completely nullified.
(*b*) Hazards of lilfe cannot be avoided but they can be made ineffective.
(*c*) To be made ineffective, hazards of life must be avoided.
(*d*) It may not be possible to nullify the hazards of life but they can quite easily be avoided.

Directions(Q. 31 –40)

Each sentence is broken into four parts a, b, c, d. Mark the part which has an error. Ignore errors of punctuation.

31. (*a*) Every man, woman
(*b*) and child
(*c*) in the house on fire
(*d*) have been saved.

32. (*a*) One of the
(*b*) best lawyers in town
(*c*) have been
(*d*) hired.

33. (*a*) I request you (*b*) kindly to
(*c*) come to me (*d*) immediately.

34. (*a*) My friend's mother
(*b*) is the principal
(*c*) of a
(*d*) girl's college.

35. (*a*) To succeed in these tests
(*b*) it is absolutely necessary
(*c*) for us
(*d*) to aim for speed and accuracy.

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36. (*a*) The aeroplane is a
(*b*) powerful instrument of war
(*c*) and their peacetime role
(*d*) is just as important.

37. (*a*) The depletion of ozone in the atmosphere
(*b*) is a reminder to all of us
(*c*) both in the developed and developing world
(*d*) that they cannot continue to use contraptions which give out noxious gases.

38. (*a*) Scotsmen are notorious
(*b*) for their thrift
(*c*) and he is the butt
(*d*) of many exaggerated jokes.

39. (*a*) A mixed economy
(*b*) is it in which
(*c*) the public sector
(*d*) and the private sector co - exist.

40. (*a*) Although the beaver's hind feet
(*b*) are webbed for swimming
(*c*) their front feet
(*d*) are small and handlike.

Directions(Q. 41 – 45)

Given below are five sentences, each of which may or may not have errors.

Mark (a), if there is only one error.

Mark (b), if there are two errors.

Mark(c), if there are more than two errors.

Mark (d), if there is no error.

41. "This feels like one big dream," says a weary American woman to her husband, as they sit in a plane flying home.

42. The worst mistake a non - profit organization can make is to take all the money it is given and be beholden to doing things on someone else's terms.

43. Although most hotels in Bhutan has a distinctly local flavour, some pander with the perceived tastes of Western tour groups - and can be rather bland.

44. The crustaceans find sanctuary in the jords, often clinging to the walls in large, gregarious groups.

45. In these tropical lowlands the Maya built an agrarian civilization that eventually supported one of the highest population densities in the pre - industrial America.

Directions (Q. 46–50)

Each of the following questions has two sentences A and B.

Mark (a), if you think sentence A has an error.

Mark (b), if you think sentence B has an error.

Mark (c), if you think both sentences A and B have errors.

Mark (d), if you think neither sentence has an error.

46. A. The boss himself or his secretary answer the phone on Saturday.
B. Neither the quality nor the prices have changed.

47. A. The members re-commended that all delinquents be fined.
B. A stone lying in one position for a long time may gather moss.

48. A. Until I received that letter, I was hoping to have had a visit from Krishnan.
B. Follow the main road for a mile; then you need to take the next road at the left.

49. A. The people to watch closely are the ones ruling behind the political scene.
B. Give the tools to whoever can use them best.

50. A. The magazine has been published continuous since 1951, and it has the funniest cartoons you can possibly imagine.
B. I feel glumly when I read his nightmarish tales.

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. Choose the statement where underlined word is used correctly
(*a*) The minister <u>insured</u> the victims that everything would be all right.
(*b*) He <u>ensured</u> that the company will not have to bear any loss.
(*c*) The actor got himself <u>ensured</u> against any accident.
(*d*) The teacher <u>insured</u> students of good results

2. The following question presents a sentence, part of which is underlined. Beneath the sentence you find four ways of phrasing the underline part. Following the requirements of the standard written English, select the answer that produces the most effective sentence.

Tuberculosis, together with its effects, <u>ranks one of the leading causes of death</u> in India.
(*a*) ranks as one of the leading causes of death
(*b*) rank as one of the leading causes of death
(*c*) has the rank of one of the leading causes of death
(*d*) are one of the leading causes of death

3. Choose the statement where underlined word is used correctly.
(*a*) When the teacher eludes to different authors, he is being <u>elusive</u>
(*b*) When the thief keeps eluding the police, he is being <u>elusive</u>
(*c*) Matters that are difficult to understand, identify or remember are <u>allusive</u>
(*d*) Mirages can be <u>allusive</u>, but a better way to express them is illusory

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4. Fill in the blank with the correct idiom/phrase.
That boy from the town was a __________ in the sleepy village.
(*a*) Dog out of herd (*b*) Sheep from the heap
(*c*) Fish out of water (*d*) Bird from the flock

5. Select the appropriate option in place of underlined part of the sentence.
<u>Increased productivity necessary</u> reflects greater efforts made by the employees.
(*a*) Increase in productivity necessary
(*b*) Increase productivity is necessary
(*c*) Increase in productivity necessarily
(*d*) No improvement required

6. Ram and Shyam shared a secret and promised to each other that it would remain between them. Ram expressed himself in one of the following ways as given in the choices below. Identify the correct way as per standard English.
(*a*) It would remain between you and me
(*b*) It would remain between me and you
(*c*) It would remain between you and I
(*d*) It would remain with me

7. What is the adverb for the given word below? Misogynous
(*a*) Misogynousness (*b*) Misogynity
(*c*) Misogynously (*d*) Misogynous

8. In the following sentence certain parts are underlined and marked P, Q and R. One of the parts may contain certain error or may not be acceptable in standard written communication. Select the part containing an error. Choose D as your Answer: if there is no error.
The student corrected $\underbrace{\text{all the errors}}_{\text{P}}$ that $\underbrace{\text{the instructor marked}}_{\text{Q}}$ on the $\underbrace{\text{answer book}}_{\text{R}}$
(*a*) P (*b*) Q
(*c*) R (*d*) No Error

9. Choose the statement where underlined word is used correctly.
(*a*) The industrialist had a <u>personnel</u> jet.
(*b*) I write my experience in my <u>personnel</u> diary.
(*c*) All <u>personnel</u> are being given the day off.
(*d*) Being religious is a <u>personnel</u> aspect.

10. Out of the following four sentences, select the most suitable sentence with respect to grammer and usage:
(*a*) Since the report lacked needed information, it was of no use to them.
(*b*) The report was useless to them because there were no needed information in it.
(*c*) Since the report did not contain the needed information, it was not real useful to them.
(*d*) Since the report lacked needed information, it would not had been useful to them.

11. Select the alternative meaning of the underlined part of the sentence.
The chain snatchers <u>took to their heels</u> when the police party arrived.
(*a*) took shelter in a thick jungle
(*b*) open indiscriminate fire
(*c*) took to flight
(*d*) unconditionally surrendered

2014

12. Which of the following options is the closest in meaning to the word underlined in the sentence below?
In a democracy, everybody has the freedom to <u>disagree</u> with the government.
(*a*) dissent (*b*) descent
(*c*) decent (*d*) decadent

13. 'Advice' is ________.
(*a*) a verb (*b*) a noun
(*c*) an adjective (*d*) both a verb and a noun

14. Which of the options given below best completes the following sentence?
She will feel much better if she ________.
(*a*) will get some rest
(*b*) gets some rest
(*c*) will be getting some rest
(*d*) is getting some rest

15. $\underbrace{\text{While trying to collect}}_{\text{I}}$ an envelope $\underbrace{\text{from under the table,}}_{\text{II}}$ $\underbrace{\text{Mr. X fell down}}_{\text{III}}$ and $\underbrace{\text{was losing consciousness.}}_{\text{IV}}$
Which one of the above underlined parts of the sentence is NOT appropriate?
(*a*) I (*b*) II
(*c*) III (*d*) IV

16. Which of the following options is the closest in meaning to the phrase underlined in the sentence below?
It is fascinating to see life forms <u>cope with</u> varied environmental conditions.
(*a*) Adopt to
(*b*) Adapt to
(*c*) Adept in
(*d*) Accept with

17. In a press meet on the recent scam, the minister said, "The buck stops here". What did the minister convey by the statement?
(*a*) He wants all the money
(*b*) He will return the money
(*c*) He will assume final responsibility
(*d*) He will resist all enquiries

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2013

18. The professor (I) ordered to (II) the students to go (III) out of the class. (IV)

Which of the above underlined parts of the sentence is grammatically incorrect?

(*a*) I (*b*) II
(*c*) III (*d*) IV

19. Choose the grammatically INCORRECT sentence:
(*a*) He is of Asian origin.
(*b*) They belonged to Africa.
(*c*) She is an European.
(*d*) They migrated from India to Australia.

20. Choose the grammatically CORRECT sentence:
(*a*) Two and two add four.
(*b*) Two and two become four.
(*c*) Two and two are four.
(*d*) Two and two make four.

21. All engineering students (I) should learn mechanics, (II) mathematics and (III) how to do computation. (IV)

Which of the above underlined parts of the sentence is not appropriate?

(*a*) I (*b*) II
(*c*) III (*d*) IV

2012

22. Choose the most appropriate alternative from the options given below to complete the following sentence :

Suresh's dog is the one ______ was hurt in the stampede.

(*a*) that (*b*) which
(*c*) who (*d*) whom

23. Choose the grammatically **INCORRECT** sentence :
(*a*) They gave us the money back less the service charges of Three Hundred rupees.
(*b*) This country's expenditure is not less than that of Bangladesh.
(*c*) The committee initially asked for a funding of Fifty Lakh rupeesr, but later settled for a lesser sum.
(*d*) This country's expenditure on educational reforms is very less.

24. One of the parts (A, B, C, D) in the sentence given below contains an ERROR. Which one of the following is **INCORRECT** ?

I requested that he should be given the driving test today instead of tomorrow.

(*a*) requested that
(*b*) should be given
(*c*) the driving test
(*d*) instead of tomorrow

ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*b*)	**2.** (*a*)	**3.** (*e*)	**4.** (*a*)	**5.** (*c*)	**6.** (*a*)	**7.** (*b*)	**8.** (*c*)	**9.** (*e*)	**10.** (*c*)
11. (*a*)	**12.** (*c*)	**13.** (*d*)	**14.** (*c*)	**15.** (*a*)	**16.** (*b*)	**17.** (*d*)	**18.** (*d*)	**19.** (*c*)	**20.** (*a*)
21. (*c*)	**22.** (*a*)	**23.** (*c*)	**24.** (*a*)	**25.** (*d*)	**26.** (*b*)	**27.** (*a*)	**28.** (*d*)	**29.** (*d*)	**30.** (*d*)
31. (*d*)	**32.** (*c*)	**33.** (*b*)	**34.** (*d*)	**35.** (*d*)	**36.** (*c*)	**37.** (*d*)	**38.** (*c*)	**39.** (*b*)	**40.** (*c*)
41. (*a*)	**42.** (*d*)	**43.** (*b*)	**44.** (*d*)	**45.** (*d*)	**46.** (*a*)	**47.** (*d*)	**48.** (*c*)	**49.** (*d*)	**50.** (*c*)

EXERCISE – II

MCQ Type Questions

1. (*b*)	**2.** (*a*)	**3.** (*b*)	**4.** (*c*)	**5.** (*c*)	**6.** (*a*)	**7.** (*c*)	**8.** (*b*)	**9.** (*c*)	**10.** (*a*)
11. (*c*)	**12.** (*a*)	**13.** (*b*)	**14.** (*b*)	**15.** (*d*)	**16.** (*b*)	**17.** (*c*)	**18.** (*b*)	**19.** (*c*)	**20.** (*d*)
21. (*d*)	**22.** (*a*)	**23.** (*d*)	**24.** (*b*)						

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EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

21. Uneasily does not denote a state, which is required in the context. 'Wearing a crown' denotes that the head is doing it, 'is wearing' denotes a continuous action, which is impossible, as the head cannot do it. Hence choice (*c*) restates the given sentence correctly.

22. All the statements have the correct combination of tenses. So, we need not check it. Let us find the errors in other statements. 'Early than now' and 'earlier' suggest that I reached early; in that case, I did not miss the train. So, choices (*b*), (*c*), and (*d*) do not state the same idea as in the question statement. Hence correct choice is (*a*).

23. The given statement states that both she and her sister are equally intelligent. This is given in statement. Hence correct choice is (*c*).

24. 'Unless' denotes condition, 'until' denotes time. Choice (*b*) has adverb clause of time.With unless, another negative is not used. 'Unless ..... don't' is wrong. So, Choices (*c*) and (*d*) are incorrect. Hence correct choice is (*a*).

25. Choices (*a*) and (*b*) state the same idea [(*a*) – passive voice; (*b*) Active voice] that the students followed the instructions, which differs from the question statement. Instructions cannot be advised. So, choice (*c*) is meaningless. Choice (*d*) states the context of the given statement aptly.

26. Despite means inspite of. Despite of is a wrong usage. 'His' need not be used in choice (*c*), as the sentence is about one person. Choice (*d*) means that the classes were ill. Hence correct choice is (*b*).

27. Choice (*a*) best restates the given sentence, though choice (*b*) is the repetition of the given statement. 'Not one can deny' in choice (*c*) and for any one to deny' in choice (*d*) make them incorrect. Hence correct choice is (*a*).

28. In this type of comparison, the pronouns are in subjective case 'Me', which is in objective case is wrongly used in choice (*a*). Equally is not used with 'old'. Of the same age is the correct expression. Hence correct choice is (*d*).

29. Choice (*a*) and (*b*) mean that the prerequisite for wisdom is age and experience, which is incorrect. Just by growing old and working for a long period, a person cannot improve his wisdom. 'The wiser' is used only when we compare wisdom with some other quality. Choice (*d*) is the correct restatement of the given one. Hence correct choice is (*d*).

30. The given statement says that one cannot completely nullify the hazards of life but they can be evaded quite easily. Choice (*a*) which states that hazards cannot be avoided is wrong (*b*) states that hazards can be made ineffective and hence this is also wrong (*c*) does not convey the meaning as stated and hence it too is incorrect. (*d*) correctly restates the idea and is our answer.

31. Every man refers to a singular noun, therefore.... has been saved.

32. has

33. drop 'kindly'

34. Girl's implies possessive noun, whereas the college is meant for girls.

35. aim at, aim for is wrong usage.

36. delete their - add : its

37. delete they - add : we

38. delete he is - add : they are.

39. delete it - add : that

40. delete their - add : its.

41. One sits 'on' a plane.

42. There are no errors.

43. Hotels 'have' and some pander 'to....'

44. There are no errors in this sentence.

45. There are no errors.

48. (A) hoping to have a visit;
(B) .... then take the next road to

50. (A) continuously; (B) glum.

EXERCISE – II

MCQ TYPE QUESTIONS

1. *Insured*–the person, group, or organization whose life or property is covered by an insurance policy.

Ensured–to secure or guarantee

3. Elusive: Difficult to answer.

4. From the statement, it appears that boy found it tough to adapt to a very different situation.

8. The is not required in 'Q'.

22. The correct answer is 'that'. In the given sentence 'was hurt in the stampede' determines which dog belongs to Suresh.

■■

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2 CHAPTER

Sentence Completion

WHAT ARE SENTENCE COMPLETION PROBLEMS?

Sentence completion problems test your vocabulary skills as well as your reading ability. Unlike the reading comprehension questions, which require you to read long passages, these problems contain a single sentence expressing a complete idea that can be understood without any additional information. Each sentence contains one or two blanks, indicating that one or two words are missing. You are then presented with five words or phrases, or five pairs of words or phrases if there are two blanks in the sentence. From these choices, you need to select the words or phrases that fit into the blanks to best complete the sentence. To make the correct choice, you will need to be able to understand the main idea of the sentence and the logical structure of the sentence. Next, you will need to know the definitions of the words in the answer choices.

BASIC STRATEGIES FOR SENTENCE COMPLETION PROBLEMS

Strategy 1

The first strategy is to break the sentence down into the main clause and introductory clause or phrase in order to identify the relationship it expresses. For example, consider the following:

In a hopsital setting, hand washing is primarily a______measure.

(*a*) prophylactic (*b*) pseudocyetic

(*c*) protensive (*d*) propaedeutic

The main clause declares that hand washing is primarily a measure. However, preceding the declaration is a condition or constraint expressed in an introductory phrase, "a hospital setting." This phrase sets the contextual field within which the main clause is to be understood. If you generalize from your knowledge, it would seem that any measure promoted in a hospital should result in good health. Only one of the answers would result in this. That is answer (*a*) in which *prophylactic* means "prevents disease."

Strategy 2

A second strategy is to attend to sets, whether a set of synonyms or antonyms or special word groups. For example, consider the following:

Even when a______ mother was made available to the infant chimpanzee, the infant detected the______and reacted with high anxiety.

(*a*) hedonistic.. contrivance

(*b*) vagrant.. apostasy

(*c*) veteran.. ruse

(*d*) pretender.. interference

Nothing in a common sense association supports the pairing of *hedonistic* and *contrivance, vagrant* and *apostasy, veteran* and *ruse,* or *pretender* and *interference*. The only set of related words is *surrogate* (meaning substitute) and *substitution.*

Strategy 3

Pay attention to the words that control the figurative field of the sentence. Consider the following passage:

In seventeenth-century France, private and public opinion portrayed the physician as a vulture of greed, circling the sick and dying, battening off the suffering of the afflicted, and ______the remains of his prey.

(*a*) spending (*b*) devouring

(*c*) retreating from (*d*) decrying

Clearly the correct answer is (*b*), devouring. The key words that control the figurative field are *vulture, circling,* and *battening.* Read the items carefully and make sure that the completed sentence is meaningful, that it makes sense.

EXERCISE – I

MCQ TYPE QUESTIONS

Directions for Q. 1 to 41 : *In each of the following sentences, a part/parts of the sentence is left unfinished. Beneath each sentence, four different ways of completing the sentence are indicated. Choose the best alternative among the four.*

1. For some of our ruling politicians, social justice has apparently come to mean that ____________.

(*a*) they have a right to plunder public money from the treasury

(*b*) they have a right to the plunder of public money from the treasury

(*c*) theirs is the right to plunder public money from the treasury

(*d*) the right is theirs to plunder public money from the treasury

2. Some critics believe that Satyajit Ray never quite came back to the great beginning he made in his path-breaking film Pather Panchali. ____________ have endured decades of well-travelled bad prints to become a signpost in cinematic history.

(*a*) The bizarre history of its misty origins

(*b*) Its haunting images

(*c*) Its compelling munificence

(*d*) The breathtaking awe it inspires

3. ____________ , the more they remain the same.

(*a*) The less the dynamism

(*b*) The more things change

(*c*) The more pronounced the transformation

(*d*) The more the merrier

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4. Revenues are likely to register a dramatic increase if ____________ on the hundred million people who are said to comprise the risingIndian middle class.
(*a*) a flat tax is collected
(*b*) a flat tax is placed
(*c*) taxes at a flat rate are charged
(*d*) tax is imposed at a flat rate

5. I am an entertainer. ____________ I have to keep smiling because, deep in my heart, laughter and sorrow have an affinity.
(*a*) Even if I have tears in me
(*b*) Despite conditions of extreme adversity
(*c*) Inspite of misery around me
(*d*) Although I have yet to make it big

6. The stock market is probably ____________ And the way the markets are plunging says a lot about investor confidence.
(*a*) an ideal indication of the health of public sentiment
(*b*) the least imperfect mechanism for judging the quantity of the sentiment of the public.
(*c*) the best indicator of public sentiment
(*d*) the best barometer to assess the sentiment of the public

7. Home is ____________ one has yet to improve.
(*a*) a discovery over which
(*b*) a certain over which
(*c*) an invention on which
(*d*) an institution upon which

8. The highest reward for a man's toil is not what he gets for it but what ___________.
(*a*) he makes out of it.
(*b*) he gets for others.
(*c*) he has overcome through it.
(*d*) he becomes by it.

9. An act of justice closes the book on a misdeed; an act of vengeance ___________.
(*a*) reopens the first chapter
(*b*) writes an epoilogue
(*c*) writes one of its own
(*d*) opens new books

10. The trail is the thing , not the end of the trail. Travel too fast and you miss __________.
(*a*) all you are travelling for.
(*b*) all the sights you are supposed to see.
(*c*) the very excitement of your travel.
(*d*) all the enjoyments of travel.

11. ________________ that in this apparent mess, two things not be interfered with.
(*a*) It is important.
(*b*) it is of cardinal important.
(*c*) It should be urgently under-stood.
(*d*) It cannot be emphasised.

12. A country's wealth is its people. But instead of drawing out the strengths of the people, instead of drawing out their talent, this use of religion debases, degrades and depresses than ____________.
(*a*) in greater and greater measure
(*b*) further
(*c*) beyond reasonable limit
(*d*) more and more

13. Wines that yielded a good commercial profit ____________ in the same limited areas of France as of now.
(*a*) seem to have been produced
(*b*) appear to have a remarkable semblance
(*c*) bear a significant similarity in terms of production to those grown
(*d*) appear to have been similarly produced

14. This is about ____________ a sociological analysis can penetrate.
(*a*) the other limits that
(*b*) just how far into the subject that
(*c*) the relative distance that
(*d*) as far as

15. The safest general characterisation of the European philosophical tradition as it has developed up to now, with all its diverse proponents, is that it consists of a_________Plato.
(*a*) series of footnotes to
(*b*) set of prologues to
(*c*) collection of chapters on
(*d*) string of commentatries to

16. The ideas that these companies used seem so simple with ___________ that their competitors will now ___________ themselves for not thinking of them first.
(*a*) advanced technology, hit
(*b*) analysis , abuse
(*c*) passage of time, curse
(*d*) hindsight, kick

17. No doubt, it was our own government but it was being run on borrowed ideas, using ____________solutions.
(*a*) worn out
(*b*) second hand
(*c*) impractical
(*d*) appropriate

18. Overall, all the recent policy changes by the government only amount to a __________ in the sugar industry.
(*a*) superficial attemp at liberalisation
(*b*) drop in the ocean
(*c*) large change
(*d*) small regulating authority

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19. In pusuance of their decision to resist what they saw as anti-labour policies, the company employees, union launched agitation to __________.

(*a*) show their virility
(*b*) reaffirm their commitment to the company.
(*c*) bring down the government
(*d*) demonstrate their strength

20. Scientists have found ________ evidence that Neanderthals, now an extinct species of humans __________ Europe, were cannibals.

(*a*) grisly, living in
(*b*) incontrovertible, a cross
(*c*) chilling inhabiting
(*d*) proper, in

21. The telephone symbolises that awkard __________ in all communication technologies ; while it ____________ to bring us together, it keeps us apart.

(*a*) paradox, needs (*b*) irony, intends
(*c*) paradox , tries (*d*) irony wishes

22. The world is fast moving into a period of 'civilisational clash' in which the primary identification of people and countries will not be ___________ as during the Cold War, but ___________.

(*a*) ideological, technological
(*b*) cultural, ethnic
(*c*) political, economic
(*d*) ideological, cultural

23. In our country, the challenges are to raise ______ incomes to reduce poverty, and to _______inefficient public sector enterprises.

(*a*) farm, liberalise
(*b*) middle-class, privatise
(*c*) workers, take over
(*d*) rural, restructure

24. The interest generated by the Soccer World Cup is ____________ compared to the way cricket ____________ the nation.

(*a*) milder, fascinates
(*b*) lukewarm, electrifies
(*c*) tepid, inspires
(*d*) unusual, grips

25. Though one eye is kept firmly on the __________, the company now also promotes _____contemporary art.

(*a*) present, experimental
(*b*) future, popular
(*c*) present, popular
(*d*) market, popular

26. The law prohibits a person from felling a sandalwood tree, even if it grows on one's own land, without prior permission from the government. As poor people cannot deal with the government, this legal provision leads to a rip-roaring business for ___________, who care neither for the ________, nor for the trees.

(*a*) middlemen, rich (*b*) the government, poor
(*c*) touts, rich (*d*) touts, poor

27. It will take some time for many South Koreans to ____________ the conflicting images of North Korea, let alone to ____________ what to make of their northern cousins.

(*a*) reconcile, decide (*b*) understand, clarify
(*c*) make out, decide (*d*) reconcile, understand

28. In these bleak and depressing times of ______ prices, non-performing governments and ____________ crime rates, Sourav Ganguly has given us, Indians, a lot to cheer about.

(*a*) escalating, increasing
(*b*) spiralling, booming
(*c*) spiralling, soaring
(*d*) ascending, debilitating

29. The manners and _________ of the nouveau riche is a recurrent ____________ in the literature.

(*a*) style, motif (*b*) morals, story
(*c*) wealth, theme (*d*) morals, theme

30. Many people suggest _________and still others would like to convince people not to buy pirated cassettes.

(*a*) to bring down audio-cassette prices to reduce the incidence of music piracy, others advocate strong legal action against the offenders,
(*b*) bringing down audio-cassette prices to reduce the incidents of music piracy, others are advocating strong legal action against offenders,
(*c*) bringing down audio-cassette prices to reduce the incidence of music piracy, others advocate strong legal action against offenders,
(*d*) audio-cassette prices to be brought down to reduce incidence of music piracy, others advocate that strong legal action must be taken against offenders,

31. The ancient Egyptians believed________so that when these objects were magically reanimated through the correct rituals, they would be able to function effectively.

(*a*) that it was essential that things they portrayed must have every relevant feature shown as clearly as possible
(*b*) it was essential for things they portray to have had every relevant feature shown as clearly as possible,
(*c*) it was essential that the things they portrayed had every relevant feature shown as clearly as possible,
(*d*) that when they portrayed things, it should have every relevant feature shown as clearly as possible

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32. Archaeologists believe that the pieces of red-ware pottery excavated recently near Bhavnagar and ________shed light on a hither to dark 600-year period in the Harappan history of Gujarat.

(*a*) estimated with a reasonable certainty as being about 3400 years old,

(*b*) are estimated reasonably certain to be about 3400 years old

(*c*) estimated at about 3400 years old with reasonable certainty,

(*d*) estimated with reasonable certainty to be about 3400 years old,

33. The genocides in Bosnia and Rwanda, apart from being mis-described in the most sinister and _______manner as 'ethnic cleansing', were also blamed, in further hand-washing rhetoric, on something dark and interior to _________ and perpetrators alike.

(*a*) innovative; communicator

(*b*) enchanting; leaders

(*c*) disingenuous; victims

(*d*) exigent; exploiters

34. As navigators, calendar makers, and other ____of the night sky accumulated evidence to the contrary, ancient astronomers were forced to______that certain bodies might move in circles about points, which in turn moved in circles about the earth.

(*a*) scrutinizers; believe

(*b*) observers; agree

(*c*) scrutinizers; suggest

(*d*) observers; concede

35. Every human being, after the first few days of his life, is a product of two factors: on the one hand, there is his _______ endowment; and on the other hand, there is the effect of environment, including _______.

(*a*) constitutional; weather

(*b*) congenital; education

(*c*) personal; climate

(*d*) genetic; pedagogy

36. Exhaustion of natural resources, destruction of individual initiative by governments, control over man's minds by central ______ of education and propaganda are some of the major evils which appear to be on the increase as a result of the impact of science upon minds suited by _______ to an earlier kind of world.

(*a*) tenets; fixation

(*b*) aspects; inhibitions

(*c*) institutions; inhibitions

(*d*) organs; tradition

37. But ________________ are now regularly written not just for tools, but well-established practices, organisations and institutions, not all of which seem to be ________________ away.

(*a*) reports, withering (*b*) stories, trading

(*c*) books, dying (*d*) obituaries, fading

38. The Darwin who _______________ is most remarkable for the way in which he ________________ the attributes of the world class thinker and head of the household.

(*a*) comes, figures

(*b*) arises, adds

(*c*) emerges, combines

(*d*) appeared, combines

39. Since her face was free of _____________ there was no way to __________ if she appreciated what had happened.

(*a*) make-up, realise

(*b*) expression, ascertain

(*c*) emotion, diagnose

(*d*) scars, understand

40. In this context, the _________ of the British labour movement is particularly __________.

(*a*) affiar, weird

(*b*) activity, moving

(*c*) experience, significant

(*d*) atmosphere, gloomy

41. Indian intellectuals may boast, if they are so inclined, of being ________________ to the most elitist among the intellectual ________________ of the world.

(*a*) subordinate, traditions

(*b*) heirs, cliques

(*c*) ancestors, societies

(*d*) heir, traditions

Directions for Q. 42 to 47.

Fill the gaps in the passages below with the most appropriate word from the options given for each gap. The right words are the ones used by the author. Be guided by the author's overall style and meaning when you choose the answers.

Von Nuemann and Morgenstern assume a decision framework in which all options are thoroughly considered, each option being independent of others, with a numerical value derived for the utility of each possible outcome (these outcomes reflecting, in turn, all possible combinations of choices). The decision is then made to maximize the expected utility.

(42) _____, such a model reflects major simplifications of the way decisions are made in the real world. Humans are not able to process information as quickly and effectively as the model assumes; they tend not to think (43) _____ as easily as the model calls for; they often deal with a particular option without really assessing its (44) _____, and when they do assess alternatives, they may be extremely nebulous about their criteria of evaluation.

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42. (*a*) Regrettably (*b*) Firstly
(*c*) Obviously (*d*) Apparently

43. (*a*) quantitatively (*b*) systematically
(*c*) scientifically (*d*) analytically

44. (*a*) implications (*b*) disadvantages
(*c*) utility (*d*) alternatives

In a large company, (45) _____ people is about as common as using a gun or a switch-blade to (46) _____ an argument. As a result, most managers have little or no experience of firing people, and they find it emotionally traumatic; as result, they often delay the act interminably, much as an unhappy spouse will prolong a bad marriage. And when the firing is done, it's often done clumsily, with far worse side effects than are necessary.

Do the world-class softward organizations have a different way of firing people ? No, but they do the deed swiftly, humanely, and professionally.

The key point here is to view the fired employees as a "failed product" and to ask how the process (47) _____ such a phenomenon in the first place.

45. (*a*) dismissing (*b*) punishing
(*c*) firing (*d*) admonishing

46. (*a*) resolve (*b*) thwart
(*c*) defeat (*d*) close

47. (*a*) derived (*b*) engineered
(*c*) produced (*d*) allowed

Directions for Q. 48 to 57 :

Fill up the blanks, numbered [48], [49]....up to [57], in the two passages below with the most appropriate word from the options given for each blank.

"Between the year 1946 and the year 1955, I did not file any income tax returns." With that [48] statement, Ramesh embarked on an account of his encounter with the Income Tax Department. "I originally owed Rs. 20,000 in unpaid taxes. With [49] and [50], the 20,000 became 60,000. The Income Tax Department then went into action, and I learned first hand just how much power the Tax Department wields. Royalties and trust funds can be [51]; automobiles may be [52], and auctioned off. Nothing belongs to the [53] until the case is settled."

48. (*a*) devious (*b*) blunt
(*c*) tactful (*d*) pretentious

49. (*a*) interest (*b*) taxes
(*c*) principal (*d*) returns

50. (*a*) sanctions (*b*) refunds
(*c*) fees (*d*) fines

51. (*a*) closed (*b*) detached
(*c*) attached (*d*) impounded

52. (*a*) smashed (*b*) seized
(*c*) dismantled (*d*) frozen

53. (*a*) purchaser (*b*) victim
(*c*) investor (*d*) offender

At that time the White House was as serene as a resort hotel out of season. The corridors were [54]. In the various offices, [55] gray men in waistcoats talked to one another in low-pitched voices. The only colour, or choler, curiously enough, was provided by President Eisenhower himself. Apparently, his [56] was easily set off; he scowled when he [57] the corridors.

54. (*a*) striking (*b*) hollow
(*c*) empty (*d*) white

55. (*a*) quiet (*b*) faded
(*c*) loud (*d*) stentorian

56. (*a*) laughter (*b*) curiosity
(*c*) humour (*d*) temper

57. (*a*) paced (*b*) strolled
(*c*) stormed (*d*) prowled.

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. Choose the most appropriate word from the options given below to complete the following sentence. The official answered ________ that the complaints of the citizen would be looked into.

(*a*) respectably (*b*) respectfully
(*c*) reputably (*d*) respectively

2. Choose the most appropriate word from the options given below to complete the following sentence.

The principal presented the chief guest with a ___________, as token of appreciation.

(*a*) momento (*b*) memento
(*c*) momentum (*d*) moment

3. Choose the appropriate word/phrase, out of the four options given below, to complete the following sentence :

Frogs ________.

(*a*) Croak (*b*) Roar
(*c*) Hiss (*d*) Patter

4. Extreme focus on syllabus and studying for tests has become such a dominant concern of Indian students that they close their minds to anything ___________ to the requirements of the exam

(*a*) related
(*b*) extraneous
(*c*) outside
(*d*) useful

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5. The Tamil version of _____ John Abraham-starrer Madras café _____ cleared by the Censor Board with no cuts last week but the film's distributors _____ no takers among the exhibitors for a release in Tamil Nadu _____ this Friday.
(*a*) Mr., was, found, on
(*b*) a, was, found, at
(*c*) the, was, found, on
(*d*) a, being, find at

6. Choose the appropriate word/phase, out of the four options given below, to complete the following sentence:
Apparent lifelessness ___________ dormant life.
(*a*) harbours (*b*) lead to
(*c*) supports (*d*) affects

7. Choose the correct verb to fill in the blank below :
Let us _____________.
(*a*) Introvert (*b*) Alternate
(*c*) Atheist (*d*) Altruist

8. Choose the most appropriate word from the options given below to complete the following sentence :
If the Athlete had wanted to come first in the race, he ___________ several hours everyday
(*a*) should practice
(*b*) should have practiced
(*c*) practiced
(*d*) should be practicing

9. Choose the appropriate word/phrase, out of the four options given below, to complete the following sentence:
Dhoni, as well as the other team members of Indian team, _____ present on the occasion.
(*a*) were (*b*) was
(*c*) has (*d*) have

10. We ________ our friend's birthday and we _______ how to make it up to him.
(*a*) completely forgot — don't just known
(*b*) forget completely — don't just know
(*c*) completely forget —just don't know
(*d*) forgot completely —just don't know

11. Which of the following options is the closest in meaning to the sentence below?
She enjoyed herself immensely at the party.
(*a*) She had a terrible time at the party
(*b*) She had a horrible time at the party
(*c*) She had a terrific time at the party
(*d*) She had a terrifying time at the party

12. Didn't you buy _______ when you went shopping?
(*a*) any paper (*b*) much paper
(*c*) no paper (*d*) a few paper

2014

13. Choose the most appropriate word from the options given below to complete the following sentence:
A person suffering from Alzheimer's disease _________ short-term memory loss.
(*a*) experienced
(*b*) has experienced
(*c*) is experiencing
(*d*) experiences

14. Choose the most appropriate word from the options given below to complete the following sentence:
_________ is the key to their happiness; they are satisfied with what they have.
(*a*) Contentment
(*b*) Ambition
(*c*) Perseverance
(*d*) Hunger

15. Choose the most appropriate word from the options given below to complete the following sentence:
One of his biggest _____ was his ability to forgive.
(*a*) vice (*b*) virtues
(*c*) choices (*d*) strength

16. After the discussion, Tom said to me, 'Please revert!' He expects me to ________.
(*a*) retract (*b*) get back to him
(*c*) move in reverse (*d*) retreat

17. The value of one U.S. Dollar is 65 Indian Rupees today, compared to 60 last year. The Indian Rupee has ________.
(*a*) Depressed (*b*) Depreciated
(*c*) Appreciated (*d*) Stabilized

18. Choose the most appropriate word from the options given below to complete the following sentence:
Communication and interpersonal skills are ________ important in their own ways.
(*a*) each (*b*) both
(*c*) all (*d*) either

19. Choose the most appropriate pair of words from the options given below to complete the following sentence:
She could not ________ the thought of ________ the election to her bitter rival.
(*a*) bear, loosing
(*b*) bare, loosing
(*c*) bear, losing
(*d*) bare, losing

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20. Choose the most appropriate phrase from the options given below to complete the following sentence:

The aircraft _________ take off as soon as its flight plan was filed.

(*a*) is allowed to (*b*) will be allowed to
(*c*) was allowed to (*d*) has been allowed to

21. Choose the most appropriate word from the options given below to complete the following sentence:

Many ancient cultures attributed disease to supernatural causes. However, modern science has largely helped ______ such notions.

(*a*) impel (*b*) dispel
(*c*) propel (*d*) repel

22. If she __________ how to calibrate the instrument, she ________ done the experiment.

(*a*) knows, will have
(*b*) knew, had
(*c*) had known, could have
(*d*) should have known, would have

23. Who ___________ was coming to see us this evening ?

(*a*) you said
(*b*) did you say
(*c*) did you say that
(*d*) had you said

24. Choose the most appropriate word from the options given below to complete the following sentence:

He could not understand the judges awarding her the first prize, because he thought that her performance was quite ___________________.

(*a*) Superb
(*b*) Medium
(*c*) Mediocre
(*d*) Exhilarating

2013

25. Friendship, no matter how ________ it is, has its limitations.

(*a*) cordial (*b*) intimate
(*c*) secret (*d*) pleasant

26. Were you a bird, you_____ in the sky.

(*a*) would fly (*b*) shall fly
(*c*) should fly (*d*) shall have flown

27. Complete the sentence:

Dare __________ mistakes.

(*a*) commit (*b*) to commit
(*c*) committed (*d*) committing

28. The Headmaster ______ to speak to you.

Which of the following options is incorrect to complete the above sentence?

(*a*) is wanting (*b*) wants
(*c*) want (*d*) was wanting

2012

29. Choose the most appropriate alternative from the options given below to complete the following sentence :

Despite several ______ the mission succeeded in its attempt to resolve the conflict.

(*a*) attempts (*b*) setbacks
(*c*) meetings (*d*) delegations

30. Choose the most appropriate alternative from the options given below to complete the following sentence :

If the tired soldier wanted to lie down, he ___ the mattress out on the balcony.

(*a*) should take
(*b*) shall take
(*c*) should have taken
(*d*) will have taken

31. Choose the most appropriate word from the options given below to complete the following sentence :

Given the seriousness of the situation that he had to face, his ___ was impressive.

(*a*) beggary (*b*) nomenclature
(*c*) jealousy (*d*) nonchalance

2011

32. Choose the most appropriate word from the options given below to complete the following sentence:

If you are trying to make a strong impression on your audience, you cannot do so by being understated, tentative or _______ .

(*a*) hyperbolic
(*b*) restrained
(*c*) argumentative
(*d*) indifferent

33. Choose the most appropriate word(s) from the options given below to complete the following sentence.

I contemplated ______ Singapore for my vacation but decided against it.

(*a*) to visit (*b*) having to visit
(*c*) visiting (*d*) for a visit

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34. Choose the most appropriate word from the options given below to complete the following sentence:

Under ethical guidelines recently adopted by the Indian Medical Association, human genes are to be manipulated only to correct diseases for which_______treatments are unsatisfactory.

(*a*) similar (*b*) most

(*c*) uncommon (*d*) available

35. Choose the most appropriate word from the options given below to complete the following sentence:

It was her view that the country's problems had been_________by foreign technocrats, so that to invite them to come back would be counter-productive.

(*a*) identified (*b*) ascertained

(*c*) exacerbated (*d*) analysed

2010

36. Choose the most appropriate word from the options given below to complete the following sentence:

If we manage to _________ our natural resources, we would leave a better planet for our children.

(*a*) uphold

(*b*) restrain

(*c*) cherish

(*d*) conserve

37. Choose the most appropriate word from the options given below to complete the following sentence:

His rather casual remarks on politics ________his lack of seriousness about the subject.

(*a*) masked

(*b*) belied

(*c*) betrayed

(*d*) suppressed

ANSWERS

EXERCISE – I

MCQ Type Questions

1. (1)	**2.** (*b*)	**3.** (*b*)	**4.** (*d*)	**5.** (*b*)	**6.** (*c*)	**7.** (*d*)	**8.** (*d*)	**9.** (*c*)	**10.** (*c*)
11. (*a*)	**12.** (*d*)	**13.** (*a*)	**14.** (*d*)	**15.** (*a*)	**16.** (*d*)	**17.** (*b*)	**18.** (*a*)	**19.** (*d*)	**20.** (*c*)
21. (3)	**22.** (*c*)	**23.** (*d*)	**24.** (*b*)	**25.** (*b*)	**26.** (*d*)	**27.** (*a*)	**28.** (*c*)	**29.** (*d*)	**30.** (*c*)
31. (*c*)	**32.** (*d*)	**33.** (*c*)	**34.** (*d*)	**35.** (*d*)	**36.** (*d*)	**37.** (*d*)	**38.** (*c*)	**39.** (*b*)	**40.** (*c*)
41. (*d*)	**42.** (*d*)	**43.** (*d*)	**44.** (*d*)	**45.** (*c*)	**46.** (*a*)	**47.** (*d*)	**48.** (*b*)	**49.** (*a*)	**50.** (*d*)
51. (*c*)	**52.** (*b*)	**53.** (*d*)	**54.** (*c*)	**55.** (*a*)	**56.** (*d*)	**57.** (*d*)			

EXERCISE – II

MCQ Type Questions

1. (*b*)	**2.** (*b*)	**3.** (*a*)	**4.** (*b*)	**5.** (*c*)	**6.** (*a*)	**7.** (*b*)	**8.** (*b*)	**9.** (*b*)	**10.** (*c*)
11. (*c*)	**12.** (*a*)	**13.** (*d*)	**14.** (*a*)	**15.** (*b*)	**16.** (*b*)	**17.** (*b*)	**18.** (*b*)	**19.** (*c*)	**20.** (*c*)
21. (*b*)	**22.** (*c*)	**23.** (*b*)	**24.** (*c*)	**25.** (*b*)	**26.** (*a*)	**27.** (*b*)	**28.** (*c*)	**29.** (*b*)	**30.** (*a*)
31. (*d*)	**32.** (*b*)	**33.** (*c*)	**34.** (*d*)	**35.** (*c*)	**36.** (*a*)	**37.** (*c*)			

EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. They in choice (*a*) means 'some of our ruling politicians' (the subject). Also grammatically 'they have a right to plunder ....' is correct wheras'...... right to the punder ......' is incorrect.

2. 'Bizarre' means 'weird'. Hence, choice (*a*) is ruled out. "Its haunting images have endured ' fits syntactically. choice (*c*) can also be ruled out as 'munificence' means generosity and this is not intended in the sentence. Also, the word 'prints' in the given sentence gets with the word 'images' of choice (*b*).

3. 'They' in the part after the comma must refer to somebody or some-thing. Hence, only choice (*b*) will fit as 'they' in that case refers to 'things'.

4. The right word to precede the proposition 'on' is imposed. Tax is not collected on .... people. Tax is not placed on people, Tax is not charged on people. Hence, choices (*a*), (*b*) and (*c*) are not the answers. Tax is imposed ..... on ..... .

5. Choice (*d*) is ruled out as it does not logically fit. Choice (*a*) is also ruled out because of the expression 'tears in me'.

6. The key words are 'plunging' and 'investor confidence'. If markets are plunging, investor confidence is low. This aspect is brought out best by choice (*c*).

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7. (*a*) 'Discovery'refers to something that existed previously and came to knowledge later. Home is not a discovery.

(*b*) Home is not a certain. It is a set up which comes into being as a result of a certain composition of various factors.

(*c*) Home is not a new invention of anyone at any stage of development.

So choice (*d*) is the best answer. Home is an institution just like 'marriage. Home is a reality only when its members accept certain principles.

8. (*a*) A man's toil may help to make out wealth, but it is not the ultimate aim.

(*b*) A man's toil may secure a lot of things for his children or friends . That too is not something great.

(*c*) A man's toil may help him to overcome many difficulties, but that does not give him much pleasure.

So, choice (*d*) is the best answer. What one becomes by one's toil is greater than what one gets.

9. (*a*) 'Reopens the first chapter' means begins from the very start.

(*b*) 'Writes an epilogue' means ends the book.

(*c*) 'Opens new books' means creates new problems.

The above three are only partly correct. An act of justice pardons for ever an act of vengeance creates new problems. It is a saying. "An act of justice closes the book on a misdeed; an act of vengeance writes one of its own."

10. Here, the trail is the important thing and hence the excitement of the trail is what you will be looking for.

11. (*b*) The expression must be "it is of cardinal importance".

(*c*) There is no urgency in understanding, "It should be understood" is enough.

(*d*) Instead of "it cannot be,' the expression should be it must be'.

So choice (*a*) is the best, It is important that the two things are not to be interfered with.

13. (*b*) It is not mentioned with what do wines bear semblance.

(*c*) The 'similarity' is only in terms of production. The similarity in production does not achieve commercial profit.

(*d*) For the same commercial profit wines need not be produced in the same of similar manner.

So, choice (*a*) is the best. It does not mention that the wines of similar profit are similarily produced or have semblance with one another.

14. (*a*) A sociological analysis does not stop at the outer limits of the subjects.

(*b*) No one can indicate how far into the subject can a sociological analysis go.

(*c*) 'Relative distance' refers to the distance of something else which is not mentioned.

So, choice (*d*) is the best. This indicates the probable distance to which the sociological analysis can go to.

30. Choices (*a*) and (*d*) are ruled out because 'suggest' does not go with 'to+ verb' form. Choice (2) cannot fit in the blank because of the error in parallelism. The appropriate structure is "Many people suggest.....and others advocate ....".

31. The tense in choices (*a*) and (*b*) is not consistent. Moreover, choice (*a*) uses both 'essential' and 'must'. thus rendering either one of these words redundant. In choice (*d*) 'things' the plural form cannot take it'.

32. Choice (*d*) fits into the blank in choice (*a*) the usage of the indefinite article is erroneous. Choices (*b*) and (*c*) are erroneous due to the incorrect ordering of words.

33. 'Disingenuous' means 'insincere' and this best fits the context as the phrase 'ethnic cleansing' is marked by apostrophes indicating a misrepresentation. Also, the word 'victims' is best in contrast to 'perpetrators'. Hence, option (*c*) is correct.

36. 'Navigators, calendar makers' are best clubbed under the category of 'observers' rather than 'scrutinizers', as the latter term is more used to refer to specialists in a particular field. 'Students' can also fit to a certain extent and this leaves us with options (*b*), (*d*). 'Concede' has more to do with defeat or revision which is more apt in this context than 'agree' or 'conclude'. Hence, option (*c*) is correct.

42. Going by options, 'regrettably' doesn't fit as there is nothing to regret. 'Firstly' is no choice and nothing 'secondly' is mentioned. Paragraph has nothing aggresive tone so 'obviously' does not suit. 'Apparently' is only and most suitable option.

43. All options are suitable for the space but to choose one, the best, is required here 'Analytically' appears to be best. It expresses to analyse something using statistically proven methods to reach a decision.

44. It is a very obvious choice determined from the paragraph giving it an access.

45. 'Firing' comes as best suited option as paragraph explains it as a subject of discussion.

46. An argument is always 'resolved', it is never thwart, defeat or closed.

47. Going by options makes choice easier. Only "how the process allowed......." suits logically.

48. 'Blunt' means `very direct' with respect to a remark. 'Devious' in choice (*d*) means 'dishonest' and is incorrect as the statement is a frank one. For the same reason 'tactful' in choice (*c*) is also not appropriate.

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49. 'Interest' is the most appropriate work because the original amout he owed to the department accumulated to the said figure. 'Taxes is redundant while. 'principal' usually refers to a loan (in this case it isn't). 'Returns' is what one gets rather than pays.

50. 'Sanctions is used for an official permission or order that limits trade and is rarely used for an amout. 'Fees' are paid for a service and 'refunds' are what one gets back. 'Fines is the best choice because of the penalty component.

51. 'Attach' is the word used for officially taking something away. 'Impound' means 'confiscate' and is generally used in the context of illegal goods or contraband. Other choices are in no way comparable.

52. 'Smashed in choice (*a*) and 'dismantled' in choice (*c*) are far from being logical. 'Frozen is a wrong choice. 'Frozen is used in the context of making bank accounts inoperable etc. Automobiles are seized.

53. Clearly the topic is about a person who has violated incom tax regulations and he should be called an 'offender' of the law.

55. Choices (*c*) and (*d*) have similar meaning and are misnomers in the given context, as the keywords 'lowpitched voices' require a word that is synonymous with 'quiet' or 'calm'. The usage of the adjective 'faded' in the sentence to describe men is untenable.

56. The given sentence depicts the President as a very angry person as suggested by the keyword 'scowled' Choices (*a*), (*b*) and (*c*) are either positive or neutral and cannot fit in the sentence.

57. 'Paced' in choice (*a*) is not consistent with the mood of the President as depicted in the first part of the sentence. It means 'to walk fast'. 'Strolled' in (*b*) means 'Walk casually' and hence is discarded. 'Stormed' in choice (*c*) refers to 'attack a place suddenly' or 'say something in a loud, angry way'. But here, neither of them fits in. 'Prowled' refers to moving in a restless manner and hence fits.

EXERCISE – II

MCQ TYPE QUESTIONS

2. The principal presented the chief guest with a memento, as token of appreciation.

3. Frogs make 'croak' sound.

4. Extraneous -irrelevant or unrelated to the subject being dealt with.

5. John-Abraham starrer Madras Cafe talks about the movie not the person, so Mr. is ruled out.

'Find no takers' is not the correct phrase. At this Friday is incorrect.

So, option (*c*) is correct.

6. Apparent: looks like

dormant: hidden

Harbour: give shelter

Effect (verb): results in

8. For condition regarding something which already happened, should have practiced is the correct choice.

13. The correct word to fill in the blank is experiences. This is because the sentence is in present tense and the sense is in present indefinite.

14. Contentment is the right word which is appropriate to the meaning of the sentence. Contentment is the sign of true happiness as it gives a sense of fulfillment.

29. The word despite tells us that mission succeeded even though there were problems. Hence the correct answer is Setbacks.

32. Tone of the sentence clearly indicates a word that is similar to understated is needed for the blank. Alternatively, word should be antonym of strong (fail to make strong impression). Therefore, best choice is restrained which means controlled / reserved / timid.

33. Contemplate is a transitive verb and hence is followed by a gerund Hence the correct usage of contemplate is verb + ing form.

34. The context seeks to take a deviation only when existing / present / current / alternative treatments are unsatisfactory. So word for the blank should be a close synonym of existing / present / current / alternative. Available is the closest of all.

35. Clues: foreign technocrats did something negatively to the problems – so it is counter-productive to invite them. All other options are non-negative. Best choice is exacerbated which means aggravated or worsened.

37. Betrayed, **means** 'showed' or revealed.

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3

CHAPTER

Synonyms

INTRODUCTION

A Synonym may be defined as a word that has the same meaning or the same general meaning as a particular word in the same Language or is in some applications a more or less satisfactory application of it. Thus the synonym of a word can be an exact equivalent in meaning or should atleast convey its meaning in one way or the other or should show in what sense it can best be used.

SOME IMPORTANT WORDS WITH THEIR SYNONYMS

A

abandon : desert, for sake, jilt, leave, evacuate, quilt, vacate, abdicate, renounce.

abduct : kidnap, run away with, seize, snatch.

abet : aid, assist, back, condone, help, promote, sanction, support, sustain, uphold.

abeyance : adjournment,deferral, discontinuation, inactivity, intermission, postponement, recess.

abide : accept, bear, brook, endure, dwell, linger, live, lodge, reside, sojourn, persist.

ability : adeptness, aptitude, capability, capacity, competence, dexterity, endowment, expertise.

ablaze : afire, aflame, alight, blazing, burning, fiery, flaming, aglow, brilliant, flashing, gleaming, illuminated, radiant, sparkling.

abstain : avoid, decline, deny, desist, renounce, refrain, withhold, forgo.

abundance : affluence, ampleness, bounty, copiousness, exuberance, pro-fusion, fortune, wealth.

abysmal : bottomless, boundless, complete, deep, extreme, incalculable, profound, vast.

accede : accept, admit, agree, comply, concede, concur, consent, endorse, assume, attain.

access : admission, admittance, approach, avenue, course, entering, outburst, paroxysm.

acclaim : applaud, approve, celebrate, eulogize, exalt, hail, praise, salute, clapping.

accompany : attend, chaperon, conduct, convoy, squire, usher, supplement.

accord : agree, assent, concur, conform, harmonize, bestow, concede, endow, render, vouchsafe.

accuse : allege, blame, cite, denounce, impench, incriminate, indict, recriminate.

accustom : acclimatize, acquaint, adapt, descipline, familiarize, habituate, inure.

acquaint : advice, announce, apprise, disclose, enlighten, notify, reveal, tell.

acrid : caustic, harsh, irritating, pungent, stinging, acrimonious, mordant, nasty, sarcastic, trenchant, vitriolic.

acrimony : bitterness, harshness, churlishness, peevishness, rancour, spleen, tartness, virulence.

adamant : determined, firm, immovable, inexorable, intransigent, obdurate, resolute, stubborn, unrelenting, flinty, steely, stony.

adept : adroit, dexterous, masterful, proficient, versed, expert, genius.

adequate : capable, commensurate, competent, requisite, tolerable, sufficient.

adhere : cleave, cling, faster, obey, observe, respect.

adieu : conge, goodbye, valediction.

adjourne : defer, delay, discontinue, prorogue.

admirable : commendable, excellent, exquisite, wonderful, worthy.

admirer : bean, lover, wooer, devotee, enthusiast, votary.

adrift : drifting, unanchored, aimless, amiss, astray.

adroit : clever, cunning, deft, dexterous, ingeneous, nimble, proficient, quick-witted, able.

audacious : courageous, daring, dauntless, intrepid, rash.

austere : exacting, forbidding, grave, grim, harsh, rigorous, severe, solemn, sterr, stringent.

averse : antipathetic, disinclined, hostile, inimical, loath, reluctant, unwilling.

avid : ardent, eager, fanatical, ferrent, passionate, zealous, avaricious,

B

baptism : immersion, purification, sprinkling, debut.

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bastion : citadel, defence, fortress, mainstay.
blaspheme : abuse, anathematize, curse, desecrate, execrate, profane, revile, swear.
breach : break, aperture, charm, cleft, fissure, infraction, infringement, estrangement.
broaden : augment, develop, fatten, spread, stretch.
brood : agonize, dwell, mope, ponder, repine, ruminate.

C

catastrophe: affliction, adversity, cataclysm, devastation, disaster, fiasco, culmination.
chaos : anarchy, bedlam, confusion, disorganization. entropy, tumult, lawlessness,
chastity : celibacy, continence, innocence, modesty, purity.
clan : band, brotherhood, fraternity, gens.
clinch : assure, conclude, confirm, determine, secure.
colloquial : conversational, demotic, everyday, familiar, idiomatic, informal, vernacular.
commotion : ado, agitation, bustle, disorder, disturbance, excitement.
complicate : confuse, entangle, involve, muddle.
contour : curve, figure, lines, profile, relief.
conviction : assurance, certainty, confidence, ferrour.
creditable : admirable, commendable, deserving, estimable, honourable, loudable.
crest : apex, zenith, pinnacle, summit, ridge.
culminate : climax, close, conclude, finish.
cynical : sarcastic, scornful, sneering, distrustful.

D

daunt : dismay, alarm, appal, frighten, intimidate, scare, subdue, terrify.
deed : achievement, act, action, exploit, feat.
deflate : collapse, confract, empty, exhaust, shrink, chasten.
deliberate : cogitate, consult, debate, mediate, ponder, reflect.
diligent : active, attentive, busy, careful, constant.
dire : alarming, appalling, awful, catastrophic, cruel, disastrous.
discreet : careful, cautious, circumspect, diplomatic, judicious, politic, prudent.
dissent : decline, differ, disagree, protest, refuse, disagreement, discord.
ditch : channel, drain, furrow, trench, discord.
dodge : dant, duck, shift, sidestep, avoid, deceive.
dubious : doubtful, hesitant, uncertain, unconvinced, undecided, unsure, ambiguous
dwell : abide, establish, inhabit, settle, sojourn.

E

edgy : anxious, irritable, nervous, touchy.
elusive : difficult to catch, shifty, slippery, tricky, baffling, fleeting, puzzling, subtle.
emblem : badge, gest, device, figure, image, representation.
enchant : beguile, bewitch, captivate, charm, delight, enamour, enrapture, fascinate, hypnotize.
enrage : aggrarate, anger incense, incite, inflame, irritate, provoke.
exalt : advance, dignify, elevate, honour, promote, acclaim, applaud, extol.
exhilarating: breathtaking, cheering, exalting, exciting, vigorating, stimulating.
expedite : accelerate, advance, assist, dispatch, facilitate, hasten, precipitate.

F

fame : celebrity, eminence, glory, honour, prominence, renown, repute, stardom.
famine : scarcity, dearth, destitution, hunger, starvation.
feeble : weak, delicate, etiolated, exhausted, infirm, languid, puny, shilpit.
fierce : barbarous, brutal, cruel, dan-gerous, ferocious, murderous, passionate, savage.
fishy : doubtful, dubious, funny, improbable, suspect, suspicious, deadpan, dull.
flawless : faultless, impeccable, perfect, unblemished.
foresee : anticipate, divine, envisage, forebode, portend, predict, prophesy, presage.
fortress : castle, citadel, fastness, redoubt, stronghold.
fraternity : brotherhood, association, company, comradship.
fretful : captious, complaining, fractions, irritable, peevish, splenetic, touchy.

G

gaiety : animation, blitheness, cheerfulness, elation, exhilaration, glee, humour.
gaudy : bright, brilliant, flashy, florid, garish, glaring, meretricious, showy, vulgar.
gestation : development, evolution, incubation, pregnancy, repening.
gracious : affable, amiable, beneficent, benevolent, cheritable, chivalrous, cordial.
grimace : face, frown, mouth, scowl, sneer.
grubby : dinty, filthy, frowzy, manky, shabby.

H

haggle : bargain, barter, beat down, dicker, higgle, dispute, quarrel, wrangle.
haphazard : accidental, arbitrary, chance, fluky, random, aimless, careless.

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hatred : abomination, animosity, animus, antagonism, antipathy, enmity.

hectic : animated, boisterous, chaotic, frantic, furious, turbulent, wild.

hilarious : amusing, comical, convivial, exhilarated, humorous, jolly, jovial, joyful, joyous.

holocaust : annihilation, carnage, destruction, devastation, massacre.

hurricane : cyclone, gale, storm, tempest, tornado, twister.

I

idle : dead, empty, gathering, dust, inactive, jobless, redundant.

immense : enormous, extensive, giant, gigantic, huge, illimitable,
: interminable, jumbo.

implicated : incriminated, involved, suspected, under suspicion.

impudence : assurance, audacity, boldness, bumptiousness, insolence.

incense : anger, enrage, excite, aroma, balm, fragrance, redolence, scent.

indent : order, request, dint, nick, notch.

inept : awkward, bumbling, bungling, clumsy, gauche, inexpert, maladroit, absurd.

insolent : abusive, bold, contemptuous, impertinent, impudent, insulting, rude, saucy.

instigate : actuate, encourage, forment, impel, incite, influence, provoke, persuade.

J, K, L, M

jaunty : airy, breezy, buoyant, carefree, dapper, gay, lively, perky, showy, smart.

jurisdiction : authority, command, control, influence, prerogative, sway.

keepsake : emblem, favour, memento, relic, reminder, souvenir, symbol, token.

latch : bar, bolt, catch, fastening, lock, sneck.

link : component, constituent, division, association, attachment.

meander : ramble, snake, straving, stroll, wander, bend, curve, loop, twist.

melancholy : dejection, depression, despondency, gloom.

N, O, P, Q

nostalgic : homesick, longing, regretful, sentimental, wistful.

obnoxious : abhorrent, abominable, detestable, disagreeable, disgusting, hateable, horrid.

onerous : burdensome, crushing, demanding, difficult, exciting, exhausting, exigent.

paw : grab, handle, roughly, manhandle, maul, molest.

percolate : drain, drip, exude, filter, leache, ooze, penetrate, permeate, pervade.

piety : devotion, dutifulness, faith, grace, piousness, religion, sanctify, veneration.

pudding : dessert, sweet.

quandary : bewilderment, stick, delicate, situation, difficult, dilemma, embarrassment.

quiver : agitate, oscillate, palpitate, pulsate, shudder, tremble, vibrate.

R, S, T

rapt : absorbed, carried away, enthralled, entranced, fascinated, gripped, intent.

reveal : announce, betray, broadcast, communicate, disclose, divulge, impart, proclaim.

rowdy : bristerous, disordely, loutish, noisy, unruly, uproarious, brawler.

sage : acute, canny, intelligent, judicious, polite, prudent, sagacious.

scorn : contempt, despite, disdain, disparagement, slight, sneer, curl, condemn.

shake : fluctuate, bump, joggle, jounce, quake, shiver, shudder, totter, tremble.

sinner : offender, reprobate, trespasser.

snigger : giggle, laugh, smirk, sneer, titter.

stimulus : encouragement, incentive, incitement, inducement, provocation.

tepid : slightly warm, luke warm.

timid : afraid, bashful, cowardly, diffident.

U, V, W, Y, Z

unmask : disclose, discover, expose, reveal, unveil.

urbane : civil, civilized, cosmopolitan, courteous, cultivated, debonair.

vehement : ardent, eager, earnest, emphatic, enthusiastic, fervent, fervid, fierce, forcible.

weired : bizarre, creepy, cerie, freakish, ghostly, mysterious, queer, spooky.

writhe : contont, distant, jerk, struggle, thrash, wiggle, twist, wriggle.

yearn : ache, covet, crave, desire, hanker, hunger, itch, languish.

zenith : acme, apex, apogee, climax, height, meridian, peak, pinnacle, summit, top.

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EXERCISE – I

MCQ TYPE QUESTIONS

Directions: Each of the questions below consists of a word followed by four lettered words or phrases. Choose the lettered word or phrase that is most nearly same in meaning to the word.

1. **Abase**
(*a*) incur (*b*) tax
(*c*) estimate (*d*) humiliate

2. **Aberration**
(*a*) deviation (*b*) abhorrence
(*c*) dislike (*d*) absence

3. **Abet**
(*a*) conceive (*b*) wager
(*c*) encourage (*d*) evade

4. **Aabeyance**
(*a*) obedience (*b*) discussion
(*c*) excitement (*d*) suspended action

5. **Abjure**
(*a*) discuss (*b*) renounce
(*c*) run off secretly (*d*) perjure

6. **Ablution**
(*a*) censure (*b*) forgiveness
(*c*) mutiny (*d*) washing

7. **Abnegation**
(*a*) blackness (*b*) self-denial
(*c*) selfishness (*d*) cause

8. **Aborigine**
(*a*) first design
(*b*) absolution
(*c*) finale
(*d*) primitive inhabitant

9. **Abortive**
(*a*) unsuccessful
(*b*) consuming
(*c*) financing
(*d*) familiar

10. **Abstinence**
(*a*) restrained (*b*) vulgar display
(*c*) deportment (*d*) reluctance

11. **Abstruse**
(*a*) profound (*b*) irrespective
(*c*) suspended (*d*) protesting

12. **Abut**
(*a*) stimulate (*b*) grasp
(*c*) oppose (*d*) adjoin

13. **Abysmal**
(*a*) bottomless (*b*) eternal
(*c*) meteric (*d*) diabolic

14. **Accede**
(*a*) fail (*b*) compromise
(*c*) correct (*d*) consent

15. **Acclivity**
(*a*) index
(*b*) report
(*c*) upslope of a hill
(*d*) negotiator

16. **Astute**
(*a*) sheer (*b*) noisy
(*c*) astral (*d*) clever

17. **Atrocity**
(*a*) endurance (*b*) fortitude
(*c*) session (*d*) heinous act

18. **Atrophy**
(*a*) capture (*b*) waste away
(*c*) govern (*d*) award prize

19. **Attenuate**
(*a*) appear (*b*) be absent
(*c*) weaken (*d*) testify

20. **Atypical**
(*a*) superfluous (*b*) fortitude
(*c*) unusual (*d*) clashing

21. **Audacity**
(*a*) boldness (*b*) asperity
(*c*) strength (*d*) stature

22. **Augment**
(*a*) make noble (*b*) anoint
(*c*) increase (*d*) harvest

23. **Auxiliary**
(*a*) righteous (*b*) prospective
(*c*) assistant (*d*) archaic

24. **Avarice**
(*a*) easiness (*b*) greed
(*c*) statement (*d*) invoice

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25. Avert
(*a*) entertain (*b*) transform
(*c*) turn away (*d*) lead toward

26. Awry
(*a*) recommended
(*b*) commiserating
(*c*) started
(*d*) crooked

27. Baleful
(*a*) doubtful (*b*) virtual
(*c*) deadly (*d*) conventional

28. Balmy
(*a*) venturesome (*b*) dedicated
(*c*) mild (*d*) fanatic

29. Banal
(*a*) philosophical (*b*) trite
(*c*) dramatic (*d*) heedless

30. Baneful
(*a*) intellectual (*b*) thankful
(*c*) decisive (*d*) poisonous

31. Boisterous
(*a*) conflicting (*b*) noisy
(*c*) testimonial (*d*) grateful

32. Bombastic
(*a*) sensitive (*b*) pompous
(*c*) rapid (*d*) sufficient

33. Bouillon
(*a*) insight
(*b*) chowder
(*c*) gold
(*d*) clear soup

34. Brackish
(*a*) careful (*b*) salty
(*c*) chosen (*d*) tough

35. Braggadocio
(*a*) weaponry (*b*) boasting
(*c*) skirmish (*d*) encounter

36. Brazen
(*a*) shameless (*b*) quick
(*c*) modest (*d*) pleasant

37. Brindled
(*a*) equine (*b*) pathetic
(*c*) hasty (*d*) spotted

38. Brochure
(*a*) opening (*b*) pamphlet
(*c*) censor (*d*) bureau

39. bucolic
(*a*) diseased (*b*) replusive
(*c*) rustic (*d*) twinkling

40. Buxom
(*a*) voluminous (*b*) indecisive
(*c*) convincing (*d*) plump

41. Cache
(*a*) lock
(*b*) hiding place
(*c*) tide
(*d*) automobile

42. Cacophony
(*a*) discord
(*b*) dance
(*c*) applause
(*d*) type of telephone

43. Callow
(*a*) youthful (*b*) holy
(*c*) mild (*d*) colored

44. Candid
(*a*) vague (*b*) outspoken
(*c*) experienced (*d*) anxious

45. Chaste
(*a*) loyal (*b*) timid
(*c*) curt (*d*) pure

46. Chide
(*a*) unite (*b*) fear
(*c*) record (*d*) scold

47. Chimerical
(*a*) developing (*b*) brief
(*c*) distant (*d*) fantastic

48. Choleric
(*a*) musical (*b*) episodic
(*c*) hot headed (*d*) global

49. Churlish
(*a*) marine (b) economical
(*c*) impolite (*d*) compact

50. Ciliated
(*a*) foolish (*b*) swift
(*c*) early (*d*) hairy

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51. Circuitous
(*a*) indirect (*b*) complete
(*c*) obvious (*d*) aware

52. City
(*a*) galvanize (*b*) visualize
(*c*) locate (*d*) quote

53. Clandestine
(*a*) abortive (*b*) secret
(*c*) tangible (*d*) doomed

54. Claustrophobia
(*a*) lack of confidence
(*b*) fear of spiders
(*c*) love of books
(*d*) fear of closed places

55. Cleft
(*a*) split (*b*) waterfall
(*c*) assembly (*d*) adherence

56. Cliche
(*a*) increase (*b*) vehicle
(*c*) morale (*d*) platitude

57. Coerce
(*a*) recover (*b*) begin
(*c*) force (*d*) license

58. Cognizance
(*a*) policy (*b*) knowledge
(*c*) advance (*d*) omission

59. Cohere
(*a*) hold together
(*b*) occur simultaneously
(*c*) recollect
(*d*) materialize

60. Conjecture
(*a*) magic (*b*) guess
(*c*) position (*d*) form

61. Connoisseur
(*a*) gourmand (*b*) lover of art
(*c*) humidor (*d*) delinquent

62. Consanguinity
(*a*) kinship (*b*) friendship
(*c*) bloodletting (*d*) relief

63. Consensus
(*a*) general agreement (*b*) project
(*c*) insignificance (*d*) sheaf

64. Construe
(*a*) explain (*b*) promote
(*c*) reserve (*d*) erect

65. Contaminate
(*a*) arrest (*b*) prepare
(*c*) pollute (*d*) beam

66. Contentious
(*a*) squealing (*b*) surprising
(*c*) quarrelsome (*d*) smug

67. Continence
(*a*) humanity (*b*) research
(*c*) embryology (*d*) self-restraint

68. Contraband
(*a*) purpose (*b*) rogue
(*c*) rascality (*d*) difficulty

69. Contrite
(*a*) smart
(*b*) penitent
(*c*) restful
(*d*) recognized

70. Controvert
(*a*) turnover (*b*) contradict
(*c*) mind (*d*) expolain

71. Convene
(*a*) propose (*b*) restore
(*c*) question (*d*) gather

72. Conversant
(*a*) ignorant
(*b*) speaking
(*c*) incorporated
(*d*) familiar

73. Copious
(*a*) plentiful (*b*) cheating
(*c*) dishonorable (*d*) adventurous

74. Corpulent
(*a*) regenerate (*b*) obese
(*c*) different (*d*) hungry

75. Decimate
(*a*) kill (*b*) disgrace
(*c*) search (*d*) collide

76. Declivite
(*a*) trap (*b*) quadrangle
(*c*) quarter (*d*) downward slope

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77. Decollete
(a) flavored (b) demure
(c) flowery (d) low-necked

78. Decorous
(a) momentary (b) emotional
(c) suppressed (d) proper

79. Decrepitude
(a) feebleness (b) disease
(c) coolness (d) melee

80. Default
(a) failure to act
(b) tendency to err
(c) desire to remedy
(d) debt

81. Defection
(a) determination (b) desertion
(c) invitation (d) affection

82. Defile
(a) maincure (b) ride
(c) pollute (d) assemble

83. Degraded
(a) surpprised (b) lowered
(c) ascended (d) learned

84. Deleterious
(a) delaying
(b) experimental
(c) harmful
(d) graduating

85. Deluge
(a) confusion (b) deception
(c) flood (d) mountain

86. Denigrate
(a) refuse (b) blacken
(c) terrify (d) admit

87. Denouement
(a) action (b) scenery
(c) resort (d) solution

88. Depravity
(a) wickedness (b) sadness
(c) heaviness (d) tidiness

89. Deranged
(a) insane (b) systematic
(c) neighborly (d) alphabetical

90. Dwindle
(a) blow (b) inhabit
(c) spin (d) lessen

91. Ecstasy
(a) joy (b) speed
(c) treasure (d) warmth

92. Edify
(a) mystify (b) suffice
(c) improve (d) erect

93. Efface
(a) countenance
(b) encourage
(c) recognize
(d) rub out

94. Effigy
(a) requisition (b) organ
(c) charge (d) dummy

95. Egregious
(a) pious (b) shocking
(c) anxious (d) sociable

96. Egress
(a) entrence (b) bird
(c) exit (d) double

97. Elated
(a) debased
(b) respectful
(c) drooping
(d) gay

98. Elusive
(a) deadly (b) eloping
(c) evasive (d) simple

99. Emaciated
(a) garrulous (b) primeval
(c) vigorous (d) thin

100. Emancipate
(a) set free (b) take back
(c) make worse (d) embolden

101. Embellish
(a) doff (b) don
(c) abscond (d) adon

102. Embroil
(a) cherish (b) overtheat
(c) entangle (d) assure

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103. Emendation
(a) correction
(b) interpretation
(c) exhumation
(d) inquiry

104. Eminent
(a) purposeful (b) high
(c) delectable (d) currious

105. Flored
(a) ruddy (b) rusty
(c) ruined (d) patient

106. Foil
(a) bury (b) frustrate
(c) shield (d) desire

107. Foment
(a) spoil (b) instigate
(c) interrogate (d) spray

108. Foolhardy
(a) strong (b) unwise
(c) brave (d) futile

109. Foppish
(a) scanty (b) radical
(c) orthodox (d) dandyish

110. Foray
(a) exursion (b) contest
(c) ranger (d) intuition

111. Formidable
(a) dangerous
(b) outlandish
(c) grandiloquent
(d) impenetrable

112. Foster
(a) accelerate (b) fondle
(c) become infected (d) raise

113. Franchise
(a) subway (b) discount
(c) license (d) reason

114. Fritter
(a) sour (b) chafe
(c) dissipate (d) cancel

115. Frugality
(a) foolishness (b) extremity
(c) indifference (d) economy

116. Fulminate
(a) fulfill (b) contemplate
(c) talk nonsense (d) protest

117. Furor
(a) excitement (b) worry
(c) flux (d) anteroom

118. Furtive
(a) underhanded (b) coy
(c) brilliant (d) quick

119. Gadfly
(a) humorist
(b) nuisance
(c) scholar
(d) bum

120. Galleon
(a) liquid measure (b) ship
(c) armada (d) company

121. Garisth
(a) sordid (b) flashy
(c) prominent (d) lusty

122. Garner
(a) prevent (b) assist
(c) collect (d) compute

123. Garnish
(a) paint (b) gamer
(c) adorn (d) abuse

124. Garrulity
(a) credulity
(b) senility
(c) loquaciousness
(d) speciousness

125. Garrulous
(a) arid (b) hasty
(c) sociable (d) talkative

126. Gauche
(a) rigid (b) swift
(c) awkward (d) taciturn

127. Gaudy
(a) holy (b) showy
(c) sentimental (d) mild

128. Gaunt
(a) victorious (b) tiny
(c) stylish (d) haggard

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129. Genuflect
(*a*) falsify (*b*) trick
(*c*) project (*d*) bend the knee

130. Germane
(*a*) bacteriological
(*b*) middle european
(*c*) prominent
(*d*) relevant

131. Germtinal
(*a*) creative (*b*) excused
(*c*) sterilized (*d*) primitive

132. Gist
(*a*) chaff (*b*) summary
(*c*) expostulation (*d*) expiation

133. Glib
(*a*) slippery (*b*) fashionable
(*c*) antiquated (*d*) articulate

134. Gnome
(*a*) fury (*b*) giant
(*c*) dwarf (*d*) native

135. Inclement
(*a*) unfavorable (*b*) abandoned
(*c*) kindly (*d*) selfish

136. Incompatible
(*a*) capable
(*b*) reasonable
(*c*) faulty
(*d*) alienated

137. Inconsequential
(*a*) disorderly (*b*) insignificant
(*c*) subsequent (*d*) insufficient

138. Incontinent
(*a*) insular (*b*) complaisant
(*c*) crass (*d*) wanton

139. Incorrigible
(*a*) narrow (*b*) straight
(*c*) inconceivable (*d*) unreformable

140. Incriminate
(*a*) exacerbate (*b*) involve
(*c*) intimidate (*d*) lacerate

141. Inculcate
(*a*) exculpate (*b*) educate
(*c*) exonerate (*d*) prepare

142. Indigent
(*a*) lazy (*b*) pusillanimous
(*c*) penurious (*d*) affluent

143. Indigniy
(*a*) pomposity (*b*) bombast
(*c*) obeisance (*d*) insult

144. Indolence
(*a*) sloth (*b*) poverty
(*c*) latitude (*d*) aptitude

145. Indubitably
(*a*) flagrantly (*b*) doubtfully
(*c*) carefully (*d*) cortainly

146. Inebriety
(*a*) revelation (*b*) drunkenness
(*c*) felony (*d*) starvation

147. Inept
(*a*) outward (*b*) spiritual
(*c*) foolish (*d*) clumsy

148. Infallible
(*a*) final
(*b*) unbelievable
(*c*) perfect
(*d*) inaccurate

149. Infirmity
(*a*) disability (*b*) age
(*c*) inoculation (*d*) hospital

150. Ingenuous
(*a*) clever (*b*) stimulating
(*c*) naive (*d*) worried

151. Inimical
(*a*) antagonistic
(*b*) anonymous
(*c*) fanciful
(*d*) accurate

152. Innocuous
(*a*) not capable (*b*) not dangerous
(*c*) not eager (*d*) not frank

153. Insinuate
(*a*) resist (*b*) suggest
(*c*) report (*d*) rectify

154. Insipid
(*a*) witty (*b*) flat
(*c*) wily (*d*) talkative

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155. Integrate
(*a*) tolerate (*b*) unite
(*c*) flow (*d*) copy

156. Inter
(*a*) bury (*b*) amuse
(*c*) relate (*d*) frequent

157. Interdict
(*a*) acclain (*b*) dispute
(*c*) prohibit (*d*) decide

158. Intermittent
(*a*) heavy (*b*) fleet
(*c*) occasional (*d*) fearless

159. Intractable
(*a*) culpable (*b*) flexible
(*c*) unruly (*d*) efficient

160. Intransigence
(*a*) lack of training
(*b*) stubbornness
(*c*) novelty
(*d*) timidity

161. Intrepid
(*a*) cold (*b*) hot
(*c*) understood (*d*) courageous

162. Intrinsic
(*a*) extrinsic (*b*) abnormal
(*c*) above (*d*) basic

163. Inundate
(*a*) abuse (*b*) deny
(*c*) swallow (*d*) flood

164. Inveigh
(*a*) speak violently
(*b*) orate
(*c*) disturb
(*d*) apply

165. Lethargy
(*a*) dynamism (*b*) orate
(*c*) disturb (*d*) apply

166. Loquacious
(*a*) verbose (*b*) tacturn
(*c*) rational (*d*) alluring

167. Loath
(*a*) tired (*b*) unwilling
(*c*) sickly (*d*) spirited

168. Mawkish
(*a*) sentimental (*b*) true
(*c*) certain (*d*) devious

169. Mediocre
(*a*) average (*b*) bitter
(*c*) medieval (*d*) industrial

170. Melee
(*a*) heat (*b*) brawl
(*c*) attempt (*d*) weapon

171. Mellifluous
(*a*) porous (*b*) honeycombed
(*c*) strong (*d*) smooth

172. Menial
(*a*) intellectual (*b*) clairvoyant
(*c*) servile (*d*) arrogant

173. Mentor
(*a*) guide (*b*) genius
(*c*) talker (*d*) philosopher

174. Mesmerize
(*a*) remember (*b*) hypnotize
(*c*) delay (*d*) bore

175. Meticulous
(*a*) steadfast (*b*) recent
(*c*) quaint (*d*) painstaking

176. Miasma
(*a*) dream (*b*) noxious fumes
(*c*) scenario (*d*) quantity

177. Militant
(*a*) combative (*b*) dramatic
(*c*) religious (*d*) quaintity

178. Minon
(*a*) monster (*b*) quorum
(*c*) majority (*d*) dependent

179. Mirage
(*a*) dessert (*b*) illusion
(*c*) water (*d*) mirror

180. Misanthrope
(*a*) benefactor
(*b*) philanderer
(*c*) man-hater
(*d*) aesthete

181. Mischance
(*a*) gamble (*b*) ordinance
(*c*) aperture (*d*) anecdote

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182. Misdemeanor
(*a*) felony (*b*) peccadillo
(*c*) indignity (*d*) flat

183. Modish
(*a*) sentimental (*b*) stylish
(*c*) vacillating (*d*) contrary

184. Mollify
(*a*) avenge (*b*) attenuate
(*c*) attibute (*d*) appease

185. Monetary
(*a*) boring (*b*) fascinating
(*c*) fiscal (*d*) stationary

186. Moratorium
(*a*) burial (*b*) gathering
(*c*) delay (*d*) refusal

187. Mordant
(*a*) dying (*b*) trenchant
(*c*) fabricating (*d*) controlling

188. Moribund
(*a*) dying (*b*) trenchant
(*c*) fabricating (*d*) controlling

189. Motley
(*a*) active (*b*) disguised
(*c*) variegated (*d*) somber

190. Muggy
(*a*) attacking (*b*) fascinating
(*c*) humid (*d*) characteristic

191. Mulct
(*a*) swindle (*b*) hold
(*c*) record (*d*) pring

192. Multilingual
(*a*) variegated (*b*) polyglot
(*c*) multilateral (*d*) polyandrous

193. Mundane
(*a*) global (*b*) futile
(*c*) spatial (*d*) worldly

194. Munificent
(*a*) grandiose (*b*) puny
(*c*) philanthropic (*d*) poor

195. Musty
(*a*) flat (*b*) necessary
(*c*) indifferent (*d*) nonchalant

196. Myopic
(*a*) visionary (*b*) nearsighted
(*c*) moral (*d*) glassy

197. Nascent
(*a*) incipient (*b*) ignorant
(*c*) loyal (*d*) treacherous

198. Phlegmatic
(*a*) calm (*b*) cryptic
(*c*) practical (*d*) salivary

199. Physiognomy
(*a*) posture (*b*) head
(*c*) physique (*d*) face

200. Piebald
(*a*) motley (*b*) coltish
(*c*) hairless (*d*) thoroughbred

201. Pillage
(*a*) hoard (*b*) plunder
(*c*) versify (*d*) denigrate

202. Pinion
(*a*) express (*b*) report
(*c*) reveal (*d*) restrain

203. Pinnacle
(*a*) foothills (*b*) card game
(*c*) pass (*d*) peak

204. Pious
(*a*) historic (*b*) devout
(*c*) multiple (*d*) fortunate

205. Pique
(*a*) pyramid (*b*) revolt
(*c*) resentment (*d*) struggle

206. Placate
(*a*) determine (*b*) transmit
(*c*) pacify (*d*) allow

207. Plagiarism
(*a*) theft of funds (*b*) theft of ideas
(*c*) belief in god (*d*) arson

208. Plaintive
(*a*) moumful (*b*) senseless
(*c*) persistent (*d*) rural

209. Platitude
(*a*) fatness (*b*) bravery
(*c*) dimension (*d*) trite remark

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210. Polemic
(a) blackness (b) lighting
(c) magnetism (d) controversy

211. Precarious
(a) priceless (b) premature
(c) primitive (d) hazardous

212. Refractory
(a) articulate (b) sinkable
(c) vaunted (d) unmanageable

213. Regal
(a) oppressive (b) royal
(c) major (d) basic

214. Reiterate
(a) gainsay (b) revive
(c) revenge (d) repeat

215. Relish
(a) desire (b) nibble
(c) savor (d) violating

216. Remiss
(a) lax (b) lost
(c) foolish (d) violating

217. Remonstrate
(a) display (b) restate
(c) protest (d) resign

218. Repartee
(a) witty retort (b) willful departure
(c) spectator (d) monologue

219. Repllent
(a) propulsive (b) unattractive
(c) porous (d) stiff

220. Rpercussion
(a) reaction (b) restitution
(c) resistance (d) magnificence

221. Replenish
(a) polish (b) repeat
(c) reinstate (d) refill

222. Replica
(a) museum piece (b) famous site
(c) battle emblem (d) facsimile

223. Reprisal
(a) reevaluation (b) assessment
(c) loss (d) retaliation

224. Reprove
(a) prevail (b) rebuke
(c) ascertain (d) prove false

225. Repudiate
(a) besmirch (b) appropriate
(c) annoy (d) reject

226. Repugnance
(a) belligerence (b) tanacity
(c) renewal (d) loathing

227. Tarry
(a) polish (b) restrain
(c) surpass (d) linger

228. Tautology
(a) memory (b) repetition
(c) tension (d) smile

229. Tawdry
(a) orderly (b) meretricious
(c) reclaimed (d) filtered

230. Temerity
(a) timidity (b) resourcefulness
(c) boldness (d) tremulousness

231. Temporal
(a) priestly (b) scholarly
(c) secular (d) sleeply

232. Tenacious
(a) fast running (b) intertional
(c) obnoxious (d) holding fast

233. Tenacity
(a) splendor (b) perseverance
(c) tendency (d) ingratitide

234. Tendentious
(a) biased (b) likely
(c) absurd (d) festive

235. Tentative
(a) prevalent (b) portable
(c) mocking (d) wry

236. Tenuous
(a) vital (b) thin
(c) careful (d) dangerous

237. Tepid
(a) boiling (b) luckewarm
(c) freezing (d) gaseous

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EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. The word similar in meaning to 'dreary' is
(*a*) cheerful (*b*) dreamy
(*c*) hard (*d*) dismal

2. Choose the word most similar in meaning to the given word :
Educe
(*a*) Exert (*b*) Educate
(*c*) Extract (*d*) Extend

3. Choose the most suitable one word substitute for the following expression
Connotation of a road or way
(*a*) Pertinacious (*b*) Viaticum
(*c*) Clandestine (*d*) Ravenous

4. Choose the word most similar in meaning to the given word:
Awkward
(*a*) Inept
(*b*) Graceful
(*c*) Suitable
(*d*) Dreadful

5. A generic term that includes various items of clothing such as a skirt, a pair of trousers and a shirt as
(*a*) fabric (*b*) textile
(*c*) fibre (*d*) apparel

6. Which of the following combinations is incorrect?
(*a*) Acquiescence – Submission
(*b*) Wheedle – Roundabout
(*c*) Flippancy – Lightness
(*d*) Profligate – Extravagant

2014

7. A student is required to demonstrate a high level of <u>comprehension</u> of the subject, especially in the social sciences.
The word closest in meaning to <u>comprehension</u> is
(*a*) understanding (*b*) meaning
(*c*) concentration (*d*) stability

8. While receiving the award, the scientist said, "I feel vindicated". Which of the following is closest in meaning to the word 'vindicated'?
(*a*) punished (*b*) substantiated
(*c*) appreciated (*d*) chastened

9. Choose the word that is opposite in meaning to the word "coherent".
(*a*) sticky (*b*) well-connected
(*c*) rambling (*d*) friendly

10. Match the columns:

	Column-1		Column-2
1.	eradicate	P.	misrepresent
2.	distort	Q.	soak completely
3.	saturate	R.	use
4.	utilize	S.	destroy utterly

(*a*) 1 : S, 2 : P, 3 : Q, 4 : R
(*b*) 1 : P, 2 : Q, 3 : R, 4 : S
(*c*) 1 : Q, 2 : R, 3 : S, 4 : P
(*d*) 1 : S, 2 : P, 3 : R, 4 : Q

2013

11. Which of the following options is the closest in meaning to the word given below:
Primeval
(*a*) Modern (*b*) Historic
(*c*) Primitive (*d*) Antique

12. Which one of the following options is the closest in meaning to the word given below?
Nadir
(*a*) Highest (*b*) Lowest
(*c*) Medium (*d*) Integration

13. They were requested not to **quarrel** with others.
Which one of the following options is the closest in meaning to the word **quarrel**?
(*a*) make out (*b*) call out
(*c*) dig out (*d*) fall out

2012

14. Which one of the following options is the closest in meaning to the word given below?
Mitigate
(*a*) Diminish (*b*) Divulge
(*c*) Dedicate (*d*) Denote

15. Which one of the following options is the closest in meaning to the word given below ?
Latitude
(*a*) Eligibility (*b*) Freedom
(*c*) Coercion (*d*) Meticulousness

2011

16. Which of the following options is the closest in the meaning to the word below :
Inexplicable
(*a*) Incomprehensible (*b*) Indelible
(*c*) Inextricable (*d*) Infallible

2010

17. Which of the following options is the closest in meaning to the word below:
Circuitous
(*a*) cyclic (*b*) indirect
(*c*) confusing (*d*) crooked

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ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*d*)	**2.** (*a*)	**3.** (*c*)	**4.** (*d*)	**5.** (*b*)	**6.** (*d*)	**7.** (*b*)	**8.** (*d*)	**9.** (*a*)	**10.** (*a*)
11. (*a*)	**12.** (*d*)	**13.** (*a*)	**14.** (*d*)	**15.** (*d*)	**16.** (*d*)	**17.** (*d*)	**18.** (*b*)	**19.** (*c*)	**20.** (*c*)
21. (*a*)	**22.** (*c*)	**23.** (*c*)	**24.** (*b*)	**25.** (*c*)	**26.** (*d*)	**27.** (*c*)	**28.** (*c*)	**29.** (*b*)	**30.** (*d*)
31. (*b*)	**32.** (*b*)	**33.** (*d*)	**34.** (*b*)	**35.** (*b*)	**36.** (*a*)	**37.** (*d*)	**38.** (*b*)	**39.** (*c*)	**40.** (*d*)
41. (*b*)	**42.** (*a*)	**43.** (*a*)	**44.** (*c*)	**45.** (*d*)	**46.** (*d*)	**47.** (*d*)	**48.** (*c*)	**49.** (*c*)	**50.** (*d*)
51. (*a*)	**52.** (*d*)	**53.** (*b*)	**54.** (*d*)	**55.** (*a*)	**56.** (*d*)	**57.** (*c*)	**58.** (*b*)	**59.** (*a*)	**60.** (*b*)
61. (*b*)	**62.** (*a*)	**63.** (*a*)	**64.** (*a*)	**65.** (*c*)	**66.** (*c*)	**67.** (*d*)	**68.** (*d*)	**69.** (*b*)	**70.** (*b*)
71. (*d*)	**72.** (*d*)	**73.** (*a*)	**74.** (*b*)	**75.** (*a*)	**76.** (*d*)	**77.** (*d*)	**78.** (*d*)	**79.** (*a*)	**80.** (*a*)
81. (*b*)	**82.** (*c*)	**83.** (*b*)	**84.** (*c*)	**85.** (*c*)	**86.** (*b*)	**87.** (*d*)	**88.** (*a*)	**89.** (*a*)	**90.** (*d*)
91. (*a*)	**92.** (*c*)	**93.** (*d*)	**94.** (*d*)	**95.** (*b*)	**96.** (*c*)	**97.** (*d*)	**98.** (*c*)	**99.** (*d*)	**100.** (*a*)
101. (*d*)	**102.** (*c*)	**103.** (*a*)	**104.** (*b*)	**105.** (*a*)	**106.** (*b*)	**107.** (*b*)	**108.** (*b*)	**109.** (*d*)	**110.** (*a*)
111. (*a*)	**112.** (*c*)	**113.** (*c*)	**114.** (*c*)	**115.** (*d*)	**116.** (*d*)	**117.** (*a*)	**118.** (*a*)	**119.** (*b*)	**120.** (*b*)
121. (*b*)	**122.** (*c*)	**123.** (*c*)	**124.** (*c*)	**125.** (*d*)	**126.** (*c*)	**127.** (*b*)	**128.** (*d*)	**129.** (*d*)	**130.** (*d*)
131. (*a*)	**132.** (*b*)	**133.** (*d*)	**134.** (*c*)	**135.** (*a*)	**136.** (*d*)	**137.** (*b*)	**138.** (*d*)	**139.** (*d*)	**140.** (*b*)
141. (*b*)	**142.** (*c*)	**143.** (*d*)	**144.** (*a*)	**145.** (*d*)	**146.** (*b*)	**147.** (*d*)	**148.** (*c*)	**149.** (*a*)	**150.** (*c*)
151. (*a*)	**152.** (*b*)	**153.** (*b*)	**154.** (*b*)	**155.** (*a*)	**156.** (*a*)	**157.** (*c*)	**158.** (*c*)	**159.** (*c*)	**160.** (*b*)
161. (*d*)	**162.** (*d*)	**163.** (*d*)	**164.** (*a*)	**165.** (*b*)	**166.** (*a*)	**167.** (*b*)	**168.** (*d*)	**169.** (*a*)	**170.** (*b*)
171. (*d*)	**172.** (*c*)	**173.** (*a*)	**174.** (*b*)	**175.** (*d*)	**176.** (*b*)	**177.** (*a*)	**178.** (*d*)	**179.** (*b*)	**180.** (*c*)
181. (*d*)	**182.** (*b*)	**183.** (*b*)	**184.** (*d*)	**185.** (*c*)	**186.** (*c*)	**187.** (*b*)	**188.** (*a*)	**189.** (*c*)	**190.** (*c*)
191. (*a*)	**192.** (*b*)	**193.** (*d*)	**194.** (*c*)	**195.** (*a*)	**196.** (*b*)	**197.** (*a*)	**198.** (*a*)	**199.** (*d*)	**200.** (*a*)
201. (*b*)	**202.** (*d*)	**203.** (*d*)	**204.** (*b*)	**205.** (*c*)	**206.** (*c*)	**207.** (*b*)	**208.** (*a*)	**209.** (*d*)	**210.** (*d*)
211. (*d*)	**212.** (*d*)	**213.** (*b*)	**214.** (*d*)	**215.** (*c*)	**216.** (*a*)	**217.** (*c*)	**218.** (*a*)	**219.** (*b*)	**220.** (*a*)
221. (*d*)	**222.** (*d*)	**223.** (*d*)	**224.** (*b*)	**225.** (*d*)	**226.** (*d*)	**227.** (*d*)	**228.** (*b*)	**229.** (*b*)	**230.** (*c*)
231. (*c*)	**232.** (*d*)	**233.** (*b*)	**234.** (*a*)	**235.** (*d*)	**236.** (*b*)	**237.** (*b*)			

EXERCISE – II

MCQ Type Questions

1. (*d*)	**2.** (*c*)	**3.** (*b*)	**4.** (*a*)	**5.** (*d*)	**6.** (*b*)	**7.** (*a*)	**8.** (*b*)	**9.** (*c*)	**10.** (*a*)
11. (*c*)	**12.** (*b*)	**13.** (*b*)	**14.** (*a*)	**15.** (*b*)	**16.** (*a*)	**17.** (*b*)			

EXPLANATIONS

EXERCISE – II

MCQ TYPE QUESTIONS

1. *dreary* - depressingly dull and bleak or repetitive.

2. The word similar in meaning to Educe is Extract.

12. **Given:** A word "Nadir"

To find: Closest meaning word to 'Nadir' in the given options.

Analysis: The dictionary meaning of 'Nadir' is 'Lowest' which is potion (*b*)

Source: Google

Given: A sequence 44, 42, 40,......

To find: Maximum sum of the sequence

Analysis: To find the maximum sum, we have to ignore all the negative terms (they will only decrease the sum). Hence, we terminate the sequence at 0.

Now, the sequence is 44, 42,......,2, 0.

This sequence is an Arithmetic progression with common difference –2. Now, the problem reduces to finding the sum of this A.P.

To find the sum of A.P., we have to find the number of terms in A.P.

Let n be the number of terms

a be the first term

l be the last term

d be the common difference.

$n = ?$

$a = 44, \quad l = 0, \quad d = -2$

Now, $l = a + (n - 1)d$

14. 'Mitigate' means to make something less harmful, serious, intense etc. Hence correct answer is 'Diminis'

16. Inexplicable means not explicable; that cannot be explained, understood, or accounted for. So the best synonym here is incomprehensible.

17. **Circuitous** means round about or not direct

So circuitous : indirect.

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4

CHAPTER

Antonyms

The basic idea of a antonym question is to find an opposite for a word. Antonym items are first and foremost a test of vocabulary.

If you have no idea of the meaning of the given word, there's not much you can do. When you don't know the meaning of an antonym, don't waste a lot of time trying to fiture it out. In other words, once you recognize that you are out of ammunition, just do a random guess, confirm it, and move on the next item.

Each antonym starts with a capitalized word, which is followed by four words of phrases in lowercase letters. The task is to find the word or phrase that is most nearly opposite in meaning to the bold word.

Plan to Answer Antonym Questions

1. Define the capitalized word.

2. Think of a meaning that is opposite to this word.

3. Read all the answer choices. Eliminate those that do not relate to the meaning you thought of. If only one choice remains, mord it and go on.

4. If more than one choice remains, go back and refine your thinking about the bold word.

5. Pick the answer choice that is most nearly opposite to the bold word.

Tedious

(*a*) unlimited (*b*) confined

(*c*) enthralling (*d*) appetizing

(*i*) The capitalized word means "boring" or "tiresome"

(*ii*) The opposite of "boring" would be something like *exciting or interesting*.

(*iii*) Looking at all the answer choices, you can immediately eliminate (*a*), and (*b*) as having nothing to do with excitement. But that leaves (*c*) *enthralling and* (*d*) *apperizing*.

(*iv*) Of the two remaining choices, *enthralling, which means "captivating"* or fascinating," is more nearly opposite in meaning to *tedious than appetizing*, which means "savory," or "delicious".

(*v*) Mark (*c*), enthralling, as the correct choice.

Note :

(*i*) **Makes sure to read all the answers :** To answer antonym questions you'll sometimes have to distinguish fine shades of meaning. That's why you have to consider very answer choice.

(*ii*) **Verb or Noun?** If the part of speech of the capitalized word is ambiguous, check the answer choices.

6. Sometimes the test writer selects a word you are likely to be familiar with, but sets up the question to test a meaning you do not ordinarily associate with the word. Here is an example:

e.g. precipitous

(*a*) pleasantly sweet

(*b*) overly ambitious

(*c*) agreeably situated

(*d*) gently sloping

The best choice is (*d*). We most often use the word precipitous to mean rash or foothardy, but its central meaning is related to precipice, a sharp drop-off.

7. Eleminate non-answers

The simple way to eleminate answer choices is to toss out any words that don't have opposite meanings.

8. Concentrate on answer choices

Words can have different meanings based upon their part of speech. If you are uncertain about the part of speech of the bold word, just check the naswer choices. If they're all verbs, for example, so is the bold word.

9. Play around with the part of speech of a word

You can alter the part of speech of the bold word and answer choices in your own mind. Sometimes an antonym will use a word you know but as a part of speech that is unfamiliar to you.

***e.g.* sublimity**

(*a*) erosion (*b*) baseness

(*c*) conciseness (*d*) insistence

The best choice is (*b*). You may know the word sublimity better as the adjective sublime, meaning "lofty, high, or noble". So you may find it easire to think about the antonym by changing *sublimity* to the more familiar form *sublime*". As you think about each answer choice, you would then change it in your mind to an adjective *Baseness*, therefore, would become *base*; and *base* is an opposite of *sublime*.

10. Use word connotations to eliminate answer choices

Even if you don't know the exact meaning of a word, you may have a vague recollection of the context in which you first encountered it. So you may know whether the word has positive overtones or negative ones. This recollection may be sufficient to get a correct answer.

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***e.g.* raffish**

(*a*) grotesque (*b*) delinquent
(*c*) uncaring (*d*) noble

The correct choice is (*d*). Let's assume that you do not know that is means "low, vulgar, the base. 'Let's further assume, however, that you have a vague knowledge of the word. You've seen it used to describe a character who is dishonest and not trustworthy. So even though you don't know the exact meaning of the word, you know that it has negative overtones. Sine you an looking for the opposite of a word with negative overtones, you would eliminate every answer choice with negative overtones. As it turns out, this strategy workd perfectly with this antonym; only one word is left. *Noble* is the only word with positive overtones.

11. Unlock word meanings by taking the word apart

Even when you encounter a word for the first time, you may be able to ascertain its meaning from its parts.

***e.g.* congnoscitive**

(*a*) courageous (*b*) expensive
(*c*) unconscious (*d*) redundant

The best answer is (*c*). This is very unusual word, but you can probably figure out its meaning by looking at its root cog-. This is the same root found in words such as *cognition and recognize*, and it has to do with knowledge. So we infer that *cognoscitive* has something to do with awareness, and (*c*) looks like a good opposite. A word of caution, however : Don't spend too much time trying to decipher the meaning of a word.

EXERCISE – I

MCQ TYPE QUESTIONS

Directions : *Each question below consists of a word printed in capital lettes, followed by four lettered words of phrases. Choosed the lettered word or phrase that is most nearly opposite in meaning to the word in bold letters.*

1. Abrogate
(*a*) transgress (*b*) signify
(*c*) allevate (*d*) ratify

2. Admonition
(*a*) premonit ion (*b*) hallucination
(*c*) escape (*d*) commendation

3. Adulation
(*a*) youth (*b*) purity
(*c*) brightness (*d*) criticism

4. Advocate
(*a*) define (*b*) oppose
(*c*) remove (*d*) inspect

5. Affable
(*a*) rude (*b*) ruddy
(*c*) needy (*d*) useless

6. Affecfted
(*a*) weary (*b*) unfriendly
(*c*) divine (*d*) unfeigned

7. Affluence
(*a*) poverty (*b*) fear
(*c*) persuasion (*d*) consideration

8. Agility
(*a*) awkwardness (*b*) solidity
(*c*) temper (*d*) harmony

9. Alacrity
(*a*) slowness (*b*) plenty
(*c*) fifth (*d*) courtesy

10. Alleviate
(*a*) endure (*b*) worsen
(*c*) enlighten (*d*) maneuver

11. Allure
(*a*) hinder (*b*) repel
(*c*) ignore (*d*) leave

12. Aloof
(*a*) triangular (*b*) gregarious
(*c*) comparable (*d*) honorable

13. Amalgamate
(*a*) equip (*b*) separate
(*c*) generate (*d*) materialize

14. Ambiguous
(*a*) salvageable (*b*) corresponding
(*c*) responsible (*d*) clear

15. Amble
(*a*) befriend (*b*) hasten
(*c*) steal (*d*) browse

16. Ambulatory
(*a*) convalescent (*b*) valedictory
(*c*) bedridden (*d*) emergency

17. Ameliorate
(*a*) make slow (*b*) make sure
(*c*) make young (*d*) make worse

18. Amicable
(*a*) penetrating (*b*) compensating
(*c*) unfriendly (*d*) zigzag

19. Amoral
(*a*) unusal (*b*) unfriendly
(*c*) ethical (*d*) suave

20. Amorphous
(*a*) nauseous (*b*) obscene
(*c*) providential (*d*) definite

21. Amplify
(*a*) distract (*b*) infer
(*c*) publicize (*d*) decrease

22. Analogous
(*a*) not comparable (*b*) not capable
(*c*) not culpable (*d*) not corporeal

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23. Anthematize
(*a*) locate (*b*) deceive
(*c*) regulate (*d*) bless

24. Anemic
(*a*) pailid (*b*) cruel
(*c*) red-blooded (*d*) ventilating

25. Animated
(*a*) worthy (*b*) dull
(*c*) humorous (*d*) lengthy

26. Animus
(*a*) pterodactyl (*b*) bastion
(*c*) giraffe (*d*) favor

27. Anomaly
(*a*) desperation (*b*) requisition
(*c*) registry (*d*) regularity

28. Anonymous
(*a*) desperate (*b*) signed
(*c*) defined (*d*) expert

29. Antediluvian
(*a*) transported (*b*) subtle
(*c*) isolated (*d*) modern

30. Antipathy
(*a*) profundity (*b*) objection
(*c*) willingness (*d*) foundness

31. Antithesis
(*a*) velocity (*b*) maxim
(*c*) similarity (*d*) acceleration

32. Aphasia
(*a*) volubility (*b*) necessity
(*c*) pain (*d*) crack

33. Baroque
(*a*) polished (*b*) constant
(*c*) transformed (*d*) simple

34. Beatific
(*a*) glorius (*b*) dreadful
(*c*) theatrical (*d*) crooked

35. Belittle
(*a*) disobey (*b*) forget
(*c*) magnify (*d*) extol

36. Bellicose
(*a*) peaceful (*b*) naval
(*c*) amusing (*d*) piecemeal

37. Benign
(*a*) tenfold (*b*) peaceful
(*c*) blessed (*d*) malignant

38. Benison
(*a*) curse (*b*) bachelor
(*c*) wedding (*d*) orgy

39. Berate
(*a*) grant (*b*) praise
(*c*) refer (*d*) purchase

40. Bestial
(*a*) animated (*b*) noble
(*c*) zoological (*d*) clear

41. Bigotry
(*a*) arrogance (*b*) approval
(*c*) mourning (*d*) tolerance

42. Bizarre
(*a*) roomy (*b*) veiled
(*c*) subduced (*d*) normal

43. Blanch
(*a*) bleach (*b*) scatter
(*c*) darken (*d*) analyze

44. Bland
(*a*) caustic (*b*) meager
(*c*) soft (*d*) uncooked

45. Blase
(*a*) fiery (*b*) clever
(*c*) intriguing (*d*) ardent

46. Bleak
(*a*) pale (*b*) sudden
(*c*) dry (*d*) cheerful

47. Blithe
(*a*) spiritual (*b*) profuse
(*c*) cheerless (*d*) hybrid

48. Capacious
(*a*) warlike (*b*) cordial
(*c*) curious (*d*) not spacious

49. Capricious
(*a*) satisfied (*b*) insured
(*c*) photographic (*d*) steadfast

50. Captious
(*a*) tolerant (*b*) capable
(*c*) frivolous (*d*) winning

51. Carnal
(*a*) impressive (*b*) minute
(*c*) spiritual (*d*) actual

52. Carnivorous
(*a*) gloomy (*b*) tangential
(*c*) productive (*d*) vegetarian

53. Carping
(*a*) rapid (*b*) uncritical
(*c*) unintellectual (*d*) illegal

54. Castigation
(*a*) commendation (*b*) patience
(*c*) generosity (*d*) understatement

55. Categorical
(*a*) negative (*b*) ironic
(*c*) impartial (*d*) qualified

56. Catholic
(*a*) religious (*b*) pacific
(*c*) narrow (*d*) weighty

57. Celerity
(*a*) assurance (*b*) state
(*c*) acerbity (*d*) delay

58. Celibate
(*a*) investing (*b*) married
(*c*) retired (*d*) commodious

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59. Censure
(*a*) process (*b*) enclose
(*c*) interest (*d*) praise

60. Centrifugal
(*a*) centripetal (*b*) ephemeral
(*c*) lasting (*d*) barometric

61. Cessation
(*a*) premium (*b*) gravity
(*c*) beginning (*d*) composition

62. Chaffing
(*a*) achieving (*b*) serious
(*c*) caitalistic (*d*) sneezing

63. Coy
(*a*) weak (*b*) airy
(*c*) brazen (*d*) old

64. Cozen
(*a*) amuse (*b*) treat honestly
(*c*) prate (*d*) shackle

65. Craven
(*a*) desirous (*b*) direct
(*c*) bold (*d*) civilized

66. Crux
(*a*) affiliction (*b*) spark
(*c*) events (*d*) trivial point

67. Cryptic
(*a*) tomblike (*b*) futile
(*c*) famous (*d*) candid

68. Cupidity
(*a*) anxiety (*b*) tragedy
(*c*) generosity (*d*) entertainment

69. Curtail
(*a*) mutter (*b*) lengthen
(*c*) express (*d*) burden

70. Cynical
(*a*) trusting (*b*) effortless
(*c*) conclusive (*d*) gallant

71. Dank
(*a*) dry (*b*) guiltless
(*c*) warm (*d*) babbling

72. Dapper
(*a*) unintelligent (*b*) untidy
(*c*) uncertain (*d*) ungrateful

73. Dauntless
(*a*) stolid (*b*) cowardly
(*c*) irrelevant (*d*) peculiar

74. Dearth
(*a*) life (*b*) abundance
(*c*) brightness (*d*) terror

75. Debacle
(*a*) progress (*b*) refusal
(*c*) masque (*d*) cowardice

76. Debilitate
(*a*) bedevil (*b*) repress
(*c*) strengthen (*d*) animate

77. Debonair
(*a*) awkward (*b*) windy
(*c*) balmy (*d*) strong

78. Derogatory
(*a*) roguish (*b*) immediate
(*c*) openionated (*d*) praising

79. Desecrate
(*a*) desist (*b*) integrate
(*c*) confuse (*d*) consecrate

80. Despicable
(*a*) streeing (*b*) worthy of esteem
(*c*) inevitable (*d*) featureless

81. Destitute
(*a*) affluent (*b*) dazzling
(*c*) stationary (*d*) characteristic

82. Devoid
(*a*) latent (*b*) eschewed
(*c*) full of (*d*) suspecting

83. Devout
(*a*) quiet (*b*) dual
(*c*) impious (*d*) straightforward

84. Diabolical
(*a*) mischievous (*b*) lavish
(*c*) seraphic (*d*) redolent

85. Diatribe
(*a*) mass (*b*) range
(*c*) eulogy (*d*) elegy

86. Diffidence
(*a*) sharapness (*b*) boldness
(*c*) malcontent (*d*) dialogue

87. Dilate
(*a*) procrastinate (*b*) contract
(*c*) conclude (*d*) participate

88. Dilatory
(*a*) narrowing (*b*) prompt
(*c*) enlarging (*d*) portentous

89. Diminution
(*a*) expectation (*b*) context
(*c*) validity (*d*) appreciation

90. Din
(*a*) lightness (*b*) safety
(*c*) silence (*d*) hunger

91. Disabuse
(*a*) crash (*b*) violate
(*c*) renege (*d*) deceive

92. Disconsolate
(*a*) unprejudiced (*b*) thankful
(*c*) threatrical (*d*) joyous

93. Enervate
(*a*) strengthen (*b*) sputter
(*c*) arrange (*d*) scrutinize

94. Enhance
(*a*) degrade (*b*) doubt
(*c*) scuff (*d*) gasp

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95. **Ennui**
(*a*) hate (*b*) excitement
(*c*) seriousness (*d*) humility

96. **Enunciate**
(*a*) pray (*b*) request
(*c*) deliver (*d*) mumble

97. **Ephemeral**
(*a*) sensuous (*b*) passing
(*c*) popular (*d*) eternal

98. **Equable**
(*a*) flat (*b*) decisive
(*c*) stormy (*d*) dishonest

99. **Equanimity**
(*a*) agitation (*b*) stirring
(*c*) volume (*d*) identity

100. **Equilibrium**
(*a*) imbalance (*b*) peace
(*c*) inequity (*d*) directness

101. **Equitable**
(*a*) able to leave (*b*) able to learn
(*c*) unfair (*d*) preferable

102. **Equivocal**
(*a*) mistaken (*b*) quaint
(*c*) azure (*d*) clear

103. **Erratic**
(*a*) unromantic (*b*) free
(*c*) popular (*d*) steady

104. **Erroneous**
(*a*) accurate (*b*) dignified
(*c*) curious (*d*) abrupt

105. **Erudite**
(*a*) professorial (*b*) stately
(*c*) short (*d*) ignorant

106. **Ethereal**
(*a*) long-lasting (*b*) earthy
(*c*) ill (*d*) critical

107. **Eulogistic**
(*a*) pretty (*b*) critical
(*c*) brief (*d*) stern

108. **Euphonious**
(*a*) strident (*b*) lethargic
(*c*) literary (*d*) significant

109. **Evasive**
(*a*) frank (*b*) correct
(*c*) empty (*d*) fertile

110. **Exasperate**
(*a*) confide (*b*) formalize
(*c*) placate (*d*) betray

111. **Excoriate**
(*a*) scandalize (*b*) encourage
(*c*) avoid (*d*) praise

112. **Exculpate**
(*a*) blame (*b*) prevail
(*c*) acquire (*d*) ravish

113. **Execrable**
(*a*) innumerable (*b*) philosophic
(*c*) physical (*d*) excellent

114. **Excrate**
(*a*) decipher (*b*) sadden
(*c*) integrate (*d*) admit

115. **Exhume**
(*a*) decipher (*b*) sadden
(*c*) integrate (*d*) inter

116. **Exodus**
(*a*) neglect (*b*) consent
(*c*) entry (*d*) gain

117. **Exonerate**
(*a*) forge (*b*) accuse
(*c*) record (*d*) doctor

118. **Exorbitant**
(*a*) moderate (*b*) partisan
(*c*) military (*d*) barbaric

119. **Extemporaneous**
(*a*) rehearsed (*b*) hybrid
(*c*) humilating (*d*) satetesmanlike

120. **Extraneous**
(*a*) modern (*b*) decisive
(*c*) essential (*d*) effective

121. **Extrinsic**
(*a*) reputable (*b*) inherent
(*c*) swift (*d*) ambitious

122. **Extrovert**
(*a*) clown (*b*) hero
(*c*) ectomorph (*d*) introvert

123. **Grandiose**
(*a*) false (*b*) ideal
(*c*) proud (*d*) simple

124. **Gratuitous**
(*a*) warranted (*b*) frank
(*c*) ingenuous (*d*) frugal

125. **Gregarious**
(*a*) antisocial (*b*) anticipatory
(*c*) glorious (*d*) horrendous

126. **Grisly**
(*a*) suggestive (*b*) doubtful
(*c*) untidy (*d*) pleasant

127. **Gullible**
(*a*) incredulous (*b*) fickle
(*c*) tantamount (*d*) easy

128. **Gusto**
(*a*) noise (*b*) panic
(*c*) atmosphere (*d*) distaste

129. **Ghusty**
(*a*) clam (*b*) noisy
(*c*) fragrant (*d*) routine

130. **Hackneyed**
(*a*) carried (*b*) original
(*c*) banned (*d*) timely

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131. Haggard
(*a*) shrewish (*b*) inspired
(*c*) plump (*d*) maidenly

132. Halcyon
(*a*) wasteful (*b*) prior
(*c*) subsequent (*d*) martial

133. Haphazard
(*a*) safe (*b*) indifferent
(*c*) deliberate (*d*) tense

134. Hapless
(*a*) cheerful (*b*) consistent
(*c*) fortunate (*d*) considerate

135. Heed
(*a*) ignore (*b*) hope
(*c*) overtake (*d*) nurture

136. Heretic
(*a*) sophist (*b*) believer
(*c*) interpreter (*d*) pacifist

137. Heterogeneous
(*a*) orthodox (*b*) pagan
(*c*) unlikely (*d*) similar

138. Hibernal
(*a*) musical (*b*) summerlike
(*c*) local (*d*) seasonal

139. Hilarity
(*a*) gloom (*b*) heartiness
(*c*) weakness (*d*) casualty

140. Hirsute
(*a*) scaly (*b*) bald
(*c*) erudite (*d*) quiet

141. Hortatory
(*a*) inquiring (*b*) denying
(*c*) killing (*d*) dissuading

142. Hovr
(*a*) commence (*b*) soothe
(*c*) leave (*d*) transform

143. Hunbbub
(*a*) calm (*b*) fury
(*c*) capital (*d*) axle

144. Hummock
(*a*) ummusical (*b*) scorn
(*c*) wakefulness (*d*) vale

145. Husbandry
(*a*) sportsmanship (*b*) dishonesty
(*c*) wastefulness (*d*) friction

146. Hybrid
(*a*) productive (*b*) special
(*c*) purebred (*d*) oafish

147. Hyperbole
(*a*) velocity (*b*) climax
(*c*) curve (*d*) understatement

148. Hypercritical
(*a*) tolerant (*b*) false
(*c*) extreme (*d*) inarticulate

149. Hypocritical
(*a*) sincere (*b*) narrow-minded
(*c*) shameful (*d*) amiable

150. Hypothetial
(*a*) rational (*b*) fantastic
(*c*) wizened (*d*) axiomatic

151. Ignoble
(*a*) produced by fire (*b*) worthy
(*c*) given to questioning (*d*) huge

152. Illusive
(*a*) not deceptive (*b*) not certain
(*c*) not obvious (*d*) not choherent

153. Irksome
(*a*) interesting (*b*) lazy
(*c*) tireless (*d*) devious

154. Irrelevant
(*a*) lacking piety (*b*) fragile
(*c*) congruent (*d*) pertinent

155. Irreparable
(*a*) legible (*b*) correctable
(*c*) proverbial (*d*) concise

156. Irtreverent
(*a*) related (*b*) mischievous
(*c*) respective (*d*) pious

157. Jaded
(*a*) upright (*b*) stimulated
(*c*) aspiring (*d*) applied

158. Jaundiced
(*a*) whitened (*b*) inflamed
(*c*) quickened (*d*) unbiased

159. Jaunty
(*a*) youthful (*b*) ruddy
(*c*) strong (*d*) sedate

160. Jeopardy
(*a*) patience (*b*) courage
(*c*) safety (*d*) willingness

161. Jettison
(*a*) salvage (*b*) submerge
(*c*) descend (*d*) decelerate

162. Jocular
(*a*) arterial (*b*) bloodless
(*c*) verbose (*d*) serious

163. Judicious
(*a*) punitive (*b*) unwise
(*c*) criminal (*d*) licit

164. Kindle
(*a*) dislike (*b*) quench
(*c*) gather (*d*) sparkle

165. Lachrymose
(*a*) cheering (*b*) smooth
(*c*) passionate (*d*) curt

166. Lackadasical
(*a*) monthly (*b*) possessing time
(*c*) ambitious (*d*) pusillanimous

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167. Laconic
(*a*) milky (*b*) verbose
(*c*) wicked (*d*) flagrant

168. Lampoon
(*a*) darken (*b*) praise
(*c*) abandon (*d*) sail

169. Languor
(*a*) vitality (*b*) length
(*c*) embarrassment (*d*) wine

170. Latent
(*a*) trim (*b*) forbidding
(*c*) execrable (*d*) obvious

171. Lavish
(*a*) hostile (*b*) unwashed
(*c*) timely (*d*) frugal

172. Laudatory
(*a*) drityinq (*b*) disclaiming
(*c*) defamatory (*d*) inflammatory

173. Lax
(*a*) salty (*b*) strick
(*c*) shrill (*d*) boring

174. Lechery
(*a*) trust (*b*) compulsion
(*c*) zeal (*d*) purity

175. Lethargic
(*a*) convalescent (*b*) beautiful
(*c*) enervating (*d*) invigorating

176. Levite
(*a*) bridge (*b*) dam
(*c*) praise (*d*) solemnity

177. Lilliputian
(*a*) destructive (*b*) proper
(*c*) gigantic (*d*) elegant

178. Limpid
(*a*) erect (*b*) turbid
(*c*) tangential (*d*) timid

179. Lithe
(*a*) stiff (*b*) limpid
(*c*) facetious (*d*) insipid

180. Livid
(*a*) alive (*b*) mundane
(*c*) positive (*d*) undiscolored

181. Loath
(*a*) loose (*b*) evident
(*c*) deliberate (*d*) eager

182. Loquacious
(*a*) taciturn (*b*) sentimental
(*c*) soporific (*d*) soothing

183. Moist
(*a*) crisp (*b*) hard
(*c*) dry (*d*) parched

184. Mask
(*a*) hit (*b*) expose
(*c*) benign (*d*) pompous

185. Moisture
(*a*) dryness (*b*) delegate
(*c*) dampness (*d*) fdefect

186. Magnify
(*a*) produce (*b*) support
(*c*) reduce (*d*) destroy

187. Mendacious
(*a*) broken (*b*) efficacious
(*c*) truthful (*d*) destructive

188. Manumit
(*a*) throw (*b*) lock
(*c*) promise (*d*) enslave

189. Nebulous
(*a*) starry (*b*) clear
(*c*) cold (*d*) fundamental

190. Nefarious
(*a*) various (*b*) lacking
(*c*) benign (*d*) pompous

191. Negation
(*a*) postulation (*b*) hypothecation
(*c*) affirmation (*d*) violation

192. Neophyte
(*a*) veteran (*b*) satellite
(*c*) desperado (*d*) handwriting

193. Niggardly
(*a*) protected (*b*) biased
(*c*) prodigal (*d*) bankrupt

194. Nocturnal
(*a*) harsh (*b*) marauding
(*c*) patrolling (*d*) daily

195. Noisome
(*a*) quiet (*b*) dismayed
(*c*) fragrant (*d*) sleepy

196. Notorious
(*a*) fashionable (*b*) renowned
(*c*) inactive (*d*) interepid

197. Obdurate
(*a*) yielding (*b*) fleeting
(*c*) finite (*d*) fascinating

198. Obese
(*a*) skillful (*b*) cadaverous
(*c*) clever (*d*) unpredictable

199. Objective
(*a*) indecisive (*b*) apathetic
(*c*) markedly inferior (*d*) emotionally involved

200. Obligatory
(*a*) demanding (*b*) optional
(*c*) facile (*d*) friendly

201. Obloquy
(*a*) praise (*b*) rectangle
(*c*) circle (*d*) dialogue

202. Obsequious
(*a*) successful (*b*) democratic
(*c*) supercilious (*d*) ambitious

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203. Obsession
(*a*) whim (*b*) loss
(*c*) phobia (*d*) delusion

204. Obsolete
(*a*) heated (*b*) desolate
(*c*) renovated (*d*) frightful

205. Obstreperous
(*a*) turbid (*b*) quiet
(*c*) remote (*d*) lucid

206. Obtuse
(*a*) sheer (*b*) transparent
(*c*) tranquil (*d*) shrewd

207. Odious
(*a*) fragrant (*b*) redolent
(*c*) fetid (*d*) delightful

208. Odium
(*a*) noise (*b*) liking
(*c*) dominant (*d*) hasty

209. Omnipotent
(*a*) weak (*b*) democratic
(*c*) despotic (*d*) passionate

210. Omniscient
(*a*) sophisticated (*b*) ignorant
(*c*) essential (*d*) trivial

211. Opiat
(*a*) distress (*b*) sleep
(*c*) stimulant (*d*) laziness

212. Opportune
(*a*) occasional (*b*) fragrant
(*c*) fragile (*d*) awkward

213. Opportunist
(*a*) man of destiny (*b*) man of principle
(*c*) changeling (*d*) adversary

214. Opporbrium
(*a*) delineation (*b*) aptitude
(*c*) majesty (*d*) praise

215. Optimum
(*a*) pessimistic (*b*) knowledgeable
(*c*) worst (*d*) minimum

216. Opulence
(*a*) pessimism (*b*) patrionsm
(*c*) potency (*d*) poverty

217. Ostentatious
(*a*) inactive (*b*) unassuming
(*c*) impolite (*d*) illicit

218. Overweening
(*a*) humble (*b*) impotent
(*c*) avid (*d*) acrimonious

219. Pejorative
(*a*) positive (*b*) legal
(*c*) determining (*d*) delighting

220. Pellucid
(*a*) logistical (*b*) philandering
(*c*) incomperhensible (*d*) vagrant

221. Penchant
(*a*) distance (*b*) imminence
(*c*) dislike (*d*) attitude

222. Penurious
(*a*) imprisoned (*b*) captivated
(*c*) generous (*d*) vacant

223. Perfunctory
(*a*) official (*b*) through
(*c*) insipid (*d*) vicarious

224. Perigee
(*a*) eclipse (*b*) planet
(*c*) apogee (*d*) refugee

225. Peripatetic
(*a*) wordly (*b*) stationary
(*c*) disarming (*d*) seeking

226. Permeable
(*a*) perishable (*b*) effective
(*c*) plodding (*d*) impenetrable

227. Pernicious
(*a*) practical (*b*) comparative
(*c*) harmless (*d*) tangible

228. Perpetual
(*a*) momentary (*b*) standard
(*c*) serious (*d*) industrial

229. Perspicuity
(*a*) grace (*b*) feature
(*c*) review (*d*) vagueness

230. Pert
(*a*) polite (*b*) perishable
(*c*) moral (*d*) deliberate

231. Pertinacious
(*a*) vengeful (*b*) consumptive
(*c*) superficial (*d*) skilled

232. Pertient
(*a*) understood (*b*) living
(*c*) discontented (*d*) irrelevant

233. Petulant
(*a*) angry (*b*) moral
(*c*) declining (*d*) uncomplaining

234. Precipitate
(*a*) dull (*b*) anticipatory
(*c*) cautious (*d*) considerate

235. Prefatory
(*a*) outstanding (*b*) magnificent
(*c*) conclusive (*d*) intelligent

236. Prelude
(*a*) intermezzo (*b*) diva
(*c*) aria (*d*) aftermath

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237. Presumption
(*a*) assertion (*b*) activation
(*c*) motivation (*d*) humility

238. Pretentious
(*a*) ominous (*b*) calm
(*c*) unassuming (*d*) futile

239. Prim
(*a*) informal (*b*) prior
(*c*) exterior (*d*) private

240. Pristine
(*a*) cultivated (*b*) condemned
(*c*) irreligious (*d*) cautious

241. Probity
(*a*) aregert (*b*) assumption
(*c*) corruptibility (*d*) extent

242. Prodigal
(*a*) large (*b*) thrifty
(*c*) consistent (*d*) compatible

243. Prodigious
(*a*) infinitesimal (*b*) indignant
(*c*) indifferent (*d*) indisposed

244. Profane
(*a*) sanctify (*b*) desecrate
(*c*) define (*d*) manifest

245. Prolific
(*a*) unworkable (*b*) backward
(*c*) barren (*d*) controversial

246. Prolix
(*a*) stupid (*b*) indifferent
(*c*) redundant (*d*) pithy

247. Prophylactic
(*a*) causing growth (*b*) causing disease
(*c*) antagonistic (*d*) brushing

248. Propinquity
(*a*) remoteness (*b*) uniqueness
(*c*) health (*d*) virtue

249. Propitious
(*a*) rich (*b*) induced
(*c*) promoted (*d*) unfavorable

250. Prosaic
(*a*) pacified (*b*) reprieved
(*c*) pensive (*d*) imaginative

251. Protean
(*a*) amateur (*b*) catholic
(*c*) unchanging (*d*) rapid

252. Protract
(*a*) make circular (*b*) shorten
(*c*) further (*d*) rectrace

253. Provident
(*a*) unholy (*b*) rash
(*c*) miserable (*d*) disabled

254. Provincial
(*a*) wealthy (*b*) crass
(*c*) literary (*d*) sophisticated

255. Psychotic
(*a*) dangerous (*b*) clairvoyant
(*c*) criminal (*d*) sane

256. Puerile
(*a*) fragrant (*b*) adult
(*c*) lonely (*d*) feminine

257. Pugnacious
(*a*) pacific (*b*) feline
(*c*) mature (*d*) angular

258. Puissant
(*a*) pouring (*b*) fashionable
(*c*) articulate (*d*) weak

259. Pulchritude
(*a*) ugliness (*b*) notoriety
(*c*) bestiality (*d*) masculinity

260. Punctilious
(*a*) happy (*b*) active
(*c*) vivid (*d*) careless

261. Punitive
(*a*) large (*b*) humorous
(*c*) rewarding (*d*) restive

262. Pusillanimous
(*a*) poverty-stricken (*b*) chained
(*c*) posthumous (*d*) courageous

263. Putative
(*a*) colonial (*b*) quarrelsome
(*c*) undisputed (*d*) powerful

264. Resilient
(*a*) pungent (*b*) foolish
(*c*) worthy (*d*) unyielding

265. Restive
(*a*) buoyant (*b*) placid
(*c*) remorseful (*d*) resistant

266. Retentive
(*a*) forgetful (*b*) accepting
(*c*) repetitive (*d*) avoiding

267. Reticence
(*a*) fatigue (*b*) fashion
(*c*) treachery (*d*) loquaciousness

268. Retrograde
(*a*) progressing (*b*) inclining
(*c*) evaluating (*d*) concentrating

269. Revere
(*a*) advance (*b*) dishonor
(*c*) age (*d*) precede

270. Rife
(*a*) direct (*b*) scant
(*c*) peaceful (*d*) grim

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271. Robust
(*a*) weak (*b*) violent
(*c*) vicious (*d*) villainous

272. Rotundity
(*a*) promenade (*b*) nave
(*c*) grotesqueness (*d*) slimness

273. Rubble
(*a*) artificial facade (*b*) unbroken stone
(*c*) pale complexion (*d*) strong defense

274. Ruddy
(*a*) robust (*b*) witty
(*c*) wan (*d*) exotic

275. Rudimentary
(*a*) pale (*b*) polite
(*c*) asinine (*d*) developed

276. Rueful
(*a*) trite (*b*) content
(*c*) capable (*d*) capital

277. Rustic
(*a*) urban (*b*) slow
(*c*) corroded (*d*) mercenary

278. Ruthless
(*a*) merciful (*b*) majestic
(*c*) mighty (*d*) militant

279. Sadistic
(*a*) happy (*b*) quaint
(*c*) kindhearted (*d*) vacant

280. Sagacious
(*a*) foolish (*b*) bitter
(*c*) voracious (*d*) veracious

281. Sallow
(*a*) salacious (*b*) ruddy
(*c*) colorless (*d*) permitted

282. Salubrious
(*a*) salty (*b*) bloody
(*c*) miasmic (*d*) maudlin

283. Salvage
(*a*) remove (*b*) outfit
(*c*) burn (*d*) lose

284. Sanctimonious
(*a*) hypothetical (*b*) paltry
(*c*) mercenary (*d*) pious

285. Sanguine
(*a*) choleric (*b*) sickening
(*c*) warranted (*d*) pessimistic

286. Satiety
(*a*) emptiness (*b*) warmth
(*c*) erectness (*d*) straightness

287. Scanty
(*a*) collected (*b*) remote
(*c*) invisible (*d*) plentiful

288. Scurrilous
(*a*) savage (*b*) scabby
(*c*) decent (*d*) volatile

289. Secular
(*a*) vivid (*b*) clerical
(*c*) punitive (*d*) positive

290. Sedentary
(*a*) vicarious (*b*) loyal
(*c*) accidental (*d*) active

291. Sedulous
(*a*) indolent (*b*) guileless
(*c*) vindictive (*d*) upright

292. Senlity
(*a*) verility (*b*) loquaciousness
(*c*) forgetfulness (*d*) youth

293. Sententious
(*a*) paragraphed (*b*) positive
(*c*) posthumous (*d*) wordy

294. Tractable
(*a*) unmanageable (*b*) irreligious
(*c*) mortal (*d*) incapable

295. Traduce
(*a*) exhume (*b*) increase
(*c*) purchase (*d*) extol

296. Tranquillity
(*a*) lack of sleep (*b*) lack of calm
(*c*) emptiness (*d*) renewal

297. Transient
(*a*) carried (*b*) close
(*c*) permanent (*d*) removed

298. Tremulous
(*a*) steady (*b*) obese
(*c*) young (*d*) healthy

299. Trenchant
(*a*) lacking bite (*b*) imperious
(*c*) inessential (*d*) unafraid

300. Trepidation
(*a*) slowness (*b*) amputation
(*c*) fearlessness (*d*) adroitness

301. Trite
(*a*) correct (*b*) original
(*c*) distinguished (*d*) premature

302. Truculent
(*a*) juicy (*b*) overflowing
(*c*) peaceful (*d*) determined

303. Tumult
(*a*) scarcity (*b*) defeat
(*c*) cooleness (*d*) secrenity

304. Turbid
(*a*) clear (*b*) improbable
(*c*) invariable (*d*) honest

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305. Turbulence
(*a*) reaction (*b*) approach
(*c*) impropriety (*d*) calm

306. Turgid
(*a*) rancid (*b*) shrunken
(*c*) cool (*d*) explosive

307. Turpitude
(*a*) amplitude (*b*) heat
(*c*) wealth (*d*) virtue

308. Tyro
(*a*) infant (*b*) rubber
(*c*) personnel (*d*) expert

309. Unearth
(*a*) conceal (*b*) gnaw
(*c*) clean (*d*) fling

310. Unfeigned
(*a*) pretended (*b*) fashionable
(*c*) wary (*d*) switched

311. Ungainly
(*a*) ignorant
(*b*) graceful
(*c*) detailed
(*d*) dancing

312. Unimpeachable
(*a*) fruitful
(*b*) rampaging
(*c*) faulty
(*d*) pensive

313. Unkempt
(*a*) bombed (*b*) washed
(*c*) neat (*d*) tawdry

314. Unruly
(*a*) chatting (*b*) obedient
(*c*) definite (*d*) lined

315. Unseemly
(*a*) effortless (*b*) proper
(*c*) conducive (*d*) pointed

316. Unsullied
(*a*) tranished (*b*) countless
(*c*) soggy (*d*) papered

317. Untenable
(*a*) supportable (*b*) tender
(*c*) sheepish (*d*) tremulous

318. Unwitting
(*a*) clever (*b*) intense
(*c*) sensitive (*d*) intentional

319. Vacillation
(*a*) remorse (*b*) relief
(*c*) respect (*d*) steadfastness

320. Valedictory
(*a*) sad (*b*) collegiate
(*c*) derivative (*d*) salutatory

321. Valor
(*a*) admonition
(*b*) injustice
(*c*) cowardice
(*d*) generosity

322. Vanguard
(*a*) regiment
(*b*) rear
(*c*) echelon
(*d*) protection

323. Vaunted
(*a*) unvanquished
(*b*) fell
(*c*) belittled
(*d*) believed

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. Which word is not a synonym for the word *vernacular*?
(*a*) regional (*b*) indigeneous
(*c*) indigent (*d*) colloquial

2011

2. Choose the word from the options given below that is most nearly opposite in meaning to the given word:

Amalgamate
(*a*) merge
(*b*) split
(*c*) collect
(*d*) separate

3. Choose the word from the options given below that is most nearly **opposite** in meaning to the given word :

Frequency
(*a*) periodicity (*b*) rarity
(*c*) gradualness (*d*) persistency

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ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*d*)	**2.** (*b*)	**3.** (*d*)	**4.** (*b*)	**5.** (*a*)	**6.** (*d*)	**7.** (*a*)	**8.** (*a*)	**9.** (*a*)	**10.** (*b*)
11. (*b*)	**12.** (*b*)	**13.** (*b*)	**14.** (*d*)	**15.** (*b*)	**16.** (*c*)	**17.** (*d*)	**18.** (*c*)	**19.** (*c*)	**20.** (*d*)
21. (*d*)	**22.** (*a*)	**23.** (*d*)	**24.** (*c*)	**25.** (*b*)	**26.** (*d*)	**27.** (*b*)	**28.** (*b*)	**29.** (*d*)	**30.** (*d*)
31. (*c*)	**32.** (*a*)	**33.** (*d*)	**34.** (*b*)	**35.** (*d*)	**36.** (*a*)	**37.** (*d*)	**38.** (*a*)	**39.** (*b*)	**40.** (*b*)
41. (*d*)	**42.** (*d*)	**43.** (*c*)	**44.** (*a*)	**45.** (*d*)	**46.** (*d*)	**47.** (*c*)	**48.** (*d*)	**49.** (*d*)	**50.** (*a*)
51. (*c*)	**52.** (*d*)	**53.** (*b*)	**54.** (*a*)	**55.** (*d*)	**56.** (*c*)	**57.** (*b*)	**58.** (*a*)	**59.** (*b*)	**60.** (*a*)
61. (*c*)	**62.** (*b*)	**63.** (*c*)	**64.** (*b*)	**65.** (*c*)	**66.** (*d*)	**67.** (*d*)	**68.** (*c*)	**69.** (*b*)	**70.** (*a*)
71. (*a*)	**72.** (*b*)	**73.** (*b*)	**74.** (*b*)	**75.** (*a*)	**76.** (*c*)	**77.** (*a*)	**78.** (*d*)	**79.** (*d*)	**80.** (*b*)
81. (*a*)	**82.** (*c*)	**83.** (*c*)	**84.** (*d*)	**85.** (*c*)	**86.** (*b*)	**87.** (*b*)	**88.** (*b*)	**89.** (*d*)	**90.** (*c*)
91. (*d*)	**92.** (*d*)	**93.** (*a*)	**94.** (*a*)	**95.** (*b*)	**96.** (*d*)	**97.** (*d*)	**98.** (*c*)	**99.** (*a*)	**100.** (*a*)
101. (*c*)	**102.** (*b*)	**103.** (*d*)	**104.** (*a*)	**105.** (*d*)	**106.** (*b*)	**107.** (*b*)	**108.** (*a*)	**109.** (*a*)	**110.** (*c*)
111. (*d*)	**112.** (*a*)	**113.** (*d*)	**114.** (*b*)	**115.** (*d*)	**116.** (*c*)	**117.** (*b*)	**118.** (*a*)	**119.** (*b*)	**120.** (*c*)
121. (*b*)	**122.** (*d*)	**123.** (*d*)	**124.** (*a*)	**125.** (*a*)	**126.** (*d*)	**127.** (*a*)	**128.** (*d*)	**129.** (*d*)	**130.** (*b*)
131. (*c*)	**132.** (*d*)	**133.** (*c*)	**134.** (*c*)	**135.** (*a*)	**136.** (*b*)	**137.** (*d*)	**138.** (*b*)	**139.** (*a*)	**140.** (*b*)
141. (*d*)	**142.** (*c*)	**143.** (*a*)	**144.** (*d*)	**145.** (*c*)	**146.** (*c*)	**147.** (*d*)	**148.** (*a*)	**149.** (*a*)	**150.** (*d*)
151. (*b*)	**152.** (*a*)	**153.** (*a*)	**154.** (*d*)	**155.** (*b*)	**156.** (*d*)	**157.** (*b*)	**158.** (*d*)	**159.** (*d*)	**160.** (*c*)
161. (*a*)	**162.** (*d*)	**163.** (*a*)	**164.** (*b*)	**165.** (*a*)	**166.** (*c*)	**167.** (*d*)	**168.** (*b*)	**169.** (*a*)	**170.** (*d*)
171. (*d*)	**172.** (*c*)	**173.** (*b*)	**174.** (*d*)	**175.** (*d*)	**176.** (*d*)	**177.** (*c*)	**178.** (*b*)	**179.** (*a*)	**180.** (*d*)
181. (*d*)	**182.** (*a*)	**183.** (*c*)	**184.** (*b*)	**185.** (*a*)	**186.** (*c*)	**187.** (*c*)	**188.** (*d*)	**189.** (*b*)	**190.** (*c*)
191. (*c*)	**192.** (*a*)	**193.** (*c*)	**194.** (*d*)	**195.** (*c*)	**196.** (*b*)	**197.** (*d*)	**198.** (*d*)	**199.** (*d*)	**200.** (*b*)
201. (*a*)	**202.** (*c*)	**203.** (*a*)	**204.** (*c*)	**205.** (*b*)	**206.** (*d*)	**207.** (*d*)	**208.** (*b*)	**209.** (*a*)	**210.** (*a*)
211. (*c*)	**212.** (*d*)	**213.** (*b*)	**214.** (*d*)	**215.** (*c*)	**216.** (*d*)	**217.** (*b*)	**218.** (*a*)	**219.** (*a*)	**220.** (*c*)
221. (*c*)	**222.** (*c*)	**223.** (*b*)	**224.** (*c*)	**225.** (*b*)	**226.** (*d*)	**227.** (*c*)	**228.** (*a*)	**229.** (*d*)	**230.** (*a*)
231. (*c*)	**232.** (*d*)	**233.** (*d*)	**234.** (*c*)	**235.** (*c*)	**236.** (*d*)	**237.** (*d*)	**238.** (*c*)	**239.** (*a*)	**240.** (*a*)
241. (*c*)	**242.** (*b*)	**243.** (*a*)	**244.** (*a*)	**245.** (*c*)	**246.** (*d*)	**247.** (*b*)	**248.** (*a*)	**249.** (*d*)	**250.** (*d*)
251. (*c*)	**252.** (*b*)	**253.** (*b*)	**254.** (*d*)	**255.** (*d*)	**256.** (*b*)	**257.** (*a*)	**258.** (*d*)	**259.** (*a*)	**260.** (*b*)
261. (*c*)	**262.** (*d*)	**263.** (*c*)	**264.** (*b*)	**265.** (*a*)	**266.** (*d*)	**267.** (*a*)	**268.** (*b*)	**269.** (*b*)	**270.** (*b*)
271. (*a*)	**272.** (*d*)	**273.** (*b*)	**274.** (*c*)	**275.** (*d*)	**276.** (*b*)	**277.** (*a*)	**278.** (*a*)	**279.** (*c*)	**280.** (*a*)
281. (*b*)	**282.** (*c*)	**283.** (*d*)	**284.** (*d*)	**285.** (*d*)	**286.** (*d*)	**287.** (*d*)	**288.** (*c*)	**289.** (*b*)	**290.** (*d*)
291. (*a*)	**292.** (*d*)	**293.** (*d*)	**294.** (*a*)	**295.** (*d*)	**296.** (*b*)	**297.** (*c*)	**298.** (*a*)	**299.** (*a*)	**300.** (*c*)
301. (*b*)	**302.** (*c*)	**303.** (*d*)	**304.** (*a*)	**305.** (*d*)	**306.** (*b*)	**307.** (*d*)	**308.** (*d*)	**309.** (*a*)	**310.** (*a*)
311. (*b*)	**312.** (*c*)	**313.** (*d*)	**314.** (*b*)	**315.** (*b*)	**316.** (*a*)	**317.** (*a*)	**318.** (*d*)	**319.** (*d*)	**320.** (*d*)
321. (*c*)	**322.** (*b*)	**323.** (*d*)							

EXERCISE – II

MCQ Type Questions

1. (*c*) **2.** (*b*) **3.** (*b*)

EXPLANATIONS

EXERCISE – II

MCQ TYPE QUESTIONS

1. *Vernacular* - expressed or written in the native language of a place

   Indigent - deficient in what is requisite
2. Amalgamate means combine or unite to form one organization or structure. So the best option here is split. Separate on the other hand, although a close synonym, it is too general to be the best antonym in the given question while Merge is the synonym; Collect is not related.
3. Best antonym is rarity which means shortage or scarcity.

■■

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5 CHAPTER

Reasoning Ability

Reasoning is the process of drawing conclusions from facts. These conclusions must follow inevitably from the facts from which they are drawn. Reasoning is not concerned with some conclusion that has a good chance of being true when facts are true. Indeed, reasoning as used here refers to logical reasoning, not of commonsense reasoning or probabilistic reasoning. The only conclusions that are acceptable are those that follow logically from the supplied facts.

TYPES OF REASONING

Reasoning can be classified in two major parts :

- Verbal Reasoning
- Non-verbal Reasoning

VERBAL REASONING

Verbal reasoning tests words, letters and numbers, and require logical reasoning and a reasonable knowledge of the English Language. It is also necessary to be familiar with simple manipulations with figures, like addition, subtractions division and multiplication. The problems of number in *Test of Reasoning* test how logical we are i.e., how well we reason and think while carrying out simple arithmetic manipulations.

CLASSIFICATION OF VERBAL REASONING

I. SERIES COMPLETION

In verbal series, words, letters or digits are given in a specific sequence/order and we have to find out next word, letter or digit to complete the given series.

Numbers or alphabetical letters, are generally called *terms of the series*. These terms follow a certain pattern throughout. In the questions we have to identify last one or two terms to continue the series or to find a missing term in between given terms to continue the sequence followed in the question. There is no set pattern and each question may follow a different pattern or sequential arrangement of letters or digits, which have to detect using common sense and reasoning ability.

TYPES OF SERIES COMPLETION QUESTIONS

Mainly following four types of question are asked.

1. Alphabet series

In this series, given alphabets follow a particular sequence or order. We have to detect the pattern from the given alphabets and find missing alphabet or the next alphabet to continue the pattern.

- There are no set rules.
- There can be omission of alphabets in an order.
- Alphabets may also be omitted in an increasing/decreasing order, which may be direct increase or decrease.
- There can also be alternate order.
- There may also be alternate sequences
- There may be several other patterns in the letter series.
- To tackle letter series questions, vary position of the alphabet and its position number in both forward and backward sequences.
- To continue the series after Z, we again begin with A. In other words, the sequence is kept in a circular order.
- In solving these questions pattern of the alphabet series should be noted.

Some Skipping Patterns.

(*i*) ***Regular Order :*** Number of alphabets skipped remains the same.

Example. A, D, G, J, ...?

Ans. M

(*ii*) ***Increasing Order :*** Each time the number of alphabets skipped increases in a given pattern.

Example. A C F J O ?

Ans. U

Here, each time number of letters skipped increases by one.

(*iii*) ***Decreasing Order :*** Each time the number of letters skipped decreases in a given pattern.

Example. A G L P S ?

Ans. U

Here number of letters skipped decreases by one each time, *i.e.,* first 5, then 4, then 3, and so on.

(*iv*) ***Interlinked Series*** **:** In this two or more different series are attached together. These different series follow their own different rules.

Example. A D F J M R ?

Ans. V

Here, there are two interlinked series.

2. Letter Series

This type of questions usually consist of a series of small letters which follow a certain pattern.

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However, some letters are missing from the series. Then these missing letters are given in a proper sequence as one of the alternatives.

Example. aaa... b...a...

(*a*) baa (*b*) abb (*c*) bab (*d*) aab (*e*) bbb

Ans. (*a*)

Method :

- First blank space should be filled in by 'b', so that we have two a's followed by two b's.
- Second blank space should be filled in either by 'a', so that we have four a's followed by two b's, or by 'b' so that we have three a's followed by three b's.
- Last space must be filled in by 'a'.
- *Thus, we have two possible answers :* 'baa' and 'bba'.

  But, only 'baa' appears in the alternatives. So, the answer is (*a*).

Note : In case, we had both the possible answers in the alternatives, we would have chosen the one that forms a more prominent pattern, which is aabb/ aaabbbaa. Thus, our answer would have been "*bba*".

3. Number series

In the number series, some numbers are arranged in a particular sequence. All the numbers form a series and change in a certain order. Sometimes, one or more numbers are wrongly put in the number series. We are required to observe the trend in which numbers change in the series and find out which number/numbers misfit into the series that number/numbers is *odd number* of the series.

Pure Series.

In this type of number series, the number itself obeys certain order so that the character of the series can be found out.

Number may be Perfect square; Perfect cube; Prime and Combination

Difference Series.

In this type of number series, change in order for the difference between each consecutive number of the series is found out.

Ratio Series.

In this type of number series, change in order for the ratios between each consecutive number of the series is found out.

4. Mixed Series

In this type of number series, numbers obeying various orders of two or more different types of series are arranged alternately in a single number series.

Rules:

1. Difference between consecutive numbers is same.
2. Differences between consecutive numbers are in Arithmetic Progression (A.P.).
3. Difference between consecutive numbers is a perfect square.
4. Differences between consecutive numbers are multiples of a number.
5. Differences between consecutive numbers are prime numbers.
6. Difference between consecutive numbers is a perfect cube.
7. Difference between consecutive numbers are in Geometric Progression (G.P.)
8. Ratio between each consecutive number is the same.
9. Ratio between each consecutive numbers is in Arithmetic Progression (A.P)
10. Ratio between consecutive number is perfect square number.
11. Ratio between consecutive number is the multiple of a number.
12. Ratio between consecutive numbers is a prime number.
13. Ratio between consecutive numbers is a perfect cube number.
14. Ratios between consecutive number are in Geometric Progression (G.P.)

5. Correspondence Series

This type of series consists of three sequences with three different elements (usually capital letters, digits and small letters). On the basis of the similarity in positions in the three sequences, a capital letter is found to correspond with a unique digit and a unique small letter, whenever it occurs. We are required to trace out this correspondence and accordingly choose the elements to be filled in at the desired places.

II. ANALOGY

'*Analogy*' means 'correspondence'.

In questions based on analogy, a particular relationship is given and another similar relationship has to be identified from the given alternatives.

Analogy means similarity or some kind of relationship. Analogy test, therefore tests the ability to see a relationship between two words and to recognize a similar relationship between two other words. It is a test of verbal reasoning ability rather than vocabulary. The key to analogy success is being able to express the relationship between the pair of words.

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KINDS OF RELATIONSHIPS WITH EXAMPLES

1. **Instrument and Measurement**
   - Thermometer : Temperature (thermometer is an instrument used to measure temperature).
   - Barometer : Pressure
   - Anemometer : Wind vane
   - Odometer : Speed
   - Scale : Length
   - Balance : Mass
   - Sphygmomanometer: Blood Pressure
   - Rain Gauge : Rain
   - Hygrometer : Humidity
   - Ammeter : Current
   - Screw Gauge : Thickness
   - Seismograph : Earthquakes
   - Taseometer : Strains
2. **Quantity and Unit**
   - Mass : Kilogram (kilogram is the unit of mass)
   - Length : Metre
   - Force : Newton
   - Energy : Joule
   - Resistance : Ohm
   - Volume : Litre
   - Angle : Radians
   - Time : Seconds
   - Potential : Volt
   - Work : Joule
   - Current : Ampere
   - Luminosity : Candela
   - Pressure : Pascal
   - Area : Hectare
   - Temperature : Degrees
   - Power : Watt
   - Conductivity : Mho
   - Magnetic field : Oersted/ Tesla
3. **Individual and Group**
   - Soldiers : Army (group of soldiers is called an army).
   - Cattle : Herd
   - Flowers : Bouquet
   - Grapes : Bunch
   - Singer : Chorus
   - Artist : Troupe
   - Fish : Shoal
   - Sheep : Flock
   - Riders : Cavalcade
   - Bees : Swarm
   - Man : Crowd
   - Sailors : Crew
   - Nomads : Horde
4. **Animal and Young one**
   - Cow : Calf (calf is the young one of cow)
   - Horse : Pony
   - Cat : Kitten
   - Sheep : Lamb
   - Butterfly : Caterpillar
   - Insect : Larva
   - Dog : Puppy
   - Cock/Hen : Chicken
   - Lion : Cub
   - Duck : Duckling
   - Man : Child
5. **Male and Female**
   - Horse : Mare (mare is the female horse)
   - Dog : Bitch
   - Stag : Doe
   - Son : Daughter
   - Lion : Lioness
   - Sorcerer : Sorceress
   - Drone : Bee
   - Gentleman : Lady
   - Nephew : Niece
   - Tiger : Tigress
6. **Individual and Class**
   - Monkey : Mammal (monkey belongs to the class of mammal).
   - Man : Mammal
   - Ostrich : Bird
   - Snake : Reptile
   - Butterfly : Insect
   - Whale : Mammal
   - Rat : Rodent
   - Lizard : Reptile
7. **Individual and Dwelling Place**
   - Dog : Kennel (dog lives in a kennel)
   - Bee : Apiary
   - Cattle : Shed
   - Lion : Den
   - Poultry : Farm
   - Monk : Monastery
   - Fish : Aquarium
   - Birds : Aviary
   - Horse : Stable
8. **Study and Topic**
   - Ornithology : Birds (ornithology is the study of birds).
   - Seismology : Earthquakes
   - Botany : Plants
   - Onomatology : Names
   - Ethnology : Human Races
   - Ontology : Reality
   - Herpetology : Amphibians
   - Pathology : Diseases
   - Astrology : Future
   - Anthropology : Man
   - Palaeography : Writings
   - Ichthyology : Fishes
   - Semantics : Language
   - Nephrology : Kidney
   - Concology : Shells
   - Haematology : Blood
   - Craniology : Skull
   - Mycology : Fungi
   - Entomology : Insects
   - Zoology : Animals
   - Occultism : Supernatural
9. **Worker and Tool**
   - Warrior : Sword (sword is the tool used by a warrior).
   - Carpenter : Saw
   - Woodcutter : Axe
   - Tailor : Needle
   - Labourer : Spade
   - Soldier : Gun
   - Sculptor : Chisel
   - Chef : Knife
   - Doctor : Stethoscope
   - Farmer : Plough
   - Author : Pen
10. **Tool and Action**
    - Needle : Sew (needle is used for sewing).
    - Knife : Cut
    - Sword : Slaughter
    - Mattock : Dig
    - Filter : Purify
    - Steering : Drive
    - Pen : Write
    - Spanner : Grip
    - Spoon : Feed
    - Microscope : Magnify
    - Gun : Shoot
    - Shovel : Scoop
    - Chisel : Carve
    - Oar : Row
    - Axe : Grind
    - Auger : Bore
    - Spade : Dig
11. **Worker and Working Place**
    - Chef : Kitchen (chef works in a kitchen)

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- Farmer : Field
- Warrior : Battlefield
- Engineer : Site
- Sailor : Ship
- Pilot : Cockpit
- Beautician : Parlour
- Artist : Theatre
- Actor : Stage
- Mechanic : Garage
- Lawyer : Court
- Scientist : Laboratory
- Teacher : School
- Doctor : Hospital
- Clerk : Office
- Servant : House
- Driver : Cabin
- Grocer : Shop
- Painter : Gallery
- Waiter : Restaurant
- Worker : Factory
- Umpire : Pitch
- Gambler : Casino

12. Worker and Product
- Mason : Wall (mason builds a wall)
- Farmer : Crop
- Teacher : Education
- Hunter : Prey
- Chef : Food
- Carpenter : Furniture
- Judge : Justice
- Author : Book
- Choreographer : Ballet
- Goldsmith : Ornaments
- Producer : Film
- Butcher : Meat
- Architect : Design
- Cobbler : Shoes
- Tailor : Clothes
- Poet : Poem
- Dramatist : Play

13. Product and Raw Material
- Prism : Glass (prism is made of glass).
- Butter : Milk
- Cloth : Fibre
- Wall : Brick
- Road : Asphalt
- Furniture : Wood
- Book : Paper
- Shoes : Leather
- Sack : Jute
- Pullover : Wool
- Omelette : Egg
- Metal : Ore
- Jewellery : Gold
- Rubber : Latex
- Linen : Flax
- Jaggery : Sugarcane
- Oil : Seed
- Wine : Grapes
- Paper : Pulp
- Fabric : Yarn

14. Part and Whole Relationship
- Pen : Nib (nib is a part of a pen).
- Pencil : Lead
- House : Kitchen
- Fan : Blade
- Class : Student
- Room : Window
- Aeroplane : Cockpit
- Book : Chapter

15. Word and Intensity
- Anger : Rage (rage is of higher intensity than Anger)
- Wish : Desire
- Kindle : Burn
- Sink : Drown
- Quarrel : War
- Error : Blunder
- Famous : Renowned
- Unhappy : Sad
- Refuse : Deny

16. Word and Synonym
- Abode : Dwelling (abode means almost the same as Dwelling)
- Blend : Mix
- Ban : Prohibition
- Assign : Allot
- Vacant : Empty
- Abduct : Kidnap
- Dearth : Scarcity
- Dissipate : Squander
- Sedate : Calm
- Brim : Edge
- House : Home
- Solicit : Request
- Presage : Predict
- Haughty : Proud
- Flaw : Defect
- Fierce : Violent
- Fallacy : Illusion
- Substitute : Replace
- Mend : Repair
- Alight : Descend
- Presume : Assume

17. Word and Antonym
- Attack : Defend (defend means opposite of Attack)
- Advance : Retreat
- Cruel : Kind
- Best : Worst
- Fresh : Stale
- Ignore : Notice
- Initial : Final
- Condense : Expand
- Chaos : Peace
- Create : Destroy
- Gradual : Abrupt
- Sink : Float
- Robust : Weak
- Gentle : Harsh
- Deep : Shallow
- Cordial : Hostile
- Affirm : Deny
- Mourn : Rejoice
- Lethargy : Alertness

18. Part
Gill : Fin (Gill and fin are two parts of a fish)

19. Type
Sword : Weapon(Sword is a type of weapon)

20. Tool & Worker
Paint Brush : Artist (Paintbrush is a tool to artist)

21. Degree
Warm : Hot (Warm is a greater degree of hot)

Fond : Doting (Fond is less extreme than doting)

22. Sign of
Grimace : Pain(Grimace is a sign of pain)

Brush : Discomfiture (A blush signifies discomfiture)

23. A place for
Miner : Quarry (A miner works in a quarry)

24. Sequence relationship
Foreward : Appendix (In a book a foreword precedes the Appendix)

25. Worker and Work
Poet : Sonnet (A sonnet is written by a poet)

Mason : Wall (A mason builds a wall)

26. Cause and Effect
Soporofic : Sleepiness (Soporofic causes sleepiness)

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COMMON TYPES OF ANALOGY QUESTIONS

1. Completing Analogous Pair

In this type of questions, given two words are, which related to each other in some way. Another word is also given and we required to find out the relationship between first two words and choose the word from the given alternatives, which bears the same relationship to the third word, as the first two bear.

Example. Doctor : Nurse : : ? : Follower

(*a*) Employer (*b*) Leader (*c*) Worker
(*d*) Manager (*e*) Union

Ans. (*b*)

Just as a nurse follows the Doctor's instructions, so also a follower works as directed by the leader.

2. Choosing Analogous Pair

In this type of questions, a pair of words is given, followed by four pairs of words as alternatives and we are required to choose the pair in which words bear the same relationship to each other as the words of the given pair bear.

Example. Sink : Float : : ?

(*a*) Brim : Edge (*b*) Mend : Repair
(*c*) Gentle : Harsh (*d*) Flow : defect
(*e*) Solicit : Request

Ans. (*b*)

Just as sink is antonym of float, so also gentle is opposite of harsh.

3. Choosing Similar word

In this type of questions, a group of three words is given, followed by four other words as alternatives. The candidate is required to choose the alternative, which is similar to the given three words.

Example. Sitar : Guitar : Tanpura

(*a*) Trumpet (*b*) Violin (*c*) Harmonium
(*d*) Mridanga (*e*) Fluet

Ans. (*b*)

Sitar, Guitar and Tanpura are all string instruments. Violin is also a string instrument.

4. Detecting Analogies

In this type of questions, we are required to trace out hidden analogy or common characteristic among the given words or to choose the word which possesses the same characteristic as the given word.

Example. Judo : Karate : Taekwando

(*a*) They are names of martial arts.
(*b*) They can be performed by obese persons.
(*c*) They are performed on stage.
(*d*) They are important items of Asian Games.
(*e*) They have origin in Kerala

Ans. (*a*)

Judo, Karate and Taekwando are martial arts and (*a*) is most suitable description for all the three.

5. Three word Analogy

In this type of questions, a group of three inter-related words is given and we are required to trace out the relationship among these three words and choose another group with similar analogy, from among the alternatives provided.

Example. Pen : Pencil : Ink

(*a*) Orange : Banana : Juice
(*b*) Table : Chair : Wood
(*c*) Cow : Milk : Curd
(*d*) Fish : Shark : Water
(*e*) Car : Engine : Circle

Ans. (*a*)

Pen contains ink and pencil belongs to the same category as pen i.e. stationery. Similarly, orange contains juice and banana belongs to the same category as orange, *i.e.* fruits.

6. Number Analogy

It includes deals two types of questions :

(*i*) Choosing a similarly related pair as the given number pair on the basis of relation between the numbers in each pair.

Example. 582 : 194 :: 258 : ?

(*a*) 82 (*b*) 86 (*c*) 92 (*d*) 58

Ans. (*b*)

Just as 582 is related to 194 as 582 is 3 times 194, in the same way 258 is 3 times the 86.

(*ii*) Choosing a number similar to a group of numbers on the basis of certain common properties that they possess.

Example. Which number belongs to given set of numbers?

2, 3, 5, 7, 11, 13, 17, 19, 23......

(*a*) 15 (*b*) 9 (*c*) 21 (*d*) 29 (*e*) 27

Ans. (*d*)

Given set of numbers belong to the prime numbers. In the given option, only prime number is 29.

7. Alphabet Analogy

In this type of questions, two groups of alphabets related to each other in same way, are given and we are required to find out this relationship and choose a group of alphabets which is related in the same way to a third group provided in the question.

Example. B C D : P Q R : : X Y Z : ?

(*a*) FGH (*b*) LMN (*c*) RST (*d*) TUV (*e*) STU

Ans. (*b*)

Each letter of the first group is moved fourteen steps forward to obtain the corresponding letter of the second group. A similar relationship will exist between the third and fourth groups.

HOW TO ANSWER ANALOGY QUESTIONS

1. Figure out how the capitalized words are related.
2. Create a sentence that expresses that connection.
3. Test the choices with your sentence and eliminate the ones that don't work.
4. If we are left with more than one answer—or no answer at all—go back and refine your sentence.
5. Choose the best answer. If none of the choices fits exactly, choose the one that works best.

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PLAN TO SOLVE ANALOGY QUESTIONS

1. Make your Sentence More Precise

The analogies get more difficult as we work our way through each group. Use the common categories as a starting point, but be prepared to refine the relationship by making your sentence more precise.

Example. Grain : Silo ::

(*a*) Pilot : plane (*b*) judge : courtroom

(*c*) water : reservoir (*d*) clock : time

If we apply the "place where" idea without thinking, here is what happens.

A silo is a place where we would find grain.

(*a*) A *plane* is a place where you would find a *pilot*.

(*b*) A *courtroom* is a place where you would find a *judge*.

(*c*) A *reservoir* is a place where you would find *water*.

(*d*) A *clock* is a place where you would find *time*.

We can eliminate (d), but that still leaves you with four possible answers. Now is express the relationship between silo and grain more precisely?

A silo is a place where grain is stored.

(*a*) A plane is a place where a pilot is stored.

(*b*) A courtroom is a place where a judge is stored.

(*c*) A reservoir is a place where water is stored.

(*d*) A highway is a place where automobiles are stored.

2. Analogies Work only if there's a clear cut connection

An analogy depends upon a necessary connection between pairs of words based on the meaning of the words. This clear connection must exist for both the original capitalized word pair and the correct answer choice, i.e. we can eliminate any answer choice for which we cannot describe a necessary relationship between the words. Use the "clear connection" test to rule out answer choices even when we don't know one of the capitalized words.

Example. Letter : Alphabet : :

(*a*) note : scale

(*b*) ocean : merchandise

(*c*) expert : automobile

(*d*) victory : farmland

We dont't know what the capitalized words are in this analogy, but you can still eliminate choices (*b*), (*c*) and (*d*). There is no clear-cut connection between the words of those choices. The actual analogy above is LETTER : ALPHABET :: note : scale.

3. Some Analogies Work Better by Working Backward

Sometimes the capitalized words fall easily into a sentence that expresses their relationship–and sometimes they don't. If we are having trouble making up a sentence that relates the two words, be prepared to shift gears. Try reversing the order of the original word pair.

Example. *Ice : Glacier ::*

(*a*) train : trestle (*b*) sand : dune

(*c*) path : forest (*d*) feather : bird

If we can't come up with a sentence relating ICE to GLACIER, try relating GLACIER to ICE :

A *glacier* is made of *ice*.

If we reverse the order of the capitalized words, we must also reverse the order of the words in each answer choice.

(*a*) A *trestle* is made up of a *train*.

(*b*) A *dune* is made up of *sand*.

(*c*) A *forest* is made up of a *path*.

(*d*) A *bird* is made up of a *feather*.

Clearly (*b*) exhibits the same relationship as the original pair.

4. Check from the Answer Choices

Many words have different meanings depending upon whether they are used as nouns or verbs or adjectives. If we are not sure how one of the capitalized words is being used, just check the answer choices. In analogies, all the answer choices will have the same grammatical structure as the capitalized words pair. That means if the answer choices are noun : noun, the capitalized pair will be NOUN : NOUN. On the other hand, if the answer choices are adjective : noun, then the capitalized pair will be ADJECTIVE : NOUN.

If we have eliminated all the answer choices with words that we know or sort of know, and we are left with a couple of choices containing words that we have never seen before, just guess and move on.

III. CLASSIFICATION

'Classification' means arrangement of given items on the basis of some common character. In this test, a group of certain items are given, out of which some are similar to in some manner and one is different from the rest. We are required to choose this one item which does not fit into the given group.

TYPES OF CLASSIFICATION QUESTIONS

1. Choosing Odd word

In this type of questions, four (or five) words are given out of which one is dissimilar to the others and students have to find that odd one from the given set.

Example. *Choose the word which is least like the other words in the group.*

(*a*) chair (*b*) cupboard (*c*) Table
(*d*) paper weight (*e*) Bed

Ans. (*d*)

Here, all except paper weight are furnitures.

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2. Choosing Odd pair of words

In this type of questions, certain pairs of words are given out of which words in all the pairs except one, bear a certain common relationship. We are required to decipher this relationship and choose the pair in which words are differently related, as the answer.

Example. *Choose the odd pair of words.*

(*a*) Loom : Cloth (*b*) Table : Drawer
(*c*) Book : Page (*d*) Car : Wheel

Ans. (*a*)

In all other pairs, second is part of the first.

3. Choosing Odd numeral

In this type of questions, certain numbers are given, out of which all except one are alike in some manner while one is different and this number is to be chosen as the answer.

Example. *Choose the number which is different from others in the group.*

(*a*) 49 (*b*) 64
(*c*) 121 (*d*) 144
(*e*) 156

Ans. (*e*)

Each of the numbers except 156 is complete square.

4. Choosing the Odd numeral pair/group

In this type of questions, certain pairs/groups of numbers are given out of which all except one are similar in some manner while one is different. The numbers in these similar pairs may have the same property or may be related to each other according to the same rule. We are is required to choose the odd pair/group.

Example. *Choose the numeral pair/group which is different from others.*

(*a*) 83 – 75 (*b*) 58 – 50 (*c*) 49 – 42
(*d*) 25 – 17 (*e*) 131– 123

Ans. (*c*)

In each of the pairs, first number is eight more than the second.

5. Choosing the Odd letter group

In this type of questions, usually four groups of letters are given. Three of them are similar to each other in some manner while one is different and we have to the answer.

Example. *Choose the group of letters which is different from others.*

(*a*) SU (*b*) PN (*c*) IK (*d*) BD

Ans. (*b*)

All other are two alternate letters

IV. CODING-DECODING

A CODE is a '*system of signals*'. Therefore, coding is a method of transmitting a message between the sender and the receiver without a third person knowing it.

Coding and Decoding Test is set up to judge the candidate's ability to decipher the rule that codes a particular word/ message and break the code to decipher the message.

TYPES OF CODING DECODING QUESTIONS

1. Letter Coding

In these questions, real alphabets in a word are replaced by certain other alphabets according to a specific rule to form its code and we are required to detect the common rule and answer the questions accordingly.

Example. If TAP is coded as SZO, then how is FREEZE coded?

(*a*) EQDFYG (*b*) ESDFYF (*c*) GQFDYF
(*d*) EQDDYD (*e*) EPDEZD

Ans. (*d*)

Each letter in the word TAP is moved one step backward to obtain the corresponding letter of the code.

$$\begin{array}{|cc|cc}\hline & S & Z & O \\ -1 & \uparrow & \uparrow & \uparrow \\ & T & A & P \\ \hline\end{array}$$

Thus, in FREEZE, F will be coded as E, R as Q, E as D and Z as Y. So, the code becomes EQDDYD.

2. Number Coding

In these questions, either numerical code values are assigned to a word or alphabetical code letters are assigned to the numbers and we are is required to analyse the code as per the directions.

Example. *If CHINA is written as 38126 and NEPAL is 25769, how is PLAINE is coded ?*

Solution : Clearly Alphabets are coded as shown:

C	*H*	*I*	*N*	*A*	*E*	*P*	*L*
3	8	1	2	6	5	7	9

Hence, PLAINE is coded as 796125.

Example. In a certain code '37' means 'which class' and '583' means 'caste and class'. What is the code for 'caste' ?

(*a*) 3 (*b*) 7 (*c*) 8
(*d*) Either 5 or 8

Ans. (*d*)

Given, which class' ≡ '37' ...(*i*)

and'caste and class' ≡ '583' ...(*ii*)

From (*i*) and (*ii*), the common word 'class' is coded as '3'.

From (*ii*), 'caste' is coded as '5' or '8'.

3. Substitution

In this type of questions, some particular objects are assigned code names. Then a question is asked that is to be answered in the code language.

Example. If cook is called *butler*, butler is called manager, manager is called teacher, teacher is called clerk and clerk is called principal, who will teach in a class ?

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(*a*) Cook (*b*) Butler (*c*) Manager
(*d*) Teacher (*e*) Clerk

Ans. (*e*)

A 'teacher' teaches in a class and as given 'teacher' is called 'clerk'. So, a 'clerk' will teach in the class.

4. Mixed letter Coding

In this type of questions, three or four complete messages are given in the coded language and the code for a particular word is asked. To analyse such codes, any two messages bearing a common word are picked up. The common code word will mean that word. Proceeding similarly by picking up all possible combinations of two, the entire message can be analysed.

Example. If 'tee see pee' means 'Drink fruit juice'; 'see kee lee'means 'Juice is sweet' and 'fee ree mee' means. "He is intelligent', which word in that language means 'sweet' ?

(*a*) see (*b*) kee (*c*) lee
(*d*) pee (*e*) None of these

Ans. (*b*)

In first and second statements, common word is 'Juice' and common code word is 'see'. So, 'see' means 'Juice'. In second and third statements, common word is 'is' and common code is lee'. So, 'lee' means 'is'. Thus, in second statement, remaining word 'sweet' is coded as 'kee'.

5. Mixed number Coding

In this type of questions, a few groups of numbers each coding a certain short message, are given. Through a comparison of the given coded messages, taking two at a time, the candidate is required to find the number code for each word and then formulate the code for the message given.

Example. In a certain code, '786' means 'study very hard', '958' means 'hard work pays' and '645' means 'study and work'. Which of the following is the code for 'very' ?

(*a*) 8 (*b*) 6 (*c*) 7 (*d*) 5

Ans. (*c*)

In first and second statements, common word is '*hard*'and common code digit is '8'. So, '8' means 'hard'.

In the first and third statements, common word is 'study' and the common code digit is '6'. So, '6' means 'study'.

Thus, in the first statement, '7' means 'very'.

6. Deciphering individual letter codes by analysis

In this type of questions, certain sample words are given along with their codes. The candidate is required to decipher individual codes for different letters by comparing, taking two words at a time, and then answer the given questions accordingly.

V. BLOOD RELATIONS

In these tests, the success of a candidate depends upon the knowledge of the blood relations.

Following Blood relations help solve these Questions :

Mother's or father's son	— Brother
Mother's or father's daughter	— Sister
Mother's or father's brother	— Uncle
Mother's or father's sister	— Aunt
Mother's or father's father	— Grand father
Mother's or father's mother	— Grand mother
Son's wife	— Daughter-in-Law
Daughter's husband	— Son-in-Law
Husband's or wife's sister	— Sister-in-Law
Husband's or wife's brother	— Brother-in-Law
Brother's son	— Nephew
Brother's daughter	— Niece
Uncle or aunt's son or daughter	— Cousin
Sister's husband	— Brother-in-Law
Brother's wife	— Sister-in-Law
Grandson's or Grand daughter's daughter	— Great grand daughter

Types of Blood Relation Questions

(*i*) Deciphering Jumbled up descriptions

In this type of questions, a round-about description is given in the form of certain small relationships and direct relationship between the persons concerned is to be deciphered.

Example. Pointing to a photograph, a man said, "*I have no brother or sister but that man's father is my father's son.*" Whose photograph was it ?

(*a*) His own (*b*) His son's
(*c*) His father's (*d*) His nephew's

Ans. (*a*)

Since man has no brother, his father's son is he himself.

(*ii*) Relation Puzzle

In this type, mutual blood relations or other informations of more than two persons are mentioned and information about any two is mentioned.

Example. A and B are brothers. C and D are sisters. A's son is D's brother. How is B related to C.

(*a*) Father (*b*) Brother (*c*) Grandfather
(*d*) Uncle (*e*) None of these

Ans.(*d*)

B is the brother of A; A's son is D's brother. This means D is the daughter of A. Since C and D are sisters, C is also the daughter of A. So, B is the uncle of C.

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(*iii*) Coded Relations

In such questions, relationships are represented by certain codes or symbols such as +, –, ×, ÷,*, Δ etc. Then relationships between certain persons, given in the form of these codes, are to be analysed.

Example. If A + B means A is the sister of B; A – B means A is the brother of B; A × B means A is the daughter of B, which of the following shows the relation that E is the maternal uncle of D?

(*a*) $D + F \times E$ (*b*) $D - F \times E$

(*c*) $D \times F + E$ (*d*) $D \times F \times E$

(*e*) None of these.

Ans.(*c*)

E is the maternal uncle of D means D is the daughter of the sister (say F) of E i.e. $D \times F + E$.

VI. PUZZLE TEST

This type of questions put in the form of puzzles involving certain number or items, be it persons or things and we are required to analyse the given information, condense it in a suitable form and answer the questions asked.

TYPE OF PUZZLE TESTS

1. Classification type Questions

This type consists of questions in which certain items belonging to different groups or possessing different qualities are given along with some clues with the help of which the candidate is required to group and analyse the given items and answer the questions accordingly.

Example. *Read the following information carefully and answer the questions that follow*:

(*i*) Five friends P, Q, R, S and T travelled to five different cities of Chennai, Calcutta, Delhi, Bangalore and Hyderabad by five different modes of transport of Bus, Train, Aeroplane, Car and Boat from Mumbai.

(*ii*) The person who travelled to Delhi did not travel by boat.

(*iii*) R went to Bangalore by car and Q went to Calcutta by aeroplane.

(*iv*) S travelled by boat whereas T travelled by train.

(*v*) Mumbai is not connected by bus to Delhi and Chennai.

1. Which of the following combinations of person and mode is not correct ?

(*a*) P — Bus (*b*) Q — Aeroplane

(*c*) R — Car (*d*) S — Boat

(*e*) T — Aeroplane

2. Which of the following combinations is true for S ?

(*a*) Delhi — Bus (*b*) Chennai — Bus

(*c*) Chennai — Boat (*d*) Data inadequate

(*e*) None of these

3. Which of the following combinations of place and mode is not correct ?

(*a*) Delhi — Bus (*b*) Calcutta — Aeroplane

(*c*) Bangalore — Car (*d*) Chennai — Boat

(*e*) Hyderabad — Bus

4. The person travelling to Delhi went by which of the following modes ?

(*a*) Bus (*b*) Train

(*c*) Aeroplane (*d*) Car

(*e*) Boat

5. Who among the following travelled to Delhi ?

(*a*) R (*b*) S

(*c*) T (*d*) Data inadequate

(*e*) None of these

Ans. *Given information can be analysed as follows :*

Mode of Transport.

R travels by Car, Q by Aeroplane, S by Boat and T by Train.

Now, only P remains. So, P travels by Bus.

Place of Travel.

R goes to Bangalore, Q to Calcutta.

Now, bus transport is not available for Delhi or Chennai. So, P who travels by Bus goes to Hyderabad.

S travels by boat and hence, S did not go to Delhi. So, S goes to Chennai.

Now, only T remains. So, T goes to Delhi.

Person	Place	Mode
P	Hyderabad	Bus
Q	Calcutta	Aeroplane
R	Bangalore	Car
S	Chennai	Boat
T	Delhi	Train

1. (*e*). Incorrect combination is T — Aeroplane

2. (*c*). Correct combination for S is Chennai — Boat

3. (*a*). Incorrect combination is Delhi — Bus

4. (*b*). T travelled to Delhi by Train

5. (*c*). T travelled to Delhi

2. Seating / Placing Arrangements

In this type of questions, some clues regarding seating or placing sequence (linear or circular) of some persons or items is given. The candidate is required to form the proper sequence using these clues and answer the questions accordingly.

Example. Four girls are sitting on a bench to be photographed. Shikha is to the left ofReena. Manju is to the right of Reena. Rita is between Reena and Manju. Who would be second from the left in the photograph ?

(*a*) Reena (*b*) Shikha (*c*) Manju

(*d*) Rita (*e*) Either Shikha or Reena

Ans. (*a*)

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Shikha is to the left of Reena and Manju is to her right.

Rita is between Reena and Manju.

Hence order is : Shikha, Reena, Rita, Manju.

In the photograph, Reena will be second from left.

3. Comparison Type Questions

In this type of questions, clues are given regarding comparisons among a set of persons or things with respect to one or more qualities and we are required to analyse the whole information, form a proper ascending/descending sequence and then answer the given questions accordingly.

Example. *Read the information given below and answer the questions that follow :*

(*i*) There is a group of five girls.

(*ii*) Kamini is second in height but younger than Rupa.

(*iii*) Pooja is taller than Monika but younger in age.

(*iv*) Rupa and Monika are of the same age but Rupa is tallest between them,

(*v*) Neelam is taller than Pooja and elder to Rupa.

1. If they are arranged in the ascending order of height, who will be in third position ?

(*a*) Monika
(*b*) Rupa
(*c*) Monika or Rupa
(*d*) Data inadequate
(*e*) None of these

2. If they are arranged in the descending order of their ages, who will be in fourth position ?

(*a*) Monika or Rupa
(*b*) Kamini
(*c*) Monika
(*d*) Data inadequate
(*e*) None of these

3. To answer the question "who is the youngest person in the group", which of the given statements is superfluous ?

(*a*) Only (*i*) (*b*) Only (*ii*) (*c*) Only (*v*)
(*d*) Either (*i*) or (*iv*) (*e*) None of these

Ans. First find sequence of heights

By (*iii*), we have $M < P$

By (*v*), we have $P < N$.

Now, Rupa is tallest and Kamini is second in height, hence sequence of heights is :

$$M < P < N < K < R.$$

Now, find age sequence :

By (*ii*), we have $K < R$.

By (*ii*), we have $P < M$.

By (*iv*), we have $R = M$.

By (*v*), we have $R < N$.

Hence sequence of ages is:

$$P < K < R = M < N \text{ or } K < P < R = M < N$$

1. (*e*). In increasing order of height, Neelam is in third position.

2. (*e*). In descending order of ages, Neelam will be in fourth position (because Monika and Rupa both lie at third position).

3. (*a*). Only statement (*i*) is not necessary.

4. Sequential Order of Things

In this type of questions, some clues are given regarding order of occurrence of certain eventsand we are required to analyse the given information, frame the right sequence and then answer the questions accordingly.

Example. *Read the following information carefully and answer the questions given below it :*

(*i*) Eight doctors P, Q, R, S, T, U, V and W visit a charitable dispensary every day except on a holiday i.e. Monday.

(*ii*) Each doctor visits for one hour from Tuesday to Sunday except Saturday. The timings are 9 a.m. to 1 p.m. and 2 p.m. to 6 p.m.; 1 p.m. to 2 p.m. is lunch break.

(*iii*) On Saturday, it is open only in the morning i.e. 9 a.m. to 1 p.m. and each doctor visits for only half an hour.

(*iv*) No other doctor visits the dispensary before doctor Q and after doctor U.

(*v*) Doctor-W comes immediately after lunch break and is followed by R.

(*vi*) S comes in the same order as P in the afternoon session.

1. Doctor P visits in between which of the following pairs of doctors ?

(*a*) S and V (*b*) U and W (*c*) R and W
(*d*) R and U (*e*) None of these

2. At what time the visit of doctor R is over on Sunday?

(*a*) 1 p.m. (*b*) 3 p.m. (*c*) 4p.m.
(*d*) 5p.m. (*e*) None of these

3. At what time the visit of Doctor T would be over on Saturday?

(*a*) 10 a.m.
(*b*) 11 a.m.
(*c*) Either 10 a.m. or 11 a.m.
(*d*) Data inadequate
(*e*) None of these

4. If the lunch break and subsequent visiting hours are reduced by 15 minutes, at what time Doctor U is expected to attend the dispensary ?

(*a*) 3.15 p.m. (*b*) 4 p.m. (*c*) 4.15 p.m.
(*d*) 4.45 p.m. (*e*) None of these

Ans. First form sequence of visit using (*iv*), (*v*) and (*vi*).

From (*iv*),Q visits first and U visits last.

From (*v*),W visits first after break and is followed by R.

From (*vi*), P visits after break.

Hence sequence of visit after break becomes W R P U

Also, S has the same position in morning session as P in afternoon session. Thus, sequence of visit before break is Q, T/V, S, V/T.

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1. (*d*). P visits between R and U.

2. (*c*). Time of visit of W is 2 p.m. to 3 p.m., Time of visit of R is 3 p.m. to 4 p.m. So, the visit of doctor R is over at 4 p.m.

3. (*c*). T visits either second or fourth. So, the time of visit on Saturday will be either 9.30 a.m. or 10.30 a.m. Thus, T's visit will be over at either 10 a.m. or 11 a.m.

4. (*b*). As mentioned, lunch break will be over and doctor W will visit at 1.45 p.m., doctor R will visit at 2.30 p.m., doctor P will visit at 3.15 p.m. and U will visit at 4 p.m.

5. Selection based on given conditions

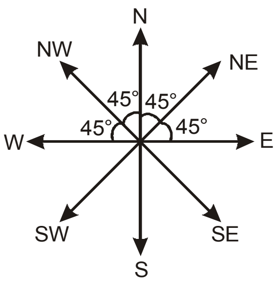
In this type of questions, a few essential criteria for selection of a group of items are given and we have to keep these conditions in mind and make the required selection as per the directions given in each question.

6. Jumbled problems

In this type of questions, some mixed clues regarding three or more qualities of given items or persons is given and we are required to analyse this mixed information with respect to different qualities and classify the items accordingly.

VII. DIRECTION SENSE

In this type of test, the questions consist of a sort of direction puzzle. A successive follow-up of directions is formulated and we are required to ascertain final direction or the distance between two points. The test is meant to judge ability to trace and follow correctly and sense the direction correctly.



Above figure shows four main directions : North N, South S, East E, West W

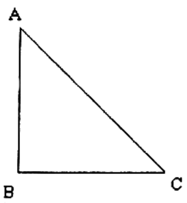
Four cardinal directions : North East NE, North West NW, South East SE, South West SW

Pythagoras theorem

The theorem is applicable only in right angle triangles, in which from theorem one angle is of 90°

$$AC^2 = AB^2 + BC^2$$

where, AC is Hypotense
AB is Perpendicular (Height)
BC is Base



Shortcut method

1. If 2 opposite turns are coming together, they will cancel each other

   Example. left, right and right, left.

2. If similar turns are coming together they will opposite the earlier direction.

   Example. if a person is going in North and taking two left turns it means now person is moving in opposite to North i.e. South.

3. Start from either left side or right side

VIII. LOGICAL VENN DIAGRAMS

This type of questions aim at analysing ability to relate a certain given group of items and illustrate it diagrammatically.

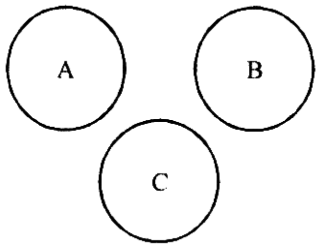
1. LOGICAL DIAGRAMS.

In this type a few types of Venn diagrams with their implications made clear. Let a group of three items are given.

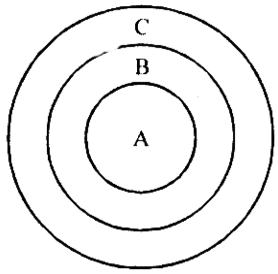
1. If items evidently belong to three different groups, then Venn diagram representing it would be asC shown alongside.

Example. Doctors, Engineers, Lawyers

These three items bear no relationship to each other. So, they are represented by 3 disjoint figures as shown:



2. If one item belongs to the class of the second and second belongs to the class of third, then representation is in the form of three concentric circles, as shown below:



Example. Seconds, Minutes, Hours

Seconds are a part of minutes and minutes are a part of hours. So, Venn diagram would be as shown in the figure with circle A representing Seconds, circle B representing Minutes and circle C representing Hours.

3. If two separate items belong to the class of the third, they are represented by two disjoint circles inside a bigger circle as shown below

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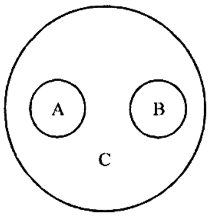
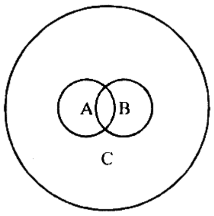


Example. Table, Chair, Furniture

Table and chair are separate items but both are items of furniture. So, they would be represented as in the figure with circle A representing Table, circle B representing Chair and circle C representing Furniture.

4. If two separate things having something common belongs to the class of third, they are represented by two joint figures inside one bigger figure is shown below.

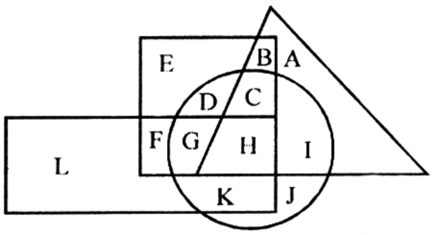


In this type of questions, generally a Venn diagram is given.

2. VENN DIAGRAMS

Each geometrical figure in the diagram represents a certain class. The candidate is required to study and analyse the figure carefully and then answer certain questions regarding the given data.

Example. In the following diagram, square represents girls, the circle represents tall persons, the triangle is for tennis players and the rectangle stands for the swimmers.



On the basis of the above diagram, answer the following questions :

1. Which letter represents tall girls who are swimmers but don't play tennis ?

(*a*) C (*b*) D (*c*) G (*d*) H (*e*) B

Ans. (*c*)

Tall girls, who are swimmers are represented by the region common to the square, circle and the rectangle i.e., G and H. But, according to the given conditions, the girls shouldn't be tennis players. So, required region should not be a part of the triangle i.e., H should be excluded. Thus, region representing the persons satisfying given conditions is G.

2. Which letter represents girls who are swimmers, play tennis but are not tall?

(*a*) B (*b*) E (*c*) H (*d*) F

(*e*) None of these

Ans. (*e*)

Girls who are swimmers and play tennis are represented by the region common to the square, triangle and rectangle i.e., H. But, it is given that the girls shouldn't be tall. So, required region should not be a part of the circle. Since H is a part of the circle. So none of these is correct

IX. ALPHABETICAL QUIBBLE

In this type of questions, we are given alphabets from A to Z. The position of a letter is given in the form of a puzzle and we are required to find this letter. However, sometimes a random letter series is given and we are required to find out how many times a letter satisfying the conditions specified in the question occurs.

Example. *Following question is based on the following alphabet series.*

ABCDEFGHIJKLMNOPQRSTUVWXYZ

Which letter is exactly midway between H and S in the given alphabet ?

(*a*) L (*b*) M (*c*) N (*d*) O

(*e*) No such letter

Ans. (*e*)

From given series we get that M & N are in midway between H and S. So, exactly midway is no such letter.

X. NUMBER, RANKING & TIME SEQUENCE

1. Number Test

In this type of questions, generally you are given a long series of numbers and we are required to find out how many times a number satisfying the conditions, specified in the question, occurs.

Example. How many 5's are there in the following sequence which are immediately followed by 3 but not immediately preceded by 7 ?

8953253855687335775365335738

(*a*) One (*b*) Two (*c*) Three

(*d*) Four (*e*) More than four

Ans. (*c*)

A number which comes after a given number is said to follow it while the one which comes before the given number precedes it. Thus, numbers satisfying the given conditions, can be shown as follows. Hence there are three such numbers.

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2. **Ranking Test**

In this type of test, generally ranks of a person both from top and bottom are mentioned and total number of persons is asked. However, sometimes this question is put in the form of a puzzle of interchanging seats by two persons.

Example. Anil ranked ninth from the top and thirty eighth from the bottom in a class. How many students are there in the class?

(*a*) 45 (*b*) 46 (*c*) 47

(*d*) 48 (*e*) 49

Ans. (*b*)

Whole class consists of :

(*i*) 8 students who have a rank higher than Anil

(*ii*) Anil; and

(*iii*) 37 students who have rank lower than Anil i.e.,(8 + 1 + 37) = 46 students.

3. **Time Sequence Test**

Example. Raman remembers that his brother's birthday is after fifteenth but before eighteenth of February whereas his sister Kamla remembers that her brother's birthday is after sixteenth but before nineteenth of February. On which day in February is Raman's brother's birthday ?

(*a*) 16th (*b*) 17th (*c*) 18th

(*d*) 19th (*e*) None of these

Ans. (*b*)

According to Raman, brother's birthday is on one of the days among 16th and 17th February

According to Kamla, the brother's birthday is on one of the days among 17th and 18th February. Hence Raman's brother's birthday is on the day common to both the above groups i.e., 17th February.

XI. MATHEMATICAL OPERATIONS

This type questions are based on simple mathematical operations.

Here, four fundamental operations :

Addition, subtraction, multiplication and division

and statements such as

'less than', 'greater than', 'equal to', 'not equal to', etc.

are represented by symbols, different from the usual ones.

The questions involving these operations are set using artificial symbols. The candidate has to substitute the real signs and solve the questions accordingly, to get the answer.

TYPE OF PROBLEMS.

1. **Solving by Substitution**

In this type of questions, we are provided with substitutes for various mathematical symbols, followed by a question involving calculation of an expression or choosing the correct/incorrect equation. We are required to put in the real signs in the given equation and then solve the questions as required.

While solving a mathematical expression, proceed according to the rule BODMAS

i.e., Brackets, Of, Division, Multiplication, Addition, Subtraction.

2. **Deriving appropriate Conclusions**

In such questions, the usual algebraic signs and mathematical symbols may be given different meanings or may even be replaced by new symbols. For instance (÷) may denote addition, (×) may denote subtraction (<) may denote division and so on. The simplest way to solve such an equation is to rewirite the problem replacing the given symbols with the signs they are supposed to stand for. For instance, if :

+ means – ÷ means +

'V' means < – means >

× means = > means ×

&, < means ÷

Then equation, 65 < 5 ÷ 2.5 — 15 will be rewritten as 65 ÷ 5 + 2.5 > 15, and problem may be solved.

In yet another type of questions included under this system, even the digits (instead of the symbols)

0, 1, 2, ... 8, 9, may be replaced by alphabets and the candidate may then be asked to perform calculations and solve the given problem. For instance, codes may be used as (a = 1, b = 2, c = 3 etc.) or in reverse (i.e., z = 1, y = 2, x = 3 and so on).

Following question employs such a code.

Q. If y × w = s and u — v = z, what is r/x ?

(*a*) y (*b*) z (*c*) s (*d*) x

Solution : If we count backwards from z, we get y as the 2nd letter, w as the 4th letter, and s as the 8th letter. In this order u is the 6th, v is the 5th and z the 1st letter of the code.

So, 2 × 4 = 8 and 6 – 5 = 1

Hence, r = 9 and x = 3.

So $\frac{r}{x}=\frac{9}{3}$ = 3. So the answer is (*d*) x.

In certain cases, arithmetical operations may not be defined. Consider the following question.

Q. If 8 + 12 = 2, and 7 + 13 = 3, then what is the value of 10 + 18 ?

(*a*) 9 (*b*) 6 (*c*) 4 (*d*) 7

Solution : $\frac{12-8}{2}$ = 2, and $\frac{13-7}{2}$ = 3. So, code employed here is special.

In x + y, '+' stands for $\frac{y-x}{2}$

$\therefore$ 10 + 18 = $\frac{18-10}{2}$ = 4, so the answer is (c).

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XII. LOGICAL SEQUENCE OF WORDS

In this type of questions, a group of words is given. The candidate is required to arrange these words in a meaningful order such as the sequence of occurrence of events, sequence from a part to the whole, sequence of increasing/decreasing size, value, intensity etc., and then choose the correct sequence accordingly.

Examples.

1. *Arrange the following in a meaningful sequence :*

1. Consultation 2. Illness
3. Doctor 4. Treatment
5. Recovery

(*a*) 2, 3, 1, 4, 5 (*b*) 2, 3, 4, 1, 5
(*c*) 4, 3, 1, 2, 5 (*d*) 5, 1, 4, 3, 2
(*e*) 2, 3, 4, 5, 1

Ans (*a*)

Illness occurs first. One then goes to the doctor and after consultation with him, undergoes treatment to finally attain recovery. Thus, the correct order is 2, 3, 1, 4, 5.

2. *Arrange the following in a logical order :*

1. Euphoria 2. Happiness
3. Ambivalence 4. Ecstasy
5. Pleasure

(*a*) 4, 1, 3, 2, 5 (*b*) 3, 2, 5, 1, 4
(*c*) 2, 1, 3, 4, 5 (*d*) 1, 4, 2, 5, 3
(*e*) 5, 4, 3, 2, 1

Ans. (*b*)

All the given words stand for 'Joy', but the intensity increases in the order — Ambivalence, Happiness, Pleasure, Euphoria, Ecstasy. Thus, the correct order is 3, 2, 5, 1, 4.

XIII. DECISION MAKING

It with questions in which we have to decide the course of action to be taken upon a candidate who has applied for a vacancy or allotment or membership to an institution, keeping in mind the essential requisites and the data given for the candidate.

TYPE I

In such type of test, a vacancy is declared. The necessary qualifications required in the candidates coming up to fill the vacancy are provided and merits of the candidates mentioned. The decision about each candidate has to be made from amongst the five choices named (*a*), (*b*), (*c*), (*d*), (*e*) which state the courses of action to be taken as per the candidate's potentials.

Example. *Read the following information to answer the given questions*

Following are the criteria for selecting & marketing officer by a company. The candidate must :

(1) be a graduate with at least 50% marks.
(2) have secured at least 40% marks in the written test.
(3) not be less than 24 years and more than 29 years as on 10th October, 2007.
(4) should have work experience of at least two years as an officer. However, if a candidate:
(5) fulfils all other criteria except at (4) above but has a diploma in Marketing Management, his/her case is to be referred to General Manager, Marketing.
(6) fulfils all other criteria except at (3) above but has worked as Marketing Officer at least for three years, his/her case is to be referred to Director, Marketing.

Based on the above criteria and information given in each of the following questions, we have to take the decision in regard to each case. We are not to assume anything. These cases are given as on 10th October, 2007.

Mark answer (*a*) if candidate is to be appointed;

Mark answer (*b*) if the candidate is not to be appointed;

Mark answer (*c*) if the data given are not sufficient to take decision;

Mark answer (*d*) if to be referred to General Manager - Marketing; and

Mark answer (*e*) if to be referred to Director – Marketing.

1. Amit, born on 5th June, 1983, has done his post-graduation in Marketing Management with first class. He has secured 50% marks in the written test. He has been working in an organisation as a Marketing Officer for the last four years.

2. Rohit has been working in an organisation as Officer for the last ten years. His date of birth is 17th February, 1964. He has secured 60% marks in the degree examination and 40% marks in the written test.

3. Manju is a first class graduate and has done a diploma in Marketing Management. She has secured 50% marks in the written test. She was 23 years old as on 5th September, 2006.

4. Nitin was born on 25th August, 1985. He has secured 60% and 50% marks in graduation and in the written test, respectively. He has been working in an organisation as Officer for the last four years.

5 Suman is a graduate with first class and has secured 60% marks in the written test. She has been working as an Officer for the last three years. She was born on 20th May, 1982.

Ans.

1. (*a*). All conditions of eligibility are satisfied.
2. (*e*). Condition (3) is violated but it can be waived by condition (6).
3. (*d*). The candidate satisfies all conditions except (4). But he fulfils condition (5) so that (4) is waived.
4. (*e*). The candidate satisfies condition (2) instead of condition (3).
5. (*a*). All conditions of eligibility are satisfied.

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TYPE II. ASSERTION AND REASON

This test is meant to judge the candidate's knowledge and his ability to reason out correctly. In this test, two statements referred to as the Assertion (A) and Reason (R) respectively are provided. Five alternative comments on these are given and the correct one is to be chosen.

Example. *For the Assertions* (*A*) *and Reasons* (*R*) *below, choose the correct alternative from the following*:

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is NOT the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

(*e*) Both A and R are false.

1. **Assertion (A) :** Moon cannot be used as a satellite for communication.

Reason (R) : Moon does not move in the equatorial plane of the earth.

Ans. (*a*)

Since R explains A.

2. **Assertion (A) :** Salt is added to cook food at higher altitudes.

Reason (R): Temperature is lower at higher altitudes.

Ans. (*b*)

Since both statements A and R are separately true but R does not explain A.

TYPE III. SITUATION REACTION TEST

In this test, certain situation is described and the candidate is required to choose the most suitable reaction to the given situation from amongst the alternatives provided. This test judges reasoning power of the candidate and his ability to act correctly and promptly to a situation that may arise in emergency.

Example. While travelling in your car, certain persons stop you on the way asking you to take an injured child to the hospital. You would :

(*a*) ask them to leave your way and then drive away.

(*b*) ask them to first call the police.

(*c*) immediately take the child to hospital.

(*d*) get out of the car and ask some other person to help them.

(*e*) None of these

Ans. (*c*)

Situation described above demands that the person should immediately render the help asked for and take the child to the hospital.

Examples

1. While playing cricket in the school, suddenly when you hit the ball, it strikes your classmate on the forehead and blood starts oozing out. You would :

(*a*) run away from the field.

(*b*) start fighting with the boy why he came in the way.

(*c*) blame somebody else for the accident.

(*d*) take the boy to the first aid room.

(*e*) None of these

Ans. (*d*)

In the above situation, urgent need is to provide first aid to the boy so that the bleeding may stop.

2. You are visiting a place for the first time and are travelling in a bus. Suddenly you realise that the driver is taking the bus to a lonely place with no right intentions. You would :

(*a*) with the help of some other passengers, try to baffle the driver and take over the bus.

(*b*) sit and wait to face the repercussions.

(*c*) jump out of the running bus.

(*d*) console the worried passengers.

Ans. (*a*)

When a wrong doing is expected, immediate action to prevent it is the need.

3. You have gone to enjoy a Diwali Mela organised by a club. Suddenly you come across a lost child crying desperately.

You would :

(*a*) neglect and walk away.

(*b*) ask the child to find his parents.

(*c*) ask him to stop crying and wait patiently for his parents.

(*d*) contact with the club authorities and make an announcement for the parents.

Ans. (*d*)

Immediate need is to find the child's parents and for this, best way is to announce the child's name and appearance so that his parents might know where the child is.

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EXERCISE – I

MCQ TYPE QUESTIONS

1. Given two numbers x and y, define

Addition	A(x, y) = x + y
Subtraction	S (x, y) = x – y
Multiplication	M (x, y) = x, y
Division	D (x, y) = x/y

S[M(D(A(a, b), 2), D(A(a, b), 2)), M(D(S(a, b), 2), D(S(a, b),2))] is equal to

(*a*) $(a - b)^2$ (*b*) ab
(*c*) $(a + b)^2$ (*d*) None of these

Directions Q. 2 – 5 :

Kamal Babu came home just after judging a beauty contest where there were four semi-finalists, Ms. Andhra Pradesh, Ms. Uttar Pradesh, Ms. Maharashtra and Ms. West Bengal. His wife was very keen on knowing who the winner was and Kamal Babu replied immediately that it was the one wearing the yellow saree. When his wife asked for more details, he gave the following information:

* The four girls were wearing sarees of different colours (yellow, red, green, white) and the runner-up was wearing green.
* The four girls were sitting in a row, and Ms. West Bengal was not sitting at either end.
* There was only one runner-up and she was sitting next to Ms. Maharashtra.
* The girls wearing yellow and white sarees occupied the seats at either end.
* Ms. West Bengal was nither the winner nor the runner-up.
* Ms. Maharashtra was wearing white.
* The winner and the runner-up were not sitting next to each other.
* The girl wearing the green saree was not Ms. Andhra Pradesh.

Answer the following questions based on the above information:

2. What was the colour of the saree that Ms. Andhra Pradesh was wearing?

(*a*) White (*b*) Yellow
(*c*) Red (*d*) Cannot be determined

3. Between which two was Ms.West Bengal sitting?

(*a*) Ms. Andhra Pradesh and Ms. Maharashtra
(*b*) Ms. Andhra Pradesh and Ms. Uttar Pradesh
(*c*) Ms. Uttar Pradesh and Maharashtra
(*d*) Cannot be deternmined

4. Who was wearing the red saree?

(*a*) Ms. Andhra Pradesh
(*b*) Ms. Uttar Pardesh
(*c*) Ms. West Bengal
(*d*) Cannot be determined

5. What was the colour of the saree that Ms. Uttar Pradesh was wearing?

(*a*) White (*b*) Green
(*c*) Red (*d*) Yellow

Directions Q. 6 – 7 :

A, B, C, D, E and F are a group of friends from a club. There are two housewives, one lecturer, one architect, one accountant and one lawyer in the group. There are two married couples in the group. The lawyer is married to D who is a housewife. No lady in the group is either an architect or an accountant C. The accountant, is married to F who is a lecturer. A is married to D and E is not a housewife.

6. What is E ?

(*a*) Lawyer (*b*) Architect
(*c*) Lecturer (*d*) Accountant

7. How many members of the group are males?

(*a*) 2 (*b*) 3
(*c*) 4 (*d*) none of these

Directions Q. 8 – 9 :

Seven university cricket players are to be honoured at a special luncheon. the players will be seated on the dais along one side of a single rectangular table.

A and G have to leave for the lunch early and must be seated at the extreme right end of the table which is closest to the exit.

B will receive the Man of the Match award and must be in the center chair.

C and D, who are bitter rivals for the position of wicket keeper, dislike each other and should be seated as far apart as possible.

E and F are best friends and want to sit together.

8. Which of the following may not be seated at either end of the table ?

(*a*) C (*b*) D
(*c*) G (*d*) F

9. Which of the following pairs may not be seated together ?

(*a*) E & A (*b*) B & D
(*c*) C & F (*d*) G & D

Directions Q. 10 –13 :

A study was conducted to ascertain the relative importance that employees in five different countries assigned to five different traits in their Chief Executive Officers. The traits were compassion (C), decisiveness (D), negotiation skills (N), public visibility (P), and vision (V). The level of dissimilarity between two countries is the maximum difference in the ranks allotted by the two countries to any of the five traits. The following table indicates the rank order of the five traits for each country.

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Country					
RANK	INDIA	CHINA	JAPAN	MALAYSIA	THAILAND
1	C	N	D	V	V
2	P	C	N	D	C
3	N	P	C	P	N
4	V	D	V	C	P
5	D	V	P	N	D

10. Which of the following pairs of countries are most dissimilar?
(*a*) China and Japan
(*b*) India and China
(*c*) Malaysia and Japan
(*d*) Thailand and Japan

11. Which of the following countries is least dissimilar to India?
(*a*) China (*b*) Japan
(*c*) Malaysia (*d*) Thailand

12. Which amongst the following countries is most dissimilar to India?
(*a*) China (*b*) Japan
(*c*) Malaysia (*d*) Thailand

13. Three of the following four pairs of countries have identical levels of dissimilarity. Which pair is the odd one out?
(*a*) Malaysia and China
(*b*) China and Thailand
(*c*) Thailand and Japan
(*d*) Japan and Malaysia

Directions Q. 14 – 17 :

Four families decided to attend the marriage ceremony of one of their colleagues. One family has no kids, while others have at least one kid each. Each family with kids has at least one kid attending the marriage.

Given below is some information about the families, and who reached when to attend the marriage.

The family with 2 kids came just before the family with no kids.

Shanthi who does not have any kids reached just before Sridevi's family.

Sunil and his wife reached last with their only kid.

Anil is not the husband of Joya.

Anil and Raj are fathers.

Sridevi's and Anita's daughters go to the same school.

Joya came before Shanthi and met Anita when she reached the venue.

Raman stays the farthest from the venue.

Raj said his son could not come because of his exams.

14. Which woman arrived third ?
(*a*) Shanthi (*b*) Sridevi
(*c*) Anita (*d*) Joya

15. Name the correct pair of husband and wife ?
(*a*) Raj and Shanthi (*b*) Sunil and Sridevi
(*c*) Anil and Sridevi (*d*) Raj and Anita

16. Of the following pairs, whose daughters go to the same school ?
(*a*) Anil and Raman (*b*) Sunil and Raman
(*c*) Sunil and Anil (*d*) Raj and Anil

17. Whose family is known to have more than one kid for certain ?
(*a*) Raman's (*b*) Raj's (*c*) Anil's (*d*) Sunil's

Directions Q. 18 – 20 :

Answer the following questions based on the statements given below:

(*i*) There are three houses on each side of the road.
(*ii*) These six houses are labeled as P, Q, R, S, T and U.
(*iii*) The houses are of different colours, namely, Red, Blue, Green, Orange, Yellow and White.
(*iv*) The houses are of different heights.
(*v*) T, the tallest house, is exactly opposite to the Red coloured house.
(*vi*) The shortest house is exactly opposite to the Green coloured house.
(*vii*) U, the Orange coloured house, is located between P and S.
(*viii*) R, the Yellow coloured house, is exactly opposite to P.
(*ix*) Q, the Green coloured house, is exactly opposite to U.
(*x*) P, the White coloured house, is taller than R, but shorter than S and Q.

18. What is the colour of the house diagonally opposite to the Yellow coloured house?
(*a*) White (*b*) Blue
(*c*) Green (*d*) Red

19. Which is the second tallest house?
(*a*) P (*b*) S
(*c*) Q (*d*) cannot be determined

20. What is the colour of the tallest house?
(*a*) Red (*b*) Blue (*c*) Green (*d*) Yellow

NUMERICAL TYPE QUESTIONS

1. Given two numbers x and y, define

Addition	A(x, y) = x + y
Subtraction	S (x, y) = x – y
Multiplication	M (x, y) = x, y
Division	D (x, y) = x/y

If x = 2, y = 3 and z = 5, then value of M[A(S (x, y), M (y, z)), S (A(x, z), S (z, y))] is ______

Directions Q. 2 – 3 :

These questions are based on the following data

A, B, C, D are standing in a queue. It is known that C and D are not standing adjacent to each other and that B is not in the third place.

2. If A is not in the second place and B is in the first place, then which is the place for D is _______

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3. If A is not standing adjacent to B and D is the last in the queue, then the place for C is ______

Directions Q. 4 – 5 :

A boy is asked to put in a basket one mango when ordered 'One', one orange when ordered 'Two', one apple when ordered 'Three' and is asked to take out from the basket one mango and an orange when ordered 'Four'. A sequence of orders is given as
1 2 3 3 2 1 4 2 3 1 4 2 2 3 3 1 4 1 1 3 2 3 4

4. The ______total organges were in the basket at the end of the above sequence.
5. The ______total fruits will be in the basket at the end of the above order sequence.

Directions Q. 6 – 7

Each of the 11 letters A, H, I, M, O, T, U, V, W, X and Z appears same when looked at in a mirror. They are called symmetric letters. Other letters in the alphabet are asymmetric letters.

6. The______ number of four-letter computer passwords can be formed using the symmetric letters (no repetition allowed)
7. The ______ number of three-letter computer passwords can be formed (no repetition allowed) with at least one symmetric letter.

Directions Q. 8 – 9 :

A string of three English letters is formed as per the following rules:

(*a*) The first letter is any vowel
(*b*) The second letter is m, n or p.
(*c*) If the second letter is m then the third letter is any vowel which is different from the first letter
(*d*) If the second letter is n then the third letter is *e* or *u*
(*e*) If the second letter is p then the third letter is the same as the first letter.

8. The ______ strings of letters can possible be formed using the above rules.
9. The ______ strings of letters can possibly be formed using the above rules such that the third letter of the string is e.

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. Mr. Vivek walks 6 meters North-East, then turns and walks 6 meters South- East, both at 60 degrees to East. He further moves 2 meters South and 4 meters West. What is the straight distance in meters between the point he started from and the point he finally reached?

(*a*) $2\sqrt{2}$ (*b*) 2
(*c*) $\sqrt{2}$ (*d*) $1/\sqrt{2}$

2. There are 16 teachers who can teach Thermodynamics (TD), 11 who can teach Electrical Sciences (ES), and 5 who can teach both TD and Engineering Mechanics (EM). There are a total of 40 teachers, 6 cannot teach any of the three subjects, i.e. EM, ES or TD. 6 can teach only ES. 4 can teach all three subjects, i.e. EM, ES and TD. 4 can teach ES and TD. How many can teach both ES and EM but not TD?

(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

3. The given question is followed by two statements: select the most appropriate option that solves the question

Capacity of a solution tank A is 70% of the capacity of tank B. How many gallons of solution are in tank A and tank B?

Statements:

I. Tank A is 80% full and tank B is 40% full
II. Tank A if full contains 14,000 gallons of solution

(*a*) Statement I alone is sufficient
(*b*) Statement II alone is sufficient
(*c*) Either statement I or II alone is sufficient
(*d*) Both the statements I and II together are sufficient

4. Operators $\square, \diamond$ and $\rightarrow$ are defined by

$$a \square b = \frac{a-b}{a+b};\ a \diamond b = \frac{a+b}{a-b};\ a \rightarrow b = ab.$$

Find the value $(66 \square 6) \rightarrow (66 \diamond 6)$

(*a*) -2 (*b*) -1
(*c*) 1 (*d*) 2

5. Humpty Dumpty sits on a wall every day while having lunch. The wall sometimes breaks. A person sitting on the wall falls if the wall breaks.

Which one of the statements below is logically valid and can be inferred from the above sentences?

(*a*) Humpty Dumpty always falls while having lunch
(*b*) Humpty Dumpty does not fall sometimes while having lunch

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(*c*) Humpty Dumpty never falls during dinner

(*d*) When Humpty Dumpty does not sit on the wall, the wall does not break

6. If ROAD is written as URDG, then SWAN should be written as :

(*a*) VXDQ (*b*) VZDQ

(*c*) VZDP (*d*) UXDQ

7. Select the pair that best expresses a relationship similar to that expressed in the pair :

Children : Pediatrician

(*a*) Adult : Orthopaedist

(*b*) Females : Gynaecologist

(*c*) Kidney : Nephrologist

(*d*) Skin : Dermatologist

8. The head of a newly formed government desires to appoint five of the six selected members P,Q,R,S,T and U to portfolios of Home, Power, Defense, Telecom, and Finance. U does not want any portfolio if S gets one of the five. R wants either Home or Finance or no portfolio. Q says that if S gets either Power or Telecom, then she must get the other one. T insists on a portfolio if P gets one.

Which is the valid distribution of portfolios?

(*a*) P-Home, Q-Power, R-Defense, S-Telecom, T-Finance

(*b*) R-Home, S-Power, P-Defense, Q-Telecom, T-Finance

(*c*) P-Home, Q-Power, T-Defense, S-Telecom, U-Finance

(*d*) Q-Home, U-Power, T-Defense, R-Telecom, P-Finance

9. Most experts feel that in spite of possessing all the technical skills required to be a batsman of the highest order, he is unlikely to be so due to lack of requisite temperament. He was guilty of throwing away his wicket several times after working hard to lay a strong foundation. His critics pointed out that until he addressed this problem success at the highest level will continue to elude him.

Which of the statement (s) below is/are logically valid and can be inferred from the above passage?

(*i*) He was already a successful batsman at the highest level

(*ii*) He has to improve his temperament in order to become a great batsman

(*iii*) He failed to make many of his good starts count

(*iv*) Improving his technical skills will guarantee success

(*a*) (*iii*) and (*iv*)

(*b*) (*ii*) and (*iii*)

(*c*) (*i*), (*ii*) and (*iii*)

(*d*) (*ii*) only

10. Alexander turned his attention towards India, since he had conquered Persia.

Which one of the statements below is logically valid and can inferred from the above sentence?

(*a*) Alexander would not have turned his attention towards India had he not conquered Persia.

(*b*) Alexander was not ready to rest on his laurels, and wanted to march to India

(*c*) Alexander was completely in control of his army and could command it to move towards India

(*d*) Since Alexander's kingdom extended to Indian borders after the conquest of Persia, he was keen to move further

11. Tanya is older than Enc.

Cliff is older than Tanya.

Eric is older than Cliff.

If the first two statements are true, then the third statement is

(*a*) True (*b*) False

(*c*) Uncertain (*d*) Data insufficient

12. Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows:

Statements:

I. No manager is a leader.

II. All leaders are executive.

Conclusions:

I. No manager is a executive.

II. All executive is a manager.

(*a*) Only conclusion I follows.

(*b*) Only conclusion II follows.

(*c*) Neither conclusion I nor II follows.

(*d*) Both conclusion I and II follow.

13. Find the missing sequence in the letter series below :

A, CD, GHI, _?___, UVWXY

(*a*) LMN (*b*) MNO

(*c*) MNOP (*d*) NOPQ

14. Ms. X will be in Bagdogra from 1/5/14 to 20/5/14 and from 22/5/14 to 31/5/14. On the morning of 21/5/14, she will reach Kochi via Mumbai.

Which one of the statements below is logically valid and can be inferred from the above sentences?

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(*a*) Ms. X will be in Kochi for only one day only in May
(*b*) Ms. X will be in Kochi for only one day in May
(*c*) Ms. X will be only in Kochi for one day in May
(*d*) Only Ms. X will be in Kochi for one day in May

15. Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows.

Statement:

I. All film stars are playback singers.
II. All film directors are film stars.

Conclusions:

I. All film directors are playback singers.
II. Some film stars are film directors.

(*a*) Only conclusion I follows
(*b*) Only conclusion II follows
(*c*) Neither conclusion I nor II follows
(*d*) Both conclusions I and II follow

16. Lamenting the gradual sidelining of the arts in school curricula, a group of prominent artists wrote to the Chief Minister last year, asking him to allocate more funds to support arts education in schools. However, no such increase has been announced in this year's Budget. The artists expressed their deep anguish at their request not being approved, but many of them remain optimistic about funding in the future.

Which of the statement(*s*) below is/are logically valid and can be inferred from the above statements?

i. The artists expected funding for the arts to increase this year.
ii. The Chief Minister was receptive to the idea of increasing funding for the arts.
iii. The Chief Minister is a prominent artist.
iv. Schools are giving less importance to arts education nowadays.

(*a*) *iii* and *iv*
(*b*) *i* and *iv*
(*c*) *i*, *ii* and *iv*
(*d*) *i* and *iii*

17. Four branches of a company are located at M, N, O, and P. M is north of N at a distance of 4 km; P is south of O at a distance of 2 km; N is southeast of O by 1 km. What is the distance between M and P in km?

(*a*) 5.34 (*b*) 6.74
(*c*) 28.5 (*d*) 45.49

18. The given statement is followed by some courses of action. Assuming the statement to be true, decide the correct option.

Statement:

There has been a significant drop in the water level in the lakes supplying water to the city.

Course of action:

I. The water supply authority should impose a partial cut in supply to tackle the situation.
II. The government should appeal to all the residents through mass media for minimal use of water.
III. The government should ban the water supply in lower areas.

(*a*) Statements I and II follow.
(*b*) Statements I and III follow.
(*c*) Statements II and III follow.
(*d*) All statements follow.

2014

19. In a group of four children, Som is younger to Riaz. Shiv is elder to Ansu. Ansu is youngest in the group. Which of the following statements is/are required to find the eldest child in the group?

Statements

1. Shiv is younger to Riaz.
2. Shiv is elder to Som.

(*a*) Statement 1 by itself determines the eldest child.
(*b*) Statement 2 by itself determines the eldest child.
(*c*) Statements 1 and 2 are both required to determine the eldest child.
(*d*) Statements 1 and 2 are not sufficient to determine the eldest child.

20. X is 1 km northeast of Y. Y is 1 km southeast of Z. W is 1 km west of Z. P is 1 km south of W. Q is 1 km east of P. What is the distance between X and Q in km?

(*a*) 1 (*b*) $\sqrt{2}$
(*c*) $\sqrt{3}$ (*d*) 2

21. Rajan was not happy that Sajan decided to do the project on his own. On observing his unhappiness, Sajan explained to Rajan that he preferred to work independently.

Which one of the statements below is logically valid and can be inferred from the above sentences?

(*a*) Rajan has decided to work only in a group.
(*b*) Rajan and Sajan were formed into a group against their wishes.
(*c*) Sajan had decided to give in to Rajan's request to work with him.
(*d*) Rajan had believed that Sajan and he would be working together.

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22. Find the odd one in the following group: ALRVX, EPVZB, ITZDF, OYEIK

(*a*) ALRVX (*b*) EPVZB
(*c*) ITZDF (*d*) OYEIK

23. Anuj, Bhola, Chandan, Dilip, Eswar and Faisal live on different floors in a six-storeyed building (the ground floor is numbered 1, the floor above it 2, and so on). Anuj lives on an even-numbered floor. Bhola does not live on an odd numbered floor. Chandan does not live on any of the floors below Faisal's floor. Dilip does not live on floor number 2. Eswar does not live on a floor immediately above or immediately below Bhola. Faisal lives three floors above Dilip. Which of the following floor-person combinations is correct?

	Anuj	*Bhola*	*Chandan*	*Dilip*	*Eswar*	*Faisal*
(*a*)	6	2	5	1	3	4
(*b*)	2	6	5	1	3	4
(*c*)	4	2	6	3	1	5
(*d*)	2	4	6	1	3	5

24. Find the next term in the sequence: 13M, 17Q, 19S, ________

(*a*) 21W (*b*) 21V
(*c*) 23W (*d*) 23V

25. If 'KCLFTSB' stands for 'best of luck' and 'SHSWDG' stands for 'good wishes', which of the following indicates 'ace the exam'?

(*a*) MCHTX
(*b*) MXHTC
(*c*) XMHCT
(*d*) XMHTC

26. "India is a country of rich heritage and cultural diversity."

Which one of the following facts best supports the claim made in the above sentence?

(*a*) India is a union of 28 states and 7 union territories.
(*b*) India has a population of over 1.1 billion.
(*c*) India is home to 22 official languages and thousands of dialects.
(*d*) The Indian cricket team draws players from over ten states.

27. In which of the following options will the expression P < M be definitely true?

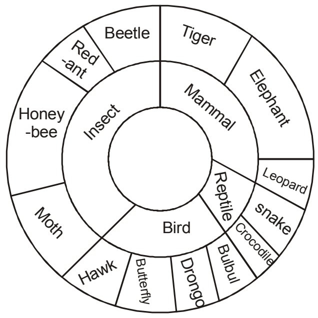
(*a*) M < R > P > S (*b*) M > S < P < F
(*c*) Q < M < F = P (*d*) P = A < R < M

28. Find the next term in the sequence:

7G, 11K, 13M, ________

(*a*) 15Q (*b*) 17Q
(*c*) 15P (*d*) 17P

29. The multi-level hierarchical pie chart shows the population of animals in a reserve forest. The correct conclusions from this information are:



(*i*) Butterflies are birds
(*ii*) There are more tigers in this forest than red ants
(*iii*) All reptiles in this forest are either snakes or crocodiles
(*iv*) Elephants are the largest mammals in this forest

(*a*) (i) and (*ii*) only
(*b*) (*i*), (*ii*), (*iii*) and (*iv*)
(*c*) (*i*), (*iii*) and (*iv*) only
(*d*) (*i*), (*ii*) and (*iii*) only

30. Find the odd one in the following group

Q,W,Z,B B,H,K,M W,C,G,J M,S,V,X

(*a*) Q, W, Z, B (*b*) B, H, K, M
(*c*) W, C, G, J (*d*) M, S, V, X

31. Lights of four colors (red, blue, green, yellow) are hung on a ladder. On every step of the ladder there are two lights. If one of the lights is red, the other light on that step will always be blue. If one of the lights on a step is green, the other light on that step will always be yellow. Which of the following statements is not necessarily correct?

(*a*) The number of red lights is equal to the number of blue lights
(*b*) The number of green lights is equal to the number of yellow lights
(*c*) The sum of the red and green lights is equal to the sum of the yellow and blue lights
(*d*) The sum of the red and blue lights is equal to the sum of the green and yellow lights

32. Read the statements:

All women are entrepreneurs.

Some women are doctors.

Which of the following conclusions can be logically inferred from the above statements?

(*a*) All women are doctors
(*b*) All doctors are entrepreneurs
(*c*) All entrepreneurs are women
(*d*) Some entrepreneurs are doctors

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33. Find the odd one from the following group:
(*a*) W,E,K,O
(*b*) I,Q,W,A
(*c*) F,N,T,X,
(*d*) N,V,B,D

34. For submitting tax returns, all resident males with annual income below ₹ 10 lakh should fill up Form P and all resident females with income below ₹ 8 lakh should fill up Form Q. All people with incomes above ₹ 10 lakh should fill up Form R, except non residents with income above ₹ 15 lakhs, who should fill up Form S. All others should fill Form T. An example of a person who should fill Form T is
(*a*) a resident male with annual income ₹ 9 lakh
(*b*) a resident female with annual income ₹ 9 lakh
(*c*) a non-resident male with annual income ₹ 16 lakh
(*d*) a non-resident female with annual income ₹ 16 lakh

35. Which number does not belong in the series below?
2, 5, 10, 17, 26, 37, 50, 64
(*a*) 17 (*b*) 37
(*c*) 64 (*d*) 26

36. A dance programme is scheduled for 10.00 a.m. Some students are participating in the programme and they need to come an hour earlier than the start of the event. These students should be accompanied by a parent. Other students and parents should come in time for the programme. The instruction you think that is appropriate for this is
(*a*) Students should come at 9.00 a.m. and parents should come at 10.00 a.m.
(*b*) Participating students should come at 9.00 a.m. accompanied by a parent, and other parents and students should come by 10.00 a.m.
(*c*) Students who are not participating should come by 10.00 a.m. and they should not bring their parents. Participating students should come at 9.00 a.m.
(*d*) Participating students should come before 9.00 a.m. Parents who accompany them should come at 9.00 a.m. All others should come at 10.00 a.m.

37. The number of people diagnosed with dengue fever (contracted from the bite of a mosquito) in north India is twice the number diagnosed last year. Municipal authorities have concluded that measures to control the mosquito population have failed in this region.
Which one of the following statements, if true, does not contradict this conclusion?
(*a*) A high proportion of the affected population has returned from neighbouring countries where dengue is prevalent
(*b*) More cases of dengue are now reported because of an increase in the Municipal Office's administrative efficiency
(*c*) Many more cases of dengue are being diagnosed this year since the introduction of a new and effective diagnostic test
(*d*) The number of people with malarial fever (also contracted from mosquito bites) has increased this year

38. Geneticists say that they are very close to confirming the genetic roots of psychiatric illnesses such as depression and schizophrenia, and consequently, that doctors will be able to eradicate these diseases through early identification and gene therapy.
On which of the following assumptions does the statement above rely?
(*a*) Strategies are now available for eliminating psychiatric illnesses
(*b*) Certain psychiatric illnesses have a genetic basis
(*c*) All human diseases can be traced back to genes and how they are expressed
(*d*) In the future, genetics will become the only relevant field for identifying psychiatric illnesses

2013

39. Select the pair that best expresses a relationship similar to that expressed in the pair:
Medicine: Health
(*a*) Science : Experiment
(*b*) Wealth : Peace
(*c*) Education : Knowledge
(*d*) Money : Happiness

40. Abhishek is elder to Savar.
Savar is younger to Anshul.
Which of the given conclusions is logically valid and is inferred from the above statements?
(*a*) Abhishek is elder to Anshul
(*b*) Anshul is elder to Abhishek
(*c*) Abhishek and Anshul are of the same age
(*d*) No conclusion follows

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41. Complete the sentence:

Universalism is to particularism as diffuseness is to _____

(*a*) specificity (*b*) neutrality

(*c*) generality (*d*) adaptation

42. After several defeats in wars, Robert Bruce went in exile and wanted to commit suicide. Just before committing suicide, he came across a spider attempting tirelessly to have its net. Time and again, the spider failed but that did not deter it to retrain from making attempts. Such attempts by the spider made Bruce curious. Thus, Bruce started observing the near-impossible goal of the spider to have the net. Ultimately, the spider succeeded in having its net despite several failures. Such act of the spider encouraged Bruce not to commit suicide. And then, Bruce went back again and won many a battle, and the rest is history.

Which one of the following assertions is best supported by the above information?

(*a*) Failure is the pillar of success.

(*b*) Honesty is the best policy.

(*c*) Life begins and ends with adventures.

(*d*) No adversity justifies giving up hope.

43. **Statement:** You can always give me a ring whenever you need.

Which one of the following is the best inference from the above statement?

(*a*) Because I have a nice caller tune.

(*b*) Because I have a better telephone facility.

(*c*) Because a friend in need is a friend indeed.

(*d*) Because you need not pay towards the telephone bills when you give me a ring.

44. **Statement:** There were different streams of freedom movements in colonial India carried out by the moderates, liberals, radicals, socialists, and so on.

Which one of the following is the best inference from the above statement?

(*a*) The emergence of nationalism in colonial India led to our Independence.

(*b*) Nationalism in India emerged in the context of colonialism.

(*c*) Nationalism in India is homogeneous.

(*d*) Nationalism in India is heterogeneous.

45. Select the pair that best expresses a relationship similar to that expressed in the pair :

water : pipe : :

(*a*) cart : road

(*b*) electricity : wire

(*c*) sea : beach

(*d*) music: instrument

46. All professors are researchers

Some scientists are professors

Which of the given conclusions is logically valid and is inferred from the above arguments :

(*a*) All scientists are researchers

(*b*) All professors are scientists

(*c*) Some researchers are scientists

(*d*) No conclusion follows

2012

47. Wanted Temporary, Part-time persons for the post of Field Interviewer to conduct personal interviews to collect and collate economic data. Requirements: High School-pass, must be available for Day, Evening and Saturday work. Transportation paid, expenses reimbursed.

Which one of the following is the best inference from the above advertisement?

(*a*) Gender-discriminatory

(*b*) Xenophobic

(*c*) Not designed to make the post attractive

(*d*) Not gender-discriminatory

48. Given the sequence of terms, AD CG FK JP, the next term is

(*a*) OV (*b*) OW

(*c*) PV (*d*) PW

2011

49. A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of the 4th day he can clear all the orders. Alternatively, if he uses only 3 trucks, then all the orders are cleared at the end of the 10th day. What is the minimum number of trucks required so that there will be no pending order at the end of the 5th day?

(*a*) 4 (*b*) 5

(*c*) 6 (*d*) 7

50. Few school curricula include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting.

Based on the above passage which topic would not be included in a unit on bereavement?

(*a*) How to write a letter of condolence

(*b*) What emotional stages are passed through in the healing process

(*c*) What the leading causes of death are

(*d*) How to give support to a grieving friend

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51. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair :

Gladiator : Arena

(*a*) dancer : stage
(*b*) commuter : train
(*c*) teacher : classroom
(*d*) lawyer : courtroom

52. The horse has played a little known but very important role in the field of medicine. Horses were injected with toxins of diseases until their blood built up immunities. Then a serum was made from their blood. Serums to fight with diphtheria and tetanus were developed this way.

It can be inferred from the passage, that horses were

(*a*) given immunity of diseases
(*b*) generally quite immune to diseases
(*c*) given medicines to fight toxins
(*d*) given diphtheria and tetanus serums

2010

53. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.

Unemployed : Worker

(*a*) fallow : land (*b*) unaware : sleeper
(*c*) wit : jester (*d*) renovated : house

54. Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (i.e. brothers and sisters). All were born on 1st January. The age difference between any two successive siblings (that is born one after another) is less than 3 years. Given the following facts:

I. Hari's age + Gita age > Irfan's age + Saira's age
II. The age difference between Gita and Saira is 1 year. However, Gita is not the oldest and Saira is not the youngest.
III. There are no twins.

In what order were they born (oldest first)?

(*a*) HSIG (*b*) SGHI
(*c*) IGSH (*d*) IHSG

55. Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for their cause.

Which of the following statements best sums up the meaning of the above passage:

(*a*) Modern warfare has resulted in civil strife
(*b*) Chemical agents are useful in modern warfare
(*c*) Use of chemical agents in warfare would be undesirable
(*d*) People in military establishments like to use chemical agents in war

56. If 137 + 276 = 435 how much is 731 + 672?

(*a*) 534
(*b*) 1403
(*c*) 1623
(*d*) 1513

NUMERICAL TYPE QUESTIONS

2015

1. Fill in the missing value

6 5 4
7 4 7 2 1
1 9 2 8 1 2 1
4 1 5 2 3
3 ? 3

2014

2. The next term in the series 81, 54, 36, 24,.... is ________

3. Fill in the missing number in the series.

2 3 6 15 ___ 157.5 630

4. What is the next number in the series?

12 35 81 173 357 ____

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ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*b*) **2.** (*b*) **3.** (*b*) **4.** (*c*) **5.** (*b*) **6.** (*b*) **7.** (*b*) **8.** (*d*) **9.** (*a*) **10.** (*a*)
11. (*b*) **12.** (*d*) **13.** (*d*) **14.** (*a*) **15.** (*b*) **16.** (*c*) **17.** (*b*) **18.** (*d*) **19.** (*d*) **20.** (*b*)

Numerical Type Questions

1. 70 **2.** 105 **3.** 2 **4.** 2 **5.** 11 **6.** 7920 **7.** 12870 **8.** 35 **9.** 10

EXERCISE – II

MCQ Type Questions

1. (*a*) **2.** (*a*) **3.** (*d*) **4.** (*c*) **5.** (*b*) **6.** (*b*) **7.** (*b*) **8.** (*b*) **9.** (*b*) **10.** (*a*)
11. (*b*) **12.** (*c*) **13.** (*c*) **14.** (*b*) **15.** (*d*) **16.** (*b*) **17.** (*a*) **18.** (*a*) **19.** (*a*) **20.** (*c*)
21. (*d*) **22.** (*d*) **23.** (*b*) **24.** (*c*) **25.** (*b*) **26.** (*c*) **27.** (*d*) **28.** (*b*) **29.** (*d*) **30.** (*c*)
31. (*d*) **32.** (*d*) **33.** (*d*) **34.** (*b*) **35.** (*c*) **36.** (*b*) **37.** (*d*) **38.** (*b*) **39.** (*c*) **40.** (*d*)
41. (*a*) **42.** (*d*) **43.** (*c*) **44.** (*d*) **45.** (*b*) **46.** (*c*) **47.** (*d*) **48.** (*a*) **49.** (*c*) **50.** (*c*)
51. (*d*) **52.** (*b*) **53.** (*d*) **54.** (*b*) **55.** (*c*) **56.** (*c*)

Numerical Type Questions

1. (3) **2.** (16) **3.** (45) **4.** (725 to 725)

EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. S[M(D(A(a, b)2, D(A(a, b)2)), M(D(S(a,b)2)) D (s(a,b)2)]

$$= S\left[\left|M\frac{a+b}{2},\frac{a+b}{2}\right|\left|M\frac{a-b}{2};\frac{a-b}{2}\right|\right]$$

$$= \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 = ab$$

6 – 7.

A → Lawyer → Male
B → Housewife → Female
C → Accountant → Male
D → Housewife → Female
E → Architect → Male
F → Lecturer → Female
Married Couple : AD & CF

8—9. The seating arrangement is shown below:

Left Right

7	6	5	4	3	2	1
D/C	E/F	F/E	B	C/D	G/A	A/G

10 – 13.

Level of Dissimilarity is equal to Maximum difference in ranks alloted to any two countries on any of the five traits.

For example, dissimilarity between India and China.

Trait	India'sRank	China's Rank	Difference
C	1	2	1
P	2	3	1
N	3	1	2
V	4	5	1
D	5	4	1

From the above table, maximum difference is 2.
So dissimilarity level between India and China is 2.

10. Calculating the level of dissimilarity for India Vs remaining countries.
China : For the parameter N (3 – 1) = 2
Japan : For the parameter D (5 – 1) = 4
Malaysia : For the parameter C (4 – 1) = 3
Thailand : For the parameter V (4 – 1) = 3
Hence least level of dissimilarity is 2 which is for China.

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11. From above calculations, highest level of dissimilarity is 4 which is for Japan.

12. *The level of dissimilarity between*

China and Japan is for the parameter D
(4 – 1) = 3

India and China is for the parameter N
(3 – 1) = 2

Malaysia and Japan is for the parameter V
(4 – 1) = 3

Thailand and Japan is for the parameter D
(5 – 1) = 4

13. *The level of dissimilarity between*

Malaysia and China is for the parameter V
(5 – 1) = 4

China and Thailand is for the parameter V
(5 – 1) = 4

Thailand and Japan is for the parameter D
(5 – 1) = 4

Japan and Malaysia is for the parameter V
(4 – 1) = 3

14 to 17 :

The given set of questions is based on distribution and sequencing.

Firstly, let us put the names of women and men,

Women : Shanthi, Sridevi, Joya, Anita

Men : Sunil, Anil, Raj, Raman

It is given that(2 kids family) $\xleftarrow{\text{came before}}$ (No kid family)

Also Shanti (0 kids) $\xrightarrow{\text{came before}}$ Sridevi

As Sunil and his wife came with their only kid, hence Sunil cannot be the husband of Shanthi.

As Anil and Raj are fathers, hence neither of them is the husband of Shanthi. Hence Raman is the husband of Shanthi.

Also, Joya came before Shanthi and met Anita on the venue, this gives the following order of their reaching the venue. [Anita, Joya, Shanthi, Sridevi]

As Sunil reached last, Sunil is the husband of Sridevi. This means that Raj is Joya's husband and Anil is Anita's husband.

Following table gives the desired details

Order	Women	Men	No. of Kids
1	Anita	Anil	at least 1
2	Joya	Raj	2
3	Shanthi	Raman	0
4	Sridevi	Sunil	1

14. Shanthi arrived third.

15. Out of the given choices. Sunil and Sridevi is the correct pair.

16. The daughters of Anita and Sridevi go to the same school. Anita's husband is Anil and Sridevi's husband is Sunil. Hence, Sunil and Anil is the correct answer.

17. Raj's family consists of two kids for certain (Anil has at least one kid, hence could be having exactly one kid also).

18 to 20 :

From statements (vii), (viii) and (ix), we get

P	U	S
	Orange	
R	Q	
Yellow	Green	

Then, T is opposite S and the colour of S is red.

From (x), the colour of house P is white, hence the colour of house T is blue.

P	U	S
White	Orange	Red
R	Q	T
Yellow	Green	Blue

Now, from (v) and (x) , we get

T > S, Q > P > R

From (vi), U is the shortest.

i.e., the order of houses in the descending order of their heights is T, S/Q, Q/S, P, R, U.

18. R is yellow coloured house and the house diagonally opposite R is S. And the colour of house S is Red.

19. The second tallest house is either S or Q.

20. The tallest house is T and its colour is blue.

NUMERICAL TYPE QUESTIONS

1. $M[A(S(x, y), M(y, z)), S(A(x, z), S(z, y))]$

$= M[A((2 - 3), (3, \times 5)), S((2 + 5), (5 - 3))]$

$= M[A(-1, 15), S(7, 2)]$

$= M[14, 5] = 70.$

2. Since B is in 1, and C and D are not adjacent, the only places possible for them are 2 and 4.

3. Since B is not in the third place and A is not adjacent to B, and D is last in the queue. The position of A will be beside D and B will be at the other corner. C will be between B and A. Hence the positions will be BCAD. Place of C is 2, hence choice (*b*)

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The sitting position is as follows.

White	Green	Red	Yellow
Maharashtra	U.P.	West Bengal	A.P.

From the above table,

Ms. Andhra Pradesh $\longrightarrow$ Yellow

4. (1) (2) 3 3 2 1 4 (2) 3 (1) 4 (2) (2) 3 3 (1) 4 1 (1) (3) (2) 3 4

Total no. oranges were in basket = 2

5. Total fruits in the basket = 11

6. Total number of four letter password formed

$$= {}^{11}C_4 \times \angle 4 = 11 \times 10 \times 9 \times 8 = 7920$$

7. Required number of passwords

$$= {}^{26}C_3 \times \angle 3 - {}^{15}C_3 \times \angle 3$$

$$= 26 \times 25 \times 24 - 15 \times 14 \times 13 = 12870$$

8. $\boxed{}$ $\boxed{}$ $\boxed{}$

1st Letter 2nd letter 3rd letter

There are 5 vowels, *a*, *e*, *i*, *o*,, *u*,.

hence 1st letter can be any of the 5 vowels ;

$\therefore$ 5 different methods. ...(*i*)

The second letter can be any of the *m, n,* or *p*. But each has its own rule for the third letter. Hence, each case to be considered separately.

$\boxed{\text{1 Vowel}}$ $\boxed{\text{m}}$

In the above case; third letter is one of the remaining 4 vowels

Hence there are $5 \times 4 = 20$ ways of forming such a string. ...(*ii*)

If the second letter is *n*, the third is either *e* or *u*, *i.e.* 2 different ways.

$\therefore$ total of 5×2 ways = 10 ways ...(*iii*)

When 2nd position is filled by P, the third letter is same as the first letter. As the 1st letter is already selected, third can be filled only one way.

Hence, there are $5 \times 1 = 5$ ways ....(*iv*)

From equations (*ii*), (*iii*) and (*iv*),

Total number of strings that can be formed

$$= 20 + 10 + 5 = 35$$

9. Third letter find is '*e*'.

Once again considering each case of the second position being filled by *m, n* or *p,* we have

$\boxed{\text{1st position}}$ $\boxed{\text{n}}$ $\boxed{\text{e}}$

2nd position 3rd position

The 3rd position shall be different form vowel in first position, *i.e.* vowel for 1st position can be selected out of the vowels *a*, *i*, *o*, *u* only.

Hence 4 different ways ....(*i*)

When 2nd possible is filled by *n*, occupying 2nd position, does not create any restriction on the vowel is position 1.

$\boxed{\text{e}}$ $\boxed{\text{n}}$ $\boxed{}$

3rd 2nd 1st

Hence, 1st position can be occupied by any of the 5 vowels.

Hence, 5 different ways ....(*ii*)

When 2nd position is filled by *p*,

$\boxed{\text{e}}$ $\boxed{\text{p}}$ $\boxed{}$

3rd 2nd 1st

In this case, letters in 3rd position out 1st position shall be the same.

Hence, 1st position shall be filled by '*e*'.

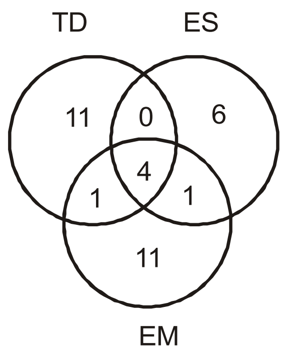
i.e. only one way of filling ....(*iii*)

Total number of possible strings = 4 + 5 + 1 = 10

EXERCISE – II

MCQ TYPE QUESTIONS

2.



3. Statement-I can be used to solve the question if capacity of both tanks is already known

Statement-II can be used if it is known what quantity of each tank is full/empty.

Therefore, by using both statements

Let capacity of tank B is *x*

$$\frac{70}{100}x = 14000$$

$$\Rightarrow \quad x = 20000 \text{ gallons}$$

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Solution in tank A = $\frac{80}{100} \times 14000$ = 11200 gallons

Solution in tank B = $\frac{40}{100} \times 20000$ = 8000 gallons

$\therefore$ Total solution = 11200 + 8000 = 19200 gallons

4. $66 \square 6 = \frac{66-6}{66+6} = \frac{60}{72} = \frac{5}{6}$

$66 \diamond 6 = \frac{66+6}{66-6} = \frac{72}{60} = \frac{6}{5}$

$(66 \square 6) \rightarrow (66 \diamond 6) = \frac{5}{6} \times \frac{6}{5} = 1$

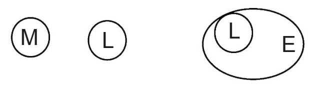
6. R + 3 = U, O + 3 = R,
A + 3 = D, D + 3 = G;
S + 3 = V, W + 3 = Z,
A + 3 = D, N + 3 = Q

7. Community of people : Doctor

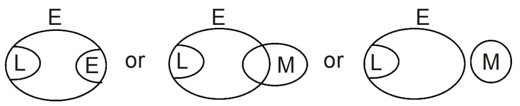
8. Since U does not want any portfolio, (*c*) and (*d*) are ruled out. R wants Home, or Finance or No portfolio, (*a*) is not valid.

Hence option (*b*) is correct.

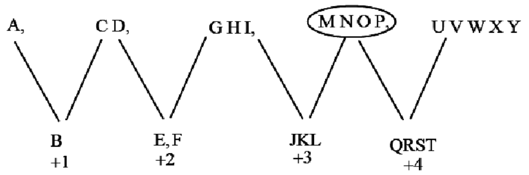
12. S + 1: S + 2 :



Therefore concluding diagram can be



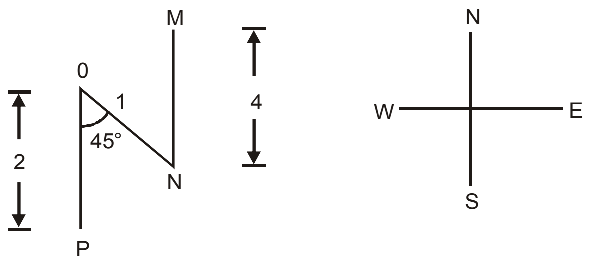
13.



14. Second sentence says that Ms. X reaches Kochi on 21/05/2014. Also she has to be in Bagdogora on 22/05/2014.

$\therefore$ She stays in Kochi for only one day in may.

17.



19. Riaz > Som ...(*i*)

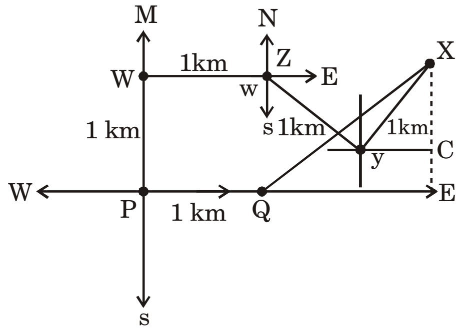
Shiv > Ansu ...(*ii*)

Ansu is youngest in the group

$\therefore$ From equations (*i*) and (*ii*)

$\therefore$ Riaz > Som > Shiv > Ansu

20.



Here XE = 1 km

$$QE = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$\therefore$ using Pythagoras theorem,

$$XQ = \sqrt{(QE)^2 + (XE)^2}$$

$$= \sqrt{2+1} = \sqrt{3} \text{ km}$$

$\therefore$ Statement I itself determines the eldest child which statement 2 is incorrect as Ansu is youngest in the group is not satisfied by statement 2

Hence option (*a*) is correct

22. ALRVX $\rightarrow$ only one vowel

EPVZB $\rightarrow$ only one vowel

ITZDF $\rightarrow$ only one vowel

OYEIK $\rightarrow$ three vowels

23. (*a*) Anuj: Even numbered floor (2, 4, 6)

(*b*) Bhola: Even numbered floor (2, 4, 6)

(*c*) Chandan lives on the floor above that of Faisal.

(*d*) Dilip: not on 2nd floor.

(*e*) Eswar: does not live immediately above or immediately below Bhola

From the options its clear, that only option (*b*) satisfies condition (*e*).

So, correct Ans is (*b*).

25. KCLFTSB: BST-Best, F-Of, LCK-Luck (Reverse order)

SHSWDG: GD-Good, WSHS-Wishes (Reverse order)

Similarly "ace the Exam'- C-Ace, T-The, XM-Exam

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26. Diversity is shown in terms of difference language.

29. It is not mentioned that elephant is the largest animal.

30. Q W Z B
17 23 26 2
6 3 2

B H K M
6 3 2

$\boxed{\text{W C G J} \quad 6 \; 4 \; 3}$

M S V X
6 3 2

33. W, E, K, O, I, Q, W, A, F, N, T, X, N, V, B, D,

+8 +6 +4 +8 +6 +4 +8 +6 +4 +8 +6 +2

−2 −2 −2 −2 −2 −2

Hence the odd one from the following group is N, V, B, D.

34. By reading the instructions, it is clear that person who fills form T is a resident female with annual income of ₹ 9 Lakh.

37. Among given option, option (*d*) *i.e.* the number of people with malarial fever has increased this year do not contradic the conclusion.

48. +2 +3 +4 +5

AD , CG , PK , JP , <u>OV</u>

+3 +4 +5 +6

49. Let each truck carry 100 units.

$$2800 = 4n + e$$

where n = normal

$$3000 = 10n + e$$

where e = excess/pending

$$\therefore \quad n = \frac{100}{3}, \quad e = \frac{8000}{3}$$

$$5 \text{ days} \Rightarrow 500\,x = \frac{5100}{3} + \frac{8000}{3}$$

$$\Rightarrow \quad 500x = \frac{8500}{3} = 17$$

$$\Rightarrow \quad x > 5$$

Minimum possible = 6

50. The given passage clearly deals with how to deal with bereavement and grief and so after the tragedy occurs and not about precautions. Therefore, irrespective of the causes of death, a school student rarely gets into details of causes—which is beyond the scope of the context. Rest all are important in dealing with grief.

51. Given relationship is worker: workplace. A gladiator is

(*i*) a person, usually a professional combatant trained to entertain the public by engaging in mortal combat with another person or a wild.

(*ii*) A person engaged in a controversy or debate, especially in public.

52. From the passage it cannot be inferred that horses are given immunity as in option (*a*), since the aim is to develop medicine and in turn immunize humans. Option (*b*) is correct since it is given that horses develop immunity after some time. Refer "until their blood built up immunities". Even option(*c*) is invalid since medicine is not built till immunity is developed in the horses. Option (*d*) is incorrect since specific examples are cited to illustrate and this cannot capture the essence.

54. $H + G > I + S$ ...(1)

and $G - S = 1$ ...(2)

G is not oldest, S is not youngest

$\therefore$ $H + 1 > I$

Irfan older than Hari

Gita older than Sarita

From given option SGHI

55. Use of chemical agents in warfare would be undesirable

56.
$$\begin{array}{rccc} 137 - & \overline{001} & \overline{011} & \overline{111} \\ +276 - & 010 & 111 & 110 \\ \hline & 100 & 011 & 101 = 435 \end{array}$$

Octal number system

$$\begin{array}{rccc} \therefore \quad 731 - & 111 & 011 & 001 \\ 672 - & 110 & 111 & 110 \\ \hline & \underline{100} & 010 & 011 \end{array}$$

Hence 1623

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NUMERICAL TYPE QUESTIONS

1. Middle number is the average of number on both sides

$\therefore$ Average of 3 and 3 is $\frac{3+3}{2} = \frac{6}{2} = 3$

2. $81 - 54 = 27;\ 27 \times \frac{2}{3} = 18$

$54 - 36 = 18;\ 18 \times \frac{2}{3} = 12$

$36 - 24 = 12;\ 12 \times \frac{2}{3} = 8$

$\therefore \quad 24 - 8 = 16$

3. $2 \quad 3 \quad 6 \quad 15 \quad \boxed{45} \quad 157.5 \quad 630$

$\underbrace{\quad}_{1.5} \quad \underbrace{\quad}_{2} \quad \underbrace{\quad}_{2.5} \quad \underbrace{\quad}_{3} \quad \underbrace{\quad}_{3.5} \quad \underbrace{\quad}_{4}$

$\frac{\text{2nd number}}{\text{1st number}}$ is in increasing order as shown above

4. The given series is

12, 35, 81, 173, 357, ...

The given series follows the following pattern

$12 \times 2 + 11 = 35$

$35 \times 2 + 11 = 81$

$81 \times 2 + 11 = 173$

$173 \times 2 + 11 = 357$

$357 \times 2 + 11 = 725$

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General Aptitude

Part II : Numerical Ability

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1

CHAPTER

Numbers and Algebra

NUMBERS

Natural Numbers

All positive integers are natural numbers.

e.g. 1, 2, 3, 4, 5,......

There are infinite natural numbers and number 1 is the *least natural number*.

Types of Natural Numbers : *Based on divisibility there are following two types of natural numbers :*

1. ***Prime Numbers:***

   A natural number larger than unity is a prime number if it does not have other divisors except itself and unity.

   Note: Unity *i.e* 1 is not a prime number.

   To Check A Number is Prime or not:

   (*i*) Take square root of the number.

   (*ii*) Round of the square root to the next highest integer and call this number as Z.

   (*iii*) Check for divisibility of the number N by all prime numbers below Z. If there is no number below the value of Z which divides N then the number will be prime.

   e.g. 241 is prime or not?

   $\sqrt{241}$ lies between 15 or 16.Hence take the value of Z =16.

   Prime numbers less than 16 are 2, 3, 5, 7, 11 and 13.

   241 is not divisible by any of these. Hence we can conclude that 241 is a *prime number.*

2. ***Composite Numbers:***

   The numbers which are not prime are known as composite numbers.

   Note: 1 is neither prime not composite

   Co-Primes: Two numbers a and b are said to be co-primes, if their H.C.F is 1.

   e.g. (2, 3), (4, 5), (7, 9), (8, 11).....

Tests for Divisibility

A number is divisible by

(1) 2, when its unit's digit is even or 0.

(2) 3, when the sum of its digits is divisible by 3.

(3) 4 when number formed by last two right hand digits is divisible by 4, or if last two digits are 0's.

(4) 5, when its unit's digit is 5 or 0.

(5) 6, when it is divisible by 2 and 3 both.

(6) 8 when number formed by last three right hand digits is divisible by 8, or when last three digit are 0's.

(7) 9, when sum of its digits is divisible by 9.

(8) 10, when its unit's digit is 0.

(9) 11, when difference between sum of the digits in the odd and sum of the digits in the even places is 0 or a multiple of 11.

Note: When any number with an even number of digits is added to its reverse, the sum is always a multiple of 11.

(10) 12, when it is divisible by 3 and 4 both.

HCF AND LCM

Greatest Common Divisor (GCD)

It is the greatest divisor common to two or more given numbers. It is also called HCF (Highest Common Factor).

Method to find HCF of given Numbers.

1. ***By method of factorization:***

   Express each given number as product of primes. Now take product of common factors, which is HCF.

   e.g., Find HCF of 136, 144, 168.

   $136 = 2 \times 2 \times 2 \times 17 = 2^3 \times 17$

   $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$

   $168 = 2 \times 2 \times 2 \times 3 \times 7 = 2^3 \times 3 \times 7$

   $\text{HCF} = 2^3 = 8$

2. ***By division method:***

   Suppose two numbers are given.

   Divide greater number by the lesser one; divide lesser by the remainder; divide first remainder by the new remainder, and so on till there is no remainder. Last divisor is the required H.C.F.

   e.g., Find H.C.F. of 12 and 15.

```
12)15(1
  -12
   3)12(4
    -12
     00
```

$\therefore \quad \text{HCF} = 3$

In case more than two numbers are given, then choose any two of them and find their HCF. HCF of these two and third gives HCF of three numbers and so on.

Note: If 'Z' is HCF of 'X' and 'Y', then Z is also a factor of aX + bY where a, b are integers.

Least Common Multiple (LCM)

LCM of two or more given number is the least number which is exactly divisible by each of the given numbers.

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Methods to find LCM of given Numbers.

1. **By method of factorization:**

Resolve each one of the given numbers into prime factors, then their LCM is product of highest powers of all factors, that occur in these numbers.

e.g., Find LCM of 136, 144, 168

$136 = 2^3 \times 17$

$144 = 2^4 \times 3^2$

$168 = 2^3 \times 3 \times 7$

$\text{LCM} = 2^4 \times 3^2 \times 17 \times 7 = 17136$

2. **Using the formula:**

Product of numbers = HCF × LCM

If two numbers are given, find their HCF.

$$\text{LCM} = \frac{\text{Product of two numbers}}{\text{HCF}}$$

e.g., Find LCM of 136, 144.

$$\text{LCM} = \frac{136 \times 144}{8} = 2448$$

If more than two numbers are given, find LCM of any two of them. Then LCM of this LCM and third number gives LCM of these three numbers and so on.

3. **Short cut method using factorization:**

e.g. Find LCM of 18, 28, 108, 105.

2	18,	28,	108,	105
2	9,	14,	54,	105
3	9,	7,	27,	105
3	3,	7,	9,	35
3	1,	7,	3,	35
5	1,	7,	1,	35
7	1,	7,	1,	7
	1,	1,	1,	1

$\text{LCM} = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 3780$

Alternatively : We can strike out 18 at once, since any multiple of 108 must contain 18 as a factor.

2	28,	108,	105
2	14,	54,	105
3	7,	27,	105
7	7,	9,	35
	1,	9,	5

$\text{LCM} = 2 \times 2 \times 3 \times 7 \times 9 \times 5 = 3780$

Note: For any given numbers, HCF is necessarily a factor of their L.C.M.

HCF and LCM of Fractions.

$$\text{HCF of Fractions} = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

$$\text{LCM of Fractions} = \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

e.g. Find HCF and LCM of $\frac{4}{5}, \frac{8}{35}, \frac{2}{15}, \frac{6}{25}$

$$\therefore \quad \text{HCF} = \frac{\text{HCF of 4, 8, 2, 6}}{\text{LCM of 5, 35, 15, 25}}$$

$$= \frac{2}{5 \times 7 \times 3 \times 5} = \frac{2}{525}$$

$$\text{LCM} = \frac{\text{LCM of 2, 8, 4, 6}}{\text{HCF of 5, 35, 15, 25}} = \frac{24}{5}$$

PROGRESSIONS

Arithmetic Progression (A.P.)

Let 'a' be the first term of an arithmetic progression; 'd' the common difference and 'n' the number of terms in the progression.

$$T_n = n^{th} \text{ term} = a + (n - 1)d$$

$$S_n = \text{Sum of n terms} = \frac{n}{2} \times [2a + (n - 1)d]$$

Progression can be represented as a, a + d, a + 2d, ..., [a + (n – 1) d]. Here, quantity d is to be added to any chosen term to get next term of the progression.

When there are n terms in an arithmetic progression, a is first term and {a + (n – 1) d} is last term.

Then $\quad S_n = \frac{n}{2} \times [\text{First Term + Last Term}]$

Geometric Progression (G.P.)

Geometric progression can be represented as a, ar, ar^2, ... where a is first term and r is common ratio of the geometric progression.

n^{th} term of the geometric progression = ar^{n-1}.

$$\text{Sum to n terms} = \frac{a(1-r^n)}{1-r} \text{ or } \frac{a(r^n-1)}{r-1}$$

$$= \frac{r \times \text{last term} - \text{First term}}{r-1}$$

Infinite Geometric Progression.

If – 1 < r < + 1 or |r| < 1, then sum of a geometric progression does not increase infinitely; it "converges" to a particular value. Such a G.P. is called infinite geometric progression.

Sum of an infinite geometric progression, $S_\infty = \frac{a}{1-r}$

Remember

Sum of first n natural numbers, $= S_n = \sum n = \frac{n(n+1)}{2}$

Sum of squares of first n natural numbers,

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of cubes of first n natural numbers,

$$\sum n^3 = \left[\frac{n(n+1)}{6}\right]^2 \frac{n^2(n+1)^2}{4} = \left[\sum n\right]^2$$

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AVERAGES, MEAN, MODE and MEDIAN

Averages

A central value around which a group of values shows a tendency to concentrate is called *average*. Thus an average is a single value that is in some way indicative of a group of values.

Measures of Central Tendency

Following are five measures of central tendency :

1. **Mode :** It is the number that occurs most frequently in a given set of numbers.

   e.g., Find mode of the set of numbers 2, 4, 6, 6, 5, 4, 3, 6, 2, 6, 4, 1, 4, 5, 4.

   Since number 4 occurs maximum number of times, hence mode is 4.

   Note : In a given set of data, if two or more values occur the same number of times, then a unique mode does not exist.

2. **Median:** It is the middle value of a group of numbers arranged in an ascending or descending order.

   If number of values (n) in a given set of data is odd, then Median is the $\left(\frac{n+1}{2}\right)^{th}$ value.

   e.g., Find median of the numbers 40, 41, 33, 48, 41, 39 and 38.

   Arrange values in ascending order, i.e. 33, 38, 39, 40, 41, 41, 48.

   Median is the $\left(\frac{7+1}{2}\right)^{th}$ value, 4th value *i.e.,* 40

   If number of values (n) in a given set of data is even, then there will be two middle values say a and b, and hence median is taken as $\frac{a+b}{2}$.

   e.g., Find median of the numbers 16, 15, 13, 14, 23, 44.

   Arrange values in descending order, i.e. 44, 23, 16, 15, 14, 13. Two middle values are 16 and 15.

   $\therefore \quad \text{Median} = \frac{16+15}{2} = \frac{31}{2} = 15.5$

MEAN

1. **Arithmetic Mean (A.M.):** The most commonly used average is the arithmetic mean (AM) or simply the average.

   Arithmetic mean of n numbers $x_1, x_2, x_3 \ldots\ldots x_n$ is denoted by $\bar{x}$ and calculated as

   $$x = \frac{x_1 + x_2 + \ldots + x_n}{n} \quad \text{or} \quad x = \frac{\sum_{i=1}^{n} x_i}{n}$$

   where Greek letter Σ(sigma) indicates "*Sum of the terms of the type*". It takes value from 1 to n.

   Arithmetic mean of a and b is $\frac{a+b}{2}$.

   e.g., Find arithmetic mean of 10, 15, 17, 20.

   $$x = \frac{10+15+17+20}{4} = 15.5$$

When large numbers are involved, an Arithmetic Mean is generally assumed to facilitate calculation. If a is assumed Arithmetic mean, then true Arithmetic mean $\bar{x}$ of group of n numbers $x_1, x_2, x_3 \ldots\ldots x_n$ is

$$\bar{x} = a + \frac{\sum_{i=1}^{n} d_i}{n},$$

where $d_i = x_i - a$ is deviation from the assumed mean.

e.g., Find the Arithmetic mean of 42, 46, 48, 50, 55, 60, 70

Let assumed mean be 50

x_i	d_i
42	42 – 50 = – 8
46	46 – 50 = – 4
48	48 – 50 = – 2
50	50 – 50 = 0
55	55 – 50 = 5
60	60 – 50 = 10
70	70 – 50 = 20

$$\therefore \quad \bar{x} = 50 + \frac{\sum_{i=1}^{7} di}{7} = 50 + \frac{21}{7} = 53.$$

Weighted Arithmetic Mean: If values $x_1, x_2, \ldots\ldots x_n$ are assigned weights $w_1, w_2, \ldots\ldots w_n$ respectively, then

$$\text{weighted arithmetic mean },\ x_w = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

The term '*weight*' stands for relative importance that is attached to the different values.

'w' can also be replaced by 'f' in a frequency distribution where 'f' denotes frequency or simply the number of times a particular observation occurs.

e.g.,

x_i	w_i	$w_i x_i$
20	3	60
30	1	30
40	5	200
50	1	50

$\Sigma w_i = 10$; $\Sigma x_i w_i = 340$

$$\therefore \quad \bar{x}_w = \frac{\sum_{i=1}^{4} w_i x_i}{\sum_{i=1}^{4} w_i} = \frac{340}{10} = 34$$

When two or more groups are combined, then combined average of the groups is

$$x_c = \frac{x_1 n_1 + x_2 n_2 + x_3 n_3 + \ldots + x_n n_n}{n_1 + n_2 + n_3 + \ldots + n_n}$$

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where x_1, x_2, x_3....x_n individual averages of the groups and n_1, n_2, ..., n_n are number of observations or values in each of the groups.

e.g., Mean marks scored by 50 boys are 70 and those scored by 100 girls are 55. Find average marks scored by this group.

$$x_c = \frac{n_1x_1 + n_2x_2}{n_1 + n_2} = \frac{70 \times 50 + 100 \times 55}{150} = \frac{9000}{150} = 60$$

2. Geometric Mean (G.M.):

G.M. of n numbers x_1, x_2, ... x_n is nth root of their products

i.e., $$\text{G.M.} = \sqrt[n]{x_1 \times x_2 \times x_3 \times ... \times x_n}$$

e.g., Geometric mean of 2, 3 and 4 is $\sqrt[3]{2 \times 3 \times 4}$

$$= \sqrt[3]{24} = 2.\sqrt[3]{3}$$

Geometric mean of a and b is $\sqrt{ab}$

e.g., Find Geometric mean of 9 and 4.

$$\text{G.M.} = \sqrt{9 \times 4} = 6$$

Geometric mean is generally used to calculate rate of growth.

If geometric mean of one group of 'a' numbers is 'x' and that of another group of 'b' numbers is 'y', then Geometric mean of the combined groups

$$= (x_a + y_b)^{\frac{1}{a+b}}$$

3. Harmonic Mean (H.M.):

Harmonic mean of n numbers x_1, x_2, ... x_n is calculated as $\dfrac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$

Harmonic mean of a and b is $\dfrac{2ab}{a+b}$.

e.g., Find Harmonic mean of 1, 0.5, 10.

x_i	$\frac{1}{x_i}$
1	1
0.5	2
10	0.1

$$\sum \frac{1}{x_i} = 3.1$$

$$\text{H.M.} = \frac{3}{3.1} = \frac{30}{31}$$

This is an important result and is generally used to find average speed when equal distances are covered at different speeds. (when different distances are covered in the same time, arithmetic mean is used to determine the average speed).

e.g., A person travels from A to B at 6 kmph and returns at 4 kmph, find his average speed.

Average Speed = Harmonic Mean of 6 and 4

$$= \frac{2 \times 6 \times 4}{6+4} = \frac{48}{10} = 4.8 \text{ kmph}$$

Note: This concept will again be covered in time, speed and distance.

Weighted Harmonic Mean: Weighted Harmonic Mean of 3 numbers x_1, x_2 and x_3 with weights w_1, w_2 and w_3 respectively is $\dfrac{\sum_{i=1}^{3} w_i}{\sum_{i=1}^{3} \frac{w_i}{x_i}}$

This is used to calculate average speed when different distances are covered at different speeds.

e.g., A cyclist covers his first 5 km at an average speed of 10 kmph, another 3 km at 8 kmph and last 2 km at 5 kmph. Find his average speed during the journey.

$$\text{Average Speed} = \text{Weighted HM}$$

$$= \frac{5+3+2}{\frac{5}{10} + \frac{3}{8} + \frac{2}{5}} = 7.84 \text{ kmph}$$

Relation between A.M., G.M. and H.M.

1. Consider two number a and b.

   Arithmetic Mean = $\dfrac{a+b}{2}$; Geometric mean = $\sqrt{ab}$

   Harmonic mean = $\dfrac{2ab}{a+b}$

   $\therefore \quad (GM)^2 = AM \times HM$

   e.g., If Arithmetic mean of two numbers is 6.5 and their Geometric mean is 6, what is their Harmonic Mean?

   $$HM = \frac{(G.M.)^2}{A.M.} = \frac{6 \times 6}{6.5} = 5.54$$

2. Arithmetic Mean > Geometric Mean > Harmonic Mean

   e.g., Arithmetic Mean of 2 and 4 is 3.

   Geometric Mean of 2 and 4 is 2.82.

   Harmonic Mean of 2 and 4 is 2.66

   $\therefore \quad 3 > 2.82 > 2.66$

ALGEBRAIC FORMULAE AND THEIR APPLICATION

An equation is a statement that *two algebraic expressions are equal*. If an equation is satisfied by any value of the variable, then equation is said to be an *identity*.

1. $(a + b)^2 = a^2 + 2ab + b^2 = (a - b)^2 + 4ab$
2. $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$
3. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
4. $(a + b)^2 - (a - b)^2 = 4ab$
5. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
6. $(a + b + c + d)^2 = a^2 + b^2 + c^2 + d^2 + 2a(b + c + d) + 2b(c + d) + 2cd$
7. $(a + b)(a - b) = a^2 - b^2$
8. $(x + a)(x + b) = x^2 + (a + b)x + ab$
9. $(x + a)(x + b)(x + c) = x^3+(a + b + c)x^2 + (ab + bc + ca)x + abc$
10. $(a + b)^3 = a^3 + 3ab(a + b) + b^3$
11. $(a - b)^3 = a^3 - 3ab(a - b) - b^3$
12. $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 - ab + b^2)$

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13. $a^3 - b^3 = (a - b)^3 + 3ab(a - b) = (a - b)(a^2 + ab + b^2)$
14. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - ac - bc)$
    If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$
15. $a^n - b^n = (a - b)(a^{n-1} + a^{n-2} b + a^{n-3} b^2 + ... + b^{n-1})$ for all n.
16. $a^n + b^n = (a + b)(a^{n-1} - a^{n-2} b + a^{n-3} b^2 - ... - b^{n-1})$ if n is even
17. $a^n + b^n = (a + b)(a^{n-1} - a^{n-2} b + a^{n-3} b^2 + ... + b^{n-1})$ if n is odd.
18. $a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$

POLYNOMIALS

Factors of polynomials: An expression is said to be resolved into factors when expressions of its product are found.

HCF of polynomials: When two or more polynomials are factorized, then product of all the common factors is HCF of the polynomials.

LCM of polynomials: When two or more polynomials are factorized, then product of all the factors with highest power is LCM of the polynomials.

INEQUATIONS

Properties

1. An inequality will still hold after each side has been increased, diminished, multiplied or divided by the same positive quantity.

i.e., if $a > b$ and $c > 0$

$a + c > b + c$

$ac > bc$

$\frac{a}{c} > \frac{b}{c}$

2. In an inequality, any term may be transposed from one side to the other if its sign is changed.

i.e., if $a - c > b$, then $a > b + c$ or $c < a - b$

3. If sides of an inequality be multiplied by same negative quantity, then sign of inequality must be reversed.

i.e., if $a > b$ and $c < 0$, then $ac < bc$

4. If $a > b$, $a, b \geq 0$ then $a^n > b^n$ and $\frac{1}{a^n} < \frac{1}{b^n}$, or $a^{-n} < b^{-n}$; if n is a positive quantity.

5. Square of every real quantity is positive and therefore must be greater than zero

i.e., for $a \neq b$, $(a - b)^2 > 0$; $a^2 + b^2 > 2ab$;

Similarly, if $x > 0$, $y > 0$, then

$$\frac{x + y}{2} \geq \sqrt{xy}$$

Hence, Arithmetic mean of two positive quantities is greater than or equal to their Geometric mean.

6. If sum of two positive quantities is given, their product is greatest when they are equal; and if product of two positive quantities is given, their sum is least when they are equal.

7. If a, b, c.... k are n unequal quantities, then

$$\left(\frac{a + b + c + ... + k}{n}\right)^n > a \times b \times c \times d ... \times k$$

i.e., $\frac{a + b + c + ... + k}{n} > (a \times b \times c \times ... \times k)^{1/n}$

Note: Arithmetic mean of any number of positive quantities is greater than their Geometric mean.

8. If a and b are positive and unequal, then

$\frac{a^m b^m}{2} > \left(\frac{a + b}{2}\right)^m$ except when m is a positiveproper fraction.

If m is a positive integer or any negative quantity, then $\frac{a^m b^m}{2} > \left(\frac{a + b}{2}\right)^m$

If m is positive and less than 1, then $\frac{a^m b^m}{2} < \left(\frac{a+b}{2}\right)^m$

If there are n positive quantities a, b, c ... k, then $\left(\frac{a^m + b^m + c^m + ... + k^m}{n}\right) > \left(\frac{a + b + c + ... + k}{n}\right)^m$

unless m is a positive proper fraction.

9. If a, b, c are positive and not all equal, then

$(a+b+c)(ab+bc+ca) > 9abc$

and, $(b + c)(c + a)(a + b) > 8abc$.

10. If x is positive and $a < b$, then $\frac{a + x}{b + x} > \frac{a}{b}$

If x is positive and $a > b$, then $\frac{a + x}{b + x} < \frac{a}{b}$

11. If x is positive and $a > b > x$, then $\frac{a - x}{b - x} > \frac{a}{b}$

If x is positive and $x < a < b$, then $\frac{a - x}{b - x} < \frac{a}{b}$

12. $\frac{a + c + e + ...}{b + d + f + ...}$ is less than greatest and greater than least of the fractions $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}, ...$

13. For positive numbers, if $a > x$, $b > y$, $c > z$, then $a + b + c + ... > x + y + z + ...$ and $abc ... > xyz ...$

14. $a^2 + b^2 + c^2 \geq bc + ca + ab$

15. $(n!)^2 > n^n$, for $n > 2$.

16. For any positive integer n, $2 \leq \left(1 + \frac{1}{n}\right)^n \leq 3$

17. $a^2b + b^2c + c^2a \geq 3abc$

18. $\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a} \geq 4$

19. $a^4 + b^4 + c^4 + d^4 \geq 4abcd$

QUADRATIC EQUATION WITH APPLICATION TO INEQUALITIES

In a quadratic equation $ax^2 + bx + c = 0$, if $b^2 - 4ac > 0$, then

$ax^2 + bx + c = a(x - \alpha)(x - \beta)$, where α and β are two roots of the equation with $\alpha < \beta$.

$(x - \alpha)(x - \beta)$ is always positive when $x < \alpha$ or $x > \beta$ i.e., x does not lie between α and β.

$(x - \alpha)(x - \beta)$ is always negative when $\alpha < x$ or $x > \beta$ i.e., x lies between α and β.

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EXERCISE – I

MCQ TYPE QUESTIONS

1. If x = b + c, y = c – a, z = a – b, then $x^2 + y^2 + z^2 - 2xy - 2xz + 2yz$ is equal to

(*a*) a + b + c (*b*) $4b^2$

(*c*) abc (*d*) $a^2 + b^2$

2. HCF of $y^3 - 4y$ and $4y(y^3 + 8)$ is

(*a*) y(y + 2) (*b*) y – 2

(*c*) 2(y + 2) (*d*) $(y + 2)(y^2 - 2y + 4)$

3. A tiled floor of a room has dimensions m × m sq.m. Dimensions of the tile used are n × n sq.m. All tiles used are green tiles except diagonal tiles which are red. After some years some green tiles are replaced by red tiles to form an alternate red and green tile pattern. How many green tiles are removed?

(m ≠ n and total number of tiles are odd).

(*a*) $\frac{m^2 - 4mn + 2n^2}{2n^2}$ (*b*) $\frac{(m - 2n)^2 - n^2}{2n^2}$

(*c*) $\frac{m^2 - 4mn - n^2}{2n^2}$ (*d*) $\frac{m^2 - 4mn - 2n^2}{2n^2}$

4. If $\frac{x - 4}{-x^2 - 6} \le \frac{2}{-x^2 - 6}$ then

(*a*) $x \le 2$ (*b*) $x \ge 2$

(*c*) $x \le 6$ (*d*) $x \ge 6$

5. Which of the following is true?

(*a*) $\sqrt{2} + \sqrt{5} = \sqrt{7}$ (*b*) $\sqrt{2} + \sqrt{5} \le \sqrt{7}$

(*c*) $\sqrt{2} + \sqrt{5} < \sqrt{7}$ (*d*) $\sqrt{2} + \sqrt{5} > \sqrt{7}$

6. If $6 \ge x \ge -2$ and $4 \ge y \ge -4$, then limits for $\frac{y}{x}$, where x and y are non zero integers, is

(*a*) $\frac{y}{x} \ge 2, \frac{y}{x} \le \frac{2}{3}$ (*b*) $\frac{y}{x} \ge \frac{-2}{3}, \frac{y}{x} \le 2$

(*c*) $\frac{y}{x} \ge \frac{-2}{3}, \frac{y}{x} \le \frac{1}{4}$ (*d*) $\frac{y}{x} \ge -4, \frac{y}{x} \le 4$

7. What are the limits of $\frac{2^n(n+1)^n}{n^n}$; where n is a positive integer?

(*a*) $2 \le \frac{2^n(n+1)^n}{n^n} \le 3$

(*b*) $2^{n+1} \le \frac{2^n(n+1)^n}{n^n} \le 3.2^n$

(*c*) $2^n \le \frac{2^n(n+1)^n}{n^n} \le 3.2^n$

(*d*) None of these

8. If $g = 10^{100}$ and $H = 10^g$, then in which interval does $g! = 1.2.3 \ldots 10^{100}$ lie?

(*a*) 10H < g! < H (*b*) $H < g! < 10^H$

(*c*) $10H < g! < 10^H$ (*d*) $10^H < g!$

9. Which of the following is true?

(*a*) Sum of four consecutive even numbers is always divisible by 8.

(*b*) Sum of four consecutive odd numbers is always divisible by 8.

(*c*) Product of any n consecutive natural numbers may not be divisible by n!.

(*d*) Product of 4 consecutive odd numbers is always divisible by 15.

10. If a and b are prime numbers, which of the following is true?

I. a^2 has three positive integer factors.

II. ab has four positive integer factors.

III. a^3 has four positive integer factors.

Codes :

(*a*) I and II only (*b*) II and III only

(*c*) All of these (*d*) None of these

11. If 7x + 6y = 420, x and y are natural numbers, then what can be said about x?

(*a*) x is always odd

(*b*) x is always even.

(*c*) x is even only if y is odd.

(*d*) x is odd if y is even.

12. If a, b, c, d, p and q are non-zero, unequal integers and $\frac{a + bi}{c + di} = \frac{p}{q}$, then $\frac{a^2 + b^2}{c^2 + d^2}$ equals

(*a*) $\frac{p}{q}$ (*b*) $\frac{p^2}{q^2}$

(*c*) 1 (*d*) None of these

13. An Egyptian fraction has a numerator equal to 1, and its denominator is a positive integer. What is the maximum number of different Egyptian fraction such that their sum is equal to 1, and their denominators are equal to 10 or less?

(*a*) 3 (*b*) 5 (*c*) 7 (*d*) 9

14. What is the least number which must be subtracted from 1936 so that the remainder when divided by 9, 10, 15 will leave in each case the same remainder 7?

(*a*) 32 (*b*) 53

(*c*) 46 (*d*) 39

15. What is the greatest number consisting of six digits which on being divided by 6, 7, 8, 9, 10 leaves 4, 5, 6, 7, 8 as remainders respectively?

(*a*) 997920 (*b*) 997918

(*c*) 999999 (*d*) 997922

16. What is the least number which on being divided by 5, 6, 8, 9, 12 leaves in each case a remainder 1 but when divided by 13 leaves no remainder?

(*a*) 2987 (*b*) 3601

(*c*) 3600 (*d*) 2986

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17. A group of four numbers has only one prime number amongst them. Which of the following must be true about the group?

I. HCF of the four numbers of the group is either 1 or equal to that prime number.

II. LCM of the four numbers of the group is same as product of the prime number and LCM of the remaining three numbers.

III. Product of four numbers is equal to product of the prime number × HCF of the group × LCM of the group.

(*a*) I only
(*b*) II only
(*c*) I and II only
(*d*) All of these

18. If (a, n)! is defined as product of n consecutive numbers starting from a, where a and n are both natural numbers, and if H is the HCF of (a, n)! and n!, then what can be said about H?

(*a*) H = a! (*b*) H = n!
(*c*) H ≥ n! (*d*) H ≥ a × n

19. HCF and LCM of two numbers is given. It is possible to find out the two numbers uniquely if

I. either sum or difference between the two numbers is known.

II. HCF of two numbers = LCM of two numbers.

III. $\frac{\text{LCM}}{\text{HCF}}$ = Prime number.

(*a*) I and II only
(*b*) II only
(*c*) II and III only
(*d*) I, II and III

20. A ball is dropped from a height of 12 m and it rebounds 1/2 of the distance it falls. If it continues to fall and rebound in this way, how far will it travel before coming to rest?

(*a*) 36 m (*b*) 30 m
(*c*) 48 m (*d*) 60 m

21. There are N questions in an exam.
For i = 1, 2, ..., N, there are 2^{N-1} students who answered 1 or more questions wrongly. If total number of wrong answers is 8191, then N will be

(*a*) 12 (*b*) 11 (*c*) 10 (*d*) 13

22. Two new charity organizations C_1 and C_2 were formed, with x members each, on January 1, 2003. On first day of each, subsequent month, in C_1, number of members increases by a certain number a, while in C_2, number of members increases in such a way that ratio of the number of members in a month to the preceding month bear a ratio equal to b. On May 1, 2003, both organizations had the same number of members. If a = 20x, then b will be

(*a*) 2 (*b*) 3 (*c*) 2.5 (*d*) 3.5

23. What is the ratio of common differences d_1 and d_2 of two arithmetic progressions if respective n^{th} terms are in the ratio of 2n + 3 : n – 11?

(*a*) 1 : 2 (*b*) 2 : 3
(*c*) 2 : 1 (*d*) 1 : 3

24. What is the sum upto 20 terms of the series $\frac{2}{3}+\frac{8}{9}+\frac{26}{27}+\frac{80}{81}+......?$

(*a*) $\frac{19.3^{20}+1}{3^{20}}$ (*b*) $\frac{19.3^{20}+1}{2.3^{20}}$

(*c*) $\frac{19.3^{20}-1}{2.3^{20}}$ (*d*) $\frac{39.3^{20}+1}{2.3^{20}}$

25. What is the sum to infinity of the series,
$3 + 6x^2 + 9x^4 + 12x^6 + ...$ given $|x| < 1$?

(*a*) $\frac{3}{(1+x^2)}$ (*b*) $\frac{3}{(1-x^2)^2}$

(*c*) $\frac{3}{(1+x^2)^2}$ (*d*) $\frac{3}{(1-x^2)}$

26. A school has 5 divisions in a class IX having 60, 50, 55, 62 and 58 students. Mean marks obtained in a History test were 56, 64, 72, 63 and 50 by each division respectively. What is overall average of the marks per students?

(*a*) 56.8 (*b*) 58.2
(*c*) 62.4 (*d*) 60.8

27. Average age of a committee of seven trustees is the same as it was five years ago, a younger man having been substituted for one of them. How much younger was he then trustee whose place he took?

(*a*) 32 years (*b*) 35 years
(*c*) 33 years (*d*) 34 years

28. Six men A, B, C, D, E, F agree with a seventh man G to provide a sum of money among them. A, B, C, D, E, F are to subscribe ₹10 each, and G is to pay ₹3 more than the average of the seven. What is the whole sum to be provided?

(*a*) ₹73.50 (*b*) ₹ 74
(*c*) ₹ 73 (*d*) ₹ 72.50

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29. Which of the following is/are true?

I. When Arithmetic Mean of a set of values and number of values in the set are known, then Geometric Mean of the set can be found out.

II. When Arithmetic Mean of a set of values and number of values in the set are known, sum of all the elements in the set can be found out.

III. When Geometric Mean of a set of values and number of values in the set are known, then Arithmetic Mean of the set can be found out.

Codes :

(*a*) Only I (*b*) Only II

(*c*) Only I and II (*d*) All except II

30. Which of the following statement/s is/are true?

I. Average of a set of values will always lie between lowest and the largest of these values.

II. If each of the values in a set is increased by a constant k, then new average of the set is increased by k.

III. If each of the values in a set is multiplied by a constant m, then new average will also be 'm' times the old average.

Codes :

(*a*) Only I and II (*b*) Only II and III

(*c*) Only I and III (*d*) I, II and III

31. Three people A, B and C weigh themselves in a particular order. First A, B, C weigh themselves individually and then AB, BC CA and ABC together respectively. Recorded weight for the last measured is 180 kgs. Then average of the 7 measures is

(*a*) 320 kgs (*b*) $\frac{360}{7}$ kgs

(*c*) $\frac{720}{7}$ kgs (*d*) Cannot be determined

32. Average age of a team of 15 employees is 36. Youngest of them is 20 years old and eldest is 36 years old. Two of them with average age 28 leave the team. If one of the two comes back on the condition that he will be made the team leader, then which of the following can possibly be average age of the new team so formed?

(*a*) 35 (*b*) 36 (*c*) 38 (*d*) 39

NUMERICAL TYPE QUESTIONS

1. If $x - y = 1$, then $x^3 - y^3 - 3xy$ equals to _______

2. When $x + y + z = 9$ and $xy + yz + zx = 11$, then $x^3 - y^3 - z^3 - 3xyz$ equals to _______

3. If $x + y + z = 0$, then $\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy}$ equals to _______

4. In an election for the President, if 261 valid votes are cast, for the 5 contestants then least number of votes a candidate requires to receive to win the election are is _______

5. A number when divided by sum of 555 and 445 gives two times their difference as quotient and 30 as remainder. The number is _______

6. Sum of three prime numbers is 100. If one of them exceeds another by 36, then one of the numbers is _______

7. Sum of all odd numbers up to 100 is_______

8. $A = 0.\ a_1a_1a_1$ ... and $B = 0.a_2a_2a_2$, where a_1 and a_2 are multiples of 3 and also, a_1 and a_2 are distinct integers from 0 to 8. Then value of A + B is _______

9. Anil wants to divide ₹100 into a number of bags so that one can ask for any amount between ₹1 and ₹100, he can give the proper amount by giving certain number of these bags without taking out the amount from them. The minimum number of bags he will require if each bag has whole number of rupees is _______

10. The remainder when $2050 \times 2071 \times 2095$ is divided by 23 is _______

11. The smallest number which when increased by 3 is divisible by 27, 35, 25 and 21 is ______

12. _______is the least number which on being divided by 12, 21 and 35 will leave in each case the same remainder 6

13. A gardener had a number of shrubs to plant in horizontal rows. At first he tried to plant 5 shrubs in each row, then 6, then 8 and then 12, but had always 1 left. On trying 13, in one row he had none left. The smallest number of shrubs that he could have had is ______

14. Sum of an infinite geometric series is 3 and sum of an infinite geometric series formed from squares of the terms of the original series is 6. First term of the first series will be____

15. The maximum sum of the series 60, 58, 56, 54, 52, ... is_______

16. On an average my income for 15 days ₹ 7, average for the first 5 days was ₹ 6 and average for the last 9 days was ₹ 8. The income on the sixth day was ________

17. The harmonic mean of two numbers whose geometric mean and arithmetic mean is 8 and 5 respectively is ________

18. Average marks of 15 students in a class is 145, maximum marks being 150. If two lowest scores are removed, the average increases by 5. Also, two lowest scores are consecutive multiples of 9. The lowest score in the class is _______

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EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. Read the following table giving sales data of five types of batteries for years 2006 to 2012

Year	Type I	Type I	Type I	Type I	Type I
2006	75	144	114	102	108
2007	90	126	102	84	126
2008	96	114	75	105	135
2009	105	90	150	90	75
2010	90	75	135	75	90
2011	105	60	165	45	120
2012	115	85	160	100	145

Out of the following, which type of battery achieved highest growth between the years 2006 and 2012?

(*a*) Type V (*b*) Type III
(*c*) Type II (*d*) Type I

2. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ac$ lies in the interval

(*a*) $\left[1, \frac{2}{3}\right]$ (*b*) $\left[\frac{-1}{2}, 1\right]$

(*c*) $\left[-1, \frac{1}{2}\right]$ (*d*) [2,–4]

3. If the list of letters, P, R, S, T, U is an arithmetic sequence, which of the following are also in arithmetic sequence?

I. 2P, 2R, 2S, 2T, 2U
II. P–3, R–3, S–3, T–3, U–3
III. P^2, R^2, S^2, T^2, U^2

(*a*) I only (*b*) I and II
(*c*) II and III (*d*) I and III

4. The number of students in a class who have answered correctly, wrongly, or not attempted each question in an exam, are listed in the table below. The marks for each question are also listed. There is no negative or partial marking.

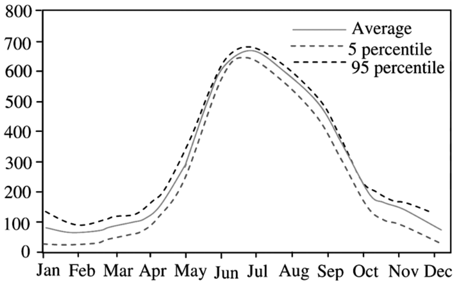
Q. No.	Marks	Answered Correctly	Answered Wrongly	Not Attempted
1	2	21	17	6
2	3	15	27	2
3	1	11	29	4
4	2	23	18	3
5	5	31	12	1

What is the average of the marks obtained by the class in the examination?

(*a*) 2.290 (*b*) 2.970
(*c*) 6.795 (*d*) 8.795

2014

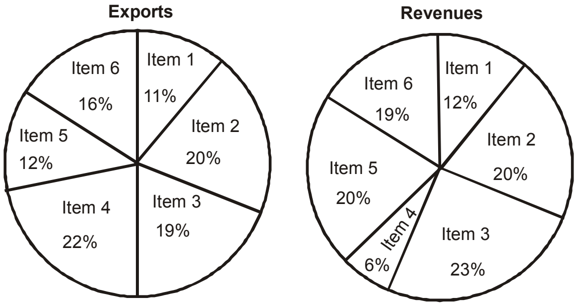
5. The monthly rainfall chart on 50 years of rainfall in Agra is shown in the following figure. Which of the following are true? (k percentile is the value such that k percent of the data fall below that value)



(*i*) On average, it rains more in July than in December
(*ii*) Every year, the amount of rainfall in August is more than that in January
(*iii*) July rainfall can be estimated with better confidence than February rainfall
(*iv*) In August, there is at least 500 mm of rainfall

(*a*) (*i*) and (*ii*) (*b*) (*i*) and (*iii*)
(*c*) (*ii*) and (*iii*) (*d*) (*iii*) and (*iv*)

6. The total exports and revenues from the exports of a country are given in the two charts shown below. The pie chart for exports shows the quantity of each item exported as a percentage of the total quantity of exports. The pie chart for the revenues shows the percentage of the total revenue generated through export of each item. The total quantity of exports of all the items is 500 thousand tonnes and the total revenues are 250 crore rupees. Which item among the following has generated the maximum revenue per kg?



(*a*) Item 2 (*b*) Item 3
(*c*) Item 6 (*d*) Item 5

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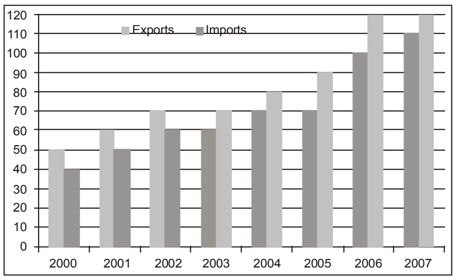
7. The statistics of runs scored in a series by four batsmen are provided in the following table. Who is the most consistent batsman of these four?

Batsman	Average	Standard deviation
K	31.2	5.21
L	46.0	6.35
M	54.4	6.22
N	17.9	5.90

(*a*) K (*b*) L
(*c*) M (*d*) N

8. The exports and imports (in crores of ₹) of a country from 2000 to 2007 are given in the following bar chart. If the trade deficit is defined as excess of imports over exports, in which year is the trade deficit 1/5th of the exports?



(*a*) 2005 (*b*) 2004
(*c*) 2007 (*d*) 2006

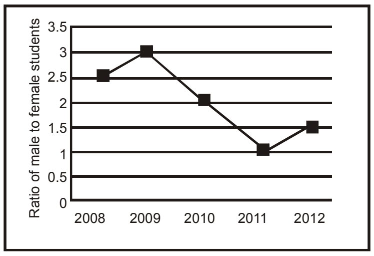
9. The table below has question-wise data on the performance of students in an examination. The marks for each question are also listed. There is no negative or partial marking in the examination.

Q.No.	Marks	Answered Correctly	Answered Wrongly	Not Attempted
1	2	21	17	6
2	3	15	27	2
3	2	23	18	3

What is the average of the marks obtained by the class in the examination?

(*a*) 1.34 (*b*) 1.74
(*c*) 3.02 (*d*) 3.91

10. The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students in 2011 and 2012 is equal, what is the ratio of male students in 2012 to male students in 2011?



(*a*) 1 : 1 (*b*) 2 : 1
(*c*) 1.5 : 1 (*d*) 2.5 : 1

11. What is the average of all multiples of 10 from 2 to 198?

(*a*) 90 (*b*) 100
(*c*) 110 (*d*) 120

12. The value of $\sqrt{12+\sqrt{12+\sqrt{12+...}}}$ is

(*a*) 3.464 (*b*) 3.932
(*c*) 4.000 (*d*) 4.444

13. If x is real and $|x^2 - 2x + 3| = 11$, then possible values of $|-x^3 + x^2 - x|$ include

(*a*) 2, 4 (*b*) 2, 14
(*c*) 4, 52 (*d*) 14, 52

2013

14. A number is as much greater than 75 as it is smaller than 117. The number is

(*a*) 91 (*b*) 93
(*c*) 89 (*d*) 96

15. Following table provides figures (in rupees) on annual expenditure of a firm for two years - 2010 and 2011.

Category	2010	2011
Raw material	5200	6240
Power & fuel	7000	9450
Salary & wages	9000	12600
Plant & machinery	20000	25000
Advertising	15000	19500
Research & Development	22000	26400

In 2011, which of the following two categories have registered increase by same percentage?

(*a*) Raw material and Salary & wages
(*b*) Salary & wages and Advertising
(*c*) Power & fuel and Advertising
(*d*) Raw material and Research & Development

16. What will be the maximum sum of 44, 42, 40,... ?

(*a*) 502 (*b*) 504
(*c*) 506 (*d*) 500

17. The current erection cost of a structure is ₹ 13,200. If the labour wages per day increase by 1/5 of the current wages and the working hours decrease by 1/24 of the current period, then the new cost of erection in ₹ is

(*a*) 16,500 (*b*) 15,180
(*c*) 11,000 (*d*) 10,120

18. Find the sum of the expression

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\dots\dots\dots+\frac{1}{\sqrt{80}+\sqrt{81}}$$

(*a*) 7 (*b*) 8
(*c*) 9 (*d*) 10

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19. In the summer of 2012, in New Delhi, the mean temperature of Monday to Wednesday was 41°C and of Tuesday to Thursday was 43°C. If the temperature on Thursday was 15% higher than that of Monday, then the temperature in °C on Thursday was

(*a*) 40 (*b*) 43
(*c*) 46 (*d*) 49

20. The set of values of p for which the roots of the equation $3x^2 + 2x + p(p - 1) = 0$ are of opposite sign is

(*a*) $(-\infty, 0)$ (*b*) $(0, 1)$
(*c*) $(1, \infty)$ (*d*) $(0, \infty)$

21. Find the sum to n terms of the series 10 + 84 + 734 + …

(*a*) $\frac{9(9^n+1)}{10}+1$ (*b*) $\frac{9(9^n-1)}{8}+1$

(*c*) $\frac{9(9^n-1)}{8}+n$ (*d*) $\frac{9(9^n-1)}{8}+n^2$

22. If $3 \le X \le 5$ and $8 \le Y \le 11$ then which of the following options is TRUE?

(*a*) $\frac{3}{5} \le \frac{X}{Y} \le \frac{8}{5}$

(*b*) $\frac{3}{11} \le \frac{X}{Y} \le \frac{5}{8}$

(*c*) $\frac{3}{11} \le \frac{X}{Y} \le \frac{8}{5}$

(*d*) $\frac{3}{5} \le \frac{X}{Y} \le \frac{8}{11}$

23. Following table gives data on tourists from different countries visiting India in the year 2011.

Country	Number of Tourists
USA	2000
England	3500
Germany	1200
Italy	1100
Japan	2400
Australia	2300
France	1000

Which two countries contributed to the one third of the total number of tourists who visited India in 2011?

(*a*) USA and Japan
(*b*) USA and Australia
(*c*) England and France
(*d*) Japan and Australia

2012

24. If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} =$

(*a*) 2.23 (*b*) 4.33
(*c*) 11.37 (*d*) 27.64

25. Raju has 14 currency notes in his pocket consisting of only ₹ 20 notes and ₹ 10 notes. The total money value of the notes is ₹ 230. The number of ₹ 10 notes that Raju has is

(*a*) 5 (*b*) 6
(*c*) 9 (*d*) 10

26. The data given in the following table summarizes the monthly budget of an average household.

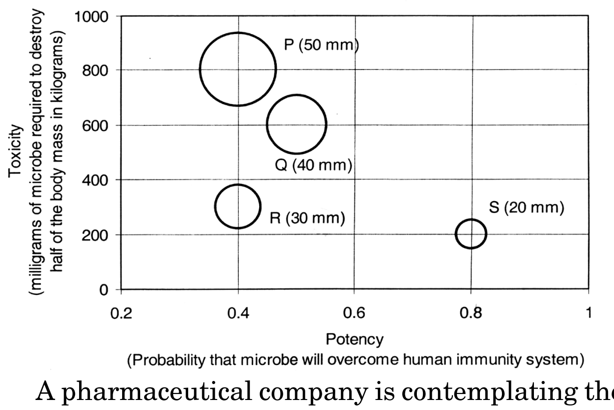
Category	Amount (Rs.)
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Other expenses	1800

The approximate percentage of the monthly budget **NOT** spent on savings is

(*a*) 10% (*b*) 14%
(*c*) 81% (*d*) 86%

2011

27. P, Q, R and S are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system within 24 hours of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure below :



A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its first attempt?

(*a*) P (*b*) Q
(*c*) R (*d*) S

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28. The sum of n terms of the series 4 + 44 + 444 + ... is

(*a*) $(4/81)[10^{n+1} - 9n - 1]$

(*b*) $(4/81)[10^{n-1} - 9n - 1]$

(*c*) $(4/81)[10^{n+1} - 9n - 10]$

(*d*) $(4/81)[10^{n} - 9n - 10]$

29. Given that $f(y) = |y|/y$, and q is any non-zero real number, the value of $|f(q) - f(-q)|$ is

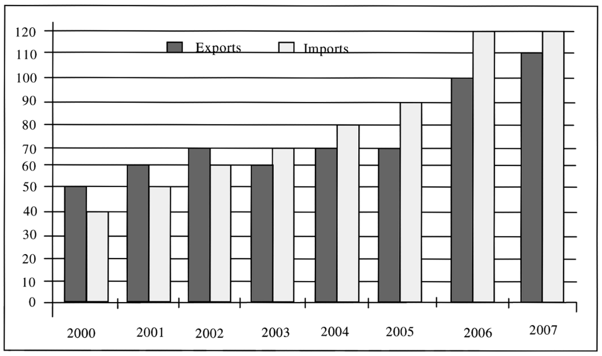
(*a*) 0 (*b*) –1

(*c*) 1 (*d*) 2

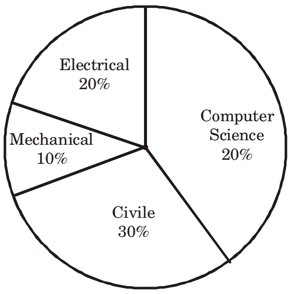
NUMERICAL TYPE QUESTIONS

2015

1. The exports and imports (in crores of Rs.) of a country from the year 2000 to 2007 are given in the following bar chart. In which year is the combined percentage increase in imports and exports the highest ?



2. The pie chart below has the breakup of the number of students from different departments in an engineering college for the year 2012. The proportion of male to female students in each department is 5 : 4. There are 40 males in Electrical Engineering. What is the difference between numbers of female students in the Civil department and the female students in the Mechanical department?



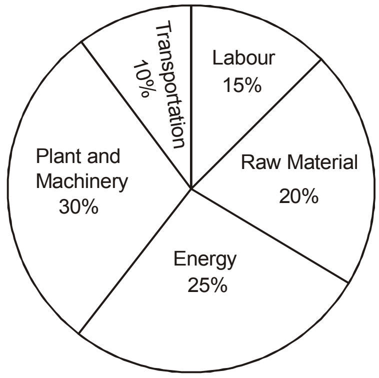
2014

3. A foundry has a fixed daily cost of ₹ 50,000 whenever it operates and variable cost of ₹ 800Q, where Q is the daily production in tonnes. What is the cost of production in ₹ per tonne for a daily production of 100 tonnes?

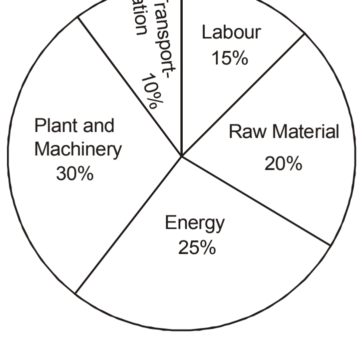
4. The smallest angle of a triangle is equal to two thirds of the smallest angle of a quadrilateral. The ratio between the angles of the quadrilateral is 3 : 4 : 5 : 6. The largest angle of the triangle is twice its smallest angle. What is the sum, in degrees, of the second largest angle of the triangle and the largest angle of the quadrilateral?

5. In a sequence of 12 consecutive odd numbers, the sum of the first 5 numbers is 425. What is the sum of the last 5 numbers in the sequence?

6. A firm producing air purifiers sold 200 units in 2012. The following pie chart presents the share of raw material, labour, energy, plant & machinery, and transportation costs in the total manufacturing cost of the firm in 2012. The expenditure on labour in 2012 is ₹ 4,50,000. In 2013, the raw material expenses increased by 30% and all other expenses increased by 20%. What is the percentage increase in total cost for the company in 2013?



7. A firm producing air purifiers sold 200 units in 2012. The following pie chart presents the share of raw material, labour, energy, plant & machinery, and transportation costs in the total manufacturing cost of the firm in 2012. The expenditure on labour in 2012 is ₹ 4,50,000. In 2013, the raw material expenses increased by 30% and all other expenses increased by 20%. If the company registered a profit of ₹ 10 lakhs in 2012, at what price (in ₹) was each air purifier sold?



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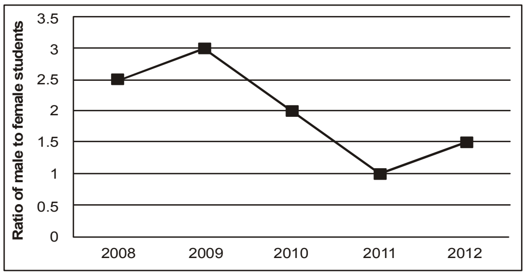
8. The sum of eight consecutive odd numbers is 656. The average of four consecutive even numbers is 87. What is the sum of the smallest odd number and second largest even number?

9. The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students doubled in 2009, by what percent did the number of male students increase in 2009?



10. If $(z + 1/z)^2 = 98$, compute $(z^2 + 1/z^2)$

11. In a survey, 300 respondents were asked whether they own a vehicle or not. If yes, they were further asked to mention whether they own a car or scooter or both. Their responses are tabulated below. What percent of respondents do not own a scooter?

		Men	Women
Own vehicle	Car	40	34
	Scooter	30	20
	Both	60	46
Do not own vehicle		20	50

12. Round–trip tickets to a tourist destination are eligible for a discount of 10% on the total fare. In addition, groups of 4 or more get a discount of 5% on the total fare. If the one way single person fare is ₹ 100, a group of 5 tourists purchasing round–trip tickets will be charged ₹ __________.

ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*b*) **2.** (*a*) **3.** (*b*) **4.** (*d*) **5.** (*d*) **6.** (*d*) **7.** (*b*) **8.** (*c*) **9.** (*b*) **10.** (*c*)
11. (*b*) **12.** (*b*) **13.** (*a*) **14.** (*d*) **15.** (*b*) **16.** (*b*) **17.** (*a*) **18.** (*b*) **19.** (*d*) **20.** (*a*)
21. (*d*) **22.** (*b*) **23.** (*c*) **24.** (*d*) **25.** (*b*) **26.** (*d*) **27.** (*b*) **28.** (*a*) **29.** (*b*) **30.** (*d*)
31. (*c*) **32.** (*b*)

Numerical Type Questions

1. 1 **2.** 432 **3.** 3 **4.** 53 **5.** 220030 **6.** 67 **7.** 2500 **8.** 1 **9.** 7 **10.** 6
11. 4722 **12.** 426 **13.** 481 **14.** 12/5 **15.** 930 **16.** 3 **17.** 12.8 **18.** 108

EXERCISE – II

MCQ Type Questions

1. (*d*) **2.** (*b*) **3.** (*b*) **4.** (*c*) **5.** (*b*) **6.** (*d*) **7.** (*a*) **8.** (*d*) **9.** (*c*) **10.** (*c*)
11. (*b*) **12.** (*c*) **13.** (*d*) **14.** (*d*) **15.** (*d*) **16.** (*c*) **17.** (*a*) **18.** (*b*) **19.** (*c*) **20.** (*b*)
21. (*d*) **22.** (*b*) **23.** (*c*) **24.** (*d*) **25.** (*a*) **26.** (*d*) **27.** (*d*) **28.** (*c*) **29.** (*d*)

Numerical Type Questions

1. (2006) **2.** (32) **3.** (1300 to 1300) **4.** (180 to 180) **5.** (495)
6. (22) **7.** (20,000) **8.** (163) **9.** (140 to 140) **10.** (96)
11. (48) **12.** (850)

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EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. $x^2 + y^2 + z^2 - 2xy - 2xz + 2yz = (x - y - z)^2$

$= (b + c - c + a - a + b)^2 = 4b^2$

2.
$$\begin{aligned} y^3 - 4y &= y(y^2 - 4) \\ &= y(y-2)(y+2) \\ 4y(y^3+8) &= 4y(y+2)(y^2-2y+4) \\ \therefore \quad \text{HCF} &= y(y+2). \end{aligned}$$

3. Total red tiles initially $= 2\left(\frac{m}{n}\right) - 1$

Total number of tiles $= \frac{m^2}{n^2}$

As number of red tiles > number of green tiles (since room has odd number of tiles and corner tiles are red)

Hence total red tiles required to form an alternate pattern

$$= \frac{\left(\frac{m}{n}\right)^2 + 1}{2} = \frac{m^2 + n^2}{2n^2}$$

∴ Number of green tiles removed

$$= \frac{m^2 + n^2}{2n^2} - 2\left(\frac{m}{n}\right) + 1$$

$$= \frac{m^2 + n^2 - 4mn + 2n^2}{2n^2}$$

$$= \frac{(m-2n)^2 - n^2}{2n^2}$$

4.
$$\frac{x-4}{-x^2-6} \le \frac{2}{-x^2-6}$$

i.e. $$\frac{x-4}{-(x^2+6)} \le \frac{2}{-(x^2+6)}$$

i.e. $x - 4 \ge 2$ ...(cancelling a negative term reverses sign of the inequality)

i.e. $x \ge 6$.

5.
$$\begin{aligned} \left(\sqrt{2} + \sqrt{5}\right)^2 &= 2 + 2\sqrt{10} + 5 \\ &= 7 + 2\sqrt{10} \end{aligned}$$

$\therefore \quad \sqrt{2} + \sqrt{5} > \sqrt{7}$

7.
$$\frac{2^n (n+1)^n}{n^n} = 2^n \left(\frac{n+1}{n}\right)^n = 2^n \left(1 + \frac{1}{n}\right)^n$$

Now, $$2 \le \left(1 + \frac{1}{n}\right)^n \le 3$$

∴ $$2.2^n \le 2^n \left(1 + \frac{1}{n}\right)^n \le 3.2^n$$

i.e., $$2^{n+1} \le 2^n \left(1 + \frac{1}{n}\right)^n \le 3.2^n$$

8. Choose k, so that

$$10^H = g^\lambda = 10^{100k},$$

$$k = \frac{H}{100}$$

∴ $$10^H = g^{\frac{H}{100}} > g^g > g!$$

On the other hand,

$10H = 10 \times 10^g = 10^{g+1} < g!$

(most factors are much bigger than 10)

$10H < g! < 10^H$

9. (*a*) Four consecutive even numbers can be written as 2n, 2n + 2, 2n + 4 and 2n + 6, where n is any natural number.

$$\begin{aligned} \therefore \quad \text{Sum} &= 2n + 2n + 2 + 2n + 4 + 2n + 6 \\ &= 8n + 12 \\ &= 4(2n + 3) \text{ not always divisible by 8.} \end{aligned}$$

Thus, (*a*) is not true.

(*b*) Four consecutive odd numbers can be written as 2n + 1, 2n + 3, 2n + 5, 2n +7 where n is a natural number.

Sum = 2n + 1 + 2n + 3 + 2n + 5 + 2n + 7 = 8n + 16 divisible by 8

Thus, (*b*) is true

(*c*) In product of n consecutive natural numbers atleast one is divisible by n, atleast one by n – 1 ... till 1.

Thus product is atleast divisible by

$n \times (n-1) \times (n-2) \ldots. 1 = n!$.

Thus, (*c*) is not true.

(*d*) Take four consecutive odd numbers as 7 × 9 × 11 × 13 which is not divisible by 15.

Thus, (*d*) is not true.

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10. Factors of a^2 are 1, a and a^2.
Factors of ab are 1, a, b and ab.
Factors of a^3 are 1, a, a^2 and a^3.

11. $7x + 6y = 420$
Equation is of the form:
$7x$ + even number = even number.
$\therefore$ 7x has to be even
Hence x has to be even.

12. $\dfrac{a+bi}{c+di} = \dfrac{p}{q}$

$\therefore \quad qa + qbi = pc + pdi$

Equating real and imaginary parts.

$qa = pc$ and $qb = pd$.

$\Rightarrow \quad a = \dfrac{p}{q}c$ and $b = \dfrac{p}{q}d$

$$\therefore \quad \frac{a^2+b^2}{c^2+d^2} = \frac{\frac{p^2}{q^2}c^2 + \frac{p^2}{q^2}d^2}{c^2+d^2} = \frac{p^2}{q^2}$$

13. We ignore $\frac{1}{7}$ and $\frac{1}{9}$ because no sum of other numbers is going to give 7th or 9th in the denominator.

Also, $\frac{1}{5}$ and $\frac{1}{10}$ are not enough to add up to anything $\left(\frac{1}{10}, \frac{2}{10}, \text{and } \frac{3}{10}\right.$ are going to leave tenths left over no matter what else you add$\left.\right)$.

What's left is $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$.

Sum total of these is $\frac{11}{8}$. So we need all of them except $\frac{1}{8}$, which means $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$. Which is the only way to do this with egyptian fractions whose denominators are 10 or less. Hence maximum number of Egyptian fractions needed is 3.

14. L.C.M. of 9, 10 and 15 = 90
$\therefore \quad 1936 = 90 \times 21 + 46$ (Remainder).
But a part of this remainder is 7.
so $\quad 46 - 7 = 39$ should be subtracted from 1936.

15. $6 - 4 = 2, 7 - 5 = 2, 8 - 6$
$= 2, 9 - 7 = 2, 10 - 8 = 2$
L.C.M. of 6, 7, 8, 9, 10 = 2520;
Greatest number of 6 digits = 999999
$2520 \times 396 + 2079 = 999999$
$\therefore$ Reminder = 2079.
Subtract 2079 from 999999, then we get
$999999 - 2079 = 997920.$
Subtract 2 from this number to get required number, which is 997918 and which will give the remainders 4, 5, 6, 7, 8 when divided by 5, 6, 7, 8, 9 respectively.

16. L.C.M. of 5, 6, 8, 9 and 12 is 360
Required number = 360 K + 1
$= (13 \times 27 + 9)\,K + 1$
$= (13 \times 27)\,K + (9K + 1)$
Now this number must be divisible by 13
$\therefore$ K = 10 and required number
= 3601.

17. I. In case all the remaining three numbers are multiples of the prime number, HCF of the group is equal to that prime number. otherwise HCF of the group is 1. Thus, I is true.

II. **Case 1:** Let numbers be 2, 4, 6 and 8, where 2 is only prime number, then LCM = 24.
But LCM of 4, 6 and 8 = 24.
LCM = 24 & 2 = 24
Case 2 : Let numbers be 2, 9, 81 and 27 Then LCM = 162 and LCM of 9, 81 and 27 = 81.
Thus, LCM = $81 \times 2 = 162$
Hence II is not always true.

III. Let numbers be 2, 4, 6 and 8
$\therefore$ LCM = 24 and HCF = 2
Product of four numbers
$= 2 \times 4 \times 6 \times 8 = 384$
Also, $384 \neq 24 \times 2 \times 2$.
Thus, III is not true.

18. (a. n)! = product of n consecutive natural number starting from 'a' which is atleast divisible by n!.
(n)! = product of n consecutive natural numbers.
For n = 2 :
(a. n)! = a(a + 1) and n! = 2
a(a + 1) is divisible by 2!.
For n = 3 :
(a n)! = a(a + 1)(a + 2) and n! = 6.
One of the factors of a(a + 1)(a + 2) is divisible by 3 and other by 2.
Thus, proceeding in this manner, (a. n)! and n! have HCF = n!
$\therefore$ H = n!.

19. Let HCF be h and LCM be l.
I. Let numbers be ah and bh.
Then $\quad abh = l$ and $(a + b)h = m$
$\Rightarrow \quad (a - b)h = n$
Using these ah and bh can be uniquely determined. Thus, I is true

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II. If HCF = LCM, then two numbers are equal and same as HCF or LCM.

Thus, II is true.

III. $\frac{\text{LCM}}{\text{HCF}}$ = a prime i.e., $\frac{\ell}{h}$ = P.

Then one of the numbers is equal to h and other is equal to ℓ. Thus, III is true.

20. Total distance travelled by the ball before coming to rest

$$= 12 + 2\,(6 + 3 + 1.5 + \ldots \text{ to } ?)$$

$$= 12 + 2.\left(\frac{6}{1-1/2}\right)$$

$$= 12 + 2(12) = 36\text{m}$$

21. Number of students who answered 1 or more questions wrongly = 2^{N-1}.

Number of students who answered 2 or more questions wrongly = 2^{N-2}

Hence number of students who answered 1 question wrongly

$$= 2^{N-1} - 2^{N-2} = 2^{N-2}$$

Similarly it can be shown that number of students who answered 2 questions wrongly

$$= 2^{N-2} - 2^{N-3} = 2^{N-3}$$

Similarly we can find number of students who answered K questions wrongly where K ≥ 3

Hence total number of questions attempted wrongly

$$s = 2^{N-2} + 2(2^{N-3}) + 3(2^{N-4}) + \ldots + (N-1)(2^{\circ}) + N(1) \ldots(i)$$

$$\therefore \quad \frac{s}{2} = 2^{N-3} + 2(2^{N-4}) + \ldots + (N-2)(2^{\circ}) + \frac{N-1}{2} + \frac{N}{2} \ldots(ii)$$

Substracting equations (*i*) from (*ii*)

$$\frac{s}{2} = 2^{N-2} + 2^{N-3} + \ldots + 2^{\circ} + \frac{1}{2}$$

$$\Rightarrow \quad s = 2^{N-1} + 2^{N-2} + 2^{N-3} + \ldots + 1$$

$$= 2^{N} - 1 = 8191$$

$$\Rightarrow \quad N = 13$$

22. Number of members in C_1 on May 1, 2003 = $x + 4a$

Number of members in C_2 on May 1, 2003 = xb^4

$$x + 4a = xb^4 \text{ and } a = 20x$$

$$\therefore \quad x(b^4 - 81) = 0$$

As $x \neq 0$, $b^4 - 81 = 0$

$$\therefore \quad b = 3$$

23. n^{th} terms are in the ratio of $\frac{2n+3}{n-11}$.

If n = 1 in 2n + 3 series, then first term is

$T_1 = 2(1) + 3 = 5$

n = 2, then second term is 2(2) + 3 = 7

$T_2 = 7$

$d = T_2 - T_1 = 7 - 5 = 2$

$\therefore \quad d_1 = 2$

If n = 1 in n –11series, then first term is 1 –11 = –10

If n = 2, then second term is 2 –11 = – 9

$\therefore \quad d_2 = T_2 - T_1 = -9 - (-10) = 1$

$\therefore \quad \frac{d_1}{d_2} = \frac{2}{1}$ or 2 : 1

24. $\frac{2}{3} + \frac{8}{9} + \frac{26}{27} + \frac{80}{81} + \ldots$ up to 20 terms

$$= \left(1-\frac{1}{3}\right) + \left(1-\frac{1}{9}\right) + \left(1-\frac{1}{27}\right) + \left(1-\frac{1}{81}\right) + \ldots \text{ up to 20 terms}$$

$$= 20 - \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \ldots + \frac{1}{3^{20}}\right)$$

$$= 20 - \frac{1}{3}\left(\frac{1-\left(\frac{1}{3}\right)^{20}}{1-\frac{1}{3}}\right)$$

$$= 20 - \frac{3^{20}-1}{2.3^{20}}$$

$$= \frac{40.3^{20} - 3^{20} + 1}{2.3^{20}} = \frac{39.3^{20}+1}{2.3^{20}}$$

25. Let $\quad s = 3 + 6x^2 + 9x^4 + 12x^6 + \ldots$

$$\therefore \quad x^2 s = 3x^2 + 6x^4 + 9x^6 + 12x^6 + \ldots$$

$$\Rightarrow \quad s - x^2 s = 3 + 3x^2 + 3x^4 + 3x^6 + \ldots$$

$$\Rightarrow \quad s(1-x^2) = 3\,[1 + x^2 + x^4 + x^6 + \ldots]$$

$$\Rightarrow \quad s(1-x^2) = 3\left[\frac{1}{1-x^2}\right]\left[\because |x| < 1\right]$$

$$\therefore \quad s = \frac{3}{\left(1-x^2\right)^2}$$

26. Average

$$= \frac{60\times56+50\times64+55\times72+62\times63+58\times50}{60+50+55+62+58}$$

$$= \frac{3360+3200+3960+3906+2900}{285} = 60.8$$

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27. Increase after 5 years should have been

$7 \times 5 = 35$ years.

But since average remains same, difference in their age is 35 years.

28. Let average of the seven men be x.

$\therefore \quad x = \dfrac{10 \times 6 + (x + 3)}{7}$

$\Rightarrow \quad 60 + x + 3 = 7x$

$\Rightarrow \quad x = \dfrac{63}{6} = 10.5$

$\therefore$ Total amount = $10.5 \times 7 = 73.5$

29. Consider a set of three numbers a, b and c.

I. $AM = \dfrac{a + b + c}{3}$

Thus, $a + b + c = 3AM$.

But GM = $\sqrt[3]{abc}$ which cannot be found out.

Thus, I is false,

II. $AM = \dfrac{a + b + c}{3}$

$\therefore$ Sum of the numbers = $a + b + c = 3AM$.

Thus, II is true.

III. GM = $\sqrt[3]{abc}$.

But AM = $\dfrac{a + b + c}{3}$, which cannot be found out.

Thus, III is false.

30. I. Average of a set of numbers is greater than smallest and smaller than the greatest number of the set. Thus, I is true.

II. Consider five numbers a, b, c, d and e whose average is $\dfrac{a + b + c + d + e}{5}$

Now if each of them is increased by k, then we have average

$= \dfrac{a + k + b + k + c + k + d + k + e + k}{5}$

$= \dfrac{a + b + c + d + e}{5} + \dfrac{5k}{5}$ = old average + k.

Thus, II is true.

III. Consider three numbers x, y and z whose average is $\dfrac{x + y + z}{3}$

Now if each of them is multiplied by m then new average = $\dfrac{xm + ym + zm}{3}$

$= m\left(\dfrac{x + y + z}{3}\right) = m \times$ odd average.

Thus, III is true.

31. Order of measures is

A, B, C, A + B, B + C, C + A, A + B + C.

Given: A + B + C = 180

Hence, average of the 7 measures

$= \left[\dfrac{[(A)+(B)+(C)+(A+B)+(B+C)+(C+A)+(A+B+C)]}{7}\right]$

$= \dfrac{4}{7}(A + B + C) = \dfrac{4}{7} \times 180 = \dfrac{720}{7}$ kgs.

32. Total age = 15×36

After two left, to tal age = $15 \times 36 - 2 \times 28$

$= 540 - 56$

$= 484$ years.

Let the age of the person who returns be x years.

Then new average = $\dfrac{484 + x}{14}$

Now x lies between 20 and 36 (both inclusive)

$\therefore$ New average min = $\dfrac{484 + 20}{14}$ = 36 years

And New average max = $\dfrac{484 + 36}{14}$

= 37.14. years.

NUMERICAL TYPE QUESTIONS

1. $x^3 - y^3 - 3xy$

$= x^3 - y^3 - 3xy(x - y)$ ...... as $(x - y) = 1$

$= (x - y)^3 = 1$.

2. $x^3 - y^3 - z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= 9[(x + y + z)^2 - 3(xy + yz + zx)]$

$= 9(81 - 33) = 432$.

3. $\dfrac{x^2}{yz} + \dfrac{y^2}{zx} + \dfrac{z^2}{xy}$

... (since, $x + y + z = 0$; $x^3 + y^3 + z^3 = 3xyz$)

$\dfrac{x^3 + y^3 + z^3}{xyz} = \dfrac{3xyz}{xyz} = 3$.

4. Worst scenario is when other four get equal number of votes.

Let the winning candidate get x votes.

$\therefore \quad x > \dfrac{261}{5}$

$\Rightarrow \quad x > 52.2$

$\therefore \quad x = 53$

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5. Number $= (555+445) \times (555-445) \times 2 + 30$
$= (555+445) \times 2 \times 110 + 30$
$= 220000 + 30 = 220030.$

6. $x + (x + 36) + y = 100$
$\Rightarrow \quad 2x + y = 64$
Therefore y must be even prime which is 2
$2x + 2 = 64 \quad \Rightarrow \quad x = 31.$
$\therefore$ Third prime number
$= x + 36 = 31 + 36 = 67.$

7. Given numbers are 1, 3, 5.........99.
This is an A.P with a = 1, d = 2.
Let it contain n terms
$\therefore \quad 1 + (n-1)2 = 99 \quad \Rightarrow \quad n = 50$
Hence required sum $= \frac{n}{2}$ (first term +last term)
$= \frac{50}{2}(1 + 99) = 2500.$

8. $A = 0.a_1a_1a_1 \ldots$
$\therefore \quad 10A = a_1.a_1a_1a_1\ldots..$
$\Rightarrow \quad 9A = a_1$
$\Rightarrow \quad A = \frac{a_1}{9}$
Similarly, $B = \frac{a_2}{9}$.
Now, a_1 and a_2 are multiples of 3 and are distinct.
Also, these values are less than 8.
either $a_1 = 3$ and $a_2 = 6$ or $a_1 = 6$ and $a_2 = 3$
$\therefore \quad a_1 + a_2 = 9.$
$\therefore \quad A + B = \frac{a_1}{9} + \frac{a_2}{9} = \frac{a_1 + a_2}{9} = \frac{9}{9} = 1.$

9. If Anil has to give 1 rupee he needs a bag with ₹1. For 2 rupees he had two bags with ₹1 each or ₹2. bag. To have minimum bags, he has a bag with ₹ 2. Now with the two bags he can give ₹3. So next he will require a bag with ₹4. With these three he can give ₹5. ₹6 and ₹7 and next bag will be one containing ₹8 and so on. Thus he would have bags with ₹1. 2. 4. 8. 16. 32. Sum of which is 63 and remaining 37 can be put in the last bag. So total number of bags is 7.

10. $2050 \times 2071 \times 2095$
$(23 \times 89 + 3) \times (23 \times 90 + 1) \times (23 \times 91 + 2)$
$= (23[89 \times 90 + 89 + 90 \times 3] + 3) \times (23 \times 91 + 2)$
$= (23 \times K + 3) \times (23 \times 91 + 2)$
$= (23 \times [K \times 91 + K \times 2 + 91 \times 3] + 6)$
$= 23 \times L + 6$ (where K and L are constants).
Hence remainder is 6.

11. L.C.M. of 27, 35, 25 and 21 = 4725
$\therefore$ Number = 4725 – 3 = 4722.

12. L.C.M. of 12, 21 and 35 = 420
$\therefore$ Number required = 420 + 6 = 426.

13. Number is 120K + 1
$= ((13 \times 9 + 3)K + 1)$
$= (13 \times 9)K + 3K + 1,$
which is divisible by 13.
3K + 1 is divisible by 13.
$\therefore \quad K = 4.$ Number = 481.

14. $\frac{a}{1-r} = 3$
$\Rightarrow \quad 3 - 3r = a$
Also, $\frac{a^2}{1-r^2} = 6$
$\Rightarrow \quad \frac{a}{1-r} \cdot \frac{a}{1+r} = 6$
$\Rightarrow \quad \frac{a}{1+r} = 2$
$\Rightarrow \quad 2 + 2r = a$
$\therefore \quad 3 - 3r = 2 + 2r$
$\Rightarrow \quad r = \frac{1}{5}$
$\therefore \quad a = 3 - \frac{3}{5} = \frac{12}{5}$

15. Since d = –2, maximum sum of the series will be sum of all the positive terms.
Least positive term of the series is 2 or 0 (both giving equal sum)
Let, 2 be the n^{th} term
$2 = 60 + (n-1)(-2)$
$\Rightarrow \quad n = 30$
$\therefore$ Required sum = 30/2 (60 + 2) = 930

16. Total income for 15 days = 15×7 = ₹105
Total income for first 5 days = 5×6 = ₹30
Total income for last 9 days = 9×8 = ₹72
$\therefore$ Income on sixth day
= 105 – (30 + 72)
= 105 – 102 = ₹3

17. $(GM)^2 = HM \times AM$
$$HM = \frac{8 \times 8}{5} = \frac{64}{5} = 12.8$$

18. Total marks of 15 students = $15 \times 145 = 2175$.
Average marks of 15 students (excluding two lowest scores) = $13 \times (145 + 5) = 1950$.
$\therefore$ Total of two lowest scores = 2175 – 1950 = 225
Given that two scores are consecutive multiples of 9
$\therefore \quad 9x + 9x + 9 = 225$
$\Rightarrow \quad 18x = 216$
$\Rightarrow \quad x = \frac{216}{18}$
$\therefore$ Lowest score = $9x = \frac{216}{18} \times 9 = 108.$

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EXERCISE – II

MCQ TYPE QUESTIONS

1. Type-I achieved a growth of 53% in the period which is higher than any other type of battery

4. Total number of students in class = 44

Average of marks,

$$= \frac{(2\times21)+(3\times15)+(1\times11)+(2\times23)+(5\times31)}{42}$$

$$= \frac{42+45+11+46+155}{44} = \frac{299}{44} = 6.795$$

$\therefore$ Option (*c*) is correct.

5. In the question the monthly average rainfall chart for 50 years has been given.

Let us check the options.

(*i*) On average, it rains more in July than in December $\Rightarrow$ correct.

(*ii*) Every year, the amount of rainfall in August is more than that in January.

$\Rightarrow$ may not be correct because average rainfall is given in the question.

(*iii*) July rainfall can be estimated with better confidence than February rainfall.

$\Rightarrow$ From chart it is clear the gap between 5 percentile and 95 percentile from average is higher in February than that in July $\Rightarrow$ correct.

(*iv*) In August at least 500 mm rainfall

$\Rightarrow$ May not be correct, because its 50 year average.

So correct option (*b*) (*i*) and (*iii*).

6. Item : 2

$$\frac{\frac{20}{100}\times250\times10^7}{\frac{20}{100}\times500\times10^3}$$

$$0.5\times10^4 = 5\times10^3 \boxed{1} = \text{Item 2}$$

Item : 3

$$\frac{23\times250\times10^7}{19\times500\times10^3}$$

1.2 = Item 3

Item : 6

$$\frac{19}{16} = 1.18 = \text{Item 6}$$

Item : 5

$$\frac{20}{12} = \frac{5}{3} = 1.6 \Rightarrow \boxed{1.6 = \text{Item 5}}$$

8. For 2005,

trade defict = (90 – 70) crores = 20 crores

Now $\frac{1}{5}^{\text{th}}$ of export = $\frac{1}{5}(70)$ crores

= 14 crores $\neq$ 20 crores.

Hence option (*a*) is wrong.

For 2004,

Trade defict =(80 – 70) crores = 10 crores

Now $\frac{1}{5}^{\text{th}}$ of export = $\frac{1}{5}(70)$ crores

= 14 crores $\neq$ 20 crores.

Hence option (*b*) is wrong.

For 2007

Trade defict = (120 – 110) crores

= 10 crores

Now $\frac{1}{5}^{\text{th}}$ of export = $\frac{1}{5}(110)$ crores

= 22 crores $\neq$ 10 crores.

Hence option (*c*) is wrong.

For 2006,

Trade defict = (120 – 100) crores

= 20 crores

Now $\frac{1}{5}^{\text{th}}$ of export = $\frac{1}{5}(100)$ crores

= 20 crores = Trade deficts

Hence option (*d*) is correct.

9. Total question

44 × 2 = 88

44 × 3 = 132

$$\frac{144}{132} = \frac{88}{308}$$

Total marks obtained = (21×2) + (15×3) + (23×2)

= 133

Total Number of students = 44

$$\text{Average} = \frac{133}{44} = 3.02$$

10. Take number of female students in 2011 = 100

$\therefore$ Number of male in 2011 = 100

No. of female in 2012 = 100

No. of male in 2012 = 150

$$\text{Ratio} = \frac{150}{100}$$

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11. All multiples of 10 from 2 to 198

$$= 10, 20, \ldots\ldots\ldots\ldots 190$$

Now $a_n = a + (n-1)d$

$$190 = 10 + (n-1) \times 10$$

$$180 = (n-1) \times 10$$

$$18 = n - 1$$

$\therefore$ $n = 19$

Hence total number of multiples are 19.

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$= \frac{19}{2}[20 + (18)(10)]$$

$$= \frac{19}{2}[20 + 180] = 1900$$

Now average of all multiples of 10 from 2 to198

$$= \frac{10 + 20 + \ldots + 190}{19}$$

$\therefore$ $= \frac{1900}{19} = 100$

12. $\sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}}$

For a particular type of question

i.e. $\sqrt{a + \sqrt{a + \sqrt{a + \ldots}}}$ we find a two consecutive number whose product is a. The greater number among the two consecutive number is the answer.

Here $a = 12$

i.e. $12 = 3 \times \underset{\substack{\downarrow \\ \text{greater number}}}{4}$

Hence $\sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}} = 4$

13. Here it is given that x is real

Now $|x^2 - 2x + 3| = 11$

$\Rightarrow$ $x^2 - 2x + 3 = 11$ or $x^2 - 2x + 3 = -11$

For $x^2 - 2x + 3 = 11 \Rightarrow x^2 - 2x - 8 = 0$

$$b^2 - 4ac = 4 - 4(1)(-8) = 36 > 0$$

Hence roots are real.

and for $x^2 - 2x + 14 = 0$

$$b^2 - 4ac = 4 - 4(1)(14) < 0$$

Hence roots are imaginary

$\therefore$ The value of x is the root of $x^2 - 2x - 8 = 0$

Now $x = \frac{+2 \pm \sqrt{4 - 4(1)(-8)}}{2}$

$$= \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2}$$

$\therefore$ $x = 4, -2.$

Now for $x = 4$,

$$|-x^3 + x^2 - x| = |-64 + 16 - 4| = 52.$$

and for $x = -2$

$$|-x^3 + x^2 - x| = |+8 + 4 + 2| = 14$$

Hence possible values of $|-x^3 + x^2 - x| = 14, 52$

16. $0 = 44 + (n-1)(-2)$

$$n - 1 = 22$$

$$n = 23$$

Sum $= \frac{n}{2}(a + l) = \frac{23}{2}(44 + 0)$

$$= 23 \times 22 = 506$$

Hence (c) is the answer

17. Let 'd' be the # of days required

Let 'h' be the daily working hours.

$\therefore$ dh can be taken as a measure of the total work be done in order to erect the structure.

Now working hours per day $h' = \frac{23}{24}h$

$\therefore$ # of days $d' = \frac{dh}{h'} = \frac{dh}{\left(\frac{23}{24}h\right)} = \frac{24}{23}d$

Let c = daily wages earlier

$\therefore$ $c = \frac{13,200}{d}$

Now daily wages $c' = \frac{6}{5}c$

$\therefore$ Total new cost $= c' \times d'$

$$= \frac{6}{5} \times \frac{13200}{d} \times \frac{24}{23}d \approx 16{,}500$$

18. **Given:** The expression

$$S = \frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{80} + \sqrt{81}}$$

To find: Sum of the expression.

Analysis: Each term of the expression is having irrational denominator. One can also see, that both the elements in the denominator of a term differ by only 1.

This gives an idea that on rationalising the denominator, each denominator will become 1, so the sum will be easier than earlier.

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Rationalising the denominator,

$$S = \frac{\sqrt{2}-\sqrt{1}}{(\sqrt{2})^2-(\sqrt{1})^2} + \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} + \ldots + \frac{\sqrt{81}-\sqrt{80}}{(\sqrt{81})^2-(\sqrt{80})^2}$$

$$= \frac{\sqrt{2}-\sqrt{1}}{1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \ldots. + \frac{\sqrt{81}-\sqrt{80}}{81-80}$$

$$= \sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+\ldots.+81-\sqrt{80}$$

Now, one can see that the terms will cancel out. Each term except $\sqrt{1}$ & $\sqrt{81}$

Occurs twice, once as positive and once a negative. So, after cancelling out, we are left with the following:

$$S = \sqrt{81}-\sqrt{1} = 9 - 1 = 8$$

So, the answer is (*b*).

19. $\frac{\text{Mon + Tues + Wed.}}{3} = 41$

Mon + Tues + Wed. = 123 ...(1)

$\frac{\text{Tues + Wed + Thurs.}}{3} = 43°$

Tue + Wed + Thu. = 129° ...(2)

(2) – (1)

Tues + Wed + Thu – (Mon + Tues. + Wed.)

= 129 – 123 = 6°

Thu. – Mon. = 6° ⇒ $\frac{115x}{100} - x = 6°$

Thus. = Mon × $\frac{115}{100}$ = $\frac{15x}{100}$ = 6°

Mon = x x = 40°

Thurs = $\frac{115x}{100}$

∴ Thurs = $\frac{115 \times 40}{100} = 46°$

24. Let 1.001 = x

$x^{1259} = 3.52$, and $x^{2062} = 7.85$

∴ $x^{3321} = x^{1259} . x^{2062}$

= 3.52 × 7.85

= 27.64

25. Let number of ₹ 20 notes be x and ₹ 10 notes be y

∴ $20x + 10y = 230$ ...(1)

and $x + y = 14$ ...(2)

Solving equations (1) and (2), we have

$x = 9$ and $y = 5$

Hence numbers of 10 rupee notes are 5.

26. Total budget = 10,500

Expenditure other than savings = 9000

Hence approximate percentage of monthly budget

$$= \frac{9000}{10500} = 86\%$$

27. By observation of the table, we can say S

	P	Q	R	S
Requirement	800	600	300	200
Potency	0.4	0.5	0.4	0.8

28. Let S = 4 (1 + 11 + 111 + ...)

$$= \frac{4}{9}(9 + 99 + 999 + \ldots)$$

$$= \frac{4}{9}\{(10-1) + (10^2-1) + (10^3-1) + \ldots\ldots\ldots\}$$

$$= \frac{4}{9}\{(10 + 10^2 + \ldots.+ 10^n) - n\}$$

$$= \frac{4}{9}\left\{10\frac{(10^n-1)}{9} - n\right\}$$

$$= \frac{4}{81}\{10^{n+1} - 9n - 10\}$$

29. Given : $f(y) = \frac{|y|}{y}$

$$\Rightarrow \quad f(q) = \frac{|q|}{q}$$

$$f(-q) = \frac{|-q|}{-q} = \frac{-|q|}{q}$$

$$|f(q) - f(q)| = \frac{|q|}{q} + \frac{|q|}{q} = \frac{2|q|}{q} = 2$$

NUMERICAL TYPE QUESTIONS

1. Increase in exports in 2006 = $\frac{100-70}{70}$ = 42.8%

Increase in imports in 2006 = $\frac{120-90}{90}$ = 33.3%

which is more than any other year.

2. According to question,

There are 40 males in Electrical Engg.

∴ Total number of females in Electrical Engg.

= 72 – 40 = 32

∴ Total number of students in Electrical Engg. = 72

⇒ 20% = 72

∴ 30% = Total number of students in Civil Engg.

∴ Total number of students in Civil Engg.

$$= \frac{30 \times 72}{20} = 36 \times 3 = 108$$

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$\therefore$ Number of female students in Civil Engg.

$$= \frac{4 \times 108}{9} = 4 \times 12 = 48$$

Now, total number of students in Mechanical Engg.

$$= \frac{10 \times 72}{20} = 36$$

$\therefore$ Number of female students in Mechanical Engg.

$$= \frac{4 \times 36}{9} = 16$$

$\therefore$ Difference between female students in Civil and Mechanical departments = 48 – 16 = 32

3. Fixed cost = ₹ 50,000

Variable cost = ₹ 800Q

Q = daily production in tones

For 100 tonnes of production daily, total cost of production = 50,000 + 800 × 100 = 130,000

So, cost of production per tonne of daily production

$$= \frac{130,000}{100} = ₹\ 1300.$$

4. Let the angles of quadrilateral are $3x$, $4x$, $5x$, $6x$

So, $3x + 4x + 5x + 6x = 360$

$$x = 20$$

Smallest angle of quadrilateral = 3 × 20 = 60°

Smallest angle of triangle = $\frac{2}{3} \times 60° = 40°$

Largest angle of triangle = 2 × 40° = 60°

Three angles of triangle are 40°, 60°, 80°

Largest angle of quadrilateral is 120°

Sum (2nd largest angle of triangle + largest angle of quadrilateral) = 60° + 120° =180°.

5. 8th observation is 7 × 2 = 14 more than 1st observation

9th observation is 14 more than 2nd observation

10th observation is 14 more than 3rd observation

11th observation is 14 more than 4th observation

12th observation is 14 more than 5th observation

Total 14 × 5 =70

Sum of the first five numbers = 425

Sum of last five numbers = 495

6.

	2012	2013
Transport (10%)	300,000	360,000
Labour (15%)	450,000	540,000
Raw material (20%)	750,000	780,000
Energy (25%)	750,000	900,000
Plant and Machinery (30%)	900,000	1,080,000
Total	**3,000,000**	**3,660,000**

Percentage increase in total cost = 22%

7. Total expenditure = $\frac{15}{100}\ x = 4{,}50{,}000$

$$x = 3 \times 10^6$$

Profit = 10 lakhs

So, total selling price = 40,00,000 ...(1)

Total purifies = 200 ...(2)

S.P. of each purifier = (1)/(2) = 20,000

8. Eight consecutive odd number = 656

$a - 6, a - 1, a - 2, a, a + 2, a + 4, a + 6$

$$a + 8 = 656$$

$$a = 81$$

Smallest $m = 75$ ...(1)

Average consecutive even numbers

$$\Rightarrow \frac{a-2+a+a+2+a+4}{4} = 87$$

$\Rightarrow a = 86$

Second largest number = 88 ...(2)

Adding equation (1) and (2),

$$75 + 88 = 163$$

9. Ratio of male to female student in 2008 = 5 : 2

In 2009, let the male student be $5x$ and female student be $2x$

Now if number of female students double in 2009

$\therefore$ Number of female student = $4x$

$\therefore$ Number of male student = $12x$

Gender	2008	2009
Male	$5x$	$12x$
Female	$2x$	$4x$

% of male student increase in 2009

$$= \left[\left(\frac{12x - 5x}{5x}\right) \times 100\right]\%$$

$$= \left[\frac{7}{5} \times 100\right]\%$$

= 140%

10. Expanding

$$\left(z + \frac{1}{z}\right)^2 = 98$$

$$\therefore \quad z^2 + \frac{1}{z^2} + 2(z)\left(\frac{1}{z}\right) = 98$$

$$\therefore \quad z^2 + \frac{1}{z^2} = 96$$

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11. Total respondents = 300

Now men respondents who do not have scooter

$= 40 + 20 = 60$

And women respondents who do not have scooter

$= 34 + 50 = 84$

$\therefore$ Total respondents who do not have scooter

$= 60 + 84 = 144$

$\therefore$ Required percent $= \left(\frac{144}{300} \times 100\right)\% = 48\%$

12. One way single person fare = ₹ 100

$\therefore$ Two way fare for single person = ₹ 200

$\therefore$ For 5 persons two way fare = ₹ 1000

Now, total discount = $(10 + 5)\% = 15\%$

Discount amount = ₹ $\left[\frac{15}{100} \times 1000\right]$ = ₹ 150

Amount to be paid = ₹$(1000 - 150)$ = ₹ 850

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2

CHAPTER

Percentage and Its Applications

The term percentage is quite frequently used in our day to day life especially to avoid the fractions less than 1. Instead of treating the complete entity as 1, we treat as 100, and take the ratios accordingly.

PERCENTAGE

Per Cent: This term means for every hundred. A fraction whose denominator is 100 is called *percentage* and numerator of the fraction is called *rate per cent*. It is denoted by the symbol %.

To find % equivalent of a fraction.

Express fraction with the denominator 100 and numerator is the required answer.

e.g., $$\frac{11}{24} = \frac{11}{24}\times 100 = 45\frac{5}{6}\%$$

To find fraction equivalent of 'a %': Divide 'a' by 100.

e.g., $$21\frac{7}{8}\% = \frac{21\frac{7}{8}}{100} = \frac{175}{800} = \frac{7}{32}$$

To increase a number by a given %:

Multiply the number by the factor $\frac{100+\text{rate}}{100}$

e.g., increase 20 by 15%,

i.e $$20\times\left(\frac{100+15}{100}\right) = 20\times\frac{115}{100} = 23$$

or $$20\times 1.15 = 23$$

To decrease a number by a given %:

Multiply the number by the factor $\frac{100-\text{rate}}{100}$

e.g., decrease 30 by 20%, i.e $30\times\left(\frac{100-20}{100}\right)$

$$= 30\times\frac{80}{100} = 24$$

or $$30\times 0.8 = 24$$

To find the % increase of a number:

$$\%\text{increase} = \frac{\text{total increase}}{\text{initial value}}\times 100$$

$$= \frac{\text{Final value} - \text{Initial value}}{\text{Initial value}}\times 100$$

e.g., Population of a village in 1980 was 3000 and in 1990 was 3200. Find the % increase.

$$\%\text{ increase} = \frac{3200-3000}{3000}\times 100$$

To find % decrease of a number:

$$\%\text{ decrease} = \frac{\text{total decrease}}{\text{Initial value}}\times 100$$

$$= \frac{\text{Initial value} - \text{Final value}}{\text{Initial value}}\times 100$$

e.g., Cost of a bike last year was ₹ 19000. Its cost this year is ₹ 17000. Find the % decrease in its cost.

$$\%\text{ decrease} = \frac{19000-17000}{19000}\times 100$$

$$= \frac{2000}{19000}\times 100 = 10.5\%$$

Note:

(1) If price of a commodity increases by r%, then reduction in consumption, so as not to increase expenditure is $\left(\frac{r}{100+r}\times 100\right)\%$

(2) If price of a commodity decrease by r%, then the increase in consumption so as not to decrease expenditure is $\left(\frac{r}{100-r}\times 100\right)\%$

(3) If A's income is r% more than that of B, then B's income is less than that of A by $\left(\frac{r}{100+r}\times 100\right)\%$

(4) If A's income is r% less than that of B, then B's income is more than that of A by $\left(\frac{r}{100-r}\times 100\right)\%$.

SIMPLE INTEREST AND COMPOUND INTEREST

When a sum of money is lent by A to B, A is called *lender* (*creditor*), B the *borrower* (*debtor*).

The sum lent is called *principal* (P).

Interest (I) is the extra money paid by the borrowed to the lender for the use of the money for a specified time.

The time for which the money is borrowed is called *period* (N).

The extra amount paid per 100 rupees in a year is called *rate per cent per annum* (R).

The sum of interest and principal is called *Amount* (A).

$$A = P + I$$

Simple Interest (S.I.)

When interest is paid as it falls due, it is called simple interest i.e., throughout the load period, interest is charged on the original sum (principal) borrowed.

$$\text{S.I.} = \frac{PNR}{100}$$

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e.g., Find interest to be paid on ₹ 1500 at 9% per annum for a period of 9 years.

Here P = 1500, R = 9%, N = 3 years

$$\therefore \quad \text{S.I.} = \frac{PNR}{100} = \frac{1500 \times 3 \times 9}{100} = ₹\ 405$$

Compound Interest (C.I.)

Money is said to be lent at compound interest when at the end of a year or other fixed period, the interest that has become due is not paid to the lender, but is added to the sum lent, and the amount thus obtained becomes the principal for the next year or period. The process is repeated until the amount for the last period has been found. The difference between final amount and the original principal is the *compound interest* (C.I.).

In compound interest, interest is calculated on the accrued interest also.

$$\text{Amount} = P\left[1 + \frac{R}{100}\right]^N$$

Compound Interest = Amount – Principal

Note:

(1) The time period after which the interest is added each time to form a new principal is called *conversion period*. It may be annually, semi annually or quarterly.

(2) In case, interest is paid semi annually (half yearly), N is number of half years and R is rate percent per half year *i.e.,*

Number of years × 2 = Number of half years.

Rate percent per annum ÷ 2 = Rate percent per half year.

(3) In case interest is paid quarterly, N is number of quarters and R is rate percent per quarter.

Number of years ÷ 4 = Number of quarters

Rate percent per quarter + 4 = Rate percent per quarter.

(4) When rates are different for different years, say R_1, R_2, R_3 percent of 1^{st}, 2^{nd} and 3^{rd} years respectively then

$$\text{Amount} = P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right)$$

(5) In case the time is a fraction of a year, say $y\frac{x}{z}$ years, then

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^y \times \left(1 + \frac{\frac{x}{z} \times R}{100}\right)$$

Population Formula: The original population of a town is P and annual increase is r%, then population in n years is $\left(1 + \frac{R}{100}\right)^n$ and if annual decrease is r%, then population in n years is given by a change of sign in the formula

i.e., $\left(1 - \frac{R}{100}\right)^n$

e.g., If annual increase in the population of a town is 4% and present population is 15625, what will be the population in 3 years?

Required population = 15625 $(1.04)^3$ = 17576

Hire Purchase: In a hire purchase plan, a customer can make use of the goods while paying for them. The amount paid at the time of purchase is called *down payment*. The remainder is paid in equal installments and each is the monthly installment. The difference between total amount to be paid and the cash price is called *installment charge*.

Monthly Installment

$$= \frac{\text{Amount to be paid} - \text{down payment}}{\text{Number of instalments}}$$

e.g., If a transistor is available at ₹ 400 each or ₹ 100 down payment and ₹ 70 per month for 5 months, find total amount paid for it, and the installment charge

Amount paid = 100 + 70 × 5 = ₹ 450

Installment charge = 450 – 400 = ₹ 50

PROFIT AND LOSS

Cost Price (CP): The price for which an article is bought is called its *cost price*.

Selling Price (SP): The price at which an article is sold is called its *selling price*.

Profit (Gain): The difference between selling price and cost price is called *profit*. For profit, selling price should be greater than cost price.

Loss: The difference between cost price and the selling price is called *loss*. When cost price is greater than the selling price, there is a loss.

Profit and loss is generally represented as a percent of the cost price, unless otherwise stated.

Overhead charges: If an individual has to spend some money on transportation etc., then this extra expenditure is called *overhead charges*.

Marked price (MP): The price on the label is called *marked price* or *list price*.

Discount: The reduction made on the '*marked price*' of an article is called *discount*. When no discount is given, '*selling price*' is the same as '*marked price*'.

List of Formulae

(1) Profit = SP – CP ... (SP > CP)

(2) Loss = CP – SP ... (CP > SP)

(3) $\% \text{ Profit} = \frac{\text{Profit}}{CP} \times 100 = \frac{SP - CP}{CP} \times 100$

(4) $\% \text{ Loss} = \frac{\text{Loss}}{CP} \times 100 = \frac{CP - SP}{CP} \times 100$

(5) $\text{Profit} = \frac{\text{Profit}\% \times CP}{100}$

(6) $\text{Loss} = \frac{\text{Loss}\% \times CP}{100}$

(7) $SP = \frac{100 + \text{Profit}\%}{100} \times CP$

(8) $SP = \frac{100 - \text{Loss}\%}{100} \times CP$

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(9) SP = Marked Price – Discount

(10) $CP = SP \times \left(\frac{100}{100+\text{Profit}\%}\right)$

(11) $CP = SP \times \left(\frac{100}{100-\text{Loss}\%}\right)$

(12) $\text{Discount }\% = \frac{\text{Discount}}{\text{Marked Price}} \times 100$

Some important point:

(1) If two items are sold, each at ₹ X, one at a gain of p% and the other at a loss of p%, there is an overall loss given by $\frac{p^2}{100}\%$. The absolute value of the loss is given by $\frac{2p^2x}{100^2 - p^2}$

(2) If CP of two items is the same and % Loss and % Gain on the two items are equal, then net loss or net profit is zero.

(3) Buy x get y free *i.e.,* if x + y articles are sold at cost price of x articles, then

$$\text{percentage discount} = \frac{x}{x+y} \times 100.$$

(4) By using false weight, if a substance is sold at cost price, then

$$\text{overall gain }\% = \frac{100+\text{Gain}\%}{100} = \frac{\text{True Scale or Weight}}{\text{False Scale or Weight}}$$

(5) In case of successive discount a% and b%, effective discount $= \left(a + b - \frac{ab}{100}\right)\%$.

PARTNERSHIP

It is an association of two or more persons who invest their money in order to carry on a certain business. A partner who manages the business is called *working partner* and the one who simply invests the money is called *sleeping partner*.

If capitals of the partners are invested for the same time, then partnership is called *simple*, and if invested for different periods, the partner is called *compound*.

If period of investment is the same for each partner, then profit or loss is divided in the ratio of their investments.

e.g., (*i*) If A and B are partners in a business, then

$$\frac{\text{Investment of A}}{\text{Investment of B}} = \frac{\text{Profit of A}}{\text{Profit of B}}$$

or $$\frac{\text{Investment of A}}{\text{Investment of B}} = \frac{\text{Loss of A}}{\text{Loss of B}}$$

(*ii*) If A, B and C are partners in a business, then Investment of A : Investment of B : Investment of C

= Profit of A : Profit of B : Profit of C

OR Loss of A : Loss of B : Loss of C

Monthly Equivalent Investment: It is the product of the capital invested and the period for which it is invested.

If period of investment is different, then profit or loss is divided in the ratio of their Monthly Equivalent Investment.

e.g., If A and B are partners in a business, then,

$$\frac{\text{Monthly Equivalent Investment of A}}{\text{Monthly Equivalent Investment of B}} = \frac{\text{Profit of A}}{\text{Profit of B}} = \frac{\text{Loss of A}}{\text{Loss of B}}$$

i.e., $$\frac{\text{Investment of A} \times \text{Period of Investment of A}}{\text{Investment of B} \times \text{Periof of Investment B}} = \frac{\text{Profit of A}}{\text{Profit of B}} = \frac{\text{Loss of A}}{\text{Loss of B}}$$

If A, B and C are partners in a business, then

Monthly Equivalent Investment of A

: Monthly Equivalent Investment of B

: Monthly Equivalent Investment of C = Profit of A

: Profit of B : Profit of C

or

Monthly Equivalent Investment of A : Monthly Equivalent Investment of B

: Monthly Equivalent Investment of C = Loss of A

: Loss of B : Loss of C

STOCKS AND SHARES

Face value: The price of a share is printed on the share certificate is called face value of the share. (Nominal value).

Dividend: Every shareholder of a company is entitled to a proportionate share of the profits of the company. The amount of profits on each share is called *dividend*.

Dividend is expressed as a percentage of the face value.

Market Value: In the market, selling price of the share may be different from its face value. Thus, market value of a share is *fluctuable price*.

Shares: *In the market, three types of shares are available* :

(*i*) **Premium share:** If market price of a share is more than nominal value, then it is called *premium share*.

(*ii*) **Par share:** If market price of a share is equal to the nominal value, then it is called *par share*.

(*iii*) **Discount share:** If market price of a share is less than the nominal value, then it is called a *discount share*.

Types of shares: *There are two types of shares:*

(*i*) **Preference shares:** On these shares a fixed rate of dividend is paid to share holders irrespective of whether the company makes any profits or whether it runs into loss. In case the company is not able to pay the agreed dividend to preference shareholders, the dividend on cumulative preference shares goes on accumulating and is paid as and when the company can do so.

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(*ii*) **Equity shares:** Profit of the company that is left over after its distribution to the preference shareholders is distributed among the equity share holders. This dividend on equity share may sometimes be more or sometimes less than the dividend on the preference share.

Stock

In order to meet expenses of a certain plan, the Government of India sometimes raises a load from the public at a certain fixed rate of interest. Bonds or promissory notes called *stocks of a fixed value* are used for sale to the public.

Note:

(1) Income per share = Rate of dividend × Nominal value of 1 share.

(2) Annual income = Income per share × Number of shares.

(3) Number of shares

$$= \frac{\text{Nominal value of all the shares}}{\text{Nominal value of one shares}}$$

$$= \frac{\text{Market value of the shares (Investment)}}{\text{Market value of one share}}$$

$$= \frac{\text{Total Dividend}}{\text{Dividend per share}}$$

(4) Rate of interest on the Invest (yield)

$$= \frac{\text{Total income } \times 100}{\text{Total investment}}$$

(5) When stoke/share is purchased, brokerage is added to cost price.

(6) When stock/share is sold, brokerage is subtracted from selling price.

(7) In most problems, face value of a share is taken to be ₹, 100 unless specified.

EXERCISE – I

MCQ TYPE QUESTIONS

1. The Manager (M) of the company along with his Assistant Manager (AM) and a Business Analyst (BA) invest ₹ 55000, ₹ 45000 and ₹ 60000 respectively in a partnership firm for six months. The Manager being the highest authority among the three gets one-tenth of the total profit and Assistant Manager being second highest authority gets $\frac{1}{20}$th of the remaining and the remaining profit is divided among the three in the ratio of their investments. If the total profit was ₹ 40000, who gets the minimum share and how much?

(*a*) BA, ₹ 11400 (*b*) AM, ₹ 11419
(c) AM, ₹ 12500 (*d*) None of these

2. In a business partnership of three people A, B and C, B's share in total profit is ₹ 250 per ₹ 2000. At this profit his return on investment is 15%. If B earns total of ₹ 900. what is the total investment made by A and C in business? Assume that each of A, B and C have invested their money for the entire year and profits are shared in the ratio of their investments.

(*a*) ₹ 48000 (*b*) ₹ 42000
(*c*) ₹ 6000 (*d*) Data insufficient

3. Abhay and Bimal started a business in partnership. Abhay invested a capital of ₹ 50000. Bimal agrees to pay Abhay an interest on half the capital at 8% per annum. Also Bimal receives a salary of ₹ 500 per month for managing the business. After six months Abhay withdraws ₹ 20000 and Bimal pitches in an equal amount. Bimal continues to pay Abhay interest at 8% per annum for the balance amount. At the end of the year they divide the profit equally and found that total earning of Bimal in the year was exactly thrice that of Abhay. The total profit ?

(*a*) ₹ 7200 (*b*) ₹ 6800
(*c*) ₹ 5400 (*d*) ₹ 4800

4. How much stock must be bought in 3 percent $89\frac{1}{8}$ in order that by selling it at $91\frac{3}{8}$ a profit of ₹ 45 may be made?

(*a*) 20 (*b*) 30 (*c*) 40 (*d*) 50

5. Which is the better investment (the security being equal), 3 percents perpetually at par, or 5 percents at 124, the latter stock being redeemable in 20 years at par? (Simple interest to be reckoned).

(*a*) 3% at par
(*b*) 5% at 124
(*c*) Both are same
(*d*) Cannot be determined

6. What rate percent per annum does a man get by buying ₹100 share (₹ 70 paid-up) at ₹ 60 when dividend is 5% per annum?

(*a*) 4.28% (*b*) 7.14%
(*c*) $8\frac{1}{3}$ % (*d*) $5\frac{5}{6}$ %

7. A rejects 0.08% of the meters as defective .How many will he examine to reject 2?

(*a*) 2500 (*b*) 1250
(*c*) 625 (*d*) 5000

8. (a% of b) + (b% of a) is equal to

(*a*) a% of b (*b*) 2% of ab
(*c*) 20% of ab (*d*) 2% of 100 ab

9. Ryan's stock of goods is worth $9462, which is 15% more than Harland's, and 15% less than Lambert's. What is the value of stock carried by Harland and Lambert respectively?

(*a*) $ 8218, $ 10032 (*b*) $ 1419, $ 10881
(*c*) $ 8042, $ 10881 (*d*) $ 8228, $ 11132

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10. Numerator of a fraction increases by 25%. What can be said about the resultant fraction?

(*a*) It is 67% less than original.

(*b*) It is equal to the original fraction.

(*c*) It is 67% more than the original fraction.

(*d*) The change cannot be determine.

11. Which of the following statement/s is/are true?

I. If two numbers are such that one is 25% more than the second, then sum of the two numbers is nine times that of the difference of the two numbers.

II. If 56% of a number A is 24% of another number B then 49% of A is 21% of B.

Codes:

(*a*) Only I (*b*) Only II

(*c*) Only I and II (*d*) None of these

12. In the Annual General Meeting of the Rotary club a chairman is to be selected for which A and B is the only two candidates. After the first round, each of them gave a speech on their policy and work plan for the coming year after which another round of election was held. The number of votes not cast for A increased by 25% in the second round over those not cast for him in the first round and A lost second round to B by twice as many votes as that by which he had won in the first round. If total 260 people voted each time what was the percent decrease in votes received by A in the second round assuming that no vote was void in any of the rounds?

(*a*) 18% (*b*) 25% (*c*) 21.43% (*d*) 30%

13. A watermelon weighs 5000 gm. 99% of its weight is water. It is kept in a drying room and after some time it turns out that it is only 98% water by weight. What is its weight now?

(*a*) 2500 gm

(*b*) 4500 gm

(*c*) 4950 gm

(*d*) None of these

14. A sum of ₹ 5000 is divided into two parts A and B such that simple interests at the rate of 20% p.a. on A and B after 2 and 3 years respectively are equal. Which of the following is false about A and B?

(*a*) Ratio of A to B is 2 : 3

(*b*) Ratio of A to B is 3 : 2

(*c*) A exceeds B by ₹ 1000

(*d*) Total interest earned on A and B is ₹ 2400

15. Which amongst the following two offers is the better one? Consider with respect to interest accrued.

I. Investing an amount compounded annually at 1% per annum for 100 years.

II. Investing the amount compounded annually at 100% per annum for 1 year.

Codes:

(*a*) First offer

(*b*) Second offer

(*c*) Both are same

(*d*) Cannot be determined

16. The population of rate in a locality x increases by 20% in one year. Observing this, the pest control committee decided to use a special kind of pesticide 'xyz' which effectively kills 160 rats in 3 months. Just after 2 years, what is the net increase or decrease in the population of rats if, initially the population of rats is 3200 and pesticide is used effectively?

(*a*) Increase of 128 rats.

(*b*) Decrease of 128 rats.

(*c*) Neither an increase nor a decrease in the population.

(*d*) None of these.

17. A man travels from A to B to buy goods which he can get 10% cheaper in B than in A. If expenses of the journey are ₹ 15 and he makes a clear saving of ₹ 10, how much does he pay for the goods?

(*a*) ₹ 225

(*b*) ₹ 200

(*c*) ₹ 150

(*d*) Cannot be determined

18. By selling 5 dozen mangoes for ₹ 156, it was found that $\frac{3}{10}$th of the outlay was gained. What should the retail price per mango be in order to gain 60%?

(*a*) ₹ 4 (*b*) ₹ 2 (*c*) ₹ 3.2 (*d*) ₹ 4.2

19. A man sells sugar to a tradesman at a profit of 20% but the tradesman becoming bankrupt pays only 80 paise in the rupee. How much percentage does the man gain or lose by his sale?

(*a*) 2.5% (*b*) 3% (*c*) 4% (*d*) 5.2%

20. A trader allows a discount of 5 percent to his customers. What price should he mark on a article the cost price of which is ₹ 800 so as to make clear profit of 25 percent on his outlay?

(*a*) ₹ 1000 (*b*) ₹ 1053

(*c*) ₹ 1200 (*d*) ₹ 1123

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21. A camera costing ₹550 is marked to be sold at a price, which gives a profit of 30%. What will be its selling price in a sale when 20% is taken off the marked price?
    (*a*) ₹ 600 (*b*) ₹ 572
    (*c*) ₹ 635 (*d*) ₹ 605

22. Buy two and get one free. Which of the following is true?
    (*a*) The person is selling at a loss.
    (*b*) The person is selling at a profit.
    (*c*) The loss or gain depends on the profit from sale of two articles.
    (*d*) The person is selling at the cost price.

Directions (Q.23–25): *Refer to the data below and answer the questions that follow.*

A profit-maximizing producer has the option of discriminating between markets A and B (i.e., charging different prices in markets A and B for the same product) for a product X. Demand for the product in each of the markets A and B is given by

$$Q_A = 42 - 0.2\, P_A$$
$$Q_B = 100 - 0.8\, P_B$$

Total cost in any market is given by

$C = 1000 + 5Q,$ *where Q is quantity sold in that market.*

Consider two cases:

Case 1 *in which she discriminates between the markets and*

Case 2 *in which she does not discriminate.*

23. What is the approximate ratio of the price in market A to price in market B in Case 1?
    (*a*) 1.65 (*b*) 2.00
    (*c*) 2.25 (*d*) 2.50

24. What is the ratio of the total quantity of product X sold in Case 1 to the total quantity sold in Case 2?
    (*a*) 0.50 (*b*) 1
    (*c*) 1.10 (*d*) 1.05

25. What is the difference in total profit in Case 1 and the total profit in Case 2?
    (*a*) ₹ 289 (*b*) ₹ 829 (*c*) ₹ – 289 (*d*) ₹ – 829

NUMERICAL TYPE QUESTIONS

1. A and B enter into a partnership. A puts in the whole capital of ₹ 45000 on the condition that the profits will be equally divided after which B will pay A interest on half the capital at 10% p.a. and receive ₹ 60 per month from A for carrying on the concern. The yearly profit in rupees, if B's income is half of A's income is ______

2. A, B and C started a business by investing ₹ 72000, ₹ 27000 and ₹ 81000. It was decided that servant's salary ₹ 2000 per month should be given from the profit. If profit is ₹ 25000 in the first month and ₹ 20000 in the second month and average profit of remaining 10 months is ₹ 21000, then share of C is _______

3. Three friends started a business of renting out air conditioners by investing ₹ 20000, ₹ 24000 and ₹ 16000, respectively. C gets 20% of total profit for repair and maintenance of the air conditioner. If in a particular year, C gets ₹ 487.50 less than total earnings of the other two, then total profit for the year in ₹ is ________

4. A person holds forty ₹ 500 shares in a concern, which pays dividend at the rate of 6 percent per annum. When the shares are at ₹ 675, he sells out and invests half the proceeds in 4 percent stock at 90. With the other half he buys a house. For which he receives an annual rent of ₹ 1440 subject to a deduction of 25P per rupees for repairs and taxes. Alteration in rupees in his annual income will be ________.

5. A person invests ₹ 34539 in three percent at 87. After receiving one year's dividend, he sells out at 89. He then invests the whole in railway stock paying 5 percent at 115.The difference in his income will be_______

6. Difference of two numbers is 1660. If 7.5 % of one number is 12.5% of the other number, then two numbers are______

7. A school has only three classes which contain 40,50,60 students respectively. The pass percent of these classes are 10, 20 and 10 respectively. Then pass percent in the school will be______

8. For a sphere of radius 10cm, the numerical value of surface area is ______ percent of the numerical value of its volume.

9. At an election, there are two candidates only, a candidate who gets 43 per cent of the votes is rejected by a majority of 420 votes. Then total number of votes recorded assuming that there was no void vote are______

10. If price of wheat be raised 30%, then by _____ percent a householder must reduce his consumption of the article so as not to increase his expenditure?

11. A fraction in reduced form is such that when it is squared and then its numerator is reduced by $33\frac{1}{3}\%$ % and denominator is reduced to 20%, its result is twice the original fraction. Sum of the numerator and the denominator is _____

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12. A person borrows two equal sums at the same time at 5 and 4 percent respectively and finds that if he repays the former sum with simple interest on certain date 6 months before the latter, he will have to pay in each case the same amount, viz., ₹ 1100. The amount borrowed in rupees is ______

13. Effective annual rate corres-ponding to a nominal rate of 6 percent per annum, payable half-yearly is ______

14. A tradesman puts two prices on his goods, one for ready money and the other for 6 month's credit interest being calculated at $12\frac{1}{2}$% per annum.If credit price on an article be ₹ 26.56, then its cash price in rupees is _______

15. An article costing ₹9000 is sold at a discount which is equal to the simple interest on ₹ 3000 for N months. If rate of discount is same as the rate of interest, the N will be_______months

16. A lent B ₹ 900 for a certain time at a certain rate percent per annum, which is equal to the square root of the number of the months of the time. After the time B wanted to return the money, but A, instead of taking the interest which amounted to ₹8 asked for a certain sum of money at the same rate for a certain number of years which is equal to the square root of the rate percent. Sum of money in rupees so that none of them should be the loser will be_______ (Take simple interest)

17. A shopkeeper sells a microwave at a discount on its marked price of ₹ 2500. But in a clearance sale he gives an additional discount of 20% thus selling it at ₹ 1500. Then first discount was_____%

18. A sports goods shop owner has 6 packets of balls each containing the same number of balls. If he sells each ball at ₹ 8 he gains ₹ 96. If there were 1 ball less in each packet and he sells each ball at ₹ 9 then he gains ₹ 90 in the transaction. The cost price of each packet of balls in rupees is ______

19. One merchant correctly calculates his percentage profit on the cost price; another wrongly calculates it on the selling price. The difference in actual profits in rupees if both claim to make $17\frac{1}{2}$% profit on goods sold at ₹ 3760 is______

20. A manufacturer marks his goods in advance at 80 percent more than the cost price, but he allows 15 articles to the dozen and also 10 percent discount for cash. The profit on his outlay does he obtain from a customer who pays cash is _____%

21. A trader buys a certain amount of goods worth ₹ 22520. He decides to make a profit of 5.36% on the sale of goods worth ₹ 5000 and increase the profit percent by 3.14% for sales upto ₹15000 and then increase the profit percent for the sale of remaining lot such that he is able to make a profit of 25% on the sale of the full lot. Then profit in rupees that he makes on the third lot of goods is______

22. Three shepherds A, B and C rented a pasture for a year. A grazed 22 sheeps for 4 months, B grazed 16 sheeps for 8 months and C grazed 32 sheeps for 6 months. If C's share of rent is ₹600, then total rent in rupees for the year was_____

23. Three bachelors, Amar, Akbar and Anthony rented a house for a year. But, Amar left after 4 months, Akbar stayed for 8 months and only Anthony stayed for the entire year. If the annual rent was ₹ 6000, then share of Akbar is______

24. Ram, Shyam and Ghanshyam invest ₹2000, ₹ 10000 and ₹ 5000 to set up a stall. Ram gets 15% of the total profit for running the stall. Ram gets ₹ 500 less than Shyam and Ghanshym together, then total profit of that year is_______

25. x, y and z start a joint venture wherein they make an annual profit of ₹ 3600. x invested one-third of the capital for one-fourth of the time, y invested one-fourth of the capital for one-half of the time, while z invested the remainder of the capital for the entire year. Then share of x in the profit is_____.

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EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2014

1. The population of a new city is 5 million and is growing at 20% annually. How many years would it take to double at this growth rate?

(a) 3 – 4 years (b) 4 – 5 years
(c) 5 – 6 years (d) 6 – 7 years

2. One percent of the people of country X are taller than 6 ft. Two percent of the people of country Y are taller than 6 ft. There are thrice as many people in country X as in country Y. Taking both countries together, what is the percentage of people taller than 6 ft?

(a) 3.0 (b) 2.5
(c) 1.5 (d) 1.25

3. Industrial consumption of power doubled from 2000-2001 to 2010-2011. Find the annual rate of increase in percent assuming it to be uniform over the years.

(a) 5.6 (b) 7.2
(c) 10.0 (d) 12.2

4. The Gross Domestic Product (GDP) in Rupees grew at 7% during 2012-2013. For international comparison, the GDP is compared in US Dollars (USD) after conversion based on the market exchange rate. During the period 2012-2013 the exchange rate for the USD increased from ₹ 50/ USD to ₹ 60/ USD. India's GDP in USD during the period 2012-2013

(a) increased by 5% (b) decreased by 13%
(c) decreased by 20% (d) decreased by 11%

2013

5. A firm is selling its product at ₹ 60 per unit. The total cost of production is ₹ 100 and firm is earning total profit of ₹ 500. Later, the total cost increased by 30%. By what percentage the price should be increased to maintained the same profit level.

(a) 5 (b) 10
(c) 15 (d) 30

2012

6. An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable.

The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is

(a) 0.288
(b) 0.334
(c) 0.667
(d) 0.720

2011

7. There are two candidates P and Q in an election. During the campaign, 40% of the voters promised to vote for P, and rest for Q. However, on the day of election 15% of the voters went back on their promise to vote for P and instead voted for Q. 25% of the voters went back on their promise to vote for Q and instead voted for P. Suppose, P lost by 2 votes, then what was the total number of voters?

(a) 100
(b) 110
(c) 90
(d) 95

NUMERICAL TYPE QUESTION

2015

1. From a circular sheet of paper of radius 30 cm, a sector of 10% area is removed. If the remaining part is used to make a conical surface, then the ratio of the radius and height of the cone is ____________.

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ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*b*) **2.** (*b*) **3.** (*a*) **4.** (*a*) **5.** (*b*) **6.** (*d*) **7.** (*a*) **8.** (*b*) **9.** (*d*) **10.** (*c*)
11. (*c*) **12.** (*c*) **13.** (*a*) **14.** (*a*) **15.** (*a*) **16.** (*c*) **17.** (*a*) **18.** (*c*) **19.** (*c*) **20.** (*b*)
21. (*b*) **22.** (*c*) **23.** (*a*) **24.** (*b*) **25.** (*a*)

Numerical Type Questions

1. 9180 **2.** 103950 **3.** 2812.50 **4.** 480 **5.** 345.2 **6.** (4150, 2490)
7. $13\frac{1}{3}$ **8.** 30 **9.** 3000 **10.** $23\frac{1}{13}$ **11.** 8 **12.** 1000
13. 6.09 **14.** 25 **15.** 4 **16.** 600 **17.** 25 **18.** 48
19. 98 **20.** 29.6 **21.** 4512 **22.** 1275 **23.** 2000 **24.** 1000
25. 480

EXERCISE – II

MCQ Type Questions

1. (*a*) **2.** (*d*) **3.** (*b*) **4.** (*d*) **5.** (*a*) **6.** (*b*) **7.** (*a*)

Numerical Type Question

1. (2.06)

EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. The ratio of their investments is 11 : 9 : 12.

So, Manager gets $\frac{1}{10}(40000) = 4000 +$ his share
Remaining amount = 36000.

Asstt.Manager gets $\frac{1}{20}(36000) = 1800 +$ his share
Remaining amount = 36000 – 1800 = 34200

$$\therefore \quad \text{M's share} = \frac{11}{32} \times 34200 + 4000$$
$$= 11756.25 + 4000$$
$$= ₹\ 15756.25$$
$$\cong ₹\ 15756$$

$$\text{AM's share} = \frac{9}{32} \times 34200 + 1800$$
$$= 9618.75 + 1800$$
$$= ₹\ 11418.75$$
$$\approx ₹\ 11419$$

$$\text{BA's share} = \frac{12}{32} \times 34200 = ₹\ 12825.$$

2. B's share in the profit = 250 per 2000
If B gets ₹ 900, then

$$\text{total profit} = 2000 \times \frac{18}{5} = 7200$$

B's return on investment = 15%

$$\therefore \quad \frac{900}{\text{B's investment}} \times 100 = 15$$

$\therefore$ B's investment = 6000
Hence profits are shared in the ratio of their investments.

$$\text{Total investments} = 6000 \times \frac{7200}{900} = 48000$$

$\therefore$ Investment made by A and C = 48000 – 6000
= 42000.

3. Interest paid by Bimal to Abhay for first six months

$$= \frac{50000}{2} \times \frac{8}{100} \times \frac{1}{2} = ₹\ 1000$$

For last six months

$$= \frac{30000 - 20000}{2} \times \frac{8}{100} \times \frac{1}{2}$$
$$= ₹\ 200$$

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$\therefore$ Total salary paid to Bimal = 12×500 = ₹ 6000

Let total profit be ₹ x

$\therefore$ Abhay's earing $= 1200 + \dfrac{x - 6000}{2}$

Bimal's earning $= 6000 + \dfrac{x - 6000}{2} - 1200$

$$\therefore \quad 3\left(1200 + \frac{x - 6000}{2}\right) = 4800 + \frac{x - 6000}{2}$$

$$\Rightarrow \quad 3600 + \frac{3x}{2} - 9000 = 4800 + \frac{x}{2} - 3000$$

$$\Rightarrow \quad \frac{3x}{2} - \frac{x}{2} = 7200;$$

$\therefore$ x = ₹ 7200

4. Profit per stocks $= 91\frac{3}{8} - 89\frac{1}{8} =$ ₹ $2\frac{2}{8}$

$$\therefore \quad \frac{45}{2\frac{1}{4}} = 20 \text{ stocks}$$

5. Income from a stock at 3% in 20 years at par

$$= \frac{100}{100} \times 3 \times 20 = 60$$

Income from a stock at 5% at 124 in 20 years

$$= \frac{100}{124} \times 5 \times 20 = 80.64.$$

6. He invests ₹ 60 and gets ₹ 100 share

Income = ₹ 5

Required % $= \dfrac{5}{60} \times 100(0.70) = 5.83\% = 5\dfrac{5}{6}\ \%$

7. Let number of meters to be examined be x.

Then $\quad 0.08\%$ of $x = 2$.

$$\Rightarrow \quad \frac{0.08}{100} \times x = 2$$

$$x = \frac{2 \times 100}{0.08} = \frac{2 \times 100 \times 100}{8} = 2500$$

8. $\left(\dfrac{a}{100} \times b\right) + \left(\dfrac{b}{100} \times a\right) = \dfrac{ab}{100} + \dfrac{ab}{100} = \dfrac{2ab}{100}$

$$= \frac{2}{100} \times ab = 2\% \text{ of ab.}$$

9. Ryan's stock = 1.15 × Harland's stock

$\therefore$ Harland's stock $= \dfrac{9462}{1.15} = \$8228$

Ryan's stock = 0.85 × Lambert's stock

$\therefore$ Lambert's stock $= \dfrac{9462}{0.85} = \$11132.$

10. Let the fraction be $\dfrac{x}{y}$.

Let $\quad x_1 = x + 0.25x = 1.25x$

$\quad\quad y_1 = y - 0.25x = 0.75x$

$$\therefore \quad \frac{x_1}{y_1} = \frac{1.25x}{0.75x} = \frac{5}{3}\frac{x}{y}$$

$$\therefore \quad \text{Increase} = \frac{\frac{5}{3}\frac{x}{y} - \frac{x}{y}}{\frac{x}{y}} \times 100 = \frac{2}{3} \times 100 = 66.67\%$$

Thus, resultant fraction is more than the original fraction by 67%. Thus III is true.

11. I. Let first number be x.

$\therefore$ Second number = 1.25x

Sum = x + 1.25x = 2.25x

Difference = 1.25x – x = 0.25x

Also, $\quad 9 \times 0.25x = 2.25x$

Thus I is true.

II. $\dfrac{56}{100} \times A = \dfrac{24}{100} \times B$

Multiplying by $\dfrac{49}{56}$ on both the sides.

$$\therefore \quad \frac{49}{56} \times \frac{56}{100} \times A = \frac{49}{56} \times \frac{24}{100} \times B$$

$$\Rightarrow \quad \frac{49}{100}A = \frac{21}{100}B$$

$\Rightarrow$ 49% of A = 21% of B.

Thus, II is true.

12. In the first round, let A get 'a' votes and B get 'b' votes.

Then, $\quad a + b = 260$

i.e., $\quad a = 260 - b \quad \ldots (i)$

In second round, B gets

$$b \times \frac{125}{100} = \frac{5b}{4}$$

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and A gets $260 - \frac{5b}{4}$

Also, $\frac{5b}{4} - \left(260 - \frac{5b}{4}\right) = 2(a - b)$

$$\frac{10b}{4} - 260 = 2(260 - b - b)$$

$$\frac{10b}{4} - 260 = 520 - 4b$$

$$4b + \frac{10b}{4} = 520 + 260$$

$$b = 120$$

Hence first round, A gets 140 and B gets 120.

In second round A gets 110 and B gets 150.

$\therefore$ % decrease in votes received by A

$$= \frac{140 - 110}{140} \times 100 = 21.43\%$$

13. Non-water matter is 1% of 5000 = 50 gm.

After drying, this matter becomes 2% of the weight of the water-melon. Hence weight of the watermelon now is 2500 gm.

14. Let A = ₹ x and B = ₹ (5000 – x)

Then, $\frac{x \times 2 \times 20}{100} = \frac{(5000 - x) \times 3 \times 20}{100}$

$\Rightarrow$ $2x = 15000 - 3x$

$\Rightarrow$ $5x = 15000$

$\Rightarrow$ $x = 3000$

A = x = ₹ 3000

and B = 5000 – x

= 5000 – 3000

= ₹ 2000

$\therefore$ $\frac{A}{B} = \frac{3000}{2000} = \frac{3}{2}$

Thus, (*a*) is not true and (*b*) is true.

A – B = 3000 – 2000 = ₹ 1000.

Thus, (*c*) is true.

(*d*) The total interest

$$= \frac{3000 \times 2 \times 20}{100} + \frac{2000 \times 3 \times 20}{100}$$

= ₹ 2400.

Thus, (*d*) is true.

15. First investment gives a return

$$P\left(1 + \frac{1}{100}\right)^{100} - P$$

First term is $P\left(1 + \frac{1}{100}\right)^{100} = X$,

such that $2P < X \le 3P$

Hence return X – P lies between P and 2P

i.e., $2P - P < X - P \le 3P - P$.

i.e., $P < X - P \le 2P$.

Whereas second offer.

$$= \text{Return} = P\left(1 + \frac{100}{100}\right)^{1} - P$$

$$= 2P - P = P.$$

Alternatively

Option of investing at 1% for 100 years is obviously better than investing at 100% for 1 year, because total compound interest for 100 years will be atleast 100% of the total amount (1% for each year), plus whatever the compounding effect will have.

16. Growth rate of rat population in 3 months

$$= 20 \times \frac{3}{12} = 5\%.$$

Increase in first 3 months = 3200 × 1.05 = 3360

Also, net decrease in 3 months = 160

$\therefore$ Rat population = 3360 – 160 = 3200

In the same way, after every 3 months, rat population remains the same.

Hence, even after 3 × 8 months i.e., 2 years, the population is maintained

17. Let CP in A be ₹ x

$\therefore$ CP in B = ₹ 0.9x

$\therefore$ $0.9x + 15 + 10 = x$

$\Rightarrow$ $0.1x = 25$

$\Rightarrow$ $x = 250$

$\therefore$ Price in B = 250 × 0.9 = ₹ 225

18. Let CP of 5 dozen mangoes be ₹ x.

SP = ₹ 156 and Gain = 0.3x

$\therefore$ $156 - x = 0.3x$

$\Rightarrow$ $x = 120$

$\therefore$ SP of 60 mangoes = 120 × 1.6 = 192

$\therefore$ SP per mango = ₹ 3.2

19. Let CP be ₹ x ; SP = 1.2x

But he gets only 1.2x × 0.8 = 0.96x

$\therefore$ Loss = 0.04x

$\therefore$ Loss = 4 %

20. Selling price = 800 × 1.25 = ₹ 1000

$\therefore$ Marked price = ₹ $\frac{1000}{0.95}$ = 1053

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21. CP = ₹ 550

$\therefore$ Marked price = 1.3×550 = ₹ 715

$\therefore$ Sales price = 0.8×715 = ₹ 572.

22. Profit or loss of the sale depends upon how much profit the person is earning on the sale of two articles.

If SP > CP of 1 article, there is a profit.

If SP < CP of 1 article, there is a loss.

If SP = CP of 1 article, there is not profit no loss.

Thus, only (*c*) is true.

23. *Following two key points need to be noted* :

(A) Producer is profit maximizing which implies she will set prices that maximize the profit function.

(B) Quantity demanded in market A is independent of quantity demanded in market B.

CASE 1:

In this case, producer charges different prices in markets A and B. This implies that she will maximize profits individually.

Equations can be re-written as

$$P_A = 210 - 5Q_A$$
$$P_B = 125 - 1.25\,Q_B$$

Profit in Market A = π_A

$= \text{Revenue} - \text{Cost}$

$= Q_A \times P_A - C_A$

$= Q_A \times (210 - 5Q_A) - (1000 + 5Q_A)$

At the maximum profit level, differential π_A with respect to Q_A is 0

or

$\therefore \quad 210 - 10Q_A - 5 = 0$

$\Rightarrow \quad Q_A = 20.5,\ P_A = 107.5$

and total revenue in market A

$= 20.5 \times 107.5$ = ₹ 2203.75

Similarly

Profit in Market B = π_B

= Revenue Cost

$= Q_B \times P_B - C_B$

$= Q_B \times (125 - 1.25\,Q_B) - (1000 + 5Q_B)$

At maximum profit level, differ-ential of π_B with respect to Q_B is 0

or $\quad 125 - 2.5Q_B - 5 = 0$

$\Rightarrow \quad Q_B = 48,\ P_B = 65$

and total revenue in market B

$= 48 \times 65$ = ₹ 3120

Using the above, we get

$$\frac{P_A}{P_B} = \frac{107.5}{65} \sim 1.65.$$

24. Data for 1 is available above. Consider case 2. In this case, prices across the two markets are the same. Hence, quantity equations can be added to give

$$Q = 142 - P$$

where Q is total quantity and P is price in both markets

$\Rightarrow \quad P = 142 - Q$

$\therefore \quad \pi = \text{Profit} = Q \times (142 - Q) - (1000 + 5Q)$

At the maximum profit level, differential of π with respect to Q is 0

$\therefore \quad Q = 68.5, \quad P = 73.5$

and $\quad$ Total Revenue = 5034.75

Given this, we note that quantities sold in both Case 1 and Case 2 are the same. Hence, the ratio is exactly 1.

25. Profit Difference between case 1 and case 2

= Revenue Difference (since quantity sold and hence, cost is the same)

$\therefore$ Profit difference $= 2203.75 + 3120 - 5034.75$

$= 289$

NUMERICAL TYPE QUESTIONS

1. Interest on ₹ 22500 = 0.1×22500 = ₹2250

Charges for managing the concern

$= 60 \times 12$ = ₹ 720

If yearly profit is ₹ x, then

$$\text{B's share} = \frac{\text{A's Share}}{2}$$

$$\therefore \quad \frac{x}{2} - 2250 + 720 = \frac{1}{2}\left(\frac{x}{2} - 720 + 2250\right)$$

$\Rightarrow \quad$ x = ₹ 9180.

2. Total annual profit

$= 25000 + 20000 + 21000 \times 10$

= ₹ 255000

Actual profit

$= 255000 - (2000 \times 12)$

= ₹ 231000

Total investment

$= 72000 + 27000 + 81000$

= ₹180000

A's share $= \dfrac{72000}{180000} \times 231000$

= ₹ 92400

B's share $= \dfrac{27000}{180000} \times 231000$

= ₹ 34650

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C's share = $\frac{81000}{180000} \times 231000$

= ₹ 103950.

3. 80% of the total profit is divided in the ratio 20000 : 24000 : 16000 = 5 : 6 : 4

80% of total profit = 5x + 6x + 4x = 15x

Total profit = $\frac{15x}{80\%}$ = 18.75x

$\therefore$ Share of C in profit = 4x + 20% of 18.75

= 4x + 3.75x = 7.75x

Share of A in profit = 5x

Share of B in profit = 6x

$\therefore$ (6x + 5x) – 7.75x = 487.50

$\Rightarrow$ 3.25x = 487.50

$\Rightarrow$ x = 150

$\therefore$ Total profit = 18.75 × 150 = ₹ 2812.50.

4. Selling price of 40 shares

= 675 ×40 = ₹ 27000

Income from half the proceeds

= $\frac{13500}{90} \times 4$ = ₹ 600

Income from house = 1440 × 0.75 = 1080

Present total income = 1080 + 600 = ₹ 1680

Earlier Income = 40 × 500 × $\frac{6}{100}$ = ₹ 1200

Increase in income = ₹480

5. One years dividend = $\frac{34539}{87} \times 3$ = ₹ 1191

Selling Price = $\frac{34539}{87} \times 89$ = ₹ 35333

Number of railway stocks bought

= $\frac{35333}{115}$ = 307.24

Income from railway stocks

= 307.24 × 5 = ₹ 1536.2

Difference in income = 345.2

6. Let two numbers be x and y.

7.5% of x = 12.5% of y

$\therefore$ 75x = 125 y

$\Rightarrow$ 3x = 5y

$\Rightarrow$ x = $\left(\frac{5}{3}\right)$y.

Now x – y = 1660

$\Rightarrow$ $\left(\frac{5}{3}\right)$y – y = 1660

$\Rightarrow$ $\left(\frac{2}{3}\right)$y = 1660

$\Rightarrow$ y = 2490

$\therefore$ x = 2490 +1660 = 4150.

Hence numbers are 4150, 2490.

7. Number of passed candidates

$= \frac{10}{100} \times 40 + \frac{20}{100} \times 500 + \frac{10}{100} \times 60$

= 4 + 10 + 6 = 20

Total students in school

= 40 + 50 + 60 =150

$\therefore$ Required percentage

$= \frac{20}{150} \times 100 = \frac{40}{3} = 13\frac{1}{3}\%$

8. Surface area = $4 \times \frac{22}{7} \times r^2$

$= \frac{3}{r}\left(\frac{4}{3} \times \frac{22}{7} \times r^3\right) = \frac{3}{r} \times$ Volume

where r = 10 cm

So we have

S = $\frac{3}{10}$ V = $\frac{3}{10} \times 100$ % of V = 30 % of V

So surface area is 30 % of volume.

9. Difference in % of votes = 57 – 43 = 14%

$\therefore$ 14% is represented by 420

$\therefore$ Total number of votes = 420 × $\frac{100}{14}$ = 3000.

10. % reduction in consumption = $\left(\frac{30}{100+30} \times 100\right)\%$

$= \left(\frac{30}{130} \times 100\right)\% = 23\frac{1}{13}\%$

11. Let the fraction be $\frac{a}{b}$

$\therefore$ $\frac{a^2}{b^2} \times \frac{\frac{2}{3}}{\frac{1}{5}} = 2\frac{a}{b}$ $\frac{a}{b} \neq 0$ if a ≠ 0,

then $\frac{a}{b} \times \frac{2}{3/5} = 2$

$\Rightarrow$ $\frac{a}{b} = \frac{3}{5}$

$\therefore$ a + b = 8.

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12. $$1100 = P + \frac{P \times 5 \times \left(T - \frac{1}{2}\right)}{100}$$

$$1100 = P + \frac{P \times 4 \times T}{100}$$

$$\therefore \quad \frac{P \times 5 \times \left(T - \frac{1}{2}\right)}{100} = \frac{P \times 4 \times T}{100}$$

$$\Rightarrow \quad 5T - \frac{5}{2} = 4T$$

$$\therefore \quad T = \frac{5}{2} \text{ years}$$

$$\therefore \quad 1100 = P\left(1 + \frac{4 \times \frac{5}{2}}{100}\right) = P\left(\frac{110}{100}\right)$$

$$\therefore \quad P = ₹\ 1000$$

13. $\left(1 + \frac{3}{100}\right)^2 = 1.0609$

$\therefore$ Effective annual rate = 6.09%

14. Let cash price be ₹P

$$\therefore \quad 26.56 = P + \frac{P \times 12.5 \times 0.5}{100}$$

$$\Rightarrow \quad P \approx ₹\ 25.$$

15. Let R be the rate of interest.

$$\text{Discount} = \frac{9000 \times R}{100}$$

$$\text{Interest} = \frac{3000 \times R \times T}{100}$$

By the given information:

$$\frac{9000 \times R}{100} = \frac{3000 \times R \times T}{100}$$

$$\Rightarrow \quad T = \frac{1}{3} \text{ years} = 4 \text{ months.}$$

16. Let N be the number of years

$$\therefore \quad R\% = \sqrt{N \times 12}$$

$$\Rightarrow \quad 48 = \frac{900 \times \sqrt{12N} \times N}{100}$$

Squaring both sides

$$N^2 = \frac{48 \times 48}{9 \times 9 \times 12} = \frac{16 \times 4}{9 \times 3} = \frac{64}{27}$$

$$\therefore \quad R = \sqrt{12 \times \frac{4}{3}} = 4\%$$

$$\therefore \quad 48 = \frac{P \times 4 \times 2}{100}$$

$$\Rightarrow \quad P = ₹\ 600$$

17. Let first discount be x%

$$\therefore \quad \frac{(100 - 20)}{100} \times \frac{100 - x}{100} \times 2500 = 1500$$

$$\Rightarrow \quad \frac{80}{100} \times \frac{100 - x}{100} \times 2500 = 1500$$

$$\Rightarrow \quad 100 - x = \frac{1500 \times 100 \times 100}{80 \times 2500}$$

$\Rightarrow$ 100 – x = 75;

$\Rightarrow$ x = 25

Alternatively,

$$\text{Successive discount} = \left(a + b - \frac{ab}{100}\right)\%$$

$$\therefore \quad \left(\frac{2500 - 1500}{2500}\right) \times 100 = a + 20 - \frac{20a}{100}$$

$$\Rightarrow \quad \frac{1000}{2500} \times 100 = a + 20 - \frac{a}{5}$$

$$\Rightarrow \quad 40 - 20 = a - 0.2a$$

$$\Rightarrow \quad a = \frac{20}{0.8} = 25\%$$

18. Let there be n number of balls in each packet.

$\therefore \quad 6n \times 8 = CP + 96$

$\Rightarrow \quad 48n = CP + 96 \quad \ldots(i)$

Also, $6(n - 1) \times 9 = CP + 90$

$\Rightarrow \quad 54n - 54 = CP + 90$

$\Rightarrow \quad 54n = CP + 144 \quad \ldots(ii)$

Solving equations (*i*) and (*ii*), we get,

$n = 8$

$\therefore \quad CP = 48 \times 8 - 96 = 384 - 96 = ₹\ 288$

Each packet cost = ₹48.

19. CP of the merchant who calculates his % profit on

$$CP = \frac{3760}{1.175} = ₹\ 3200$$

$\therefore \quad$ His profit $= 0.175 \times 3760 = ₹\ 560$

Profit of the merchant who calculates his % profit on SP

$= 0.175 \times 3760 = ₹\ 658$

$\therefore$ Difference in profit = 658 – 560 = ₹ 98.

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20. Let CP be ₹ x

∴ MP = 1.8x

SP of 15 articles = 12 × 1.8x = 21.6x

∴ Discount = 0.9 × 21.6x = 19.44x

∴ Profit on 15 articles = (19.44 – 15)x = 4.44x

∴ Profit % = $\frac{4.44x}{15x} \times 100 = 29.6\%$.

21. C.P. of goods for the trader = ₹ 22520

$$S.P. = \frac{125 \times 22520}{100} = ₹\ 28150$$

∴ Profit = ₹ 5630

Now, $5630 = 0.0536 \times 5000 + 0.085 \times 10000 + \frac{x}{100} \times 7520$

⇒ 5630 = 268 + 850 + 75.2x

⇒ 75.2x = ₹ 4512,

Which is nothing but profit from sale of third lot.

22. A's Monthly Equivalent Rent = 22 × 4

B's Monthly Equivalent Rent = 16 × 8

C's Monthly Equivalent Rent = 32 × 6

$$\frac{\text{B's Monthly Equivalent Rent}}{\text{C's Monthly Equivalent Rent}} = \frac{\text{Rent paid by B}}{\text{Rent paid by C}}$$

Rent paid by B = $\frac{16 \times 8}{32 \times 6} \times 600 = ₹\ 400$

Similarly, rent paid by A

$$= \frac{22 \times 4 \times 600}{32 \times 6} = ₹\ 275$$

∴ Total rent = 400 + 600 + 275 = ₹1275

23. Rate in which the rent is to be divided = 4 : 8 : 12

∴ Akbar's share of rent

$$= \frac{8}{24} \times 6000$$

= ₹ 2000.

24. 85% of the total profit is divided in the ratio 2 : 10 : 5.

85% of total profit = 2x + 10x + 5x = 17x

$$\text{Total profit} = \frac{17x}{85\%} = 20x$$

∴ Share of Ram in the profit

= 2x + 15% of 20x

= 2x + 3x = 5x

Share of Shyam in the profit = 10x

Share of Ghanshyam in the profit = 5x

Now, (10x + 5x) – 5x = 500

⇒ 10x = 500

⇒ x = 50

∴ Total profit = 20 × 50 = ₹ 1000.

25. Let capital invested be C.

Monthly Equivalent Investment of

$$x = \frac{C}{3} \times \frac{T}{4}$$

Monthly Equivalent Investment of

$$y = \frac{C}{4} \times \frac{T}{2}$$

Monthly Equivalent Investment of

$$z = \frac{5C}{12} \times T$$

∴ Profit is to be shared in the ratio

$$\frac{CT}{12} : \frac{CT}{8} : \frac{5CT}{12}$$

i.e., 2 : 3 : 10

∴ Share of x in the profit

$$= \frac{2}{15} \times 3600 = ₹\ 480$$

EXERCISE – II

MCQ TYPE QUESTIONS

1. $$A = P\left[1 + \frac{r}{100}\right]^t$$

$$10 \text{ million} = 5 \text{ million} \left[1 + \frac{20}{100}\right]^t$$

$$2 = \left[1 + \frac{1}{5}\right]^t$$

$$2 = \left(\frac{6}{5}\right)^t = \log 2 = t \log\left(\frac{6}{5}\right)$$

$$0.301 = t\ 0.0791$$

$$t = \frac{0.301}{0.0791} = 3.8 \text{ years}$$

∴ Hence around 3–4 years would it take to double the growth rate

2. Let number of people in country y = 100

So, number of people in country x = 300

Total number of people taller than 6ft in both the countries $= 300 \times \frac{1}{100} + 100 \times \frac{2}{100} = 5$

% of people taller than 6ft in both the countries

$$= \frac{5}{400} \times 100 = 1.25\%.$$

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4. Per ₹ 100 final value ₹ 107

$\Rightarrow$ per $\dfrac{100}{50}$ Dollars final value $\dfrac{107}{60}$

for 100 dollars _____?

$$= \frac{100 \times 50}{100} \times \frac{107}{60} = 89.16$$

Descreased by 11%

6.

	X	Y
Supply	60%	40%
Reliable	96%	72%
Overall	0.576	0.288

$\therefore \quad P(x) = \dfrac{0.288}{0.576 + 0.288} = 0.334$

7.

P	Q
40%	60%
–6%	+6%
+15%	–15%
49%	51%

$\therefore \quad 2\% = 2 \quad \Rightarrow \quad 100\% = 100$

NUMERICAL TYPE QUESTION

1. 90% of area of sheet = Cross sectional area of cone

$\Rightarrow\ 0.9 \times \pi \times 30 \times 30 = \pi \times r_1 \times 30$

($\because$ Slant height of cone (l) = 30)

$\Rightarrow\ 27 \text{ cm} = r_1$

$\therefore$ Height of the cone = $\sqrt{30^2 - 27^2}$ = 13.08 cm

Hence ratio of radius and height is 2.06.

■■

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3 CHAPTER
Time and Work

TIME, SPEED AND DISTANCE

Speed.

Speed of a body is defined as the distance covered by it in unit time.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Average Speed: If a body travels $d_1, d_2, d_3 \dots d_n$ distances, with speeds $s_1, s_2, s_3 \dots s_n$ in time $t_1, t_2, t_3 \dots t_n$ respectively, then average speed of the body through the total distance is given by

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$
$$= \frac{d_1 + d_2 + d_3 + \dots d_n}{t_1 + t_2 + t_3 + \dots t_n}$$
$$= \frac{s_1t_1 + s_2t_2 + s_3t_3 + \dots + s_nt_n}{t_1 + t_2 + t_3 + \dots + t_n}$$
$$= \frac{d_1 + d_2 + d_3 + \dots + d_n}{\frac{d_1}{s_1} + \frac{d_2}{s_2} + \frac{d_3}{s_3} + \dots + \frac{d_n}{s_n}}$$

Some Important Points:

(1) If a body covers the distance d_1 and d_2 at a speed of s_1 and s_2 km/hr, respectively, in time t_1 and d_2, then

Total time taken, $T = t_1 + t_2 = \frac{d_1}{s_1} + \frac{d_2}{s_2}$.

Total distance covered, $d = d_1 + d_2 = s_1t_1 + s_2t_2$.

(2) While travelling a certain distance d, if a man changes his speed in the ratio m: n, then ratio of time taken becomes n : m.

(3) If a certain distance (d), say from A to B, is covered at 'a' km/hr and same distance is covered again say from B to A in 'b' km/hr, then average speed during the whole journey is given by

$$\text{Average speed} = \left(\frac{2ab}{a+b}\right)\frac{\text{km}}{\text{hr}} \dots$$

(which is harmonic mean of a and b)

Also, if t_1 and t_2 is taken to travel from A to B and B to A respectively, then distance 'd' from A to B is given by

$$d = (t_1 + t_2)\left(\frac{ab}{a+b}\right); \; d = (t_1 - t_2)\left(\frac{ab}{a-b}\right);$$
$$d = (a - b)\left(\frac{t_1t_2}{t_1 - t_2}\right)$$

(4) If a body travels a distance 'd' from A to B with speed 'a' in time t_1 and travels back from B to A i.e., same distance with $\frac{m}{n}$ of the usual speed 'a', then change in time taken to cover the same distance
is given by

$$\text{Change in time} = \left(\frac{n}{m} - 1\right) \times t_1; \text{ for } n > m$$
$$= \left(1 - \frac{n}{m}\right) \times t_1; \text{ for } m > n$$

(5) If two persons A and B start at the same time in opposite directions from two points and meet at any
point in 'a' and 'b' hr, respectively after having met, then,

$$\frac{A'\text{s Speed}}{B'\text{s Speed}} = \frac{\sqrt{b}}{\sqrt{a}}$$

Relative speed.

(1) Time taken by a moving object 'x' meters long in passing a stationary object of negligible length from the time they meet is same as the time taken by the moving object to cover 'x' meters with its own speed.

(2) Time taken by a moving object 'x' meters long in passing a stationary object 'y' meters long from the time they meet, is same as the time taken by the moving object to cover 'x + y' meters with its own speed.

(3) If two objects of length 'x' and 'y' meters move in the same direction at 'a' and 'b' m/s, then
time taken to cross each other from the time they meet

$$= \frac{\text{Sum of their length}}{\text{Relative speed}}$$

i.e., $\frac{x+y}{a-b}$ if $a > b$ or else, $\frac{x+y}{b-a}$

(4) If two objects of length 'x' and 'y' meters, move in the opposite direction at 'a' and 'b' m/s, then
time taken to cross each other from the time they meet

$$= \frac{\text{Sum of their length}}{\text{Relative speed}} = \frac{x+y}{a+b}$$

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(5) If speed of a boat in still water is x km/hr and speed of the stream is y km/hr, then

Relative speed while travelling with the stream i.e., speed downstream = (x + y) km/hr.

Relative speed while travelling against the stream i.e., speed upstream = (x – y) km/hr.

(6) Also speed of the boat in still water = $\frac{1}{2}$ (Speed with stream + Speed against stream)

Speed of the river = $\frac{1}{2}$ (Speed with stream – Speed against stream)

RACES

Some Important Points:

(1) 'A gives B a start of x meters', implies that, if distance between starting point and finishing point is L meters, then A covers L meters while B covers L – x metres.

e.g., In a 100 metre race. A gives B a start of 10 metres means, while A runs 100 metres, B runs 100 –10 = 90 metres.

(2) '*A beats B by x metres*', implies that, if distance between starting point and finishing point is L meters, then A wins the race by covering L metres, while B covers L – x metres only.

(3) 'A gives B a start of t seconds', implies that A starts the race t seconds after B starts from the starting point.

(4) '*A beats B by t seconds*', implies that, A and B start together from the starting point, but A reaches the finishing point t seconds before B finishes.

Note: (*iii*) and (*iv*) both imply that B takes t seconds more than A to finish the distance.

(5) A beats B by 'x' metres or 't' seconds means, B runs 'x' metres in 't' seconds.

(6) Winner's distance = Length of the race.

(7) Distance covered by loser = Winner's distance – (Beat distance + Start distance)

(8) Time taken by winner = Time taken by loser – (Beat time + Start time)

(9) $$\frac{\text{Winner's Time}}{\text{Loser's Distance}} = \frac{\text{Loser's time}}{\text{Winner's distance}}$$

$$= \frac{\text{Beat time + Start time}}{\text{Beat distance + Start distance}}$$

(10) If a race ends in a dead heat, then beat time = 0 and beat distance = 0.

(11) Two persons staring at the same time and from the same point along a circular path will be together again for the first time. When faster gains one complete round over the other, then time taken by faster person to complete one round over the other

$$= \frac{\text{Length of race course}}{\text{Relative Speed}}$$

(12) Two persons, starting at the same time from the same point along a circular path, will be together again for the first time at the same starting point, at a time which is LCM of the time taken by each to complete a round.

(13) Three persons, staring at the same time and from the same point along a circular path, will be together for the first time after the start at a time which is equal to LCM of the time taken by the fastest to gain a complete round over each of the other two.

(14) A overtakes B $\frac{1}{n}$ th of x^{th} round means, when A has completed $\left(x - \frac{1}{n}\right)$ rounds, B has completed $\left[(x-1) - \frac{1}{n}\right]$ round.

Also, $\frac{\text{A's Speed}}{\text{B's Speed}}$

$$= \frac{\text{Rounds completed by A in a given time}}{\text{Rounds completed by B in the same time}}$$

e.g., A overtakes B in the middle of the 4^{th} round implies, when A has completed $3\frac{1}{2}$ rounds, B has completed $2\frac{1}{2}$ rounds. 2 $\frac{\text{A's Speed}}{\text{B's Speed}} = \frac{3\frac{1}{2}}{2\frac{1}{2}}$

WORK

Some Important Points:

(1) If a can do a work in 'a' number of days, then in one day $\frac{1}{a}^{th}$ work is done.

Conversely, if a man does $\frac{1}{a}^{th}$ of a work in 1 day, then he can complete the work in $1 \div \frac{1}{a}$ = a days.

(2) If A is 'x' time as good a workman as B, then he will take $\frac{1}{x}^{th}$ of the time taken by B to do the same work.

(3) If A and B can do a piece of work in 'x' and 'y' days respectively. then working together, they will take $\frac{xy}{x+y}$ days to finish the work and in one day. They finish $\left(\frac{x+y}{xy}\right)^{th}$ part of the work.

(4) To compare the work done by different people, first find the amount of work each can do in the same time.

(5) If number of men to do a job is changed in the ratio a : b, then time required to do the work will be in the ratio b : a, assuming amount of work done by each of them in the given time is the same, or they are identical.

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(6) If two men A and B together can finish a job in 'x' days and if A working alone takes 'a' days more than A and B working together and B working alone takes 'b' days more than A and B working together, then $x = \sqrt{ab}$.

(7) To do a piece of work, the number of men employed and the number of days required to do the work are in inverse proportion. Also, the number of men employed and the hours worked per day are in inverse proportion.

PIPES AND CISTERNS

A pipe connected with a cistern is called an *inlet*, if it fills the cistern.

A pipe connected with a cistern is called an *outlet*, if it empties the cistern.

filling or emptying a cistern can be considered as work done.

Some Important Points:

(1) If an inlet pipe fills a cistern in 'a' hours, then $\frac{1}{a}^{th}$ part is emptied in 1 hour.

Similary, if an outlet pipe empties a cistern in 'a' hours, then $\frac{1}{a}^{th}$ part is emptied in 1 hour.

(2) If pipe A is 'x' times bigger than pipe B, then pipe A will take $\frac{1}{x}^{th}$ of the time taken by pipe B to fill the cistern.

(3) If A and B fill a cistern in 'm' and 'n' hours, respectively, then together they will take $\frac{mn}{m+n}$ hours to fill the cistern and in one hour $\frac{m+n}{mn}^{th}$ part of the cistern will be filled.

Similarly, 'A' and 'B' empty a cistern in 'm' and 'n' hours respectively, then, together they will take $\frac{mn}{m+n}^{th}$ hours to empty the cistern and in one hour $\frac{m+n}{mn}^{th}$ part of the cistern will be empty.

(4) If an inlet pipe fills a cistern in 'm' hours and an outlet pipe empties the cistern in 'n' hours, then the net part filled in 1 hour when both the pipes are opened is $\left(\frac{1}{m}-\frac{1}{n}\right)$ i.e., $\frac{n-m}{mn}$ and the cistern will get filled in $\left(\frac{mn}{n-m}\right)$ hours.

For the cistern to get filled, m < n. If m > n, the cistern will never get filled.

In general,

Net part filled of a cistern = (Sum of work done by inlets) – (Sum of work done by outlets)

(5) If an inlet pipe fills a cistern in 'a' minutes, takes 'x' minutes longer to fill the cistern due to a leak in the cistern, then the time in which the leak will empty the cistern is given by $a \times \left(1+\frac{a}{x}\right)$.

(6) If two pipes A and B can fill a cistern in 'x' minutes and if A alone can fill it in 'a' minutes more than 'x' minutes and B alone can fill it in 'b' minutes more than 'x' minutes, then $x = \sqrt{ab}$.

EXERCISE – I

MCQ TYPE QUESTIONS

1. A barrel full of beer has 2 taps, one midway which draws a litre in 6 minutes and the other at the bottom which draws a litre in 4 minutes. The lower tap is normally used after the level of beer in the barrel is lower than midway. The capacity of the barrel is 36 litres. A new assistant opens the lower tap when the barrel is full and draws out some beer. As a result the lower tap has been used 24 minutes before the usual time. For how long was the beer drawn out by the new assistant?

(*a*) 10 mins. (*b*) 8 mins.
(*c*) 16 mins. (*d*) 32 mins.

2. Pipe A can fill a cistern in 36 minutes and B in 48 minutes. If both the pipes are opened together, when should pipe B be closed so that the cistern may be just full in 24 minutes?

(*a*) 8 mins. (*b*) 9 mins.
(*c*) 12 mins. (*d*) 16 mins.

3. Pipes P, Q and R are attached to a tank and each can act as either an inlet or outlet pipe. Pipes P, Q and R respectively take 8, 10 and 12 hours to fill the empty tank or empty the full tank. In the first hour, pipes P and R work as inlet and Q work as outlet. In the second hour, pipes P and Q work as inlet and pipe R as outlet. In the third hour pipes Q and R work as inlet and pipe P as outlet and the process goes on like this. When will the cistern be filled?

(*a*) In the 8th hour. (*b*) In the 9th hour.
(*c*) In the 10th hour. (*d*) In the 11th hour.

4. There are 2 inlet pipes and an outlet pipe. The efficiency of one of the inlet pipes is double than that of the other. Also, the efficiency of the outlet pipe is half that of the lesser efficient inlet pipe. The empty tank gets filled in 16 hours when all the pipes are opened. How many hours will be taken to fill the empty tank when the 'lesser' efficiency inlet pipe is plugged and the rest kept opened?

(*a*) $33\frac{1}{3}$ hours (*b*) $26\frac{2}{3}$ hours
(*c*) 100 hours (*d*) 80 hours

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5. A train overtakes 2 persons walking at 2 km/hr and 4 km/hr respectively in the same direction and completely passes them in 9 seconds and 10 seconds. Length of the train and its speed in km/hr respectively are

(*a*) 75 m, 18 km/hr (*b*) 80 m, 21 km/hr
(*c*) 60 m, 20 km/hr (*d*) 50 m, 22 km/hr

6. A policeman goes after a thief who has a 100 m start. The policeman runs 1 km. in 6 mins. and the thief 1 km in 10 mins. How far did the thief go before he was overtaken?

(*a*) 100 m (*b*) 125 m
(*c*) 150 m (*d*) 250 m

7. Everyday Sachin exercises for $2\frac{1}{2}$ hours. He exercises by walking, jogging and running. His speeds while walking, jogging and running in the ratio 1 : 2 : 4. Which of the following statements are true?

I. If he spends equal time on the three activities, then he runs twice the distance that he jogs.

II. If he runs at 8 km per hour and distance covered by running and jogging together is 12 km, then he walks a distance of 1 km in 1 hour.

III. If the time for which he walks, jogs and runs is in the ratio 2 : 2 : 1 and he covers a total distance of 10 km, then his speeds while walking, jogging and running are 2 km/hr, 4 km/hr and 8 km/hr res-pectively.

Codes:

(*a*) I and II (*b*) II and III
(*c*) I and III (*d*) I, II and III

8. A monkey climbing up a greased pole ascends 10 metres and slips down 2 metres in alternate minutes. If the pole is 64 metres high, how long will it take him to reach the top?

(*a*) 16 min
(*b*) 14 min and 48 sec
(*c*) 12 min
(*d*) 14 min

9. A train travelling at 10 m/sec from A to B at 7 a.m. meets a train leaving B at 7:20 a.m. and coming to A at a speed $\frac{1}{3}$ times faster than the first train. If the distance from A to B is 68 km. then, at what distance from A will the two trains meet?

(*a*) 72 km (*b*) 36 km
(*c*) 60 km (*d*) 50 km

10. Two trains A and B start from stations X and Y towards each other. B leaves station Y half an hour after train A leaves station X. Two hours after train A has started, the distance between trains A and B is $\frac{19}{30}$th of the distance between stations X and Y. How much time would it take each train (A and B) to cover the distance X to Y, if train A reaches half an hour later to its destination as compared to B?

(*a*) 8 hrs, 6 hrs (*b*) 5 hrs, 4 hrs
(*c*) 10 hrs, 9 hrs (*d*) 9 hrs, 8 hrs

11. Two men, A and B, run a 4 km race on a circular course of $\frac{1}{4}$ km. If their speeds are in the ratio of 5 : 4, how often does the winner pass the other?

(*a*) Once (*b*) Twice
(*c*) Thrice (*d*) Four times

12. A and B start from the same point to run in opposite directions round a circular path 550 yards in length, A giving B a start of 100 yards. They pass each other when A has run 250 yards. Who will come first to the starting point and at what distance will they be apart?

(*a*) A, 20 yards (*b*) B, 10 yards
(*c*) B, 20 yards (*d*) A, 10 yards

13. Two runners run in the same direction along a circular track 2 km long. The faster runner overtakes the slower one every 30 minutes. What are their speeds, if faster one completes a round one minute sooner than the other?

(*a*) 24 km/hr and 20 km/hr
(*b*) 44 km/hr and 28 km/hr
(*c*) 20 km/hr and 18 km/hr
(*d*) 20 km/hr and 16 km/hr

14. To do certain work alone 'A' takes 4 hrs, 'B' takes 5 hrs and 'C' takes 6 hrs. How long would A and B together take to do a work which C can do in $4\frac{1}{2}$ hrs.?

(*a*) 2 hrs 10 mins. (*b*) 2 hrs 30 mins.
(*c*) 1 hr 20 mins. (*d*) 1 hr 40 mins.

15. A can do piece of work in 8 days which B and C can do in 12 and 16 days respectively. If they work on alternate days, then which of the following is true?

(*a*) If A started the work on the first day followed by B and c, then the work will be finished on the 10th day.
(*b*) If B started the work on the first day followed by C and A, then the work will be finished on the 11th day.
(*c*) If C started the work on the first day followed by A and B, then the work will be finished on the 11th day.
(*d*) None of these

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NUMERICAL TYPE QUESTIONS

1. Meera paints walls for a living. She sometimes has Ashok to work with her. Meera takes 2 days to paint a 10 ft by 10 ft wall. Ashok takes 3 days to do the same job. Meera has to pay Ashok for his work on a per day basis, so she'd rather do without him. Meera has recently got a contract to paint the walls and ceiling of a room of dimensions 10 ft by 15 ft and a height of 10 ft. She had promised to do the job within 9 days. The ______ number of days does she need to employ Ashok?

2. 3 pipes can fill a reservoir in 10, 15 and 20 hours respectively. If the three taps are opened one after another in the given order, with a certain fixed time gap between them, the reservoir fills in 5 hours. The time gap will be _____ hr.

3. Two pipes A and B can fill a tank in 8 hours. If only pipe A is open, then it would take 4 hours longer to fill the tank. The _______ hours it would take if only pipe B is open.

Directions (Q.4 – 5): *Refer to the data below and answer the questions that follow.*

The boiler tank in a chemical factory holds 10^5 litres. 5 tanks each having one-fifth the capacity of the boiler tank fill in 'hard water' at same rates in the boiler tank in 2 hours. The outlet of the two of smaller tanks work as inlet pipes and other two work as outlet and the fifth tank fill in the main 'boiler' at half its efficiency.

4. The number of hours taken to fill in the main boiler when all the pipes are opened at once is ________

5. Only three pipes are working, two at their full efficiency and the third one with half of its efficiency and all three are acting as an inlet. If all the three are opened alternately with the latter pipe being first to be opened to fill the boiler such that each smaller tank fills 'hard water' in the main boiler for equal time, then number of hours taken to fill the boiler to half its capacity is______

6. A contract is to be completed in 46 days and 117 men were set to work, each working 8 hours a day. After 33 days, 4/7 of the work is completed. The _______ number of additional men may be employed so that the work may be completed in time, each man now working 9 hours a day?

7. A train travelling at 42 km/hr. passes a cyclist going in the same direction in 9 secs.; if cyclist had been going in the opposite direction, the train would have passed him in 5 secs. Length of the train is______ metres

8. A man rows upstream 13 km. and downstream 28 km. taking 5 hrs. each time. Velocity of the current in km/hr_______ is

Directions (Q. 9 – 11): *Refer to the data below and answer the questions that follow.*

The variation in the speed of a car on a particular day at the respective times is shown in the table below:

s(km/hr)	t(hr)
0	11.00 am
40	11.30 am
50	1.00 pm
85	1.30 pm
10	3.30 pm
10	4.30 pm

9. The distance travelled by the car from 11 a.m. to 1 p.m. is _______km

10. If car maintains the speed it has at 4.30 p.m. then at _______ time will the car cover the same distance as it had covered from 11.00 a.m. to 1.00 p.m.?

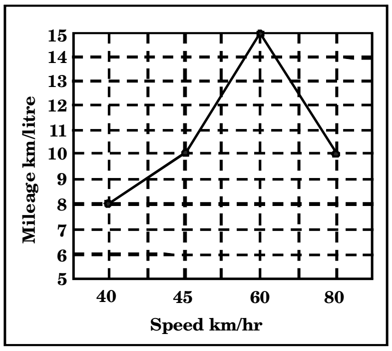
11. Average speed of the car from 11.00 a.m. to 4.30 p.m. is________ kmph

12. A cheetah chase a deer which is 100 m ahead. The time in which the deer takes 10 leaps the cheetah takes only 6 leaps. In one leap, the deer covers 1 m while the cheetah covers 2 m. In ______ leaps would the cheetah catch up the deer.

13. A hare pursued by a hound who is 50 of her own leaps before him. When the hare takes 4 leaps, the hound takes 3. In one leap, the hare goes $1\frac{3}{4}$ metres and the hound $2\frac{3}{4}$ metres. In how _______ leaps will the hound overtake the hare.

Directions (Q. 14 – 15): *Refer to the graph below and answer the questions that follow.*

The engine of the new car Palio brought by Fiat in the market follows certain mileage characteristics as shown in the diagram below:



Vijay bought the new car Palio and filled exactly 10 litres in his car. He goes at a speed of 40 km/hr for first 20 km and then at a speed of 60 km/hr for next 60 km and driving at a constant speed reaches his destination which was 115 km away from his starting point.

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14. The time in hours taken by Vijay if he reaches his destination in minimum possible time and finishes all the fuel.

15. In the above question, if Vijay had travelled at a constant average speed rather than changing his speed, taking the same minimum time, then the approximate saving in the fuel consumed is ______ litres

16. In a 2 km race on a circular course of $\frac{1}{4}$ of a km. A overlaps B in the middle of his 6th round. Then_____ distance (in km) will A win at the same rate of running

17. A runs $1\frac{3}{8}$ times faster than B. A gives B a start of 120 metres. _______ metre must the winning post be so that it may be a dead heat.

18. The work done by a man, a woman and a child is in the ratio of 3 : 2 : 1. There are 20 men, 30 women and 36 children in a factory. Their weekly wages amount of ₹ 780, which is divided in the ratio of work done by the men, women and children. The wages of 15 men, 21 women and 30 children for 2 weeks is _______

19. A can do a work in 6 days and B in 8 days. With the help of a boy, the three complete the work in 3 days and get ₹ 200. The boy's share in rupees, if the money is distributed in the ratio of the work done is _______

20. A can finish a job in 12 days and B when working at twice his efficiency finishes a job in 9 days. The ______ number of days will they take if they work for two days alternately working at their standard rate given that A starts first.

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. An electric bus has onboard instruments that report the total electricity consumed since the start of the trip as well as the total distance covered. During a single day of operation, the bus travels on stretches M, N, O and P, in that order. The cumulative distances travelled and the corresponding electricity consumption are shown in the table below.

Stretch	Comulative distance(km)	Electricity used (kWh)
M	20	12
N	45	25
O	75	45
P	100	57

The stretch where the electricity consumption per km is minimum is

(*a*) M (*b*) N

(*c*) O (*d*) P

2014

2. It takes 30 minutes to empty a half-full tank by draining it at a constant rate. It is decided to simultaneously pump water into the half-full tank while draining it. What is the rate at which water has to be pumped in so that it gets fully filled in 10 minutes?

(*a*) 4 times the draining rate

(*b*) 3 times the draining rate

(*c*) 2.5 times the draining rate

(*d*) 2 times the draining rate

2013

3. A tourist covers half of his journey by train at 60 km/h, half of the remainder by bus at 30 km/h and the rest by cycle at 10 km/h. The average speed of the tourist in km/h during his entire journey is

(*a*) 36

(*b*) 30

(*c*) 24

(*d*) 18

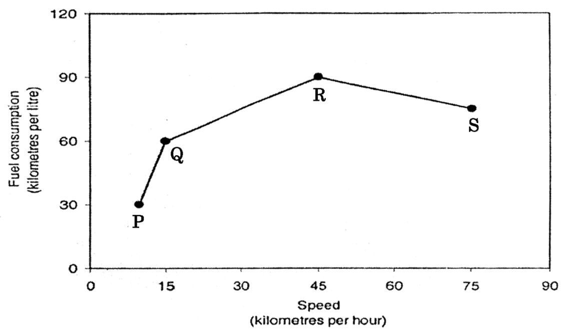
4. A car travels 8 km in the first quarter of an hour, 6 km in the second quarter and 16 km in the third quarter. The average speed of the car in km per hour over the entire journey is

(*a*) 30 (*b*) 36

(*c*) 40 (*d*) 24

2011

5. The fuel consumed by a motorcycle during a journey while traveling at various speeds is indicated in the graph below.



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The distances covered during four laps of the journey are listed in the table below

Lap	Distance (kilometers)	Average speed (kilometers per hour)
P	15	15
Q	75	45
R	40	75
S	10	10

From the given data, we can conclude that the fuel consumed per kilometre was least during the lap

(*a*) P (*b*) Q
(*c*) R (*d*) S

2010

6. 5 skilled workers can build a wall in 20 days; 8 semi-skilled workers can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall?

(*a*) 20 days (*b*) 18 days
(*c*) 16 days (*d*) 15 days

NUMERICAL TYPE QUESTIONS

2015

1. A tiger is 50 leaps of its own behind a deer. The tiger takes 5 leaps per minute to the deer's 4. If the tiger and the deer cover 8 metre and 5 metre per leap respectively, what distance in meters will the tiger have a run before it catches the deer?

2014

2. A man can row at 8 km per hour in still water. If it takes him thrice as long to row upstream, as to row downstream, then find the stream velocity in km per hour.

3. A train that is 280 metres long, travelling at a uniform speed, crosses a platform in 60 seconds and passes a man standing on the platform in 20 seconds. What is the length of the platform in metres?

ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*c*) **2.** (*d*) **3.** (*c*) **4.** (*b*) **5.** (*d*) **6.** (*c*) **7.** (*c*) **8.** (*b*) **9.** (*b*) **10.** (*c*)
11. (*c*) **12.** (*d*) **13.** (*a*) **14.** (*d*) **15.** (*c*)

Numerical Type Questions

1. 6 **2.** $\frac{1}{2}$ **3.** 16 **4.** 20 **5.** 6 **6.** 81 **7.** 75 **8.** 1.5 **9.** 77.5
10. 12.15 a.m. **11.** 39.3 **12.** 300 **13.** 210 **14.** 2 **15.** $2\frac{1}{3}$ **16.** $\frac{4}{11}$ **17.** 440
18. 1170 **19.** 25 **20.** 14

EXERCISE – II

MCQ Type Questions

1. (*b*) **2.** (*a*) **3.** (*c*) **4.** (*c*) **5.** (*a*) **6.** (*d*)

Numerical Type Questions

1. (800) **2.** (4) **3.** (560 to 560)

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EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. Top tap is operational till 18 litres is drawn out.

$\therefore$ Time after which the lower tap is usually open
$= 18 \times 6 = 108$ minutes.

$\therefore$ Time after which it is open now
$= 108 - 24 = 84$ minutes.

$\therefore$ Litres drawn $= \frac{84}{6} = 14$ litres

$\therefore$ $18 - 14 = 4$ litres

were drawn by the new assistant.

$\therefore$ Time $= 4 \times 4 = 16$ minutes.

Alternatively

Upper tap was not used for 24 minutes.

$\therefore$ Litres drawn $= \frac{24}{6} = 4$ litres.

$\therefore$ Beer was drawn for $4\times 4 = 16$ minutes.

2. Let pipe B be turned-off after x minutes.

Part filled in x minutes $= x\left(\frac{1}{36}+\frac{1}{48}\right) = \frac{7x}{144}$.

Part filled in 24 – x minutes $= \frac{24-x}{36}$

$$\therefore \quad \frac{7x}{144}+\frac{24-x}{36} = 1$$

$$\Rightarrow \quad \frac{7x+96-4x}{144} = 1$$

$$\Rightarrow \quad x = 16 \text{ minutes}$$

3. In a cycle of 3 hours : pipes P, Q and R are working as inlet pipes for 2 hours each and they are working as outlet pipes for an hour each. So part of tank filled in 3 hours.

$$= 2x\left(\frac{1}{8}+\frac{1}{10}+\frac{1}{12}\right)-\left(\frac{1}{8}+\frac{1}{10}+\frac{1}{12}\right)$$

$$= \frac{1}{8}+\frac{1}{10}+\frac{1}{12} = \frac{15+12+10}{120} = \frac{37}{120}$$

Hence they will take $\frac{120\times 3}{37} = 9\frac{27}{37}$ hours.

Thus, tank will be filled in 10th hour.

4. Let number of hours taken by the outlet and the 2 inlet pipes be 4x, 2x and x respectively. So in 1 hour the part of empty tank filled is

$$\frac{1}{x}+\frac{1}{2x}-\frac{1}{4x} = \frac{1}{16}$$

$$\frac{5}{4x} = \frac{1}{16}$$

$$\Rightarrow \quad x = 20$$

Hence, inlet pipe with higher efficiency fills the tank in 20 hours.

So in one hour, the desired pipes fill

$$\frac{1}{x}-\frac{1}{4x} = \frac{3}{4x} \text{ th of the tank.}$$

Tank gets filled in

$$\frac{4x}{3} = \frac{80}{3} \text{ hours} = 26\frac{2}{3} \text{ hours.}$$

5. In each case, train has to travel its own length, to pass each man.

If x km/hr is speed of the train, then

$$\text{Length} = \text{Relative speed} \times \text{Time}$$

$$= (x-2)\times\frac{9}{60\times 60}$$

$$= (x-4)\times\frac{10}{60\times 60}$$

$$\therefore \quad 9x - 18 = 10x - 40$$

$$\Rightarrow \quad x = 22 \text{ km/hr.}$$

$$\therefore \quad \text{Length} = (22-2)\times\frac{9}{3600}$$

$$= \frac{20\times 9}{3600} \text{ km} = 50 \text{ m.}$$

6. Policeman gains $= \frac{1}{6}-\frac{1}{10} = \frac{4}{60}$ km/min

$$= 66\frac{2}{3} \text{ m/min.}$$

Hence to gain100 m, time required

$$= \frac{100}{66\frac{2}{3}} = 1\frac{1}{2} \text{ min.}$$

Hence thief has gone ahead by

$$1\frac{1}{2}\times\frac{1}{10}\times 1000 \text{ metres} = 150 \text{ m.}$$

7. I. Ratio of speed = 1 : 2 : 4

If equal time is spent on each of these, then ratio of distance = 1 : 2 : 4

So, Sachin runs twice the distance that he jogs.

Thus, I is true.

II. He runs at 8 km/hr and therefore walks at 2 km/hr.

Hence II is not true.

III. Ratio of speed= 1 : 2 : 4

Ratio of time = 2 : 2 : 1

Ratio of distance

$= 2 \times 1 : 2 \times 2 : 4 \times 1 = 2 : 4 : 4$

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If Sachin covers a total distance of 10 km, then he

Walks for $\frac{2}{2+4+4}\times 10 = 2$ km,

Joggs for $\frac{4}{2+4+4}\times 10 = 4$ km.

Runs for $\frac{4}{2+4+4}\times 10 = 4$ km.

Total time taken is $2\frac{1}{2}$ hours = $\frac{5}{2}$ hours.

Thus, he walks for $\frac{2}{2+2+1}\times\frac{5}{2} = 1$ hour.

jogs for $\frac{2}{2+2+1}\times\frac{5}{2} = 1$ hour.

runs for $\frac{1}{2+2+1}\times\frac{5}{2} = \frac{1}{2}$ hour.

So speed while,

Walking = $\frac{2}{1} = 2$ km/hr

Jogging = $\frac{4}{1} = 4$ km/hr

Running = $\frac{4}{\frac{1}{2}} = 8$ km/hr.

Thus, III is true.

8. In 1 minute, monkey ascends 10 metres but he takes 1 minute to slip down 2 metres. Thus, at the end of 2 minutes, net ascending of the monkey is $=10 - 2 = 8$ metres.

Thus, to have a net ascending of 8 metres, process of ascending and then slipping happens once. So, to cover 64 metres, above process is repeated $\frac{64}{8}$ or 8 times.

It is clear that in 8 such happenings, the monkey will slip 7 times, because 8th time, he will ascend to the top. Thus, in climbing 7 times and slipping 7 times, he covers (7×8) or 56 metres.

Time taken to cover 56 metres

$$= \frac{56\times 2}{8} = 14 \text{ minutes}$$

Remaining distance = 64 – 56 = 8 metres

Time taken to ascend 8 metres

$$= \frac{8}{10} \text{ min} = \frac{4}{5} \text{ min.}$$

Total time taken = 14 minutes + $\frac{4}{5}$ min

= 14 mins. 48 sec.

9. If they meet t hrs after 7 a.m,

$$10 \times \frac{18}{5} \times t + 10 \times \frac{18}{5} \times \frac{4}{3}\left(t - \frac{1}{3}\right) = 68$$

$\Rightarrow \quad 36t + 48t - 16 = 68$

$\Rightarrow \quad t = 1$ hr.

They meet at a distance of

$10 \times 60 \times 60 = 36000$ m

i.e., 36 km from A.

10. Two equations are

$$T_1 - T_2 = \frac{1}{2} + \frac{1}{2} = 1 \qquad ...(i)$$

where T_1 and T_2 are time taken by trains A and B to cover the whole distance

and $$\frac{2}{T_1} + \frac{1.5}{T_2} = \frac{11}{30} \qquad ...(ii)$$

Solving equations (*i*) and (*ii*) ,we get

$T_1 = 10$ hrs. and $T_2 = 9$ hrs.

11. When A runs 5 rounds, B runs 4 rounds (ratio of speeds)

$\therefore$ A passes B each time A has run 5 rounds or

$5 \times \frac{1}{4} = \frac{5}{4}$ km. $= 1\frac{1}{4}$ km.

$1\frac{1}{4}$ km is contained in 4 km.

3 times.

Hence A passes B thrice.

12. When A has run 250 yards, B should have run

$550 - 250 = 300$ yards.

$\therefore$ B runs $300 - 100 = 200$ yards in the same time taken by A to run 250 yards.

$\therefore$ Ratio of the speeds of A and B

= 250 : 200 = 5 : 4

If speed of A is 5x yards per minute, then speed at B = 4x yards/min.

Time taken by B to reach the starting point

$$= \frac{450}{4x} = \frac{112.5}{x} \text{ min.}$$

Time taken by A to reach the starting point

$$= \frac{550}{5x} = \frac{110}{x} \text{ min.}$$

$\therefore$ A wins by $\frac{2.5}{x}$ min

$\therefore$ Distance travelled by B in $\frac{2.5}{x}$ min.

$$= \left(\frac{2.5}{x}\right) \times (4x) = 10 \text{ yards}$$

Hence B is 10 yards behind A.

13. Let n_1 and n_2 be the speeds in km/hr and let n_1 be the greater speed i.e., $n_1 > n_2$.

$$\therefore \quad \frac{2}{n_2} = \frac{2}{n_1} + \frac{1}{60}.$$

$$\Rightarrow \quad n_1 n_2 = 120(n_1 - n_2)$$

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Faster runner covers $(n_1 - n_2)$ km. more than the slower runner in 1 hour.

Hence to cover one round of 2 km, he will take $\frac{2}{n_1 - n_2}$ or $\frac{1}{2}$ hour.

$$\therefore \quad \frac{2}{n_1 - n_2} = \frac{1}{2}$$

$$\Rightarrow \quad n_1 - n_2 = 4$$

$$\Rightarrow \quad n_1 = 4 + n_2$$

$$\Rightarrow \quad n_2(4 + n_2) = 120 \times 4$$

$$\Rightarrow \quad n_2^2 + 4n_2 - 480 = 0$$

$$\Rightarrow \quad n_2 = 20 \text{ or } -24 \text{ (rejected)}$$

$$\therefore \quad n_1 = 4 + n_2 = 24 \text{ km/hr}$$

Hence speeds are 24 km/hr and 20 km/hr

Alternatively

Options may also be substituted to find the result.

14. Work done by A and B in 1 hr

$$= \frac{1}{4} + \frac{1}{5} = \frac{9}{20}^{\text{th}} \text{ part.}$$

$\therefore$ Ratio of work done by (A + B)

and $\quad C = \frac{9}{20} : \frac{1}{6}$

$\therefore$ Ratio of time taken

$$= \frac{1}{6} : \frac{9}{20} = 10 : 27$$

If C takes 27 hrs. (A + B) will take 10 hrs.

If C takes $4\frac{1}{2}$ hrs. (A + B) will take

$$\frac{10 \times 9}{2 \times 27} \cong 1 \text{ hr. } 40 \text{ mins.}$$

15. I. Work done by A, B and C together in 3 days

$$\frac{1}{8} + \frac{1}{12} + \frac{1}{16} = \frac{13}{48} \text{th of the work.}$$

Thus, in 9 days, total work done will be

$$3 \times \frac{13}{48} = \frac{39}{48} \text{th of the work}$$

$\therefore$ Work remaining $= 1 - \frac{39}{48} = \frac{9}{48}$ th of the work is left of which A will finish $\frac{6}{48}$ th of work on 10th day and B will finish remaining work on 11th day.

Thus, I is false

II. Work done by A, B, C together in 9 days $= \frac{39}{48}$

$$\text{Work left } = 1 - \frac{39}{48}$$

$$= \frac{9}{48} \text{ th of the whole.}$$

On the 10th day B will finish $\frac{1}{12}$th of the whole.

$$\therefore \quad \text{Work left } = \frac{9}{48} - \frac{1}{12}$$

$$= \frac{5}{48} \text{ th of the whole.}$$

On 11th day C will finish $\frac{1}{16}$th of the work

$$\therefore \quad \text{Work left } = \frac{5}{48} - \frac{1}{16}$$

$$= \frac{2}{48} \text{ th of the whole}$$

which will be done by A on the 12th day. Thus, work will be finished on the 12th day. Thus, II is false.

III. Work done by A, B and C in 9 days $= \frac{39}{48}$

Work left after 9 days

$$= 1 - \frac{39}{48} = \frac{9}{48} \text{ th of the whole.}$$

On the tenth day $\frac{1}{16}$ th will be finished by C.

$$\therefore \quad \text{Work left } = \frac{9}{48} - \frac{1}{16}$$

$$= \frac{6}{48} \text{ th of the whole,}$$

which will be finished by A on the eleventh day. Thus, III is true.

NUMERICAL TYPE QUESTIONS

1. Total area to be painted

$= 10 \times 15$ (ceiling) $+ 2 \times 10 \times 10$ (wall) $+ 2 \times 10 \times 15$ (wall)

$= 650$ sq.ft

In one day Meera can paint

$$10 \times \frac{10}{2} = 50 \text{ sq.ft}$$

Ashok can paint

$$10 \times \frac{10}{3} = \frac{100}{3} \text{ sq.ft,}$$

So, together they can paint $\frac{250}{3}$ sq.ft per day.

Let number of days Ashok works be x.

Work done in 9 days $= 50(9 - x) + \frac{250}{3}x = 650$

$$\therefore \quad 450 - 50x + \frac{250x}{3} = 650$$

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$$\Rightarrow \quad \frac{100}{3}x = 200$$

$$\Rightarrow \quad x = 6 \text{ days.}$$

2. Let fixed interval be x

$$\therefore \quad \frac{5}{10} + \frac{5-x}{15} + \frac{5-2x}{20} = 1$$

$$\Rightarrow \quad (5)6 + (5-x)4 + (5-2x)3 = 60$$

$$\Rightarrow \quad 30 + 20 - 4x + 15 - 6x = 60$$

$$\Rightarrow \quad x = \frac{1}{2} \text{ hour.}$$

3. Let pipe B alone take b hours longer than if both A and B were open.

$$8 = \sqrt{4 \times x}$$

$$\Rightarrow \quad 64 = 4 \times x$$

$$\Rightarrow \quad x = 16 \text{ hours.}$$

Alternatively

Part filled if both A and B are open $= \frac{1}{8}$

Part filled if only A is open $= \frac{1}{12}$

$\therefore$ Part filled if only B is open $= \frac{1}{8} - \frac{1}{12} = \frac{1}{24}$

$\therefore$ B alone would take 24 hours, i.e., B alone would take 24 – 8 = 16 hours longer to fill the tank.

4. Since 5 tanks of same efficiency fill the boiler in 2 hours each tank individually takes

$2 \times 5 = 10$ hours to fill it.

Tank with half the efficiency will take

$10 \times 2 = 20$ hours.

In one hour, fraction of boiler getting filled when 4 fully efficient pipes with two as inlet and two as outlet and 5th acts at 50% efficiency as an inlet is

$$\frac{1}{10} + \frac{1}{10} + \frac{1}{20} - \frac{1}{10} - \frac{1}{10} = \frac{1}{20}$$

Hence it will take 20 hours to fill the boiler tank.

5. Here fraction of tank filled in 1 hour

$$\frac{1}{10} + \frac{1}{10} + \frac{1}{20} = \frac{5}{20} = \frac{1}{4}\text{th}$$

of the tank.If they are opened alternately, then tank gets filled in $4 \times 3 = 12$ hours

Hence half of the tank gets filled in $\frac{12}{2} = 6$ hours.

6. 4/7 of work is completed.

Remaining work $= 1 - \frac{4}{7} = \frac{3}{7}$

Remaining period = 46 – 33 = 13 days

Less work, less men(direct proportion)

Less days, more men(Indirect proportion)

More hours/day, less men(Indirect proportion)

work	4/7 : 3/7	
Days	13 : 33	:: 117 : x
hrs/day	9 : 8	

$$\frac{4}{7} \times 13 \times 9 \times x = \left(\frac{3}{7}\right) 33 \times 8 \times 117$$

$x = (3 \times 33 \times 8 \times 117) / (4 \times 13 \times 9)$

$= 198$ men

So, additional men to be employed

$= 198 - 117 = 81$

7. Let L is length of train in km and x is speed of the cyclist, Then

(*i*) $\frac{3600}{9}$ L = 42 – x = 400L

(*ii*) $\frac{3600}{5}$ L = 42 + x =720 L

On adding, we get

1120 L = 84

$$\Rightarrow \quad L = \frac{84 \times 1000}{1120} = 75\text{m}$$

8.

$$\frac{13}{\text{Speed upstream}} = 5$$

and

$$\frac{28}{\text{Speed downstream}} = 5$$

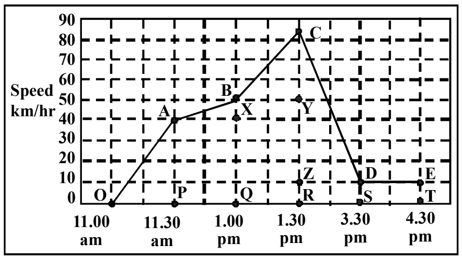
$\therefore$ Speed upstream $= \frac{13}{5}$ km/hr.

and Speed downstream $= \frac{28}{5}$ km/hr.

$\therefore$ Speed of river $= \frac{1}{2}\left(\frac{28}{5} - \frac{13}{5}\right)$

$= \frac{1}{2} \times \frac{15}{5} = 1.5$ km/hr

9. Graph of speed vs time is plotted as shown. Area under the graph and time axis gives the distance.



Required distance

$= A(\Delta\ OAP) + A(\Delta\ BXA) + A(\square\ APQX)$

$$= \frac{1}{2} \times \left(\frac{1}{2}\right) \times 40 + \frac{1}{2} \times 10 \times \left(\frac{3}{2}\right) + \left(\frac{3}{2}\right) \times 40$$

$$= 10 + \frac{30}{4} + 60 = 77.5 \text{ km.}$$

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10. Distance covered are the same.

Speed in the latter is maintained at 10 kmph.

$\therefore \quad 77.5 = 10 \times x$

$\Rightarrow \quad x = 7.75$ hours.

Hence, clock time will be 4.30 + 7 hours 45 mins

= 12.15 a.m.

11. Average speed = $\dfrac{\text{total distance travelled}}{\text{total time taken}}$

Distance travelled from 1.00 pm to 4.30 p.m.

= A(□BQRY) + A(ΔBYC) + A(ΔCZD) + A(□ZRTE)

$= 50 \times \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times 35 + \frac{1}{2} \times (2) \times 75 + 3 \times 10$

$= \frac{50}{2} + \frac{35}{4} + 75 + 30$

= 130 + 8.75

= 138.75 km.

Total distance travelled = 77.5 + 138.75

= 216.25 km.

$\therefore$ Average speed $= \dfrac{216.25}{5.5} = 39.3$ kmph.

12. While deer makes 10 leaps, cheetah makes 6 leaps.

$\therefore$ Distance covered in 10 leaps by deer

= 10 × 1 = 10m

Distance covered in 6 leaps by cheetah

= 6 × 2 = 12 m

Hence in six leaps, cheetah gains

12 – 10 = 2 m over the deer.

Cheetah has to gain 100 m over the deer.

So, total leaps required by cheetah

$= \frac{6}{2} \times 100 = 300.$

13. 50 leaps of the hare $50 \times 1\frac{3}{4} = \frac{175}{2} = 87.5$ m.

So, hound should gain $\frac{175}{2}$ m over the hare.

When hound travels $3 \times 2\frac{3}{4}$ m, then

hare travels $4 \times 1\frac{3}{4}$ m.

Hence in 3 leaps of the hound the hound gains

$\frac{33}{4} - \frac{28}{4} = \frac{5}{4}$ metres

$\therefore$ Number of leaps required $= \dfrac{\frac{175}{2}}{\frac{5}{4}} \times 3$

$= \frac{175}{2} \times \frac{4}{5} \times 3 = 210$ leaps.

14. Fuel required for first 20 km $= \frac{20}{8} = 2.5$ litres.

Fuel required for next 60 km $= \frac{60}{15} = 4$ litres.

$\therefore$ Petrol left = 10 – 6.5 = 3.5 litres.

Now, he has to travel = 115 – (60 + 20)

= 35 km in 3.5 litres.

So he can go at a speed of either 45 km/hr or 80 km/hr as mileage is 10 km/litre in both cases. But since he reaches in minimum time, he travels at 80 km/hr

$\therefore$ Time taken $= \frac{20}{40} + \frac{60}{60} + \frac{35}{80} = \frac{155}{80}$

$= 1 + \frac{75}{80}$ hours

= 1 hours 56 minutes and 25 seconds.

15. Total distance travelled = 115 km

Total time taken $= \frac{155}{80}$ hours

$\therefore$ Average speed $= \dfrac{115}{\frac{155}{80}} = 60$ km/hr

$\therefore$ Fuel consumption $= \frac{115}{15} = 7\frac{2}{3}$ litres.

So, fuel saved $= 10 - 7\frac{2}{3} = 2\frac{1}{3}$ litres.

16. A overlaps B in the middle of the 6th round

So, when A runs $5\frac{1}{2}$ rounds, B runs only $4\frac{1}{2}$ rounds.

$\therefore \quad \dfrac{\text{A's Speed}}{\text{B's Speed}} = \dfrac{5\frac{1}{2}}{4\frac{1}{2}} = \dfrac{11}{9}$

So when A runs 11 km B runs 9 km.

Thus, when A runs 2 km, B runs $2 \times \frac{9}{11} = \frac{18}{11}$ km.

$\therefore$ A wins by $\left(2 - \frac{18}{11}\right) = \frac{4}{11}$ km.

17. Speeds of A and B are as

$1\frac{3}{8} : 1 = \frac{11}{8} : 1 = 11 : 8$

In a race of 11 metres, A gain

11 – 8 = 3 metres over B.

To gain 120 metres, race must be of

$120 \times \frac{11}{3} = 440$ metres.

18.

	Work	Number
Men	3	20
Women	2	30
Children	1	36

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Ratio of wages = (3 × 20) : (2 × 30) : (1 × 36)

= 5 : 5 : 3

Total wages of 20 men = $\frac{5}{13} \times 780$ = ₹ 300

∴ Wages of a man = ₹ 15.

Similarly, wages of 30 women

$\frac{5}{13} \times 780$ = ₹ 300

Wages of a woman = ₹ 10

Wages of 36 children $\frac{3}{13} \times 780$ = ₹ 180

Wages of a child = ₹ 5

∴ Total wages of 15 men, 21 women and 30 children

= 15 × 15 + 21 × 10 + 30 × 5

= ₹ 585

∴ Total wages for 2 weeks = ₹ 1170.

19. Work done by A in 3 days

$= 3 \times \frac{1}{6} = \frac{1}{2}$ of the work.

Work done by B in 3 days

$= 3 \times \frac{1}{8} = \frac{3}{8}$ of the work.

∴ Work done by the boy in 3 days

$= 1 - \left(\frac{1}{2} + \frac{3}{8}\right)$

$= \frac{1}{8}$ of the work.

∴ Ratio of work done

$= \frac{1}{2} : \frac{3}{8} : \frac{1}{8} = 4 : 3 : 1$

∴ Boy's share = $\frac{1}{8}$ × ₹ 200 = ₹ 25.

20. A finishes the job at standard rate in 12 days,

So in 1 day, the part of job done by A = $\frac{1}{12}$

Similarly the part of job done by B in 1 day

$= \frac{1}{2 \times 9} = \frac{1}{18}$

If a works for 2 days followed by 2 days of B at standard rate.

Part of work done = $\frac{2}{12} + \frac{2}{18} = \frac{5}{18}$.

So, in (2 alternate days work of 'A' and 'B')

3 × 4 = 12 days

They will finish $3 \times \frac{5}{18} = \frac{15}{18}$th of the work.

Work remaining = $1 - \frac{15}{18} = \frac{3}{18} = \frac{1}{6}$

This is finished by A alone in $\dfrac{\frac{1}{6}}{12}$ = 2 days.

Hence, total number days = 12 + 2 = 14 days.

EXERCISE – II

MCQ TYPE QUESTIONS

1. For $M \Rightarrow \frac{12}{20} = 0.6$

$N \Rightarrow \frac{25}{45} = 0.55$

$O \Rightarrow \frac{45}{75} = 0.6$

$P \Rightarrow \frac{57}{100} = 0.57$

2. $V_{half} = 30(s)$ drawing rate = s

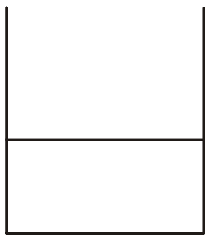
Total volume = 60 S tank

$(s^1)(10) - (s)10 = 30s$

$s^1(s) - s = 3s$

$s1 = 4s$

$s^1 = 4$ drawing rate



3. Given: A tourist covers half the journey at 60 km/h, one fourth at 30 km/h and the remaining one fourth at 10 km/h

To find: Avearge speed of the whole journey.

Analysis: One approach to this is to find individual time consumed in each of the three parts and use that value to get total time, and find average speed.

So,

Lets assume total length of journey = x km

∴ First phase at 60 km/h

Time $t_1 = \frac{\text{Distance}}{\text{Speed}} = \frac{x}{2}\frac{1}{60} = \frac{x}{120} h$

Second phase at 30 km/h

Distance = $\frac{x}{4}$ km

Time $t_2 = \frac{x}{4.30} = \frac{x}{120}$

Third phase at 10 km/h

Distance = $\frac{x}{4}$ km

Time $t_3 = \frac{x}{4.10} = \frac{x}{40} h$

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$\therefore$ Total time $= \frac{x}{120}+\frac{x}{120}+\frac{x}{40} = \frac{x}{40}\left(\frac{1}{3}+\frac{1}{3}+1\right)$

$= \frac{x}{40}\frac{(1+1+3)}{3} = \frac{5x}{120}$

$\therefore$ Average speed $= \frac{\text{Total Distance}}{\text{Total time}}$

$= \frac{x\cdot 120}{5x} = 24$ km/h

Hence, the answer is (c)

4. Average Speed $= \frac{\text{Total distance}}{\text{total time taken}}$

Total Distance $= \frac{8+6+16}{15+15+15} = \frac{30\times 60}{45}$

= 40km/h.

5.

	Fuel consumption	Actual
P	60 km/l	$\frac{15}{60}=\frac{1}{4}l$
Q	90 km/l	$\frac{75}{90}=\frac{5}{6}l$
R	75 km/l	$\frac{40}{75}=\frac{8}{15}l$
S	30 km/l	$\frac{10}{30}=\frac{1}{3}l$

6. 5 skilled workers build wall in 20 days

1 skilled worker build wall in 20 × 5 days

Hence in 1 day, part of work done by skilled work

$= \frac{1}{100}$

Similarly in 1 day part of work done by semi-skilled workers

$= \frac{1}{25\times 8}$

and in 1 day part of work done by un-skilled worker

$= \frac{1}{30\times 10}$

So part of work done in 1 day by 2 skilled, 6 semi-skilled and 5 unskilled

$= \frac{2}{100}+\frac{6}{200}+\frac{5}{300} = \frac{1}{15}$

So work done by given workers in days = 15

NUMERICAL TYPE QUESTIONS

1. Tiger – 1 leap ⇒ 8 meter

Speed = 5 leap/hr = 40m/min

Deer → 1 leap = 5 meter

speed = 4hr = 20m/min

Let at time 't' the tiger catches the deer.

$\therefore$ Distance travelled by deer + initial distance between them

50 × 8 ⇒ 400m = distance covered by tiger.

$\Rightarrow$ 40 × t = 400 + 20t

$\Rightarrow$ $t = \frac{400}{200}$ = 20 min

$\Rightarrow$ total distance ⇒ 400 + 20 × t = 800 m

2. Speed of man = 8;

Left distance = d

Time taken $= \frac{d}{8}$

Upstream: Speed of stream = s

$\Rightarrow$ speed upstream = S' = (8 – s)

$$t' = \left(\frac{d}{8-s}\right)$$

Downstream:

Given speed downstream $= t'' = \frac{d}{8+s}$

$\Rightarrow$ $3t' = t'' \Rightarrow \frac{3d}{8-s} = \frac{d}{8+s}$

$\Rightarrow$ $\frac{3d}{8-s} = \frac{d}{8+s}$

$\Rightarrow$ s = 4 km/hr

3. Let the length of platform = x.m

and train length = 280 m (given)

According to Question

$$\frac{x+280}{60} = \frac{280}{20}$$

$\therefore$ x = 560 m.

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4 CHAPTER

Ratio, Proportion and Mixtures

RATIO

Ratio of two terms 'a' and 'b' is denoted by a : b (read as 'a is to b') and is measured by $\frac{a}{b}$.

Numerator 'a' is called *'antecedent'* and denominator 'b' is called 'consequent'.

While comparing two quantities in terms of ratio :

(*i*) Two quantities must be of the same kind.

(*ii*) Units of measurement of the two quantities must be the same.
e.g., if a = ₹ 2 and b = 50 paise,
then a : b = 200 : 50 = 4 : 1.

(*iii*) Ratio is a pure number, i.e., without any unit of measurement as it denotes how many times a given quantity is in comparison to another.

(*iv*) Ratio would stay unaltered even if both antecedent and the consequent are multiplied or divided by the same number.

e.g., $a : b = \frac{a}{b} = \frac{a \times m}{b \times m} = \frac{\frac{a}{m}}{\frac{b}{m}}$.

Properties of Ratios

1. **Invertendo :** If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$ i.e., inverse ratios of two equal ratios are equal.

2. **Alternendo :** If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$ i.e., ratios of antecedents and consequents of two equal ratios are equal.

3. **Componendo :** If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$ i.e., adding 1 to both sides.

4. **Dividendo :** If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$ i.e., subtracting 1 from both sides.

5. **Componendo-Dividendo :** If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ i.e., dividing results of componendo by dividend.

Equal Ratio : If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$; then each ratio

$$= \frac{\text{Sum of Numerators}}{\text{Sum of Denominators}} = \frac{a+c+e...}{b+d+f...}$$

The principle can also be applied after multiplying Numerator and Denominator of any fraction by the same number.

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ...$, then each ratio is equal to $\left(\frac{pa^n + qc^n + re^n + ...}{pb^n + qd^n + rf^n + ...}\right)^{1/n}$,where p, q, r, n ... may have any values except that they must not all be zeros.

PROPORTION

When ratio of two terms is equal to the ratio of two other terms, then these four terms are said to be in **proportion,** i.e., if a : b = c : d, then a, b, c and d are in proportion.

a, b, c and d are called first, second, third and fourth proportionals respectively. The terms a and d are called **extremes,** while b and c are called **means.**

If a : b = c : d, then ad = bc i.e., product of extremes is equal to product of means.

Continued Proportion

When $\frac{a}{b} = \frac{b}{c}$, then a, b and c are said to be in continued proportion and b is called *geometric mean* or *mean proportional* between a and c.

Also, $b^2 = a \times c$, $\Rightarrow$ $b = \sqrt{ac}$

If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = ...$, then a, b, c, d, e ... are said to be in continued proportion.

Concept of continued proportion is useful in the following situation.

(*i*) If four quantities a, b, c and d and a : b, b : c and c : d, are know, then we can find a : d as $\frac{a}{d} = \frac{a}{b} \times \frac{b}{c} \times \frac{c}{d}$

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(*ii*) If a, b, c and d are four quantities and a : b, b : c, c : d are known, then we can find a : b : c : d.

e.g., If a : b = 2 : 3; b : c = 4 : 5, c : d = 7 : 9, find a : b : c : d.

a : b = 8 : 12, b : c = 12 : 15 (LCM of 3 and 4 is 12)

∴ a : b : c = 8 : 12 : 15

a : b : c = 56 : 84 : 105, c : d = 105 : 135 (LCM of 7 and 15 is 105)

∴ a : b : c : d = 56 : 84 : 105 : 135

MIXTURES AND ALLIGATIONS

Simple Mixture : When two different ingredients are mixed together, it is called *simple mixture.*

Compound Mixture : When two or more simple mixtures are mixed together to form another mixture, it is called compound mixture.

Alligation Rule

Alligation rule states that "When different quantities of the same or different ingredients, of different costs (one cheap and other dear) are mixed together to produce a mixture of a mean cost, then ratio of their quantities is inversely proportional to the difference in their cost from the mean cost."

$$\frac{\text{Quantity of Cheap}}{\text{Quantity of Dear}} = \frac{\text{Price of Dear} - \text{Mean Price}}{\text{Mean Price} - \text{Price of Cheap}}$$

Application of Alligation Rule :

(1) To find mean value of a mixture when prices of two or more ingredients, which are mixed together and the proportion in which they are mixed are given.

(2) To find proportion in which ingredients at given prices must be mixed to produce a mixture at a given price.

Some Important Results :

(1) When x_1 quantity of ingredient A of cost C_1 and x_2 quantity of ingredient B of cost C_2 are mixed, the, cost of the mixture,

$$C_m = \frac{C_1x_1 + C_2x_2}{x_1 + x_2}$$

(2) Similarly, when more than two ingredients are mixed, then $C_m = \dfrac{C_1x_1 + C_2x_2 + C_3x_3 + \ldots C_nx_n}{x_1 + x_2 + x_3 + \ldots x_n}$

(3) When two mixtures M_1 and M_2 each containing ingredient A and B in the ratio a : b and x : y, respectively, are mixed, then proportion of the ingredients A and B i.e., $q_A : q_B$, in the compound mixture is given by.

$$\frac{q_A}{q_B} = \frac{M_1 \times \left(\frac{a}{a+b}\right) + M_2 \times \left(\frac{x}{x+y}\right)}{M_1 \times \left(\frac{b}{a+b}\right) + M_2 \times \left(\frac{y}{x+y}\right)}$$

and, quantity in which M_1 and M_2 is to be mixed when quantity of A and B i.e., q_A and q_B in the compound mixture is known is given by

$$\frac{\text{Quantity of } M_1}{\text{Quantity of } M_2} = \frac{\left(\frac{x}{x+y}\right) - \left(\frac{q_A}{q_A + q_B}\right)}{\left(\frac{q_A}{q_A + q_B}\right) - \left(\frac{a}{a+b}\right)}$$

(4) When a mixture of three ingredients A, B and C is given, take any two ingredients such that cost of the mixture is between costs of the two chosen ones and find the ratio. Once again, take two more ingredients and find their ratio. Then find combined ratio. This will give infinite number of solutions.

(5) If a vessel contains 'a' litres of liquid A, and 'b' litres be withdrawn and replaced by liquid B, then if 'b' litres of mixture be withdrawn and replaced by liquid B, and operation repeated 'n' times in all, then

$$\frac{\text{Liquid A left after } n^{th} \text{ operation}}{\text{Initial quantity of liquid A in vessel}} = \frac{\left(\frac{a-b}{a}\right)^n}{1 - \left(\frac{a-b}{a}\right)^n}$$

$$\frac{\text{Liquid A left after } n^{th} \text{ operation}}{\text{Liquid B left after } n^{th} \text{ operation}} = \frac{\left(\frac{a-b}{a}\right)^n}{1 - \left(\frac{a-b}{a}\right)^n}$$

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EXERCISE – I

MCQ TYPE QUESTIONS

1. A cask of wine when fully filled holds 10 litres. 2 litres of wine is removed and filled with water. Then 4 litres in the solution is replaced with water. Then, 6 and 8 litres respectively. At the end of the 4th operation, the ratio of wine to water is

(*a*) $\frac{4!}{(5)^4}$ (*b*) $\frac{8!}{(10)^4}$

(*c*) $\frac{4!}{5^4 - 4!}$ (*d*) $\frac{8!}{(10)^4 - 8!}$

2. A 20 litre vessel is filled with alcohol. Some of the alcohol is poured out into another vessel of an equal capacity, which is then completely filled by adding water. Then mixture obtained is poured into the first vessel to capacity. Then $6\frac{2}{3}$ litres is poured from the first vessel into second. Both vessels now contain an equal amount of alcohol. How much alcohol was originally poured from the first vessel into the second?

(*a*) 9 litres

(*b*) 10 litres

(*c*) 12 litres

(*d*) 12.5 litres

3. What is the ratio compounded of x – y : x + y and $y^2 + xy : x^2 - xy$?

(*a*) $y^2 : x^2$ (*b*) xy : 1

(*c*) y : x (*d*) $\sqrt{x} : \sqrt{y}$

4. If ratio of two natural numbers x and y is 'a' and that of y and x is 'b', then value of 'a + b' is

(*a*) greater than 2

(*b*) lesser than 1

(*c*) greater than 1

(*d*) data insufficient.

5. The charges of a goldsmith is partly fixed and partly variable with the amount in grams of gold. The charge is ₹ 300 for 20 grams of gold and ₹ 550 for 45 grams of gold. Find the charge of work for 100 grams of gold.

(*a*) ₹ 900

(*b*) ₹ 1000

(*c*) ₹ 1100

(*d*) ₹ 9000

Directions (Q.6–7) : *Refer to the data below and answer the questions that follow.*

The ratio of white collar to blue collar employees in a firm is 8 : 3. Also ratio of male employees to female employees is 7 : 4. It is observed that 60% of the white collar employees are males.

6. What is the ratio of female white collar employees to male blue collar employees?

(*a*) 16 : 11 (*b*) 24 : 11

(*c*) 3 : 1 (*d*) Data insufficient

7. If there are 48 female blue collar workers, what is the difference between male white collar employees and female blue collar employees?

(*a*) 288 (*b*) 240

(*c*) 220 (*d*) Data insufficient

8. In a chemical experiment, two NaOH solution bottles are used. Bottle A contains salt and water in the ratio 7 : 3 and bottle B contains salt and water in the ratio 4 : 3. In what proportion should the quantities be taken from A and B to give the 2 : 1 NaOH solution?

(*a*) 2 : 1 (*b*) 10 : 7

(*c*) 20 : 7 (*d*) 1 : 2

9. A triswitch mixer grinder operates in three stages of speed in revolutions per min i.e., rpm. The speed of the motor with some load in the second stage varies as a square of the speed in stage 1 and that in stage 3 varies as a cube of that in stage 2. If mixer motor makes 50 revolutions in 3 minutes in stage 1, its speed in stage 2 and 3 with the same load is 2500 and 12500 rpm respectively. What will be the speed of motor (in rpm) in stage 3, if at the same load motors speed in rpm is 25 revolutions in 6 minutes in stage 1?

(*a*) $\frac{3125}{16}$ rpm (*b*) $\frac{5^5}{2^{10}}$ rpm

(*c*) $\frac{5^{10}}{2^5}$ rpm (*d*) None of these

10. Two bags have certain number of mangoes. If half the mangoes from bag 1 are transferred to bag 2, the ratio of mangoes in bag 1 to bag 2 gets reversed. Now one-third and two-third of the mangoes from the first and the second bag are transferred to the second and first bag respectively. The ratio of mangoes in bag 1 to bag 2 now is

(*a*) 2 : 1 (*b*) 2 : 2

(*c*) 4 : 5 (*d*) 5 : 4

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NUMERICAL TYPE QUESTIONS

1. Five litres are drawn from a cask full of wine and it is then filled with water. Five litres of the mixture are drawn and the cask is again filled with water. The quantity of wine now left in the cask to that of the water in it is in the ratio 361 : 39. The cask hold ______ litres.

2. In a mixture of wheat and barley, wheat is 60%. To 400 kg of the mixture a quantity of barley is added and then the wheat is $53\frac{1}{3}\%$. The_____kg of barley are added?

3. Two jars of capacity 3 and 5 litres are filled with mixtures of alcohol and water. In the smaller jar 25% of the mixture is alcohol and in the larger 25% of the mixture is water. The jars are emptied into a 9 litre cask and remaining volume is filled up with water. Then percentage of alcohol in the cask will be ________

4. A man buys milk at ₹ 5 a litre and after adding water, sells it at ₹ 6 a litre, thereby making a profit of $33\frac{1}{3}\%$. The proportion of water to milk in the mixture is_______

5. The percentage of alcohol in two wine tanks is 20 and 40 respectively and rest is grape juice. If a wine glass is filled to its half by the tank 1 and 2 in the ratio 2 : 3 and the rest is filled again by pure alcohol, then the ratio of the grape juice to alcohol in the wine glass is _______

6. In ______ ratio must a grocer mix two varieties of tea worth ₹ 60 a kg and ₹ 65 a kg so that by selling the mixture at ₹ 68.20 a kg he may gain 10%?

7. A container contains 40 lit of milk. From this container 4. lit of milk was taken out and replaced by water. This process was repeated further two times. The ______litres of milk is now contained by the container?

8. A merchant has 1000kg of sugar,part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is_____ kg

9. Volumes in the proportion 3 : 4 : 7 of three different substances are mixed together. The densities of equal volume of the substances are in the ratio 5 : 2 : 6 respectively. The weight of the third substance contained in 130 lbs. of the mixture is _______ lbs

10. In a bag, there are 50 paise coins, 25 paise coins and rupee coins, proportional to the numbers 3, 4 and $2\frac{1}{2}$. If they amount to ₹ 210, then number of rupee coins is______

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. A cube of side 3 units is formed using a set of smaller cubes of side 1 unit. Find the proportion of the number of faces of the smaller cubes visible to those which are NOT visible.

(*a*) 1 : 4

(*b*) 1 : 3

(*c*) 1 : 2

(*d*) 2 : 3

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2. A container originally contains 10 litres of pure spirit. From this container 1 litre of spirit is replaced with 1 litre of water. Subsequently, 1 litre of the mixture is again replaced with 1 litre of water and this process is repeated one more time. How much spirit is now left in the container?

(*a*) 7.58 litres

(*b*) 7.84 litres

(*c*) 7 litres

(*d*) 7.29 litres

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ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*c*) **2.** (*b*) **3.** (*c*) **4.** (*c*) **5.** (*c*) **6.** (*a*) **7.** (*b*) **8.** (*c*) **9.** (*b*) **10.** (*a*)

Numerical Type Questions

1. 100 **2.** 50 **3.** 50 **4.** 1 : 9 **5.** 17 : 33 **6.** 3 : 2 **7.** 29.16 **8.** 600 **9.** 84 **10.** 105

EXERCISE – II

MCQ Type Questions

1. (*c*) **2.** (*d*)

EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. Stage $\dfrac{\text{Wine left}}{\text{Original quantity}}$

0 $\dfrac{10}{10}$

1 $\dfrac{8}{10}$

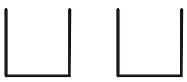
2 $\dfrac{8}{10}\times\dfrac{6}{10}$

3 $\dfrac{8}{10}\times\dfrac{6}{10}\times\dfrac{4}{10}$

4 $\dfrac{8}{10}\times\dfrac{6}{10}\times\dfrac{4}{10}\times\dfrac{2}{10} \Rightarrow \dfrac{4\times3\times2\times1}{5^4} = \dfrac{4!}{5^4}$

$$\therefore \quad \frac{\text{Wine}}{\text{Water}} = \frac{\frac{4!}{5^4}}{1-\frac{4!}{5^4}} = \frac{4!}{5^4-4!}$$

2.



(1) (2)

	C_1	C_2
Operation1	20 litres alcohol	Empty
Operation 2	(20 – x) alcohol	x litres alcohol (20 – x) litres water
Operation 3	(20 – x) litres alcohol + xlitres mixture i.e., $\left(\frac{x^2}{20}\text{ litres alcohol}\right)$	(20 – x) litres mixture

Now $\frac{1}{3}$ rd of C_1 poured into C_2

Quantity of alcohol in $C_1 = C_2 = 10$ litres

In C_1 : $\dfrac{2}{3}\left[20 - x + (x)\left(\dfrac{x}{20}\right)\right] = 10$

Solving we get, $x = 10$ litres

Hence10 litres of alcohol was initially poured from C_1 to C_2.

3. Required ratio $= \dfrac{x-y}{x+y}\times\dfrac{y^2+xy}{x^2-xy} = \dfrac{y}{x} = y : x$

4. *Given* : $\dfrac{x}{y} = a$ and $\dfrac{y}{x} = b$

If $\dfrac{x}{y} > 1$, then $\dfrac{y}{x} < 1$,

$$\Rightarrow \quad \frac{x}{y}+\frac{y}{x} > 1\,.$$

Hence option (*c*) is correct.

5. Total charge = fixed + variable

$$T_C = m + kx$$

where m and k are constants.

$\therefore$ $300 = m + k \times 20$

Also, $550 = m + k \times 45$

Solving for m and k; we get

$$m = 100;\ k = 10.$$

Hence, for 100 grams,

total charge = 100 + 10 × 100 = ₹ 1100.

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Q. 6 and 7.

Using given data, we can draw following table

	Male	Female	
White Collar	4.8x	3.2x	8x
Blue Collar	2.2x	0.8x	3x
	7x	4x	11x

Now, all the questions can be answered.

6. $\dfrac{\text{Female white collar employess}}{\text{Male blue collar employees}} = \dfrac{3.2x}{2.2x} = \dfrac{16}{11}$

7. Since $0.8x = 48$

$\Rightarrow$ $x = 60$

$\therefore$ Required difference $= 4.8x - 0.8x$

$= 4.8 \times 60 - 0.8x = 240$

8. Let 10x be taken from bottle A and 7y be taken from bottle B.

$$\text{Total salt taken} = \left(\frac{7}{10} \times 10x\right) + \left(\frac{4}{7} \times 7y\right)$$

$$= 7x + 4y$$

Total water taken = 3x + 3y.

$$\therefore \quad \frac{7x + 4y}{3x + 3y} = \frac{2}{1}$$

$\Rightarrow$ $7x + 4y = 6x + 6y$

$\Rightarrow$ $x = 2y$

$$\therefore \quad \frac{x}{y} = \frac{2}{1}$$

$$\therefore \text{ Ratio of quantities} = \frac{10x}{7y}$$

$$= \frac{10}{7} \times \frac{2}{1} = \frac{20}{7}$$

9. Let speed of the motor in stage 1 be x rpm. Then its speed in stage 2 be $(x^2) = ax^2$ (say) and in stage 3 be $(ax^2)^3 = b.a^3x^6$ (say)

Given that $x = \dfrac{50}{3}$ ...(i)

$ax^2 = 2500$ ...(ii)

$ba^3x^6 = 12500$ ...(iii)

Substituting value of x in (ii), we get

$$a = \frac{2500}{(50)^2} \times 3^2 = 9$$

Again, substituting values of a and x, we get

$$b = \frac{12500}{(2500)^3} = \frac{1}{125} \times 10^{-4}$$

Given : $x = \dfrac{25}{6}$ rpm

$$\therefore \text{ Speed in stage 3} = \frac{10^{-4}}{125} \times (9)^3 \times \left(\frac{25}{6}\right)^6$$

$$= \frac{10^{-4}}{5 \times 25} \times (3)^6 \times \frac{(25 \times 5) \times (5 \times 25^4)}{(2)^6 \cdot (3)^6}$$

$$= \frac{5 \times 25^4}{2^6 \times 10^4} = \frac{5^5}{2^{10}} \text{ rpm}$$

10. Let bag 1 and 2 have

	x mangoes	y mangoes
Step 1	$\dfrac{x}{2}$	$y + \dfrac{x}{2}$

But $\dfrac{x}{2} : \; y + \dfrac{x}{2} = y : x$

Step 2	$\dfrac{2}{3}y + \dfrac{2}{3}x$	$\dfrac{1}{3}y + \dfrac{1}{3}x$

$$\therefore \quad \text{Required ratio} = \frac{\frac{2}{3}y + \frac{2}{3}x}{\frac{1}{3}y + \frac{1}{3}x} = \frac{2\left(\frac{x+y}{3}\right)}{1\left(\frac{x+y}{3}\right)}$$

$$= 2 : 1$$

NUMERICAL TYPE QUESTIONS

1. Let quantity of wine be a, i.e., capacity of the cask be 'a'

$$\text{i.e., } \frac{\text{Quantity of wine}}{\text{Quantity of water}} = \frac{\left(1 - \frac{5}{a}\right)^2}{1 - \left(1 - \frac{5}{a}\right)^2}$$

$$\therefore \quad \frac{361}{39} = \frac{\left(1 - \frac{5}{a}\right)^2}{1 - \left(1 - \frac{5}{a}\right)^2}$$

$$\Rightarrow \quad \frac{361}{39} - \frac{361}{39}\left(1 - \frac{5}{a}\right)^2 = \left(1 - \frac{5}{a}\right)^2$$

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$$\Rightarrow \quad \left(1-\frac{5}{a}\right) = \sqrt{\frac{361}{400}} = \frac{19}{20}$$

$$\Rightarrow \quad \frac{5}{a} = \frac{1}{20}$$

$\therefore$ a = 100 litres

2. Quantity of wheat in 400 kg of mixture

$= 0.6 \times 400 = 240$ kg

Quantity of barley = 400 – 240 = 160 kg

Let x kg of barley be added to 400 kg of the mixture

$$\therefore \quad \frac{240}{160+x} = \frac{53\frac{1}{3}}{46\frac{2}{3}} = \frac{160}{140} = \frac{8}{7}$$

$$\Rightarrow \quad 240 \times 7 = 8 \times 160 + 8x$$

$$\Rightarrow \quad x = 50 \text{ kg}$$

3. Alcohol content in 1st jar = $0.25 \times 3 = 0.75$ litre

Alcohol content in 2nd jar = $0.75 \times 5 = 3.75$ litres

$\therefore$ Total alcohol in 9 litre cask = 0.75 + 3.75

= 4.5 litres

$\therefore$ Percentage of alcohol in the cask

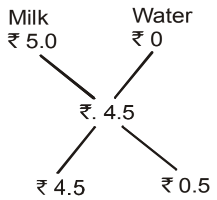
$$= \frac{4.5}{9} \times 100 = 50\%$$

4. Cost price of milk = ₹ 5/litre

$$\text{Cost price of mixture} = \frac{100}{133\frac{1}{3}} \times 6 = ₹\ 4.5/\text{litre}$$

$$\therefore \quad \frac{\text{Quantity of water}}{\text{Quantity of milk}} = \frac{5-4.5}{4.5-0} = \frac{1}{9}$$

i.e.,



Milk : Water = 9 : 1

5.

	Alcohol	Grape juice	Alcohol : Grape
Tank 1	20%	80%	1 : 4
Tank 2	40%	60%	2 : 3

Ratio of grape juice to alcohol in half filled wine glass

$$= \frac{2\times\frac{4}{5}+3\times\frac{3}{5}}{2\times\frac{1}{5}+3\times\frac{2}{5}} = \frac{17}{8}$$

So, half capacity of glass = 17 + 8 = 25

So, full capacity = $25 \times 2 = 50$

Now rest 25 parts has to be filled with alcohol.

So, ratio of the grape juice to alcohol

= 17 : (8 + 25) = 17 : 33.

Alternatively,

As the wine glass is filled to half by taking two parts from tank 1 and three parts from tank 2 and second half is equivalent to five parts which is pure alcohol,

percentage concentration of grape juice

$$= \frac{2\times 80+3\times 60+5\times 10}{2+3+5} = 34\%$$

Hence ratio of grape juice to alcohol

= 34 : 66 = 17 : 33.

6. S.P of 1 kg of mixture = ₹ 68.20, Gain =10%

$$\text{S.P of 1 kg of mixture} = ₹\left(\frac{100}{110}\times 68.2\right) = ₹\ 62.$$

Cost of 1 kg tea of 1st kind 60   Cost of 1 kg tea of 2nd kind 65

Mean Price ₹ 62

3   2

$\therefore$ Required ratio = 3 : 2.

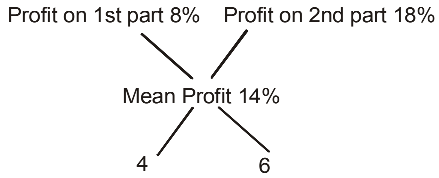
7. Amount of milk left after 3 operations

$$= 40\left(1-\frac{4}{40}\right)^3$$

$$= 40 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$$

= 29.16 lit

8.



Ratio of 1st and 2nd parts = 4 : 6 = 2 : 3.

Quantity of 2nd part = $\frac{3}{5} \times 1000$ kg = 600 kg.

9. Ratio of volumes = $\frac{3}{14} : \frac{4}{14} : \frac{7}{14}$

Ratio of densities = $\frac{5}{13} : \frac{2}{13} : \frac{6}{13}$

$\therefore$ Ratio of weights = $\frac{3}{14}\times\frac{5}{13} : \frac{4}{14}\times\frac{2}{13} : \frac{7}{14}\times\frac{6}{13}$

= 15 : 8 : 42

... (Mass = Volume × Density)

Weight of the third substance = $\frac{42}{65} \times 130 = 84$ ℓbs.

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10. Let number of 50p coins be 3x, number of 25p coins be 4x and number of rupee coins be $2\frac{1}{2}$x.

$$\therefore \quad 2\frac{1}{2}x + \frac{1}{2} \times 3x + \frac{1}{4} \times 4x = 210$$

$$\Rightarrow \quad \frac{5}{2}x + \frac{3}{2}x + x = 210$$

$$\Rightarrow \quad 5x = 210$$

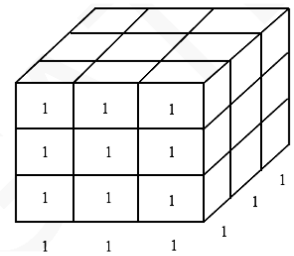
$$\Rightarrow \quad x = 42$$

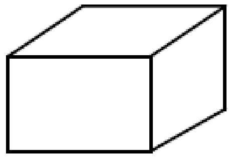
$\therefore$ Number of rupee coins = $\frac{5}{2} \times 42 = 105$.

EXERCISE – II

MCQ TYPE QUESTIONS

1.





Number of faces per cube = 6

Total number of cubes = 9 × 3 = 27

$\therefore$ Total number of faces = 27 × 6 = 162

$\therefore$ Total number of non visible faces = 162 – 54 = 108

$$\therefore \frac{\text{Number of visible faces}}{\text{Number of non visible faces}} = \frac{54}{108} = \frac{1}{2}$$

2. $10\left(\frac{10-1}{10}\right)^3 = 10\left(\frac{9}{10}\right)^3 = \frac{729}{1000}$

$\therefore \frac{729}{1000} \times 1 = 7.29$ litres

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5

CHAPTER

Permutations and Combinations & Probability

PERMUTATIONS AND COMBINATION

PERMUTATIONS

Each of the arrangements which can be made by taking some or all of a number of items is called a *Permutation*. Permutation implies "*arrangement*" or that "*order of the items*" is important.

Permutations of three items a, b and c taken two at a time are ab, ba, ac, ca, cb and bc. Since the order in which items are taken is important, ab and ba are counted as two different permutations. The words "permutation" and "arrangement" are synonymous and can be used interchangeably."

Number of permutations of n things taking r at a time is denoted by ^nP_r (and read as "nPr")

COMBINATIONS

Each of the groups or selections which can be made by taking some or all of a number of items is called *Combination*. In combinations, order in which the items are taken is not considered as long as the specific things are included.

Combinations of three items a, b and c taken two at a time are ab, bc and ca. Here, ab and ba are not considered separately because the order in which a and b are taken is not important but it is only required that a combination including a and b is what is to be counted. Words "*combination*" and "*selection*" are synonymous.

Number of combinations of n things taking r at a time is denoted by ^nC_r (and read as "nCr")

$$^nP_r = \frac{n!}{(n-r)!},\quad {}^nC_r = \frac{n!}{r!(n-r)!}$$

Number of ways in which n things may be arranged taking them all at a time, when p of the things are exactly alike of one kind, q of them exactly alike of another kind, r of them exactly alike of a third kind, and the rest all distinct is $\frac{n!}{p!q!r!}$

Number of ways of selecting one or more items from n given items = $2^n - 1$

Number of ways of dividing (p + q) items into two groups of p and q items respectively is $\frac{(p+q)!}{p!q!}$.

Number of ways of dividing 2p items into two equal groups of p each is $\frac{(2p)!}{(p!)^2}$ where two groups have distinct identity.

Number of ways of dividing 2p items into two equal groups of p each is $\frac{(2p)!}{2!(p!)^2}$ where two groups do not have distinct identity.

Number of ways in which (p + q + r) things can be divided into three groups containing p, q and r things respectively is $\frac{(p+q+r)!}{p!q!r!}$

Number of circular arrangements of n distinct items is (n – 1)! If there is a DIFFERENCE between clockwise and anticlockwise arrangements and (n – 1)!/2 if there is NO DIFFERENCE between clockwise and anticlockwise arrangements.

For $x_1 + x_2 + x_3 + ------ + x_n = s$ where $s \geq 0$, number of positive integral solutions (when $s \geq n$) is $^{s-1}C^{n-1}$ and number of non-negative integral solution is $^{n+s-1}C^{n-1}$.

PROBABILITY

If in an experiment there are 'n' occurrences out of which 'm' occurrences are favourable to a particular event E, then probability of the event E [written as P(E)] is defined as P(E) = m/n.

For the event E, non-occurrence of the event is denoted by $\overline{E}$. Out of a total n occurrences, if m occurrences are favourable to the event E, it means that (n – m) occurrences are not favourable to the event $\overline{E}$, i.e. (n - m) occurrences are favourable to the event $\overline{E}$.

Hence, probability of the event $\overline{E}$ will be $P(\overline{E})\ \frac{n-m}{n}$ (complementary event).

We can also see that $P(\overline{E}) + P(E) = 1$.

In some problems when we are asked to find out the probability of an event $\overline{E}$, it becomes easier to calculate probability of E and subtract that probability from 1 to get $P(\overline{E})$.

Note:

(*i*) Probability [P(E)] of the happening of an event E is called *probability* of success and probability $\left[P(\overline{E})\right]$ of non-happening of the event is called *probability of failure*.

(*ii*) If P(E) = 1, then event is called a *certain event* and if P(E) = 0, then event is called *impossible event*.

(*iii*) Instead of saying that the chance of happening of an event is m/n, we can also say that odds in favour of the event are m (to n – m). Similarly, if m/n is probability of the occurrences of an event, since (n – m)/n is probability of non-occurrence of the event, we can also say that the odds against the event are (n – m) to m.

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Addition Theorem on Probability

If there are two sets A and B, we know that the number of elements in $A \cup B$ is given by

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

A similar relationship exists in probability theory (called Addition Theorem of Probability) as $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, where A and B are two events.

If A and B are mutually exclusive events (i.e, A and B are disjoint sets), then $P(A \cap B) = 0$

If there are two mutually exclusive events A and B, then $P(A \cup B) = P(A) + P(B)$.

Independent Events

Two events, E_1 and E_2 are said to be independent, if occurrence of the event E_2 is not affected by the occurrence or non-occurrence of the event E_1.

If A and B are two independent events, then

$$P(A \cap B) = P(A) \times P(B)$$

EXERCISE – I

MCQ TYPE QUESTIONS

Directions (Q. 1 – 2) :

If A and B are two possible events of an experiment such that $P(A \cap B) = 0.6$ and $P(A) = 0.3$ then find $P(B)$ given that

1. A and B are mutually exclusive events

(*a*) $\frac{7}{10}$ (*b*) $\frac{4}{10}$

(*c*) $\frac{3}{10}$ (*d*) $\frac{1}{2}$

2. A and B are independent events

(*a*) $\frac{4}{7}$ (*b*) $\frac{9}{10}$ (*c*) $\frac{3}{10}$ (*d*) $\frac{3}{7}$

3. Raju throws a fair dice. He is promised an amount (in rupees), which is twice the number showing up if that number is odd and an amount thrice the number showing up, if it is even. What is the maximum amount Raju would be willing to pay each time to throw the dice, if in the long run he wants to make an average profit of ₹ 7 per throw?

(*a*) ₹ 6 (*b*) ₹ 7 (*c*) ₹ 3 (*d*) ₹ 2

Directions (Q. 4 – 5) :

7 letters are to be placed in seven addressed envelops. If the letters are placed at random into the envelopes, the probability that

4. All of them are placed in the corresponding envelops is

(*a*) 1

(*b*) $\frac{1}{6!}$

(*c*) $\frac{1}{7!}$

(*d*) None of these

5. Exactly six letters are placed in their corresponding envelopes is

(*a*) $1-\frac{1}{7!}$ (*b*) $\frac{1}{7!}$ (*c*) 1 (*d*) 0

Directions (Q. 6 – 8) :

If three cards are drawn at random, from a well shuffled pack of cards, then what is the probability that

6. All of them are from the same suit?

(*a*) $\frac{4\left({}^{13}C_3\right)}{{}^{52}C_3}$ (*b*) $\frac{\left({}^{13}C_3\right)^3}{{}^{52}C_3}$

(*c*) $\frac{4\left({}^{13}C_3\right)^2}{{}^{52}C_3}$ (*d*) $\frac{{}^{13}C_3}{{}^{52}C_3}$

7. All of them are from different suits?

(*a*) $\frac{13}{{}^{52}C_3}$ (*b*) $\frac{4(13)^3}{{}^{52}C_3}$

(*c*) $\frac{{}^{39}C_3}{{}^{52}C_3}$ (*d*) $\frac{(13)^4}{{}^{52}C_3}$

8. Two of them are number cards from the same suit and the remaining number card is from a different suit?

(*a*) $\frac{{}^{9}C_2\,{}^{9}C_1}{{}^{52}C_3}$ (*b*) $\frac{{}^{9}C_2\,{}^{27}C_1}{{}^{52}C_3}$

(*c*) $\frac{4\left({}^{9}C_2\,{}^{27}C_1\right)}{{}^{52}C_3}$ (*d*) $\frac{4\left({}^{9}C_2\,{}^{9}C_1\right)}{{}^{52}C_3}$

9. A bag contains 5 white balls and 7 red balls. If 5 balls are drawn at random, then probability that 3 balls are red and the rest are white is

(*a*) $\frac{175}{198}$ (*b*) $\frac{175}{396}$

(*c*) $\frac{38}{132}$ (*d*) $\frac{221}{396}$

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10. A bag contains 6 five rupee coins, 5 two rupee coins and 4 one rupee coins. If 5 coins are selected at random from the bag, then odds in favour of the draw yielding minimum possible amount are

(a) 1:3002 (b) 5:2998
(c) 2:3003 (d) 5:3003

11. In how many ways can 6 girls and 6 boys sit around a circular table so that no two boys sit together?

(a) $(5!)^2$ (b) $(6!)^2$
(c) 5! 6! (d) 11!

12. How many ten-digit numbers can be formed using all the digits of 1324642119 such that the even digits appear only in odd places?

(a) $(5!)^2$ (b) $\frac{(5!)^2}{3!}$
(c) 5! 3! 2! (d) $\frac{(5!)^2}{(2!)^2\,3!}$

Directions (Q. 13 – 14) :

A man has 12 friends whom he wants to invite for lunch. In how many ways can he invite

13. At least one of them?

(a) 4096 (b) 4095
(c) 2047 (d) 2048

14. At least 10 of them?

(a) 66 (b) 140
(c) 153 (d) None of these

15. In how many ways can 6 prizes be distributed among 3 boys, if each boy is eligible to receive one or more prizes?

(a) 3^6 (b) 6^3
(c) ${}^6C_3$ (d) ${}^6P_3$

Directions (Q. 16 – 17) :

Find the number of ways of dividing 16 different books equally

16. Among 4 boys

(a) $\frac{16!}{(4!)^4}$ (b) $\frac{16!}{(4!)^3}$
(c) $\frac{16!}{(4!)^5}$ (d) $(4!)^4$

17. Into 4 parcels (Parcels are not distinct)

(a) $\frac{16!}{(4!)^4}$ (b) $\frac{16!}{(4!)^3}$
(c) $\frac{16!}{(4!)^5}$ (d) $(4!)^4$

18. How many linear arrangements can be made using all the letters of the word ILLUSTRATE which begin with S and end with a vowel?

(a) $\frac{8!}{(2!)^2}$ (b) 8!
(c) $\frac{8!}{2!}$ (d) $\frac{8!}{(2!)^3}$

19. From a group of 10 professors and 6 assistant professors, a management institute desires to send to delegation of 8 persons consisting of 5 professors and 3 assistant professors to the IIMs annual meet. If Prof. Balamurali, a science professor refuses to be in the delegation if assistant Prof. Sheshdari, an arts professor is included in the delegation, then in how many ways can the delegation be formed?

(a) ${}^9C_4\,{}^4C_3$
(b) ${}^9C_5\,{}^4C_2$
(c) ${}^{10}C_5\,{}^6C_3 - {}^9C_4\,{}^5C_2$
(d) ${}^9C_4\,{}^4C_3 + {}^9C_5\,{}^4C_2$

20. Two red pencils, three black pencils and two white pencils are to be arranged in a row such that

1. No two adjacent pencils are of the same colour and
2. Pencils at the two ends of the row are of same colour.

(a) 12 (b) 8 (c) 9 (d) 10

NUMERICAL TYPE QUESTIONS

Directions (Q. 1 – 2) :

The odds against an event are 4 to 5 and the odds in favour of another independent event are 3 to 7. The probability that

1. Exactly one of them will occur is _______

2. Neither of them will occur is _______

Directions (Q. 3 – 5) :

Three persons Shiva, Jagan and Rohit aim at a target. Their respective probabilities of hitting the target are 2/3, 5/7 and 3/8. The probability that

3. None of them hits the target is _______

4. At least two of them hit the target is _______

5. Exactly one of them hits the target si _______

6. If three consecutive letters are selected at random from the English alphabet, then the probability that at least one letter is a vowel is _______

7. Eight unbiased coins are tossed together. The probability that the number of heads is equal to the number of tails is _______

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8. In a biased coin, Head occurs three times as frequently as tail occurs. If the coin is tossed 3 times, then the probability of getting two heads is _______

Directions (Q. 9 – 10) :

Two cards are drawn at random from a well shuffled pack of cards. Given that both are black, the probability that the cards have

9. Same honour on them is_______

10. Different honours and belong to different suits is _______

Directions (Q. 11 – 13) :

Three mountaineers Akil, Nikil, and Sunil are climbing up a mountain with their respective probability of reaching the summit being $\frac{2}{3}, \frac{5}{8}$ and $\frac{4}{7}$, *respectively. The probability that*

11. None of them reach the summit is _______

12. Exactly two of them reaches the summit is _______

13. Atleast two of them reach the summit is _______

Directions (Q. 14 – 15) :

There are 12 points in a plane of which 4 are on a straight line and no three of the other points lie on a straight line.

14. The _______number straight lines can be formed by joining these points

15. The _______number of triangles can be formed by joining these points

16. The _______ number of non-negative integral solutions does the equation $x_1 + x_2 + x_3 + x_4 = 10$ have?

17. A certain number of students of a school participated in the chess tournament of their annual sports meet. Each player played 1 game against each of the other player. It was found that in 66 games both the players were girls, and in 240 games one was a girl and the other was a boy. The number of games in which both the players were boys is _______

18. Nine points are marked on a straight line and 10 points are marked on another line which is parallel to the first line. The_______ number of triangles can be formed with these points as vertices?

19. The _______ number of positive integers less than 100,000 and divisible by 125 can be formed using the digits 0, 1, 2, 5 and 8, if repetition is allowed?

20. There is an unlimited supply of identical red, blue, and green coloured balls. In _______ number of ways can 12 balls be selected from the supply.

21. Number of positive integral solutions of the equation $a + b + c = 15$ is _______

22. If six unbiased coins are tossed together, then probability that the number of heads exceeds the number of tails is_______

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. Four cards are randomly selected from a pack of 52 cards. If the first two cards are kings, what is the probability that the third card is a king?

(*a*) 4/52 (*b*) 2/50

(*c*) 1/52 × (1/52) (*d*) 1/52 × (1/52) × (1/50)

2. Five teams have to compete in a league, with every team playing every other team exactly once, before going to the next round. How many matches will have to be held complete the league round of matches?

(*a*) 20 (*b*) 10 (*c*) 8 (*d*) 5

3. Right triangle PQR is to be constructed in the *xy* – plane so that the right angle is at P and line PR is parallel to the-axis. The *x* and *y* coordinates of P, Q, and R are to be integers that satisfy the inequalities: $-4 \le x \le 5$ and $6 \le y \le 16$. How many different triangles could be constructed with these properties?

(*a*) 110 (*b*) 1,100

(*c*) 9,900 (*d*) 10,000

4. A coin is tossed thrice. Let X be the event that head occurs in each of the first two tosses. Let Y be the event that a tail occurs on the third toss. Let Z be the event that two tails occurs in three tosses. Based on the above information, which one of the following statements is TRUE?

(*a*) X and Y are not independent

(*b*) Y and Z are dependent

(*c*) Y and Z are independent

(*d*) X and Z independent

5. Ram and Ramesh appeared in an interview for two vacancies in the same department. The probability of Ram's selection is 1/6 and that of Ramesh is 1/8. What is the probability that only one of them will be selected?

(*a*) $\frac{47}{48}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{13}{48}$ (*d*) $\frac{35}{48}$

6. Given set A = {2, 3, 4, 5} and Set B = {11, 12, 13, 14, 15}, two numbers are randomly selected, one from each set. What is probability that the sum of the two numbers equals 16 ?

(*a*) 0.20 (*b*) 0.25

(*c*) 0.30 (*d*) 0.33

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7. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the student has 75% chance of passing in at least one, a 50% chance of passing in at least two and a 40% chance of passing in exactly two. Following relations are drawn in $m, p.c$:

I. $p + m + c = \frac{27}{20}$

II. $p + m + c = \frac{13}{20}$

III. $(p) \times (m) \times (c) = \frac{1}{10}$

(*a*) Only relation I is true.
(*b*) Only relation II is true.
(*c*) Relations II and III are true.
(*d*) Relations I and III are true.

2014

8. A five digit number is formed using the digits 1,3,5,7 and 9 without repeating any of them. What is the sum of all such possible five digit numbers?

(*a*) 6666660
(*b*) 6666600
(*c*) 6666666
(*d*) 6666606

9. You are given three coins: one has heads on both faces, the second has tails on both faces, and the third has a head on one face and a tail on the other. You choose a coin at random and toss it, and it comes up heads. The probability that the other face is tails is

(*a*) 1/4
(*b*) 1/3
(*c*) 1/2
(*d*) 2/3

2013

10. Out of all the 2-digit integers between 1 and 100, a 2-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7?

(*a*) 13/90
(*b*) 12/90
(*c*) 78/90
(*d*) 77/90

11. What is the chance that a leap year, selected at random, will contain 53 Saturdays?

(*a*) 2/7 (*b*) 3/7
(*c*) 1/7 (*d*) 5/7

12. In a factory, two machines M1 and M2 manufacture 60% and 40% of the autocomponents respectively. Out of the total production, 2% of M1 and 3% of M2 are found to be defective. If a randomly drawn autocomponent from the combined lot is found defective, what is the probability that it was manufactured by M2?

(*a*) 0.35 (*b*) 0.45
(*c*) 0.5 (*d*) 0.4

2012

13. A and B are friends. They decide to meet between 1 PM and 2 PM on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{16}$
(*c*) $\frac{7}{16}$ (*d*) $\frac{9}{16}$

2010

14. Given digits 2, 2, 3, 3, 3, 4, 4, 4, 4 how many distinct 4 digit numbers greater than 3000 can be formed?

(*a*) 50 (*b*) 51
(*c*) 52 (*d*) 54

NUMERICAL TYPE QUESTIONS

2015

1. How many four digit numbers can be formed with the 10 digits 0, 1, 2, ... 9 if no number can start with 0 and if repetitions are not allowed?

2014

2. In any given year, the probability of an earthquake greater than Magnitude 6 occurring in the Garhwal Himalayas is 0.04. The average time between successive occurrences of such earthquakes is _____ years.

3. 10% of the population in a town is HIV^+. A new diagnostic kit for HIV detection is available; this kit correctly identifies HIV^+ individuals 95% of the time, and HIV^- individuals 89% of the time. A particular patient is tested using this kit and is found to be positive. The probability that the individual is actually positive is ______

4. A batch of one hundred bulbs is inspected by testing four randomly chosen bulbs. The batch is rejected if even one of the bulbs is defective. A batch typically has five defective bulbs. The probability that the current batch is accepted is ________

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ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*c*) **2.** (*d*) **3.** (*d*) **4.** (*c*) **5.** (*d*) **6.** (*a*) **7.** (*b*) **8.** (*c*) **9.** (*b*) **10.** (*b*)

11. (*c*) **12.** (*d*) **13.** (*b*) **14.** (*d*) **15.** (*a*) **16.** (*a*) **17.** (*c*) **18.** (*b*) **19.** (*c*) **20.** (*d*)

Numerical Type Questions

1. $\frac{47}{90}$ **2.** $\frac{14}{45}$ **3.** $\frac{5}{84}$ **4.** $\frac{107}{168}$ **5.** $\frac{17}{56}$ **6.** $\frac{13}{24}$ **7.** $\frac{35}{128}$ **8.** $\frac{27}{64}$ **9.** $\frac{4}{325}$ **10.** $\frac{12}{325}$

11. $\frac{3}{56}$ **12.** $\frac{37}{84}$ **13.** $\frac{19}{28}$ **14.** 61 **15.** 216 **16.** 286 **17.** 190 **18.** 765 **19.** 99 **20.** 91

21. 91 **22.** $\frac{57}{64}$

EXERCISE – II

MCQ Type Questions

1. (*b*) **2.** (*b*) **3.** (*c*) **4.** (*d*) **5.** (*b*) **6.** (*a*) **7.** (*d*) **8.** (*b*) **9.** (*b*) **10.** (*d*)

11. (*a*) **12.** (*c*) **13.** (*c*) **14.** (*b*)

Numerical Type Questions

1. 4536 **2.** (25 to 25) **3.** (0.47 to 0.48) **4.** (0.8145)

EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1 and 2

Given : $P(A \cup B) = 0.6$

and $P(A) = 0.3$

1. If A and B are mutually exclusive events, then

$$P(A \cap B) = 0$$

$$\therefore \quad P(A \cup B) = P(A) + P(B)$$

$$0.6 = 0.3 + P(B)$$

$$\Rightarrow \quad 0.3 = P(B)$$

Hence $P(B) = 0.3$

2. If A and B are independent events, then

$$P(A \cap B) = P(A).P(B)$$

$$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - P(A).\, P(B)$$

$$P(A \cap B) = P(A) + P(B)\,[1 - P(A)]$$

$$\Rightarrow \quad 0.6 = 0.3 + P(B)\,(1 - 0.3)$$

$$0.3 = P(B)(0.7)$$

$$\Rightarrow \quad \frac{0.3}{0.7} = P(B)$$

$$\therefore \quad P(B) = \frac{3}{7}$$

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3. When a die is rolled, for each number the probability of getting it is $\frac{1}{6}$.

When number on the dice is 1, 3 or 5 Raju receives ₹ 2, ₹ 6 and ₹10 respectively.

When the number on the die is 2, 4 or 6, then Raju receives ₹ 6, ₹ 12 and ₹ 18 respectively.

$\therefore$ Expected value $= \frac{1}{6}$ [2 + 6 + 10 + 6 + 12 + 18]

$$= \frac{54}{6} = ₹\ 9$$

Hence to make an average profit of ₹ 7 per throw, Raju must pay ₹ 2 (9 – 7) for each time to throw the die.

4. 7 letters can be arranged in 7 addressed envelopes in 7! ways.

$\therefore$ n(S) = 7!

We can arrange 7 letters into corresponding 7 addressed envelopes in only one way.

$\therefore$ Required probability $= \frac{1}{7!}$.

5. If six letters can be placed in their corresponding addressed envelopes, then seventh letter is also in the correct envelope.

Hence required probability is 0.

6, 7 and 8.

Three cards can be drawn from 52 cards in $^{52}C_3$ ways

n(S) = $52c_3$

6. Since we have 4 suits i.e., Diamonds, Spades, Clubs and Hearts and each suit contains 13 cards.

Three cards can be selected from 13 cards in $^{13}C_3$ ways.

$\therefore$ Number of favorable outcomes = 4. $^{13}C_3$

Required probability $= 4.\frac{^{13}C_3}{^{52}C_3}$

7. From four suits we can select 3 suits in $^4C_3$ ways.

One card can be selected from each suit in $^{13}C_1$. $^{13}C_1$. $^{13}C_1$

$\therefore$ Number of favorable outcomes

$= {}^4C_3 . {}^{13}C_1 . {}^{13}C_1 . {}^{13}C1$

$\therefore$ Required probability $= \frac{4(13)^3}{^{52}C_3}$

8. Each suit contains 9 number cards. 2 number cards can be drawn from 9 cards in $^9C_2$ ways..

Third card can be selected from the remaining 27 number cards in $^{27}C_1$ ways.

Hence total number of favorable outcomes is $4(^9C_2)(^{27}C_1)$

Required probability $= \frac{4\left(^9C_2\right)\left(^{27}C_1\right)}{^{52}C_3}$

9. Total number of balls in the bag = 5 + 7 = 12.

5 balls can be drawn from 12 balls in $^{12}C_5$ ways

Number of ways of drawing 3 red balls and 2 white balls from 5 white balls and 7 red balls is $^5C_2 . {}^7C_3$

$\therefore$ Required probability $= \frac{^5C_2 . {}^7C_3}{^{12}C_5}$

$$= \frac{10 \times 35}{792} = \frac{175}{396}$$

10. Total number of coins in bag = 6 + 5 + 4 = 15.

5 coins can be drawn from 15 coins in $^{15}C_5$ ways.

When 4 coins are one rupee coins and fifth coin is a two rupee coin the amount will be minimum.

$\therefore$ Number of favourable cases = $^4C_4 . {}^{45}C_1 = 5$

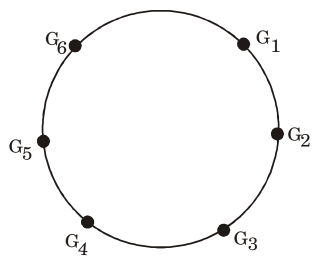
Probability that the amount will be minimum is

$$P(E) = \frac{5}{^{15}C_5} = \frac{5}{3003}$$

$\therefore$ $P(\bar{E}) = 1 - P(E) = 1 - \frac{5}{3003} = \frac{2998}{3003}$

$\therefore$ Odds in favour of the required event = 5: 2998

11.



Six girls can be arranged around a circle in 5! ways.

Now, the six boys can be arranged in the six places in between the girls in 6! ways.

Hence, required number of ways = 5 ! 6!

12. Of the 10 places available, there are 5 even places. Digits 2, 4, 6, 4 and 2 can be placed in these 5 even places in $\frac{5!}{2!2!}$ ways.

Remaining digits 1, 3, 1, 1 and 9 can be placed in the remaining 5 places in $\frac{5!}{3!}$ ways.

Hence, required number of ways

$$= \frac{5!}{2!2!}\frac{5!}{3!} = \frac{(5!)^2}{(2!)^2\, 3!}$$

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13. Number of ways of inviting at least one friend is
$$2^{12} - 1 = 4095$$

14. Number of ways of inviting at least 10 friends
$$= {}^{12}C_{10} + {}^{12}C_{11} + {}^{12}C_{12}$$
$$= 66 + 12 + 1 = 79$$

15. Each prize can be distributed in 3 ways, so, 6 prizes can be distributed in 3^6 ways.

16. Number of ways of dividing 16 books equally among 4 boys
$$= \frac{16!}{4!4!4!4!} = \frac{16!}{(4!)^4}$$

17. Number of ways of dividing 16 books equally into 4 parcels
$$= \frac{16!}{4!(4!)^4} = \frac{16!}{(4!)^5}$$

18. There are four vowels viz., U, A, I, E in the word "ILLUSTRATE".

First letter must be S and the last letter can be any one of the vowels U, A, I and E.

Remaining 8 places can be filled with the remaining 8 letters in $\frac{8!}{2!2!}$ ways.

Hence, required number of arrangements is
$$= 4 \times \frac{8!}{2!2!} = 8!$$

19. Prof. Balamurali and Asst. B of Sheshadri cannot be included together in the delegation.

Hence, number of ways of forming delegation
$$= {}^{10}C_5 \, {}^6C_3 - {}^9C_4 \, {}^5C_2$$

20. Possible ways of arranging the pencils are:

RBWBWBR, WBRBRBW, BWBRWRB, BRBWRWB, BWRBWRB, BWRRBRWB, BRWBRWB, BRWBWRB, BWRWBRB and BRWBBWB

i.e. a total of 10 possible arrangements.

NUMERICAL TYPE QUESTIONS

1 and 2 :

Let A and B be the two events.

Given: $P(\bar{A}) : P(A) = 4 : 5$

$$\Rightarrow \quad P(\bar{A}) = \frac{4}{9}; \quad P(A) = \frac{5}{9}$$

Also, $P(B) : P(\bar{B}) = 3 : 7$

$$\Rightarrow \quad P(B) = \frac{3}{10} \quad \text{and} \quad P(\bar{B}) = \frac{7}{10}$$

1. $A \cap \bar{B}$ and $\bar{A} \cap B$ denote events that exactly one of them will occur.

$$P(A \cap \bar{B}) = P(A)\, P(\bar{B}) = \frac{5}{9} \cdot \frac{7}{10} = \frac{7}{18}$$

$$= P(\bar{A} \cap B) = P(\bar{A}).P(B)$$

$$= \frac{4}{9} \cdot \frac{3}{10} = \frac{2}{15}$$

Required probability is

$$= \frac{7}{18} + \frac{2}{15} = \frac{35 + 12}{90} = \frac{47}{90}$$

2. P (neither of them occurs)

$$= P(\bar{A} \cap \bar{B}) = P(\bar{A}).P(\bar{B}) = \frac{7}{10}\left(\frac{4}{9}\right) = \frac{14}{45}$$

3, 4 and 5.

Let A, B and C be the events that Shiva, Jagan and Rohit hit the target respectively.

$$\therefore \quad P(A) = \frac{2}{3}; \qquad P(\bar{A}) = \frac{1}{3}$$

$$P(B) = \frac{5}{7}; \qquad P(\bar{B}) = \frac{2}{7}$$

$$P(C) = \frac{3}{8}; \qquad P(\bar{C}) = \frac{5}{8}$$

3. P (None of them hit the target)

$$= P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}).P(\bar{B}).P(\bar{C})$$

$$= \left(\frac{1}{3}\right) \cdot \left(\frac{2}{7}\right) \cdot \left(\frac{5}{8}\right) = \frac{5}{84}$$

4. Event $(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)$ denotes that exactly two persons hit the target, and $A \cap B \cap C$ is the event that all the three persons hit the target.

$$P(A \cap B \cap \bar{C}) = P(A)\, P(B)\, P(\bar{C})$$

$$= \frac{2}{3} \cdot \left(\frac{5}{7}\right) \cdot \left(\frac{5}{8}\right) = \frac{25}{84}$$

$$P(A \cap \bar{B} \cap C) = P(A)\, P(\bar{B})\, P(C)$$

$$= \frac{2}{3} \cdot \left(\frac{2}{7}\right) \cdot \left(\frac{3}{8}\right) = \frac{1}{14}$$

$$P(\bar{A} \cap B \cap C) = P(\bar{A})\, P(B)\, P(C)$$

$$= \frac{1}{3} \cdot \left(\frac{5}{7}\right) \cdot \left(\frac{3}{8}\right) = \frac{5}{56}$$

$$P(A \cap B \cap C) = P(A)\, P(B)\, P(C)$$

$$= \frac{2}{3} \cdot \left(\frac{5}{7}\right) \cdot \left(\frac{3}{8}\right) = \frac{5}{28}$$

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Hence required probability

$$= P\left(A \cap B \cap \bar{C}\right) + P\left(A \cap \bar{B} \cap C\right) + P\left(\bar{A} \cap B \cap C\right) + P\left(A \cap B \cap C\right)$$

$$= \frac{25}{84} + \frac{1}{14} + \frac{5}{56} + \frac{5}{28} = \frac{50+12+15+30}{168} = \frac{107}{168}$$

5. Exactly one of them hits the target

$\left(A \cap \bar{B} \cap \bar{C}\right) \cup \left(\bar{A} \cap B \cap \bar{C}\right) + \left(\bar{A} \cap \bar{B} \cap C\right)$

denotes that exactly one of them hits the target.

$$P\left(A \cap \bar{B} \cap \bar{C}\right) = P(A) \,.\, P(\bar{B})\,.\, P(\bar{C})$$

$$= \frac{2}{3} \times \frac{2}{7} \times \left(\frac{5}{8}\right) = \frac{5}{42}$$

$$P\left(\bar{A} \cap B \cap \bar{C}\right) = P\left(\bar{A}\right).P(B).P\left(\bar{C}\right) = \frac{1}{3}.\frac{2}{7}.\frac{3}{8} = \frac{1}{28}$$

$$P\left(\bar{A} \cap \bar{B} \cap C\right) = P(\bar{A}).P(\bar{B})P(C) = \frac{1}{3}.\frac{5}{7}.\frac{5}{8} = \frac{25}{168}$$

Hence required probability is

$$= P\left(A \cap \bar{B} \cap \bar{C}\right) + P\left(\bar{A} \cap B \cap \bar{C}\right) + P\left(\bar{A} \cap \bar{B} \cap C\right)$$

$$= \frac{5}{42} + \frac{25}{168} + \frac{1}{28} = \frac{20+25+26}{168} = \frac{51}{168} = \frac{17}{56}$$

6. Three consecutive letters can be selected from English alphabet in 24 ways.

$\therefore$ $n(S) = 24$

Since, no two consecutive letters are vowels, the number of ways that among three letters one letter is vowel is 13.

$\therefore$ Required probability $= \frac{13}{24}$

7. Eight coins are tossed, so $n(S) = 2^8$

Let E be the favorable event. It is required to get 4 heads and 4 tails which is possible in the favorable out come HHHHTTTT.

This can be arranged in $\frac{8!}{4!4!}$ ways.

$$n(E) = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4 \times 3 \times 2 \times 1 \times 4!} = 70$$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{70}{2^8} = \frac{35}{128}$

8. *Given* : $P(H) = 3P(T)$

We know

$$P(H) + P(T) = 1$$
$$3P(T) + P(T) = 1$$

$$P(T) = \frac{1}{4}$$

$$P(H) = \frac{3}{4}$$

Since the coin is tossed 3 times, two heads may occur in 1st and 2nd trial, 2nd and 3rd trial or 1st and 3rd trial.

$\therefore$ Required probability

$$= \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{1}{4} \times \frac{3}{4}$$

$$= \frac{9}{64} + \frac{9}{64} + \frac{9}{64} = \frac{27}{64}$$

9. Two black cards can be selected in $26c_2$ ways.

A pack contain 8 black honours.

Hence number ways of selecting two same honours is 4.

$\therefore$ Required probability $= \frac{4}{26c_2} = \frac{4}{325}$

10. Let one card is king, then second card can be any of the other 3 cards from the other suit and it can be selected in $4 \times 3 = 12$ ways. Since there are four honour cards, hence

$\therefore$ Required probability $= \frac{12}{325}$

11, 12 and 13

Probability that Akil reach the summit

$= P(A) = 2/3$

$\therefore$ $P\left(\bar{A}\right) = 1/3$

Probability that Nikil reach the summit is

$P(N) = 5/8$

$\therefore$ $P\left(\bar{N}\right) = 3/8$

Probability that Sunil reach the summit

$$P(S) = \frac{4}{7}$$

$\therefore$ $P\left(\bar{S}\right) = \frac{3}{7}$

11. Probability that none of them reaches the summit

$$= P\left(\bar{A} \cap \bar{N} \cap \bar{S}\right) = P\left(\bar{A}\right)P\left(\bar{N}\right)P\left(\bar{S}\right)$$

$$= \frac{1}{3} \times \frac{3}{8} \times \frac{3}{7} = \frac{3}{56}$$

12. Probability that exactly two of them reaches the summit

$$= P(A \cap S \cap \bar{N}) + P(A \cap N \cap \bar{S}) + P(\bar{A} \cap N \cap S)$$

$$= P(A)P(S)P(\bar{N}) + P(A)P(N)P(\bar{S}) + P(N)P(S)P(\bar{A})$$

$$= \frac{2}{3}.\frac{4}{7}.\frac{3}{8} + \frac{2}{3}.\frac{5}{8}.\frac{3}{7} + \frac{1}{3}.\frac{5}{8}.\frac{4}{7} = \frac{37}{84}$$

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13. Probability that atleast two of them reaches the summit

= Exactly two of them reach summit
+ three of them reach the summit

$$= \frac{37}{84} + \frac{5}{21} = \frac{57}{84} = \frac{19}{28}$$

14. 2 points determine a line.

Hence12 points can form $^{12}C_2$ straight lines.

But 4 points are collinear, i.e they give only one line.

Number of straight lines that can be formed

$$= {}^{12}C_2 - {}^{4}C_2 + 1 = 66 - 6 + 1 = 61$$

15. Three points determine a triangle.

$\therefore$ 12 non-collinear points can give $^{12}C_3$ triangles.

But 4 points are collinear, so, these 4 points do not give any triangles.

Hence, number of triangles = $^{12}C_3 - {}^{4}C_3 = 216$

16. Number of non-negative integral solutions of the equation

$$x_1 + x_2 + ... + x_k = n \text{ is } {}^{(n+k-1)}C_{k-1}.$$

Hence number of non-negative integral solutions of

$$x_1 + x_2 + x_3 + x_4 = 10 \text{ is } {}^{13}C_3 = 286.$$

17. Let m, n be the number of girls and the number of boys respectively. Then

$$^{m}C_2 = 66$$

$$\Rightarrow \quad \frac{m(m-1)}{2!} = 66 \quad \Rightarrow \quad m = 12$$

Also, $\quad {}^{m}C_1 \times {}^{n}C_1 = 240 \Rightarrow n = 20$

Hence, number of games in which both the players were boys = $^{20}C_2 = \dfrac{20(19)}{2} = 190$

18. Points lying on a straight line do not form triangles. Hence, number of triangles that can be formed by using the given points

$$= {}^{19}C_3 - ({}^{9}C_3 + {}^{10}C_3) = 765$$

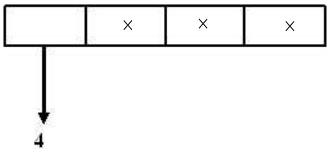
19. A number is divisible by 125 if and only if number formed by its last three digits is divisible by 125.

So, possible combinations for last three digits are: 000, 125, 250 and 500.

Now, number of three-digit number divisible by 125 is 3.

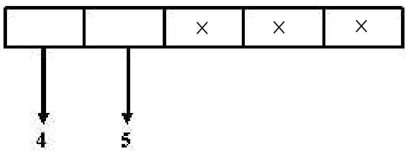
Number of four-digit numbers divisible by 125

= 4 (4) = 16.



Number of five-digit numbers divisible by 125

= 4 (4) (5) = 80.



Hence, required number of numbers

= 3 + 16 + 80 = 99.

20. Let the number of red, blue and green balls selected from the supply be x_1, x_2 and x_3 respectively.

Then, $\quad x_1 + x_2 + x_3 = 12.$

Number of non-negative integral solutions of the above equation

$$= {}^{(n+k-1)}C_{k-1} = {}^{14}C_2 = 91.$$

21. Number of positive integral solutions of equation

$$x_1 + x_2 + ... + x_k = n \text{ is } {}^{n-1}C_{k-1}.$$

Number of positive integral solutions of the equation

Hence $\quad a + b + c = 15$

$$^{15-1}C_{3-1} = {}^{14}C_2 = 91$$

22. When 'n' coins are tossed together, then probability of getting exactly 'r' tails = $\dfrac{^{n}C_r}{2^n}$

Hence probability of getting atleast two tails

= 1 – (probability of getting no tail
+ probability of getting exactly one tail)

$$= 1 - \left\{\frac{^{6}C_0}{64} + \frac{^{6}C_1}{64}\right\} = 1 - \frac{7}{64} = \frac{57}{64}$$

EXERCISE – II

MCQ TYPE QUESTIONS

1. There are 4 kings in a pack of 52 cards.

If 2 cards are selected and both are kings, remaining cards will be 50 out of which 2 will be kings.

2. For a match to be played, we need 2 teams

No. of matches = No. of ways of selections 2 teams out of 5

$$= {}^{5}C_2 = 10$$

4. $\quad x$ = {HHT, HHH}

y depends on x

z = {TTH, TTT}

$\therefore$ 'd' is the correct choice.

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5. P (Ram) = 1/6; p(Ramesh) = 1/8

p(only at) = p (Ram) × p (not ramesh) + p(Ramesh) × p(n_0 × R_{am})

$$= 1/6 + \frac{7}{8} + \frac{1}{8} \times \frac{5}{6}$$

$$\Rightarrow \frac{12}{40} = 1/4$$

6. Here the given sets are

A = {2, 3, 4, 5}

and B = {11, 12, 13, 14, 15}

desired outcome = {(2, 14), (3, 13), (4, 12), (5, 11)}

$\therefore$ n(E) = 4

and total outcome = 4 × 5 = 20

$\therefore$ $n(s)$ = 20

$\therefore$ Probability of an event

$$= \frac{n(E)}{n(s)} = \frac{4}{20} = \frac{1}{5} = 0.2$$

8. The digit in unit place is selected in 4! Ways

The digit in tens place is selected in 4! Ways

The digit in hundreds place is selected in 4! Ways

The digit in thousands place is selected in 4! Ways

The digit in ten thousands place is selected in 4! Ways

Sum of all values for 1

$4! \times 1 \times (10^0 + 10^1 + 10^2 + 10^3 + 10^4)$

$= 4! \times 11111 \times 1$

Similarly for '3' 4! × (11111) × 3

Similarly for '5' 4! × (11111) × 5

Similarly for '7' 4! × (11111) × 7

Similarly for '9' 4! × (11111) × 9

$\therefore$ sum of all such numbers

4! ×(11111)× (1+ 3 + 5 + 7 + 9)

= 24 × (11111) × 25

= 6666600

9. The probability that the other face is tail is $\frac{1}{3}$.

10. **Given:** All the 2-digit numbers

To find: Probability of selecting a number at random not divisible by 7

Analysis: Total number of 2-digit numbers

10, 11, ..., 99 → 90 numbers

Unfavourable numbers *i.e.* numbers divisible by 7 : 14, 21, ..., 98

Number of unfavourable numbers = 13

This can be hand counted or we can use the following properties:

Last term = First time + (no. – 1) Diff

98 = 14 + (no. – 1) 7

$\Rightarrow$ no. = 13

$$\Rightarrow \text{Probability} = \frac{\text{Total – unfavourable}}{\text{Total}}$$

$$= \frac{90-13}{90} = 77/90$$

Hence, the solution is (d)

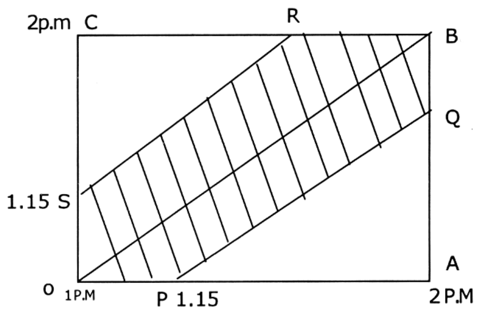
11. at a leap year

52 weeks, and 2 extra day they are (Mon, Tues) (Tues, Wed) (Wed Thu.) (Thus. Fri) (Fri. Sat) (Sat. Sun) (Sun Mond.)

$n(s)$ = 7

n(E) = 2 P(E) = $\frac{2}{7}$

13.



OB is the line when both A and B arrive at same time.

Total sample sapce = 60 × 60 = 3600

Favourable cases = Area of OABC – 2(Area of SRC)

$$= 3600 - 2 \times \left(\frac{1}{2} \times 45 \times 45\right)$$

= 1575

$$\therefore \text{Required probability} = \frac{1575}{3600} = \frac{7}{16}$$

14. 2, 2, 3, 3, 4, 4, 4, 4 $x > 3000$

322 — 3, 4	344 — 2, 3, 4
323 — 2, 3, 4	422 — 3, 4
324 — 2, 3, 4	423 — 2, 3, 4
332 — 2, 3, 4	424 — 2, 3, 4
333 — 2, 4	432 — 2, 3, 4
334 — 2, 3, 4	433 — 2, 3, 4
342 — 2, 3, 4	434 — 2, 3, 4
343 — 2, 3, 4	442 — 2, 3, 4
	443 — 2, 3, 4
	444 — 2, 3, 4

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NUMERICAL TYPE QUESTIONS

1. In thousands place, 9 digits except 0 can be placed

In hundreds place, 9 digits can be placed (including 0, excluding the one used in thousands place)

In tens place, 8 digits can be placed (excluding the ones used in thousands and hundreds place)

In ones place, 7 digits can be placed (excluding the one used in thousands, hundreds and tens place)

Total number of combinations

$$= 9 \times 9 \times 8 \times 7 = 4536$$

2. Given, in one year, probability of an earthquake > 6 magnitude = 0.04

So, average time between successive earthquakes = 1/0.04 = 25 years

3. Given, 10% of the population is HIV^+, so probability = 0.1 for HIV^+ and 0.9 for HIV^-

The diagnostic kit identifies $\rightarrow$ 0.95 HIV^+ and 0.89 HIV^- correctly

So, the required probability that the person is actually HIV^+ = 0.95/2 = 0.475

4. Probability for one bulb to be non-defective is $\frac{95}{100}$

$\therefore$ Probabilities that none of the bulbs is defectives $\left(\frac{95}{100}\right)^4 = 0.8145$

■■

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6

CHAPTER

Miscellaneous

EXERCISE – I

MCQ TYPE QUESTIONS

1. If one root of a quadratic equation is $3 + \sqrt{2}$, then quadratic equation is

(*a*) $x^2 - 6x - 7 = 0$ (*b*) $x^2 + 6x + 7 = 0$

(*c*) $x^2 + 6x - 7 = 0$ (*d*) $x^2 - 6x + 7 = 0$

2. Three consecutive positive integers such that square of their sum exceeds the sum of their squares by 214, are

(*a*) 5, 6, 7 (*b*) 10, 11, 12

(*c*) 9, 10, 11 (*d*) 7, 8, 9

3. A and B attempt to solve a quadratic equation of the form $ax^2 + bx + c = 0$. A starts with a wrong value of b and gets the roots as –3 and –5. B starts with a wrong value of C and gets the roots as 6 and 2. The correct roots are

(*a*) 3, 4 (*b*) 3, 5

(*c*) 4, 6 (*d*) 2, 3

4. What is the value of

$$\cfrac{1}{5+\cfrac{1}{5+\cfrac{1}{5+.....}}}\ ?$$

(*a*) $\frac{5+\sqrt{29}}{2}$ (*b*) $\frac{5-\sqrt{29}}{2}$

(*c*) $\frac{5\pm\sqrt{29}}{2}$ (*d*) 7

5. A man buys two horses for ₹ 86. By selling one for three – quarters of its cost price, and the other for four-thirds of its cost price, he makes a profit of ₹ 3 on the whole transaction. The cost price (in rupees) of each horse is

(*a*) ₹ 39, ₹ 47 (*b*) ₹ 34, ₹ 52

(*c*) ₹ 48, ₹ 38 (*d*) ₹ 44, ₹ 42

6. What is the number of triplets (a, b, c) where a, b, c are positive integers, which satisfy the simultaneous equations ?

(*i*) ab + bc = 44

(*ii*) ac + bc = 23 is/are

(*a*) 1

(*b*) 2

(*c*) 4

(*d*) Indefinitely many

7. A man sitting beside me at a hotel ate idlis one after the other by ordering plate by plate. He said to me after drinking some water the last one I ate was my hundredth idli in last five days. Each day I ate 6 more than the previous day. How many he ate yesterday?

(*a*) 26 (*b*) 24 (*c*) 22 (*d*) 20

8. A man, who went out between five and six o'clock and returned between six and seven o'clock, found that the hands of the watch had exchanged places. When did he go out?

(*a*) $32\frac{4}{13}$ minutes past 5 o'clock.

(*b*) $27\frac{2}{13}$ minutes past 5 o'clock.

(*c*) $37\frac{3}{13}$ minutes past 5 o'clock.

(*d*) $31\frac{5}{13}$ minutes past 5 o'clock.

9. What is the time between 7 a.m. and 8 a.m. when hands of a clock will be in the same straight line but not coincide?

(*a*) 7 hours $5\frac{5}{11}$ min

(*b*) 7 hours $3\frac{5}{11}$ min

(*c*) 7 hours $7\frac{3}{11}$ min

(*d*) 7 hours $7\frac{7}{11}$ min

NUMERICAL TYPE QUESTIONS

1. When $x^5 + x^4 + 5x^2 - 3$ is divided by (x + 2), the remainder will be_______

2. Pressure of a gas is inversely proportional to the volume. If pressure of oxygen is 84 atm when the volume is 1000 cm^3, then volume when the pressure is 28 atm will be_______ cm^3

3. The value of 'a' will make $x^2 - ax + 25$ a perfect square is _______

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4. The value of $\alpha^3 + \beta^3$ if α and β are roots of the equation : $2x^2 - 7x - 4 = 0$ is _______

5. Sum of Anita's and Sunita's age is 43 years. 11 years hence, Anita's age will be $\frac{7}{6}$ times Sunita's age then. Sunita's present age is___ years

6. In a school trip, the students were accommodated in two rooms A and B. If 5 students are shifted from room A to room B then there will be equal number of students in both the rooms. But if 5 students from room B is shifted to room A, then the resulting number of students in room A is double the number of students left in room B. Total number of students are _______

7. A band of workers unloaded a certain number of boxes from their ship. The number of workers were between 50 and 60 (both inclusive). Each worker handled exactly 11 boxes, and each box was handled by exactly 7 workers. Exactly_______number of boxes were there?

8. If ax = b,by = c ,cz = a, then value of xyz is _______

9. The value of

$$\frac{1}{\left(1+\frac{xb}{xa}+\frac{xc}{xa}\right)}+\frac{1}{\left(1+\frac{xa}{xb}+\frac{xc}{xb}\right)}+\frac{1}{\left(1+\frac{xb}{xc}+\frac{xa}{xc}\right)}$$

is _______

10. A palm tree was 90 cm high, when it was planted. It grows by an equal number of cm each year, and at the end of the seventh year it was one ninth taller than at the end of the sixth year. The height of tree in cm at the end of the twelfth year is _______

11. A bottle and its cork together cost ₹ 1.10, and the bottle costs ₹ 1.00 more than its cork. The price of the bottle is ______

12. The value of is (100 – 1) (100 – 2) (100 – 3) .... (100 + 1) (100 + 2) (100 + 3) is _______

13. The value of $\dfrac{2+\dfrac{1}{3+\dfrac{4}{5}}}{2+\dfrac{1}{3+\dfrac{1}{1+\dfrac{1}{4}}}}$ is _______

14. Three integers, a, b and c, are such that $\frac{b}{c} = 3$. The fraction $\frac{b}{a}$ is in its lowest form. The_______ is the possible value of a.

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. If $\log_x\left(\frac{5}{7}\right) = -\frac{1}{3}$, then the value of x is

(a) $\frac{343}{125}$ (b) $\frac{125}{343}$

(c) $-\frac{25}{49}$ (d) $-\frac{49}{25}$

2. A function $f(x)$ is linear and has a value of 29 at $x = -2$ and 39 at $x = 3$. Find its value at $x = 5$.

(a) 59 (b) 45

(c) 43 (d) 35

3. If $x > y > I$, which of the following must be true?

(i) $\ln x > \ln y$ (ii) $e^x > e^y$

(iii) $y^x > x^y$ (iv) $\cos x > \cos y$

(a) i and ii (b) i and iii

(c) iii and iv (d) ii and iv

4. log tan1° + log tan2° +-------+ log tan89° is ____.

(a) 1 (b) $1/\sqrt{2}$

(c) 0 (d) –1

5. Based on the given statements, select the most appropriate option to solve the given question.

What will be the total weight of 10 poles each of same weight?

Statements:

(I) One fourth of the weight of a pole is 5 kg

(II) The total weight of these poles is 160 kg more than the total weight of two poles.

(a) Statement II alone is not sufficient

(b) Statement II alone is not sufficient

(c) Either I or II alone is sufficient

(d) Both statements I and II together are not sufficient.

6. Consider a function $f(x) = 1 - |x|$ on $-1 \le x \le 1$. The value of x at which the function attains a maximum, and the maximum value of function are:

(a) 0, –1

(b) –1, 0

(c) 0, 1

(d) –1, 2

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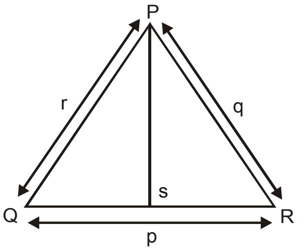
7. In a triangle PQR, PS is the angle bisector of $\angle$QPR and $\angle$QPS = 60°. What is the length of PS?



(*a*) $\frac{(q+r)}{qr}$ (*b*) $\frac{qr}{(q+r)}$

(*c*) $\sqrt{(q^2+r^2)}$ (*d*) $\frac{(q+r)^2}{qr}$

8. Based on the given statements, select the most appropriate option to solve the given question.

If two floors in a certain building are 9 feet apart, how many steps are there in a set of stairs that extends from the first floor to the second floor of the building?

Statements:

I. Each step is ¾ foot high.

II. Each step is 1 foot wide.

(*a*) Statement I alone is sufficient, but statement II alone is not sufficient.

(*b*) Statement II alone is sufficient, but statementI alone is not sufficient.

(*c*) Both statements together are sufficient, but neither statement alone is sufficient.

(*d*) Statement I and II together are not sufficient.

2014

9. If $y = 5x^2 + 3$, then the tangent at $x = 0, y = 3$

(*a*) passes through $x = 0, y = 0$

(*b*) has a slope of +1

(*c*) is parallel to the x-axis

(*d*) has a slope of –1

10. Let $f(x, y) = x^n y^m$ = P. If x is doubled and y is halved, the new value of f is

(*a*) 2^{n-m} P (*b*) 2^{m-n} P

(*c*) $2(n - m)$ P (*d*) $2(m - n)$ P

11. A regular die has six sides with numbers 1 to 6 marked on its sides. If a very large number of throws show the following frequencies of occurrence: $1 \to 0.167$; $2 \to 0.167$; $3 \to 0.152$; $4 \to 0.166$; $5 \to 0.168$; $6 \to 0.180$. We call this die

(*a*) irregular (*b*) biased

(*c*) Gaussian (*d*) insufficient

12. Consider the equation : $(7526)_8 - (Y)_8 = (4364)_8$, where $(X)_N$ stands for X to the base N. Find Y.

(*a*) 1634 (*b*) 1737

(*c*) 3142 (*d*) 3162

13. At what time between 6 *a.m.* and 7 *a.m.* will the minute hand and hour hand of a clock make an angle closest to 60°?

(*a*) 6 : 22 *a.m.* (*b*) 6 : 27 *a.m.*

(*c*) 6 : 38 *a.m.* (*d*) 6 : 45 *a.m.*

14. The roots of $ax^2 + bx + c = 0$ are real and positive a, b and c are real. Then $ax^2 + b|x| + c = 0$ has

(*a*) No roots (*b*) 2 real roots

(*c*) 3 real roots (*d*) 4 real roots

2012

15. The cost function for a product in a firm is given by $5q^2$, where q is the amount of production. The firm can sell the product at a market price of ₹ 50 per unit. The number of units to be produced by the firm such that the profit is maximized is

(*a*) 5 (*b*) 10

(*c*) 15 (*d*) 25

16. Which of the following assertions are **CORRECT ?**

P : Adding 7 to each entry in a list adds 7 to the mean of the list

Q : Adding 7 to each entry in a list adds 7 to the standard deviation of the list

R : Doubling each entry in a list doubles the mean of the list

S : Doubling each entry in a list leaves the standard deviation of the list unchanged

(*a*) P, Q (*b*) Q, R

(*c*) P, R (*d*) R, S

17. A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation $y = 2x - 0.1x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is

(*a*) 8 meters (*b*) 10 meters

(*c*) 12 meters (*d*) 14 meters

18. There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using this balance, the minimum number of weighings required to identify the heavier bag is

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 8

2011

19. If log (P) = (1/2) Log (Q) = (1/3) Log (R), then which of the following options is TRUE?

(*a*) $P^2 = Q^3 R^2$ (*b*) $Q^2 = PR$

(*c*) $Q^2 = R^3 P$ (*d*) $R = P^2 Q^2$

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20. The variable cost (V) of manufacturing a product varies according to the equation V = 4q, where q is the quanity produced. The fixed cost (F) of production of same product reduces with q according to the equation F = $100/q$. How many units should be produced to minimize the total cost (V + F)?

(*a*) 5 (*b*) 4
(*c*) 7 (*d*) 6

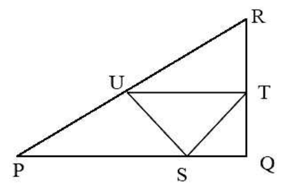
21. Three friends R, S and T shared toffee from a bowl. R took 1/3rd of the toffees, but returned four to the bowl. S took 1/4th of what was left but returned three toffees to the bowl. T took half of the remainder but returned two back into the bowl. If the bowl had 17 toffees left, how many toffees were originally there in the bowl?

(*a*) 38 (*b*) 31
(*c*) 48 (*d*) 41

NUMERICAL TYPE QUESTIONS

2015

1. In the given figure angle Q is a right angle, PS : QS = 3:1, RT : QT = 5:2 and PU : UR = 1:1. If area of triangle QTS is 20 cm^2, then the area of triangle PQR in cm^2 is ______.



2. If p, q, r, s are distinct integers such that:

$f(p, q, r, s) = \max(p, q, r, s)$

$g(p, q, r, s) = \min(p, q, r, s)$

$h(p, q, r, s)$ = remainder of $(p \times q)/(r \times s)$ if $(p \times q) > (r \times s)$ or remainder of $(r \times s)/(p \times q)$ if $(r \times s) > (p \times q)$

Also a function $fgh(p, q, r, s) = f(p, q, r, s) \times g(p, q, r, s) \times h(p, q, r, s)$

Also the same operations are valid with two variable function of the form $f(p, q)$.

What is the value of $fg(h(2, 5, 7, 3), 4, 6, 8$

2014

3. When a point inside of a tetrahedron (a solid with four triangular surfaces) is connected by straight lines to its corners, how many (new) internal planes are created with these lines? _________.

ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*d*) **2.** (*a*) **3.** (*b*) **4.** (*a*) **5.** (*d*) **6.** (*b*) **7.** (*a*) **8.** (*a*) **9.** (*a*)

Numerical Type Questions

1. 1 **2.** 3000 **3.** 10 **4.** $\frac{511}{8}$ **5.** 19 **6.** 60 **7.** 88 **8.** 1 **9.** 1

10. 450 **11.** 1.05 **12.** 0 **13.** 1 **14.** 104

EXERCISE – II

MCQ Type Questions

1. (*a*) **2.** (*c*) **3.** (*a*) **4.** (*c*) **5.** (*c*) **6.** (*c*) **7.** (*b*) **8.** (*a*) **9.** (*c*) **10.** (*a*)

11. (*b*) **12.** (*c*) **13.** (*a*) **14.** (*d*) **15.** (*a*) **16.** (*c*) **17.** (*b*) **18.** (*a*) **19.** (*b*) **20.** (*a*)

21. (*c*)

Numerical Type Questions

1. 280 **2.** (8) **3.** (6)

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EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. If one root = $3 + \sqrt{2}$, then other root = $3 - \sqrt{2}$

$\therefore$ Sum of roots = $3 + \sqrt{2} + 3 - \sqrt{2} = 6$

$\therefore$ Product of roots

$= (3 + \sqrt{2})(3 - \sqrt{2}) = 9 - 2 = 7$

Quadratic equation is

x^2 – (Sum of roots) x + Product of roots = 0

i.e., $x^2 - 6x + 7 = 0$

2. Let consecutive positive integers be $x - 1$, x and $x + 1$

$\therefore [x - 1 + x + x + 1]^2$

$= (x - 1)^2 + x^2 + (x + 1)^2 + 214$

$(3x)^2 = x^2 + 1 - 2x + x^2 + x^2 + 2x + 1 + 214$

$9x^2 = 3x^2 + 216$

$6x^2 = 216$

$x^2 = 36$; $x = \pm 6$

But x is a positive integer, hence x = 6

So, numbers are 5, 6, and 7.

3. Equation formed by A with the roots –3 and –5 is

$x^2 - (-8x) + 15 = 0$

i.e., $x^2 + 8x + 15 = 0$

But value of B is wrong.

Hence coefficient of x is wrong.

Equation formed by B with roots 6 and 2 is

$x^2 - 8x + 12 = 0$

But the value of C is wrong.

hence constant term is wrong.

So, correct equation is

$x^2 - 8x + 15 = 0$.

Hence, real roots are 3 and 5.

4. Let

$$x = 5 + \frac{1}{\left[5 + \cfrac{1}{5 + \cfrac{1}{5 + ...}}\right]}$$

i.e., $x = 5 + \dfrac{1}{x} = \dfrac{5x + 1}{x}$

$\Rightarrow x^2 = 5x + 1$

$\Rightarrow x^2 - 5x - 1 = 0$

$$\Rightarrow x = \frac{5 \pm \sqrt{(-5)^2 - 4(-1)(1)}}{2}$$

$$= \frac{5 \pm \sqrt{25 + 4}}{2} = \frac{5 \pm \sqrt{29}}{2}$$

Since, all the terms are positive. The answer has to be $\dfrac{5 + \sqrt{29}}{2}$.

5. Let cost price of horses by ₹ x and ₹ y.

$\therefore \quad x + y = 86 \quad$ ...(1)

and $\quad \dfrac{3x}{4} + \dfrac{4y}{3} = 89$

$\Rightarrow \quad 9x + 16y = 1068 \quad$ ...(2)

Multiplying (1) by 9, $\quad 9x + 9y = 774 \quad$ ...(3)

Subtracting (3) from (2),

$7y = 294$

$\Rightarrow \quad y = 42$

Substituting value of 'y' in equation (1),

$x + 42 = 86$

$\Rightarrow \quad x = 44$.

6. Equation (ii) is $c(a + b) = 23$, a prime number.

So, 2 factors must be 1, 23.

Since a, b are positive, therefore $a + b > 1$

$\Rightarrow$ c = 1; a + b = 23

Put c = 1; b = 23 – a in (i)

$\Rightarrow a^2 - 22a + 21 = 0$

$\Rightarrow$ a = 1 or 21

$\Rightarrow$ b = 22 or 2

$\therefore$ Solution sets are (1, 22, 1); (21, 2, 1).

7. First day the number of idlis he ate be x

Second day the count is (x+6)

Third day ---------------------- (x+12)

Fourth day --------------------(x+18)

Fifth day -----------------------(x+24)

Total is $5x + 6(1+2+3+4) = 100$

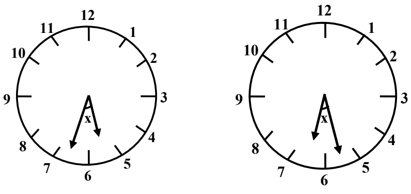
$5x + 60 = 100$

$x = 8$

Day	Idlis
1	8
2	14
3	20
4	26
5	32

So, on fourth day, number of idlis the man ate were 26.

8.



When hour hand moves 1 minute space, then minute hand moves 12 minute spaces.

Hence when hour hand moves x minutes, then minute hand moves 12x minutes.

$\therefore \quad 12x = 60 - x$

$\Rightarrow \quad x = \dfrac{60}{13}$

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Interchangeable positions occur when original interval between the hands is $4\frac{8}{13}$ minute spaces or a multiple of this.

Hour hand moves through $\frac{60}{13}$ minute spaces.

At 5 O'clock, minute hand is 25 minutes behind time.

It has to gain $25\frac{60}{13} = \frac{385}{13}$ minutes.

$\frac{385}{13}$ minutes will be gained in

$\frac{385}{13} \times \frac{60}{55} = \frac{420}{13} = 32\frac{4}{13}$ minutes.

Hence man went out at $32\frac{4}{13}$ minutes past 5 O'clock.

9. When they are in the same straight line, in opposite direction. Minute hand and hour hand are 30 minute spaces apart.

At 7 p.m. they are 25 minute spaces apart.

So, minute hand has to gain only (30 – 25) = 5 minute spaces.

i.e., $\frac{60}{55} \times 5 = \frac{60}{11} = 5.\frac{5}{11}$ minutes.

Hence hands are in the same straight line at $5\frac{5}{11}$ minutes past 7.

NUMERICAL TYPE QUESTIONS

1. $p(x) = x^5 + x^4 + 5x^2 - 3$

$= (-2)^5 + (-2)^4 + 5(-2)^2 - 3$

$= (-32) + 16 + 5 \times 4 - 3 = 1$

2. $p \propto \frac{1}{v}$

$\therefore \quad p = \frac{k}{v}$

where, $k = 84 \times 1000$

$\therefore \quad v = \frac{k}{p} = \frac{84 \times 1000}{28} = 3000 \text{ cm}^3$

3. Middle term = $2\sqrt{\text{first term} \times \text{last term}}$

$ax = 2 \times x \times 5 = 10x$

$\therefore \quad a = 10$

4. $\alpha + \beta = \frac{7}{2}$

$\alpha\beta = -\frac{4}{2} = -2$

$(\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta)$

$\left(\frac{7}{2}\right)^3 = \alpha^3 + \beta^3 + 3(-2)\left(\frac{7}{2}\right)$

$\frac{343}{8} = \alpha^3 + \beta^3 - 21$

$\frac{343}{8} + 21 = \alpha^3 + \beta^3$

$\therefore \quad \frac{511}{8} = \alpha^3 + \beta^3$

5. Let Anita's present age be 'x' years and that of Sunita by 'y' years.

$\therefore \quad x + y = 43$ ...(1)

and $(x + 11) = \frac{7}{6}(y + 11)$

$\Rightarrow \quad 6x + 66 = 7y + 77$

$\Rightarrow \quad 6x - 7y = 11$ ...(2)

Multiplying (1) by 6.

$6x + 6y = 258$ ...(3)

Subtracting (3) from (2),

$-13y = -247$

$\therefore \quad y = 19$ years.

6. Let the number of students in room A be x and number of students in room B be y.

$\therefore \quad x - 5 = y + 5$

$x - y = 10$ ...(1)

Also, $x + 5 = 2(y - 5)$

$\Rightarrow \quad x - 2y = -15$ ...(2)

Solving (1) and (2) we get,

$x = 35$ and $y = 25$

$\therefore \quad x + y = 60$

7. Let P be the number of workers, and let C be the number of boxes. Then, by counting the number of boxes handling in 2 different ways yields 11P = 7C.

Since P and C are integers, it follows that P must be divisible by 7.

Since $50 \le P \le 60$,

P must equal 56

$\therefore \quad C = 11 \times \frac{56}{7} = 88$

8. $ax = b$

$\Rightarrow \quad (cz)\ x = b$ ....[since cz = a]

$\Rightarrow \quad (by)\ xz = b$ ....[since by = c]

$\Rightarrow \quad xyz = 1$

9. $\frac{1}{\left(1 + \frac{xb}{xa} + \frac{xc}{xa}\right)} + \frac{1}{\left(1 + \frac{xa}{xb} + \frac{xc}{xb}\right)} + \frac{1}{\left(1 + \frac{xb}{xc} + \frac{xa}{xc}\right)}$

$= \frac{xa}{(xa + xb + xc)} + \frac{xb}{(xa + xb + xc)} + \frac{xc}{(xa + xb + xc)}$

$= \frac{(xa + xb + xc)}{xa + xb + xc} = 1$

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10. Let tree grows x cm each year

Height of the tree at the end of the sixth year

$= (90 + 6x)$ cm

Growth in seventh year,

$$x = \frac{1}{9}(90 + 6x) \text{ cm}$$

$$x = 10 + \frac{2x}{3} \qquad \therefore \quad x = 30$$

Hence height of tree at the end of the twelfth year $= (90+12 \times 30) = 450$cm.

11. Let B = price of the bottle

C = price of the cork

Given : B + C = ₹ 1.10 ...(1)

and B – C = ₹ 1.00 ...(2)

From equations (1) and (2)

B = ₹ 1.05

C = ₹ 0.05.

12. $(100-1)(100-2)(100-3)\ldots\ldots\ldots$

$(100-100)(100+1)(100+2)(100+3)$

$\Rightarrow (100-1)(100-2)(100-3)\ldots\ldots\ldots$

$(0)(100+1)(100+2)(100+3) = 0.$

13. $$\frac{2+\frac{1}{\frac{19}{5}}}{2+\frac{1}{3+\frac{4}{5}}} = \frac{2+\frac{5}{19}}{2+\frac{5}{19}} = 1$$

14. $\frac{b}{c} = 3 \Rightarrow b = 3c$, i.e., b is divisible by 3.

Also, $\frac{b}{c}$ cannot be reduced,

i.e., a is not divisible by 3.

EXERCISE – II

MCQ TYPE QUESTIONS

1. $\frac{5}{7} = x^{-1/3} \Rightarrow \frac{7}{5} = x^{1/3} \Rightarrow \left(\frac{7}{5}\right)^3 = x \Rightarrow x = \frac{343}{125}$

2. $f(x) = 2x + 33 = 2(5) + 33 = 43$

3. For whole numbers, greater the value greater will be its log.

Same logic for power of e.

4. $\log \tan 1° + \log \tan 89° = \log(\tan 1° \times \tan 89°)$

$= \log(\tan 1° \times \cot 1°)$

$= \log 1 = 0$

Using the same logic total sum is '0'.

6. The given function is

$$f(x) = 1 - |x| \text{ on } -1 \le x \le 1$$

In order to find maximum and minimum value of function in interval [–1, 1]

At $x = -1$, $f(-1) = 1-|-1| = 1-1 = 0$

At $x = -0.5$, $f(-0.5) = 1-|-0.5| = 1-0.5 = 0.5$

At $x = 0$, $f(0) = 1-|0| = 1$

At $x = 0.5$, $f(0.5) = 1-|0.5| = 0.5$

At $x = 1$, $f(1) = 1-|1| = 1-1 = 0$

$\therefore$ Hence, maximum value occurs at $x = 0$ and its maximum value is 1.

$\therefore$ Option (c) is correct.

9. $y = 5x^2 + 3,\ \frac{dy}{dx} = 10x$

Slope of tangent $= \left(\frac{dy}{dx}\right)_{x=0,y=3} = 10 \times 0 = 0$

Slope = 0 $\Rightarrow$ tangent is parallel to x-axis.

10. $P' = 2^n X^n \left(\frac{1}{2}\right)^m y^m = 2^{n-m} X^n Y^m = 2^{n-m} P$

11. For a very large number of throws, the frequency should be same for unbiased throw. As it not same, then the die is baised.

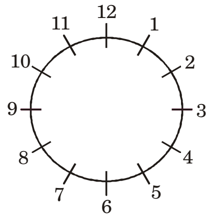
12. $(7526)_8 - (y)_8 = (4364)_8$

$\Rightarrow y_8 = (7526)_8 - (4364)_8$

$$\begin{array}{cccc} 7 & \overset{4}{\cancel{5}} & \overset{(8+2=10)}{2} & 6 \\ 4 & 3 & 6 & 4 \\ \hline 3 & 1 & 4 & 2 \\ \hline \end{array}$$

When we have base 8, we borrow 8 instead of 10 as done in normal subtraction.

13.



From option

At 6 a.m.

Both hand – 180° aparts,

At 6 : 20 a.m. – both hand apart

$\Rightarrow \quad 180° + 10° - 120° = 70°$

At 6 : 22 a.m. – both hand apart

$\Rightarrow \quad 180° + 12° - 132° = 60°$ (closest)

14. $ax^2 + bx + c = 0$

Roots are real and positive

$\therefore \quad b^2 - 4ac > 0$

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By the above condition we get two positive real roots

Now, $ax^2 + b|x| + c = 0$

This can be written as;

$ax^2 + bx + c$

$$\Delta = b^2 - 4ac > 0$$

Now for $ax^2 - bx + c$

$$(-b)^2 - 4ac \quad \Rightarrow \quad b^2 - 4ac$$

Again $b^2 - 4ac > 0$

So again we get real roots

Thus we have 4 real roots of $ax^2 + b|x| + c = 0$

15. $P = 50q - 5q^2$

$$\frac{dp}{dq} = 50 - 10q; \ \frac{d^2p}{dq^2} < 0$$

$\therefore$ p is maximum at $50 - 10q = 0$ or, $q = 5$

Else check with options

16. P and R always holds true

Else consider a sample set {1, 2, 3, 4} and check accordingly

17. $y = 2x - 0.1x^2$

$$\frac{dy}{dx} = 2 - 0.2x$$

$$\frac{d^2y}{dx^2} < 0$$

$\therefore$ y maximises at $2 - 0.2x = 0$

$\Rightarrow x = 10$

$\therefore$ $y = 20 - 10 = 10\text{m}$

18. Let us categorize bags in three groups as

$A_1 A_2 A_3$ $\quad$ $B_1 B_2 B_3$ $\quad$ $C_1 C_2$

1st weighing A vs B

Case-1	**Case-2**
$A_1 A_2 A_3 = B_1 B_2 B_3$	$A_1 A_2 A_3 \neq B_1 B_2 B_3$
Then either C_1 or C_2 is heavier	Either A or B would be heavier (Say A > B)
2nd weighing	
C_1 vs C_2	A_1 vs A_2
If $C_1 > C_2$, then C_1	If $A_1 = A_2$, then A_3
If $C_1 < C_2$, then C_2	If $A_1 > A_2$, then A_1
If $A_1 < A_2$, then A_2	

19. $\log P = \frac{1}{2} \log Q = \frac{1}{3} \log (R) = k$

$\therefore$ $P = b^k$, $Q = b^{2k}$, $R = b^{3k}$

Hence, $Q^2 = b^{4k} = b^{3k} b^k = PR$

20. Checking with all options in formula: $(4q+100/q)$ i.e. (V+F). Option (*a*) gives the minimum cost.

21. Let total number of toffees in bowl be x

R took $\frac{1}{3}$ of toffees and returned 4 to the bowl

$\therefore$ Number of toffees with R $= \frac{1}{3}x - 4$

Remaining of toffees in bowl $= \frac{2}{3}x + 4$

Number of toffees with S $= \frac{1}{4}\left(\frac{2}{3}x + 4\right) - 3$

Remaining toffees in bowl $= \frac{3}{4}\left(\frac{2}{3}x + 4\right) + 4$

Number of toffees with T $= \frac{1}{2}\left(\frac{3}{4}\left(\frac{2}{3}x + 4\right) + 4\right) + 2$

Remaining toffees in bowl

$$= \frac{1}{2}\left[\frac{3}{4}\left(\frac{2}{3}x + 4\right) + 4\right] + 2$$

Given : $\quad \frac{1}{2}\left[\frac{3}{4}\left(\frac{2}{3}x + 4\right) + 4\right] + 2 = 17$

$\Rightarrow \quad \frac{3}{4}\left(\frac{2}{3}x + 4\right) = 27$

$\Rightarrow \quad x = 48$

NUMERICAL TYPE QUESTIONS

1. Let area of triangle PQR be 'A'

$$\frac{SQ}{PQ} = \frac{1}{1+3} = \frac{1}{4}$$

$$\frac{QT}{QR} = \frac{2}{2+5} = \frac{2}{7}$$

$\therefore$ Area of $\Delta QTS = \frac{1}{2} \times SQ \times QT$

$$= \frac{1}{2} \times \left(\frac{1}{4}PQ\right) \times \left(\frac{2}{7}QR\right)$$

$$= \frac{1}{4} \times \frac{2}{7} \times \left(\frac{1}{2} \times PQ \times QR\right)$$

$$= \frac{1}{14} \times \text{Area of } \Delta PQR$$

Given $20 \text{ cm}^2 = \frac{1}{14} \times A$

$\therefore$ $A = 14 \times 20 = 280 \text{ cm}^2$

2. $fg(h(2, 5, 7, 3), 4, 6, 8) = fg(1, 4, 6, 8)$

$$= f(1, 4, 6, 8) \times g(1, 4, 6, 8)$$

$$= 8 \times 1 = 8$$

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Engineering Mathematics

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1

CHAPTER

Linear Algebra

DETERMINANTS

A determinant of order n has n rows and n columns. It has $n \times n$ elements.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{vmatrix}$$

MINOR

If a_{ij} is an element which is in the i^{th} row and j^{th} column of a squre matrix A, then determinant of the matrix obtained by deleting i^{th} row and j^{th} column of A is called minor of a_{ij} and is denoted by M_{ij}

e.g. If $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, then

$$M_{11} = \text{minor of } a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}\,a_{33} - a_{32}\,a_{23}$$

$$M_{12} = \text{minor of } a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$= a_{21}\,a_{33} - a_{31}\,a_{23} \text{ and so on.}$$

9 minors can be obtained corresponding to 9 elements of Δ.

e.g. Minor of the element

$$a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = M_{11}$$

$$a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = M_{12} \text{ and so on.}$$

Determinant of a Square Matrix

Only square matrix can have determinants because in a determinant, number of rows is equal to number of columns.

Let $A = [a_{ij}]_{\max}$ be a square matrix of order n.

$$\text{Then} \quad |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{vmatrix}$$

is called *determinant of matrix A.*

Properties of Determinants

(*i*) If all the elements of one column or one row of a determinant are multiplied by the same number "C", then value of the new determinant is "C" times the value of the given determinant.

(*ii*) Value of a determinant does not change when rows and columns are interchanged.

(*iii*) If any two columns or two rows of a determinant are interchanged, then value of the determinant is multiplied by – 1.

(*iv*) In a determinant, sum of the products of the element of any column (or row) with the cofactors of the corresponding elements of any other column (or row) is zero.

(*v*) If two columns (or two rows) of a determinant are identical, then value of the determinant is zero.

(*vi*) If in a determinant each element in any column (or row) consists of sum of two terms, then determinant can be expressed as the sum of two determinants of the same order.

(*vii*) If the elements of a column (or row) of a determinant are added K times the corresponding elements of another column (or row), then value of the determinant obtained is equal to value of the original determinant.

COFACTOR

If a_{ij} is an element which is in the i^{th} row and j^{th} column of a square matrix A, then product of $(-1)^{i+j}$ and minor of a_{ij} is called cofactor of a_{ij} and is denoted by A_{ij}.

e.g. If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then

$$A_{11} = \text{cofactor of } a_{11} = (-1)^{1+1} M_{11}$$

$$= +1 \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}\,a_{33} - a_{32}\,a_{23}$$

$$A_{12} = \text{cofactor of } a_{12} = (-1)^{1+2} M_{11}$$

$$= -1 \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = (a_{31}\,a_{23} - a_{21}\,a_{33})$$

and so on.

and if $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, then

$$A_1 = \text{cofactor of } a_1 = (-1)^{1+1} \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$$

$$= b_2c_3 - b_3c_2$$

$$B_1 = \text{cofactor of } b_1 = (-1)^{1+2} \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}$$

$$= -(a_2c_3 - a_3c_2) = a_3c_2 - a_2c_3$$

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C_1 = cofactor of c_1

$$= (-1)^{1+3}\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} = a_2b_3 - a_3b_2 \text{ and so on.}$$

Example. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, then find all cofactors of element of A.

Solution. $A_{11} = (-1)^{1+1}\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} = +(45-48) = -3$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} = -(36-42) = 6$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} = +(32-35) = -3$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} = -(18-24) = 6$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 7 & 9 \end{vmatrix} = +(9-21) = -12$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = -(8-14) = 6$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = +(12-15) = -3$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = -(6-12) = 6$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} = +(5-8) = -3$$

ALGEBRA OF MATRICES.

MATRIX

A set of m,n numbers (real or complex) arranged in the form of a rectangular may have m rows and n columns is called an $m \times n$ *matrix.*

An $m \times n$ matrix is written as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

Element a_{ij} belongs to i^{th} row and j^{th} column and is sometimes called as $(i, j)^{th}$ element of the matrix.

Note : In a matrix, number of rows and columns need not be equal.

Types of matrix :

1. Square matrix

In this matrix, the number of rows is equal to the number of columns.

e.g. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$ and $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}_{3\times 3}$

In these square matrices, $m = n$

Elements a_{ij} of a square matrix $A = [a_{ij}]_{n\times n}$ for which $i = j$ are called *diagonal elements* and line along which they lie is called *principal diagonal* of the matrix.

2. Null matrix or Zero matrix

The $m \times n$ matrix whose All elements are 0 is called *null matrix* (or zero matrix) of the type $m \times n$. It is denoted by 0.

e.g. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{3\times 4}$ is zero matrix of type 3×4

Above matrix can be written as

$$A = [a_{11}\, a_{22}\, a_{33}\, a_{44}]$$

3. Diagonal matrix

In this matrix, elements above and below the principal diagonal are all zero.

e.g. $A = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$

4. Scalar matrix

In this matrix, all diagonal elements are equal.

e.g. $A = \begin{bmatrix} C & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & C \end{bmatrix}$

- If A be a matrix and $A^2 = A$, then A is called *idempotent*.

  If $AB = A$ and $BA = B$, then A and B are called *idempotent*.
- If A be a matrix and satisfies $A^2 = I$, then A is called *invalutory*.

5. Rectangular matrix

A matrix of order $m \times n$ is called a *rectangular matrix* if $m \neq n$

e.g. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \end{bmatrix}_{2\times 3}$, $\begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 2 \end{bmatrix}_{3\times 2}$, $\begin{bmatrix} 1 & 2 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{bmatrix}_{2\times 4}$

6. Unit matrix or Identity matrix

A square matrix each of whose diagonal elements is 1 and each of whose non-diagonal elements is equal to zero is called a *unit matrix or an identity matrix*. It is denoted by I_n when n is order of the matrix.

e.g. $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

are unit matrices of order 2, 3, 4, respectively.

7. Column matrix

Any $m \times 1$ matrix which has m rows and only one column is a column matrix or a column vector.

Row matrix

Any $1 \times n$ matrix which has only one row and n columns is called a row matrix or as row vector.

e.g. $X_1 = [1\ 2\ 3\ 4\ 5\ 6]_{1\times 6}$ is a row matrix of the type 1×6

$X_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}_{4\times 1}$ is a column matrix of type 4×1

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8. Triangular matrix

It is of two types

(*i*) Lower triangular matrix. In this matrix, all the elements above the principal diagonal are zero.

e.g. $\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$

The diagonal and lower triangular elements may or may not be zero

(*ii*) Upper triangular matrix. In this matrix, all the elements below the principal diagonal are zero.

e.g. $\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}_{4\times4}$

The diagonal and upper triangular elements may or may not be zero

Types of Square Matrices.

(*i*) Nilpotent matrix.

A square matrix A is called a *nilpotent matrix* if there exists a positive integer n such that $A^n = 0$ and $A^{x-1} \neq 0$

If n is least positive integer such that $A^n = 0$, then n is called *index of the nilpotent* matrix A.

(*ii*) Symmetrical matrix.

It is a square matrix in which $a_{ij} = a_{ji}$ for all i, j. A symmetric matrix is necessarily a square one.

If A is symmetric**, then $A^T = A$**

e.g. $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

(*a*) A A^T is always symmetric matrix

(*b*) $\dfrac{A+A^T}{2}$ is always symmetric matrix

(c) If A and B are symmetric then A + B and A – B also symmetric AB, BA may or May not by symmetric.

(*iii*) Skew-symmetrical matrix.

It is a square matrix in which $a_{ij} = -a_{ji}$ for all i and j.

In a skew symmetrical matrix, all elements along the diagonal are zero.

e.g. $\begin{bmatrix} 0 & h & g \\ h & 0 & f \\ g & f & 0 \end{bmatrix}$

For any matrix A, the matrix $\dfrac{A-A^T}{2}$ is always skew symmetric.

Theorem:

Every square matrix can be uniquely expressed as a sum of a symmetric matrix and a skew symmetric matrix.

(*iv*) Hermitian matrix.

It is a square matrix A in which $(i, j)^{\text{th}}$ element is equal to conjugate complex of the $(j, i)^{\text{th}}$ element.

i.e. $a_{ij} = \bar{a}_{ji}$ for all i and j

e.g. $\begin{bmatrix} x & 3-4i & 1+2i \\ 3+4i & 0 & 5-4i \\ 1-2i & 5+4i & 1 \end{bmatrix}$

A necessary and sufficient condition for a matrix A to be hermitian is that

$$A = A^\theta$$

where A^θ is transposed conjugates of A.

(*v*) Skew-Hermitian matrix.

It is a square matrix $A = [a_{ij}]$ in which $a_{ij} = -\bar{a}_{ji}$ for all i and j.

The diagonal elements of a skew-Hermitian matrix must be pure imaginary numbers or zero.

A necessary and sufficient condition for a matrix A to be skew-Hermitian is that

$$A^\theta = -A$$

(*vi*) Orthogonal matrix.

A square matrix A is called *orthogonal matrix if*

$$AA^T = A^TA = I$$

e.g. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and

$$A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

then $AA^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

(*vii*) Adjoint matrix.

Let A be the square matrix given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}$$

Let A_{ij} be the cofactor a_{ij} then transpose of the matrix $[A_{ij}]$ is called *adjoint of the matrix A.*

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$$\text{Adjoint } A = \begin{bmatrix} A_{11} & A_{21} & \cdots & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & \cdots & A_{n2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ A_{1n} & A_{2n} & \cdots & \cdots & A_{nn} \end{bmatrix}$$

If A be an n-rowed square matrix, then $(\text{Adj } A)\, A = A\,(\text{Adj } A) = |A|\, I_n$

where I_n is the $n \times n$ identity matrix.

SUB-MATRIX

Any matrix obtained by omitting some rows and columns from a given $(m \times n)$ matrix B is called *submatrix of B*.

The matrix B itself is a sub-matrix of B as it can be obtained from B by omitting no rows or columns.

Principal Sub-matrix

A square sub-matrix of a square matrix B whose diagonal elements are also diagonal elements of matrix B is called *principal sub-matrix*.

ADDITION OF MATRICES

Let A and B be two matrices of the same type $m \times n$.

If $A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{m \times n}$, then

$$A + B = [a_{ij} + b_{ij}]_{m \times n}$$

e.g. If
$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

and
$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & b_{22} & b23 & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{m1} & b_{m2} & b_{m3} & \cdots & b_{mn} \end{bmatrix}$$

then,
$$A + B = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} & \ldots\ldots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} & \ldots\ldots & a_{2n}+b_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & a_{m3}+b_{m3} & \ldots\ldots & a_{mn}+b_{mn} \end{bmatrix}$$

Properties of Matrix Addition

(*i*) Matrix addition is Commulative.

If A and B be two $m \times n$ matrices, then

$$A + B = B + A$$

(*ii*) Matrix addition is Associative.

If A, B, C be three matrices each of type $m \times n$, then

$$(A + B) + C = A + (B + C)$$

(*iii*) Existence of Additive identity.

If 0 be the $m \times n$ matrix each of whose element is zero, then $A + 0 = A = 0 + A$

where A is a $m \times n$ matrix

(*iv*) Existence of additive inverse.

If $B = [b_{ij}]_{m \times n}$, then negative of matrix B is defined as he matrix $[-b_{ij}]_{m\times n}$ and is denoted by –B.

$$-B + B = 0 = B + (-B)$$

0 is null matrix of order m× n

(*v*) Cancellation Law.

If A, B, C are three main matrices, then

$$B + A = C + A$$
$$\Rightarrow \quad B = C \quad \text{(right cancellation law)}$$
$$A + B = A + C$$
$$\Rightarrow \quad B = C \quad \text{(left cancellation law)}$$

These cancellation laws hold good in the case of addition of matrices.

(*vi*) The equation B + X = 0 has a unique solution in the set of all $m \times n$ matrices.

EQUALITY OF TWO MATRICES

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if

(*i*) both are the same size, and

(*ii*) $a_{ij} = b_{ij}$ for each pair of subscripts i and j.

If two matrices A and B are equal, then we write

$$A = B$$

MULTIPLICATION OF A MATRIX

1. By a scalar

Let C any real number called scaler and B be any $m \times n$ matrix, then

$$CB = BC = [cb_{ij}]_{m \times n}$$

where, $[b_{ij}]_{m \times n} = B$

Properties :

(*i*) If p and q are two scalars and B is any $m \times n$ matrix, then

$$p\,(qB) = (pq)\,B$$

(*ii*) $(p + q)\,B = pB + qB$

(*iii*) If A and B are two matrices each of the type $m \times n$, then

$$p\,(A + B) = p\,(A) + p(B)$$

where p is any scalar.

(*iv*) If B be any $m \times n$ matrix, then

$$(-p)\,B = (-pB) = p\,(-B)$$

(*a*) 1B = B

(*b*) (–1) B = –B

2. By a matrix

Let $A = [a_{ij}]_{l \times m}$ and $B = [b_{jk}]_{m \times n}$ be two matrices such that number of columns in A is equal to number of rows in B. Then in $l \times n$ matrix

$$C = [C_{ik}]_{l \times n} \text{ such that } C_{ik} = \sum_{j=1}^{n} a_{ij}\, b_{jk}$$

is called *product of the matrices A and B.*

We can write $C = AB$

Properties :

(*i*) Associative laws.

If A, B, C are $l \times m$, $m \times n$ and $n \times p$ matrices respectively, then

$$A\,(BC) = (AB)\,C$$

(*ii*) Distributive laws.

$$A\,(B + C) = AB + AC$$

where A, B, C are any three $l \times m$, $m \times n$ and $n \times p$ matrices respectively.

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(*iii*) Commutative.

Multiplication of matrices is not always commutative.

For commutative AB = BA

For anti commutative AB = – BA

(*iv*) Product of two matrices can be zero matrix while neither of them is a zero matrix.

(*v*) If A be an $l \times m$ matrix. and I_m denotes m rowed unit matrix, then

$$AI_m = A = I_l A$$

TRANSPOSE OF A MATRIX

If row and column are interchanged in a matrix A, then matrix obtained is called *transpose matrix A* and is denoted by A^T.

- If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then $A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$
- If $A = [a_{ij}]$, then $A^T = [a_{ji}]$
- If A is a square matrix, then $|A| = |A^T|$

Properties of transpose

Theorem 1.

If A is any matrix, then

$$(A^T)^T = A.$$

Theorem 2.

If A and B are two matrices of same type, then

$$(A + B)^T = A^T + B^T.$$

Note : If A is any matrix and k is any complex number, then

$$(kA)^T = kA^T.$$

Theorem 3.

If A and B are two matrices for which conformability for multiplication is assured, then

$$(AB)^T = B^T A^T.$$

Corollary 1 : If $A_1, A_2, \ldots\ldots\ldots A_n$ are n matrices which are conformable for multiplication, then

$$(A_1, A_2, \ldots\ldots A_n)^T = A_n^T A_{n-1}^T \ldots\ldots\ldots\ldots A_1^T.$$

Note : If A is any matrix, then AA^T and A^TA are defined. They are square matrices but their order may be different.

e.g. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 0 & 3 \end{bmatrix}$ then $A^T = \begin{bmatrix} 1 & -2 \\ 2 & 0 \\ 0 & 3 \end{bmatrix}$

and $AA^T = \begin{bmatrix} 5 & -2 \\ -2 & 13 \end{bmatrix}$

$$A^TA = \begin{bmatrix} 5 & 2 & -6 \\ 2 & 4 & 0 \\ -6 & 0 & 9 \end{bmatrix}$$

INVERSE OF MATRIX

Let A and B are two square matrices of the same order, such that AB = BA = I

then B is called *inverse of A* and is denoted by A^{-1}.

$$A^{-1} = \frac{(adj A)}{|A|}$$

Properties

(*i*) A^{-1} exists only when A is non singular, i.e. $|A| \neq 0$

(*ii*) The inverse of a matrix is unique.

(*iii*) $(AB)^{-1} = B^{-1} A^{-1}$

(*iv*) $(A^T)^{-1} = (A^{-1})^T$

(*v*) $(A^{-1})^\theta = (A^\theta)^{-1}$

INVERSE OF A SQUARE MATRIX

A square matrix A is said to be an invertible matrix if there exists a square matrix B such that

AB = BA = I, then matrix B is called inverse of A.

- A rectangular matrix can not be invertible.
- Every square matrix need not be invertible.

Theorem 1.

An invertible matrix has unique inverse.

If A is an invertible matrix, then its inverse is denoted by A^{-1}

Hence $AA^{-1} = A^{-1}A = I.$

Theorem 2.

If A is an invertible matrix, and A^{-1} is also invertible then $(A^{-1})^{-1} = A$

Theorem 3.

If A and B are two invertible matrices of same type, then AB is also invertible and $(AB)^{-1} = B^{-1} A^{-1}$.

Corollary 1: If $A_1, A_2, \ldots A_n$ are invertible matrices of same type, then $A_1A_2 \ldots A_n$ is also invertible and

$$(A_1 A_2 \ldots A_n)^{-1} = A_n^{-1} A_{n-1}^{-1} \ldots A_1^{-1}$$

If A is an invertible matrix and k is a nonzero complex number, then kA is invertible and

$$(kA)^{-1} = k^{-1} A^{-1}$$

Theorem 4.

If A is an invertible matrix, then A^T is also invertible and $(A^T)^{-1} = (A^{-1})^T$

Theorem 5.

If A is a nonsingular matrix, then A is invertible and

$$A^{-1} = \frac{\text{Adj A}}{\det A}$$

If A is a square matrix, then

$$A\,(\text{Adj A}) = (\det A)\, I$$

Theorem 6.

If A is an invertible matrix, then A is nonsingular.

If A is an invertible matrix, then

$$\det (A^{-1}) = \frac{1}{\det A}$$

Theorem 7.

A square matrix A is non singular if A is invertible.

Corollary 2 : If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad \neq bc$, then

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & b \\ c & a \end{bmatrix}$$

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ORTHOGONAL MATRIX

An invertible square matrix is called orthogonal, if

$$A^T = A^{-1}$$

$\therefore \quad AA^T = I$

$\Rightarrow \quad |A| = |A^T|$

$\Rightarrow \quad |A| = \pm 1$

Example. If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, then find A^{-1}.

Solution. $A^{-1} = \dfrac{1}{14-15}\begin{bmatrix} 7 & -3 \\ -5 & 2 \end{bmatrix}$

$$= -1\begin{bmatrix} 7 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -5 & 2 \end{bmatrix}$$

Theorem 8 .

Let A, B, C be three square matrices of same type and A be a non-singular matrix. Then

$AB = AC \quad \Rightarrow B = C$

and $\quad BA = CA \quad \Rightarrow B = C.$

Theorem 9 .

If A and B are nonsingular matrices of same type, then

$$\text{Adj}(AB) = (\text{Adj } B)(\text{Adj } A)$$

Theorem 10 .

If A is a square matrix of type n, then

$\det(\text{Adj } A) = (\det A)^{n-1}$

Example. If A is a square matrix of type 3 and $\det A = 10$, then find (Adj A).

Solution. $\det(\text{Adj } A) = (\det A)^{3-1}$

$$= 10^2 = 100$$

RANK OF A MATRIX

If A is not a null matrix; then rank of the matrix A is positive integer, if it possesses following two properties :

(i) Each minor of order $n + 1$ of the matrix A vanishes (i.e. is zero).

(ii) There exists at least one minor of order n of the matrix A which is not zero.

If A is a null matrix, then rank of A is defined as *zero*.

A need not be square matrix.

Properties.

- Rank of A is denoted by $p(A)$ and is UNIQUE.
- Every matrix will have a rank.
- If A is a matrix of order $m \times n$, then
  $p(A) \le m$ and n (smaller of the two).
- If $P(A) = n$, then every minor of order $n + 1$, $n + 2$, etc. is zero.
- If A is a matrix of order $n \times n$, then
  A is non-singular $\Leftrightarrow p(A) = n$.
- Rank of $I_n = n$.
- A is a matrix of order $m \times n$. If every kth order minor $(k < m, k < n)$ is zero, then
  $$P(A) < k$$
- A is a matrix of order $m \times n$. If there is a minor of order $k(k < m, k < n)$ which is not zero, then
  $$p(A) \ge k$$
- Rank of a zero matrix is defined to be zero. The rank of a unit matrix is defined to be one.
- If x is a non-zero column matrix and y is a non-zero row matrix, then $p(x\,y) = 1$
- Rank of a matrix is greater than or equal to (not less than) rank of every sub-matrix thereof.
- If A is any n-rowed square matrix of rank, $n - 1$, then $\quad \text{adj } A \neq 0$.
- Rank of the transpose of a matrix is equal to rank of the original matrix.
- Rank of a matrix does not change by pre-multiplication or post-multiplication with a non-singular matrix.
- If $A \sim B$, then $\quad p(A) = p(B)$.
- Rank of a product of two matrices cannot exceed rank of either matrix.
- Rank of the sum of two matrices cannot exceed sum of their ranks.
- Elementary transformations do not change the rank of a matrix.

ECHELON FORM OF A MATRIX

Let A be the given matrix of order m × n, then it can be reduced to Echelon form by row tranformation.

If we can make all the entries equal to zero, then

$$R(A) = \text{Number of none zero rows}$$

Consistency and Solution of Algebraic equations

In argument matrix A*, we reduce echelon form.

(*i*) If $\quad$ Rank of A^* = Rank of A,
then equation are consistent.

(*ii*) If $\quad$ Rank of $A^* \neq$ Rank of A,
then system is inconsistent (i.e. no solution)

Homogenous system of equations AX = 0

(*i*) If $\quad$ rank = number of unknowns,
then it has zero solution.

(*ii*) If $\quad$ rank < number of unknowns,
then it has infinitely may solution.

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ELEMENTARY TRANSFORMATION OF A MATRIX

Following operations on a matrix are called *elementary transformations*.

(*i*) Interchange of any two rows (or columns).

(*ii*) Multiplication of elements of any row (or column) by any non-zero number.

(*iii*) Addition to elements of any other row (or column) the corresponding elements of any other row (or column) multiplied by any number.

SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS BY MATRIX.

Simultaneous linear equations of two or three variables are solved by using matrices.

1. Homogeneous linear equations

Let system of m homogeneous equations in n unknown $x_1, x_2, \ldots, x_n$ be

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n = 0 \end{array}\right\} \quad \ldots(i)$$

$$\text{Let A} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix}_{m \times n}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \ldots \\ \ldots \\ \ldots \\ x_n \end{bmatrix}_{n \times 1} ; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \ldots \\ \ldots \\ 0 \end{bmatrix}_{m \times 1}$$

where A, X and B are $m \times n$, $n \times 1$ and $m \times 1$ matrices respectively.

Equation *(i)* can be written in the form of a single matrix equation

$$AX = B \quad \ldots(ii)$$

The matrix A is called *coefficient matrix* of the system of equation *(i)*

Solutions of equation AX = B.

Working Rule:

Reduce the coefficient matrix A to Echelon form by applying elementary row transformations only.

From this we know rank of the matrix A.

Let matrix A is of the type $m \times n$ and its rank is r.

- If $r < m$, then $(m - n)$ equations will be eliminated. Thus given system of m equations will be replaced by an equivalent system of r equations. Solving these r equations we can express values of some unknown in terms of remaining $n - r$ unknowns. These $n - r$ unknown can be given any arbitrarily chosen values.
- If $r = n$, then zero solution (trivial solution) will be the only solution.
- If $r < n$, there will be an infinity of solutions.

2. Linear non-homogeneous equations

Let system of m simultaneous equations in n unknowns $x_1, x_2, \ldots, x_n$ be

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = K_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = K_2$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n = K_m$$

$$\text{Let A} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ \ldots \\ x_n \end{bmatrix}_{n \times 1},$$

$$K = \begin{bmatrix} K_1 \\ K_2 \\ \ldots \\ \ldots \\ K_m \end{bmatrix}_{m \times 1}$$

where A, X, K are $m \times n$, $n \times 1$ and $m \times 1$ matrices respectively.

Above equations can be written in the form of a single matrix equation AX = K

Consistent and Inconsistent equations

If above system has a solution, then equations are *consistent* otherwise equations are *called inconsistent equations*.

- The system of equation AX = K is consistent, i.e. possesses a solution, if and only if coefficient matrix A and augmented matrix [A K] are of the same rank.
- If A be an n-rowed non-singular matrix, X be an $n \times 1$ matrix, K be an $m \times 1$ matrix, then system of equation AX = K has a unique solution.

Solution of equation AX = K.

Working rule :

Let coefficient matrix A is of the type $m \times n$.

Write augmented matrix [A K] and reduce it to a Echelon form by applying only elementary row

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transformation on it. This Echelon form will give ranks of the augmented matrix [A K] and coefficient matrix A.

Different cases :

Case I : *Rank A < Rank [A K]*

In this case, equation AX = K are inconsistent, i.e. they have no solution.

Case II : *Rank A = Rank [A K] = r (suppose)*

In this case, equation AX = K are consistent.

- If $r < m$, then in the process of reducing the matrix [A K] to Echelon form, $(m - r)$ equations will be eliminated. Then given system of m equations will be replaced by an equivalent system of r equations. From these r equations we shall be able to express the values of some r unknowns in terms of the remaining $n - r$ unknowns which can be given any arbitrary chosen values.
- If $r = n$, there will be a unique solution.
- If $r < n$, then $n-r$ variables can be assigned arbitrary values. So there will be an infinite number of solutions. Only $n - r + 1$ solutions will be linearly independent and rest of the solutions will be linear combinations of them.
- If $m < n$, then $r \leq m < n$.

  Thus $n - r > 0$.

Therefore when number of equations is less than the number of unknowns, equations will always have an infinite number of solutions, provided they are consistent.

MATRIX INVERSION METHOD

Theorem 1 .

If A is a nonsingular matrix, then $A^{-1}B$ is a solution AX = B.

Proof : A is nonsingular

$\Rightarrow$ A^{-1} exists and $AA^{-1} = A^{-1}A = I$

$\Rightarrow$ $A(A^{-1}B) = (AA^{-1})B = IB = B$

$\therefore$ $X = A^{-1}B$ is a solution of AX = B.

Theorem 2 .

If A is a nonsingular matrix, then AX = B has a unique solution.

Proof : A is nonsingular

$\Rightarrow$ A^{-1} exists and $AA^{-1} = A^{-1}A = I$

By theorem 1, $X = A^{-1}B$ is a solution of AX = B

Let $X = X_1$, $X = X_2$ be two solutions of AX = B.

$$\therefore \quad AX_1 = B, AX_2 = B$$

$$\Rightarrow \quad AX_1 = AX_2$$

$$\Rightarrow \quad A^{-1}(AX_1) = A^{-1}(AX_2)$$

$$\Rightarrow \quad (A^{-1}A)X_1 = (A^{-1}A)X_2$$

$$\Rightarrow \quad IX_1 = IX_2$$

$$\Rightarrow \quad X_1 = X_2$$

Hence AX = B has a *unique solution*.

Theorem 3.

If A is a nonsingular matrix, then $X = A^{-1}B$ is the unique solution of AX = B.

Method of finding solution $X = A^{-1}B$ of AX = B is called **matrix inversion method.**

Example. Solve following equations by matrix inversion method

$$x + y + z = 6$$
$$2x - y + z = 3$$
$$x + 2y - z = 2.$$

Solution.

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$

Given equations can be written as

$$AX = B$$

By matrix inversion method solution is,

$$X = A^{-1}B$$

$$\det A = 1(1-2) - 1(-2-1) + 1(4+1)$$
$$= -1 + 3 + 5 = 7.$$

Cofactors of elements of A are

$$A_{11} = +(1-2) = -1$$
$$A_{12} = -(-2-1) = 3$$
$$A_{13} = +(4+1) = 5$$
$$A_{21} = -(-1-2) = 3$$
$$A_{22} = +(-1-1) = -2$$
$$A_{23} = -(2-1) = -1$$
$$A_{31} = +(1+1) = 2$$
$$A_{32} = -(1-2) = 1$$
$$A_{33} = +(-1-2) = -3$$

$$\therefore \quad \text{Adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -1 & 3 & 2 \\ 3 & -2 & 1 \\ 5 & -1 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{\det A} = \frac{1}{7}\begin{bmatrix} -1 & 3 & 2 \\ 3 & -2 & 1 \\ 5 & -1 & -3 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{7}\begin{bmatrix} -1 & 3 & 2 \\ 3 & -2 & 1 \\ 5 & -1 & -3 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} -6 & +9 & +4 \\ 18 & -6 & +2 \\ 30 & -3 & -6 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ 14 \\ 21 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore$ $x = 1, y = 2, z = 3$ is the solution.

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CRAMER'S RULE

Theorem 4 .

Let
$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2 \qquad ...(i)$$
$$a_3x + b_3y + c_3z = d_3$$
be a system of linear equations.

If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$, $\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$,

$$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

then solution of equation, (i) is

$$x = \frac{\Delta_1}{\Delta}, y = \frac{\Delta_2}{\Delta}, z = \frac{\Delta_3}{\Delta} \text{ is}$$

Proof : Let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$.

Then $\quad \det A = \Delta \neq 0$

$\therefore \quad AX = B$ has unique solution

Hence equation (i) has unique solution.

Let, A_1, A_2, A_3 be the cofactors of a_1, a_2, a_3 respectively. Thus

$$(a_1x + b_1y + c_1z)\,A_1 + (a_2x + b_2y + c_2z)\,A_2 + (a_3x + b_3y + c_3z)\,A_3$$
$$= d_1A_1 + d_2A_2 + d_3A_3$$
$$\Rightarrow \quad x[a_1A_1 + a_2A_2 + a_3A_3] + y[b_1A_1 + b_2A_2 + b_3A_3] + z[c_1A_1 + c_2A_2 + c_3A_3] = \Delta_1$$

$$\Rightarrow \quad x\Delta + y0 + z0 = x = \Delta_1 \quad \Rightarrow \quad x = \frac{\Delta_1}{\Delta}$$

Similarly, $y = \dfrac{\Delta_2}{\Delta}$, and $z = \dfrac{\Delta_3}{\Delta}$

Hence solution of (i) is $x = \dfrac{\Delta_1}{\Delta}, y = \dfrac{\Delta_2}{\Delta}, z = \dfrac{\Delta_3}{\Delta}$

- $\Delta_1, \Delta_2, \Delta_3$ are also denoted as $\Delta_x, \Delta_y, \Delta_z$ respectively.
- $\dfrac{x}{\Delta_1} = \dfrac{y}{\Delta_2} = \dfrac{z}{\Delta_3} = \dfrac{1}{\Delta}$

Example. Solve following equations by Cramer's rule.

$$x + y + z = 6$$
$$2x - y + z = 3$$
$$x + 2y - z = 2.$$

Solution. $\quad \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$

$$= 1(1-2) - 1(-2-1) + 1(4+1)$$
$$= -1 + 3 + 5 = 7$$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 3 & -1 & 1 \\ 2 & 2 & -1 \end{vmatrix}$$
$$= 6(1-2) - 1(-3-2) + 1(6+2)$$
$$= -6 + 5 + 8 = 7$$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$
$$= 1\,(-3-2) - 6(-2-1) + 1(4-3)$$
$$= -5 + 18 + 1 = 14$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 2 & -1 & 3 \\ 1 & 2 & 2 \end{vmatrix}$$
$$= 1(-2-6) - 1(4-3) + 6(4+1)$$
$$= -8 - 1 + 30 = 21$$

By Cramer's rule,

$$\frac{x}{\Delta_1} = \frac{y}{\Delta_2} = \frac{z}{\Delta_3} = \frac{1}{\Delta}$$

$$\therefore \quad \frac{x}{7} = \frac{y}{14} = \frac{z}{21} = \frac{1}{7}$$

Hence solution is, $\quad x = 1, y = 2, z = 3$

Coefficient and Augmented matrix.

Let
$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2 \qquad ...(i)$$
$$a_3x + b_3y + c_3z = d_3$$
be a system of linear equations.

Then matrix, $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is called **coefficient matrix**

and matrix $K = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix}$ is called **augmented matrix.**

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EIGEN VALUES AND EIGEN VECTORS.

Let $A = [a_{ij}]$ be an $n \times n$ matrix.

- Matrix $A - \lambda I$ is called *characteristic matrix* of A, where I is identity matrix.
- Determinant $|A - \lambda I|$ is called *characteristic polynomial of A.*
- Equation $|A - \lambda I| = 0$ is called *characteristic equation* of A and its roots are called *eigen values.*
- Problem of finding the eigen values of a matrix is called *eigen-value problem*.
- If λ is a characteristic root or eigen values of an $n \times n$ matrix A, then a non-zero vector X such that $AX = \lambda X$ is called *characteristic vector* or *eigen vector of* A corresponding to the characteristic root λ.

  The set of eigen values of A is called the spectrum of A.

Relations between Eigen values and Eigen vectors

- λ is a eigen values of a matrix A if and only if there exists a non-zero vector X such that

$$AX = \lambda X$$

- If X is a characteristic vector of a matrix A corresponding to the characteristic value λ, then KX is also a characteristic vector of A corresponding to the same characteristic value λ. Here K is any non-zero scalar.
- If X is a eigen vector of a matrix A, then X cannot correspond to more than one eigen value of A.
- Eigen vectors corresponding to distinct characteristic roots of a matrix are linearly independent.

Properties of Eigen values

(*i*) Sum of all the eigen values of a matrix is equal to sum of the elements of the principal diagonal.

(*ii*) Product of the eigen values of a matrix is equal to its determinant.

(*iii*) If λ is an eigen value of a matrix A, then $\frac{1}{\lambda}$ is the eigen value of A^{-1}.

(*iv*) If λ is an eigen value of an orthogonal matrix, then $\frac{1}{\lambda}$ is also its eigen value.

(*v*) If $\lambda_1, \lambda_2 \ldots\ldots\ldots \lambda_m$ are the eigen values of a matrix A, then A^n has the eigen values $\lambda_1^n, \lambda_2^n \ldots\ldots\ldots \lambda_m^n$.

(*vi*) Eigen value of A = Eigen value of A^T

(*vii*) Maximum number of distinct eigen values = size of A

(*viii*) In a triangular and diagonal matrix, Eigen values are diagonal elements themselves.

(*ix*) If A and B are two matrices of same order then the matrix AB and BA will have same characteristic roots.

Cayley-Hamilton theorem.

Every square matrix satisfies its characteristic equation, i.e. if for a square matrix A of order n

$$|A - \lambda I| = (-1)^n [\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \ldots\ldots + a_n]$$

then matrix equation

$$X^n + a_1 X^{n-1} + a_2 X^{n-2} + a_3 X^{n-3} + \ldots\ldots + a_n I = 0$$

is satisfied by X = A

i.e., $A^n + a_1 A^{n-1} + a_2 A^{n-2} + \ldots\ldots + a_n I = 0$

LU Decomposition Method

1. LU Decomposition Method

Let us suppose that 'A' is a matrix.

Now we can write A = LU

where 'L' is Lower Triangular matrix and 'U' is Upper Triangular matrix.

An LU decomposition of any given matrix A is product of a lower triangular matrix and an upper triangular matrix. The product of L and U is equal to A.

$$\text{Let } A = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} = LU$$

Our aim is to find L and U, So that product of it result into A.

$$\text{where} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}$$

$$\text{and} \quad U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

Now $\quad A = LU$

$$\therefore \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

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$$\therefore \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12}+U_{22} & L_{21}U_{13}+U_{23} \\ L_{31}U_{11} & L_{31}U_{12}+L_{32}U_{22} & L_{31}U_{13}+L_{32}U_{23}+U_{33} \end{bmatrix}$$

$\therefore \quad U_{11} = 25$

$U_{12} = 5$

$U_{13} = 1$

Now $\quad L_{21}U_{11} = 64$

$$\therefore \quad L_{21} = \frac{64}{U_{11}} = \frac{64}{25} = 2.56$$

Now $\quad L_{31}U_{11} = 144$

$$\therefore \quad L_{31} = \frac{144}{U_{11}} = \frac{144}{25} = 5.76$$

and $\quad L_{21}U_{12} + U_{22} = 8$

$\therefore \quad (2.56)(5) + U_{22} = 8$

$\therefore \quad U_{22} = -4.8$

and $\quad L_{21}U_{13} + U_{23} = 1$

$\therefore \quad (2.56)(1) + U_{23} = 1$

$\therefore \quad U_{23} = -1.56$

and $\quad L_{31}U_{12} + L_{32}U_{22} = 12$

$\therefore \quad (5.76)(5) + L_{32}(-4.8) = 12$

$\therefore \quad 28.8 + L_{32}(-4.8) = 12$

$$\therefore \quad L_{32} = \frac{12-28.8}{-4.8} = 3.5$$

and $\quad L_{31}U_{13} + L_{32}U_{23} + U_{33} = 1$

$\therefore \quad (5.76)(1) + (3.5)(-1.56) + U_{33} = 1$

$\therefore \quad U_{33} = 1 - 5.76 + 5.46$

$\therefore \quad U_{33} = 0.7$

Hence Lower triangular matrix

$$L = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix}$$

and

$$U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

Cross check

$$[L][U] = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} = [A]$$

Hence Verified

Note : LU decomposition method is not possible for such matrix whose leading submatrices has zero value as determinant.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 1 & 3 & 4 \end{bmatrix}$$ does not have LU decomposition

as the leading submatrix has determinant equal to

$$\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = (4 \times 1) - (2 \times 2) = 0$$

Hence LU decomposition of the above matrix 'A' is not possible.

LU decomposition method to solve system of equation

Once a matrix A is decomposed into L and U we can obtain solution to

$$AX = B \qquad ...(i)$$

Steps to solve system of equation

1. For given matrix A, find L and U.

   So that $\quad A = LU$

   $\therefore \quad LUX = B$ (from (*i*)) ...(*ii*)

2. Let $\quad Y = UX$

   $\therefore \quad LY = B$ (from (*ii*))

Solve the triangular system for Y.

3. Lastly solve the triangular system

   $UX = Y$ for X

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SOLVED PROBLEM

1. Find inverse of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

Solution.

Determinant of given matrix A is

$$\Delta = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

If A_1, A_2 ... be the co-factors of a_1, a_2... in Δ, then

$$A_1 = -24, \quad A_2 = -8, \quad A_3 = -12,$$
$$B_1 = 10, \quad B_2 = 2, \quad B_3 = 6,$$
$$C_1 = 2, \quad C_2 = 2, \quad C_3 = 2$$

Thus, $D = a_1A_1 + a_2A_2 + a_3A_3 = -8$

$$\therefore \quad \text{Adj. A} = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} -24 & -8 & -12 \\ 10 & 2 & 6 \\ 2 & 2 & 2 \end{bmatrix}$$

Hence inverse of matrix A

$$= \frac{\text{adj.A}}{\Delta} = \frac{1}{-8} \begin{bmatrix} -24 & -8 & -12 \\ 10 & 2 & 6 \\ 2 & 2 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & 1 & \frac{3}{2} \\ -\frac{5}{4} & -\frac{1}{4} & -\frac{3}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$$

2. Determine rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

Solution.

Operate $R_2 - R_1$ and $R_3 - 2R_1$, therefore

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{bmatrix} = A \text{ (say)}$$

Obviously, 3rd oder minor of A vanishes. All its second order minors formed by 2nd and 3rd rows are zero.

But another minor $\begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix} = -1 \neq 0$

Hence rank of A is 2

3. Investigate value of λ and μ, so that the equations

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

have

(*i*) no solution,

(*ii*) a unique solution, and

(*iii*) an infinite number of solutions

Solutions :

We have $\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$

The system admits of unique solution if, and only if, coefficent matrix is of rank 3. This requires in

$$\begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{vmatrix} = 15\,(5 - \lambda) \neq 0$$

Thus for a unique solution $\lambda \neq 5$ and m may have any value.

If $\lambda = 5$, the system wil have no solution for those values of μ for which the matrices

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{bmatrix}$$

and $\quad K = \begin{bmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & 5 & \mu \end{bmatrix}$

are not of the same rank. But A is of rank 2 and K is not of rank 2 unless $\mu = 9$.

Thus if $\lambda = 5$ and $\mu \neq 9$, the system will have no solution.

If $\lambda = 5$ and $\mu = 9$, the system will have an infinite number of solutions.

4. Find eigen vlaues and eigen vectors of the matrix.

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Solution.

Characteristic equation is

$$|A - \lambda I| = \begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & 4 \\ 2 & -4 & 3-\lambda \end{bmatrix}$$

$$= \lambda^3 + 18\lambda^3 - 45\lambda = 0$$

$$\Rightarrow \quad \lambda\,(\lambda - 3)\,(\lambda - 15) = 0$$

$$\therefore \quad \lambda = 0, 3, 15$$

Thus eigen values of A are 0, 3, 15.

If x, y, z be the component of an eigen vectors corresponding to the eigen value λ, then

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$$[A-\lambda I]X = \begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Putting $\lambda = 0$, we have

$$8x - 6y + 2z = 0,$$
$$-6x + 7y + 4z = 0,$$
$$2x - 4y + 3z = 0$$

These equations determine a single linearly independent solution which may be takne as (1, 2, 2), so that every non-zero multiple of this vector is an eigen vector corresponding to $\lambda = 0$.

Similarly, eigen vectors corresponding to $\lambda = 3$ and $\lambda = 15$ are arbitrary non-zero multiples of the vectors (2, 1, –2) and (2, –2, 1) which are obtained from above equation.

Hence three vectors may be taken as (1, 2, 2), (2, 1, – 2), (2, –2, 1)

5. If $A\begin{bmatrix} -2 & 1 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 7 \\ -1 & 20 \end{bmatrix}$, find matrix A

Solution. $A = \begin{bmatrix} -1 & 7 \\ -1 & 20 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 3 & 5 \end{bmatrix}^{-1}$

$$= \begin{bmatrix} -1 & 7 \\ -1 & 20 \end{bmatrix}\frac{1}{-10-3}\begin{bmatrix} 5 & -1 \\ -3 & -2 \end{bmatrix}$$

$$= \frac{-1}{13}\begin{bmatrix} -1 & 7 \\ -1 & 20 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ -3 & -2 \end{bmatrix}$$

$$= \frac{-1}{13}\begin{bmatrix} -26 & -13 \\ -65 & -39 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

6. If ω is cube root of –1, then find vlaue of

$$\begin{bmatrix} 1 & -\omega & \omega^2 \\ -\omega & \omega^2 & 1 \\ \omega^2 & 1 & -\omega \end{bmatrix}$$

Solution.

Since ω is cube root of –1, therefore

$$\omega^3 = -1$$

i.e., $\omega^3 + 1 = 0 \Rightarrow (\omega + 1)(\omega^2 - \omega + 1) = 0$

Hence either $\omega = -1$ or $\omega^2 - \omega + 1 = 0$

Let $A = \begin{bmatrix} 1 & -\omega & \omega^2 \\ -\omega & \omega^2 & 1 \\ \omega^2 & 1 & -\omega \end{bmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$$A = \begin{bmatrix} 1-\omega+\omega^2 & -\omega & \omega^2 \\ -\omega+\omega^2+1 & \omega^2 & 1 \\ \omega^2+1-\omega & 1 & -\omega \end{bmatrix} = \begin{bmatrix} 0 & -\omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & -\omega \end{bmatrix}$$

$= 0$ (since $1 - \omega + \omega^2 = 0$)

7. If $A = \begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}$, find $|A^{-1}|$.

Solution.

$$A = \begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}$$

$\therefore$ $|A| = -3 - 10 = -13 \neq 0$

and $A^{-1} = \dfrac{\text{Adj}\,A}{|A|} = \dfrac{\begin{bmatrix} 1 & -5 \\ -2 & -3 \end{bmatrix}}{-13}$

$$|A^{-1}| = \frac{-1}{13}(-3-10) = 1$$

8. If α, β are roots of the equation $\begin{vmatrix} 2x+3 & 1 \\ x & x+2 \end{vmatrix} = 0$, then find $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$

Solution.

Given: $\begin{vmatrix} 2x+3 & 1 \\ x & x+2 \end{vmatrix} = 0$

$\therefore$ $(2x + 3)(x + 2) - x = 0$

$\Rightarrow$ $2x^2 + 6x + 6 = 0$

$\Rightarrow$ $x^2 + 3x + 3 = 0$

Since α, β are roots of this equation, therefore

$$\alpha + \beta = -3, \quad \alpha\beta = 3$$

$\therefore$ $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta} = \dfrac{-3}{3} = -1$

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EXERCISE – I

MCQ TYPE QUESTIONS

1. Eigen values of the matrix $\begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$ are

(*a*) 1, 1, 1 (*b*) 1, 1, 2 (*c*) 1, 4, 4 (*d*) 1, 2, 4

2. Rank of the following $(n+1)\times(n+1)$ matrix, where a is a real number, is

$$\begin{bmatrix} 1 & a & a^2 & . & . & . & a^n \\ a & a & a^2 & . & . & . & a^n \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 1 & a & a^2 & . & . & . & a^n \end{bmatrix}$$

(*a*) 1
(*b*) 2
(*c*) *n*
(*d*) dependent upon the value of A

3. Matrices $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ commute under multiplication

(*a*) if $a = b$ or $\theta = n\pi$, where n is an integer
(*b*) always
(*c*) never
(*d*) if $a \cos\theta \neq b \sin\theta$

4. If A = $\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 5 \\ -3 & -5 & 0 \end{bmatrix}$, then

(*a*) A' = – A (*b*) A' = A
(*c*) A' = $-\dfrac{1}{A}$ (*d*) none of these

5. Eigen values of the system represented by

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} X \text{ are}$$

(*a*) 0, 0, 0, 0 (*b*) 1, 1, 1, 1
(*c*) 0, 0, 0, –1 (*d*) 1, 0, 0, 0

6. Matrix, A = $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is not

(*a*) orthogonal
(*b*) non–singular
(*c*) singular
(*d*) have A^{-1} exists

7. In the matrix equation Px = q, which of the following is a necessary condition for the existence of at least one solution for the unknown vector x ?

(*a*) Augmented matrix [Pq] must have the same rank as matrix P
(*b*) Vector q must have only non-zero elements
(*c*) Matrix P must be singular
(*d*) Matrix P must be square

8. Eigen vectors of a real symmetric matrix corresponding to different eigen values are

(*a*) orthogonal (*b*) singular
(*c*) non-singular (*d*) none of these

9. An arbitrary vector X is an eigen vector of the matrix A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix}$, if (a, b) =

(*a*) (0, 0) (*b*) (1, 1) (*c*) (0, 1) (*d*) (1, 2)

10. For which value of k, following system is consistent ?

$$\begin{aligned} 2x - 5ky + 6z &= 0 \\ kx + 2y - 2z &= 0 \\ 2x + 2y - kz &= 0 \end{aligned}$$

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 5

11. The system of equations

$$a_1x + a_2y = 0$$
$$b_1x + b_2y = 0$$

where a_1, a_2, b_1, b_2, are real numbers, has a non-trivial solution if

(*a*) $a_1b_1 = a_2b_2$ (*b*) $a_1b_2 = b_1a_2$
(*c*) $a_1a_2 = b_1b_2$ (*d*) none of these

12. Solution of the given matrix equation is

$$\begin{bmatrix} 2 & 2 & 2 \\ 1 & 5 & -3 \\ -1 & -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(*a*) $x_1 = 0, x_2 = 0, x_3 = 0$
(*b*) $x_1 = 1, x_2 = 1, x_3 = 0$
(*c*) $x_1 = 0, x_2 = 1, x_3 = 1$
(*d*) $x_1 = 2, x_2 = -5, x_4 = -1$

13. If product of matrix

$$A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

is a null matrix, then θ and ϕ differ by an

(*a*) odd multiple of π (*b*) even multiple of π
(*c*) odd multiple of π/2 (*d*) even multiple of π/2

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14. Inverse of the matrix $\begin{Bmatrix} ab & 0 \\ 0 & 1 \end{Bmatrix}$ is

(a) $\begin{Bmatrix} a & 0 \\ 0 & a \end{Bmatrix}$ (b) $\begin{Bmatrix} b & 0 \\ a & b \end{Bmatrix}$

(c) $\begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix}$ (d) $\begin{Bmatrix} a & 0 \\ 0 & 1/b \end{Bmatrix}$

15. The system of equations

$$2x + 4y = 10,$$
$$5x + 10y = 25$$

has

(a) no unique solution
(b) only one solution
(c) only two solutions
(d) infinite solutions

16. Sum of the eigen values of the matrix $\begin{pmatrix} 3 & 4 \\ x & 1 \end{pmatrix}$ for real and negative values of x is

(a) greater than zero
(b) less than zero
(c) zero
(d) dependent on the value of x

17. The system of equations

$$4x + 6y + 8z = 0$$
$$7x + 8y + 9z = 0$$
$$3x + 2y + z = 0$$

has

(a) no solution
(b) only one solution
(c) two solutions
(d) infinite number of solutions

18. Eigen values of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are

(a) 0, 0 0 (b) 0, 0 1 (c) 0, 0, 3 (d) 1, 1, 1

19. For following set of simulataneous equations

$$1.5x - 0.5y = 2$$
$$4x + 2y + 3z = 9$$
$$7x + y + 5z = 10$$

(a) solution is unique
(b) infinitely many solutions exist
(c) equations are incompatible
(d) finite number of multiple solutions exist

20. If $R = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}$, then top row of R^{-1} is

(a) [5 6 4] (b) [5 –3 1]
(c) [2 0 –1] (d) [2 –1 1/2]

21. The system of simultaneous equations

$$x + 2y + z = 6$$
$$2x + y + 2z = 6$$
$$x + y + z = 5$$

has

(a) unique solution
(b) infinite number of solutions
(c) no solution
(d) exactly two solutions

22. Eigen vectors of the matrix $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ is (are)

(a) (1, 0) (b) (0, 1) (c) (1, 1) (d) (1, –1)

23. Inverse of the matrix $\begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} \frac{5}{13} & \frac{-1}{13} \\ \frac{2}{13} & \frac{3}{13} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{2}{13} & \frac{5}{13} \\ \frac{-1}{13} & \frac{3}{13} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{-1}{13} & \frac{5}{13} \\ \frac{2}{13} & \frac{3}{13} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{13} & \frac{-5}{13} \\ \frac{2}{13} & \frac{3}{13} \end{bmatrix}$

24. If A and B are square matrices of size $n \times n$, then which of the following statement is not true ?

(a) det (AB) = det (A) det (B)
(b) det (kA) = k^n det (A)
(c) det (A + B) = det (A) + det (B)
(d) det (A^T) = 1/det (A^{-1})

25. If matrix A is $m \times n$ and B is $n \times p$, then number of multiplication operations and addition operations needed to calculate the matrix AB, respectively are

(a) mn^2p, mpn (b) $mpn, (n-1)$
(c) mpn, mpn (d) $mn^2 p, (m+p) n$

26. If A be an invertible matrix and inverse of 7A is $\begin{bmatrix} -1 & 2 \\ 4 & -7 \end{bmatrix}$, then matrix A is

(a) $\begin{bmatrix} 1 & 2/7 \\ 4/7 & 1/7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -4/7 \\ -2/7 & 1/7 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix}$

27. Matrix $B = A^T$, where A is any matrix is

(a) skew symmetric
(b) symmetric about the secondary diagonal
(c) always symmetric
(d) another general matrix

28. If $A^T = A^{-1}$, where A is a real matrix, then A is

(a) normal (b) symmetric
(c) Hermitian (d) orthogonal

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29. If A and B are non-zero squarematrices, then AB = I implies

(*a*) A and B are orthogonal
(*b*) A and B are singular
(*c*) B is singular
(*d*) A is singular

30. Eigen values of a square symmetric matrix are always

(*a*) positive
(*b*) real and imaginary
(*c*) negative
(*d*) real

31. If A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then state transition matrix e^{At} is

(*a*) $\begin{bmatrix} 0 & e^{-t} \\ e^{-t} & 0 \end{bmatrix}$ (*b*) $\begin{bmatrix} e^{t} & 0 \\ 0 & e^{t} \end{bmatrix}$

(*c*) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix}$ (*d*) $\begin{bmatrix} 0 & e^{t} \\ e^{t} & 0 \end{bmatrix}$

32. For matrix $p = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, if one of the eigen values is equal to –2, then which of the following is an eigen vector ?

(*a*) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ (*b*) $\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$

(*c*) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ (*d*) $\begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix}$

33. Eigen values and corresponding eigen vectors of a 2 × 2 matrix are given by

Eigenvalue	Eigenvector
$\lambda_1 = 4$	$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\lambda_2 = 8$	$v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The matrix is

(*a*) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$ (*b*) $\begin{bmatrix} 4 & 6 \\ 6 & 4 \end{bmatrix}$

(*c*) $\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ (*d*) $\begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix}$

34. A second order system starts with an initial condition of $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ without any external input. The state transition matrix for the system is given by $\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix}$. The state of the system at the end of 1 second is given by

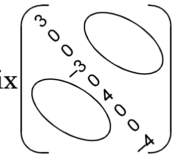
(*a*) $\begin{bmatrix} 0.271 \\ 1.100 \end{bmatrix}$ (*b*) $\begin{bmatrix} 0.135 \\ 0.368 \end{bmatrix}$

(*c*) $\begin{bmatrix} 0.271 \\ 0.736 \end{bmatrix}$ (*d*) $\begin{bmatrix} 0.135 \\ 1.100 \end{bmatrix}$

NUMERICAL TYPE QUESTIONS

1. Rank of the matrix A = $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 0 \\ 1 & 0 & 0 & 0 \\ 4 & 0 & 3 & 0 \end{bmatrix}$ is _______

2. Rank of the diagonal matrix



is _____

3. Rank of the matrix A = $\begin{bmatrix} 1 & 3 & 5 & 1 \\ 2 & 4 & 8 & 0 \\ 3 & 1 & 7 & 5 \end{bmatrix}$ is______

4. Rank of the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ is_______

5. The value of λ for which the equations

$$2x + y + 2z = 0$$
$$x + y + 3z = 0$$
$$4x + y + \lambda z = 0$$

have a non-zero solution, is ________

6. The system of linear equations

$$(4d - 1)\,x + y + z = 0$$
$$-\,y + z = 0$$
$$(4d - 1)\,z = 0$$

has a non-trivial solution, if d equals _____

7. Rank of a 3 × 3 matrix C (= AB), found by multiplying a non-zero column matrix A of size 3 × 1 and a non-zero row matrix B of size 1 × 3, is_____

8. Sum of eigen values of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ is_____

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9. ______value of x, will the matrix $\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$ become singular?

10. Rank of the matrix, $A = \begin{bmatrix} 1 & -2 & -1 \\ -3 & 3 & 0 \\ 2 & 2 & 4 \end{bmatrix}$ is______

11. Solution(s) to the equations

$$2x + 3y = 1$$
$$x - y = 4$$
$$4x - y = a$$

will exist if a is equal to________

12. If $A = \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{1}{2} & a \\ 0 & b \end{bmatrix}$, then (a + b) equals to _________

13. For matrix $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$, eigen value corresponding to the eigen vector $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$ is________

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. Consider the following 2 × 2 matrix A where two elements are unknown and are marked by a and b. The eigen values of this matrix are –1 and 7. What are the values of a and b ?

$$A = \begin{pmatrix} 1 & 4 \\ b & a \end{pmatrix}$$

(*a*) $a = 6, b = 4$ (*b*) $a = 4, b = 6$
(*c*) $a = 3, b = 5$ (*d*) $a = 5, b = 3$

2. In the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$, one of the eigen values is 1. The eigenvectors corresponding to the eigen value 1 are

(*a*) $\{\alpha(4,2,1) | \alpha \neq 0, \alpha \in R\}$

(*b*) $\{\alpha(4,2,1) | \alpha \neq 0, \alpha \in R\}$

(*c*) $\{\alpha(\sqrt{2},0,1) | \alpha \neq 0, \alpha \in R\}$

(*d*) $\{\alpha(-\sqrt{2},0,1) | \alpha \neq 0, \alpha \in R\}$

2014

3. Which one of the following statements is TRUE about every $n \times n$ matrix with only real eigenvalues?

(*a*) If the trace of the matrix is positive and the determinant of the matrix is negative, at least one of its eigenvalues is negative.

(*b*) If the trace of the matrix is positive, all its eigenvalues are positive.

(*c*) If the determinant of the matrix is positive, all its eigenvalues are positive.

(*d*) If the product of the trace and determinant of the matrix is positive, all its eigenvalues are positive.

2013

4. Which one of the following does NOT equal $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$?

(*a*) $\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix}$ (*b*) $\begin{vmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{vmatrix}$

(*c*) $\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$ (*d*) $\begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}$

2012

5. Let A be the 2 × 2 matrix $a_{11} = a_{12} = a_{21} = +1$ and $a_{22} = -1$. Then the eigenvalues of the matrix A^{19} are

(*a*) 1024 and – 1024

(*b*) $1024\sqrt{2}$ and $-1024\sqrt{2}$

(*c*) $4\sqrt{2}$ and $-4\sqrt{2}$

(*d*) $512\sqrt{2}$ and $-512\sqrt{2}$

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2011

6. Consider the matrix as given below.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}$$

Which one of the following options provides the CORRECT values of the eigenvalues of the matrix?

(*a*) 1, 4, 3 (*b*) 3, 7, 3

(*c*) 7, 3, 2 (*d*) 1, 2, 3

7. Four matrices M_1, M_2, M_3 and M_4 of dimensions $p \times q$, $q \times r$, $r \times s$ and $s \times t$ respectively can be multiplied in several ways with different number of total scalar multiplications. For example when multiplied as $((M_1 \times M_2) \times (M_3 \times M_4))$, the total number of scalar multiplications is $pqr + rst + prt$. When multiplied as $(((M_1 \times M_2) \times M_3) \times M_4)$, the total number of scalar multiplications is $pqr + prs + pst$.

If $p = 10$, $q = 100$, $r = 20$, $s = 5$, and $t = 80$, then the minimum number of scalar multiplications needed is

(*a*) 248000 (*b*) 44000

(*c*) 19000 (*d*) 25000

2010

8. Consider the following matrix

$$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$$

If the eigenvalues of A are 4 and 8, then

(*a*) $x = 4, y = 10$ (*b*) $x = 5, y = 8$

(*c*) $x = -3, y = 9$ (*d*) $x = -4, y = 10$

2008

9. The following system of equations

$$x_1 + x_2 + 2x_3 = 1$$
$$x_1 + 2x_2 + 3x_3 = 2$$
$$x_1 + 4x_2 + \alpha x_3 = 4$$

has a unique solution. The only possible value (s) for α is/are

(*a*) 0 (*b*) either 0 or 1

(*c*) one of 0, 1 or – 1 (*d*) any real number

10. How many of the following matrices have an eigenvalue 1?

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

(*a*) one (*b*) two

(*c*) three (*d*) four

2006

11. Consider the polynomial

$$p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3, \text{ where } a_i \neq 0, i.$$

The minimum number of multiplications needed to evaluate p on an input x is

(*a*) 3 (*b*) 4

(*c*) 6 (*d*) 97

NUMERICAL TYPE QUESTIONS

2015

1. In the LU decomposition of the matrix $\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$, if the diagonal elements of U are both 1, then the lower diagonal entry l_{22} of L is ______.

2. The larger of the two eigenvalues of the matrix $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ is _____

3. Perform the following operations on the matrix

$$\begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix}.$$

i. Add the third row to the second row.

ii. Subtract the third column from the first column.

The determinant of the resultant matrix is _____.

2014

4. Consider the following system of equations :

$$3x + 2y = 1$$
$$4x + 7z = 1$$
$$x + y + z = 3$$
$$x - 2y + 7z = 0$$

The number of solutions for this system is ______.

5. The value of the dot product of the eigenvectors corresponding to any pair of different eigen values of a 4-by-4 symmetric positive definite matrix is ____________.

6. There are 5 bags labelled 1 to 5. All the coins in a given bag have the same weight. Some bags have coins of weight 10 gm, others have coins of weight 11 gm. I pick 1, 2, 4, 8, 16 coins respectively from bags 1 to 5. Their total weight comes out to 323 gm. Then the product of the labels of the bags having 11 gm coins is ___.

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7. If the matrix A is such that

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} [1 \quad 9 \quad 5]$$

then the determinant of A is equal to _______.

8. The product of the non-zero eigenvalues of the matrix

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{vmatrix} \text{ is } \_\_\_\_\_\_.$$

ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*c*) **2.** (*a*) **3.** (*a*) **4.** (*a*) **5.** (*d*) **6.** (*b,d*) **7.** (*a*) **8.** (*a*) **9.** (*b*) **10.** (*b*)
11. (*b*) **12.** (*a*) **13.** (*c*) **14.** (*c*) **15.** (*d*) **16.** (*a*) **17.** (*b*) **18.** (*c*) **19.** (*c*) **20.** (*b*)
21. (*c*) **22.** (*c,d*) **23.** (*c*) **24.** (*c*) **25.** (*b*) **26.** (*a*) **27.** (*c*) **28.** (*d*) **29.** (*a*) **30.** (*d*)
31. (*b*) **32.** (*d*) **33.** (*a*) **34.** (*a*)

Numerical Type Questions

1. 3 **2.** 4 **3.** 3 **4.** 1 **5.** 8 **6.** $\frac{1}{4}$ **7.** 1 **8.** 7 **9.** 4 **10.** 2
11. 12 **12.** $\frac{7}{20}$ **13.** 6

EXERCISE – II

MCQ Type Questions

1. (*d*) **2.** (*b*) **3.** (*a*) **4.** (*a*) **5.** (*d*) **6.** (*a*) **7.** (*c*) **8.** (*d*) **9.** (*d*) **10.** (*a*)
11. (*b*)

Numerical Type Questions

1. 5 **2.** 6 **3.** 0 **4.** 1 **5.** 0 **6.** 12 **7.** 0 to 0 **8.** 6 to 6

EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. Characteristic equation is

$$[A - \lambda I] = 0$$

$$\Rightarrow \begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 3-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (3-\lambda)\{(3-\lambda)^2 - 1\} - 1(3-\lambda) + 1\} - 1\{(3-\lambda)+1\} = 0$$

$$\Rightarrow \{(3-\lambda)+1\}\{(3-\lambda)(3-\lambda-1)\} - 2\} = 0$$

$$\lambda = 1, 4, 4$$

$\therefore$ $\lambda =$ **1, 4, 4** are eigen values.

4. $$A' = \begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -5 \\ 3 & 5 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 5 \\ -3 & -5 & 0 \end{bmatrix} = -A.$$

This is a skew-symmetric matrix.

6. Determinant $A = 1(\cos^2\theta - \sin^2\theta)$

Hence A is non-singular and A^{-1} exists.

7. According to *Rouche's theorem,* the system is consistent if and only if coefficient matrix and augmented matrix K are of the same rank, otherwise the system is inconsistent.

8. Let A be a real symmetric matrix, therefore

$$A^T = A$$

Let α_1 and α_2 be different eigen values of the matrix A, and X_1 and X_2 be the corresponding vectors, then

$$AX_1 = \alpha_1 X_1 \quad \text{and} \quad AX_2 = \alpha_2 X_2$$

Taking transpose of the second equation

$$(AX_2)^T = (\alpha_2 X_2)^T$$

$$\Rightarrow \quad X_2^T A^T = \alpha_2 . X_2^T$$

But $\quad A^T = A,$

$$\therefore \quad X_2^T A = \alpha_2 . X_2^T$$

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Post multiply by X_1, we get

$$X^T_2 AX_1 = \alpha_2 X_2^T X_1$$

But $AX_1 = \alpha_1 X_1$

$\therefore$ $X^T_2 \alpha_1 X_1 = \alpha_2 X_2^T X_1$

$\Rightarrow$ $(\alpha_1 - \alpha_2) X_2^T X_1 = 0$

Since $\alpha_1 \neq \alpha_2$, $\alpha_1 - \alpha_2 \neq 0$

$\therefore$ $X_2^T X_1 = 0$

i.e. X_2 and X_1 are orthogonal.

9. Since matrix is triangular, hence eigen values are α, a, b.

If (X_1, X_2, X_3) is an arbitrary eigen vector, say corresponding to 1, then

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = 1 \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

X_2, X_3 being not zero, we have

$$X_1 = X_1 ; aX_2 = X_2$$

which gives $a = 1$

and $bX = X_3$

$\Rightarrow$ $b = 1$

$\therefore$ $(a, b) = (1, 1)$.

10. $A = \begin{bmatrix} 2 & -5k & 6 \\ k & 2 & -2 \\ 2 & 2 & -k \end{bmatrix}$

Equations are consistent, if rank of A and that of k are equal. But in this case it is always true. Hence equations will have a trivial solution if $|A| \neq 0$ Therefore only non-trivial solution will exist if $|A| = 0$

i.e. $\begin{bmatrix} 2 & -5k & 6 \\ k & 2 & -2 \\ 2 & 2 & -k \end{bmatrix} = 0$

$\Rightarrow$ $2(-2k + 4) + 5k(-k^2 + 4) + 6(2k - 4)$

$\Rightarrow$ $-5k^3 + 20k - 4k + 8 + 12k - 24 = 0$

$\Rightarrow$ $5k^3 - 28k + 16 = 0$

$\Rightarrow$ $5k^3 - 10k^2 + 10k^2 - 20k - 8k + 16 = 0$

$\Rightarrow$ $(5k^2 + 10k - 8)(k - 2) = 0$

$$k = \frac{-10 \pm \sqrt{100 + 160}}{2} \text{ or } 2$$

$$= -1 \pm \sqrt{13/5} \text{ or } 2$$

11. $\dfrac{x}{y} = -\dfrac{a_2}{a_1}$ and $\dfrac{x}{y} = -\dfrac{b_2}{b_1}$

Equations are consistent only if

$$-\frac{a_2}{a_1} = -\frac{b_2}{b_1} \Rightarrow a_1 b_2 = b_1 a_2$$

12. Let matrics, $A = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 5 & -3 \\ -1 & -3 & 1 \end{bmatrix}$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ and } B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $AX = B$

Multiplying both sides by A^{-1},

$$X = A^{-1} B$$

But as B = 0; therefore $X = 0$

Hence $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\therefore$ $x_1 = 0, \quad x_2 = 0, \quad x_3 = 0.$

13. AB

$$= \begin{bmatrix} \cos\theta.\cos\phi\cos(\theta-\phi) & \cos\theta.\sin\phi.\cos(\theta-\phi) \\ \cos\phi\sin\theta.\cos(\theta-\phi) & \sin\theta.\sin\phi\cos(\theta-\phi) \end{bmatrix}$$

= A null matrix, when $\cos(\theta - \phi) = 0$

i.e. if $(\theta - \phi)$ is an odd multiple of $\left(\dfrac{\pi}{2}\right)$.

16. Eigen values are given by the solution of equation,

$$\begin{vmatrix} 3-\lambda & 4 \\ x & 1-\lambda \end{vmatrix} = 0$$

Since x is real and negative, put $x = -k$, where k is positive constant

$\therefore$ $(3 - \lambda)(1 - \lambda) + 4k = 0$

$\Rightarrow$ $\lambda^2 - 4\lambda + 3 + 4k = 0$

If λ_1 and λ_2 be the solutions of above equation, then λ_1 and λ_2 are eigen values.

Now, Sum of eigen values

= Sum of roots of above equation

i.e. $\lambda_1 + \lambda_2 = \dfrac{-(-4)}{1} = 4 (> 0)$

17. For given system of equations,

$$\Delta = \begin{vmatrix} 4 & 6 & 8 \\ 7 & 8 & 9 \\ 3 & 2 & 1 \end{vmatrix} = 4(8-18) - 6(7-27) + 8(14-24)$$

$$= -40 + 120 - 80 = 0$$

Since $\Delta = 0$, hence given system of equations has unique solution, *i.e.* only one solution.

18. Given matrix is $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Now $\begin{bmatrix} (1-\lambda) & 1 & 1 \\ 1 & (1-\lambda) & 1 \\ 1 & 1 & (1-\lambda) \end{bmatrix} = 0$

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$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{bmatrix} (3-\lambda) & (3-\lambda) & (3-\lambda) \\ 1 & (1-\lambda) & 1 \\ 1 & 1 & (1-\lambda) \end{bmatrix} = 0$$

$$\Rightarrow \quad (3-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 1 & (1-\lambda) & 1 \\ 1 & 1 & (1-\lambda) \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1$
$C_3 \to C_3 - C_1$

$$\Rightarrow \quad (3-\lambda)\begin{vmatrix} 1 & 0 & 0 \\ 1 & -\lambda & 1 \\ 1 & 0 & -\lambda \end{vmatrix} = 0$$

Hence eigen values are 0, 0, 3

20. $$\text{Adj } R = \begin{bmatrix} 5 & -6 & 4 \\ -3 & 4 & -3 \\ 1 & -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 5 & -3 & 1 \\ -6 & 4 & -1 \\ 4 & -3 & 1 \end{bmatrix}$$

Since, $|R| = 1$, therefore top row $\to [5\ -3\ 1]$

21. Given equations are :

$x + 2y + z = 6$
$2x + y + 2z = 6$
$x + y + z = 5$

Given system can be written as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 5 \end{bmatrix}$$

Applying row operation

$R_2 = R_2 - 2R_1$, $R_3 = R_3 - R_1$, we get

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ -1 \end{bmatrix}$$

Applying $R_3 = 3R_3 - R_2$, we get

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ 3 \end{bmatrix}$$

Since rank of co-efficient matrix is 2 and rank of argument matrix is 3, which is not equal. Hence system has no solution.

22. Eigen value are roots of the equation

$$\begin{bmatrix} 1-\alpha & -1 \\ -1 & 1-\alpha \end{bmatrix} = 0$$

i.e. $\quad (1-\alpha)^2 - 1 = 0$

$\Rightarrow \quad \alpha^2 - 2\alpha = 0$

$\Rightarrow \quad \alpha = 0 \quad \text{or} \quad \alpha = 2$

When $\alpha = 0$, $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This gives, $\quad x = y$

When $\alpha = 2$, $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}.$$

This gives, $\quad x = -y$

Hence, eigen vectors corresponding to 0 and 2 are $\begin{bmatrix} x \\ x \end{bmatrix}$ and $\begin{bmatrix} x \\ -x \end{bmatrix}$, or $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

23. For matrix $\quad A = \begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}$

cofactors are :

$a_{11} = 1, a_{21} = -5, \ a_{12} = -2, a_{22} = -3$

$\therefore$ Matrix by cofactors = $\begin{bmatrix} 1 & -2 \\ -5 & -3 \end{bmatrix}$

$$\text{Adjoint A} = \begin{bmatrix} 1 & -5 \\ -2 & -3 \end{bmatrix}$$

$|A| = -3\,(1) - 5\,(2) = -3 - 10 = -13$

$$\therefore \quad A^{-1} = \frac{\text{Adjoint A}}{|A|} \begin{bmatrix} -\frac{1}{13} & \frac{5}{13} \\ \frac{2}{13} & \frac{3}{13} \end{bmatrix}$$

26. $\quad$ Inverse of $7A = \begin{bmatrix} -1 & 2 \\ 4 & -7 \end{bmatrix}$

$$\det |\Delta| = \begin{vmatrix} -1 & 2 \\ 4 & -7 \end{vmatrix} = 7 - 8 = -1$$

$\therefore$ Matrices of cofactor = $\begin{bmatrix} -7 & -4 \\ -2 & -1 \end{bmatrix}$,

and it's transpose, $\quad A^T = \begin{bmatrix} -7 & -2 \\ -4 & -1 \end{bmatrix}$

$$\therefore \quad \text{Inverse} = \frac{A^T}{|\Delta|} = \begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix}$$

$$\therefore \quad A = \frac{1}{7}\left[\frac{A^T}{|\Delta|}\right] = \begin{bmatrix} 1 & \frac{2}{7} \\ \frac{4}{7} & \frac{1}{7} \end{bmatrix}$$

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31. Here $[s\,I - A]^{-1} = \begin{bmatrix} s-1 & 0 \\ 0 & s-1 \end{bmatrix}^{-1}$

Then, $2^{-1}[sI - A]^{-1} = 2^{-1}\left\{\begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s-1} \end{bmatrix}\right\}$

$= \begin{bmatrix} e^{t} & 0 \\ 0 & e^{t} \end{bmatrix}$

32. $|P - \lambda I| = \begin{vmatrix} 3-\lambda & -2 & 2 \\ 0 & -2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$

$\therefore \quad \lambda = -2, 3, 1$

Putting $\lambda = -2$, we get

$5x - 2y + 2z = 0$

$z = 0$

So. $\quad 5x = 2y$ eigen vector $= \begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix}$

33. Given matrix : $\quad A = \begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$

We know $\quad |\lambda I - A| = 0$

$\Rightarrow \quad \begin{vmatrix} \lambda-6 & -2 \\ -2 & \lambda-6 \end{vmatrix} = 0$

$\Rightarrow \quad \lambda^2 - I2\lambda + 32 = 0$

$\Rightarrow \quad \lambda = 4, 8$ (eigen values)

For $\lambda_1 = 4$,

$(\lambda_1 I - A) = \begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix}, \nu_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

For $\lambda_1 = 8$,

$(\lambda_2 I - A) = \begin{bmatrix} 2 & -6 \\ -6 & 2 \end{bmatrix}, \nu_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

34. State of the system at time t is,

$X(t) = [sI - A]^{-1}X(0) = \phi(t)X(0)$

$= \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2e^{-2t} \\ 3e^{-t} \end{bmatrix}$

At $\quad t = 1, \quad X(1) = \begin{bmatrix} 2e^{-2} \\ 3e^{-1} \end{bmatrix} = \begin{bmatrix} 0.271 \\ 1.100 \end{bmatrix}$

NUMERICAL TYPE QUESTIONS

1. Determinant A = 0

$\Rightarrow \quad \begin{vmatrix} 4 & 2 & 3 \\ 1 & 0 & 0 \\ 4 & 0 & 3 \end{vmatrix} = (-1) \times 6 = -6 \neq 0$

Hence rank of A = 3

2. Since the number of non-zero elements on this diagonal matrix is four, hence the rank is four.

3. Given matrix A possesses a minor of order 3

viz. $\quad \begin{vmatrix} 1 & 3 & 1 \\ 2 & 4 & 0 \\ 3 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ 2 & 4 & 0 \\ -2 & -14 & 5 \end{vmatrix}$

replacing C_1 and C_2 by $C_1 - C_3$ and $C_2 - 3C_3$.

$= \begin{vmatrix} 2 & 4 \\ -2 & -14 \end{vmatrix}$ expanding with respect to R_1

$= 2(-14) - (4)(-2) = -28 + 8 \neq 0$

$\therefore \quad p(A) \geq 3 \qquad \ldots (i)$

Also A does not possess any minor of order 4, *i.e.* 3 + 1

$\therefore \quad p(A) \leq 3 \qquad \ldots (ii)$

From equations (*i*) and (*ii*), we get

$p(A) = 3$ *i.e.* rank of A is 3.

5. Equivalent matrix equation is

$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

In order that given system of equations may have non-zero solution, rank of A should be less than 3. This requires that

$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & \lambda \end{bmatrix} = 0$$

Interchanging R_1 and R_2,

$$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & 2 \\ 4 & 3 & \lambda \end{bmatrix} = 0$$

By $(R_2 - 2R_1)$ and $(R_3 - 4R_1)$,

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & -1 & -4 \\ 0 & -1 & \lambda-12 \end{bmatrix} = 0$$

$\Rightarrow \begin{bmatrix} -1 & -4 \\ -1 & \lambda-12 \end{bmatrix} \Rightarrow -1(\lambda - 12) - (-1)(4) = 0$

$\Rightarrow \quad \lambda = \mathbf{8}$

6. The system of homogeneous linear equations has a non-trivial solution if

$$\begin{bmatrix} 4d-1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 4d-1 \end{bmatrix} = 0$$

$\Rightarrow -(4d-1)^2 = 0 \qquad \Rightarrow d = \frac{1}{4}$

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7. Let $A = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$; $B = \begin{bmatrix} a_2 & b_2 & c_2 \end{bmatrix}$

$\therefore \quad C = \begin{bmatrix} a_1a_2 & a_1b_2 & a_1c_2 \\ b_1a_2 & b_1b_2 & b_1c_2 \\ c_1a_2 & c_2b_2 & c_1c_2 \end{bmatrix}$

$= a_1\, a_2\, [b_1\, b_2\, c_1\, c_2 - b_1 b_2\, c_1 c_2] = 0$

Cofactor $= \begin{vmatrix} a_1a_2 & a_1b_2 \\ b_1a_2 & b_1a_b \end{vmatrix} = 0$

so rank = 1

8. Sum of eigen values of given matrix

= Sum of element of diagonal of given matrix

= 1 + 5 + 1 = 7

9. For singular matrix,

$$\begin{vmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{vmatrix} = 0$$

$\Rightarrow \quad 8[\,0 - 12] - x\,[\,0 - 2 \times 12] = 0$

$\Rightarrow \quad -96 + 24x = 0$

$\Rightarrow \quad x = 4$

10. Since, *det* $A = \begin{vmatrix} 1 & -2 & -1 \\ -3 & 3 & 0 \\ 2 & 2 & 4 \end{vmatrix} = 12 - 24 + 12 = 0$

and $M_{11} = 12 \neq 0$. Hence rank is 2.

11. Equations are consistent when

$$\begin{vmatrix} 2 & 3 & -1 \\ 1 & -1 & -4 \\ 4 & -1 & -a \end{vmatrix} = 0$$

$\Rightarrow \quad 2(a - 4) - 3\,(-a + 16) - 1\,(-1 + 4) = 0$

$5a - 8 - 48 - 3 = 0$

$5a = 59$

$a = \dfrac{59}{5} \approx 12$

12. We know, $AA^{-1} = I$

$\therefore \quad \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & a \\ 0 & b \end{bmatrix} = \begin{bmatrix} 1 & 2a - 0.1b \\ 0 & 3b \end{bmatrix}$

$\Rightarrow \quad 3b = 1 \qquad \Rightarrow \quad b = \dfrac{1}{3}$

or $\quad 2a - 0.1b = 0$

$\Rightarrow \quad 2a = \dfrac{b}{10} \qquad \Rightarrow \quad a = \dfrac{1}{60}$

$\therefore \quad a + b = \dfrac{1}{3} + \dfrac{1}{60} = \dfrac{7}{20}$

13. Given : Matrix $M = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

$\therefore \quad M - \lambda I = \begin{bmatrix} 4-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix}$

For the eigen vector $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$

$(4 - \lambda)\,(101) + 2\,(101) = 0$

$\Rightarrow \quad 4 - \lambda + 2 = 0 \quad \Rightarrow \quad \lambda = 6$ (eigen value)

EXERCISE – II

MCQ TYPE QUESTIONS

1. Sum of Eigen value $= 1 + a$

$\therefore \quad 6 = 1 + a$

$\therefore \quad \boxed{a = 5}$

and product of Eigen value = det (A)

$-7 = a - 4b$

$\therefore \quad -7 = 5 - 4b$

$\therefore \quad -7 - 5 = -4b$

$\therefore \quad \boxed{b = 3}$

$\therefore$ Option (*d*) is correct.

2. X be an eigen vector corresponding to eigen value $\lambda = 1$, then $AX = \lambda X \Rightarrow (A - I)X = 0$

$$\begin{bmatrix} 0 & -1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = 0$$

$\Rightarrow \quad -y + 2z = 0 \quad$ and $\quad x + 2y = 0$

$\Rightarrow \quad y = 2z \quad$ and $\quad \dfrac{x}{-2} = y$

$\therefore \quad \dfrac{x}{-2} = y = 2z$

$\Rightarrow \quad \dfrac{x}{-4} = \dfrac{y}{2} = \dfrac{Z}{1} = \alpha$ (say)

$\Rightarrow \quad X = \begin{pmatrix} -4 \\ 2 \\ 1 \end{pmatrix} \alpha;\ \alpha \neq 0$

$\therefore$ Eigen vectors are $\{\alpha(-4, 2, 1) | \alpha \neq 0, \alpha \in R|\}$

3. If the trace of the matrix is positive and the determinant of the matrix is negative then atleast one of its eigen values is negative.

Since determinant = product of eigen values.

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4. Given :

$$\text{A matrix} \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

To find: Which of the options are not equal to given matrix?

Analysis: Lets try to transform over the given matrix.

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \xrightarrow{C_2 \to C_2 + C_1} \begin{vmatrix} 1 & x+1 & x^2 \\ 1 & y+1 & y^2 \\ 1 & z+1 & z^2 \end{vmatrix}$$

$$\text{option } (b) : \to \begin{vmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{vmatrix} \; C_3 \to C_3 + C_1$$

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \xrightarrow[R_2 \to R_2 + R_3]{R_1 \to R_1 + R_2} \begin{vmatrix} 1 & x+y & x^2+y^2 \\ 1 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}_{:\to \text{option (D)}}$$

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 - R_2} \begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}_{\text{option (C)}}$$

We are able to generate option B, C, and D from given matrix.

Hence, option a can be the possible solution, so lets try to operate on option A to see if we can prove it.

$$\begin{vmatrix} . & 0(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix} \to C_3 \to C_3 - C_1$$

$$\begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix} \xleftarrow{C_2 \to C_2 - C_3} \begin{vmatrix} 1 & x^2+x & x \\ 1 & y^2+y & y \\ 1 & z^2+z & z \end{vmatrix}$$

We know, if we interchange any two columns of a matrix its determinan gets multiplied by '–1'.

Option (a) is formed from given matrix by interchanging column 2 and 3

$$\text{Hence} \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = -\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix}$$

Hence (a) is the answer.

5. From the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} A^2 = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 2I$$

Where $I = 2 \times 2$ unit matrix

So $A^{19} = (A^2)^9 . A$

$= (2I)^9 A = 2^9 A$

Now, to get Eigen values, we must solve the equation.

$$[\lambda I - A^{19}] = 0$$

$$\begin{vmatrix} \lambda - 2^9 & -2^9 \\ -2^9 & \lambda + 2^9 \end{vmatrix} = 0$$

$\Rightarrow \quad \lambda^2 - 2^{18} - 2^{18} = 0$

$\Rightarrow \quad \lambda = \pm 512\sqrt{2}$

6. Given matrix is upper triangular matrix and its diagonal elements are its eigen values

$= 1, 4, 3$

7. Multiply as $(M_1 \times (M_2 \times M_3)) \times M_4$

Total number of scalar multiplication is

$= qrs + pqs + pst$

$= 10000 + 5000 + 4000 = 19000$

8. Given : $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$

$\therefore \quad [A - \lambda I] = 0$

where, I = unit matrix

$$\therefore \quad \begin{bmatrix} 2-\lambda & 3 \\ x & y-\lambda \end{bmatrix} = 0$$

$\Rightarrow \quad (2 - \lambda)(y - \lambda) - 3x = 0$

Given eign values, $\lambda = 4, 8$

When $\lambda = 4$,

$-2(y - 4) - 3x = 0$

$\Rightarrow \quad 3x + 2y = 8$ ...(i)

When $\lambda = 8$,

$-6(y - 8) - 3x = 0$

$\Rightarrow \quad x + 2y = 16$ ...(ii)

Solving equations (i) and (ii), we get

$x = -4, y = 10$

9. For unique solution, $\Delta \neq 0$

$$\Delta_1 = \Delta_2 = \Delta_3 = 0$$

$$\Delta = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 4 & \alpha \end{bmatrix} \neq 0$$

$$\Delta_1 = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 3 \\ 4 & 4 & \alpha \end{bmatrix} = 0$$

$\Rightarrow \quad 0 = 0$ (columns same)

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so, can't be determined by Δ_1

$$\Delta_2 = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 4 & \alpha \end{bmatrix} = \Delta,$$

so, can't be equated to zero

otherwise $0 = 0$

$$\Delta_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 4 & 4 \end{bmatrix} = 0$$

No α

10. (*i*) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$\therefore \begin{bmatrix} 1-\lambda & 0 \\ 0 & -\lambda \end{bmatrix} = 0$$

$$\Rightarrow (1-\lambda)(-\lambda) = 0$$

Hence eigen values are

$$\lambda = 0, 1$$

(*ii*) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$\therefore \begin{bmatrix} 0-\lambda & 1 \\ 0 & -\lambda \end{bmatrix} = 0$$

$$\Rightarrow (-\lambda)(-\lambda) = 0$$

Hence eigen values are $\lambda = 0, 0$

(*iii*) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$$\therefore \begin{bmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (1-\lambda)^2 + 1 = 0$$

$$\lambda^2 - 2\lambda + 2 = 0$$

$$\Rightarrow \lambda \neq 1$$

(*iv*) $\begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$

$$\therefore \begin{bmatrix} -1-\lambda & 0 \\ 1 & -1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (-1-\lambda)(-1-\lambda) - 1 = 0$$

$$\Rightarrow (1+\lambda)^2 - 1 = 0$$

$$\Rightarrow \lambda^2 + 2\lambda + = 0$$

Hence eigen values are $\lambda = 0, -2$

11. The expression is $a_0 + a_1x + a_2x^2 + a_3x^3$

The deciding term is the last term which requires most number of multiplications.

For any value of x, the computation of $a_3 \times x \times x \times x$ is essential.

Hence minimum number of multiplications is 3.

NUMERICAL TYPE QUESTIONS

2. The given matrix is

$$A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$$

Inorder to find eigen values of above matrix

$$|A - \lambda I| = 0$$

$$\therefore \begin{vmatrix} 4-\lambda & 5 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$\therefore$ $(4-\lambda)(1-\lambda) - 10 = 0$

$\therefore$ $4 - 5\lambda + \lambda^2 - 10 = 0$

$\therefore$ $\lambda^2 - 5\lambda - 6 = 0$

$\therefore$ $\lambda^2 - 6\lambda + 1\lambda - 6 = 0$

$\therefore$ $\lambda(\lambda - 6) + 1(\lambda - 6) = 0$

$\therefore$ $\lambda + 1 = 0$ or λ

$\therefore$ $\lambda = -1$ or $\lambda = 6$

$\therefore$ Larger eigen value of matrix is 6

3. The given matrix is

$$A = \begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix}$$

Now according to question

(*i*) $R_2 \to R_2 + R_3$ (Adding third row to the second row)

$$\therefore B = \begin{bmatrix} 3 & 4 & 45 \\ 20 & 11 & 300 \\ 13 & 2 & 195 \end{bmatrix}$$

(*ii*) $C_1 \to C_1 - C_3$ (subtracting third column from first column)

$$\therefore C = \begin{bmatrix} -42 & 4 & 45 \\ -280 & 11 & 300 \\ -182 & 2 & 195 \end{bmatrix}$$

Now $|C| = -42(2145 - 600) - 4(-54600 + 54600) + 45(-560 + 2002)$

$$\therefore |C| = -42(1545) - 4(0) + 45(1442)$$

$$= -64890 + 64890 = 0$$

4. $3x + 2y = 1$

$4x + 7z = 1$

$x + y + z = 3$

$x - 2y + 7z = 0$

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Augmented matrix is $\begin{bmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 7 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & -2 & 7 & 0 \end{bmatrix}$

$R_1 \leftrightarrow R_3 \begin{bmatrix} 1 & 1 & 1 & 3 \\ 4 & 0 & 7 & 1 \\ 3 & 2 & 0 & 1 \\ 1 & -2 & 7 & 0 \end{bmatrix}$

$R_2 \to R_2 - 4R_1\ R_3 \to R_3 - 3R_1,\ R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -4 & 3 & -11 \\ 0 & -1 & -3 & -8 \\ 0 & -3 & 6 & -3 \end{bmatrix}$$

$$\begin{matrix} R_3 \to 4R_3 - R_2 \\ R_4 \to 4R_4 - 3R_2 \end{matrix} \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -4 & 3 & -11 \\ 0 & 0 & -15 & -21 \\ 0 & 0 & 15 & 21 \end{bmatrix}$$

$$R_4 \to R_4 + R_3 \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -4 & 3 & -11 \\ 0 & 0 & -15 & -21 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

ρ (A : B) = ρ(A) = 3 = no. of variables

$\therefore$ Unique solution exists

5. The eigen vectors corresponding to distinct eigen values of real symmetric matrix are orthogonal.

6. Let the weight of coins in the respective bags (1 through 5) be a,b,c,d and e-each of which can take one of two values namely 10 or 11 (gm). Now, the given information on total weight can be expressed as the following equation:

$1.a + 2.b + 4.c + 8.d + 16.e = 323$

$\Rightarrow$ a must be odd $\Rightarrow a = 11$

The equation then becomes:

$11 + 2.b + 4.c + 8.d + 16.e = 323$

$\Rightarrow \quad 2.b + 4.c + 8.d + 16.e = 312$

$\Rightarrow \quad b + 2.c + 4.d + 8.e = 156$

$\Rightarrow \quad b$ must be even $\Rightarrow b = 10$

The equation then becomes:

$10 + 2.c + 4.d + 8.e = 156$

$\Rightarrow \quad 2.c + 4.d + 8.e = 146$

$\Rightarrow \quad c + 2.d + 4.e = 73$

$\Rightarrow \quad c$ must be odd $\Rightarrow c = 11$

The equation now becomes:

$11 + 2.d + 4.e = 73$

$\Rightarrow \quad 2.d + 4.e = 62$

$\Rightarrow \quad d + 2.e = 31$

$\Rightarrow \quad e = 11$ and $e = 10$

Therefore, bags labelled 1, 3 and 4 contain 11 gm coins

$\Rightarrow$ Required Product = 1*3*4* = 12.

7. $A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} [1\ 9\ 5]$

$A = \begin{bmatrix} 2 & 18 & 10 \\ -4 & -36 & -20 \\ 7 & 63 & 35 \end{bmatrix}$

$$|A| = 2(-36 \times 35 + 20 \times 63) - 18(-4 \times 35 + 140) + 10(-4 \times 63 + 36 \times 7)$$
$$= 2(-1260 + 1260) - 18(-140 + 140) + 10(-252 + 252) = 0$$

8. $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$

$Am = Xm \Rightarrow x_1 + x_5 = m\,x_1 = m\,x_5$

$\Rightarrow x_2 + x_3 + x_4 = m\,x_2 = m\,x_4$

For $m \neq 0$

Say, $x_1 = x_5 = a$

$x_2 = x_3 = x_4 = b$

$\Rightarrow \quad x_1 + x_5 = m\,x_1$

$\Rightarrow \quad 2a = m\,a$

$\Rightarrow \quad m = 2$

$\Rightarrow \quad x_2 + x_3 + x_4 = \mathrm{m}\,x_2$

$\therefore \quad 3b = mb$

$\therefore \quad m = 3$

For $m = 0 \Rightarrow$ eigenvalue $m = 0$

$\therefore$ There are 3 distinct eigen value = 0, 2, 3.

Product of non zero eigen value = 2 × 3 = 6.

■■

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2
CHAPTER

Calculus

FUNCTIONS OF SINGLE VARIABLE

LIMITS.

By limit we are able to find out behaviour of a function $f(x)$ near a point c, when distance between x and c is very small i.e. $|x-c|$ is small.

Specifically, we would like to know whether there is some real number k such that $f(x)$ approaches k as x approaches c.

We write as $\lim_{x\to c} f(x) = k$

Right hand and Left hand limits

Left-hand limit: $\lim_{x\to c^-} f(x) = k$

Right-hand limit: $\lim_{x\to c^+} f(x) = k$

If $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$, then limit of $f(x)$ as $x \to c$ exists and is written as $\lim_{x\to c} f(x)$.

The limit of a function f(x) may not exist in any of the following situations :

(*i*) $\lim_{x\to c^-} f(x)$ does not exist

(*ii*) $\lim_{x\to c^+} f(x)$ does not exist

(*iii*) Both $\lim_{x\to c^-} f(x)$, and $\lim_{x\to c^+} f(x)$ exist but are unequal

Theorems on Limits

If $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist, then

(1) $\lim_{x\to c} \left(f(x) \pm g(x)\right) = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$

(2) $\lim_{x\to c} \left(f(x).g(x)\right) = \lim_{x\to c} f(x).\lim_{x\to c} g(x)$

(3) $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$ where $\left(\lim_{x\to c} g(x) \neq 0\right)$

Some useful Limits

(1) $\lim_{x\to 0} \dfrac{\sin x}{x} = 1 = \lim_{x\to 0} \cos x = \lim_{x\to 0} \dfrac{\tan x}{x}$

(2) $\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e = \lim_{x\to\infty} \left(1+\dfrac{1}{x}\right)^x$

(3) $\lim_{x\to 0} \dfrac{\log_a (1+x)}{x} = \log_a e \quad (a > 0, a \neq 0)$

(4) $\lim_{x\to 0} \dfrac{\log(1+x)}{x} = 1$

(5) $\lim_{x\to 0} \dfrac{a^x - 1}{x} = \log a \quad (a > 0)$

(6) $\lim_{x\to 0} \dfrac{(1+x)^m - 1}{x} = m$

(7) $\lim_{x\to\infty} \dfrac{\log x}{x^m} = 0 \quad (m > 0)$

(8) $\lim_{x\to 0} \dfrac{x^n - a^n}{x - a} = na^{n-1}$

Theorem (L'Hospital's Rule) : Suppose f and g are differentiable and $g'(x) \neq 0$ near a (except possibly at a). Suppose that

$$\lim_{x\to a} f(x) = 0 \text{ and } \lim_{x\to a} g(x) = 0$$

or $\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$

(In other words, we have an indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$.) Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

if the limit on the right side exists (or is ∞ or $-\infty$)

Indeterminate Forms of Type $\frac{0}{0}$ and $\frac{\infty}{\infty}$

Examples :

1. Find $\lim_{x\to\infty} \dfrac{5x-2}{7x+3}$

Solution :

We have, $\lim_{x\to\infty} \dfrac{5x-2}{7x+3} \quad \left[\dfrac{\infty}{\infty} \text{ form}\right]$

$$= \lim_{x\to\infty} \frac{\frac{5x-2}{x}}{\frac{7x+3}{x}}$$

$$= \lim_{x\to\infty} \frac{\frac{5x}{x} - \frac{2}{x}}{\frac{7x}{x} + \frac{3}{x}}$$

$$= \lim_{x\to\infty} \frac{5 - \frac{2}{x}}{7 + \frac{3}{x}} = \frac{5-0}{7+0} = \frac{5}{7}$$

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2. Find $\lim\limits_{x\to\infty} \dfrac{x^5+x^4+x^3+x^2+x+1}{2x^5+x^4+x^3+x^2+x+1}$

Solution : We have

$$\lim_{x\to\infty} \frac{x^5+x^4+x^3+x^2+x+1}{2x^5+x^4+x^3+x^2+x+1} \quad \left[\frac{\infty}{\infty}\text{form}\right]$$

$$= \lim_{x\to\infty} \frac{\dfrac{x^5+x^4+x^3+x^2+x+1}{x^5}}{\dfrac{2x^5+x^4+x^3+x^2+x+1}{x^5}}$$

$$= \lim_{x\to\infty} \frac{\dfrac{x^5}{x^5}+\dfrac{x^4}{x^5}+\dfrac{x^3}{x^5}+\dfrac{x^2}{x^5}+\dfrac{x}{x^5}+\dfrac{1}{x^5}}{\dfrac{2x^5}{x^5}+\dfrac{x^4}{x^5}+\dfrac{x^3}{x^5}+\dfrac{x^2}{x^5}+\dfrac{x}{x^5}+\dfrac{1}{x^5}}$$

$$= \lim_{x\to\infty} \frac{1+\dfrac{1}{x}+\dfrac{1}{x^2}+\dfrac{1}{x^3}+\dfrac{1}{x^4}+\dfrac{1}{x^5}}{2+\dfrac{1}{x}+\dfrac{1}{x^2}+\dfrac{1}{x^3}+\dfrac{1}{x^4}+\dfrac{1}{x^5}}$$

$$= \frac{1+0+0+0+0+0}{2+0+0+0+0+0}$$

$$= \frac{1}{2}$$

3. Find $\lim\limits_{x\to-2} \dfrac{x+2}{\ln(x+3)}$

Solution : We have

$$\lim_{x\to-2} \frac{x+2}{\ln(x+3)} \left[\frac{0}{0}\text{form}\right]$$

$$= \lim_{x\to-2} \frac{(x+2)'}{(\ln(x+3))'}$$

$$= \left\{ \lim_{x\to-2} \frac{(x+2)'}{\frac{1}{x+3}\cdot(x+3)'} = \lim_{x\to-2} \frac{x'+2'}{\frac{1}{x+3}\cdot(x'+3')} = \lim_{x\to-2} \frac{1+0}{\frac{1}{x+3}\cdot(1+0)} \right\}$$

$$= \lim_{x\to-2} \frac{1}{\dfrac{1}{x+3}}$$

$$= \lim_{x\to-2} \frac{1\cdot(x+3)}{\dfrac{1}{x+3}\cdot(x+3)}$$

$$= \lim_{x\to-2} \frac{x+3}{1}$$

$$= \lim_{x\to-2} (x+3)$$

$$= -2+3 = 1$$

4. Find $\lim\limits_{x\to\infty} \dfrac{3^x}{x^2+x-1}$

Solution : We have

$$\lim_{x\to\infty} \frac{3^x}{x^2+x-1} \left[\frac{\infty}{\infty}\text{form}\right]$$

$$= \lim_{x\to\infty} \frac{(3^x)'}{(x^2+x-1)'}$$

$$= \lim_{x\to\infty} \frac{3^x\ln 3}{2x+1} \left[\frac{\infty}{\infty}\text{form}\right]$$

$$= \lim_{x\to\infty} \frac{(3^x\ln 3)'}{(2x+1)'}$$

$$= \lim_{x\to\infty} \frac{\ln 3(3^x)'}{2}$$

$$= \lim_{x\to\infty} \frac{\ln 3\cdot 3^x\cdot\ln 3}{2} = \infty \text{ (D.N.E.)}$$

5. Find $\lim\limits_{x\to\infty} \dfrac{\ln x}{\sqrt{x}}$

Solution : We have

$$\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} \left[\frac{\infty}{\infty}\text{form}\right]$$

$$= \lim_{x\to\infty} \frac{(\ln x)'}{(x^{1/2})'}$$

$$= \lim_{x\to\infty} \frac{x^{-1}}{\frac{1}{2}x^{-1/2}}$$

$$= \lim_{x\to\infty} \frac{x^{-1}\cdot x}{\frac{1}{2}x^{-1/2}\cdot x} = \lim_{x\to\infty} \frac{1}{\frac{1}{2}x^{1/2}} = 0$$

6. Find $\lim\limits_{x\to\infty} \dfrac{x+\sin x}{x+\cos x}$

Solution : We have

$$\lim_{x\to\infty} \frac{x+\sin x}{x+\cos x} \left[\frac{\infty}{\infty}\text{form}\right]$$

$$= \lim_{x\to\infty} \frac{\dfrac{x+\sin x}{x}}{\dfrac{x+\cos x}{x}}$$

$$= \lim_{x\to\infty} \frac{\dfrac{x}{x}+\dfrac{\sin x}{x}}{\dfrac{x}{x}+\dfrac{\cos x}{x}}$$

$$= \lim_{x\to\infty} \frac{1+\dfrac{\sin x}{x}}{1+\dfrac{\cos x}{x}}$$

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It is easy to show that

$$\lim_{x\to\infty}\frac{\sin x}{x} = 0$$

and $$\lim_{x\to\infty}\frac{\cos x}{x} = 0$$

$$\lim_{x\to\infty}\frac{x+\sin x}{x+\cos x} = \lim_{x\to\infty}\frac{1+\frac{\sin x}{x}}{1+\frac{\cos x}{x}}$$

$$= \frac{1+0}{1+0} = 1$$

Indeterminate Forms of Type $\infty - \infty$ and $0 \cdot \infty$

Examples :

1. Find $\lim_{x\to\infty}(x-\ln x)$.

Solution : We have

$$\lim_{x\to\infty}(x-\ln x)\ [(\infty-\infty)\text{ form}]$$

$$= \lim_{x\to\infty}\left(x\cdot 1 - x\cdot\frac{\ln x}{x}\right)$$

$$= \lim_{x\to\infty} x\left(1-\frac{\ln x}{x}\right)$$

Note that $$\lim_{x\to\infty}\frac{\ln x}{x}\ \left[\frac{\infty}{\infty}\text{form}\right]$$

$$= \lim_{x\to\infty}\frac{(\ln x)'}{x'}$$

$$= \lim_{x\to\infty}\frac{x^{-1}}{1}$$

$$= \lim_{x\to\infty}\frac{1}{x} = 0$$

$$\therefore \quad \lim_{x\to\infty}(x-\ln x) = \lim_{x\to\infty} x\left(1-\frac{\ln x}{x}\right)$$

$$= \lim_{x\to\infty} x(1-0) = \infty$$

2. Find $\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$.

Solution : We have

$$\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)[(\infty-\infty)\text{ form}]$$

$$= \lim_{x\to 1}\left(\frac{1\cdot(x-1)}{\ln x\cdot(x-1)}-\frac{\ln x\cdot 1}{\ln x\cdot(x-1)}\right)$$

$$= \lim_{x\to 1}\frac{x-1-\ln x}{\ln x(x-1)}\ \left[\frac{0}{0}\text{form}\right]$$

$$= \lim_{x\to 1}\frac{(x-1-\ln x)'}{(\ln x(x-1))'}$$

$$= \lim_{x\to 1}\frac{x'-1'-(\ln x)'}{(\ln x)'\cdot(x-1)+\ln x\cdot(x-1)'}$$

$$= \lim_{x\to 1}\frac{1-\frac{1}{x}}{\frac{x-1}{x}+\ln x}$$

$$= \lim_{x\to 1}\frac{\left(1-\frac{1}{x}\right)x}{\left(\frac{x-1}{x}+\ln x\right)x}$$

$$= \lim_{x\to 1}\frac{1\cdot x-\frac{1}{x}\cdot x}{\frac{x-1}{x}\cdot x+\ln x\cdot x}$$

$$= \lim_{x\to 1}\frac{x-1}{x-1+x\ln x}\ \left[\frac{0}{0}\text{form}\right]$$

$$= \lim_{x\to 1}\frac{(x-1)'}{(x-1+x\ln x)'}$$

$$= \lim_{x\to 1}\frac{x'-1'}{x'-1'+x'\ln x+x(\ln x)'}$$

$$= \lim_{x\to 1}\frac{1-0}{1-0+1\cdot\ln x+x\cdot\frac{1}{x}}$$

$$= \lim_{x\to 1}\frac{1}{2+\ln x}$$

$$= \frac{1}{2+0} = \frac{1}{2}$$

3. Find $\lim_{x\to\infty} x^{1/x}$

Solution :

Note that $\lim_{x\to\infty} x^{1/x}$ is ∞^0 type of an indeterminate form.

Put $$y = x^{1/x}$$

then $$\ln y = \ln x^{1/x}$$

$$= \frac{1}{x}\ln x = \frac{\ln x}{x}$$

We have $$\lim_{x\to\infty}\frac{\ln x}{x}\ \left[\frac{\infty}{\infty}\text{form}\right]$$

$$= \lim_{x\to\infty}\frac{(\ln x)'}{x'}$$

$$= \lim_{x\to\infty}\frac{x^{-1}}{1}$$

$$= \lim_{x\to\infty}\frac{1}{x} = 0$$

$$\therefore \quad \lim_{x\to\infty} x^{1/x} = e^0 = 1$$

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4. Find $\lim\limits_{x\to\pi/2}(\tan x)^{2x-\pi}$

Solution :

Note that $\lim\limits_{x\to\pi/2}(\tan x)^{2x-\pi}$ is ∞^0 type of an indeterminate form. Put

$$y = (\tan x)^{2x-\pi}$$

Then $\ln y = \ln\left((\tan x)^{2x-\pi}\right)$

$$= (2x-\pi)\ln(\tan x)$$

$$= \left\{\frac{(2x-\pi)\ln(\tan x)}{1} = \frac{(2x-\pi)^{-1}\cdot(2x-\pi)\ln(\tan x)}{(2x-\pi)^{-1}\cdot 1}\right\}$$

$$= \frac{\ln(\tan x)}{(2x-\pi)^{-1}}$$

We have $\lim\limits_{x\to\pi/2}\dfrac{\ln(\tan x)}{(2x-\pi)^{-1}}\ \left[\dfrac{\infty}{\infty}\text{form}\right]$

$$= \lim_{x\to\pi/2}\frac{[\ln(\tan x)]'}{\left[(2x-\pi)^{-1}\right]'}$$

$$= \lim_{x\to\pi/2}\frac{\frac{1}{\tan x}\cdot(\tan x)'}{(-1)(2x-\pi)^{-2}\cdot(2x-\pi)'}$$

$$= \lim_{x\to\pi/2}\frac{\frac{1}{\tan x}\cdot\sec^2 x}{(-1)(2x-\pi)^{-2}\cdot 2}$$

$$= -\frac{1}{2}\lim_{x\to\pi/2}\frac{\frac{\frac{1}{\sin x}}{\cos x}\cdot\frac{1}{\cos^2 x}}{(2x-\pi)^{-2}}$$

$$= -\frac{1}{2}\lim_{x\to\pi/2}\frac{\frac{1}{\frac{\sin x}{\cos x}\cdot\cos^2 x}}{(2x-\pi)^{-2}}$$

$$= -\frac{1}{2}\lim_{x\to\pi/2}\frac{\frac{1}{\sin x\cos x}}{(2x-\pi)^{-2}}$$

$$= -\frac{1}{2}\lim_{x\to\pi/2}\frac{\frac{1}{\sin x\cos x}\cdot\sin x\cos x(2x-\pi)^2}{(2x-\pi)^{-2}\cdot\sin x\cos x(2x-\pi)^2}$$

$$= -\frac{1}{2}\lim_{x\to\pi/2}\frac{(2x-\pi)^2}{\sin x\cos x}$$

$$= -\frac{1}{2}\lim_{x\to\pi/2}\frac{(2x-\pi)^2}{\sin\left(\frac{\pi}{2}\right)\cos x}$$

$$= -\frac{1}{2}\lim_{x\to\pi/2}\frac{(2x-\pi)^2}{\cos x}\ \left[\frac{0}{0}\text{form}\right]$$

$$= -\frac{1}{2}\lim_{x\to\pi/2}\frac{\left[(2x-\pi)^2\right]'}{(\cos x)'}$$

$$= -\frac{1}{2}\lim_{x\to\pi/2}\frac{2(2x-\pi)\cdot(2x-\pi)'}{-\sin x}$$

$$= -\frac{1}{2}\lim_{x\to\pi/2}\frac{2(2x-\pi)\cdot 2}{-\sin x}$$

$$= -\frac{1}{2}\cdot\frac{2\cdot\left(2\cdot\frac{\pi}{2}-\pi\right)\cdot 2}{-\sin\frac{\pi}{2}} = 0$$

Therefore $\lim\limits_{x\to\pi/2}(\tan x)^{2x-\pi} = e^0 = 1$

5. Find $\lim\limits_{x\to 0^+} x^x$

Solution :

Note that $\lim\limits_{x\to 0^+} x^x$ is 0^0 type of indeterminate form.

Put $y = x^x$

then $\ln y = \ln x^x = x\ln x$

$$= \left\{\frac{x\ln x}{1} = \frac{x^{-1}\cdot x\ln x}{x^{-1}\cdot 1}\right\}$$

$$= \frac{\ln x}{x^{-1}}$$

We have $\lim\limits_{x\to 0^+}\dfrac{\ln x}{x^{-1}}\ \left[\dfrac{\infty}{\infty}\text{form}\right]$

$$= \lim_{x\to 0^+}\frac{(\ln x)'}{(x^{-1})'}$$

$$= \lim_{x\to 0^+}\frac{x^{-1}}{-x^{-2}}$$

$$= \lim_{x\to 0^+}\frac{x^{-1}\cdot x^2}{-x^{-2}\cdot x^2}$$

$$= \lim_{x\to 0^+}\frac{x}{-1}$$

$$= \frac{0}{-1} = 0$$

Therefore $\lim\limits_{x\to 0^+} x^x = e^0 = 1$

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6. $\lim_{x\to 0} \frac{\sin x}{x} = \frac{0}{0}$

By L' Hospital Rule

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{x} = \frac{1}{1} = 1$$

7. $\lim_{x\to 0} \frac{x^2 + x - \sin x}{x^2} = \frac{0}{0}$

By L' Hospital Rule

$$\Rightarrow \lim_{x\to 0} \frac{x^2 + x - \sin x}{x^2} = \lim_{x\to 0} \frac{2x + 1 - \cos x}{2x} = \frac{0}{0}$$

$$\Rightarrow \lim_{x\to 0} \frac{x^2 + x - \sin x}{x^2} = \lim_{x\to 0} \frac{2 + \sin x}{2} = 1$$

8. $\lim_{x\to 0} \frac{x^n}{x^m} = \frac{0}{0}$

By L' Hospital Rule

$$\lim_{x\to 0} \frac{x^n}{x^m} = \frac{nx^{n-1}}{mx^{m-1}} = \frac{0}{0}$$

$$\Rightarrow \quad \lim_{x\to 0} \frac{x^n}{x^m} = \frac{n(n-1)x^{n-2}}{m(m-1)x^{m-2}} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{x^n}{x^m} = \frac{n(n-1).......1}{m(m-1).....1} = \frac{n!}{m!}$$

so on

CONTINUITY AND DISCONTINUITY

Continuity

- The function f is said to be *continuous* at $x = c$ if $\lim_{x\to c} f(x) = f(c)$
- The function f is said to be continuous on an open interval (a, b) if it is continuous at each point of (a,b)
- The function is said to be continuous on a closed interval $[a, b]$ if
  - (*i*) f is continuous at each point of (a, b)
  - (*ii*) $\lim_{x\to a^+} f(x) = f(a)$
  - (*iii*) $\lim_{x\to b^-} f(x) = f(b)$

Note : Graphically, $f(x)$ is continuous if its graph can be drawn on paper without raising the pencil.

Discontinuity

A function f is said to be discontinuous at c if it is not continuous at that point. A function f is said to be discontinuous on an interval if it is discontinuous at least one point of the interval.

A function is discontinuous at c under any of the following circumstances:

(*i*) f is not defined at c

(*ii*) $\lim_{x\to c} f(x)$ does not exist

(*iii*) f is defined at c and $\lim_{x\to c} f(x)$ exists but $\lim_{x\to c} f(x) \neq f(c)$

Note : If f and g are continuous at $x = c$, then $f + g$, $f.g$, αf, and $\frac{f}{g}$ is also continuous at $x = c$ $\quad (g(c) \neq 0)$

DIFFERENTIABILITY

A function $f(x)$ is said to be differentiable at $x = a$ if

both $\quad \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$, $h > 0$

and $\quad \lim_{h\to 0} \frac{f(a-h) - f(a)}{-h}$, $h > 0$

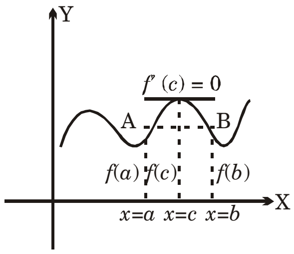
exist and have common value (finite). This common value is called *derivative of* $f(x)$ at the point $x = a$ and is denoted by $f'(a)$.

MEAN VALUE THEOREMS.

1. Rolle's theorem

If a function $f(x)$ is continuous in closed interval $[a, b]$, derivable in the open interval (a, b) and $f(a) = f(b)$, then there exists atleast one real c in (a, b) such that $f'(c) = 0$.

2. Lagrange's mean value theorem.



If $f(x)$ be a function such that $f(x)$ is continuous in $[a, b]$, and $f'(x)$ exists in (a, b), then there exist at least one c, such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

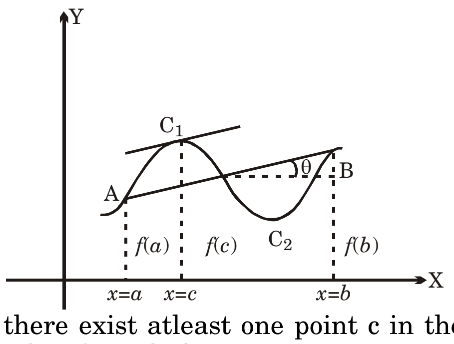
3. Cauchy's Mean value theorem

If $f(x)$ and $g(x)$ be two functions such that

(*i*) $f(x)$ and $g(x)$ both are continuous in $[a \;\; b]$

(*ii*) $f'(x)$ and $g'(x)$ both exist in $(a, \;\; b)$

$g'(x) \neq 0 \;\forall\; x \in (a \;\; b)$



then there exist atleast one point c in the closed interval $(a \;\; b)$ such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

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4. Taylor's theorem

If $f(x)$ be a continuous function such that

$$f'(x), f''(x), f'''(x) \ldots f^{n-1}(x)$$

all are continuous in $[a, a+h]$

$f^n(x)$ exist in $(a, a+h)$, where $h = b - a$

then $$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a)$$

$$+ \ldots\ldots \frac{h^{n-1}}{n-1!}f^{n-1}(a) + \frac{h^n}{n!}f^n(a)$$

If we put $a = 0$, $h = x$, then the series becomes

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \ldots.$$

FUNCTIONS OF TWO VARIABLES

Let $f(x,y)$ be any function of two independent varibles x and y.

Limit

A function f tends to a limit l, when (x, y) tends to (a, b)

$$\lim_{(x,y)\to(a,b)} f(x,y) = l$$

Note : If $\lim_{(x,y)\to(a,b)} f(x,y) = l$ and if $y = \phi(x)$ is any function such that $\phi(x) \to b$ when $x \to a$, then $\lim_{(x,y)\to(a,b)} f(x,\phi(x))$ must exist and be equal to l.

Repeated Limits

If a function f is defined in some neigbourhood of (a, b), then limit $\lim_{y\to b} f(x,y)$ if exists, is a function of x, say $\phi(x)$.

If limit $\lim_{x\to a}\phi(x)$ exists and is equal to K, we write

$$\lim_{x\to a}\lim_{y\to b} f(x,y) = \text{K}$$

Here K is repeated limits of f as $y \to b$, $x \to a$

If we change order of taking the limit, we get other repeated limit.

$$\lim_{y\to b}\lim_{x\to a} f(x,y) = \text{K}'$$

when first $x \to a$ and then $y - b$

These two limits may or may not be equal.

Note :

(*i*) If repeated limits are not equal, then simultaneous limits cannot exist.

(*ii*) In case simultaneous limit exists, these two repeated limits if they exist are necessarily equal but the converse is not true.

Continuity

A function f is said to be *continuous* at a point (a,b) of its domain to definition if

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a, b)$$

COMPUTING THE DERIVATIVE

Rules of Differentiation

$(f+g)' = f' + g'$ $\to$ Sum Rule

$(f-g)' = f' - g'$ $\to$ Difference Rule

$(f.g)' = fg' + gf'$ $\to$ Product Rule

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2} \to \text{Quotient Rule}$$

$$\frac{1}{dx}(f(g(x))) = \frac{df}{dg}\cdot\frac{dg}{dx} \to \text{Chain Rule}$$

Some Standard Derivative

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
x^n	nx^{n-1}	$\sin h\ x$	$\cos h\ x$
$ln\ x$	$1/x$	$\cos h\ x$	$\sin h\ x$
$log_a x$	$log_a e.\frac{1}{x}$	$\sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
e^x	e^x	$\cos^{-1}x$	$\frac{-1}{\sqrt{1-x^2}}$
a^x	$a^x \log_a e$	$\tan^{-1}x$	$\frac{1}{1+x^2}$
$\sin x$ $\cos x$	$\cos x$ $\sin x$	$\text{cosec}^{-1}x$	$\frac{-1}{x^2\sqrt{x^2-1}}$
$\tan x$	$\sec^2 x$	$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$
$\sec x$	$\sec x \tan x$	$\cot^{-1}x$	$\frac{1}{1+x^2}$
$\text{cosec}\ x$	$-\text{cosec}\ x \tan x$	$\lvert x\rvert$ $\frac{x}{\lvert x\rvert}$	
$\cot x = -\text{cosec}^2 x$			$(x \neq 0)$

DIFFERENTIATION BY SUBSTITUTION

If a function contain as expression of the form

(1) $a^2 - x^2$; put $x = a \sin t$ or $x = a \cos t$

(2) $a^2 + x^2$; put $x = a \tan t$ or $x = a \cot t$

(3) $x^2 - a^2$; put $x = a \sec t$ or $x = a\ \text{cosec}\ t$

(4) $\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$; put $x = a \cos t$

(5) $a \cos x \pm b \sin x$; put $a = r\cos\theta$, and $b = r \sin\theta$, $r > 0$

PARTIAL DERIVATIVES

The partial differential coefficient of $z = f(x, y)$ with respect to x is defined as

$$\underset{\delta x\to 0}{\text{Lt}} \frac{f(x+\delta x, y) - f(x, y)}{\delta x}$$

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Provided this limit exists and is denoted by

$$\frac{\partial f}{\partial x} \text{ or } \frac{\partial z}{\partial x} \text{ or } f_x \text{ or } z_x$$

Similarly, partial derivatives of $f(x, y)$ with respect to y is defined as

$$\underset{\delta y \to 0}{\text{Lt}} \frac{f(x, y + \delta y) - f(x, y)}{\delta y}$$

Provided this limit exists and is denoted by

$$\frac{\partial f}{\partial y} \text{ or } \frac{\partial z}{\partial y} \text{ or } f_y \text{ or } z_y$$

$\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ are called *first order partial derivatives of z*.

- *The partial derivation of f(x, y) with respect to y is the ordinary derivative of f(x, y) when x is regarded as constant.*

Example. If $z = \log (x^2 + y^2)$, find $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$

Solution:

Given : $z = \log (x^2 + y^2)$

$$\therefore \quad \frac{\partial z}{\partial x} = \frac{2x}{x^2 + y^2}$$

$$\Rightarrow \quad x\frac{\partial z}{\partial x} = \frac{2x^2}{x^2 + y^2}$$

$$\frac{\partial z}{\partial y} = \frac{2y}{x^2 + y^2}$$

$$\Rightarrow \quad y\frac{\partial z}{\partial y} = \frac{2y^2}{x^2 + y^2}$$

$$\therefore \quad x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{2(x^2 + y^2)}{(x^2 + y^2)} = 2$$

Partial derivatives of higher order

Since f_x and f_y are also functions of x and y and so we can further differentiate f_x and f_y partially with respect to x or y. The further partial derivatives of f_x or f_y are called *partial derivatives f higher order of u*.

- Partial derivatives of $\frac{\partial u}{\partial x}$ with respect to x and y are $\frac{\partial^2 u}{\partial^2 x}$ or $\frac{\partial^2 u}{\partial y\, \partial x}$ respectively.
- Partial derivatives $\frac{\partial^2 u}{\partial^2 x^2}$, $\frac{\partial^2 u}{\partial y\, \partial x}$, $\frac{\partial^2 u}{\partial xy}$, $\frac{\partial^2 u}{\partial y^2}$ are also denoted by $f_{xx}, f_{yx}, f_{xy}, f_{yy}$ respectively.

Example. If $u = e^{xyz}$, then find $\frac{\partial^3 u}{\partial x \partial y \partial z}$.

Solution:

Given : $\quad u = e^{xyz}$

$$\therefore \quad \frac{\partial u}{\partial z} = e^{xyz}\, xy = uxy, \ \frac{\partial u}{\partial y} = uxz$$

and $\quad \frac{\partial u}{\partial x} = uyz$

Now, $\quad \frac{\partial^2 u}{\partial y \partial z} = ux + xy$

$$\frac{\partial u}{\partial y} = ux + (xy)\,(uxz) = ux + ux^2 yz$$

$$\therefore \quad \frac{\partial^3 u}{\partial x \partial y \partial z} = u + x\frac{\partial u}{\partial x} + 2u\, xyz + x^2yz\, \frac{\partial u}{\partial x}$$

$$= u + x(uyz) + 2uxyz + x^2yz\,(uyz)$$

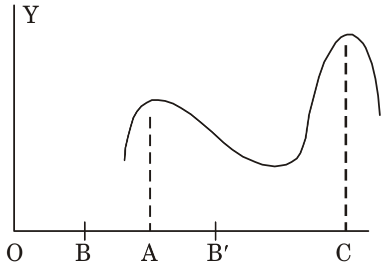
$$= e^{xyz}(1 + 3xyz + x^2\, y^2z^2)$$

MAXIMA AND MINIMA OF FUNCTIONS OF TWO VARIABLES.

A function $f\,(x, y)$ is said to have a maximum or minimum at $x = a, y = b$, according as $f\,(a + h, b + k)$ is less or greater than $f\,(a, b)$ for all positive or negative small values of h and k.

In other words, if $\Delta = f\,(a + h, b + k) - f\,(a, b)$ is of the same sign for all small values of h, k and if this sign is negative, then $f\,(a, b)$ is a maximum. If this sign is positive, then $f\,(a, b)$ is a minimum.

A maximum or minimum value of a function is called its *extreme value*.



Above figure shows a graph of the function $f(x)$ and $OA = a$, *i.e.,* $f\,(x)$ has a maximum value for $x = a$ because $f\,(a)$ has a value more than the values of $f\,(x)$ for every value of x between B and B′. $f(x)$ is said to be a *maximum* at $x = a$, even though value of $f\,(x)$ at $x = a$ should be greater than all other values of $f\,(x)$ in some small neighbourhood. Thus a maximum value of $f\,(x)$ is not necessarily the greatest value of $f(x)$. In fact, a curve might have several maxima (and minima).

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Condition for Maxima and Minima

Necessary conditions for existence of a maximum or a minimum of $f(x)$ at $x = a, y = b$ is

$$\left(\frac{\partial f}{\partial x}\right)_{\substack{x=a\\y=b}} = 0; \text{ and } \left(\frac{\partial f}{\partial y}\right)_{\substack{x=a\\y=b}} = 0$$

Sufficient condition for Maxima and Minima :

Let $r = \left(\frac{\partial^2 f}{\partial x^2}\right)_{\substack{x=a\\y=b}}, s = \left(\frac{\partial^2 f}{\partial x \partial y}\right)_{\substack{x=a\\y=b}}$

and $t = \left(\frac{\partial^2 f}{\partial y^2}\right)_{\substack{x=a\\y=b}}$

Case I: $rt - s^2 > 0$

$f(x, y)$ will have maximum or a minimum of $x = a, y = b$.

Again, $f(x, y)$ is a maximum or a minimum according as r is negative or positive

Case II: $rt - s^2 < 0$

$f(x,y)$ will have neither a maximum nor a minimum at $x = a, y = b$.

Case III: $rt - s^2 = 0$

This case is doubtful and further investigation is needed to determine whether

(x, y) is maximum or a minimum at $x = a, y = b$ or not.

Finding absolute maxima and minima values

Working rules :

If f is a differentiable function in $[a\ b]$ except at finitely many points, then to find absolute maximum and absolute minimum values, adopt following procedure :

(*i*) Evaluate $f(x)$ at the points, where $f'(x) = 0$.

(*ii*) Evaluated $f(a)$ and $f(b)$.

Then maximum of these values is the **absolute maxima** and minimum of these values is called **absolute minima.**

Finding local maxima and local minima.

Working rules :

Condition for maxima :

- Function $f(x)$ has a maximum value $f(a)$ if $f'(a) = 0$ and $f'(x)$ changes sign from +ve to –ve as x passes through 'a' from left to right.
- In general, for any even number n, $f'(a) = f''(a) = f'''(a) \ \ldots = f^{n-1}(a) = 0$ and $f^n(a) < 0$, then $f(a)$ is a maximum value of $f(x)$.
- In particular, if $n = 2$, then $f'(a) = 0$ and $f''(a) < (o)$, and the function is maximum at $x = a$.

Condition for minima :

- Function $f(x)$ has a minimum value $f(a)$ if $f'(a) = 0$ and $f'(x)$ changes sign from –ve to +ve as x passes through $x = a$ from left to right.
- In general, for any even number n, if $f'(a) = f''(a) = f'''(a) = \ldots f^{n-1}(a) = 0$, and $f^n(a) > 0$, then $f(a)$ is minimum value of $f(x)$
- In particular, if $n = 2$, then $f'(a) = 0$ and $f''(a) > 0$, the function is minimum at $x = a$.

Note : If n is a even number such that

$$f'(a) = f''(a) = \ldots f^n(a) = 0, f^{n+1}(a) \neq 0,$$

then $f(a)$ is neither maximum nor minimum of the function $f(x)$.

In particular if $n = 2$ and $f'(a) = f''(a) = 0$. $f'''(a) \neq 0$, then $f(a)$ is neither maximum nor minimum values of $f(x)$. Then the point $x = a$ is called **point of inflexion.**

Extreme value.

Either a maximum value or minimum value $f(a)$ of the function $f(x)$ is called *extreme value*.

Stationary value.

If $f'(a) = 0$, then $f(a)$ is called *stationary value*.

Every extreme value is stationary but every stationary value of the function need not to be extreme value

Example. Find the extreme value of

$$f(x) = x^5 - 5x^4 + 5x^3 - 1$$

Solution:

$$f'(x) = 5x^4 - 20x^3 + 15x^2$$

and $f'(0) = 0$

Hence, $f(0)$ is stationary value but $f(0)$ is not an extreme value because $f''(0) = 0$ but $f'''(0) \neq 0$.

Greatest value.

The greatest value of the function in an interval (a, b) is either a maximum value in an interval or end value (i.e. at $x = a$ or $x = b$) of $f(x)$ whichever is greater.

Least values.

The least value of $f(x)$ in an interval (a, b) is either a minimum value of $f(x)$ at a point inside the interval or an end value (i.e. at $x = a$ or $x = b$) of $f(x)$ whichever is less.

Example. Find greatest and least value of the function $2x^2 - 9x - 1$, in the internal [2, 5]

Solution:

Let $f(x) = 2x^2 - 9x - 1$

$\Rightarrow$ $f'(x) = 4x - 9$

At $x = \frac{9}{4}$ $f'(x) = 0$

$\therefore$ $f''(x) = 4 > 0$

$\therefore$ $f\left(\frac{9}{4}\right) = -\frac{89}{8}$ is a minimum value.

Also, $f(2) = -11, f(5) = 4$

$\therefore$ Greatest value = 4, and least value = $-\frac{89}{8}$

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SOME STANDARD INTEGRATIONS

1. $\int x^n = \frac{x^{n+1}}{n+1}$, $(n \neq -1)$

2. $\int \frac{1}{x} dx = \log x$

3. $\int e^x dx = e^x$

4. $\int a^x dx = \frac{a^x}{\log a}$

5. $\int \sin x \, dx = -\cos x$

6. $\int \cos x \, dx = \sin x$

7. $\int \sec^2 x \, dx = \tan x$

8. $\int \text{cosec}^2 dx = -\cot x$

9. $\int \sec x \tan x \, dx = \sec x$

10. $\int \text{cosec}\, x \cot x \, dx = -\text{cosec}\, x$

11. $\int \tan x \, dx = \log \sec x = -\log \cos x$

12. $\int \cot x \, dx = \log \sin x$

13. $\int \sec x \, dx = \log(\sec x + \tan x)$

14. $\int \text{cosec}\, x \, dx = -\log(\text{cosec}\, x + \cot x) = \log \tan \frac{x}{2}$

15. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x$ or $-\cos^{-1} x$

16. $\int \frac{1}{\sqrt{1+x^2}} dx = \sin h^{-1} x$

17. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$

18. $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x = \cos^{-1} \frac{1}{x}$

19. $\int \sin hx \, dx = \cos hx$

20. $\int \cos hx \, dx = \sin hx$

21. $\int \frac{1}{\sqrt{x^2-a^2}} dx = \log\left(x + \sqrt{x^2-a^2}\right) = \cos h^{-1} \frac{x}{a}$

22. $\int \frac{1}{\sqrt{a^2+x^2}} dx = \log\left(x + \sqrt{a^2+x^2}\right)$

23. $\int \frac{1}{\sqrt{a^2+x^2}} dx = \sin h^{-1}\left(\frac{x}{a}\right)$

24. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} = \frac{1}{a}\cos$

25. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

26. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)$ if $x > a$

27. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)$ if $x < a$

28. $\int u \,.\, v \, dx = u \int v \, dx - \int\left[\left(\frac{du}{dx}\right)\left(\int v dx\right)\right] dx$

where u and v are the function of x. u is a first function and v as a second function.

29. $\int e^x [f(x) + f'(x)] \, dx = \boldsymbol{e^x f(x) + c}$

30. $\int \frac{f'(x)}{f(x)}$ **dx** $= \boldsymbol{\log f(x) + C}$

Note : I L A T E

I $\rightarrow$ Inverse
L $\rightarrow$ Logarithmic function
A $\rightarrow$ Algebraic
T $\rightarrow$ Trigonometric
E $\rightarrow$ Exponential

Example. Calculate $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx$

Solution:

Let $\tan^{-1} x = y \Rightarrow \frac{1}{1+x^2} dx = dy$

then $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx = \int e^y dy = e^y + C = e^{\tan^{-\frac{1}{x}}} + C$

Example. Calculate $\int 1.\sin^{-1} x \, dx$

Solution:

Take $u = \sin^{-1} x$ and $v = 1$

Then $\int \sin^{-1} x dx = \sin^{-1} x \int dx - \int\left[\frac{d(\sin^{-1} x)}{dx}\int dx\right] dx$

$= x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} \; x \, dx$

$= x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$

$= x \sin^{-1} x + \sqrt{1-x^2} + C$

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Example. Calculate $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\, dx$

Solution:

Let $\quad y = e^x - e^{-x} \quad \Rightarrow \quad dy = (e^x + e^{-x})\, dx$

$$\therefore \int \frac{\left(e^x + e^{-x}\right)}{e^x - e^{-x}} = \int \frac{dy}{y} = \log y + \text{C} = \log\,(e^x + e^{-x}) + \text{C}$$

DEFINITE INTEGRAL

If a function is defined in the interval $[a, b]$, then definite integral of $f(x)$ is

$$\int_a^b f(x)\, dx = \left[F(x)\right]_a^b = F(b) - F(a)$$

where F(x) is an integral of $f(x)$

$f(b)$ is the value of $f(x)$ at $x = b$, and

$f(a)$ is the value of $f(x)$ at $x = a$.

a and b are called *lower upper limit respectively.*

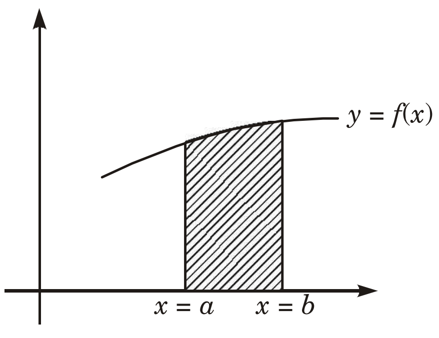
Geometrically, definite integral represents the area bounded by curve $y = f(x)$, x-axis and the lines $x = a$ and $x = b$.

Properties of Definite Integrals

Let $\int f(x)\, dx = F(x)$. Then

1. $\int_a^b f(x)\, dx = \int_a^b f(y)\, dy$

i.e. in definite integral the variable of integration is dummy.



2. $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

3. $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

where c is a point lies between the interval $[a, b]$. and $f(x)$ integrable over $[a, c]$ and $[c, b]$.

4. $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$

5. $\int_{-a}^a f(x)\, dx = \int_0^a f(x)\, dx + \int_0^a f(-x)\, dx$

If function is even, i.e. $f(-x) = f(x)$, then

$$\int_{-a}^a f(x)\, dx = 2\int_0^a f(x)\, dx$$

If function is odd, i.e. $f(-x) = -f(x)$, then

$$\int_{-a}^a f(x)\, dx = 0$$

6. $\int_0^{2a} f(x)\, dx = \int_0^a f(x)\, dx + \int_0^a f(2a - x)\, dx$

If $f(x) = f(2a - x)$, then

$$\int_0^{2a} f(x)\, dx = 2\int_0^a f(x)\, dx$$

and if $f(x) = -f(2a - x)$, then

$$\int_0^{2a} f(x)\, dx = 0$$

Definite Integral as Limit of sum

If a function $f(x)$ is either increasing or decreasing on $[a, b]$, then

$$\int_a^b f(x)\, dx = \underset{h \to 0}{\text{Lt}}\, h[f(a) + f(a + h) + \ldots + f(a+(n-1)\,h]$$

$$= \underset{h \to 0}{\text{Lt}}\, h\,[f(a+h) + f(a + 2h) + \ldots + f(a + nh)]$$

where, $h = \frac{b - a}{n}$

Example. From definition of a definite integral as the limit of a sum, evaluate $\int_1^2 (x + x^2)\, dx$.

Solution:

$$\int_1^2 (x + x^2)\, dx$$

$$= \underset{h \to 0}{\text{Lt}} \sum_{i=1}^n h\,[(1 + ih) + (1 + ih)^2 \ldots\ldots] \text{ where} \left(h = \frac{1}{n}\right)$$

$$= \underset{h \to 0}{\text{Lt}}\, h \sum_{i=1}^n [1 + ih + 1 + 2\,ih + i^2 h^2]$$

$$= \underset{h \to 0}{\text{Lt}}\, h \sum_{i=1}^n (2 + 3hi) + i^2 h^2$$

$$= \underset{h \to 0}{\text{Lt}} \left[2h \sum_{i=1}^n 1 + 3h^2 \sum_{i=1}^n i + h^3 \sum_{i=1}^n i^2\right]$$

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$$= \underset{h \to 0}{\text{Lt}} \left[2nh + \frac{3}{2} h^2 n\,(n+1) + \frac{1}{6} h^3 n\,(n+1)(2n+1) \right]$$

$$= \underset{h \to 0}{\text{Lt}} \left[2 + \frac{3}{2}(1+h) + \frac{1}{6}(1+h)\,(2+h) \right]$$

...since $nh = 1$

$$= 2 + \frac{3}{2} + \frac{1}{3}$$

$$= \mathbf{\frac{23}{6}}$$

Improper Integral

Definite intgral $\int_0^a f(x)\,dx$ is *called improper integral* if

(i) range of integration is infinite and integrand is bounded.

(ii) range of integration is definite and integrand is unbounded.

(iii) neither range of integration is finite nor integrand is bounded over it.

MULTIPLE INTEGRALS

Let a single-valued and bounded function $f(x,y)$ of two independent variables x,y be defined in a closed region R of the xy-plane. Divide the region R into sub-regions by drawing lines parallel to co-ordinate axes. Number the rectangles which lie entirely inside the region R from 1 to n.

Let (x_r, y_r) be any point inside the rth rectangle whose area is δA_r

$$f(x_1,y_1)\,\delta A_1 + f(x_2,y_2)\,\delta A_2 + ...f(x_n,y_n)\,\delta A_n$$

$$= \sum_{r=1}^{n} f(x_r,y_r)\delta A_r \qquad .... \text{(i)}$$

Let number of these sub-regions increases indefinitely such that the largest linear dimension (i.e.diagonal) of δA_r approaches zero. The limit of the sum(i), if it exists, irrespective of the mode of sub-division, is called *double integral of f(x,y)* over the region R and is denoted by $\iint_R f(x,y)\,dA$.

In other words, $\underset{\substack{n \to \infty \\ \delta A_r \to 0}}{Lt} \sum_{r=1}^{n} f(x_r, y_r)\,\delta A_r = \iint_R f(x,y)\,dA$

which is also expressed as $\iint_R f(x,y)\,dxdy$

or $\iint_R f(x,y)\,dy\,dx$

Let the region R be bounded by the curves

$x = x_1$ *and* $y = y_2$

$x = x_2$ *and* $y = y_2$

Properties of a Double Integral

(i) When x_1, x_2 are functions of y and y_1, y_2 are constants.

Let AB and CD be the curves $x_1=\phi_1(y)$ and $x_2=\phi_2(y)$. Then

$$\iint_R f(x,y)\,dxdy = \int_{y_1}^{y_2} \int_{x_1=\phi_1(y)}^{x_2=\phi_2(y)} f(x,y)\,dx\;\;dy$$

(ii) When y_1, y_2 are functions of x and x_1, x_2 are constants.

Let AB and CD be the curves

$y_1 = \phi_1(x)$ and $y_2=\phi_2(x)$. Then

$$\iint_R f(x,y)\,dx\,dy = \int_{x_1}^{x_2} \int_{y_1=\phi_1(x)}^{y_2=\phi_2(x)} f(x,y)\,dy\;\;dx$$

***(iii)* When x_1, x_2, y_1,y_2 are constants.**

Here the region of integration R is the rectangle. Therefore

$$\iint_R f(x,y)\,dxdy = \int_{y_1}^{y_2} \int_{x_1}^{x_2} f(x,y)\,dx\;\;dy$$

$$= \int_{x_1}^{x_2} \int_{y_1}^{y_2} f(x,y)\,dy\,dx$$

(i) If region R is partitioned into two parts, say R_1 and R_2, then

$$\iint_R f(x,y)\,dx\,dy = \iint_{R1} f(x,y)\,dx\,dy$$

$$+ \iint_{R2} f(x,y)\;dx\,dy$$

(ii) Double integral of the algebraic sum of a fixed number of functions is equal to algebraic sum of the double integrals taken far each term. Thus

$$\iint_R [f_1(x,y) + f_2(x,y) + f_3(x,y) + ...]\,dx\,dy$$

$$= \iint_R f_1(x,y)\,dx\,dy + \iint_R f_2(x,y)\,dx\,dy$$

$$+ \iint_R f_3(x,y)\,dx\,dy + ...$$

(iii) A constant factor may be taken outside the integral sign. Thus

$$\iint_R K\;f(x,y)\,dx\,dy = K\iint_R f(x,y)\,dx\,dy$$

where K is constant

Evaluation of Double Integrals

During evaluating double intergrals, first integrate with respect to the variable limits, treating other varibles as constant, and then integrate with respect to the variable with constant limit.

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CHANGE OF ORDER OF INTEGRATION

In a double integral, if limits of integration are constant, then order of integration is immaterial, provided limits of integration are changed accordingly. Thus

$$\int_c^d \int_a^b f(x,y)\ dx\ dy = \int_a^b \int_c^d f(x,y)\ dy\ dx$$

But if limits of integration are variable, a change in the order of integration changes in the limits of integration.

TRIPLE INTEGRALS

Consider a function $f(x,y,z)$ which is continuous at every point of the 3-dimensional finite region V.

Divide V into n elementary volumes $\delta V_1, \delta V_2, \ldots \delta V_n$.

Let (x_r, y_r, z_r) be any point within rth sub-division δV_r.

Consider the sum $\sum_{r=1}^{\infty} f(x_r, y_r, z_r)\ \delta V_r$

The limit of this sum, if it exists, as $n \to \infty$ and $\delta V_r \to 0$ is called *triple integral* of $f(x,y,z)$ over the region V and is denoted by $\iiint f(x,y,z)\ dV$

It can also be expressed as the repeated integral $\int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2} f(x,y,z)\ dx\ dy\ dz.$

FOURIER SERIES

If numbers $a_0, a_1, \ldots, a_n, \ldots, b_1, b_2, \ldots, b_n, \ldots$ are derived from a function f by means of Euler-Fourier formulas

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\, dx \qquad (n = 0,1,2,\ldots)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\, dx \qquad (n = 1,2,\ldots)$$

then series $a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ is called *Fourier series*.

Dirichlet's Condition

Assumptions for expansion in a Fourier's series :

(*i*) Given function $f(x)$ is assumed to be defined and single valued in the given range $(-l, l)$.

(*ii*) $f(x)$ is periodic outside $(-l, l)$ with period $2l$.

(*iii*) Series $a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ is uniformly convergent, so that term by term integration of the series is possible.

Even and Odd function of *x*(sine series and cosine series)

Case I: *When f(x) is an even function*

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx$$

where, $a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cos nx\ dx$$

$$b_n = 0$$

Case II: *When f(x) is an odd function*

$$f(x) = \sum_{n=1}^{\infty} (b_n \sin nx)$$

where, $b_n = \frac{2}{\pi}\int_0^{\pi} f(x) \sin nx\ dx$

$$a_0 = 0$$

$$a_n = 0$$

Half Range Expansions

A series which contains only sine terms or cosine terms is called *half range Fourier sine or cosine series* respectively.

In this case, function $f(x)$ is generally defined in the interval $(0, l)$.

Now if $f(x)$ is an even function, then Fourier cosine series is

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{nx\pi}{l}$$

where $a_0 = \frac{1}{l}\int_0^{l} f(x)\ dx$;

$$a_n = \frac{2}{l}\int_0^{l} f(x) \cos \frac{n\pi x}{l}\,dx$$

If f(x) is an odd function of x, then Fourier sine series is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{nx\pi}{l}$$

where, $a_0 = 0$ $b_n = \frac{2}{l}\int_0^{l} f(x) \sin \frac{n\pi x}{l}\,dx$

Fouriers series in the interval (a, b)

If function $f(x)$ satisfies Dirichlet's conditions in the interval (a, b), then

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2n\pi x}{b-a} + b_n \sin\frac{2n\pi x}{b-a}\right)$$

where, $a_0 = \frac{1}{b-a}\int_a^b f(x)\,dx$

$$a_n = \frac{2}{b-a}\int_a^b f(x) \cos\frac{2n\pi x}{b-a}\,dx$$

$$b_n = \frac{2}{b-a}\int_a^b f(x) \sin\frac{2n\pi x}{b-a}\,dx$$

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CONVERGENCE SERIES

SEQUENCE.

A set of numbers $u_1, u_2 .... u_n .....$ formed according to some definite rule is called a *sequence*. A sequence is generally denoted by $\{u_n\}$.

Finite sequence.

A sequence is called a *finite sequence* if it has a finite number of terms.

Infinite sequence.

If sequene has infinite number of terms, then it is called an *infinite sequence.*

Positive term sequence.

A series $u_1 + u_2 + u_3 .....$ is called *positive term series*, if $u_n > 0 \ \forall\ n$.

SERIES.

Sum of the corresponding terms of the sequence $u_1, u_2, u_3 .....$, *i.e.*, $u_1 + u_2 + u_3 .....$ is called *series*. A series is called a *finite* or *infinite* according as the corresponding sequence is *finite* or *infinite*.

Infinites series $u_1 + u_2 + u_3 + ... + u_n + ...$ is denoted by $\sum u_n$ and sum of the first n terms series is denoted by S_n.

$\therefore \qquad S_n = u_1 + u_2 + u_3 + ... + u_n$

Alternating series.

A series $u_1 - u_2 + u_3 .....$ is called *alternating series*, if $u_n > 0 \ \forall\ n$.

Nature of Series

(1) Convergent.

Infinite series $u_1 + u_2 + u_3 + ...$ is said to be convergent, if

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}(u_1 + u_2 + ... + u_n) \text{ is finite.}$$

(2) Divergent.

Infinite series $u_1 + u_2 + u_3 + ...$ is said to be divergent, if

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}(u_1 + u_2 + ... u_n) = +\infty \text{ or } -\infty$$

(3) Oscillatory.

Infinite series $u_1 + u_2 + u_3 + ...$ is said to be an oscillatory series, if

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}(u_1 + u_2 + ... u_n)$$

is not defined or oscillates between two limites.

Properties of Convergent and Divergent Series:

(*i*) Nature of an infinite series remains unaltered by adding or removing a finite number of terms.

(*ii*) Nature of an inifinite series remains unaltered by multiplying or dividing the terms by some fixed non zero constant.

(*iii*) If $\sum u_n$ and $\sum v_n$ are two series such that $0 \le u_n \le v_n$ for all n and $\sum v_n$ is convergent, then $\sum u_n$ is also convergent.

(*iv*) If $\sum u_n$ and $\sum v_n$ are two series such that $u_n \ge v_n \ge 0$ for all n and $\sum v_n$ is divergent, then $\sum u_n$ is also divergent.

(*v*) If infinite series $\sum u_n$ and $\sum v_n$ both are convergent, then $\sum(u_n + v_n)$, $\sum(u_n - v_n)$ and $\sum(\lambda u_n + \lambda v_n)$ are also convergent.

(*vi*) If series $\sum u_n$ of positive terms is convergent, then $\lim_{n\to\infty} u_n = 0$ (only necessary not sufficient).

(*vii*) The series $\sum u_n$ is not convergent, $\lim\limits_{n\to\infty} u \ne 0$

Absolutely Convergent Series

Series $\sum u_n$ is called *absolutely convergent* if series $\sum |u_n|$ is convergent.

Conditionally (or semi) Convergent series

A convergent series Σu_n is called conditionlly convergent series, if $\sum |u_n|$ is divergent.

Series Tests

1. Leibnitz's test (Alternating series test)

Infinite series $u_1 - u_2 + u_3 - u_4 + ...$ is convergent, if

(*i*) $u_1 > u_2 > u_3 > u_4 + ...$, and

(*ii*) $u_n \to 0$ as $n \to \infty$

2. Geomteric Series test

Geometric series $a + ar + ar^2 + ...$ is

(*i*) convergent, if $|r| < 1$

(*ii*) divergent, if $r \ge 1$

(*iii*) osillatory, when $r \le -1$

3. Hyperharmonic or *p*-Series test

Inifinite series $\sum \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + ...$ is

(*i*) convergent if $p > 1$

(*ii*) divergent if $p \le 1$

4. Comparison test

If $\sum u_n$ and $\sum v_n$ are two series of positive terms such that $\lim_{n\to\infty} \frac{u_n}{v_n}$ is finite ($\ne 0$), then $\sum u_n$ and $\sum v_n$ behave alike.

Note : Test series $\sum v_n$ of the comparison test is called *auxiliary series* and

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$$v_n = \frac{\text{highest power term in numerator of } u_n}{\text{highest power term in denominator in } u_n}$$

5. D' Alembert's ratio test

If $\sum u_n$ is a series of positive terms and, if

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \text{L},$$

then the series is

(*i*) convergent, if L > 1

(*ii*) divergent if, L < 1

(*iii*) the test fails, if L = 1

Note:

I. If $\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = 1$ and $\frac{u_n}{u_{n+1}}$ contains e, then Logarithmic test is used.

II. If $\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = 1$ and $\frac{u_n}{u_{n+1}}$ does not contains e, then either Raabe's test of Gauss's test is used.

6. Raabe's test

$\sum u_n$ is a series of positive terms and, if

$\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}} - 1\right) = \text{L}$, then the series is

(*i*) convergent, if L > 1

(*ii*) divergent, if L < 1

(*iii*) The test fails, if L = 1

7. Gauss's test

If $\sum u_n$ is a series of positive terms and if ratio $\frac{u_n}{u_{n+1}}$ is expressed in the form

$$\frac{u_n}{u_{n+1}} = 1 + \frac{\lambda}{n} + \frac{\mu}{n^2} + \frac{\alpha}{n^3} + \ldots..$$

then the series is

(*i*) convergent, if $\lambda > 1$

(*ii*) divergent if $\lambda \le 1$

8. Logrithmic test

If $\sum u_n$ is series of positive terms and if

$\lim_{n\to\infty}\left(n \log \frac{u_n}{u_{n+1}}\right) = \text{L}$, then the series is

(*i*) convergent, if L > 1

(*ii*) divergent, if L < 1

(*iii*) the test fails, if L = 1

9. De Morgan and Bertrand's test

If $\sum u_n$ is series of positive terms and if

$$\lim_{n\to\infty}\left[\left\{n\left(\log \frac{u_n}{u_{n+1}} - 1\right) - 1\right\}\log n\right] = \text{L},$$

then the series is

(*i*) convergent, if L > 1

(*ii*) divergent if, L < 1

(*iii*) the test fails, if L = 1

10. Higher Logarithmic test

If $\sum u_n$ is series of positive terms and if

$$\lim_{n\to\infty}\left\{\left(n \log \frac{u_n}{u_{n+1}} - 1\right)\log n\right\} = \text{L},$$

then the series is

(*i*) convergent, if L > 1

(*ii*) divergent, if L < 1

(*iii*) the test fails, if L = 1

11. Cauchy's Root test

If $\sum u_n$ is a series of positive terms and if $\lim_{n\to\infty}(u_n)^{1/n} = \text{L}$, then the series is

(*i*) convergent, if L < 1

(*ii*) divergent, if L > 1

(*iii*) the test fails, if L = 1

12. Cauchy's Condensation test

If $\sum u_n$ is series of positive terms and if $u_{n+1} < u_n$ then series $\sum u_n$ and $\sum a^n u_n$ (a > 1 any integer) behave alike.

13. Cauchy's Integral test

If $\sum u_n$ is series of positive terms and $f(x)$ be a continuous positive decreasing function of $(x > 0)$, then $\sum u_n$ and $\int_1^\infty f(x)\,dx$ behave alike.

14. Kummer's test

Let a divergent series of positive terms $\sum (d_n)^{-1}$ is given, and let a terms of a series of positive terms $\sum u_n$ be such that

$$\lim_{n\to\infty}\left(d_n \frac{u_n}{u_{n+1}} - d_n + 1\right) = \text{L},$$

then the series is

(*i*) convergent, if L > 0

(*ii*) divergent, if L < 0

(*iii*) the test fails, if L = 1

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EXERCISE – I

MCQ TYPE QUESTIONS

1. If $\lfloor x \rfloor$ denotes the greatest integer not greater than 2, then $\lim\limits_{x\to 2}[x]$ is

(*a*) 0 (*b*) 1
(*c*) 2 (*d*) does not exist

2. If two functions $f(x) = \begin{cases} x\sin\dfrac{1}{x} & ,x\neq 0 \\ 0 & ,x\neq 0 \end{cases}$ $g(x) = |x|$ are given, then at $x = 0$

(*a*) both f and g continuous
(*b*) f is continuous and g is discontinuous
(*c*) f is discontinuous and g is continuous
(*d*) both f and g are discontinuous

3. If $f(x) = |x|$, then $f(x)$ is

(*a*) discontinuous at $x = 0$
(*b*) continuous only at $x = 0$
(*c*) continuous at all values of x
(*d*) discontinuous at $x = 1$

4. The function $f(x) = \begin{cases} ax^2+b & if \quad x>2 \\ 2 & if \quad x=2 \\ 2ax-b & if \quad x<2 \end{cases}$

is continuous at $x = 2$, if

(*a*) $a = 0, b = \dfrac{1}{2}$
(*b*) $a = \dfrac{1}{2}, b = 0$
(*c*) $a = -\dfrac{1}{2}, b = 0$
(*d*) $a = 0, b = -\dfrac{1}{2}$

5. If $f(x) = x - [x]$, then $f(x)$ is discontinuous

(*a*) $\forall\ x \in$ Z
(*b*) $\forall\ x \in$ Q
(*c*) $\forall\ x \in$ R
(*d*) at very real number except integers

6. The interval in which Lagrange's theorem is applicable for the function $f(x) = \dfrac{1}{x}$ is

(*a*) $[-3, 3]$ (*b*) $[-2, 2]$
(*c*) $[2, 3]$ (*d*) $[-1, 1]$

7. If $f(x) = 3x^4 - 4x^2 + 5$, then the interval for which $f(x)$ satisfied all the condition of Rolle's theorem is

(*a*) $[0, 2]$ (*b*) $[-1, 1]$
(*c*) $[-1, 0]$ (*d*) $[1, 2]$

8. If $f(x) = |x|$, then In the interval $[-1, 1]$, $f(x)$ is

(*a*) satisfied all the conditions of Rolle's theorem
(*b*) satisfied all the conditions of Mean Value theorem
(*c*) does not satisfied the conditions of Mean Value theorem
(*d*) none of these

9. If f and F be both continuous in $[a, b]$ and are derivable in (a, b) and if $f'(x) = F'(x)$ for all x in (a, b), then $f(x)$ and $F(x)$ differ

(*a*) by 1 in $[a, b]$
(*b*) by x in $[a, b]$
(*c*) by constant in $[a, b]$
(*d*) none of these

10. $x^{1/x}$ is a decreasing function if

(*a*) $x < e$
(*b*) $x > e$
(*c*) $x = e$
(*d*) $x > \dfrac{1}{e}$

11. $1 - \dfrac{x^2}{\underline{|2}} + \dfrac{x^4}{\underline{|4}} + \ldots + (-1)^n \dfrac{x^{2n}}{\underline{|2n}} + (-1)^{n+1} \dfrac{x^{2n+1}}{\underline{|2n+1}} \sin\theta x$ is equal to

(*a*) $\sin x$
(*b*) $\cos x$
(*c*) $\tan x$
(*d*) $\log x$

12. Taylor's expansion of the function $f(x) = \dfrac{1}{1+x^2}$ is

(*a*) $\sum\limits_{n=0}^{\infty} (-1)^n x^{2n}$ where $-1 < x < 1$
(*b*) $\sum\limits_{n=0}^{\infty} (-1)^n x^{2n}$ for all real x
(*c*) $\sum\limits_{n=0}^{\infty} x^{2n}$ where $-1 < x < 1$
(*d*) $\sum\limits_{n=0}^{\infty} (-1)^n x^n$ where $-1 < x < 1$

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13. If $f(x)$, $\phi(x)$ and $\psi(x)$ have derivatives when $a \le x \le b$, then $\exists$ a value of c of x lying between a and b such that

(*a*) $\begin{vmatrix} f'(c) & \phi'(c) & \psi'(c) \\ f(b) & \phi(b) & \psi(b) \\ f(a) & \phi(a) & \psi(a) \end{vmatrix} = 0$

(*b*) $\begin{vmatrix} f(c) & \phi(c) & \psi(c) \\ f'(a) & \phi'(a) & \psi'(a) \\ f'(b) & \phi'(b) & \psi'(b) \end{vmatrix} = 0$

(*c*) $\begin{vmatrix} f(a) & \phi(a) & \psi(a) \\ f'(b) & \phi'(a) & \psi'(b) \\ f''(c) & \phi''(c) & \psi''(c) \end{vmatrix} = 0$

(*d*) none of these

14. If $f(x) = ax + b$, $x \in [-1, 1]$, then the point $c \in (-1, 1)$ where $f'(c) = \dfrac{f(1) - f(-1)}{2}$

(*a*) does not exist (*b*) can be any $c \in (-1, 1)$

(*c*) can be only $\dfrac{1}{2}$ (*d*) can be only $\dfrac{-1}{2}$

15. Let $f(x) = x(x-1)(x-2)$, $a = 0$, $b = \dfrac{1}{2}$.

The value of '*c*' in the Lagrange's mean value theorem is

(*a*) $\dfrac{6+\sqrt{11}}{6}$ (*b*) $\dfrac{6-\sqrt{11}}{6}$

(*c*) $\dfrac{6+\sqrt{-12}}{6}$ (*d*) $\dfrac{6-\sqrt{21}}{6}$

16. Maxima and Minima occur

(*a*) simultaneously (*b*) once

(*c*) alternately (*d*) rarely

17. If $x + y = k$, $x > 0$, $y > 0$, then xy is the maximum when

(*a*) $x = ky$ (*b*) $kx = y$

(*c*) $x = y$ (*d*) none of these

18. The triangle of maximum area inscribed in a circle of radius r is

(*a*) a right angled triangle with hypotenuse measuring $2r$

(*b*) an equilateral triangle

(*c*) an isosceles triangle of height r

(*d*) does not exist

19. Sum of the perimeters of a circle and a square is l. If the sum of area is least, then

(*a*) side of the square is double the radius of the circle

(*b*) side of the square is half of the radius of the circle

(*c*) side of the square is equal to radius of the circle

(*d*) none of these

20. Which of the following is false?

(*a*) $f(a)$ is an extreme value of $f(x)$ if $f'(a) = 0$

(*b*) If $f(a)$ is an extreme value of $f(x)$, then $f'(a) = 0$

(*c*) If $f'(a) = 0$, then f(a) is an extreme value of $f(x)$

(*d*) None of these

21. Let $f(x) = |x|$, then

(*a*) $f'(0) = 0$

(*b*) $f(x)$ is maximum at $x = 0$

(*c*) $f(x)$ is minimum at $x = 0$

(*d*) none of these

22. The greatest and the least value of $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 1$ in $[0, 2]$ are

(*a*) 0, 8 (*b*) 0, – 8

(*c*) 1, 8 (*d*) 1, – 8

23. The function $f(x) = x^5 - 5x^4 + 5x^3 - 1$ has

(*a*) one minima and two maxima

(*b*) two minima and one maxima

(*c*) two minima and two maxima

(*d*) one minima and one maxima

24. The maximum value of $\dfrac{\log x}{x}$ in $(0, \infty)$ is

(*a*) e (*b*) $\dfrac{1}{e}$

(*c*) 1 (*d*) none of these

25. Maximum volume of a cylinder in a cone with semi vertical angle 'α' and height 'b' is

(*a*) $\dfrac{4}{27}\pi b^3 \tan^2 \alpha$ (*b*) $\dfrac{4}{27}\pi b^2 \tan \alpha$

(*c*) $\dfrac{4}{27}\pi b \tan^3 \alpha$ (*d*) none of these

26. The maximum point on the curve $x = e^x y$ is

(*a*) $(1, e)$

(*b*) $(1, e^{-1})$

(*c*) $(e, 1)$

(*d*) $(e^{-1}, 1)$

27. The value of function $f(x) = x + \dfrac{1}{x}$ at the points of minimum and maximum are respectively

(*a*) – 2 and 2

(*b*) 2 and – 2

(*c*) – 1 and 1

(*d*) 1 and – 1

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28. The series $1-\frac{1}{2}+1-\frac{3}{4}+1-\frac{7}{8}+\ldots$ is

(*a*) convergent

(*b*) conditionally convergent

(*c*) absolutely convergent

(*d*) oscillatory

29. The series $1 + 2x + 3x^2 + 4x^3 + \ldots$ where $0 < x < 1$, is

(*a*) convergent (*b*) divergent

(*c*) oscillatory (*d*) semi-convergent

30. For geometric series $1 - k + k^2 - k^3 + \ldots$ which of the following is false?

(*a*) The series is convergent if $k = -\frac{1}{3}$

(*b*) The series is divergent if $k = -3$

(*c*) The series is oscillatory if $k = -1$

(*d*) The series is divergent if $k = -2$

31. The series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\ldots$ is

(*a*) convergent (*b*) divergent

(*c*) oscillatory (*d*) none of these

32. The series $\frac{2}{1^P}+\frac{3}{2^P}+\frac{4}{3^P}+\ldots$ is

(*a*) convergent if $p \geq 2$ and divergent if $p < 2$

(*b*) convergent if $p > 2$ and divergent if $p \leq 2$

(*c*) convergent if $p \leq 2$ and divergent if $p > 2$

(*d*) convergent if $p < 2$ and divergent if $p \geq 2$

33. The series $\sum \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$ is

(*a*) convergent if $x < 1$ and divergent if $x \geq 1$

(*b*) convergent if $x \leq 1$ and divergent if $x > 1$

(*c*) convergent if $x > 1$ and divergent if $x \leq 1$

(*d*) convergent if $x \geq 1$ and divergent if $x < 1$

34. The series $\frac{x}{1}+\frac{1}{2}.\frac{x^3}{3}+\frac{1.3}{2.4}.\frac{x^5}{5}+\frac{1.3.5}{2.4.6}.\frac{x^7}{7}+\ldots$ is

(*a*) convergent if $x^2 > 1$ and divergent is $x^2 \leq 1$

(*b*) convergent if $x^2 \geq 1$ and divergent if $x^2 < 1$

(*c*) convergent if $x^2 < 1$ and divergent if $x^2 \geq 1$

(*d*) convergent if $x^2 \leq 1$ and divergent is $x^2 > 1$

35. Sum of the alternating harmonic series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots \text{ is}$$

(*a*) zero

(*b*) infinite

(*c*) log 2

(*d*) not defined as the series is not convergent

36. The series $x+\frac{2^2x^2}{\lfloor 2}+\frac{3^3x^3}{\lfloor 3}+\frac{4^4x^4}{\lfloor 4}+\ldots$ is convergent if

(*a*) $0 < x < \frac{1}{e}$ (*b*) $x > \frac{1}{e}$

(*c*) $\frac{2}{e} < x < \frac{3}{e}$ (*d*) $\frac{3}{e} < x < \frac{4}{e}$

37. The alternating series $u_1 - u_2 + u_3 - u_4 + \ldots (0 < u_n < n)$ is convergent if

(*a*) $u_n < u_{n+1}$ and $u_n \to 0$ as $n \to \infty$

(*b*) $u_n < u_{n+1}$ and $u_n \to \infty$ as $n \to \infty$

(*c*) $u_n > u_{n+1}$ and $u_n \to 0$ as $n \to \infty$

(*d*) $u_n > u_{n+1}$ or $u_n \to \infty$ as $n \to \infty$

38. The series $\frac{a}{b}+\frac{a(a+1)}{b(b+1)}+\frac{a(a+1)(a+2)}{b(b+1)(b+2)}+\ldots$ is

(*a*) convergent if $b - a < 1$ and divergent if $b - a \geq 1$

(*b*) convergent if $b \leq 1 + a$ and divergent if $b > 1 + a$

(*c*) convergent if $b > 1 + a$ and divergent if $b \leq 1 + a$

(*d*) convergent if $b \geq 1 + a$ and divergent if $b < 1 + a$

39. The series $x + x^{1+1/2} + x^{1+1/2+1/3} + x^{1+1/2+1/3+1/4} + \ldots$ is

(*a*) convergent if $x \geq \frac{1}{e}$

(*b*) divergent if $x \geq \frac{1}{e}$

(*c*) convergent if $x > \frac{1}{e}$

(*d*) divergent if $x \leq \frac{1}{e}$

40. The series $\frac{1}{n(\log n)^P}$ is divergent if

(*a*) $p > 1$ (*b*) $p \geq 1$

(*c*) $p < 1$ (*d*) $p \leq 1$

41. Let Σu_n be a series of positive terms and let $\sum \frac{1}{d^n}$ be a divergent series of positive terms such that $\lim_{n\to\infty}\left(d_n \frac{u_n}{u_{n+1}} - d_{n+1}\right) = k$, then the series is

(*a*) convergent if $k > 0$ (*b*) divergent if $k \geq 0$

(*c*) divergent if $k < 0$ (*d*) both (*a*) and (*c*)

42. The series $\sum_{n=3}^{\infty} \frac{1}{n \log n (\log \log n)^p}, p > 0$ is

(*a*) convergent if $p < 1$ and divergent if $p \geq 1$

(*b*) convergent if $p \leq 1$ and divergent if $p > 1$

(*c*) convergent if $p > 1$ and divergent if $p \leq 1$

(*d*) convergent if $p \geq 1$ and divergent if $p < 1$

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43. The series $x + \frac{x(x+1)}{\lfloor 2} + \frac{x(x+1)(x+2)}{\lfloor 3} + \dots$ is

(*a*) convergent if $x > 0$ (*b*) divergent if $x > 0$

(*c*) convergent if $x > 1$ (*d*) divergent if $-1 < x < 0$

44. Which of the following series is convergent?

(*a*) $\sum \sin\frac{1}{n}$ (*b*) $\sum \cos\frac{1}{n}$

(*c*) $\sum \frac{1}{n}\sin\frac{1}{n}$ (*d*) $\sum \sqrt{n^2+1} - \sqrt{n^2}$

45. The series $\sum \frac{\lfloor n.2^n}{n^n}$ is

(*a*) convergent

(*b*) divergent

(*c*) conditionally convergent

(*d*) none of these

46. The series $\sum \frac{4.7\dots(3n+1)}{1.2\dots n} x^n$ is convergent, if

(*a*) $|x| < 1$ (*b*) $|x| < \frac{1}{3}$

(*c*) $|x| < \frac{1}{4}$ (*d*) $|x| < \frac{1}{2}$

47. The series $\frac{2}{1^2} - \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + \dots$ is

(*a*) conditionally convergent

(*b*) absolutely convergent

(*c*) divergent

(*d*) none of these

48. The sequence $\left(\frac{n}{e^n}\right)_{n=1}^{\infty}$ is

(*a*) divergent

(*b*) convergent

(*c*) monotonic increasing

(*d*) oscillating

49. Which of the following series is divergent?

(*a*) $1 + \frac{2.1}{3.4} + \frac{2.4.1}{3.5.6} + \frac{2.4.6.1}{3.5.7.8} + \dots$

(*b*) $\sum \frac{n^3+a}{2^n+a}$

(*c*) $\sum \frac{n^2}{\lfloor n}$

(*d*) $\sum \frac{\sqrt{n}}{n+\sqrt{n}}$

50. The series $\sum_{n=1}^{\infty} \sin\frac{\pi}{n^p}$ is

(*a*) convergent for all value of p

(*b*) convergent for $p \le 1$ and divergent for $p > 1$

(*c*) convergent for $p > 1$ and divergent for $p \le 1$

(*d*) divergent for all values of p

51. The series $\sum \cos\left(\frac{1}{n}\right)$ is

(*a*) convergent (*b*) divergent

(*c*) oscillatory (*d*) none of these

52. The series $x \log x + x^2 \log 2x + \dots + x^n \log nx + \dots$ is convergent if

(*a*) $x < 1$ (*b*) $x \le 1$

(*c*) $x \ge 1$ (*d*) none of these

53. If p and q are positive real numbers, then the series $\frac{2^p}{1^q} + \frac{3^p}{2^q} + \frac{4^p}{3^q} + \dots$ *ad inf*, is converges for

(*a*) $p < q - 1$ (*b*) $p < q + 1$

(*c*) $p \ge q - 1$ (*d*) $p \ge q + 1$

54. The value of $\int_0^{5\pi} (2 - sin\,x)dx$ is

(*a*) > 0 (*b*) 2

(*c*) 0 (*d*) undefined

55. The function $f(x) = 3x(x-2)$ has a

(*a*) minimum at $x = 1$

(*b*) maximum at $x = 1$

(*c*) minimum at $x = 2$

(*d*) maximum at $x = 2$

56. The value of integral $\int_{-2}^{2} \frac{dx}{x^2}$ is

(*a*) 0 (*b*) 0.25

(*c*) 1 (*d*) ∞

57. The value of ξ in the mean value theorem of $f(b) - f(a) = (b-a) f'(\xi)$

for $f(x) = Ax^2 + Bx + C$ in (a, b) is

(*a*) $b + a$ (*b*) $b - a$

(*c*) $\frac{(b+a)}{2}$ (*d*) $\frac{(b-a)}{2}$

58. The function $f(x) = x^3 - 6x^2 + 9x + 25$ has

(*a*) a maxima at $x = 1$ and a minima at $x = 3$

(*b*) a maxima at $x = 3$ and a minima at $x = 1$

(*c*) no maxima, but a minima at $x = 1$

(*d*) a maxima at $x = 1$, but no minima

59. If $\Phi(x) = \int_0^{x^2} \sqrt{t}\, dt$, then $\frac{d\Phi}{dx}$ is

(*a*) $2x^2$

(*b*) $\sqrt{x}$

(*c*) 0

(*d*) 1

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60. What is the derivative of $f(x) = |x|$ at $x = 0$?

(*a*) 1 (*b*) –1

(*c*) 0 (*d*) Does not exist

61. The minimum point of the function $f(x) = (x^3/3) - x$ is at

(*a*) $x = 1$ (*b*) $x = -1$

(*c*) $x = 0$ (*d*) $x = \frac{1}{\sqrt{3}}$

62. Angle between two unit-magnitude coplanar vectors P(0.866, 0.500, 0) and Q(0.259, 0.966, 0) will be

(*a*) 0° (*b*) 30°

(*c*) 45° (*d*) 60°

63. Volume of an object expressed in spherical coordinates is given by

$$V = \int_0^{2\pi}\int_0^{\pi/3}\int_0^1 r^2 \sin\phi \, dr \, d\phi \, d\theta .$$

Value of the integral is

(*a*) $\frac{\pi}{3}$ (*b*) $\frac{\pi}{6}$

(*c*) $\frac{2\pi}{3}$ (*d*) $\frac{\pi}{4}$

64. The function, $y = x^2 + \frac{250}{x}$ $x = 5$ attains

(*a*) maximum (*b*) minimum

(*c*) neither (*d*) 1

65. Volume generated by revolving the area bounded by the parabola $y^2 = 8x$ and the line $x = 2$ about y-axis is

(*a*) $\frac{128\,\pi}{5}$ (*b*) $\frac{5}{128\,\pi}$

(*c*) $\frac{127}{5\,\pi}$ (*d*) none of these

66. The integration of $\int \log x \, . \, dx$ has the value

(*a*) $(x \log x - 1)$

(*b*) $\log x - x$

(*c*) $x (\log x - 1)$

(*d*) None of these

67. If a function is continuous at a point, then its first derivative

(*a*) may or may not exist

(*b*) exists always

(*c*) will not exist

(*d*) has a unique value

68. The integral $\lim_{a\to\infty}\int_1^a x^{-4} dx$

(*a*) diverges (*b*) converges to $\frac{1}{3}$

(*c*) converges to $-\frac{1}{a^3}$ (*d*) converges to 0

NUMERICAL TYPE QUESTIONS

1. $\lim_{x\to 0}\frac{e^x - 1}{x}$ is equal to ______

2. If $\lim_{x\to 1}\frac{f(x)-2}{f(x)+2} = 0$, then $\lim_{x\to 1} f(x)$ is equal to ______

3. $\lim_{x\to 0}\frac{1-\cos x}{2}$ is equal to ______

4. $\lim_{x\to\infty} x^n e^{-x}$ is equal to ______

5. $\lim_{x\to 1}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$ is given by

6. $\lim_{n\to 0} e^{\left(\frac{1}{n\log n}\right)}$ is

7. $\lim_{x\to\infty} x\sin\frac{1}{x}$ is ______

8. $\lim_{x\to\infty}\left[\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\ldots+\frac{1}{n}\right]$ is ______

9. The '*c*' of the mean value theorem for the function $f(x) = x(x - 2)$, when $a = 0$, $b = \frac{3}{2}$ is ______

10. The minimum value of $|x^2 - 5x + 2|$ is ______

11. The maximum value of $\frac{1}{\sqrt{2}}(\sin x - \cos x)$ is ______

12. Maximum slope of the curve $-x^3 + 6x^2 + 2x + 1$ is ______

13. A rectangular box with square base in open at the top. The maximum volume of the box made from 300 m³ wood is ______

14. A series $\sum_1^\infty u_n$ of positive terms is convergent, if $\lim_{n\to\infty}\frac{u_n}{u_{n+1}} = l$ and l is greater than ______

15. $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x + y)\, dx\, dy$ is ______

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EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. $\lim x_{\to\infty} x^{1/x}$ is

(a) ∞ (b) 0

(c) 1 (d) Not defined

2. Let $f(x) = x^{-(1/3)}$ and A denote the area of the region bounded by f(x) and the X-axis, when x varies from – 1 to 1. Which of the following statements is/are TRUE?

I. f is continuous in [–1, 1]

II. f is not bounded in [–1, 1]

III. A is nonzero and finite

(a) II only (b) III only

(c) II and III only (d) I, II and III

3. The value of $\lim\limits_{x\to\infty}(1+x^2)e^{-x}$ is

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) ∞

4. In for non-zero x, $af(x)+bf\left(\frac{1}{x}\right)=\frac{1}{x}-25$ where $a \neq b$ then $\int_1^2 f(x)dx$ is

(a) $\frac{1}{a^2-b^2}\left[a(\ln 2-25)+\frac{47b}{2}\right]$

(b) $\frac{1}{a^2-b^2}\left[a(2\ln 2-25)-\frac{47b}{2}\right]$

(c) $\frac{1}{a^2-b^2}\left[a(2\ln 2-25)+\frac{47b}{2}\right]$

(d) $\frac{1}{a^2-b^2}\left[a(\ln 2-25)+\frac{47b}{2}\right]$

2014

5. Let the function

$$f(\theta) = \begin{vmatrix} \sin\theta & \cos\theta & \tan\theta \\ \sin(\pi/6) & \cos(\pi/6) & \tan(\pi/6) \\ \sin(\pi/3) & \cos(\pi/3) & \tan(\pi/3) \end{vmatrix}$$

Where $\theta \in \left[\frac{\pi}{6},\frac{\pi}{3}\right]$ and $f'(\theta)$ denote the derivative of f with respect to θ. Which of the following statements is/are TRUE?

(I) There exists $\theta \in \left(\frac{\pi}{6},\frac{\pi}{3}\right)$ such that $f'(\theta) = 0$

(II) There exists $\theta \in \left(\frac{\pi}{6},\frac{\pi}{3}\right)$ such that $f'(\theta) \neq 0$

(a) I only (b) II only

(c) Both I and II (d) Neither I nor II

6. A function $f(x)$ is continuous the interval [0, 2]. It is known that $f(0) = f(2) = -1$ and $f(1) = 1$. Which one of the following statements must be true?

(a) There exists a y in the interval (0, 1) such that $f(y) = f(y + 1)$

(b) For every y in the interval (0, 1), $f(y) = f(2 - y)$

(c) The maximum value of the function in the interval (0, 2) is 1

(d) There exists a y in the interval (0, 1) such that $f(y) = -f(2 - y)$

7. The value of the integral given below is

$$\int_0^{\pi} x^2 \cos x\, dx$$

(a) -2π (b) π

(c) $-\pi$ (d) 2π

2013

8. Which one of the following functions is continuous at $x = 3$?

(a) $f(x) = \begin{cases} 2, & \text{if } x=3 \\ x-1, & \text{if } x>3 \\ \frac{x+3}{3}, & \text{if } x<3 \end{cases}$ (b) $f(x) = \begin{cases} 4, & \text{if } x=3 \\ 8-x, & \text{if } x\neq 3 \end{cases}$

(c) $f(x) = \begin{cases} x+3, & \text{if } x\le 3 \\ x-4, & \text{if } x>3 \end{cases}$ (d) $f(x) = \frac{1}{x^3-27}$, if $x \neq 3$

2012

9. Consider the function $f(x) = \sin(x)$ in the interval $x \in \left[\frac{\pi}{4},\frac{7\pi}{4}\right]$. The number and location(s) of the local minima of this function are

(a) One, at $\frac{\pi}{2}$ (b) One, at $\frac{3\pi}{2}$

(c) Two, at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ (d) Two, at $\frac{\pi}{4}$ and $\frac{3\pi}{2}$

2011

10. Given $i = \sqrt{-1}$, what will be the evaluation of the definite integral $\int_0^{\pi/2} \frac{\cos x + i\sin x}{\cos x - i\sin x}dx$?

(a) 0 (b) 2

(c) $-i$ (d) i

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2010

11. What is the value of $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n}$?

(*a*) 0 (*b*) e^{-2}

(*c*) $e^{-1/2}$ (*d*) 1

2009

12. $\int_0^{\pi/4}\left(\frac{1-\tan x}{1+\tan x}\right)$ evaluates to

(*a*) 0 (*b*) 1

(*c*) $ln\ 2$ (*d*) $\frac{1}{2}\ ln\ 2$

2008

13. $\lim_{x\to\infty}\frac{x-\sin x}{x+\cos x}$ equals

(*a*) 1 (*b*) -1

(*c*) ∞ (*d*) $-\infty$

14. A point on a curve is said to be an extremum if it is a local minimum or a local maximum. The number of distinct extrema for the curve $3x^4 - 16x^3 + 24x^2 + 37$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

2007

15. Consider the following two statements about the function $f(x) = |x|$

P. $f(x)$ is continuous for all real values of x

Q. $f(x)$ is differentiable for all real values of x

Which of the following is **TRUE?**

(*a*) P is true and Q is false.

(*b*) P is false and Q is true

(*c*) Both P and Q are true

(*d*) Both P and Q are false

NUMERICAL TYPE QUESTIONS

2015

1. $\int_{/\pi}^{2/\pi}\frac{\cos\left(\frac{1}{x}\right)}{x^2}dx =$ ________.

2014

2. The function $f(x) = x \sin x$ satisfies the following equation. $f''(x) + f(x) + t\cos x = 0$. The value of t is __.

3. If $\int_0^{2\pi}|x\sin x|\,dx = k\pi$, then the value of k is equal to ______.

ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*d*)	**2.** (*a*)	**3.** (*c*)	**4.** (*b*)	**5.** (*d*)	**6.** (*c*)	**7.** (*c*)	**8.** (*c*)	**9.** (*c*)	**10.** (*b*)
11. (*b*)	**12.** (*b*)	**13.** (*a*)	**14.** (*b*)	**15.** (*d*)	**16.** (*c*)	**17.** (*d*)	**18.** (*b*)	**19.** (*a*)	**20.** (*d*)
21. (*c*)	**22.** (*d*)	**23.** (*d*)	**24.** (*b*)	**25.** (*a*)	**26.** (*b*)	**27.** (*d*)	**28.** (*a*)	**29.** (*a*)	**30.** (*c*)
31. (*a*)	**32.** (*b*)	**33.** (*a*)	**34.** (*c*)	**35.** (*c*)	**36.** (*a*)	**37.** (*a*)	**38.** (*c*)	**39.** (*b*)	**40.** (*d*)
41. (*d*)	**42.** (*c*)	**43.** (*b*)	**44.** (*c*)	**45.** (*a*)	**46.** (*b*)	**47.** (*a*)	**48.** (*b*)	**49.** (*d*)	**50.** (*c*)
51. (*b*)	**52.** (*a*)	**53.** (*a*)	**54.** (*a*)	**55.** (*a*)	**56.** (*d*)	**57.** (*c*)	**58.** (*a*)	**59.** (*a*)	**60.** (*b*)
61. (*a*)	**62.** (*c*)	**63.** (*a*)	**64.** (*b*)	**65.** (*a*)	**66.** (*c*)	**67.** (*d*)	**68.** (*b*)		

Numerical Type Questions

1. (1)	**2.** (2)	**3.** (0)	**4.** (0)	**5.** (1)	**6.** (0)	**7.** (1)	**8.** ($\frac{1}{2}$)	**9.** ($\frac{3}{4}$)	**10.** (0)
11. (1)	**12.** (14)	**13.** (500)	**14.** (1)	**15.** (2)					

EXERCISE – II

MCQ Type Questions

1. (*c*)	**2.** (*c*)	**3.** (*c*)	**4.** (*a*)	**5.** (*c*)	**6.** (*a*)	**7.** (*a*)	**8.** (*a*)	**9.** (*d*)	**10.** (*d*)
11. (*b*)	**12.** (*d*)	**13.** (*a*)	**14.** (*b*)	**15.** (*a*)					

Numerical Type Questions

1. (–1)	**2.** (–2)	**3.** (4)

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EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. $\text{L.}\lim_{x\to 2}[x] = \lim_{h\to 0}[2-h] = \lim_{h\to 0}1 = 1$

$\text{R.}\lim_{x\to 2}[x] = \lim_{h\to 0}[2+h] = \lim_{h\to 0}2 = 2$

$\therefore \quad \text{L.}\lim_{x\to 2}[x] \ne \text{R.}\lim_{h\to 2}[x]$

2. $\text{L.}\lim_{x\to 0}f(x) = \lim_{h\to 0}f(0-h) = \lim_{h\to 0}-h\sin\left(-\frac{1}{h}\right)$

$= \lim_{h\to 0}+h\sin\frac{1}{h} = 0$

$\text{R.}\lim_{x\to 0}f(x) = \lim_{h\to 0}f(0+h)$

$= \lim_{h\to 0}-h\sin\left(\frac{1}{h}\right) = 0$

Also $\quad f(0) = 0$

Since $\text{L.}\lim_{x\to 0}f(x) = \text{R.}\lim_{x\to 0}f(x) = f(0) = 0$

Hence $f(x)$ is continuous at $x = 0$

Now, $\text{L.}\lim_{x\to 0}g(x) = \lim_{h\to 0}g(0-h)$

$= \lim_{h\to 0}|-h| = \lim_{h\to 0}h = 0$

$\text{R.}\lim_{x\to 0}g(x) = \lim_{h\to 0}g(0+h)$

$= \lim_{h\to 0}|h| = \lim_{h\to 0}h = 0$

$g(0) = |0| = 0$

$\therefore \quad \text{L.}\lim_{x\to 0}g(x) = \text{R.}\lim_{x\to 0}g(x) = g(0) = 0$

Hence $g(x)$ is also continuous at $x = 0$

3. Let $a \in$ R (real numbers)

$\text{L.}\lim_{x\to a}f(x) = \lim_{h\to 0}|a-h| = \lim_{h\to 0}(a-h) = a$

$\text{R.}\lim_{x\to a}f(x) = \lim_{h\to 0}|a+h| = \lim_{h\to 0}a+h = a$

and $\quad f(a) = |a| = a$

$\therefore \quad \lim_{x\to a}|x| = a = f(a)$

Hence $\lim_{x\to a}|x|$ continuous for all values of x.

4. $\text{L.}\lim_{x\to 2}f(x) = \lim_{h\to 0}2a(2-h) - b = 4a - b$

$\text{R.}\lim_{x\to 2}f(x) = \lim_{h\to 0}a(2+h)^2 + b = 4a + b$

The function will be continuous if

$\text{L.}\lim_{x\to 2}f(x) = \text{R.}\lim_{x\to 2}f(x) = f(2)$

$\therefore \quad 4a - b = 2$

$4a + b = 2$

Solving we get, $a = \frac{1}{2}, \quad b = 0$

5. Given function is, $f(x) = x - [x]$

Let a is a real number except integer and h be a very small real number, then

$[a-h] = b$ (an integer less than to a)

$[a+h] = b$ (an integer greater than to a)

Also $\quad [a] = b$

$\therefore \quad \text{L.}\lim_{x\to 0}f(x) = \lim_{h\to 0}(a-h) - [a-h]$

$= \lim_{h\to 0}(a-h) - b = a - b$

Also $\quad f(a) = (a) - [a] = a - b$

$\therefore \quad \text{L.}\lim_{x\to a}f(x) = \text{R.}\lim_{x\to a}f(x) = f(a) = a - b$

$\therefore \quad f(x) = x - [x]$

is continuous all real except integers.

6. Since $f(x) = \frac{1}{x}$ is not continuous in [– 3, 3] [– 2, 2] or, [– 1, 1], hence the point of discontinuity is '0'. Only in [2, 3] the function is continuous, and differentiable. Hence mean value theorem is applicable in [2, 3].

7. Given function is

$$f(x) = 3x^4 - 4x^2 + 5$$

(*i*) $f(x)$ is continuous in every real interval

(*ii*) $f'(x)$ exist in any real interval

(*iii*) $\quad f(-1) = 3(-1)^4 - 4(-1)^2 + 5 = 4$

$f(1) = 3(1)^4 - 4(1)^2 + 5 = 4$

$f(-1) = f(1)$

Also $\quad f'(c) = 12c^3 - 8c = 0$

$\Rightarrow \quad c = 0, c = \pm\sqrt{\frac{2}{3}}$

$\therefore \quad c \in [-1, 1]$

Hence all the condition of Rolle's Theorem are satisfied in the interval [–1, 1]

8. Since $f(x) = |x|$ is continuous in [–1, 1] but it is not differentiable at $x = 0 \in (-1, 1)$

9. Since f and F are two continuous function in [a, b] such that

$$f'(x) = F'(x)$$

Let $\quad \phi(x) = f(x) - F(x)$

$\therefore \quad \phi'(x) = f'(x) - F'(x)$

$= 0 \quad$ [from equation]

$\therefore \quad \phi(x) = \text{constant}$

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10. Let $f(x) = x^{1/x}$

$\therefore \quad f'(x) = x^{1/x}\left(\dfrac{1-\log_e x}{x^2}\right) < 0$

$f(x)$ is decreasing if $f'(x) < 0$

i.e., $x^{1/x}\left(\dfrac{1-\log x}{x^2}\right) < 0$

$\Rightarrow\ 1 - \log_e x < 0 \quad \Rightarrow\ \log_e x > 1 \quad \Rightarrow\ x > e$

11. Let $f(x) = \cos x$

$$f^n(x) = \cos\left(x + \frac{n\pi}{2}\right)$$

$$\therefore \quad f'(0) = 0$$

$$f''(0) = -1$$

$$f'''(0) = 0$$

$$f''''(0) = 1$$

....

....

and so on

By Maclaurin's theorem, we get

$$\cos(x) = f(0) + xf'(0) + \frac{x^2}{\lfloor 2}f''(0) + \frac{x^2}{\lfloor 3}f'''(0)$$

$$+ \ldots + \frac{x^{2n}}{\lfloor 2n}f^{2n}(0) + \frac{x^{2n+1}}{\lfloor 2n+1}f^{2n+1}(\theta x)$$

$$= 1 + 0 - \frac{x^2}{\lfloor 2} + 0 + \frac{1}{\lfloor 4}\ldots + (-1)^{n+1}\frac{x^{2n+1}}{\lfloor 2n+1}\sin\theta x$$

$$= 1 - \frac{x^2}{\lfloor 2} + \frac{1}{\lfloor 4}x^4 + \ldots + (-1)^{n+1}\frac{x^{2n+1}}{\lfloor 2n+1}\sin\theta x$$

12. Given: $f(x) = \dfrac{1}{1+x^2}$

$\therefore \quad (1 + x^2) f(x) = 1; \qquad f(0) = 1$

$(1 + x^2) f'(x) + 2x f(x) = 0; \qquad f'(0) = 0$

$(1 + x^2) f''(x) + 4x f'(x) + 2f(x) = 0; \quad f''(0) = -2$

$(1 + x^2) f'''(x) + 6x f''(x) + 6f'(x) = 0; \quad f'''(0) = 0$

$(1 + x^2) f^{1v}(x) + 8x f'''(x) + 12 f''(x) = 0; f^{1v}(0)= 24$

and so on

$$\therefore \quad f(x) = f(0) + x f'(0) + \frac{x^2}{\lfloor 2}f'(0) + \ldots$$

$$\therefore \quad \frac{1}{1+x^2} = 1 + x.\,0 + \frac{x^2}{\lfloor 2}(-2) + \frac{x^2}{\lfloor 3}.0 + \frac{x^4}{\lfloor 4}.24$$

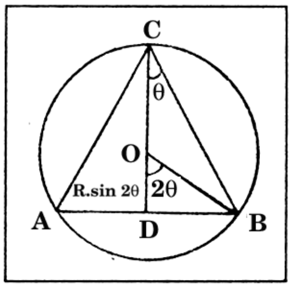
$$+ \ldots + (-1)^n.\frac{x^{2n}}{\lfloor 2n}.\lfloor 2n$$

$$= 1 - x^2 + x^4 + \ldots + (-1)^n.\,x^{2n}$$

$$= \sum_{n=0}^{\infty}(-1)^n.x^{2n} \text{ for all real } x.$$

18. Let ABC be a triangle inscribed in the circle with centre O and radius r.

If area of this triangle is maximum, then vertex C should be at a maximum distance from the base AB i.e., CD must be perpendicular to AB. Hence ABC is an isosceles triangle.



If $\angle BCD = \theta$,

where D is the mid-point of AB, then

$\angle BOD = 2\theta$

$\therefore \quad AB = 2BD = 2r \sin 2\theta$

$CD = CO + OD = r + r\cos 2\theta$

If S be the area of the triangle ABC, then

$$S = \frac{1}{2}AB \times CD = \frac{1}{2} \times 2r \sin 2\theta\ (r + r\cos 2\theta)$$

$$\therefore \quad \frac{ds}{d\theta} = r^2[\sin 2\theta\,(-2\sin 2\theta] + (1+\cos 2\theta)(2\cos 2\theta)]$$

For maximum and minimum,

$$\frac{dS}{d\theta} = 0$$

$\therefore \quad \cos 4\theta + \cos 2\theta = 0$

$\Rightarrow \quad 2\cos 3\theta \cos\theta = 0$

Hence either $\cos 3\theta = 0$, or $\cos\theta = 0$

If $\cos\theta = 0$, then $\theta = \dfrac{\pi}{2}$

If $\cos 3\theta = 0$, then $3\theta = \dfrac{\pi}{2}$

$\Rightarrow \quad \theta = \dfrac{\pi}{6}$

$\left(\dfrac{d^2S}{d\theta^2}\right)_{\theta=\pi/6}$ is negative.

Hence S is maximum for $\theta = \dfrac{1}{6}\pi$

$$\angle ACB = 2\theta = 2\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$$

$$= \angle ABC = \angle BAC$$

Hence ΔABC is an equalateral triangle.

19. Let radius of circle is r and side of a sqaure is x.

$\therefore \quad \text{Perimeter} = 2\pi r + 4x = l \qquad \ldots(i)$

Let area be A

$\therefore \quad A = \pi r^2 + x^2$

$$= \pi r^2 + \left(\frac{l - 2\pi r}{4}\right)^2 \qquad \text{[from equation } (i)]$$

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$$\frac{dA}{dr} = 2\pi r + \frac{2}{16}(1-2\pi r).(-2\pi)$$

and $\dfrac{d^2A}{dr^2} = 2\pi + \dfrac{\pi^2}{2}$

For maximum and minimum,

$$\frac{dA}{dr} = 0$$

$\Rightarrow \quad 2\pi r - \dfrac{\pi}{4}(l-2\pi r) = 0$

$\Rightarrow \quad l = 8r + 2\pi r \quad ...(ii)$

$\dfrac{d^2A}{dr^2} > 0$, hence area is least

From equations (i) and (ii),

$$2\pi r + 4x = 8r + 2\pi r$$

$\Rightarrow \; 4x = 8r \quad \Rightarrow \quad x = 2r$

Hence side of a square is double the radius of circle.

20. Since statements (a), (b), (c) are all correct, hence the statement (d) is false.

22. $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 1$

$f(0) = 1$

$f(2) = 2^4 - 8.2^3 + 22.2^2 - 24.2 + 1 = -7$

Now $\; f'(x) = 4x^3 - 24x^2 + 44x - 24$

For maximum and minimum,

$f'(x) = 0$

$\Rightarrow \quad 4x^3 - 24x^2 + 44x - 24 = 0$

$\Rightarrow \quad 4(x-1)(x-2)(x-3) = 0$

$\therefore \quad x = 1, 2, 3$

Since $x = 3$ does not lie in $[0, 2]$

Hence, consider only $x = 1$ and $x = 2$.

We have

$f(1) = 1^4 - 8.1^3 + 22.1^2 - 24.1 + 1 = -8$

Greatest of $\; f(x) =$ largest of $\{1, -7, -8\} = 1$

Least of $\; f(x) =$ smallest of $\{1, -7, -8\} = -8$

23. $f(x) = x^5 - 5x^4 + 5x^3 - 1$

$f'(x) = 5x^4 - 20x^3 + 15x^2$

$f''(x) = 20x^3 - 60x^2 + 30x$

$f'''(x) = 60x^2 - 120x + 30$

For maximum and minimum

$f'(x) = 0$

$\therefore \quad x^4 - 4x^3 + 3x^2 = 0$

$\Rightarrow \quad x^2(x-1)(x-3) = 0$

$\therefore \quad x = 0, 1, 3$

$f''(0) = 0$

$f'''(0) = 30 \neq 0$

$\therefore f(x)$ is neither maximum nor minimum at $x = 0$

$f''(1) = 20 - 60 + 30 = -10 < 0$

$\therefore f(x)$ is maximum at $x = 1$

$f''(3) = 540 - 540 + 90 = 90 > 0$

$\therefore f(x)$ is maximum at $x = 3$

Hence there is one maximum and one minimum for $f(x)$.

24. Given function is

$$f(x) = \frac{\log x}{x}$$

$$\therefore \quad f'(x) = \frac{x.\frac{1}{x} - \log x.1}{x^2} = \frac{1-\log x}{x^2}$$

and $$f''(x) = \frac{x^2.\left(-\frac{1}{x}\right) - (1-\log x).2x}{x^4}$$

$$= \frac{2\log x - 3}{x^3}$$

For maximum and minimum of $f(x)$,

$f'(x) = 0 \quad \Rightarrow 1 - \log x = 0 \quad \Rightarrow \quad x = e$

$$\therefore \quad f''(e) = \frac{2-3}{e^3} = -\frac{1}{e^3} < 0$$

Hence $f(x)$ is maximum at $x = e$

$\therefore$ Maximum value, $f(e) = \dfrac{\log e}{e} = \dfrac{1}{e}$

54. $\displaystyle\int_0^{5\pi}(2-\sin x)dx = [2x - \cos x]_0^{5\pi}$

$= 10\pi - 1 - 1 = 10\pi - 2 > 0$

55. $f(x) = 3x(x-2) = 3x^2 - 6x$

Differentiating, we get

$$\frac{df(x)}{dx} = f'(x) = 6x - 6$$

Again differentiating,

$$\frac{d^2f(x)}{dx} = f''(x) = 6 > 0$$

For maxima or minima,

$f'(x) = 0$

$\therefore \quad 6x - 6 = 0,$

$\Rightarrow \quad x = 1$

As $f'(x) = 6 > 0$, $f(x)$ is minimum at $x = 1$

56. $$\int_{-2}^{2}\frac{dx}{x^2} = \int_{-2}^{2}\frac{1}{x^2}dx$$

$f(x) = \dfrac{1}{x^2}$ = even function

For even function,

$$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx = 2\int_0^2 \frac{1}{x^2}dx = 2\left[-1\frac{1}{x}\right]_0^2 = \infty$$

$$\therefore \quad \int_{-2}^{2}\frac{1}{x^2} = \infty$$

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57. Given : $f(x) = Ax^2 + Bx + C$

$\Rightarrow \quad f'(x) = 2Ax + B \quad$ and $\dfrac{f(b)-f(a)}{b-a} = f'(\xi)$

$$\Rightarrow 2A\xi + B = \frac{(Ab^2+Bb+C)-(Aa^2+Ba+C)}{b-a}$$

$$= \frac{A(b^2-a^2)+B(b-a)}{b-a} = A(b+a)+B$$

$\therefore \quad \xi = \dfrac{b+a}{2}$

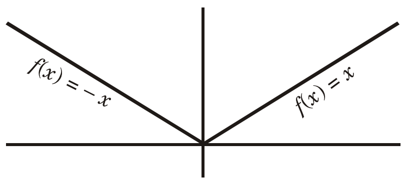
58. $f'(x) = 3(x^2 - 4x + 3), f''(x) = 6x - 12$

At $x = 1; f''(1) = -$ve; and, at $x = 3, f''(3) = +$ ve

59. $$\phi(x) = \int_0^{x^2} \sqrt{t}\ dt = \left|\frac{2}{3}t^{3/2}\right|_0^{x^2} = \frac{2}{3}x^3$$

$\therefore \quad \dfrac{d\phi}{dx} = \dfrac{2}{3} \times 3x^2 = 2x^2$

60. $f(x) = |x|$, and $f(x) = x$ for $x > 0$



$\Rightarrow \quad f(x) = -x$ for $x < 0$

Hence from graph it is clear that derivative does not exist for $x = 0$.

61. $$f(x) = \frac{x^3}{3} - x$$

$\Rightarrow \quad f'(x) = x^2 - 1$

For maximum and minimum value, $f'(x) = 0$

$\therefore \quad x^2 - 1 = 0$

$\Rightarrow \quad x = \pm 1$

Again $\quad f''(x) = 2x$

At $x = 1, \quad f''(x) = 2 > 0$

$x = 1, \quad f''(x) = -2 < 0$

Hence minimum at $x = 1$

62. $\vec{P} = 0.866i + 0.500j + 0k, \ \vec{Q} = 0.259i + 0.966j + 0k$

$\therefore \quad \vec{P} \cdot \vec{Q} = |P|\,|Q|\cos\theta$

$= (0.866i + 0.5j + 0k)\ (0.259i + 0.966j + 0k)$

$= \sqrt{(0.866)^2 + (0.5)^2} \times \sqrt{(0.259)^2 + (0.966)^2} \cos\theta$

$\therefore \quad \cos\theta = \dfrac{0.866 \times 0.259 + 0.5 \times 0.966}{\sqrt{0.9999} \times \sqrt{1.001}} = 0.707$

$\Rightarrow \quad \theta = 45°$

63. $$V = \int_0^{2\pi}\int_0^{\pi/3}\int_0^1 r^2 \sin\phi\, dr\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/3}\left[\frac{r^3}{3}\right]_0^1 \sin\phi\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/3} \frac{1}{3}\sin\phi\, d\phi\, d\theta$$

$$= \frac{1}{3}\int_0^{2\pi} [-\cos\phi]_0^{\pi/3}\, d\theta$$

$$= \frac{1}{3}\int_0^{2\pi} \frac{1}{2}\, d\theta = \frac{1}{6} \times 2\pi = \frac{\pi}{3}.$$

64. Given: $\quad y = x^2 + \dfrac{250}{x}$

$\therefore \quad \dfrac{dy}{dx} = 2x - \dfrac{250}{x^2}$,

and $\quad \dfrac{dy^2}{dx^2} = 2 + \dfrac{250 \times 2}{x^3}$

When $x = 5$, $\quad \dfrac{dy}{dx} = 10 - 10 = 0$

and $\dfrac{d^2y}{dx^2}$ is positive

Therefore function is minimum at $x = 5$.

65. Given, $\quad y^2 = 8x$ and $P(x, y)$ is any point

Volume of the element

$$= \pi[2^2 - x^2]\, dy = \pi\left[4 - \frac{y^4}{64}\right] dy$$

$\therefore$ Required volume $= 2\pi \displaystyle\int_0^4 \left[4 - \frac{y^4}{64}\right] dy$

$$= 2\pi\left[4y - \frac{y^5}{320}\right]_{y=0}^4 = 2\pi\left[16 - \frac{4^5}{320}\right] = \frac{128\pi}{5}$$

66. $\int \log x \,.\, dx = \log x \,.\, x - \int x \,.\, \dfrac{d}{dx}(\log x)\, dx$

$= x \log x - \int 1 \,.\, dx = x \log x - x = x(\log x - 1)$

68. $$\lim_{a\to\infty}\int_1^a x^{-4} dx = \lim_{a\to\infty}\left[-\frac{1}{3}\left(x^{-3}\right)_1^a\right]$$

$$= \lim_{a\to\infty} -\frac{1}{3}\left[a^{-3} - 1\right] = -\frac{1}{3}[0-1] = \frac{1}{3}$$

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NUMERICAL TYPE QUESTIONS

1. $$\lim_{x\to 0}\frac{e^x-1}{x} = \lim_{x\to 0}\frac{\left(1+x+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\ldots\right)-1}{x}$$

$$= \lim_{x\to 0}\left(1+\frac{x}{\lfloor 2}+\frac{x^2}{\lfloor 3}+\ldots\right) = 1$$

By L' Hospital's Rule

$$\lim_{x\to 0}\frac{e^x-1}{x}\left[\text{form}\frac{0}{0}\right] = \lim_{x\to 0}\frac{e^x}{1} = 1$$

4. $$\lim_{x\to\infty} x^n e^{-x} = \lim_{x\to\infty}\frac{x^n}{e^x} \qquad \left[\text{form}\frac{\infty}{\infty}\right]$$

By L' Hospital Rule Differentiating denominator and numerator n times, we get

$$\lim_{x\to\infty}\frac{x^n}{e^x} = \lim_{x\to\infty}\frac{\lfloor n}{e^x} = 0$$

5. $$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$$

$$= \lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\times\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$$

$$= \lim_{x\to 0}\frac{1+x-1+x}{x(\sqrt{1+x}+\sqrt{1+x})}$$

$$= \lim_{x\to 0}\frac{2x}{x(\sqrt{1+x}+\sqrt{1-x})}$$

$$= \frac{2}{\sqrt{1+0}+\sqrt{1-0}} = \frac{2}{1+1} = 1$$

6. Let $$A = \lim_{n\to 0} e^{\frac{1}{n\log n}}$$

$$\therefore \quad \log_e A = \lim_{n\to 0}\frac{1}{n\log n}.$$

$$\Rightarrow \quad \log_e A = \lim_{n\to 0}\frac{1/n}{\log n} \qquad \left[\text{form}\frac{\infty}{\infty}\right]$$

$$= \lim_{n\to 0}\frac{-1/n^2}{1/n}$$

$$= \lim_{n\to 0}-\frac{1}{n}$$

$$\log_e A = -\infty$$

$$A = e^{-\infty} = 0$$

7. $$\lim_{x\to\infty} x\sin\frac{1}{x} = \text{Let }\frac{1}{x} = n$$

So, $x\to\infty,\ n\to 0$

$$\lim_{n\to 0}\frac{\sin n}{n} = 1$$

8. $$\lim_{n\to\infty}\left[\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\ldots+\frac{1}{n}\right]$$

$$= \lim_{n\to\infty}\left[\frac{1+2+3+\ldots+n}{n^2}\right]$$

$$= \lim_{n\to\infty}\frac{n(n+1)/2}{n^2}$$

$$= \lim_{n\to\infty}\frac{1}{2}\left(1+\frac{1}{n}\right)$$

$$= \frac{1}{2}(1+0) = \frac{1}{2}$$

9. Given function is

$$f(x) = x(x-2) = x^2-2x$$

$f(x)$ is continuous function in $\left[0,\frac{3}{2}\right]$ and differentiable in $\left(0,\frac{3}{2}\right)$. Therefore, mean value thorem is applicable in $\left[0,\frac{3}{2}\right]$

$$f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2-2\left(\frac{3}{2}\right)$$

$$= \frac{-3}{4}$$

$$f(0) = 0$$

By Lagrange's mean value theorem, there exist $c\in\left(0,\frac{3}{2}\right)$ such that

$$\therefore \quad f'(c) = \frac{f\left(\frac{3}{2}\right)-f(0)}{\frac{3}{2}-0}$$

$$\Rightarrow \quad 2c-2 = \frac{-\frac{3}{4}-0}{\frac{3}{2}} \text{ or } -\frac{1}{2}$$

$$\Rightarrow \quad c = +\frac{3}{4}\in\left(0,\frac{3}{2}\right)$$

10. Value of a mod function cannot be less than zero, therefore

$$f(x) = |x^2-5x+2| \text{ is zero.}$$

11. Let $$f(x) = \frac{1}{\sqrt{2}}(\sin x-\cos x)$$

$$= \sin x\cos\frac{\pi}{4}-\cos x\sin\frac{\pi}{4} = \sin\left(x-\frac{\pi}{4}\right)$$

Since maximum value of $\sin\theta$ is 1, hence maximum value of $f(x) = 1$

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12. Let $f(x) = -x^3 + 6x^2 + 2x + 1$

Slope of the function is,

$$f'(x) = -3x^2 + 12x + 2 = F(x) \text{ (say)}$$

Now to find minimum or maximum of $F(x)$.

$\therefore$ $F'(x) = -6x + 12$

and $F''(x) = -6$

For maxima and minima

$F'(x) = 0$

$\Rightarrow$ $x = 2$

$\therefore$ $F''(2) = -6 < 0$

Hence $F(x)$ (slope) is maximum at $x = 2$

$\therefore$ Maximum slope $= F(2) = -3(2)^2 + 12(2) + 2 = 14$

13. Volume is maximum at $x = 10$

At $x = 10$, $y = \dfrac{300-100}{40} = 5$

$\therefore$ Maximum volume $= x^2y = (10)^2 \times 5 = 500$

15. $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x+y)\, dx.dy = [\ \cos(x+y)]_0^{\pi/2} dy$

$$= \left[\sin y - \sin\left(\frac{\pi}{2}+y\right)\right]_0^{\pi/2}$$

$$= (1-0) - (0-1) = 2$$

EXERCISE – II

MCQ TYPE QUESTIONS

1. $\lim\limits_{x\to\infty} x^{1/x} = (\infty)^0 = 1$

$\therefore$ Option (c) is correct.

4. $af(x) + b.f\left(\dfrac{1}{x}\right) = \dfrac{1}{x} - 25$ ...(1)

$\Rightarrow$ $af\left(\dfrac{1}{x}\right) + b.f(x) = x - 25$ ...(2)

Solving, we get

$$f(x) = \frac{1}{a^2-b^2}\left[a\left(\frac{1}{x}-25\right) - b(x-25)\right]$$

$$\therefore \int_1^2 f(x)dx = \frac{1}{a^2-b^2}\left[a\{\ln x - 25x\}_1^2 - b\left\{\frac{x^2}{2} - 25x\right\}_1^2\right]$$

$$= \frac{1}{a^2-b^2}\left[a\{\ln 2 - 25\} - b\left\{\frac{3}{2} - 25\right\}\right]$$

$$= \frac{1}{a^2-b^2}\left[a\{\ln 2 - 25\} + \frac{47}{2}b\right]$$

5. By Mean value theorem.

6. Define $g(x) = f(x) - f(x+1)$ in $[0,1]$. $g(0)$ is negative and $g(1)$ is positive. By intermediate value theorem there is $y€(0,1)$ such that $g(y) = 0$

That is $f(y) = f(y+1)$.

Thus answer is (a)

7. $\int_0^{\pi} x^2 \cos x\, dx = x^2(\sin x) - 2x(-\cos x) + 2(-\sin x)\Big|_0^{\pi}$

$$= (\pi^2 \sin\pi + 2\pi\cos\pi - 2\sin\pi) - (0+0+0)$$

$$= -2\pi$$

8. Continuous at $x = 3$?

(a) $f(x) = \begin{cases} 2 & \text{if } x = 3 \\ x-1 & \text{if } x > 3 \\ \dfrac{x+3}{3} & \text{if } x < 3 \end{cases}$

$$\underset{x\to 3-}{\text{Lt}} f(x) = \underset{x\to 3-}{\text{Lt}} \frac{x+3}{3} = 2$$

$$\underset{x\to 3+}{\text{Lt}} f(x) = \underset{x\to 3+}{\text{Lt}}\ x - 1 = 2$$

$$\therefore \underset{x\to 3-}{\text{Lt}} f(x) = f(x)\big|_{x=3} = \underset{x\to 3+}{\text{Lt}} f(x)$$

$\Rightarrow$ This is continuous at $x = 3$

(b) $f(x) = \begin{cases} 4 & \text{if } x = 3 \\ 8-x & \text{if } x \neq 3 \end{cases}$

Option (b) is false

$$\underset{x\to 3^-}{\text{Lt}} = \underset{x\to 3^-}{\text{Lt}}\ 8 - x = 5 \neq f(x)\big|_{x=3}$$

(c) $f(x) = \begin{cases} x+3 & \text{if } x \le 3 \\ x-4 & x > 3 \end{cases}$

$$\underset{x\to 3^+}{\text{Lt}} f(x) = \underset{x\to 3^+}{\text{Lt}}\ x - 4$$

$$f(x)\big|_{x=3} = 3 + 3 = 6 = -1$$

$$\underset{x\to 3^+}{\text{Lt}} f(x) \neq f(x)\big|_{x=3}$$

Option (c) is false

(d) $f(x) = \dfrac{1}{x^3 - 27}$ if $x \neq 3$

10. $\int_0^{\pi/2} \dfrac{e^{ix}}{e^{-ix}}\, dx = \int_0^{\pi/2} e^{2ix} dx$

$$= \left(\frac{e^{2ix}}{2i}\right)_0^{\pi/2} = \frac{1}{2i}\,[e^{i\pi} - 1]$$

$$= \frac{1}{2i}\,[\cos\pi + i\sin\pi - 1]$$

$$= \frac{1}{2i}[-1 + 0 - 1]$$

$$= \frac{-2}{2i}$$

$$= \frac{-1}{i} \times \frac{i}{-1} = i$$

11. $\lim\limits_{n\to\infty}\left(1 - \dfrac{1}{n}\right)^{2n} = \left(\lim\limits_{n\to\infty}\left(1 - \dfrac{1}{n}\right)^{-n}\right)^2$

$$= (e)^{-2} = e^{-2}$$

[because $\lim\limits_{n\to\infty}\left(1 - \dfrac{1}{f(x)}\right)^{-f(x)}$

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12. $$\frac{1-\tan x}{1+\tan x}dx = \frac{\cos x - \sin x}{\cos x + \sin x}dx$$

Let $\cos x + \sin x = t$

$\Rightarrow$ $(-\sin x + \cos x)\, dx = dt$

$$\therefore \quad \left(\frac{1}{t}\right)dt = ln\ t$$

$$= \ln(\sin x + \cos x)$$

$$= \ln\left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right)$$

$$= \ln\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{1}{2}\ \ln 2$$

13. $$\lim_{x\to\infty}\frac{x-\sin x}{x+\cos x} = \lim_{x\to\infty}\frac{x(1-\sin x/x)}{x(1+\cos x/x)}$$

$$= \lim_{x\to\infty}\frac{x1-\sin x/x}{x1+\cos x/x}$$

$$= \frac{1-\lim\limits_{x\to\infty}\dfrac{\sin x}{x}}{1+\lim\limits_{x\to\infty}\dfrac{\cos x}{x}} = \frac{1-0}{1+0} = 1$$

14. Given : $y = 3x^4 - 16x^3 + 24x^2 + 37$

For maximum value or minimum

$$\frac{dy}{dx} = 12x^3 - 18x^2 + 43x = 0$$

$\Rightarrow$ $x(x^2 - 4x + 4) = 0$

$\Rightarrow$ $x(x-2)^2 = 0$

$\Rightarrow$ $x = 0, 2, 2$

$$\frac{d^2y}{dx^2} = 36x^2 - 96x + 48$$

At $x = 0$, $\dfrac{d^2y}{dx^2} = +48$

At $x = 2$, $\dfrac{d^2y}{dx^2} = 36 \times 4 - 96 \times 2 + 48$

$= 0$

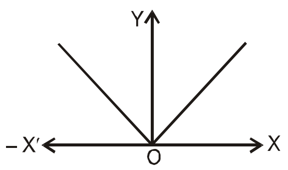
so only 1 local minima.

either $x = 0$ or complex root

so only value = 0

15. $$f(x) = |x|,\ f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

Graph of $f(x)$ will be



From above graph for $f(x)$, we can easily see that $f(x)$ is continuous for all real values of x.

But $f(x)$ is not differentiable for all real values of x.

For example at $x = 0$,

$$\text{Left hand derivative} = \lim_{h\to o}\frac{f(O-h)-f(O)}{-h}$$

$$= \lim_{h\to o}\frac{-(O-h)}{-h} = -1 = -1$$

$$\text{Right hand derivative} = \lim_{h\to o}\frac{f(O+h)-(O)}{h}$$

$$= \lim_{h\to o}\frac{h}{h} = 1$$

Since, Left hand derivative

$\neq$ Right hand derivative.

Therefore, $f(x)$ is not differentiable at $x = 0$, hence notz differentiable for all real values of x.

NUMERICAL TYPE QUESTIONS

2. Given $f''(x) + f(x) + t\cos x = 0$

and $f(x) = x\sin x$

$f'(x) = x\cos x + \sin x$

$f''(x) = x(-\sin x) + \cos x + \cos x$

$= 2\ \cos x - x\sin x$

$= 2\cos x - f(x)$

$\therefore$ $2\cos x - f(x) + f(x) + t\cos x = 0$

$\Rightarrow$ $2\cos x = -t\cos x$

$\Rightarrow$ $t = -2$

3. $$\int_0^{2\pi}|x\sin x|dx = \text{K}\pi$$

$$\Rightarrow \int_0^{\pi}x\sin x\, dx + \int_{\pi}^{2\pi}-(x\sin x)dx = \text{K}\pi$$

$$\left[\begin{matrix}\because |\sin x| = -\sin x \\ \pi < x < 2\pi\end{matrix}\right]$$

$$\Rightarrow \quad x(-\cos x) - 1(-\sin x)\Big|_0^{\pi} - (-x\cos x + \sin x)\Big|_{\pi}^{2\pi} = \text{K}\pi$$

$$\Rightarrow \left(-\pi\cos\pi + \sin\pi\right) - 0 - \left[\left(-2\pi\cos 2\pi + \sin 2\pi\right)\right.$$

$$\left. - \left(-\pi\cos\pi + \sin\pi\right)\right] = \text{K}\pi$$

$$\Rightarrow \pi + 0 - \left[-2\pi + 0 - (\pi + 0)\right] = \text{K}\pi$$

$\Rightarrow$ $4\pi = \text{K}\pi$

$\Rightarrow$ $\text{K} = 4$

■■

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3

CHAPTER

Probability

PROBABILITY

It is a concept of mathematics which measures the degree of certainty or uncertainty of the occurrence of events.

- If any event can happen in m ways and fails in n ways and each of the $(m + n)$ ways are equally likely to occur, then probability of the happening of the events is defined as the ratio, $\frac{m}{m+n}$ and that of its failing as $\frac{n}{m+n}$.

  If probability of the happening is denoted by p and not happening by q, then $p + q = 1$.
- If event is certain to happen, its probability is unity.
- If happening is impossible, then its probability is zero.

CONDITIONAL PROBABILITY

Consistency of the definition with the relative frequency concept of probability can be obtained from the following construction.

Let an experiment is repeated a large number, of times, N resulting in

both A and B, $A \cap B$, n_{11} times;

A, not B, $A \cap \bar{B}$, n_{21} times;

B, not A, $\bar{A} \cap B$, n_{12} times;

and neither A nor B, $\bar{A} \cap \bar{B}$, n_{22} times.

These results are:

	A	$\bar{A}$	
B	n_{11}	n_{12}	$n_{11}+n_{12}$
$\bar{B}$	n_{21}	n_{22}	$n_{21}+n_{22}$
	$n_{11}+n_{21}$	$n_{12}+n_{22}$	N

$n_{11} + n_{12} + n_{21} + n_{22} = N.$

Then it follows that with large N

$$P(A) \approx \frac{n_{11}+n_{21}}{N}, \quad P(B) \approx \frac{n_{11}+n_{12}}{N},$$

$$P(A \cup B) \approx \frac{n_{11}+n_{12}+n_{21}}{N}$$

and conditional probabilities

$$P(B|A) \approx \frac{n_{11}}{n_{11}+n_{21}} \quad \text{and} \quad P(A \cap B) \approx \frac{n_{11}}{N}$$

$$P(A/B) \approx \frac{n_{11}}{n_{11}+n_{12}}$$

Here definitions of P(B | A) and P(A | B) is verified.

Example. Let a box contains r red balls labeled 1, 2, 3, ..., r and b black balls labeled 1, 2, 3, ..., b. Assume that the probability of drawing any particular ball is $\frac{1}{b+r}$

If a ball from the box is known to be red, what is the probability it is the red ball labeled 1?

Solution.

First find probability that it is red (event A).

$$P(A) = \frac{r}{b+r}$$

Then compute probability that the ball is the red ball with the number 1 on it.

$$P(A \cap B) = \frac{1}{b+r}$$

where B is the event that the ball has a 1 on it.

Then probability that the ball is red and labeled 1 given that it is red is given by

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{b+r}}{\frac{r}{b+r}} = \frac{1}{r}$$

This differs from the unconditional probability of B, which is given by

$$P(B) = \frac{2}{b+r}$$

Summation Rule

Let $B_1, B_2, ..., B_n$ be (pairwise) disjoint events of positive probability whose union is $\cap$.

Then $\quad A = \bigcup_{j=1}^{n} (A \cap B_j)$

Using multiplication rule, we have

$$P(A) = P\left(\bigcup_{j=1}^{n} (A \cap B_j)\right) = \sum_{j=1}^{n} P(A \cap B_j)$$

$$= \sum_{j=1}^{n} P(A|B_j) P(B_j)$$

e.g. Consider rolling a die. Let the sample space be given by $\Omega = \{1, 2, 3, 4, 5, 6\}$

Now consider a partition B_1, B_2, B_3, given by

$B_1 = \{1, 2\}$
$B_2 = \{3, 4\}$
$B_3 = \{5, 6\}$

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Let A be the event that the number on the die is equal to one or greater than or equal to four. This gives

$$A = \{1, 4, 5, 6\}$$

$$P(A) = \frac{4}{6} = \frac{2}{3}.$$

This can also be computed using the formula as follows :

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3)$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + (1)\left(\frac{1}{3}\right)$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{3} = \frac{2}{3}$$

Bayes Rule

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A)}$$

Now, if $B_1, B_2, ..., B_n$ are disjoint events of positive probability whose union is Ω, then

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_{j=1}^{n} P(A|B_j)P(B_j)}$$

Consider same example as before.

Probability for B_1,

$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A)} = \frac{\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}{\frac{2}{3}} = \frac{\frac{1}{6}}{\frac{2}{3}} = \frac{1}{4}$$

Probability for B_2,

$$P(B_2|A) = \frac{P(A|B_2)P(B_2)}{P(A)} = \frac{\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}{\frac{2}{3}} = \frac{\frac{1}{6}}{\frac{2}{3}} = \frac{1}{4}$$

Probability for B_3,

$$P(B_3|A) = \frac{P(A|B_3)P(B_3)}{P(A)} = \frac{(1)\left(\frac{1}{3}\right)}{\frac{2}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

Conditional Probability for given Multiple events

Conditional probability of A given $B_1, B_2, ...B_n$, is written $P(A|B_1, B_2, ..., B_n)$ and is defined as

$$P(A|B_1, B_2, ..., B_n) = P(A|B_1 \cap B_2 \cap ... \cap B_n)$$

for any events A, $B_1, B_2, ..., B_n$ such that

$$P(B_1 \cap B_2 \cap ... \cap B_n) > 0$$

More Complex Multiplication Rule

$$P(B_1 \cap B_2 \cap B_3 ... \cap B_n)$$

$$= P(B_1)P(B_2 | B_1)P(B_3 | B_1B_2)...P(B_n | B_1, B_2, ...B_{n-1})$$

ELEMENTS OF PROBABILITY.

1. **Experiment**

   It is a process in which a certain work is repeated under the same conditions, the outcomes (or results) of which need not be the same.

   e.g. tossing a coin or rolling a die are experiments

2. **Sample space.**

   The set S of all possible outcomes of a given experiment is called *sample space* for the experiment. An outcome an element of S, is called a *sample point*.

   e.g. for experiment of tossing a fair (or unbiased) coin

   sample space S = (H, T)

   where H, T refer to head, tail respectively.

3. **Event**

   A subset of the sample space S is called an *event*. The set (a) consisting of a single sample point a in S is called *elementary event*.

   Since empty set ϕ and S are always subsets of S, they are also events ϕ is called *impossible event* or *null event*, and S is called *sure* or *certain event*.

   Given two events A, B. form New events can be formed using the set operations of union, intersection and complementation.

   (*i*) $A \cup B$ is the event that occurs if and only if A occurs or B occurs (or both).

   (*ii*) $A \cap B$ is the event that occurs if and only if both A and B occurs.

   (*iii*) A^c or $\overline{A}$, the complement of A, is the event that occurs if and only if A does not occur.

Mutually exclusive events

Two events A and B are said to be mutually exclusive if and only if they cannot occur simultaneously, i.e. if $A \cap B = \phi$.

Three or more events are called *mutually exclusive*, if every two of them are mutually exclusive.

Example. Find total number of possible events that can occur for an experiment.

Solution.

Let sample space S consist of n sample points. Then total number of possible events

= total number of subsets of S

= $|p(S)|$

where $p(S)$ is power set of $S = 2^n$.

Example. In an experiment, a fair coin is tossed 4 times. Describe the sample

Solution.

The sample space S consists of 16 (= 2^4) sample points.

S = [HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, THTH, HTHT, THHT, HTTT, THTT, TTHT, TTTH, TTTT]

Finite sample space

A sample space S is said to be a finite sample space, if S is a finite set.

Finite probability space

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Let S = (a_1, a_2 ...., a_n) be a finite sample space. S is said to be a finite probability space or probability model, if each sample point a_i in S we can assign a real number p_i, called probability of a_i such that

(*i*) $p_i \geq 0$, for each i

(*ii*) $p_1 + p_2 + \ldots\ldots + p_n = 1$

If A is an event, then probability of A, denoted by P(A) is sum of the probabilities of the sample points in A. For the elementary event $\{a_i\}$ we write P(a_i) instead of P (a_i).

Equiprobable space

A finite probability space S in which each sample point has the same probability is called an *equiprobable space*. If E is an event, then

$$P(E) = \frac{\text{number of elements in E}}{\text{number of elements in S}} = \frac{|E|}{|S|}$$

$$= \frac{\text{number of outcomes favourable to E}}{\text{total number of possible outcomes}}$$

- The word "*at random*" is used only when dealing with a equiprobable space. By the statement, "*a ball is drawn at random from a bag condisining 10 balls*" we mean that each ball in the bag has the same probability of being chosen.
- If p = probability of happening of an event E = P(E), q = probability of not happening of event E = $P(\overline{E})$

then $\quad q = P(\overline{E}) = \dfrac{|\overline{E}|}{|S|} = \dfrac{|S|-|E|}{|S|} = 1 - \dfrac{|E|}{|S|} = 1 - p$

$\Rightarrow \quad p + q = 1$

LAW OF TOTAL PROBABILITY.

Let B_1 B_2, ... B_n be mutually exclusive and let an event A occur only if anyone of B_i occurs. Then

$$P(A) = \sum_{i=1}^{n} P(A/B_i)P(B_i)$$

Baye's Theorm

Let E_1, E_2, ... E_n be mutually exclusive events such that $P(E_i) > 0$ for each i.

Then for any event $A \subseteq \bigcup_1^n E_i$ such that P(A) > 0, we have

$$P(E_i/A) = \frac{P(A/E_i)P(E_i)}{\sum_{i=1}^{n} P(A/E_i)P(E_i)}, \; i = 1, 2, ,., n$$

Example. Three urns A, B and C have 1 white , 2 black, 3 red balls, 2 white 1 black, 1 red balls and 4 white, 5 black, 3 red balls respectively. One urn is chosen at random and two balls are drawn. They happen to be white and red balls. What is the probability that they came from urn B ?

Solution.

Let E_1, E_2 and E_3 be the events of choosing urns A, B and C respectively.

Then $\quad P(E_1) = P(E_2) = P(E_3) = \dfrac{1}{3}$ ... (*i*)

Let X be the event of choosing two balls, white and red. To find $P(E_2/X)$.

By Baye's theorem, $\quad (E_2/X)$

$$= \frac{P(X/E_2)P(E_2)}{P(X/E_1)P(E_1) + P(X/E_2)P(E_2) + P(X/E_3)P(E_3)}$$

$$= \frac{P(X/E_2)}{P(X/E_1) + P(X/E_2) + P(X/E_3)} \quad \{\text{from } (i)\}$$

Now, $\quad P(X/E_1) = \dfrac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \dfrac{1}{5}$;

$P(X/E_2) = \dfrac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \dfrac{1}{3}$ and $P(X/E_3)$

$$= \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{2}{11}$$

From (*ii*), $\quad P(E_2/X) = \dfrac{\frac{1}{3}}{\frac{1}{5} + \frac{1}{3} + \frac{2}{11}} = \dfrac{55}{118}$

MEASURES OF CENTRAL TENDENCY

1. **Mean** (Arithmetic Mean A.M. or the average),
2. **Median** (positional average)
3. **Mode** (value of the variable which occurs most, frequently)
4. **Geometric Mean** (G.M. used extensively in finding the rate of population growth)
5. **Harmonic Mean** (H. M.)

1. Arithmetic mean $(\overline{x})$

Let x_1, x_2, ... x_n be the observed values and let f_i be the frequency of x_i (t_i) value of x_i repeated f_i times.

i.e., if the data is 3, 1, 3, 3, 2, 1, 3, 2, 2, then $x_1 = 1$, $x_2 = 2$ and $x_3 = 3$, and $f_1 = f_2 = 3$ and $f_3 = 4$.

$$\overline{x} = \frac{1}{n}(x_1 + x_2 + \ldots. + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

and for frequency distribution

$$\overline{x} = \frac{1}{N}(f_1 x_1 + f_2 x_2 + \ldots. + f_n x_n)$$

$$\overline{x} = \frac{1}{N}\sum_{i=1}^{n} f_i x_i \quad \text{where} \quad N = \sum_{i=1}^{n} f_i$$

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2. Median

Median is the middle score. If number of events are even then average of the two middles are taken. The median is better for describing the typical value.

Example. At a ski rental shop, data was collected on number of rentals on each of ten consecutive Saturdays:

44, 50, 38, 96, 42, 47, 40, 39, 46, 50.

To find sample mean, add them and divide by 10:

$$\frac{44+50+38+96+42+47+40+39+46+50}{10} = 49.2$$

Note: Mean value is not a value of the sample.

To find the median, first sort the data:

38, 39, 40, 42, 44, 46, 47, 50, 50, 96

There are two middle numbers 44 and 46. To find the median, we take average of the two.

$$\text{Median} = \frac{44+46}{2} = 45$$

Also mean is larger than all but three of the data points. The mean is influenced by outliers while the median is robust.

$$\text{Median} = L + \left(\frac{\frac{N}{2} - F_{i-1}}{f_m}\right)h$$

where, L = lower limited of Median class
N = Total Number of data items
h = width of Median class
f_m = frequency of Median class.

3. Mode.

Mode is the value which occurs most frequently.

$$\text{Mode} = a + \frac{C(f_i - f_{i-1})}{2f_i - f_{i-1} - f_{i+1}}$$

where a = modal class
f_i = maximum frequency
C = constant difference for each class.

Example. Find median of the following data.

Cost	10–20	20–30	30–40	40–50	50–60
Items in a group	4	5	3	6	3

Solution.

Cost	Number of items in the group	Cumulative frequency
10 – 20	4	4
20 – 30	5	9
30– 40	3	12
40 – 50	6	18
50 – 60	3	21

Here, N = 21.

$$\therefore \quad \frac{N}{2} = 10.5$$

The median class is 30 – 40.

$$\text{From formula, Median} = 30 + \frac{10}{12}(10.5 - 9) = 30 + 1.25 = \mathbf{31.25}$$

Example. Find mode for the following distribution:

Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	5	9	8	12	28	20	12	11

Solution.

Maximum frequency = 28

$\therefore$ Modal class = 40 – 50

$$\text{Mode} = a + \frac{C(f_i - f_{i-1})}{2f_i - f_{i-1} - f_{i+1}} = 40 + \frac{10(28-12)}{(2 \times 28) - 12 - 20} = 40 + 6.666 = \mathbf{46.67}$$

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Example. The expenditure of 100 families is given in the following frequency table.

Monthly Expenditure (in hundreds of rupees)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of families	14	?	27	?	15

If mode for the distribution is 24, calculate the missing frequencies.

Solution.

Let missing frequencies for the classes 10 – 20 and 30 – 40 be f_1 and f_2 respectively.

Expenditure	**Number of families**	**Cumulative frequency**
0 – 10	14	14
10 – 20	f_1	$14 + f_1$
20 – 30	27	$41 + f_1$
30 – 40	f_2	$41 + f_1 + f_2$
40 – 50	15	$56 + f_1 + f_2$

$$N = 100 = 56 + f_1 + f_2$$

$$\Rightarrow \quad f_1 + f_2 = 100 - 56 = 44 \qquad \text{....}(i)$$

Mode is given to be 24, which lies in the class 20 – 30. So 20 – 30 is the modal class.

$$\text{Mode} = a + \frac{C(f_i - f_{i-1})}{2f_i - f_{i-1} - f_{i+1}}$$

$$\therefore \quad 24 = 20 + \frac{10(27 - f_1)}{(2 \times 27) - f_1 - f_2}$$

$$\Rightarrow \quad 24 - 20 = \frac{270 - 10f_1}{54 - f_1 - f_2} = \frac{270 - 10f_1}{54 - 44} \qquad (\because f_1 + f_2 = 44)$$

$$\therefore \quad f_1 = 23$$

Substituting in (i), we get $f_2 = 21$.

Relation between Mean, Mode and Median.

For a symmetrical distribution

$$\text{Mean} = \text{Median} = \text{Mode}$$

For a moderately asymmetrical distribution

$$\text{Mean} - \text{Mode} = 3\,(\text{Mean} - \text{Median})$$

4. Geometric mean (G)

$$G = \sqrt[n]{x_1 x_2 \ldots\ldots x_n} = (x_1 x_2 \ldots\ldots\ldots x_n)^{1/n}$$

For a frequency distribution,

$$G = \left(x_1^{f_1} x_2^{f_2} \ldots\ldots x_n^{f_n}\right)^{1/N}, \quad \text{where } N = \sum_{i=1}^{n} f_i$$

For both cases, taking log

$$\log G = \frac{1}{n} \sum_{i=1}^{n} \log(x_i)$$

For a frequency distribution

$$\log G = \frac{1}{N} \sum_{i=1}^{n} f_i \log(x_i)$$

- If n_1 and n_2 are sizes and G_1 and G_2 are geometric means of two series, then geometric mean of the combined series is given by

$$\log G = \frac{n_1 \log G_1 + n_2 \log G_2}{n_1 + n_2}$$

- Geometric mean is used to find the rate of population growth and the rate of interest and it is also used in the construction of index numbers.

5. Harmonic mean.

It is the reciprocal of mean of reciprocals, i.e.

$$H = \frac{1}{\left(\frac{1}{n} \sum_{i=1}^{n} (1/x_i)\right)}$$

For a frequency distribution,

$$H = \frac{1}{\left(\frac{1}{N} \sum_{i=1}^{n} (f_i / x_i)\right)}; \text{ where } N = \sum_{i=1}^{n} f_i$$

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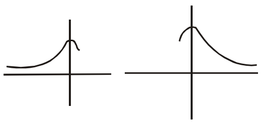
Skewness.

It is defined as lack of symmetry.

$$\text{Coefficient of skewness} = \frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}}$$



Negatively skewed Positively skewed

MEASURES OF DISPERSION

The set of constant which would in a concise way explain the "*variability*" or "*spread*" in a data is called *measures of dispersion or variability*.

The average for two groups of the same number of measurements may be equal, but one group may be more variable than the other.

e.g. set of five values 5, 6, 7, 8, 9 has the mean as 7; while another set of five values 1, 6, 4, 10, 14 also has the same mean 7. The second set has, obviously more variability than the first.

Usually four measures of dispersion or variability are defined:

1. **Range.**

   This is the difference between two extreme observations in the data given.

   In a frequency distribution ,

   R = (largest x value) – (smallest x value).

   Obviously, this is not a very satisfactory measure except in cases where the number of observations are small and a quick calculation for the scatter is needed. It is used in statistical quality control studies rather widely.

2. **Quartile deviation (Q.D.)**

   Median bisects the distribution. If distribution divided into four parts, *quartiles* are obtained. *First quartile*, Q_1, would have 25 percent of the values below it and the rest above it;

   Third quartile, Q_3 would have 75 percent of values below it and the rest above it. The method of calculation of quartiles is similar to that of the median with slight variations.

   $$Q_1 = l + \frac{\frac{N}{4} - f_{Q_1}}{f} \times C; \qquad Q_3 = l + \frac{\frac{3N}{4} - f_{Q_3}}{f} \times C$$

   where we locate Q_1 – class and Q_3 – class properly

   l = lower limit of the quartile class

   C = common factor

   Quartile deviation is defined as

   $$\text{Q.D.} = \frac{1}{2}(Q_3 - Q_1)$$

3. **Average deviation (A).**

   If average chosen is A (say), then average deviation about A is average deviation

   $$\text{A. D. (A)} = \frac{1}{n}\sum |(x_i - A)| \text{ for discrete data}$$

   $$= \frac{1}{n}\sum f_i |x_i - A| \text{ for a frequency distribution.}$$

   Usually, either median or the mean are takes as the average.

4. **Standard deviation (Root mean square deviation).**

   Standard deviation,

   $$\sigma = \sqrt{\frac{1}{n}\Sigma(x_i - \bar{x})^2} \quad \text{for discrete data}$$

   $$= \sqrt{\frac{1}{N}\Sigma f_i(x_i - \bar{x})^2} \quad \text{for frequency distribution.}$$

Square of the standard deviation , σ^2 is defined as the *variance* (V).

$$\therefore \qquad V = \sigma^2 = \frac{1}{N}\Sigma f_i(x_i - \bar{x})^2$$

Out of these measures, σ is widely used as a companion to $\bar{x}$ on which is based, when dealing with dispersion or scatter.

Calculation of V or σ.

$$V = \left[\frac{\Sigma f_i d_1^2}{N} - \frac{(\Sigma f_i d_1^2)^2}{N}\right] \times C^2$$

Some important results.

- Average deviation is least when taken from the median.
- Standard deviation is not less than the average deviation from the mean.
- For raw data $x_1, x_2, ...x_n$,

$$V = \frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2 \quad \text{or} \quad V = \frac{\Sigma x^2}{n} - (\bar{x})^2$$

If n_1, n_2 are sizes of two groups, $\bar{x}_1$, $\bar{x}_2$ their means; σ_1, σ_2 their standard deviations, then standard deviation of the combined group is determined from

$$(n_1 + n_2)\,\sigma_2 = n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)$$

where, $d_1 = \bar{x} - \bar{x}_1$ and $d_2 = \bar{x} - \bar{x}_2$,

$\bar{x}$ being the combined mean.. This result can be extended to more than two groups.

Determination of Variance and Standard Deviation

Steps :

1. Calculate the mean, $\bar{x}$.
2. Write a table that subtracts the mean from each observed value.
3. Square each of the differences.
4. Add this column.
5. Divide by n –1, where n is number of items in the sample This is the *variance*.
6. To get the *standard deviation,* take square root of the variance.

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Example. The owner of a restaurant is interested in how much people spend at the restaurant. He examines 10 randomly selected receipts for parties of four and writes down the following data :

44, 50, 38, 96, 42, 47, 40, 39, 46, 50

Find mean, standard deviation and variance.

Solution.

Mean is calculated by adding and dividing by 10

Mean, $\bar{x}$ = 49.2

Following table is used for getting standard deviation :

p	***x*–49.2**	**(*x*–49.2)^2**
44	–5.2	27.04
50	0.8	0.64
38	11.2	125.44
96	46.8	2190.24
42	–7.2	51.84
47	–2.2	4.84
40	–9.2	84.64
39	–10.2	104.04
46	–3.2	10.24
50	0.8	0.64
Total		**2600.4**

Now $V = \left(\dfrac{2600.1}{10-1}\right) = 288.7 \approx 289$

Hence variance is 289 and standard deviation is square root of 289 = 17.

Since standard deviation can be thought of measuring how far the data values lie from the mean, we take mean and move one standard deviation in either direction. The mean for this example was about 49.2 and the standard deviation was 17.

We have, $49.2 - 17 = 32.2$

and $49.2 + 17 = 66.2$

What is means is that most of the patrons probably spend between 32.20 and 66.20.

Coefficients to Dispersion

When two series of measurements have to be compared, then averages may be different and the units which the measurements are recorded may also be different. Hence, coefficients of dispersion are provided as

- $\dfrac{\text{Range}}{\text{A}+\text{B}}$ *,where A is largest and B is smallest of the values.*
- $\dfrac{Q_3 - Q_1}{Q_3 + Q_1}$
- $\dfrac{\text{Average deviation about} A}{A}$
- *Each of these is free from units of measure*ment and is a pure number.

Coefficient of variation (C.V.) $= \dfrac{\sigma}{x} \times 100$

Thus coefficient of variation is a percentage.

RANDOM VARIABLE.

Many variables of interest in a system are not expressible on explicit function of time, all such variable on explicit function of time, all such variable are classified as random variable. These are implying uncertainty about their exact nature.

Random Variable

Discrete | Continuous

Random variable deal with probability theory

Discrete Probability Theory | Continuous Probability Theory

Let X be a random variable.

- Function $F(x) = P(X \le x)$ is called *distribution function of X.*
- Mean or Expectation of $X = \mu = E(X)$

$$= X(a_1)\,P(a_1) + \ldots\ldots\ldots\ldots + X(a_n)\,P(a_n)$$

$$= \sum_{i=1}^{n} X(a_i)P(a_i),$$

where S is probability space $(a_1, a_2, \ldots.. a_n)$

- Variance of X = Var (X) = $\sigma^2 = E(X^2) - [E(X)]^2$

Standard deviation of X = σ

Example. Find expectation of the number of points when a fair die is rolled.

Solution.

Let X be the random variable showing number of points.

Then X = 1, 2, 3, 4, 5, 6

a_i	$P(X = a_i) = P(a_i)$	**Product**
1	$\frac{1}{6}$	$\frac{1}{6}$
2	$\frac{1}{6}$	$\frac{2}{6}$
3	$\frac{1}{6}$	$\frac{3}{6}$
4	$\frac{1}{6}$	$\frac{4}{6}$
5	$\frac{1}{6}$	$\frac{5}{6}$
6	$\frac{1}{6}$	$\frac{6}{6}$
		E(X) = $\frac{21}{6} = \frac{7}{2}$

$\therefore$ Expectation = $\dfrac{7}{2}$.

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Example. What is the expectation of the number of failure preceding the first success in an infinite series of independent trials with constant probability of success p in each trial ?

Solution.

Let p = probability of succes

$\therefore$ q = probability of failure = $1 - p$

Let X be the random variable representing the number of failure preceding the first success.

Since an infinite series of independent trials are conducted, X = 0 , 1, 2

P(X = x) = probability that there are x failures preceding the first success.

$= q \times q \times ....\times q \times p = q^x p$

$$\therefore \quad E(X) = \sum_{x=0}^{\infty} xp(X = x) = \sum_{x=0}^{\infty} xq^x p$$

$= 1\, qp + 2q^2p + 3q^3p + ...$

$= qp\,(1 + 2q + 3q^2 + ...)$

$= qp\,(1 - q)^{-2}$ [$\because (1-q)^{-2}$ by Binomial theroem]

$$= \frac{qp}{(1-q)^2} = \frac{qp}{p^2} = \frac{q}{p}$$

Example. The Bernouilli probability law with parameter p in which $0 \le P \le 1$ is defined by a random variable X taking the value 1 with probability P and the value 0 with probability q. Find the mean and variance of X.

Solution.

a_i	P (a_i)	Product
1	p	p
0	q	0
		E(X) = p

$\therefore$ Mean = E (X) = p.

Now to find Var (X) :

a_i	a_i^2	P(a_i)	Product $[a_i^2 \times P(a_i)]$
1	1	p	p
0	0	q	0
			E(X²) = p

$\therefore$ Var (X) $= E(X^2) - [E(X)]^2$

$= p - p^2 = (1 - p)\, p = pq$

Absolute Frequency

Let a value x_i is repeated f_i times, then f_i is called *frequency or absolute frequency.*

Let N = sum of all frequencies f_i, then

$\frac{f_i}{N}$ is called *relative frequency* , i = 1, 2 ....n.

Sum of all relative frequencies

$$= \frac{f_i}{N} + \frac{f_2}{N} + ...+ \frac{f_n}{N}$$

$$= \frac{f_1 + f_2 ....f_n}{N} = \frac{N}{N} = 1.$$

PROBABILITY DISTRIBUTION

Let a random experiment is repeated n times. If an event A occurs n_A times, then its probability is defined as

$$p(A) = p(x_k) = \lim_{n\to\infty} \frac{n_A}{n}$$

= Probability of outcome A.

where, n_A = times occur A

n = total time experiment

- This limit may not exist.
- This definition presumes that all outcomes are equally likely to occur.

$p(A + B) = p(A) + p(B) - p(AB)$

Here, p(AB) = joint probability.

Type of Probability Distribution

The are of two types :

1. Discrete Distribution.

It is of two types.

(*i*) Binomial Distribution.

It is concerned with trials of a repetitive nature in which only the occurrence or non-occurrence, success or failure, acceptance or erejection, yes or no of a particular event is of interest.

If a series of independent trials are perfomed such that for each trial, p is the probability of success and q that of a failure, then probability of r successes in a series of n trials is given by $^nC_r p^r q^{n-r}$, where r takes any integral value from 0 to n.

The probabilities of 0, 1, 2,......r,.....,n successes are therefore, given by

$q^n, {}^nC_1 pq^{n-1}, {}^nC_2 p^2q^{n-2}, ...{}^nC_r p^r q^{n-r}, ..., p^n$

Probability of the number of successes so obtained is called **binomial distribution** because probabilities are successive terms in the expansion of the binominal $(q + p)^n$.

Hence sum of the probabilities

$= q^n + {}^nC_1\, pq^{n-1} + {}^nC_2\, p^2q^{n-2} +....+ p^n$

$= (q + p)^n = 1$

Constants of Binomial distribution.

Moment generating function about the origin,

$M_0(t) = E(e^{tx}) = \sum {}^nC_1\, p^x q^{n-x}\, e^{tx}$

$= \sum^n C_x\, (pe^t)^x\, q^{n-x} = (q + pe^t)^n$

Differentiating with respect to t and putting $t = 0$, we get mean given by

$M_m(t) = e^{-npt}\,(q + pe^t)^n = (qe^{-pt} + pe^{qt})^n$

$$= 1 + pq\frac{t^2}{2!} + pq(q^2 - p^2\frac{t^3}{3!}$$

$$+ pq(q^3 + p^3)\frac{t^4}{4!} + ...$$

$$\Rightarrow \quad 1 + \mu_1 t + \mu_2\frac{t^2}{2!} + \mu_3\frac{t^3}{3!} + \mu_4\frac{t^4}{4!} + ...$$

$$= 1 + npq\frac{t^2}{2!} + npq(q - p)\frac{t^3}{3!}$$

$$+ npq\,[1 + 3(n - 2)\,pq]\frac{t^4}{4!} + ...$$

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Equating coefficients of like power of t on either side, we have

$$\mu_2 = npq,\ \mu_3 = npq(q-p),\ \mu_4 = npq\,[1+3(n-2)pq]$$

Also, $$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(q-p)^2}{npq} = \frac{(1-2p)^2}{npq}$$

$$\beta_1 = \frac{\mu_4}{\mu_2^2} = 3 + \frac{1-6pq}{npq}$$

∴ Mean = np

Standard deviation = √(npq)

$$\text{Skewness} = \frac{1-2p}{\sqrt{npq}}$$

Kurtosis = β_2

Obs. Skewness is

positive for $p < \left(\frac{1}{2}\right)$

negative for $p > \left(\frac{1}{2}\right)$

zero for $p = \left(\frac{1}{2}\right)$, i.e. when probability curve of binomial distribution will be symmetrical (bell-shaped)

As number of trials n increase indefinitely $\beta_1 \to 0$ and $\beta_2 \to 3$

Binomial frequency distribution:

If n independent trials constitute one experiment and this experiment be repeated N times, then frequency of r successes = $N^n\ C_r\ p^r\ q^{n-r}$.

The possible number of successes together with these expected frequencies constitute binomial frequency distribution.

Applications of Binomial distribution:

This distribution is applied to problems concerning

(*a*) number of defectives in a sample from production line.

(*b*) estimation of reliability of system.

(*c*) number of rounds fired from a gun hitting a target.

(*d*) rader detection.

Bernouilli Trials.

Independent repeated trials of an experiment with two outcomes only are called *Bernouilli trials,* call One of the outcomes is called *success* and the other outcome is called *failure*.

Let p = probability of success in a Bernouilli trial

q = probability of failure = $1 - p$.

A binomial experiment consisting of a fixed number n of trials is denoted by B (n, p).

Probability of r success in the experiment B (n, p) is given by

$$P(r) = {}^nC_r\, p^r\, q^{n-r}$$

The function P(r) for r = 0 , 1, 2 ...., n for B (n, p) is called **binomial distribution.**

Example. If four coins are tossed , find the chance that there should be two heads and two tails.

Solution.

$$p = \text{probability of a head} = \frac{1}{2}$$

$$q = \text{probability of a tail} = 1 - p = \frac{1}{2}$$

Hence probability of getting 2 heads (and 2 tails) when four coins are tossed

$$= P(2) = {}^4C_2\, p^2\, q^{4-2}$$

$$= 6 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^2 = \frac{3}{8}$$

(*ii*) Poisson Distribution.

It is a distribution related to the probabilities of events which are extremely rare, but which have a large number of independent opportunities for occurrence.

e.g. number of persons born blind per year in a large city is the phenomena, in which this law is followed.

This distribution can be derived as limiting case of the binomial distribution by making n very large and p very small keeping np fixed (= m, say)

The probability of r successes in a binomial distribution

$$P(r) = {}^nC_r p^r q^{n-r}$$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)\,p^r q^{n-r}}{r!}$$

$$= \frac{np(np-p)(np-2p)..(\overline{np-r}-1p)(1-p)^{n-r}}{r!}$$

As $n \to \infty$, $p \to 0$ $(np = m)$,

$$p(r) = \frac{m^r}{r!} \underset{n\to\infty}{\text{Lt}} \frac{(1-m/n)^2}{(1-m/n)^r} = \frac{m^r}{r!}\ e^{-m}$$

so that probabilities of 0, 1, 2, ..., r,...... success in poisson distributio is given by

$$e^{-m},\ me^{-m},\ \frac{m^2\,e^{-m}}{2!},\ldots\ \frac{m^r\,e^{-m}}{r!},\ldots$$

Sum of these probabilities is unity as it should be.

2. Continuous Probability Distribution

When a variate X takes every value in an interval, it gives rise to continuous distribution of X. e.g. distributions defined by the variates like heights or weights are continuous distributions.

A major conceptual difference, however, exists between discrete and continuous probabilities. When thinking in discreate terms, the probability associated with an event is meaningful. With continuous event, however, where number of events is infinitely large, the probability that a specific event will occur is practically zero. For this reason, continuous probability statements must be worded somewhat, differently from discrete ones. Instead of finding the

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probability that z equals some value, we find the probability of x falling in a small interval.

Thus probability distribution of a continuous variate x is defined by a function f(x) such that probability of the variate x falling in the small interval

$$[x - \frac{1}{2}dx] \text{ to } [x + \frac{1}{2}dx] \text{ is } f(x)\,dx.$$

Let X be a continuous random variable. If $f(x)$ is probability density functions of X, then

- $P(X \le a) = \int_{-\infty}^{a} f(x)dx$
- $P(a \le X \le b) = \int_{a}^{b} f(x)dx$
- Expectation of X = E (X) = Mean = $\int_{-\infty}^{\infty} xf(x)dx$
- Variance of X = y (X) = $\int_{-\infty}^{b} [x - E(x)^2 f(x)dx]$
- $\int_{-\infty}^{\infty} f(x)dx = 1$

Types of Continuous Probability Distribution

(*i*) Uniform Distribution.

A continuous random variable X is said to follow uniform distribution or rectangular distribution if its probability density function is given by

$$f(x) = \frac{1}{b-a},\ a \le x \le b = 0, \text{ otherwise}$$

The uniform distribution generally arises in the study of round off errors where measurements are recorded upto a certain level of accuracy.

Cumulative distribution function of X is given by

$f(x) = p\ (X \le x).$

$$= \int_{-\infty}^{x} f(x)dx = \begin{cases} 0 & if\ x \le a \\ \dfrac{x-a}{b-a}, & if\ a \le x \le b \\ 1 & if\ x > b \end{cases}$$

$$\text{Mean} = \frac{a+b}{2}, \text{ and Variance} = \frac{(b-a)^2}{12}$$

Example. The melting point X of a certain specimen be assumed to be a continuous random variable which is uniformly distributed over the interval [110, 120]. Find density function of X, mean of X, variance of X and P $(112 \le x \le 115.)$.

Solution.

Here, a = 100, b = 120

$$\therefore \quad f(x) = \begin{cases} \dfrac{1}{120-110} & 110 \le x \le 120 \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \dfrac{1}{10} & 110 \le x \le 120 \\ 0, & \text{otherwise} \end{cases}$$

$$\text{Mean of } X = \frac{a+b}{2} = \frac{110+120}{2} = 115$$

$$\text{Variance of } X = \frac{(b-a)^2}{12} = \frac{(120-110)^2}{12} = \frac{25}{3}$$

$$P(112 \le x \le 115) = \int_{112}^{115} f(x)dx = \int_{112}^{115} \frac{1}{10}dx = \frac{3}{10}$$

(*ii*) Exponential Distribution.

A continuous random variable X is said to have the exponential distribution if its probability density function is given by

$$f(x) = \begin{cases} ae^{-ax}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

a being some positive number, called its *parameter*

This distribution arises in the study of life-length of electronic components.

Also the inter-arrival time and service time in queuing theory have exponential distributions.

$$\text{Mean} = \frac{1}{a}, \text{ Variance} = \frac{1}{a^2}$$

Example. The sales tax return of a salesman is exponentially distributed with parameter $\frac{1}{4}$. What is the probability that his sale will exceed ₹ 10,000 assuming that sales tax is charged at the rate of 5% on the sales ?

Solution.

$$\text{Here,} \qquad f(x) = \begin{cases} \dfrac{1}{4}e^{-\frac{x}{4}}, & x > 0 \\ 0\ , & \text{otherwise} \end{cases}$$

Sales tax for the sale of ₹ 10,000

$= 10000 \times \frac{5}{100}$ = ₹ 500

Now, P $(x > 500)$ = 1 P $(x \le 500)$

$$= 1 - \int_{0}^{500} f(x)dx = 1 - \frac{1}{4}\int_{0}^{500} e^{-x/4}dx$$

$$= 1 + \left(e^{-x/4}\right)_0^{500} = e^{-125}$$

(*iii*) Normal Distribution.

A continuous random variable X is said to follow normal distribution if its probability density function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$-\infty \le x \le \infty, \sigma > 0, -\infty < \mu < \infty$$

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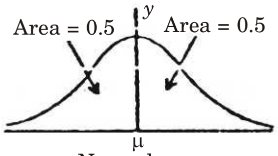
In such a situation, write X – N (μ, σ^2)

The distribution involves two parameters μ and σ.

Properties.

(*a*) Distribution is symmetrical.

(*b*) Mean = μ, Variance = σ^2.



Normal curve

(*c*) Mean, median and mode coincide.

(*d*) $f(x) \geq 0$ for all x.

(*e*) $\int_{-\infty}^{\infty} f(x)dx = 1$, *i.e.* total area under the curve $y = f(x)$ bounded by the axis of x is 1.

Note:

Curve $y = f(x)$, called *normal curve* is a *bell-shaped curve.* It is symmetrical about $x = \mu$. The two tails on the left and right sides of the mean extend to infinity.

Put $z = \dfrac{x-\mu}{\sigma}$. Then z is called a standard *normal variate* and its probability density function is given by

$$p(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, -\infty \leq z \leq \infty.$$

Mean of the standard normal distribution is 0 and variance is 1. Write Z – N (0, 1).

Area under the standard normal curve.

(*a*) Between $z = -1$ and $z = 1$ is 0.6827 (since total area under standard normal curve is 1)

i.e. $p(-1 < z < 1) = 0.6827$

(*b*) Between $z = -2$ and $z = 2$ is 0.9545

i.e., $p(-2 < z < 2) = 0.9545$

(*c*) Between $z = -3$ and $z = 3$ is 0.9973

i.e., $p(-3 < z < 3) = 0.9973$

In other words,

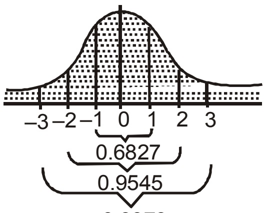
$p(\mu - \sigma < x < \mu + \sigma) = 0.6827$

$p(\mu - 2\sigma < x < \mu + 2\sigma) = 0.9545$

$p(\mu - 3\sigma < x < \mu + 3\sigma) = 0.9973$

$p(\mu - 1.96\,\sigma < x < \mu + 1.96\sigma) = 0.95$

$p(\mu - 2.58\sigma < x < \mu + 2.58\,\sigma) = 0.99$



Since distribution is symmetric, we consider only positive values of z.

Then $\phi(z) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_0^z e^{-\frac{z^2}{2}}\, dz$

gives area under the normal curve between $z = 0$ and $z = z$.

For different values of z, $\phi(z)$ can be calculated. These values of $\phi(z)$ have been tabulated.

There is another table called *table of ordinates*. We read

$$p(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

corresponding to a value of z from this table.

Many distributions tend to a normal distribution in the limit.

When a variable is not normal, it can be made normal using some suitable transformation.

When the sample size is large, distributions of the sample mean, sample variance etc. approach normality. Thus distribution forms a basis for tests of significance.

Normal distribution is also called *distribution of errors*.

Normalized Gaussian random variable

It mean of the standard normal distribution is 0 and variance is 1, write Z – N (0, 1).

The distribution function of X are plotted respectively.

Mean and variance of X are

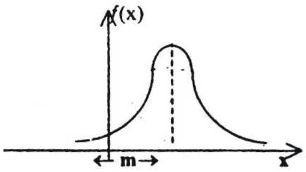
$$\mu_x = E[X] = \mu$$
$$\sigma_x^2 = E[(X - \mu_x)^2] = \sigma^2$$

Notation N(μ ;σ^2) are used to denote that X is normal (or Gaussian) with mean m and variance σ^2.

In particular, X = N (0 ; 1), i.e. X with zero mean and unit variance is defined as a *normalized Gaussian random variable.*

Central Unit Theorem

Normal (or Gaussian) distribution has played a significant role in the study of random phenomena in nature. Many naturally occurring random phenomena are approximately normal. *Central–limit theore* states that *sum of a large number of independent random variables, under certain conditions can be approximated by a normal distribution.*



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp - \frac{(x-\mu)^2}{2\sigma^2}$$

If mean = 0, i.e. $\mu = 0$,

then $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} \exp - \dfrac{(x)^2}{2\sigma^2}$

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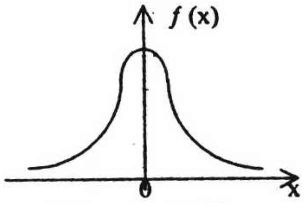
Gaussian Process

$$f(y) = \frac{1}{\sqrt{(2\pi\sigma_y^2)}} \exp\frac{[-(y-\mu_y)^2}{2\sigma_y^2}$$



A Gaussian Pulse

where, μ_y = mean

σ_y^2 = variance

Properties of Gaussian process:

(*i*) If a Gaussian process x(t) is applied to a stable linear filter, then random process y(t) developed. The output of the filter is also Gaussian.

(*ii*) Consider the set of random variables or samples $x(t_1), x(t_2), x(t_3)$ ----- obtained by observing a random process x(t) at time t_1, t_2, -----.

Second moment or Mean square value

Consider random variable X which assume the possible value x_1, x_2, x_3 ----- x_m. In a sequence of n experiment, let event x_1 occur n_1 time, x_2 occur n_2 times etc. Arithmetic average or mean of x_k is

$$X_{avg} = \frac{n_1x_1 + n_2x_2 + ----+x_m\ n_m}{n} = \frac{1}{n}\sum_{k=1}^{m} x_k n_k$$

If $n \to \infty$, then $\frac{n_1}{n} \to p(x_1)$

$$\therefore \quad X_{avg} = p_1x_1 + p_2x_2 + ---- p_m x_m = \sum_{k=1}^{m} p_k x_k$$

and $$X^2_{avg} = \frac{n_1x_1^2 + n_2 x_2^2 + ----}{N}$$

SOLVED EXAMPLES

1. A bag contains 8 white and 6 red balls. Find the probability of drawing two balls of the same colour.

Solution.

Two balls out of 14 can be drawn in $^{14}C_2$ ways which is total number of outcomes.

Two white balls out of 8 can be drawn in $^8C_2$ ways. Thus probability of drawing 2 white balls

$$= \frac{^8C_2}{^{14}C_2} = \frac{28}{91}$$

Similarly 2 red balls out of 6 can be drawn in $^6C_2$ ways.

Thus probability of drawing 2 red balls

$$= \frac{^6C_2}{^{14}C_2} = \frac{15}{91}$$

Hence probability of drawing 2 balls of the same colour (either both white or both red)

$$= \frac{28}{91} + \frac{15}{91} = \frac{43}{91}$$

2. Find probability of drawing an ace or a spade or both from a deck of cards?

Solution.

Probability of drawing an ace from a deck of 52 cards $= \frac{4}{52}$

Similarly probability of drawing a card of spades

$$= \frac{13}{52}$$

and probability of drawing an ace of spades $= \frac{1}{52}$.

Since two events (i.e., a card being an ace and a card being of spades) are not mutually exclusive. Hence probability of drawing an ace or a spade

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{4}{13}.$$

3. A pair of dice is tossed twice. Find the probability of scoring 7 points

(*a*) once (*b*) at least once (*c*) twice.

Solution.

In a single toss of two dice, the sum 7 can be obtained as (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) i.e., in 6 ways

Hence probability of getting 7 $= \frac{6}{36} = \frac{1}{6}$.

Thus, probability of not getting 7 $= 1 - \frac{1}{6} = \frac{5}{6}$.

(*a*) Probability of getting 7 in the first toss and getting 7 in the second toss

$$= \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}.$$

Similarly, probability of not getting 7 in the first toss and getting 7 in the second toss

$$= \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}.$$

Since these are mutually exclusive events, addition law of probability applies.

$\therefore$ Required probability $= \frac{5}{36} + \frac{5}{36} = \frac{5}{18}$

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(*b*) Probability of not getting 7 in either toss

$$= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$$

Hence probability of getting 7 at least once

$$= 1 - \frac{25}{36} = \frac{11}{36}.$$

(*c*) Probability of getting 7 twice

$$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}.$$

4. There are two groups of subjects; one of which consists of 5 science and 3 engineering subjects, and the other consists of 3 science and 5 engineering subjects. An unbiased die is cast. If the number 3 or number 5 turns up, a subject is selected at randam from the first group, otherwise the subject is selected at randam from the second group. Find probability that an engineering subject is selected ultimately.

Solution.

Probability of turning up 3 or 5 = $\frac{2}{6} = \frac{1}{3}$.

Probability of selecting an engineering subject from first group = $\frac{3}{8}$

Hence probability of selecting an engineering subject from first group on turning up 3 or 5

$$= \frac{1}{3} \times \frac{3}{8} = \frac{1}{8}$$

Now probability of not turning 3 or 5

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

Probability of selecting an engineering subject from second group = $\frac{5}{8}$

Hence probability of selecting an engineering subject from second group on turning up 3 or 5

$$= \frac{2}{3} \times \frac{5}{8} = \frac{5}{12}$$

Thus probability of selecting an engineering subject

$$= \frac{1}{8} + \frac{5}{12} = \frac{13}{24}$$

5. A box A contains 2 white and 4 black balls. Another box B contains 4 white and 7 black balls. A ball is transferred from the box A to the box B. Then a ball is drawn from the box B. Find the probability that it is white.

Solution.

Probability of drawing a white ball from box B will depend on whether the transferred ball is black or white.

If a black ball is transferred, its probability is $\frac{4}{6}$.

There are now 5 white and 8 black balls in the box B. Then probability of drawing white ball from box B is $\frac{5}{13}$.

Thus probability of drawing a white ball from urn B, if transferred ball is black $= \frac{4}{6} \times \frac{5}{13} = \frac{10}{39}$

Similarly the probability of drawing a white ball from urn B, if transferred ball is white

$$= \frac{2}{6} \times \frac{6}{13} = \frac{2}{13}$$

$\therefore$ Required probability $= \frac{10}{39} + \frac{2}{13} = \frac{16}{39}$.

6. Two persons A and B toss an unbiased coin alternately on the understanding that the first who gets the head wins. If A starts the game, find their respective chances of winning.

Solution.

Probability of getting a head = $\frac{1}{2}$.

Then A can win in 1st, 3rd, 5th, ... throws.

Hence chances of A's winning

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^2 \frac{1}{2} + \left(\frac{1}{2}\right)^4 \frac{1}{2} + \left(\frac{1}{2}\right)^6 \frac{1}{2} + \ldots..$$

$$= \frac{\frac{1}{2}}{1 - \left(\frac{1}{2}\right)^2} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3} \ldots..$$

Hence chance of B's winning $= 1 - \frac{2}{3} = \frac{1}{3}$.

7. (*i*) Is the function defined as follows a density function?

$$f(x) = e^{-x}, x \geq 0 = 0, \quad x < 0$$

(*ii*) If so, determine probability that the variate having this density will fall in the interval (1, 2)?

(*iii*) Also find cumulative probability function F(2)?

Solution.

(*i*) $f(x)$ is clearly ≥ 0 for every x in (1, 2) and

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} 0 . dx + \int_{0}^{\infty} e^{-x} dx = 1$$

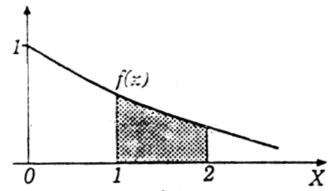
Hence function $f(x)$ satisfies the requirements for a density function.

(*ii*) Required probability = P $(1 \leq x \leq 2)$

$$= \int_{1}^{2} e^{-x} dx = e^{-1} - e^{-2}$$

$$= 0.368 - 0.135 = 0.233.$$

This probability is equal to the shaded area in Fig. (*a*).



(*a*)

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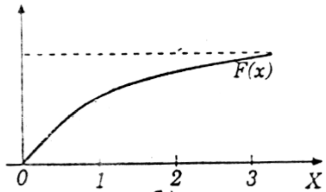
(*iii*) Cumulative probability function F(2)

$$\int_{-\infty}^{2} f(x)dx = \int_{-\infty}^{0} 0.dx + \int_{0}^{2} e^{-x}dx$$
$$= 1 - e^{-2} = 1 - 0.135 = 0.865$$



(b)

which is shown in Fig.(*b*).

8. X is a continuous random variable with probability density function given by

$$f(x) = kx\ (0 \le x < 2) = 2k\ (2 \le x < 4)$$
$$= -kx + 6k\ (4 \le x < 6)$$

Find k and mean value of X.

Solution.

Since total probability is unity

$$\therefore \quad \int_0^6 f(x)dx = 1$$

i.e., $\int_0^2 kx\,dx + \int_2^4 2k\,dx + \int_4^6 (-kx+6k)dx = 1$

$$\Rightarrow k\left|\frac{x^2}{2}\right|_0^2 + 2k|x|_2^4 + \left(-k\frac{x^2}{2} + 6kx\right)_4^6 = 1$$

$$\Rightarrow 2k + 4k + (-10k + 12k) = 1 \Rightarrow k = \frac{1}{8}.$$

Mean of X

$$= \int_0^6 xf(x)dx$$
$$= \int_0^2 kx^2dx + \int_2^4 2kx\,dx + \int_4^6 x(-kx+6k)dx$$
$$= k\left|\frac{x^3}{3}\right|_0^2 + 2k\left|\frac{x^2}{2}\right|_2^4 + \left(-k\left|\frac{x^3}{3}\right|_4^6 + 6k\left|\frac{x^2}{2}\right|_4^6\right)$$
$$= k\left(\frac{8}{3}\right) + k(12) - k\left(\frac{152}{3}\right) + 3k(20) = \frac{1}{8}(24) = 3$$

9. The frequency distribution of a measurable characteristic varying between 0 and 2 is as under

$$f(x) = x^3,\ 0 \le x \le 1 = (2-x)^3,\ 1 \le x \le 2.$$

Calculate standard deviation and also mean deviation about the mean.

Solution.

$$\text{Total frequency, N} = \int_0^1 x^3\,dx + \int_1^2 (2-x)^3\,dx$$
$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$\therefore$ μ_1' (about the origin)

$$= \frac{1}{N}\left[\int_0^1 x.x^3\,dx + \int_1^2 x(2-x)^3\,dx\right]$$
$$= 2\left\{\left|\frac{x^5}{5}\right|_0^1 + \left|-x.\frac{(2-x)^4}{4}\right|_1^2 - \left|\frac{(2-x)^5}{20}\right|_1^2\right\}$$
$$= 2\left(\frac{1}{5} + \frac{1}{4} + \frac{1}{20}\right) = 1$$

μ_2' (about the origin)

$$= \frac{1}{N}\left[\int_0^1 x^2.x^3dx + \int_1^2 x^2(2-x)^3\,dx\right]$$
$$= 2\left\{\left|\frac{x^6}{6}\right|_0^1 + \left|-x^2\frac{(2-x)^4}{4}\right|_1^2 + \frac{1}{2}\int_1^2 x(2-x^4)dx\right\}$$
$$= 2\left\{\frac{1}{6} + \frac{1}{4} + \frac{1}{2}\left[\frac{1}{5} + \frac{1}{30}\right]\right\} = \frac{16}{15}$$

$$\therefore \quad \sigma^2 = \mu_2 = \mu_2' - (\mu_1')^2 = \frac{1}{15}$$

i.e., standard deviation $\sigma = \dfrac{1}{\sqrt{15}}$

Mean deviation about the mean

$$= \frac{1}{N}\left\{\int_0^1 |x-1|x^3dx + \int_1^2 |x-1|(2-x)^3\,dx\right\}$$
$$= 2\left\{\int_0^1 (1-x)x^3dx + \int_1^2 (x-1)(2-x)^3\,dx\right\}$$
$$= 2\left\{\left(\frac{1}{4} - \frac{1}{5}\right) + \left(0 + \frac{1}{20}\right)\right\} = \frac{1}{5}$$

10. The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that

(*a*) exactly two will be defective.

(*b*) at least two will be defective.

(*c*) none will be defective

Solution.

Probability of a defective pen $= \dfrac{1}{10} = 0.1$

Hence probability of a non-defective pen

$$1 - 0.1 = 0.9$$

(*a*) Probability that exactly two will be defective

$= {}^{12}C_2\,(0.1)^2\,(0.9)^{10} = 0.2301.$

(*b*) Probability that at least two will be defective

$= 1 -$ (probability that either none or one is non-defective)

$= 1 - [{}^{12}C_0(0.9)^{12} + {}^{12}C_1(0.1)(0.9)^{11}] = 0.3412.$

(*c*) Probability that none will be defective

$= {}^{12}C_{12}\,(0.9)^{12} = 0.2833$

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11. In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads and 4 tails.

Solution.

$$P(\text{head}) = \frac{1}{2} \text{ and } P(\text{tail}) = \frac{1}{2}$$

By Binomial distribution, probability of 8 heads and 4 tails in 12 trials,

$$P(X = 8) = {}^{12}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^4 = \frac{12!}{8!4!} \cdot \frac{1}{2^{12}} = \frac{495}{4096}$$

$\therefore$ Expected number of such cases in 256 sets

$$= 256 \times P(X = 8) = 256 \frac{495}{4096} = 30.9 \approx 31$$

12. If probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2,000 individuals more than two will get a bad reaction.

Solution.

It follows a Poisson distribution as probability of occurrence is very small.

Mean, $m = np = 2000\ (0.001) = 2$

Probability that more than 2 will get a bad reaction
= 1 – [probability that no one gets a bad reaction
+ probability that one gets a bad reaction
+ probability that two get bad reaction]

$$= 1 - \left[e^{-m} + \frac{m^1 e^{-m}}{1!} + \frac{m^2 e^{-m}}{2!}\right]$$

$$= 1 - \left[\frac{1}{e^2} + \frac{2}{e^2} + \frac{2}{e^2}\right] \qquad \ldots[\because m = 2]$$

$$= 1 - \frac{5}{e^2} = 0.32 \qquad [\because e = 2.718]$$

EXERCISE – I

MCQ TYPE QUESTIONS

1. If A, B, C are three events, then $P[A \cap (B \cup C)] =$
(*a*) $P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$
(*b*) $P(A) + P(B) + P(C) - P(B)\,P(C)$
(*c*) $P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$
(*d*) $P(B \cap C) + P(A \cap B) - P(A \cap B \cap C)$

2. If A and B are two arbitrary events, then $P(A \cap B)$ cannot be
(*a*) less than $P(A) + P(B) - 1$
(*b*) greater than $P(A) + P(B)$
(*c*) equal to $P(A) + P(B) - P(A \cup B)$
(*d*) equal to $P(A) + P(B) + P(A \cup B)$

3. If M and N are two events, then probability that exactly one of them occurs is not equal to
(*a*) $P(M) + P(N) - 2P(M \cap N)$
(*b*) $P(M) + P(N) - P(M \cap N)$
(*c*) $P(\overline{M}) + P(\overline{N}) - 2P(\overline{M} \cap \overline{N})$
(*d*) $P(M \cap \overline{N}) + P(\overline{M} \cap N)$

4. If A and B are any two events then which one of the following pairs of events are mutually exclusive ?
(*a*) $A \cup B$ and $A \cap \overline{B}$ (*b*) $A \cup B$ and $\overline{A} \cap B$
(*c*) $A \cup B$ and $\overline{A} \cap \overline{B}$ (*d*) $A \cup B$ and $A \cap B$

5. There are three events A, B, C one of which must, and only one can happen, the odds are 8 to 3 against A, 5 to 2 against B. The odds against C are
(*a*) 43 to 34 (*b*) 32 to 23
(*c*) 34 to 13 (*d*) none of these

6. An urn contains 5 red and 10 black balls. Eight of them are placed in another urn. The chance that the latter then contains 2 red and 6 black balls is
(*a*) $\frac{140}{429}$ (*b*) $\frac{129}{440}$
(*c*) $\frac{139}{420}$ (*d*) none of these

7. A has one share in a lottery in which there is 1 prize and 2 blanks; B has three shares in a lottery in which there are 3 prizes and 6 blanks; compare the probability of A's success to that of B's success is
(*a*) 7 : 16 (*b*) 16 : 7 (*c*) 6 : 14 (*d*) 14 : 6

8. If A and B are independent, then A and $\overline{B}$ are also
(*a*) independent
(*b*) dependent
(*c*) both (*a*) and (*b*)
(*d*) none of these

9. Let $S = \left\{1, \frac{1}{2}, \left(\frac{1}{2}\right)^2, \ldots, \left(\frac{1}{2}\right)^n\right\}$.

Let A and B be two events defined as

$A = \left\{1, \frac{1}{2}\right\}$ and $B = \left\{\left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^4, \ldots\right\}$.

Then $P(A \cap B)$ will be
(*a*) 0 (*b*) 1
(*c*) ∞ (*d*) none of these

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10. If $E_1, E_2, ..., E_n$ are n mutually exclusive events, then the probability of the happening of any one of them is equal to the sum of the probabilities of the happening of separate events.

i.e., $P(E_1 \cup E_2 \cup E_3 ..... \cup E_n)$
$= P(E_1) + P(E_2) + ... + P(E_n)$.

This is

(*a*) Law of total probability
(*b*) Law of probability
(*c*) Both (*a*) and (*b*)
(*d*) None of these

11. What is the probability of correctly choosing an unknown integer between 0 to 9 within three chances ?

(*a*) $\frac{963}{1000}$ (*b*) $\frac{973}{1000}$
(*c*) $\frac{983}{1000}$ (*d*) None of these

12. If $P(A/C) \geq P(B/C)$ and $P(A/\overline{C_1}) \geq P(B/\overline{C_1})$, then

(*a*) $P(A) \geq P(B)$ (*b*) $P(A) \leq P(B)$
(*c*) $P(A) < P(B)$ (*d*) $P(A) > P(B)$

13. If A and B are independent and $P(C) = 0$, then A, B and C are independent.

(*a*) True
(*b*) False

14. Six dice are thrown simultaneously. The probability that all will show different faces is

(*a*) $\frac{5!}{6^5}$ (*b*) $\frac{5!}{6^4}$
(*c*) $\frac{5!}{6^3}$ (*d*) none of these

15. Given two events A and B and

$P(A) = \frac{1}{4}$, $P(B/A) = \frac{1}{2}$ and $P(A/B) = \frac{1}{4}$.

State that following is true?

(*a*) A is sub-event of B (*b*) $P(A/B) = \frac{3}{4}$
(*c*) $P(A/B) + P(A/\overline{B}) = 1$
(*d*) None of these

16. Which of the following is correct?

(*a*) If $P(A) > 0$, $P(B) > 0$ and $P(A/B) = P(B/A)$, then
$P(A) = P(B)$.
(*b*) If A and B mutually exclusive, then
$P(A/\overline{B}) = P(A)/(1 - P(B))$.
(*c*) If A and B are independent, then
$P(A \cup B) = 1 - P(\overline{A}) \cdot P(\overline{B})$
(*d*) All of these

17. A and B throw alternately with a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, then his chance of winning is

(*a*) $\frac{31}{60}$ (*b*) $\frac{30}{61}$
(*c*) $\frac{31}{61}$ (*d*) none of these

18. If $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, then

(*a*) $P(A \cup B) \geq \frac{3}{4}$ (*b*) $\frac{3}{8} < P(A \cap B) \leq \frac{5}{8}$
(*c*) $\frac{1}{8} \leq P(A \cap \overline{B}) \leq \frac{3}{8}$ (*d*) All of these

19. Three machines M_1, M_2 and M_3 produce identical items. Of their respective output 5%, 4% and 3% of items are faulty. On a certain day, M_1 has produced 25% of the total output, M_2 has produced 30% and M_3 the remainder. An item selected at random is found to be faulty. What are the chances that it was produced by the machine with the highest output?

(*a*) 0.155 (*b*) 0.255 (*c*) 0.355 (*d*) 0.455

20. For any two events A and B

(*a*) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$
(*b*) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(*c*) $P(A/B) \leq P(A)$.
(*d*) All of these

21. Which of the following is correct?

(*a*) $P(A \cup B/C) = P(A/C) + P(B/C) - P(A \cap B/C)$
(*b*) $P(A \cap B/C) + P(A \cap B/C) = P(A/C)$
(*c*) If A and B are independent, then A and $\overline{B}$ are also independent.
(*d*) All of these

22. Which of the following is correct?

(*a*) If A, B and C are mutually independent, then $A \cup B$ and C are also independent.
(*b*) The events $E_1, E_2, ..., E_n$ are mutually exclusive. Let $E = \bigcup_{i=1}^{n} E_1$.
If $P(A/E_i) = P(B/E_i)$, i = 1, 2, ..., n, then
$P(A/E) = P(B/E)$.
(*c*) Both (*a*) and (*b*)
(*d*) None of these

23. Let p be the probability that a man aged y will get into an accident in a year. What is the probability that a man among n men of all aged y will get into an accident first ?

(*a*) $\frac{1}{n}\left(1-(1+p)^n\right)$ (*b*) $\frac{1}{n}\left(1-(1+p)^n\right)$
(*c*) $n\left(1-(1+p)^n\right)$ (*d*) None of these

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24. What would be the expectation of the number of failures preceding the first success in an infinite series of independent trials with the constant probability of success p ?

(*a*) $\frac{1}{p}$ (*b*) $\frac{1}{q}$

(*c*) $\frac{q}{p}$ (*d*) None of these

25. A restaurant serves two special dishes A and B to its customers consisting of 60% men and 40% women. 80% of men order dish A and the rest B and 70% women order dish A and the rest B. In what ratio of A to B should the restaurant prepare the dishes?

(*a*) 19 : 6 (*b*) 6 : 9 (*c*) 16 : 7 (*d*) 7 : 16

26. If A and B are mutually exclusive and $P(A \cup B) = P(A) + P(B)$, then $P(A/A \cup B) =$

(*a*) $\frac{P(A)}{P(A)+P(B)}$ (*b*) $\frac{P(A)}{P(A)-P(B)}$

(*c*) $\frac{P(B)}{P(A)+P(B)}$ (*d*) None of these

27. If $P(A) = P(\bar{B})$, then

(*a*) $A = \bar{B}$ (*b*) $B = 0$

(*c*) $A \neq B$ (*d*) None of these

28. If P (A) = 0, then

(*a*) $A = \phi$ (*b*) $A = 0$

(*c*) $A = \infty$ (*d*) none of these

29. The mean of a set of number is $\bar{x}$. If each number is increased by λ, then mean of the new set is

(*a*) $\bar{x}$ (*b*) $\bar{x} + \lambda$

(*c*) $\lambda\bar{x}$ (*d*) none of these

30. If each observation of raw data whose variance is σ^2, is increased by λ, then variance of the new set is

(*a*) σ^2 (*b*) $\lambda^2\sigma^2$

(*c*) $\lambda + \sigma^2$ (*d*) $\lambda^2 + \sigma^2$

31. If μ is mean of distribution, then $\sum f_i(\gamma_i - \mu)$ is equal to

(*a*) M.D

(*b*) standard deviation

(*c*) 0

(*d*) none of these

32. $b_{xy} \times b_{yx}$ is equal to

(*a*) $\rho(X, Y)$ (*b*) $\text{cov}(x, y)$

(*c*) $\{\rho(x, y)\}^2$ (*d*) None of these

33. If cov (X, Y) = 0, then which of the following is negative?

(*a*) b_{yx} (*b*) b_{xy}

(*c*) $\rho(X, Y)$ (*d*) none of these

34. The mean of a set of observations is $\bar{x}$. If each observation is divided by α, $\alpha \neq 0$ and then is increased by 10, then the mean of new set is

(*a*) $\frac{\bar{x}}{\alpha}$ (*b*) $\frac{\bar{x}+10}{\alpha}$

(*c*) $\frac{\bar{x}+10\alpha}{\alpha}$ (*d*) $\alpha\bar{x} + 10$

35. S.D. of n observation $a_1, a_2, a_3, \ldots\ldots a_n$ is σ, then If S.D. of the observations $\lambda a_1, \lambda a_2, \lambda a_3, \ldots\ldots \lambda a_n$ is

(*a*) $\lambda\sigma$ (*b*) $-\lambda\sigma$

(*c*) $|\lambda|\sigma$ (*d*) σ

36. If two lines of regression are $Y = 3x - 5$ and $Y = 2x - 4$, then $\rho(X, Y)$ is equal to

(*a*) $\sqrt{\frac{2}{3}}$ (*b*) $\sqrt{\frac{1}{6}}$

(*c*) $\sqrt{\frac{3}{2}}$ (*d*) none of these

37. If two lines regression are $3x - y - 5 = 0$ and $2x - y - 4 = 0$, then $\bar{x}$ and $\bar{y}$ are respectively

(*a*) 1 and –2 (*b*) –1 and 2

(*c*) 2 and –1 (*d*) –2 and –1

38. If regression equation of Y on X is $Y = \lambda x + 4$ and that of X on Y be $4x = Y - 5$, then

(*a*) $0 < 4\lambda \leq 1$ (*b*) $0 \leq \lambda \leq 1$

(*c*) $0 \leq \lambda \leq 4$ (*d*) none of these

39. If cov (X,Y) = 0, then two lines of the regression are

(*a*) parallel (*b*) concident

(*c*) at right angles (*d*) none of these

40. If in a regression analysis problem, $b_{yx} = -0.9$ and $b_{xy} = 0.4$, then $\rho(x, y)$ is

(*a*) 0.6 (*b*) –0.6

(*c*) 0 (*d*) none of these

41. It standard deviation for two variables X and Y are 3 and 4 respectively and their covariance is 8, then correlation coefficient between them is

(*a*) $\frac{2}{3}$ (*b*) $\frac{8}{3\sqrt{2}}$ (*c*) $\frac{9}{8\sqrt{2}}$ (*d*) $\frac{2}{9}$

42. Mean deviation of the data $a, a + d, a + 2d, \ldots a + 2nd$ from the mean is equal to

(*a*) $\frac{n(n+1)|d|}{2n+1}$ (*b*) $\frac{n(n+1)d}{2n+1}$

(*c*) $\frac{(n+1)|d|}{2n}$ (*d*) none of these

43. If $\rho > 0$ and $m = \frac{b_{yx} + b_{xy}}{2}$, then

(*a*) $m \geq \rho$ (*b*) $m = \rho$

(*c*) $m \leq \rho$ (*d*) none of these

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44. If two lines of regression are $x + 2y = 7$ and $2x + y = 7$, then regression equation of Y and X is
(*a*) $2x + y = 7$ (*b*) $x + 2y = 7$
(*c*) $x + 2y = 0$ (*d*) $2x + y = 0$

45. Two variables X and Y are connected by the relation $ax + by + c = 0$ where $ab < 0$, then $\rho(x, y)$ is equal to
(*a*) $\frac{a}{b}$ (*b*) $\frac{b}{a}$ (*c*) –1 (*d*) 1

46. Mean of the numbers 0, 1, 2, 3, ...n with respective weight ${}^nC_0$, ${}^nC_1$, ${}^nC_2$,......., ${}^nC_n$ is
(*a*) $\frac{2^n}{n+1}$ (*b*) $\frac{2^{n+1}}{n+1}$
(*c*) $\frac{n}{2}$ (*d*) none of these

47. Seven (distinct) car accidents occurred in a week. What is the probability that they all occurred on the same day ?
(*a*) $\frac{1}{7^7}$ (*b*) $\frac{1}{7^6}$ (*c*) $\frac{1}{2^7}$ (*d*) $\frac{7}{2^7}$

48. Consider two events E_1 and E_2 such that probability of E_1, $P_r(E_1) = \frac{1}{2}$, probability of E_2, $P_r(E_2) = \frac{1}{3}$ and probability of E_1 and E_2, P_r (E_1 and E_2) = $\frac{1}{5}$.
Which of the following statement(s) is/are TRUE?
(*a*) P_r (E_1 or E_2) is $\frac{2}{3}$
(*b*) Events E_1 and E_2 and E_3 are independent
(*c*) Events E_1 and E_2 are not independent
(*d*) $P_r\left(\frac{E_1}{E_2}\right) = \frac{4}{5}$

49. The probability that top and bottom cards of a randomly shuffled deck are both aces is
(*a*) $\frac{4}{52} \times \frac{4}{52}$ (*b*) $\frac{4}{52} \times \frac{3}{52}$
(*c*) $\frac{4}{52} \times \frac{3}{51}$ (*d*) $\frac{4}{52} \times \frac{4}{51}$

50. The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7 is
(*a*) $\frac{16}{25}$ (*b*) $\left(\frac{9}{10}\right)^3$ (*c*) $\frac{27}{75}$ (*d*) $\frac{18}{25}$

51. If 20 per cent managers are technocrats, the probability that a random committee of 5 managers consists of exactly 2 technocrats is
(*a*) 0.2048 (*b*) 0.4000 (*c*) 0.4096 (*d*) 0.9421

52. A regression moodel is used to express a variable Y as a function of another variable X. This implies that
(*a*) there is a causal relationship between Y and X
(*b*) a value of X may be used to estimate a value of Y
(*c*) values of X exactly determine values of Y
(*d*) there is no causal relationship between Y and X

53. The following data about the flow of liquid was observed in a continuous chemical process plant

Flow rate (litres/sec)	Frequency
7.5 to 7.7	1
7.7 to 7.9	5
7.9 to 8.1	35
8.1 to 8.3	17
8.3 to 8.5	12
8.5 to 8.7	10

Mean flow rate of the liquid is
(*a*) 8.00 litres/sec (*b*) 8.06 litres/sec
(*c*) 8.16 litres/sec (*d*) 8.26 litres/sec

54. From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be Kings, if the first card is NOT replaced?
(*a*) $\frac{1}{26}$ (*b*) $\frac{1}{52}$ (*c*) $\frac{1}{169}$ (*d*) $\frac{1}{221}$

55. A box contains 10 screws, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be
(*a*) 100% (*b*) 50%
(*c*) 49% (*d*) None of these

56. If probabilities that A and B will die withina year are p and q respectively, then probability that only one of them will be alive at the end of the year is
(*a*) pq (*b*) $p(1-q)$
(*c*) $q(1-p)$ (*d*) $p + q - 2pq$

57. If sum and product of the mean and variance of a binomial distribution are 24 and 18 respectively, then distriution is
(*a*) $\left(\frac{1}{7}+\frac{1}{8}\right)^{12}$ (*b*) $\left(\frac{1}{4}+\frac{3}{4}\right)^{16}$
(*c*) $\left(\frac{1}{6}+\frac{5}{6}\right)^{24}$ (*d*) $\left(\frac{1}{2}+\frac{1}{2}\right)^{32}$

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NUMERICAL TYPE QUESTIONS

1. A card is taken out of a pack of 52 cards numbered 2 to 53. The probability that the numbers on the card is a prime number less than 20 is ______

2. The probability of getting a number between 1 and 100. Which is divisible by one and itself only is _______

3. If A and B are two elements such that $P(A) = \frac{2}{3}$, $P(B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{2}$, then $P(A \cup B)$ is _______

4. If A and B are two events such that $P(A \cup B) = 0.65$, $P(A \cap B) = 0.15$, then $P(\overline{A}) + P(\overline{B})$ is ______

5. There are two bags one of which contains 3 black and 4 white balls while other contains 4 black and 3 white balls. A die is cast, if the face 1 or 3 turns up, a ball is taken from first bag and if any other face turns up, a ball is chosen from second bag. The probability of choosing of black ball is _______

6. The chance that a leap year selected at random will contain 53 Sundays is ______

7. A five figure number is formed by the digits 0, 1, 2, 3, 4 without repetition. Then the probability that the number formed is divisible by 4 is _____

8. A man alternately tosses a coin and throws a dice, beginning with the coin. Then the probability that he will get a head before he gets a 5 or 6 on dice is_____

9. A box A contains 2 white and 4 black balls. Another box B contains 5 white and 7 black balls. A ball is transferred from the box A to the box B. Then a ball is drawn from the box B. The probability that it is white is ________

10. If $P(A) = 0$, then $P(A \cap B)$ is _______

11. A problem in mechanics is given to three students A, B and C whose chances of slogging it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Then probability that the problem will be solved is _______

12. In an experiment a coin is tossed 4 times. What is. The size of the sample space is ______

13. If mode of a data is 18 and mean is 24, then median is _______

14. Regression equation of Y on X is

$$8x - 10y + 66 = 0 \text{ and } \sigma_x = 3$$

Hence cov (X, Y) is equal is _______

15. Coefficient of correlation between two variates X and Y is ________

X	1	2	3	4	5
Y	5	4	3	2	1

16. If mean = (3 median – mode) x, then value of x is_________

17. If median = (mode + 2 mean) μ, then μ is equal to _______

18. If two lines of regression are at right angles, then ρ (X, Y) is equal to _______

19. 25% of the items of data are less than 35 and 25% of the items are more than 75. Q.D of the data is________

20. If X and Y are two independent variables such that $\bar{x} = 5$, $\bar{y} = 10$ $\sigma_x^2 = 4$ $\sigma_y^2 = 9$ and if $u = 3x + 4y$, $v = 3x - y$, then $\rho(u, v)$ is equal to ______

21. Four fair coins are tossed simultaneously. The probability that at least one head and one tail turn up is _______

22. The ______ number of four digit EVEN numbers have all 4 digits distinct ?

23. The minimum number of cards to be dealt from an arbitrarily shuffled deck of 52 cards to guarantee that three cards are from same suit is ________

24. A die is rolled three times. The probability that exactly one ODD number turns up among the three outcomes is _______

25. E_1 and E_2 are events in a probability space satisfying the following constraints :

- $P_r(E_1) = P_r(E_2)$
- $P_r(E_1 \cup E_2) = 1$
- E_1 and E_2 are independent

The value of $P_r(E_1)$, the probability of event E_1 is________

26. Two girls have picked 10 Roses, 15 Sunflowers and 14 Daffodils. The number of ways they can divide the flowers amongst themselves ?

27. The probability that it will rain today is 0.5. The probability that it will rain tomorrow is 0.6. The probability that it will rain either today or tomorrow is 0.7. The probability that it will rain today and tomorrow is _______

28. The probability that two friends share the same birth-month is________

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29. Arrivals at a telophone booth are considered to be Poisson, with an average time of 10 minutes between successive arrivals. The length of a phone call is distributed exponentially with mean 3 minutes. The probability that an arrival does not have to wait before service is ________

30. A box contains 5 black and 5 red balls. Two balls are randomly picked one after another from the box, without replacement. The probability for both balls being red is_______

31. A fair coin is tossed three times in succession. If the first toss poduces a head, then the probability of getting exactly two heads in three tosses is _____

32. In a frequencey distribution, mid value of a class is 15 and class interval is 4. The lower limit of the class is______

33. Following marks were obtained by the students in a test :
81, 72, 90, 90, 86, 85, 92, 70, 71, 83, 89, 95, 85, 79, 62. Range of the marks is_________

34. Standard deviation for 7, 9, 11, 13, 15 is______

35. In a binomial distribution, mean is 4 and variance is 3. Then, its mode is________

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You given are fair coin to a person in that room, without knowing which type he is from and tell him to loss it and hide the result from you till you ask for it. Upon asking, the person replies the following.

"The result of the toss is head if and only if I am telling the truth."

Which of the following options are correct ?

(*a*) The result is head
(*b*) The result is tail
(*c*) If the person is of Type 2, then the result is tail
(*d*) If the person is of Type 1, then the result is tail

2013

2. Suppose p is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and p has a Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval?

(*a*) $8/(2e^3)$ (*b*) $9/(2e^3)$
(*c*) $17/(2e^3)$ (*d*) $26/(2e^3)$

2012

3. Suppose a fair six-sided die is rolled once. If the value on the die is 1, 2, or 3, the die is rolled a second time. What is the probability that the sum total of values that turn up is at least 6?

(*a*) $\frac{10}{21}$ (*b*) $\frac{5}{12}$
(*c*) $\frac{2}{3}$ (*d*) $\frac{1}{6}$

2011

4. If the difference between the expectation of the square of a random variable $(E[X]^2)$ and the square of the expectation of the random variable $(E[X])^2$ is denoted by R, then

(*a*) $R = 0$ (*b*) $R < 0$
(*c*) $R \geq 0$ (*d*) $R > 0$

5. If two fair coins are flipped and at least one of the outcomes is known to be a head, what is the probability that both outcomes are heads?

(*a*) $\frac{1}{3}$ (*b*) $\frac{1}{4}$
(*c*) $\frac{1}{2}$ (*d*) $\frac{2}{3}$

6. A deck of 56 cards (each carrying a distinct number from 1 to 5) is shuffled thoroughly. Two coards are then removed one at a time from the deck. What is the probability that the two cards are selected with the number on the first card being one higher than the number on the second card?

(*a*) $\frac{1}{5}$ (*b*) $\frac{4}{25}$
(*c*) $\frac{1}{4}$ (*d*) $\frac{2}{5}$

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7. Consider a finite sequence of random values X = [x_1, x_2, ... x_n]. Let μ_x be the mean and σ_x be the standard deviation of X. Let another finite sequence Y of equal length be derived from this as $y_i = a^*x_i + b$, where a and b are positive constants. Let μ_y be the mean and σ_y be the standard deviation of this sequence. Which one of the following statements is INCORRECT?

(*a*) Index position of mode of X in X is the same as the index position of mode of Y in Y.

(*b*) Index position of median of X in X is the same as the index position of median of Y in Y.

(*c*) $\mu_y = a\mu_x + b$

(*d*) $\sigma_y = a\sigma_x + b$

2010

8. Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probalility of q. What is the probability of a computer being declared faulty?

(*a*) $pq + (1-p)(1-q)$ (*b*) $(1-q)p$

(*c*) $(1-p)q$ (*d*) pq

9. What is the probability that a divisor of 10^{99} is a multiple of 10^{96}?

(*a*) 1/625 (*b*) 4/625

(*c*) 12/625 (*d*) 16/625

2009

10. An unbalanced dice (with 6 faces, numbered from 1 to 6) is thrown. The probability that the face value is odd is 90% of the probability that the face value is even. The probability of getting any even numbered face is the same.

If the probability that the face is even given that it is greater than 3 is 0.75, which one of the following options is closest to the probability that the face value exceeds 3 ?

(*a*) 0.453 (*b*) 0.468

(*c*) 0.485 (*d*) 0.492

2008

11. Aishwarya studies either computer science or mathematics everyday. If she studies computer science on a day, then the probability that she studies mathematics the next day is 0.6. If she studies mathematics on a day, then the probability that she studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probability that she studies computer science on Wednesday?

(*a*) 0.24 (*b*) 0.36

(*c*) 0.4 (*d*) 0.6

12. Let X be a random variable following normal distribution with mean + 1 and variance 4. Let Y be another normal variable with mean – 1 and variance unknown. If P (X ≤ –1) = P (P ≥ 2), the standard deviation of Y is

(*a*) 3 (*b*) 2

(*c*) $\sqrt{2}$ (*d*) 1

2007

13. Suppose we uniformly and randomly select a permutation from the 20! permutations of 1, 2, 3...., 20. What is the probability that 2 appears at an earlier position than any other even number in the selected permutation ?

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{10}$

(*c*) $\frac{9!}{20!}$ (*d*) None of these

2006

14. For each element in a set of size $2n$, an unbiased coin is tossed. The $2n$ coin tosses are independent. An element is chosen if the corresponding coin toss were head. The probability that exactly n elements are chosen is

(*a*) $\frac{\binom{2n}{n}}{4^n}$ (*b*) $\frac{\binom{2n}{n}}{2^n}$

(*c*) $\frac{1}{\binom{2n}{n}}$ (*d*) $\frac{1}{2}$

NUMERICAL TYPE QUESTIONS

2015

1. Let X and Y denote the sets containing 2 and 20 distinct objects respectively and F denote the set of all possible functions defined from X to Y. let f be randomly chosen from F .The probability of f being one-to-one is _______.

2014

2. Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is ________.

3. For fair six-sided dice are rolled. The probability that the sum of the results being 22 is $X/1296$. The value of X is _________.

4. The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without

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replacement). The system is deemed functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by p. Then $100p$ = ____________.

5. Each of the nine words in the sentence "*The quick brown fox jumps over the lazy dog*" is written on a separate piece of paper. These nine pieces of paper are kept in a box. One of the pieces is drawn at random from the box. The *expected* length of the word drawn is ________. (The answer should be rounded to one decimal place.)

6. The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is ______ .

7. Let S be a sample space and two mutually exclusive events A and B be such that $A \cup B = S$. If P(.) denotes the probability of the event, the maximum value of P(A)P(B) is ___________.

ANSWERS

EXERCISE – I

MCQ Type Questions

1.(*c*)	**2.**(*c*)	**3.**(*b*)	**4.**(*c*)	**5.**(*a*)	**6.**(*a*)	**7.**(*a*)	**8.**(*a*)	**9.**(*a*)	**10.**(*a*)
11.(*b*)	**12.**(*a*)	**13.**(*a*)	**14.**(*b*)	**15.**(*d*)	**16.**(*a*)	**17.**(*b*)	**18.**(*d*)	**19.**(*c*)	**20.**(*d*)
21.(*a*)	**22.**(*a*)	**23.**(*a*)	**24.**(*c*)	**25.**(*a*)	**26.**(*a*)	**27.**(*a*)	**28.**(*a*)	**29.**(*b*)	**30.**(*b*)
31.(*c*)	**32.**(*c*)	**33.**(*d*)	**34.**(*c*)	**35.**(*c*)	**36.**(*a*)	**37.**(*a*)	**38.**(*c*)	**39.**(*c*)	**40.**(*d*)
41.(*a*)	**42.**(*a*)	**43.**(*a*)	**44.**(*b*)	**45.**(*d*)	**46.**(*c*)	**47.**(*b*)	**48.**(*c*)	**49.**(*c*)	**50.**(*d*)
51.(*a*)	**52.**(*b*)	**53.**(*c*)	**54.**(*d*)	**55.**(*d*)	**56.**(*d*)	**57.**(*d*)			

Numerical Type Questions

1. $\frac{2}{13}$	**2.** $\frac{25}{98}$	**3.** $\frac{11}{12}$	**4.** 1.2	**5.** $\frac{11}{21}$	**6.** $\frac{2}{7}$	**7.** $\frac{5}{16}$	**8.** $\frac{3}{4}$	**9.** $\frac{16}{39}$
10. 0	**11.** $\frac{3}{4}$	**12.** 16	**13.** 22	**14.** 7.2	**15.** –1	**16.** $\frac{1}{2}$	**17.** $\frac{1}{3}$	**18.** 0
19. 50	**20.** 0	**21.** $\frac{7}{8}$	**22.** 2296	**23.** 9	**24.** $\frac{1}{8}$	**25.** $\frac{1}{2}$	**26.** 2640	**27.** 0.4
28. $\frac{1}{12}$	**29.** 0.3	**30.** 2/9	**31.** $\frac{3}{4}$	**32.** 13	**33.** 33	**34.** 2.8	**35.** 6	

EXERCISE – II

MCQ Type Questions

1. (*a*)	**2.** (*c*)	**3.** (*b*)	**4.** (*c*)	**5.** (*a*)	**6.** (*a*)	**7.** (*d*)	**8.** (*b*)	**9.** (*a*)	**10.** (*b*)
11. (*c*)	**12.** (*a*)	**13.** (*d*)	**14.**(*a*)						

Numerical Type Questions

1. (0.95) **2.** (0.25) **3.** (10) **4.** (11.85 to 11.95) **5.** (3.8 to 3.9)

6. (0.259 to 0.261) **7.** (0.25)

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EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

5. Since odds are 8 to 3 against A

$\therefore \quad P(A) = \frac{3}{11}\ ; \quad (P(B) = \frac{2}{7}\ )$

since odds are 5 to 2 aganist B

and $\quad P(A) + P(B) + P(C) = 1$

$\therefore \quad P(C) = \frac{1}{1} - \frac{3}{11} - \frac{2}{7} = \frac{34}{77}$

Hence odds against C are 43 to 34.

6. Number of ways in which 8 balls can be drawn out of 15 is ${}^{15}C_8$.

Number of ways of drawing 2 red balls is ${}^5C_2$ and corresponding to each of these ${}^5C_2$ ways of drawing a red ball, there are ${}^{10}C_6$ ways of drawing 6 black balls.

Hence total number of ways in which 2 red and 6 black balls can be drawn = ${}^5C_2 \times {}^{10}C_6$.

$\therefore \quad$ Required probability $= \frac{{}^5C_2 \times {}^{10}C_6}{{}^{15}C_8}$

$= \frac{140}{429}$

7. A can draw a ticket in ${}^3C_1 = 3$ ways.

Number of cases in which A can get a prize is clearly 1.

$\therefore \quad$ Probability of A's success $= \frac{1}{3}$.

Again B can draw a ticket in ${}^9C_3 = \frac{9\times8\times7}{3\times2\times1}$

= 84 way.

Number of ways in which B gets all blanks

$= {}^6C_3 = \frac{6\times5\times4}{3\times2\times1} = 20.$

$\therefore \quad$ Number of ways of getting a prize

$= 84 - 20 = 64$

Thus, probability of B's success $= \frac{64}{84} = \frac{16}{21}$

$\therefore \quad$ A's probability of success : B's probability of success

$= \frac{1}{3} : \frac{16}{21} = 7 : 16$

8. $P(A\cap\overline{B}) = P(A) - P(A\cap B)$

$= P(A) - P(A)\,P(B)$

$= P(A)\,(1 - P(B)) = P(A)\,P(\overline{B})$

Hence, **A** and $\overline{\mathbf{B}}$ are independent.

9. $A\cap B = \phi$

$\therefore \quad P(A\cap B) = 0$

11. $S = \{0, 1, 2, ..., 9\}$

Suppose we want a particular integer "3" to be chosen.

Probability of not choosing "3" is $\frac{9}{10}$ (in one chance)

Probability of not choosing 3 in all the three chances $= \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$

Hence probability of choosing it in at least one of the three chances

$= 1 - \frac{27}{1000} = \frac{973}{1000}$

12. $\frac{P(A\cap C)}{P(C)} > \frac{P(B\cap C)}{P(C)}$

$\Rightarrow \quad P(A\cap C) > P(B\cap C)$

and $\quad \frac{P(A\cap\overline{C})}{P(\overline{C})} > \frac{P(B\cap\overline{C})}{P(\overline{C})}$

$\Rightarrow \quad P(A\cap\overline{C}) > P(B\cap\overline{C})$

Adding

$P(A\cap C) + P(A\cap\overline{C}) > P(B\cap C) + P(B\cap\overline{C})$

$P(A) > P(B)$ since $(A\cap C)$ and $A\cap\overline{C}$ are disjoint.

13. $P(C) = 0$

$\Rightarrow \quad C = \phi$

$P(A\cap B\cap C) = P(A\cap B\cap\phi)$

$= P(\phi) = 0$

$P(A)\times P(B)\times P(C) = 0 \quad$ ... since $P(C) = 0$

$\therefore \quad P(A\cap B\cap C) = P(A) - P(B) - P(C)$

Hence A, B, C are independent.

14. Number of ways different numbers can occur on six dice = 6!

Total number of ways of occurrence = 6^5

$\therefore \quad$ Required probability, $\quad P = \frac{6!}{6^5} = \frac{5!}{6^4}$

15. $P(A) = \frac{1}{4}$

and $\quad P(B/A) = \frac{P(B\cap A)}{P(A)} = \frac{1}{2}$

$\Rightarrow \quad P(A\cap B) = \frac{1}{8},\ P(A/B) = \frac{1}{4}$

$\Rightarrow \quad \frac{P(A\cap B)}{P(B)} = \frac{1}{4} \quad \Rightarrow P(B) = \frac{1}{2}$

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(*a*) If A is a sub-event of B, $A \subset B$, then $A \cap B = A$

$\therefore$ $P(A \cap B)$ should be equal to P(A). This is false.

(*b*) $P(A/B) = \frac{3}{4}$ is false.

(*c*) $P(A \cap \overline{B}) = P(A) - P(A \cap B) = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$

$$\therefore \; P(A/\overline{B}) = \frac{P(A \cap \overline{B})}{P(\overline{B})} = \frac{\frac{1}{8}}{1-\frac{1}{2}} = \frac{1}{4}$$

$$\therefore \; P(A/B) + P(A/\overline{B}) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Hence given statement is false.

17. The sum 6 can be obtained as follows :
(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), *i.e.* in 5 ways.

Probability of *A*'s throwing 6 with 2 dice = $\frac{5}{36}$

$\therefore$ Probability of *A*'s not throwing 6 = $\frac{31}{36}$

Similarly, probability of *B*'s throwing 7 = $\frac{6}{36}$, i.e. $\frac{1}{6}$

$\therefore$ Probability of *B*'s not throwing 7 = $\frac{5}{6}$

Now A can win, if he throws 6 in the first, third, fifth, seventh etc. throws.

$\therefore$ Chance of *A*'s winning

$$= \frac{5}{36} + \frac{31}{36}\times\frac{5}{6}\times\frac{5}{36} + \frac{31}{36}\times\frac{5}{6}\times\frac{31}{36}\times\frac{5}{6}\times\frac{5}{36} + ...$$

$$= \frac{5}{36}\times\left[1+\left(\frac{31}{36}\times\frac{5}{6}\right)+\left(\frac{31}{36}\times\frac{5}{6}\right)^2+\left(\frac{31}{36}\times\frac{5}{6}\right)^3+.....\right]$$

$$= \frac{5}{36}\times\frac{1}{1-(31/36)\times(5/6)} = \frac{5}{36}\times\frac{36\times 6}{61} = \frac{30}{61}$$

18. $(A \cup B) \supset A$

(*a*) $P(A \cup B) \geq P(A) = \frac{3}{4}$

(*b*) $A \cap B \subset B$

$$\therefore \quad P(A \cap B) \leq P(B) = \frac{5}{8}$$

Also
$$\begin{aligned} P(A \cap B) &= 1 - P(A \cap B)^c \\ &= 1 - P(A^c \cup B^c) \\ &> 1 - \{P(A^c) + P(B^c)\} \\ &= 1 - \left\{\frac{1}{4}+\frac{3}{8}\right\} = \frac{3}{8} \end{aligned}$$

(*c*) $A \cap \overline{B} \subset \overline{B}$

$$\therefore \; P(A \cap \overline{B}) \leq P(\overline{B}) = \frac{3}{8}$$

Also $A \cap \overline{B} = \left(\overline{\overline{A} \cup B}\right)$

$$\therefore \; P(A \cap \overline{B}) = 1 - P(\overline{A} \cup B) \geq 1 - \{P(\overline{A}) + P(B)\}$$

$$= 1 - \left\{\frac{1}{4}+\frac{5}{8}\right\} = \frac{1}{8}$$

19. Let the event of drawing a faulty item from any of the machines be *A*, and the event that an item drawn at random was produced by M_i be B_i. To $P(B_i/A)$ proceed as follows :

	M_1	M_2	M_3	Remarks
$P(B_i)$	0.25	0.30	0.45	$\because$ sum = 1
$P(A/B_i)$	0.05	0.04	0.03	
$P(B_i)P(A/B_i)$	0.0125	0.012	0.0135	sum = 0.38
$P(B_i/A)$	$\frac{0.0125}{0.038}$	$\frac{0.0125}{0.038}$	$\frac{0.0135}{0.038}$	By Baye's theorem

The highest output being from M_3, required probability = $\frac{0\cdot 0135}{0\cdot 038}$ = 0.355.

20. (*a*) $B = (A \cap B) \cup (\overline{A} \cap B)$

$\Rightarrow$ $P(B) = P(A \cap B) + P(\overline{A} \cap B)$ as $A \cap B$ and $\overline{A} \cap B$ are mutually exclusive.

$\Rightarrow$ $P(\overline{A} \cap B) = P(B) - P(A \cap B)$.

(*b*)
$$\begin{aligned} P(A \cup B) &= P[A \cup (\overline{A} \cap B)] \\ &= P(A) + P(\overline{A} \cap B) \\ &= P(A) + P(B) - P(A \cap B) \end{aligned}$$

(*c*)
$$\begin{aligned} P(A/B) &= P(A)\,P(B/A) \\ &\leq P(A) \quad \text{as } P(B/A) \leq 1. \end{aligned}$$

21. (*a*) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore$ $P((A \cap C) \cup (B \cap C)) = P(A \cap C) + P(B \cap C) - P(A \cap C \cap B \cap C)$

and $P((A \cup B) \cap C) = P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$

$$\Rightarrow \quad \frac{P((A \cup B) \cap C)}{P(C)} = \frac{P(A \cap C)}{P(C)} + \frac{P(B \cap C)}{P(C)} \; \frac{P(A \cap B \cap C)}{P(C)}$$

$\Rightarrow P(A \cup B/C) = P(A/C) + P(B/C) - P(A \cap B/C)$

(*b*) $P\left(A \cap \overline{B}/C\right) + P(A \cap B/C)$

$$= \frac{P(A \cap \overline{B} \cap C)}{P(C)} + \frac{P(A \cap B \cap C)}{P(C)}$$

$$= \frac{P(A \cap \overline{B} \cap C) + P(A \cap B \cap C)}{P(C)}$$

$$= \frac{P(A \cap \overline{B} \cap C) \cup (A \cap B \cap C)}{P(C)}$$

$$= \frac{P(A \cap C)}{P(C)} = \mathbf{P(A/C)}$$

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(c) $P(A \cap \overline{B})$
$= P(A) - P(A \cap B) = P(A) - P(A)P(B)$
$= P(A)(1 - P(B)) = P(A)P(\overline{B})$

Hence A and $\overline{B}$ are independent.

22. (a) $P((A \cup B) \cap C) = P((A \cap C) \cup (B \cap C))$
$= P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$
$= P(A)P(C) + P(B)P(C) - P(A)P(B)P(C)$
$= P(C)(P(A) + P(B) - P(A \cap B))$
$= P(C)P(A \cup B)$

(b) $P(A/E) = P(B/E)$

$$\frac{P(A \cap E)}{P(E)} = \frac{P(B \cap E)}{P(E)}$$

if $P(A \cap E) = P(B \cap E)$

if $P\left(A \cap \bigcup_{i=1}^{n} E_i\right) = P\left(B \cap \bigcup_{i=1}^{n} E_i\right)$

if $P\bigcup_{i=1}^{n}(A \cap E_i) = P\left(\bigcup_{i=1}^{n}(B \cap E_i)\right)$

$$\sum_{i=1}^{n} P(A \cap E_i) = \sum_{i=1}^{n} P(B \cap E_i)$$

This is true since $P(A \cap E_i) = P(B \cap E_i)$ for each i.

23. Let E_i be the event of a person to get into an accident. Then

$P(E_i) = p \;\forall\; i$

P (at least one man meet with an accident)
$= P(E_1 \cup E_2 \cup \ldots \cup E_n)$
$= 1 - P(\overline{E}_1 \cup \overline{E}_2 \cup \ldots \cup \overline{E}_n)$
$= 1 - P(\overline{E}_1 \cap \overline{E}_2 \cap \ldots \cap \overline{E}_n)$
$= 1 - P(\overline{E}_1)P(\overline{E}_2) \ldots P(\overline{E}_n)$
$= 1 - (1-p)(1-p) \ldots (1-p)$
$= 1 - (1-p)^n$

Hence P (at least one man meets with an accident/a person is chosen) $= \frac{1}{n}(1 - (1-p)^n)$.

24. Expectation of X = E(X) = average = mean

$$= \frac{1}{n}\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} p_i x_i$$

where p_i denote probability of occurrence of x_i.
Let $q = 1 - p$.
Then probabilities of success in 1st, 2nd, 3rd trials are

$p, qp, q^2p, \ldots$

$\therefore$ $E(X) = (0) \cdot p + (1) \cdot qp + (2) \cdot q^2 p + \ldots$
$= qp(1 + 2q + 3q^2 + \ldots + nq^{n+1} + \ldots)$

$$= \frac{qp}{(1-q)^2} = \frac{qp}{p^2} = \frac{q}{p}$$

25. Ratio of A and B
$= 60 \times 80 + 40 \times 70 : 60 \times 20 + 40 \times 30$
$= 76 : 24 =$ **19 : 6**

26. $$P(A/A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$$

$$= \frac{P(A)}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B)}$$

27. $P(A) = P(\overline{B})$
$\Rightarrow$ $P(A) = 1 - P(B)$ $\Rightarrow$ $P(A) + P(B) = 1$
$\Rightarrow$ $P(A \cup B) - P(A \cap B) = 1 = P(S)$
Hence $A \cup B = S$ only if $A \cap B = \phi$
i.e. $A = \overline{B}$ only if $A \cap B = \phi$

28. $P(A) = 0$

$$\Rightarrow P\left(\frac{A}{S}\right) = 0 \Rightarrow \frac{n(A)}{n(S)} = 0 \Rightarrow n(A) = 0$$

$\Rightarrow$ $A = \phi$

29. When each item is increased by λ, the mean is also increased by λ.

30. When each item is increased by λ, the S.D. is multiplied by $|\lambda|$ and variance is multiplied by λ^2.

31. $\Sigma f_i (y_i - \mu) = f_i y_i - M\,\Sigma f_i = \Sigma f_i y_i - \Sigma f_i Y_i = 0$

32. $$b_{yx}\, b_{xy} = \frac{\text{cov}(X,Y)}{\sigma_x^2}\;\frac{\text{cov}(X,Y)}{\sigma_y^2}$$

$$= \left\{\frac{\text{cov}(X,Y)}{\sigma_x \sigma_y}\right\}^2 = \{\rho\, X, Y\}^2$$

33. Cov (X, Y), b_{xy}, b_{xy} and ρ(X, Y) all are either positive, or 0 or negative simultaneously.

34. When each item is divided by α, *i.e.* it becomes $\frac{\overline{x}}{\alpha}$

When each item is increased by 10, mean also increased by 10.

$$\therefore \quad \text{New mean} = \frac{\overline{x}}{\alpha} + u = \frac{x + 10\alpha}{\alpha}$$

35. Standard Deviation of new data

$$= \sqrt{\frac{\Sigma(\lambda a_1)^2}{n} - \left(\frac{\Sigma \lambda a_i}{n}\right)^2} = \sqrt{\lambda^2}\sqrt{\frac{\Sigma a_1^{\,2}}{n} - \left(\frac{\Sigma a_i}{n}\right)^2} = |\lambda|\sigma$$

36. If we take $y = 3x - 5$ as regression equation of Y on X and $Y = 2x - 4$ as that of X on Y, then

$$b_{yx} = 3\; b_{xy} = \frac{1}{2}$$

$\Rightarrow b_{yx}\, b_{xy} = \frac{3}{2}$, which is not possible.

Hence, equation $y = 3x - 5$, *i.e.* $x = \frac{1}{3}y + \frac{5}{3}$ is the regression equation of X on Y
and equation, $Y = 2x - 4$ is that Y on X.

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Then $\quad b_{yx} = 2$ and $b_{xy} = \dfrac{1}{3}$

$\Rightarrow \quad b_{xy}\, b_{xy} = \dfrac{3}{2}$

$\Rightarrow \quad \{\rho\,(X, Y)\}^2 = \dfrac{3}{2}$

$\Rightarrow \quad \rho(XY) = \sqrt{\dfrac{2}{3}}$

37. Solving equations
$3x - y - 5 = 0$ and $2x - y - 4 = 0$,
we get $\quad x = 1$ and $y = -2$.

38. Here $b_{yx} = \lambda$ and $b_{xy} = \dfrac{1}{4}$.

Since $0 \le b_{yx}\, b_{xy} \le 1$, therefore we must have

$0 \le \lambda\left(\dfrac{1}{4}\right) \le 1 \quad \Rightarrow \quad 0 \le \lambda \le 4$

As $b_{xy} \ne 0$, $\therefore\ b_{yx} \ne 0 \quad \Rightarrow \lambda \ne 0$.
Hence, we must have $\quad 0 < \lambda \le 4$.

39. $\quad \text{Cov}\,(X, Y) = 0 \quad \Rightarrow \quad b_{yx} = b_{xy} = 0$
So, two lines of regression are $y - \bar{y} = 0$ and $x - \bar{x} = 0$, which are at right angles.

40. Given : $b_{yx} = -0.9$ is negative and $b_{xy} = 0.4$ is positive which is not possible as b_{yx} and b_{xy} are always of same sign. Hence given data is inconsistent.

41. $\quad \rho(X, Y) = \dfrac{\text{cov}(X,Y)}{\sigma_x \sigma_y} = \dfrac{8}{3 \times 4} = \dfrac{2}{3}$

42. Mean, $\bar{x} = \dfrac{a + (a + d) + (a + 2d) + \ldots + (a + 2nd)}{2n + 1}$

$$= \frac{2n+1}{2}\{a + (a + (a + 2nd))\} = a + nd$$

Sum of numerical deviations from the mean

$$= \sum_{i=1}^{2n+1} |\{a + (i-1)d\} - \{a + nd\}|$$
$$= |a - a - nd| + |a + d - a - nd| + \ldots + |a + nd - a - nd| + |a + (n+1)\ d - a - nd| + \ldots + |a + 2nd - a - nd|$$
$$= |d|\{n + (n-1) + (n-2) + \ldots + 1 + 0 + 1 + 2 + \ldots + n\}$$
$$= |d| \times 2\{1 + 2 + 3 + \ldots + n\} = n(n + 1)|d|$$

$\therefore \quad$ M.D. (about mean) $= \dfrac{n(n+1)|d|}{2n+1}$

43. Since $\rho > 0$, therefore b_{yx} and b_{xy} are also positive

Hence $\quad \dfrac{b_{yx} + b_{xy}}{2} \ge \sqrt{b_{yx}\ b_{xy}}$

$\Rightarrow \quad \dfrac{b_{yx} + b_{xy}}{2} \ge \sqrt{\rho^2}$

$\Rightarrow \quad \dfrac{b_{yx} + b_{xy}}{2} \ge \rho \quad \Rightarrow \quad m \ge \rho$

44. If first time $x + 2y = 7$ is taken as regression equation of Y on X, then we write it as

$$y = -\frac{1}{2}x + \frac{7}{2} \qquad \Rightarrow \qquad b_{yx} = -\frac{1}{2}$$

Also the second equation (which will be regression equation X on Y) can be written as

$$x = -\frac{1}{2}y + \frac{7}{2} \qquad \Rightarrow \qquad b_{xy} = -\frac{1}{2}$$

$\therefore \quad b_{yx}\, b_{xy} = \left(\dfrac{-1}{2}\right)\left(\dfrac{-1}{2}\right) = \dfrac{1}{4} \le 1$

Hence regression equation of Y on X is
$$x + 2y = 7$$

45. Since X and Y connected by linear relationship, therefore they are in perfect correlation.
Also $\quad ab < 0$

$$\frac{a}{b} < 0.$$

$\therefore \quad y = \left(-\dfrac{a}{b}\right)x - \dfrac{c}{b}$ shows that when x increases, y also increases; so X and Y are in perfect direct correlation

$\Rightarrow \quad \rho(X, Y) = 1.$

46. Required mean

$$= \frac{0^n C_0 + 1^n C_1 + 2^n C_2 + \ldots + n^n C_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n} = \frac{n2^{n-1}}{2^n}$$

$$= \frac{n}{2} \qquad (\because C_1 + 2C_2 + 3C_3 + \ldots nC_n = n2^{n-1})$$

51. Probability of technocrat manager $= \dfrac{20}{100} = \dfrac{1}{5} = p$

Probability of non-technocrat manager

$$= \frac{80}{100} = \frac{4}{5} = q$$

Probability of a random committee of 5 with exactly 2 technocrats

$$= {}^5C_2 p^2 q^3 = \frac{5 \times 4}{1 \times 2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3 = 0.2048$$

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54. Probability of both cards being kings

$$= \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$$

56. Required probability

= P [(A dies and B is alive) or (A is alive and B dies)]

$= p(1-q)+(1-p)q$

$= p+q-2pq$

57. Here, $m+\sigma^2 = 24$

and $m\sigma^2 = 128$

Solving we get

$m = 16$ or 8.

If $m = 16$ then

$\sigma^2 = 8$

If $m = 8$, then

$\sigma^2 = 56$.

Case I : $np = 16$ and $npq = 8$

$\Rightarrow \quad p = \frac{1}{2},\ q = \frac{1}{2}$ and $n = 32$.

Case II : $np = 8$ and $npq = 56$

$\Rightarrow \quad q = 7$, which is not possible.

Hence distribution is $(q+p)^n = \left(\frac{1}{2}+\frac{1}{2}\right)^{32}$.

NUMERICAL TYPE QUESTIONS

5. Let E_1 be the event that a ball is drawn from first bag, E_2 the event that a ball is drawn from second bag and E the event a black ball is chosen, therefore

$$P(E) = P(E_1)\, P\left(\frac{E}{E_1}\right) + P(E_2)\, P\left(\frac{E}{E_2}\right)$$

$$= \frac{2}{6}\times\frac{3}{7}+\frac{4}{6}\times\frac{4}{7} = \frac{11}{21}$$

6. A leap year consists of 366 days, so that there are 52 full weeks (and hence 52 Sundays) and two extra days. These two days can be

(i) Monday, Tuesday

(ii) Tuesday, Wednesday,

(iii) Wednesday, Thursday

(iv) Thursday, Friday

(v) Friday, Saturday

(vi) Saturday, Sunday

(vii) Sunday, Monday

Of these 7 cases, the last two are favourable

$\therefore$ Required probability $= \frac{2}{7}$.

7. The five digits can be arranged in 5! ways, out of which 4! will begin with zero.

$\therefore$ Total number of 5-figure numbers formed

$= 5! - 4! = 96.$

Those numbers formed will be divisible by 4 which will have two extreme right digits divisible by 4,

i.e., numbers ending in 04, 12, 20, 24, 32, 40.

Now, numbers ending in 04 = 3! = 6,

numbers ending in 12 = 3! – 2! = 4,

numbers ending in 20 = 3! = 6,

numbers ending in 24 = 3! – 2! = 4,

numbers ending in 32 = 3! – 2! = 4,

andnumbers ending in 40 = 3! = 6.

[Numbers having 12, 24, 32 in the extreme right are (3! – 2!), since the numbers having zero on the extreme left are to excluded.]

$\therefore$ Total number of favourable ways

$= 6 + 4 + 6 + 4 + 4 + 6 = 30$

Hence, required probability $= \frac{30}{96} = \frac{5}{16}$.

8. $P(H) = \frac{1}{2},\ P(T) = \frac{1}{2}$

Let A be the event of getting 5 or 6.

$\therefore$ $P(A) = \frac{1}{3}$ and B be the event of getting 1, 2, 3, 4;

$\therefore$ $P(B) = \frac{2}{3}$

In order to get a 'Head' before "5 or 6", the following events have to occur:

H, TBH, TBTBH, TBTBTBH, ...

$$\therefore \quad P = \frac{1}{2}+\frac{1}{2}\times\frac{2}{3}\times\frac{1}{2}+\left(\frac{1}{3}\right)^2\cdot\frac{1}{2}+\left(\frac{1}{3}\right)^3\cdot\frac{1}{2}$$

$$= \frac{\frac{1}{2}}{1-\frac{1}{3}} = \mathbf{\frac{3}{4}}$$

9. Probability of drawing a white ball from box B will depend on whether transferred ball is black or white.

If a black ball is transferred, then its probability is $\frac{4}{6}$.

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There are now 5 white and 8 black balls in the box B. Then probability of drawing white ball from box $B = \frac{5}{13}$

Hence probability of drawing a white ball from urn B, if the transferred ball is black

$$= \frac{4}{6} \times \frac{5}{13} = \frac{10}{39}$$

Similarly, probability of drawing a white ball from urn B, if transferred ball is white

$$= \frac{2}{6} \times \frac{6}{13} = \frac{2}{13}.$$

$\therefore$ Required probability $= \frac{10}{39} + \frac{2}{13} = \mathbf{\frac{16}{39}}$.

10. $P(A) = 0$

$\Rightarrow$ $A = \phi$

$\therefore$ $A \cap B = \phi$

$\therefore$ $P(A \cap B) = P(\phi) = 0$

12. Sample space, S = {HHHH, HHHT, HHTH, HHTT, ... , TTTT}

Clearly, $|S| = 2^4 = 16$.

14. Given regression limit is

$$8x - 10y + 66 = 0$$

$$\Rightarrow \quad y = \frac{8}{10}x + \frac{66}{10} = \frac{4}{5}x + \frac{33}{5}$$

Hence $b_{yx} = \frac{4}{5}$

$$\Rightarrow \quad \frac{\text{cov}(X, Y)}{\sigma_x^2} = \frac{4}{5}$$

$$\Rightarrow \quad \text{cov}(X, Y) = \frac{9 \times 4}{5}$$

15. Since, $x + y = 6$ for all the pairs of observations,

$\therefore$ $b_{xy} = b_{yx} = -1$

$\Rightarrow$ $\rho(X, Y) = -1$

16. Mode $= 3$ Median $- 2$ Mean

$$\Rightarrow \quad \text{Mean} = \frac{3}{2}\text{Median} - \frac{1}{2}\text{Mode}$$

$$= \frac{1}{2}(3\,\text{Median} - \text{Mode})$$

17. Mode = 3 Median – 2 Mean

$\Rightarrow$ 3 Median = Mode + 2 Mean

$\Rightarrow$ Median $= \frac{1}{3}$(Mode + 2 Mean)

$\Rightarrow$ $M = \frac{1}{3}$

18. The two lines of regression are at right angles only when

$$b_{yx}\left(\frac{1}{b_{xy}}\right) = -1 \quad \Rightarrow \quad b_{yx} = -b_{xy}$$

But both b_{yx} and b_{xy} are of same sign, therefore we must have

$$b_{yx} = b_{xy} = 0 \Rightarrow \rho(X, Y) = 0$$

19. Given : Lower quartile, $Q_1 = 35$

Upper quartile $Q_3 = 75$.

$$\therefore \text{ Quartile deviation} = \frac{Q_3 - Q_1}{2} = \frac{75 - 35}{2} = 20$$

20. Here, $\bar{u} = 3\bar{x} + 4\bar{y}$ and $\bar{v} = 3\bar{x} - \bar{y}$

$$u - \bar{u} = 3(x - \bar{x}) + 4(y - \bar{y})$$

$$v - \bar{v} = 3(x - \bar{x}) - (y - \bar{y})$$

$$\Rightarrow \quad \text{cov}(u, v) = \frac{1}{n}\Sigma(u - \bar{u})(v - \bar{v})$$

$$= \frac{1}{n}\Sigma\{9(x - \bar{x})^2 - 4(y - \bar{y})^2 + 9(x - \bar{x})(y - \bar{y})\}$$

$$= 9\left(\frac{\Sigma(x - \bar{x})^2}{n}\right) - 4\left(\frac{\Sigma(y - \bar{y})^2}{n}\right) + 9\left(\frac{\Sigma(x - \bar{x})(y - \bar{y})}{n}\right)$$

$$= 9\,\sigma_x^2 - 4\,\sigma_y^2 + 9\,\text{cov.}(x, y)$$

$$= 9 \times 4 - 4 \times 9 + 9(0) = 0$$

{since X, Y are independent, therefore cov (X, Y) = 0}

$$\Rightarrow \quad \rho(u, v) = \frac{\text{cov}(u, v)}{\sigma_u \sigma_v} = 0$$

28. Probability that first friend is born in any month $= 100\% = 1$

Probability that second friend is born in the same month as that of first friend $= 1 \times \frac{1}{12} = \frac{1}{12}$

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30. Probability of drawing a red ball = $\frac{5}{10}$

If the ball is not replaced, then box will have a ball, so probability of drawing the red ball in next chance = $\frac{4}{9}$

Hence probability of drawing 2 balls

$$= \frac{5}{10} \times \frac{4}{9} = \frac{2}{9}$$

31. After first heads in first toss

probability of tails in 2nd and 3rd toss

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$\therefore$ Probability of exactly two heads = $1 - \frac{1}{4} = \frac{3}{4}$

32. Let lower limit be x.

Then, upper limit = x + 4

$$\therefore \quad \frac{x + (x+4)}{2} = 15$$

$$\Rightarrow \quad x = 13$$

33. Range = Difference between largest values

= 95 – 62 = 33

34.
$$m = \frac{7+9+11+13+15}{5} = \frac{55}{5} = 11$$

$$\sum \delta^2 = |7-11|^2 + |9-11|^2 + |11-11|^2 + |13-11|^2 + |15-11|^2$$

$$= 40$$

$$\therefore \quad \sigma = \sqrt{\frac{\sum \delta^2}{n}} = \sqrt{\frac{40}{5}} = \sqrt{8}$$

$$= 2\sqrt{2} = 2 \times 1.41 = 2.8$$

35. We have $np = 4$ and $npq = 3$

$$\Rightarrow \quad q = \frac{3}{4} \text{ and } p = (1-q) = \frac{1}{4}$$

Mode is an integer such that

$$np + p > x > np - q$$

$$\Rightarrow \quad 4 + \frac{1}{4} > x > 4 - \frac{3}{4}$$

$$\Rightarrow \quad \frac{13}{4} < x < \frac{17}{4}$$

$$\Rightarrow \quad 3.25 < x < 4.25$$

$$\Rightarrow \quad x = 4$$

EXERCISE – II

MCQ TYPE QUESTIONS

2. Given: P # of cars passing every minute

P is a random variable following Poisson's Distribution

$$\therefore \quad \text{Prob. } (P = K) = \frac{\lambda^k e^{-\lambda}}{K!}$$

$$\text{Mean} = E[P] = l$$

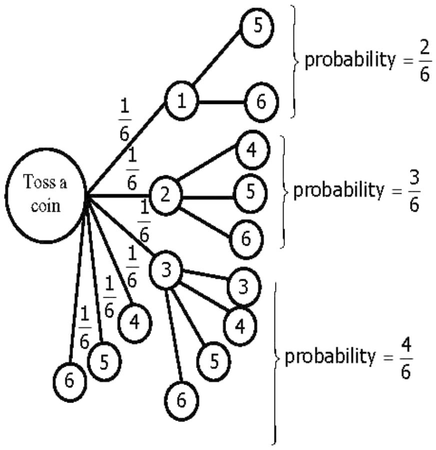
$$\therefore \quad \lambda = 3$$

Probability of observing fewer than 3 cars

= Prob. (P = 0) + Prob (P = 1) + Prob (P = 2)

$$= \frac{3^0.e^{-3}}{0!} + \frac{3^1.e^{-3}}{1!} + \frac{3^2.e^{-3}}{2!} = \frac{1}{e^3}\left[1 + 3 + \frac{9}{2}\right] = \boxed{\frac{17}{2e^3}}$$

3.



$\therefore$ Required probability

$$= \frac{1}{6} \times \frac{2}{6} + \frac{1}{6} \times \frac{3}{6} + \frac{1}{6} \times \frac{4}{6} + \frac{1}{6} = \frac{15}{36} = \frac{5}{12}$$

5. Sample space = {HH,HT, TH}

Required probability = $\frac{1}{3}$

6. (2, 1), (3, 2), (4, 3), (5, 4)

Required probability = $\frac{4}{5 \times 4} = \frac{4}{20} = \frac{1}{5}$

8. Probability of faulty declared

= (already faulty) × (declared faulty after testing)

Let probability of already faulty = p

Probability that after testing it declares faulty

= $(1-q)$

$\therefore$ Required probability = $(1-q)\,p$

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9. Total number of divisors of $10^{99} = 2^{99} \times 5^{99} = 2 \times 5$ and that are most direct of 10^{96}

$$\equiv (99 + 1)(991 + 1)$$

$$10^{96} \times 10^3$$

$\Rightarrow 10^{96}, 2 \times 10^{96}, 4 \times 10^{96}, 9 \times 10^{96}, 8 \times 10^{96}$

$$\left|\frac{1000}{n}\right| = 0$$

$\Rightarrow 10 \times 10^{96}, 20 \times 10^{96}, 29 \times 10^{96}, 40 \times 10^{96}, 50 \times 10^{96}$

$n = 1$—100 $\quad 125 \times 10^{96},\ 250 \times 10^{96},\ 500 \times 10^{96},$

$100 \times 10^{96}, 100 \times 10^{96}, 200 \times 10^{96}$

= total 16

$\therefore$ Required probability $= \dfrac{10}{10000} = \dfrac{1}{625}$

10. Given : P(ODD) = 0.9*P(EVEN) $\neq$ P(2) = P(4)

= P(6) $\neq$ P(EVEN | Exceeds 3) = 0.75

Now we know

P(ODD) = 1 – P (EVEN)

or 0.9*P (EVEN) = 1 – P (EVEN)

{Given P(ODD) = 0.9*P(EVEN)}

or P(EVEN) = 1/1.9

Because of independent events

P(2) = P(4) = P(6) = P(2 $\cup$ 4 $\cup$ 6)/3

= P(EVEN)/3 = 1/(3*1.9)

Now, P(EVEN | Exceeds 3)

= P(EVEN $\cap$ Exceeds 3)/P(Exceeds 3)

P(Exceeds 3) = P(EVEN $\cap$ Exceeds 3)/P(EVEN | Exceeds 3)

P(Exceeds 3) = P(4 $\cup$ 6)/0.75

P(Exceeds 3) = (P(4) + P(6))/0.75

P(Exceeds 3) = (1/3*1.9) + 1/(1.9 * 3))/0.75

P(Exceeds 3) = 0.468

11. Combinations are CMC, CMM, CCC, CCM.

Probability of

CMC = 0.6 × 0.4 = 0.24 —Favourable

CMM = 0.6 × 0.6 = 0.36

CCC = 0.4 × 0.4 = 0.16 —Favourable

CCM = 0.4 × 0.6 = 0.24

Total probability = 1

Favourable = 0.24 + 0.16 = 0.4

$\therefore$ P = 0.4/1

12. Here standard deviation is 3.

13. Number of permutations with '2' in the first position = 19!

Number of permutations with '2' in the second position = 10 × 18!

(fill the first space with any of the 10 odd numbers and the 18 spaces after the 2 with 18 of the remaining numbers in 18! ways)

Number of permutations with '2' in 3rd position = 10 ×´ 9 × 17!

(fill the first 2 places with 2 of the 10 odd numbers and then the remaining 17 places with remaining 17 numbers)and so on until '2' is in 11th place. After that it is not possible to satisfy the given condition, since there are only 10 odd numbers available to fill before the '2'. So the desired number of permu-tations which satisfied the given condition is

19! + 10 × 18! + 10 × 9 × 17! + 10 × 9 × 8 × 16! + ... + 10! × 9!

Now the probability of this happenning is given by

$$\frac{19!+10\times18!+10\times9\times17!...+10!\times9!}{20!}$$

Which is clearly not choices (*a*), (*b*) or (*c*)

$\therefore$ Answer is (*d*)

14. Required favourable cases

= Number of ways of selecting n elements out of the set of size $2n = \dbinom{2n}{n}$

Since each toss may result in either Head or Tail (2 possibilities), hence for $2n$ tosses,

total combinations $= 2^{2n} = 4^n$

$\therefore$ Required probability $= \dfrac{\text{Favourable cases}}{\text{Total cases}}$

$$= \frac{\binom{2n}{n}}{4^n}$$

NUMERICAL TYPE QUESTIONS

1. $|x| = 2$

and $|y| = 20$

Total number of function from set x to set y is

$20^2 = 400$

and total number of one-one function from set x to set y is

$$^{20}P_2 = \frac{20!}{(20-2)!} = \frac{20!}{18!} \qquad \left(\because {}^nP_r = \frac{n!}{(n-r)!}\right)$$

$= 20 \times 19 = 380$

$\therefore$ Probabiltiy the F chosen to be one-one function

$$= \frac{380}{400} = \frac{38}{40} = \frac{19}{20} = 0.95$$

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2. The smaller sticks, therefore, will range in length from almost 0 meters up to a maximum of 0.5 meters, with each length equally possible. Thus, the average length will be about 0.25 meters, or about a quarter of the stick.

3. 22 occurred in following ways

6 6 6 4 $\rightarrow$ 4 ways

6 6 5 5 $\rightarrow$ 6 ways

Required probability = $\frac{6+4}{2296} = \frac{10}{2296}$

$\Rightarrow \quad x = 10$

4. p = P [at least three computers are working]

= P (3 or 4 computers working)

$$= \frac{\left(4_{C_3}\right)\times\left(6_{C_1}\right)}{10_{C_4}} + \frac{4_{C_4}}{10_{C_4}} = \frac{5}{42} \Rightarrow 100\text{ p} = 11.9.$$

5. Given words are

THE, QUICK, BROWN, FOX, JUMPS, OVER, THE, LAXY, DOG

LET X be the random variable such that X =length of the word

The Length of the words THE, FOX, THE, DOG is 3

The Length of the words OVER, LAXY is 4

The length of the words QUICK, BROWN, JUMPS, is 5

The corresponding probabilities are given below

x	3	4	5
P(X)	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{3}{9}$

Expected length of the word

$$= \sum \text{xp(x)} = 3\left(\frac{4}{9}\right) + 4\left(\frac{2}{9}\right) + 5\left(\frac{3}{9}\right) = 3.8889$$

6. Positive number between 1 to 100 (inclusive) not divisible by 2, 3 or 5 are

{1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89, 91, 97}

$$P(E) = \frac{26}{100} = 0.26$$

7. Given $A \cup B = S$

$\Rightarrow$ P (A $\cup$ B) = P (S) = 1

$\Rightarrow$ P(A) + P(B) = 1

($\because$ A & B are mutually exclusive)

$\Rightarrow$ P (B) = 1 – P(A)

Maximum value of P(A) P(B) = ?

Maximum value of P (A) [1 – P (A)] = ?

Let P(A) = X

Let $f(x) = x\,(1 - x) = x - x^2$

for $f(x)$ maximum $\Rightarrow f'(x) = 0$

$\Rightarrow \; 1 - 2x = 0$

$\Rightarrow \; x = \frac{1}{2}$

$f''(x) = -2; f''\left(\frac{1}{2}\right) < 0$

$\therefore \; f(x)$ has maximum

At $x = \frac{1}{2}$ and maximum value

$$= f\left(\frac{1}{2}\right) = \frac{1}{2}\left(1-\frac{1}{2}\right) = \frac{1}{2}.\frac{1}{2} = \frac{1}{4} = 0.25$$

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4
CHAPTER

Discrete Mathematics

MATHEMATICAL LOGIC

STATEMENTS

Atomic statements.

The simple statements without connectives are called *atomic or primary statements*.

Molecular statements.

The new statements formed by joining atomic statements through connectives are called *molecular or compound statements.*

Truth table.

A table showing the truth value of a statement formula for each possible combinations of truth values of component statements is called its *truth table*.

A statement formula having n distinct components will have 2^n rows in its Truth table.

Truth values.

A statement is a declarative sentence which has one and only one of two possible values, called *truth values*. These two truth values are *true* and *false* denoted by the symbol T and F respectively; sometimes it is also denoted by the symbols 1 and 0.

The object language contains such declarative sentences. Since we allow only two possible truth values in our study, the logic is called *two-valued logic.*

The sentence, " *This statement is false*" is not a statement because we cannot assign a definite truth value to it. If we assign the value T, then sentence says that the '*statement is false*'; if we assign F, then the 'statement is true'. This is an example of *semantic paradox.* The sentence like

$$101 + 1 = 110$$

is a statement. It has a truth value, but the truth value depends on the context. If we take decimial system, the statement is false, but in binary system the statement is true. Even then we consider such sentences as statements.

CONNECTIVES

If we form new sentences from given sentences using 'and', 'but', 'if' etc., we can get new propositions from given propositions using *connectives*.

(1) Propositional connectives.

The new sentence obtained will be a proposition only when the new sentence has a truth value T or F (but not both).

(2) Logical connectives.

Truth value of the new sentence depends on the (logical) connectives and the truth value of the given propositions.

SOME CONNECTIVES.

(1) Negation (NOT) .

If P is a proposition, then negation P or NOT P (denoted by P) is a proposition whose truth value is T if P has truth value F, and F if P has truth value T.

Table. Truth Table for Negation

P	**P**
T	F
F	T

(2) Conjunction (AND).

If P and Q are two propositions, then conjunction of P and Q (read as "P and Q") is a proposition whose truth values are given in the Truth Table and denoted by $P \wedge Q$.

Table. Truth Table for Conjunction

P	**Q**	**$P \wedge Q$**
T	T	T
T	F	F
F	T	F
F	F	F

(3) Disjunction (OR).

If P and Q are two propositions, then disjunction of P and Q (read as P or Q) is a proposition whose truth values are given in the truth table and denotd by $P \vee Q$.

$P \vee Q$ is true if P is true or Q is true or both are true.

Table. Truth Table for Disjunction

P	**Q**	**$P \vee Q$**
T	T	T
T	F	T
F	T	T
F	F	F

Example. If P represents "*This book is nice*" and Q represents "*This book is cheap*", write the following sentences in symbolic form

(*a*) This book is good and cheap.

(*b*) This book is not good but cheap.

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(*c*) This book is costly but good.
(*d*) This book is neither good nor cheap.
(*e*) This book is good or cheap.

Solution:

(*a*) $P \wedge Q$ (*b*) $(\rceil P) \wedge Q$ (*c*) $(\rceil Q) \wedge P$ (*d*) $(\rceil P \wedge (\rceil Q))$ (*e*) $P \vee Q$

Table. Representation and meaning of Logical connectives

Connective	Resulting proposition	Read as
Negation $\rceil$	$\rceil P$	NOT P
Conjunction $\wedge$	$P \wedge Q$	P AND Q
Disjunction $\vee$	$P \vee Q$	P OR Q (or both)
Implication $\rightarrow$	$P \rightarrow Q$	IF P THEN Q (P Implies Q)
IF AND ONLY IF $\rightleftarrows$	$P \rightleftarrows Q$	P IF AND ONLY IF Q

(**4**) **Implication (IF ... THEN .....).**

If P and Q are two propositions, then "IF P THEN Q" is a proposition whose truth values are given in the truth table and is denoted by $P \rightarrow Q$.

We also read $P \rightarrow Q$ as *"P implies Q"*.

Table. Truth Table for Implication

P	**Q**	**$P \rightleftarrows Q$**
T	T	T
T	F	F
F	T	T
F	F	T

$P \rightarrow Q$ assumes the truth value F only if P has truth value T and Q has truth value F. In all the other cases, $P \rightarrow Q$ assumes the truth value T. In the case of natural languages, we are concerned about the truth values of the sentence "IF P THEN Q" only when P is true. When P is false, we are not concerned about the truth value of "IF P THEN Q". But in the case of mathematical logic, we have to definitely specify the truth value of $P \rightarrow Q$ in all cases. So truth value of $P \rightarrow Q$ is defined as T when P has truth value F (irrespective of the truth value of Q).

(**5**) **IF and Only IF.**

If P and Q are two statements, then P if and only if Q (denoted by $P \rightleftarrows Q$) is a statement whose truth value is T when the truth values of P and Q are the same and whose truth value is F when the statements differ. It is also called B conditional and means that P implies Q and Q implies P

i.e. $(P \rightarrow Q) \wedge (Q \rightarrow P)$

Table. Truth Table for IF and Only IF

P	**Q**	**$P \rightleftarrows Q$**
T	T	T
T	F	F
F	T	F
F	F	T

Example. Translate the following sentences into propositional forms :

(*a*) If it is not raining and I have time then I will go to a movie.
(*b*) It is raining and I will not go to a movie.
(*c*) It is not raining.
(*d*) I will not go to a movie.
(*e*) I will go to movie only if it is not raining.

Solution:

Let P be proposition *"It is raining"*,
Q be the proposition *"I have time"*,
R be proposition *"I will go to a movie"*.

Then

(*a*) $\rceil P \wedge Q \rightarrow R$ (*b*) $P \wedge \rceil R$ (*c*) $\rceil P$

(*d*) $\rceil R$ (*e*) $R \rightleftarrows \rceil P$

WELL-FORMED FORMULAS

Definition 1.

A propositional variable is a symbol representing any proposition. Usually a real variable is represented by the symbol x. This means that x is not a real number but can take a real value. Similarly, a propositional variable is not a proposition but can be replaced by a proposition.

Definition 2.

A well-formed formula (wff) is defined recursively as follows :

I. *If P is a propositional variable, then it is a wff.*

II. *If α is a wff, then $\rceil\alpha$ is a wff.*

III. *If α and β are well-formed formulas, then*

$(\alpha \vee \beta), (\alpha \wedge \beta), (\alpha \rightarrow \beta), (\alpha \rightleftarrows \beta)$ *are well-formed formulas.*

IV. *A string of symbols is a wff if and only if it is obtained by finitely many applications of (I)–(III).*

A well formed formula is not a proposition, but if we substitute the proposition in place of propositional variable, we get a proposition.

e.g.(*i*) $[\rceil(P \vee Q) \wedge (\rceil Q \wedge R) \rightarrow Q)$ is a wff.

(*ii*) $(\rceil P \wedge Q) \rightleftarrows Q$ is a wff.

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We can drop parentheses when there is no ambiguity.

e.g. in propositions we can remove the outermost parentheses. We can also specify hierarchy of connectives and avoid parentheses.

For the convenience, we can refer to a wff as a formula.

Duality law.

Two formulas A & A′ are said to be duals of each other if either can be obtained form the other by replacing $\wedge$ by $\vee$ and vice-versa and T by F and vice-versa.

e.g. Dual of $(P \wedge Q) \vee T$ is $(P \vee Q) \wedge F$

Let A and A′ are duals consisting of $P_1, P_2 \ldots\ldots P_n$ propositional variables. By repeated application of De Morgans Law, it can be shown that

$$\neg A(P_1, P_2 \ldots P_n) = A'(\neg P_1, \neg P_2 \ldots\ldots \neg P_n)$$

Theorem.

If two formulas are equivalent, then their duals are also equivalent.

Truth Table For Well-Formed Formula.

If we replace the propositional variables in a formula α by propositions, we get a proposition involving connectives. The table giving truth value of such proposition obtained by replacing the propositional variables by arbitrary propositions is called *truth table of α.*

If α involves n propositional constants, we have 2^n possible combinations of truth values of propositions replacing the variables.

Example. Obtain the truth table for $\alpha = (P \vee Q) \wedge (P \to Q) \wedge (Q \to P)$.

Solution:

P	Q	$P \vee Q$	$P \to Q$	$(P \vee Q) \wedge (P \to Q)$	$(Q \to P)$	α
T	T	T	T	T	T	T
T	F	T	F	F	T	F
F	T	T	T	T	F	F
F	F	F	T	F	T	F

Example. Construct the truth table for $\alpha = (P \vee Q) \to ((P \vee R) \to (R \vee Q))$.

Solution:

P	Q	R	$P \vee R$	$R \vee Q$	$(P \vee R) \to (R \vee Q)$	$(P \vee Q)$	α
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	T	T	T	F	T
F	F	F	F	F	T	F	T

Some formulas have truth value T for all possible assignments of truth values to the propositional variables. e.g., $P \vee \neg P$ has the truth value T irrespective of the truth value of P. Such formulas are called *tautologies.*

Definition 3.

A tautology or a universally true formula is a well-formed formula whose truth value is T for all possible assignments of truth values to the propositional variables.

e.g. $P \vee \neg P$, $(P \wedge Q) \to P$ and $((P \to Q) \wedge (Q \to R)) \to (P \to R)$ *are tautologies*.

Example. Show that $\alpha = (P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$ is a tautology.

Solution:

Truth Table for α

P	Q	R	$Q \to R$	$P \to (Q \to R)$	$P \to Q$	$P \to R$	$(P \to Q) \to (P \to R)$	α
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

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Definition 4.

A contradiction (or absurdity) is a wff whose truth value is F for all possible assignments of truth values to the propositional variables.

e.g. $P \wedge \urcorner P$ and $(P \wedge Q) \wedge \urcorner Q$ are contradictions.

Note : α is a contradiction if and only $\urcorner\alpha$ is a tautology.

Equivalence of Well Formed Formulas.

Definition 5.

Two wffs α and β in propositional variables $P_1, P_2, \ldots P_n$ are equivalent (or logically equivalent) if the formula $\alpha \rightleftarrows \alpha$ is a tautology. When α and β are equivalent, we write $\alpha \equiv \beta$.

α and β are equivalent if the truth tables for α and β are the same.

e.g. $P \wedge Q \equiv Q \wedge P$ and $P \wedge P \equiv P$

Note : The difference betwen $\alpha \rightleftarrows \beta$ and $\alpha \equiv \beta$, $\alpha \rightleftarrows \beta$ is a formula, where as $\alpha \equiv \beta$ is not a formula, but it denotes the relation between α and β.

Example. Show that $(P \to (Q \vee R)) \equiv ((P \to Q) \vee (P \to R))$.

Solution:

Let $\alpha = (P \to Q \vee R)$ and $\beta = ((P \to Q) \vee (P \to R))$.

We construct the truth values of α and β for all assignments of truth values to the variables P, Q and R, given in the table below.

P	**Q**	**R**	**$Q \vee R$**	**$P \to Q \vee R$ (α)**	**$P \to Q$**	**$P \to R$**	**$(P \to Q) \vee (P \to R)(\beta)$**
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

As the columns corresponding to α and β coincide, $\alpha \equiv \beta$.

As truth value of a tautoloy is T, irrespective of the truth values of propositional variables, we denote any tautology by T. Similarly, we denote any contradiction by F.

LOGICAL IDENTITIES

Some equivalences are useful for deducing other equivalences, and are called *identities*

The identities $I_1 - I_{12}$ can be used to simplify formulas. If a formula β is part of another formula α and β is equivalent to β', then we can replace β and β' in α and resulting wff is equivalent to α.

Table : Logical identities

I_1	**Idempotent laws**	$P \vee P \equiv P$, $P \wedge P \equiv P$
I_2	**Commutative laws**	$P \vee Q \equiv Q \vee P$, $P \wedge Q \equiv Q \wedge P$
I_3	**Associative laws**	$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$, $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
I_4	**Distributive laws**	$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$, $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
I_5	**Absorption laws**	$P \vee (P \wedge Q) \equiv P$, $P \wedge (P \vee Q) \equiv P$
I_6	**De Morgan's laws**	$\urcorner(P \vee Q) \equiv \urcorner P \wedge \urcorner Q$, $\urcorner(P \wedge Q) \equiv \urcorner P \vee \urcorner Q$
I_7	**Involution law**	$P \equiv \urcorner(\urcorner P)$
I_8	$P \vee \urcorner P \equiv T$, $P \wedge \urcorner P \equiv F$	
I_9	$P \vee T \equiv T$, $P \wedge T \equiv P$, $P \vee F \equiv P$	$P \wedge F \equiv F$
I_{10}	$(P \to Q) \wedge (\urcorner Q) \equiv \urcorner P$	
I_{11}	**Contrapositive**	$P \to Q \equiv \urcorner Q \to \urcorner P$
I_{12}	$P \to Q \equiv (\urcorner P \vee Q)$	

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Example. Show that $(P \wedge Q) \vee (P \wedge \neg Q) \equiv P$

Solution:

$$\begin{aligned} \text{L.H.S.} &= (P \wedge Q) \vee (P \wedge \neg Q) & \\ &\equiv P \wedge (Q \vee \neg Q) & \text{using Distributive law (i.e. } I_4) \\ &\equiv P \wedge T & \text{using } I_8 \\ &\equiv P & \text{using } I_8 \end{aligned}$$

Example. Show that $(P \to Q) \wedge (R \to Q) \equiv (P \vee R) \to Q$

Solution:

$$\begin{aligned} \text{L.H.S.} &= (P \to Q) \wedge (R \to Q) & \\ &\equiv (\neg P \vee Q) \wedge (\neg R \vee Q) & \text{using } I_{12} \\ &\equiv (Q \vee \neg P) \wedge (Q \vee \neg R) & \text{using Commutative law} \\ &\equiv Q \vee (\neg P \wedge \neg R) & \text{using Distributive law} \\ &\equiv Q \vee (\neg (P \vee R)) & \text{using De Morgan's law} \\ &\equiv (\neg (P \vee R)) \vee Q & \text{using Commutative law} \\ &\equiv (P \vee R) \to Q & \text{using } I_{12} \\ &= \text{R.H.S.} & \end{aligned}$$

Normal Forms Of Well-Formed Formulas.

Various well-formed formulas are seen in terms of two propositional variables, say P and Q. We also know that two such formulas are equivalent if and only if they have the same truth table. The number of distinct truth tables for formulas in P and Q is 2^4 (as possible combinations of truth values of P and Q are TT, TF, FT, FF, truth table of any formula in P and Q has 4 rows. So number of distinct truth tables is 2^4). Thus there are only 16 distinct (nonequivalent) formulas, and any formula in P and Q is equivalent to one of these 16 formulas.

Here we give a method of reducing a given formula to an equivalent form called a *normal form*. We also use '*sum*' for disjunction. '*product*' for conjunction, and '*literal*' either for P or for $\neg P$, where P is any propositional variable.

Definition 6 .

An elementary product is a product of literals. An elementary sum is a sum of literals.

e.g. $P \wedge \neg Q$, $\neg P \wedge \neg Q$, $P \wedge Q$, $\neg P \wedge Q$ *are elementary products.*

$P \vee \neg Q$, $P \vee \neg R$ *are elementary sums.*

Definition 7.

A formula is in disjunctive normal form if it is a sum of elementary products.

e.g. $P \vee (Q \wedge R)$ and $P \vee (\neg Q \wedge R)$ *are in disjunctive normal form.*

$P \wedge (Q \vee R)$ *is not in disjunctive normal form.*

Construction to obtain Disjunctive Normal form of a given formula.

Step 1: Eliminate $\to$ and $\to$ using logical identities.
(We can use I_{12}, *i.e.*, $P \to Q \equiv (\neg P \vee Q)$.

Step 2: Use De Morgan's law (I_6) to eliminate $\neg$ before sums or products. The resulting formula has $\neg$ only before propositional variables, *i.e.* it involves sum, product and literals.

Step 3: Apply distributive laws (I_4) repeatedly to eliminate product of sums. The resulting formula will be a sum of products of literals, *i.e.* sum of elementary products.

Example. Obtain a disjunctive normal form of
$$P \vee (\neg P \to (Q \vee (Q \to \neg R)))$$

Solution:

$$\begin{aligned} & P \vee (\neg P \to (Q \vee (Q \to \neg R))) & \\ & \equiv P \vee (\neg P \to (Q \vee (\neg Q \vee \neg R))) & (\text{step 1 using } I_{12}) \\ & \equiv P \vee (P \vee (Q \vee (\neg Q \vee \neg R))) & (\text{step 1 using } I_{12} \text{ and } I_7) \\ & \equiv P \vee P \vee Q \vee \neg Q \vee \neg R & \text{using } I_3 \\ & \equiv P \vee Q \vee \neg Q \vee \neg R & \text{using } I_1 \end{aligned}$$

Thus, $P \vee Q \vee \neg Q \vee \neg R$ is a disjunctive normal form of the given formula.

Advantages of constructing principal disjunctive normal forms

(*i*) For a given formula, its principal disjunctive normal form is unique.

(*ii*) Two formulas are equivalent if and only if their principal disjunctive normal forms coincide.

Definition 8.

A min term in n propositional variables $P_1, \ldots. P_n$ is $Q_1 \wedge Q_2 \ldots \wedge Q_n$, where each Q_i is either P_i or $\neg P_i$ ....

e.g. min terms in P_1 and P_2 are $P_1 \wedge P_2$, $\neg P_1 \wedge P_2$, $\wedge \neg P_2$, $P_1 \wedge \neg P_2$, $\neg P_1 \wedge \neg P_2$. The number of min terms in n variables is 2^n.

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Definition 9.

A formula α is in principal disjunctive normal form if α is a sum of min terms.

Construction to obtain principal Disjunctive Normal form of a given formula.

Step 1 : Obtain a disjunctive normal form.

Step 2 : Drop elementary products which are contradictions (such *s* $P \wedge \rceil P$).

Step 3 : If P_i and $\rceil P_i$ are missing in an elementary product α replace α by

$$(\alpha \wedge P_i) \vee (\alpha \wedge \rceil P_i).$$

Step 4 : Repeat step 3 until all elementary products are reduced to sum of min terms. Use idempotent laws to avoid repetition of min terms.

Example. Obtain the canonical sum-of-products form (i.e. principal disjunctive normal form) of

$$\alpha = P \vee (\rceil P \wedge \rceil Q \wedge R).$$

Solution:

α is already in disjunctive normal form. There are no contradictions. So we have to introduce missing variables (step 3). $\rceil P \wedge \rceil Q \wedge R$ in α is already a min term.

$$\begin{aligned} \text{Now, } P &\equiv (P \wedge Q) \vee (P \wedge \rceil Q) \\ &\equiv ((P \wedge Q \wedge R) \vee (P \wedge Q \wedge \rceil R)) \vee (P \wedge \rceil Q \wedge R) \\ &\quad \vee (P \wedge \rceil Q \wedge \rceil R) \\ &\equiv ((P \wedge Q \wedge R) \vee (P \wedge Q \wedge \rceil R)) \vee (P \wedge \rceil Q \wedge R) \\ &\quad \vee (P \wedge \rceil Q \wedge \rceil R) \end{aligned}$$

Therefore, canonical sum-of-products form of α is

$$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \rceil R) \vee (P \wedge \rceil Q \wedge R) \vee (P \wedge \rceil Q \wedge \rceil R) \vee (\rceil P \wedge \rceil Q \wedge R)$$

Example. Obtain the principal disjunctive normal form of

$$\alpha = (\rceil P \vee \rceil Q) \rightarrow (\rceil P \wedge R)$$

Solution.

$$\begin{aligned} \alpha &= (\rceil P \vee \rceil Q) \rightarrow (\rceil P \wedge R) \\ &\equiv (\rceil(\rceil P \vee \rceil Q)) \vee (\rceil P \wedge R) && \text{using } I_{12} \\ &\equiv (P \wedge Q) \vee (\rceil P \wedge R) && \text{using De Morgan's law} \\ &\equiv ((P \wedge Q \wedge R) \vee (P \wedge Q \wedge \rceil R)) \vee ((\rceil P \wedge R \wedge Q) \\ &\quad \vee (\rceil P \wedge R \wedge \rceil Q)) \\ &\equiv (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \rceil R) \vee (\rceil P \wedge Q \wedge R) \\ &\quad \vee (\rceil P \wedge \rceil Q \wedge R) \end{aligned}$$

So principal disjunctive normal form of α is

$$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \rceil R) \vee (\rceil P \wedge Q \wedge R) \vee (\rceil P \wedge \rceil Q \wedge R)$$

A min term of the form $Q_1 \wedge Q_2 \ldots. \wedge Q_n$ can be represented by $a_1 a_2 \ldots a_n$,

where, $a_i = 0$ if $Q_i = \rceil P_i$ and $a_i = 1$ if $Q_i = P_i$.

So, principal disjunctive normal form can be represented by a 'sum' of binary strings.

e.g. $(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \rceil R) \vee (\rceil P \wedge \rceil Q \wedge R)$ is represented by $111 \vee 110 \vee 001$.

The main terms in two variables P and Q are 00, 01, 10, and 11. Each wff is equivalent to its principal disjunctive normal form. Every principal disjunctive normal form corresponds to the min terms in it, and hence to a subset of {00, 01, 10, 11}. As the number of subsets is 2^4, the number of distinct formulas is 16.

Functionally complete set of connectives.

Any set of connectives in which every formula can be expressed in terms of as equivalent formula containing connectives from this set is called *functionally complete set of connectives*.

e.g. $\{\wedge, \rceil\}$ and $\{\vee, \rceil\}$ are functionally complete

$\therefore \quad P \vee Q = \rceil(\rceil P \wedge \rceil Q)$

and $\quad P \wedge Q = \rceil(\rceil P \vee \rceil Q)$

$\{\rceil\}, \{\wedge\}, \{\vee\}, \{\wedge, \vee\}$ and not functionally complete.

Other connectives.

(1) Exclusive OR ($\overline{\vee}$)

$P \,\overline{\vee}\, Q$ is true when either of P and Q is true, but not both

$\overline{\vee}$ is associative and commulative

$$P \,\overline{\vee}\, Q \equiv \rceil(P \rightleftarrows Q)$$

(2) NAND ($\uparrow$)

$$\begin{aligned} P \uparrow Q &\equiv \rceil(P \wedge Q) \\ \rceil P &\equiv P \uparrow P \\ P \vee Q &\equiv (P \uparrow P) \uparrow (Q \uparrow Q) \\ P \wedge Q &\equiv (P \uparrow Q) \uparrow (P \uparrow Q) \end{aligned}$$

(3) NOR ($\downarrow$)

$$\begin{aligned} P \downarrow Q &= \rceil(P \vee Q) \\ \rceil P &= P \downarrow P \\ P \vee Q &= (P \downarrow Q) \downarrow (P \downarrow Q) \\ P \wedge Q &= (P \downarrow P) \downarrow (Q \downarrow Q) \end{aligned}$$

$\therefore \quad \{\uparrow\}, \{\downarrow\}$ are functionally complete.

Example. For a given formula α, truth table is given in the Table below. Find the principal disjunctive normal form.

P	Q	R	α
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

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Solution:

We have T in the α-column corresponding to the rows 1, 4, 5 and 8.

The min term corresponding to the first row is

$$P \wedge Q \wedge R.$$

Similarly, main terms corresponding to rows 4, 5 and 8 are

$$P \wedge \neg Q \wedge \neg R,\ \neg P \wedge Q \wedge R \text{ and } \neg P \wedge \neg Q \wedge \neg R.$$

Therefore, principal disjunctive normal form of α is

$$(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$$

We can form 'dual' of disjunctive normal form which is termed as **conjunctive normal form.**

Definition 10.

A formula is in conjunctive normal form if it is a product of elementary sums.

If α is in disjunctive normal form, then $\neg\alpha$ is in conjunctive normal form. (This can be seen by applying De Morgan's laws). So to obtain conjunctive normal form of α, we construct the disjunctive normal of $\neg\alpha$ and use negation.

Definition 11.

A max term in n propositional variables $P_1, P_2, \ldots. P_n$ is

$Q_1 \vee Q_2 \ldots \vee Q_n$, where each Q_i is either P_i or $\neg P_i$.

Definition 12.

A formula α is in principal conjunctive normal form if α is a product of max terms. For obtaining principal conjunctive normal form of α, we can construct principal disjunctive normal form of $\neg\alpha$ and apply negation ($\neg$).

Example. Find the principal conjunctive normal form of

$$\alpha = P \vee (Q \to R).$$

Solution:

$$\begin{aligned} \neg\alpha &= \neg(P \vee (Q \to R)) \\ &\equiv \neg(P \vee (\neg Q \vee R)) && \text{using } I_{12} \\ &\equiv \neg P \wedge (\neg(\neg Q \vee R)) && \text{using De Morgan's law} \\ &\equiv \neg P \wedge (Q \wedge \neg R) && \text{using De Morgan's law and } I_7 \end{aligned}$$

$\neg P \wedge Q \wedge \neg R$ is the principal disjunctive normal form of $\neg\alpha$.

Hence, principal conjunctive normal form of α is

$$\neg(\neg P \wedge Q \wedge \neg R) = P \vee \neg Q \vee R$$

The logical identities given in Table above and normal forms of well-formed formulas bear a close resemblance of identities in Boolean algebras and normal forms of Boolean functions. Actually, the propositions under $\vee, \wedge$ and $\neg$ form a Boolean algebra if equivalent propositions are identified. T and F act as bounds (*i.e.* 0 and 1 of a Boolean algebra). Also, statement formulas form a Boolean algebra under $\vee, \wedge$ and $\neg$ if equivalent formulas are identified.

The normal forms of well-formed formulas correspond to normal form of Boolean functions and we can 'minimise' a formula in a similar manner.

PROPOSITIONAL CALCULUS

(Statement calculus)

In a logical reasoning, certain propositions are assumed to be true, and based on that assumption some other propositions are derived (deduced or inferred).

Hypothesis or Premises.

The propositions that are assumed to be true are called *hypotheses* or *premises*.

Conclusion.

The proposition derived by using the rules of inference is called a *conclusion.*

Valid argument.

The process of deriving conclusions based on the assumption of premises is called *a valid argument.*

So in a valid argument, we are concerned with the process of arriving at the conclusion rather than obtaining the conclusion.

Rules of inference.

These are simply tautologies in the form of implication (i.e. $P \to Q$).

e.g. $P \to (P \vee Q)$ is such a tautology, and it is a rule of inference.

We write this in the form $\dfrac{P}{P \vee Q}$. Here P denotes a premise. The proposition below the line,

i.e. $P \vee Q$ is the conclusion.

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For valid arguments, we can use the rules of inference given in the table below.

Rules of Inference		*Implication Form*
RI_1 :	Addition	
	$\dfrac{P}{\therefore P \vee Q}$	$P \rightarrow (P \vee Q)$
RI_2 :	Conjunction	
	$\dfrac{Q}{\therefore P \wedge Q}$	$Q \rightarrow P \wedge Q$
RI_3 :	Simplification	
	$\dfrac{P \wedge Q}{\therefore Q}$	$(P \wedge Q) \rightarrow Q$
RI_4 :	Modus ponens	
	P $\dfrac{P \rightarrow Q}{\therefore Q}$	$(P \wedge (P \rightarrow Q)) \rightarrow Q$
RI_5 :	Modus tollens	
	$\neg Q$ $\dfrac{P \rightarrow Q}{\therefore \neg P}$	$(\neg Q \wedge (P \rightarrow Q)) \rightarrow \neg P$
RI_6 :	Disjunctive syllogism	
	$\neg P$ $\dfrac{P \vee Q}{\therefore Q}$	$(\neg P \wedge (P \vee Q)) \rightarrow Q$
RI_7 :	Hypothetical syllogism	
	$P \rightarrow Q$ $\dfrac{Q \rightarrow R}{\therefore P \Rightarrow R}$	$((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$
RI_8 :	Constructive dilemma	
	$(P \rightarrow Q) \wedge (R \rightarrow S)$ $\dfrac{P \vee R}{\therefore Q \vee S}$	$(P \rightarrow Q) \wedge (R \rightarrow S) \wedge (P \vee R) \rightarrow (Q \vee S)$
RI_9 :	Constructive dilemma	
	$(P \rightarrow Q) \wedge (R \rightarrow S)$ $\dfrac{\neg Q \vee \neg S}{\therefore \neg P \vee \neg R}$	$(P \rightarrow Q) \wedge (R \rightarrow S) \wedge (\neg Q \vee \neg S) \rightarrow (\neg P \vee \neg R)$

Example. Can we conclude S from the following premises ?

I. $P \rightarrow Q$ II. $P \rightarrow R$

III. $\neg(Q \wedge R)$ IV. $S \vee P$

Solution:

The valid argument for deducing S from the given for premises is given as a sequence. On the left, well-formed formula are given and on the right indicate whether the proposition is a premises (hypothesis) or a conclusion. If it is a conclusion, indicate premises and the rules of inference or logical identities used for deriving the conclusion.

1.	$P \rightarrow Q$	Premise I
2.	$P \rightarrow R$	Premise II
3.	$(P \rightarrow Q) \wedge (P \rightarrow R)$	Lines 1, 2 and R I_2
4.	$\neg(Q \wedge R)$	Premise III
5.	$\neg Q \vee \neg R$	Line 4 and De Morgan's law (I_6)
6.	$\neg P \vee \neg P$	Line 3, 5 and destructive dilemma (RI_9)
7.	$\neg P$	Idempotent law I_1
8.	$S \vee P$	Premise IV
9.	S	Lines 7, 8 and disjunctive syllogism RI_6

Thus S can be concluded from the given premises.

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Exmaple. Check validity of the following argument:

If Ram has completed B.E. Computer Science or M.B.A., then he is assured of a good job. If Ram is assured of a good job, he is happy. Ram is not happy. So Ram has not completed M.B.A.

Solution:

The propositions can be named in the following ways:

P denotes "Ram has completed B.E. Computer Science".

Q denotes "Ram has completed M.B.A.".

R denotes "Ram is assured of a good job".

S denotes "Ram is happy".

The given premises are :

I. $(P \vee Q) \rightarrow R$

II. $R \rightarrow S$

III. $\rceil S$

The conclusion is $\rceil Q$

1. $(P \vee Q) \rightarrow R$ Premise (I)

2. $R \rightarrow S$ Premise (II)

3. $(P \vee Q) \rightarrow S$ Lines 1, 2 and hypothetical syllogism R I_7

4. $\rceil S$ Premise (III)

5. $\rceil(P \vee Q)$ Lines 3, 4 and modus tollens

6. $\rceil P \wedge \rceil Q$ De Morgan's law I_6

7. $\rceil Q$ Line 6 and simplification RI_3

Thus the argument is valid.

Example. Test validity of the following argument :

If milk is black, then every crow is white. If every crow is white, then it has four legs. If every crow has four legs, then every buffalo is white and brisk. The milk is black'. Therefore, the buffalo is white.

Solution:

We name the proposition in the following way :

P denotes "The milk is black"

Q denotes "Every crow is white"

R denotes "Every crow has four legs"

S denotes ""Every buffalo is white"

T denotes "Every buffalo is brisk"

The given premises are:

I. $P \rightarrow Q$

II. $Q \rightarrow R$

III. $R \rightarrow S \wedge T$

IV. P

The conclusion is S

1. P Premise (IV)

2. $P \rightarrow Q$ Premise (I)

3. Q Modus ponens RI_4

4. $Q \rightarrow R$ Premise (II)

5. R Modus ponens RI_4

6. $R \rightarrow S \wedge T$ Premise (III)

7. $S \wedge T$ Modus ponens RI_4

8. S Simplification RI_3

Thus the argument is valid.

PREDICATE CALCULUS

Consider two propositions "*Rita is a student*", and "*Sita is a student*".

As propositions, there is no relation between them, but we know they have something in common. Both Rita and Sita share the property of being a student. We can replace the two propositions by a single statement "x is a student". The common feature expressed by "is a student" is called a *predicate*. In predicate calculus we deal with sentences involving predicates.

Statements involving predicates occur in Mathematics and programming languages. *e.g.* "$2x + 3y = 4z$", "IF (D.GE. 0.0) GO TO 20" are statements in Mathematics and FORTRAN, respectively involving predicates. Some logical deductions are possible only by separating predicates'.

Predicates.

Sentences involving predicates describing property of the objects are denoted by P(x), where P denotes the predicates and x is a variable denoting any object.

e.g. P(x) can denote "x is a student".

In this sentence, x is a variable and P denotes the predicate "is a student".

The sentence "x is the father of y" also involves a predicate "is the father of". Here the predicate describes the relation between two persons. We can write this sentence as F(x, y).

Similarly, $2x + 3y = 4z$ can be described by S(x, y, z).

Note : Although, P(x), involving a predicate looks like a proposition, it is not a proposition. As P(x) involves a variable x, we cannot assign a truth value to P(x). However, if we replace x by an individual object, we get a proposition.

e.g. if we replace x by Rita in P(x), we get the proposition "Rita is a student". (we can denote this proposition by P(Rita).

If we replace x by 'A can', then also we get a proposition (whose truth value is F).

S (2, 0, 1) is the proposition 2.2 + 3.0 = 4.1 (whose truth value is T).S (1, 1, 1) is the proposition 2.1 + 3.1 = 4.1 (whose truth value is F).

The following definition is regarding possible 'values" which can be assigned to variables.

Definition 13.

For a declarative sentence involving a predicate, the universe discourse, or simply the universe, is the set of all possible values which can be assigned to variables.

e.g. universe of discourse for P(x) : "x is a student", can be taken as the set of all human names; the universe of discourse for E(n); "n is an even integer", can be taken as the set of all integers (or the set of all real numbers).

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Quantifier.

Universal Quantifier:

The phrase 'for all' (denoted by $\forall$) is called *universal quantifier.*

Using this symbol, we can write

"For all x, $x^2 = (-x)^2$" as $\forall x\, Q(x)$, where $Q(x)$ is" $x^2 = (-x)^2$"

Existential Quantifier:

The phrase 'there exists' (denoted by $\exists$) is called *existential quantifier*.

The sentence. "There exists x such that $x^2 = 5$" can be written as $\exists x R(x)$, where $R(x)$ is '$x^2 = 5$'.

Scope of the Quantifier:

$P(x)$ in $\forall x P(x)$ or in $\exists x P(x)$ is called the *scope of the quantifier* $\forall$ or $\exists$.

Note : The symbol $\forall$ can be read as 'for every', 'for any', 'for each', 'for arbitrary'. The symbol $\exists$ can be read as 'for some' for 'at least one'.

When we use quantifiers, we should specify the universe of discourse. If we change universe of discourse, the truth value may change.

e.g. consider $\exists x R(x)$ where $R(x)$ is $x^2 = 5$.

If universe of discourse is the set of all integers, then $\exists x R(x)$ is false.

If universe of discourse is the set of all real numbers, then $\exists x R(x)$ is true (when $x = \pm\sqrt{5}, x^2 = 5$).

The logical connectives involving predicates can be used for declarative sentences involving predicates.

Example. *Express the following sentences involving predicates in symbolic form :*

(*a*) All students are clever.

(*b*) Some students are not successful.

(*c*) Every clever student is successful.

(*d*) There are some successful students who are not clevers.

(*e*) Some students are clever and successful.

Solution:

As quantifiers are involved, we have to specify the universe of discourse. we can take the universe of discourse as the set of all students.

Let $C(x)$ denote "x is clever"

Let $S(x)$ denote "x is successful".

Then sentence 1 can be written as $\forall x C(x)$. Sentences (*b*) – (*e*) can be written as

$$\exists x(\neg S(x))$$
$$\forall x\, (C(x) \rightarrow S(x))$$
$$\exists x(S(x) \wedge \neg C(x))$$
$$\exists x\, (C(x) \wedge S(x))$$

Well-formed formulas of predicate calculus.

A well-formed formula (wff) of predicate calculus is a string of variables such as x_1, x_2, ...x_n, connectives, parentheses and quantifiers defined recursively by following rules :

I. $P(x_1, ..., x_n)$ is a wff, when P is a predicate involving n variables $x_1\, x_2, ..., x_n$.

II. If α is a wff, then $\neg\alpha$ is a wff.

III. If α and β are wffs, then $\alpha \vee \beta$, $\alpha \wedge \beta$, $\alpha \rightarrow \beta$, $\alpha \rightarrow \beta$ are also wffs.

IV. If α is a wff and x is any variable, then $\forall x(\alpha)$, $\exists x(\alpha)$ are wffs.

V. A string is a wff if and only if it is obtained by finitely many applications of rules (I) – (IV).

Note : A proposition can be viewed as a sentence involving a predicate with 0 variables. So propositions are wffs of predicate calculus by rule (I).

Definition 14.

Let α and β be two predicate formulas in variables x_1,.... x_n, and let U be a universe of discourse of α and β. Then α and β are equivalent to each other over U if for every possible assignment of values to each variable in α and β, the resutling statements have the same truth values. We can write $\alpha \equiv \beta$ over U.

We say that α and β are equivalent to each other $(\alpha \equiv \beta)$ if $\alpha \equiv \beta$ over U for every universe of discourse U.

Remark : In predicate formulas, the predicate variables may or may not be quantified. We can classify the predicate variables in a predicate formula, depending on whether they are quantified or not. This gives to the following definitions.

Definition 15.

If a formula of the form $\exists x(Px)$ or $\forall x P(x)$ occurs as part of a predicate formula α, then such part is called an x-bound part of α, and occurrence of x is called bound occurrence of x. An occurrence of x is free if it is not a bound occurrence. A predicate variable in α is free if its occurrence is free in any part of α.

e.g. in $\alpha = (\exists x_1 P(x_1, x_2)) \wedge (\forall x_2 Q(x_2, x_3))$, the occurrence of x_1 in $\exists x_1 P(x_1, x_2)$ is a bound occurrence and that of x_2 is free.

In $\forall x_2 Q(x_2, x_3)$, the occurrence of x_2 is a bound occurrence the occurrence of x_3 in α is free.

Note : Quantified parts of a predicate formula such as $\forall x P(x)$ or $\exists x P(x)$ are propositions. We can assign values from the universe of discourse only to free variables in a predicate formula α.

Definition 16.

A predicate formula is valid if for all possible assignments of values from any universe of discourse to free variables, the resulting propositions have truth value T.

Definition 17.

A predicate formula is satisfiable if for some assignment of values to predicate variables the resulting proposition has ruth value T.

Definition 18.

A predicate formula is unsatisfiable if for all possible assignments of values from any universe of discourse to predicate variables the resulting propositions have truth value F.

Note: valid predicate formulas correspond to tautologies among proposition formulas and unsatisfiable predicate formulas correspond to contradictions.

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Rules of inference for predicate calculus.

(1) Proposition formulas are also predicate formulas:

(2) Predicate formulas where all the variables are quantified are proposition formulas. Therefore, all the rules of inference for proposition formulas are also applicable for predicate calculus wherever necessary.

For predicate formulas not involving connectives such as $A(x)$, $P(x, y)$, we can get equivalences and rules of inference.

e.g. corresponding to I_6 in Table of logical identities, we get

$$\neg(P(x) \vee Q(x)) \equiv \neg(P(x)) \wedge \neg(Q(x)).$$

Corresponding to RI_3 in Table of rules of reference $P \wedge Q \rightarrow R$, we get

$$P(x) \wedge Q(x) \rightarrow P(x).$$

Thus we can replace propositional variables by predicate variables in previous tables.

Table. Equivalences involving two quantifiers and valid implications

I_{13}	**Distributivity of $\exists$ over $\vee$**
	$\exists x(P(x) \vee Q(x)) = \exists xP(x)\ \exists xQ(x)$
	$\exists x(P \vee Q(x)) \equiv P \vee (\exists xQ(x))$
I_{14}	**Distributivity of $\forall$ over $\wedge$**
	$\forall x(P(x) \wedge Q(x)) \equiv \forall xP(x) \wedge \forall xQ(x)$
	$\forall x(P \wedge Q(x)) \equiv P \wedge (\forall xQ(x))$
I_{15}	$\neg(\exists xP(x) \equiv \forall x\ \neg P(x))$
I_{16}	$\neg(\forall xP(x)) \equiv \exists x \neg(P(x))$
I_{17}	$\exists x(P \wedge Q(x)) \equiv P \wedge (\exists xQ(x))$
I_{18}	$\forall x(P \vee Q(x)) \equiv P \vee (\forall xQ(x))$
RI_{10}	$\forall xP(x) \rightarrow \exists xP(x)$
RI_{11}	$\forall xP(x) \vee \forall\ xQ(x) \rightarrow \forall x(P(x) \vee Q(x))$
RI_{12}	$\exists x(P(x) \wedge Q(x) \rightarrow \exists xP(x) \wedge \exists xQ(x)$

Sometimes it is required to derive some conclusions from a given set of premises involving quantifiers, we may have to eliminate the quantifiers before applying the rules of inference for proposition formulas. Also, when the conclusion involves quantifiers, we may have to introduce quantifiers. The necessary rules of inference for addition and deletion of quantifiers are given in Table below.

Table. Rules of inference for addition and deletion of quantifiers

RI_{13} :	**Universal instantiation**
	$\dfrac{\forall xP(x)}{\therefore P(c)}$
	c is some element of the universe.
RI_{14} :	**Existential instantiation**
	$\dfrac{\exists xP(x)}{\therefore P(c)}$
	c is some element for which $P(c)$ is true
RI_{15} :	**Universal generalisation**
	$\dfrac{P(x)}{\forall xP(x)}$
	x should not be free in any of the given premises.
RI_{16} :	**Existential generalisation**
	$\dfrac{P(c)}{\therefore \exists xP(x)}$
	c is some element of the universe.

Example. Discuss the validity of the following arguments

I. All graduates are educated.

II. Rita is a graduate. Therefore,

III. Rita is educated.

Solution:

Let $G(x)$ denote "x is a graduate".

Let $E\ (x)$ denote "x is educated".

Let R denote "Rita".

So the premises are $\forall x(G(x) \Rightarrow E(x))$ and $G(R)$. The conclusion is $E(R)$.

1.	$\forall x(G(x) \Rightarrow E(x))$	Premise (I)
2.	$G(R) \Rightarrow E(R)$	Universal instantiation RI_{13}
3.	$G(R)$	Premise (II)
4.	$\therefore E(R)$	Modus ponens RI_{14}

Thus the conclusion, viz. (III), is valid.

Example. Discuss the validity of the following argument :

All educated persons are well behaved. Rita is educated. No well-behaved person is quarrelsome. Therefore, Rita is not quarrelsome.

Solution:

Let the universe of discourse be the set of all educated persons.

Let $P(x)$ denote x is well-behaved".

Let y denote "Rita".

Let $Q(x)$ denote "x is equarrelsome".

So the premises are :

I. $\forall xP(x)$

II. y is a particular element of the universe of discourse.

III. $\forall x(P\ (x) \Rightarrow \neg Q(x))$.

To obtain the conclusion, we have the following arguments :

1.	$\forall xP(x)$	Premise (I)
2.	$P(y)$	Universal instantiation RI_{13}
3.	$\forall x(P(x) \Rightarrow \neg Q(x))$	Premise (III)
4.	$P(y) \Rightarrow \neg Q(y)$	Universal instantiation RI_{13}
5.	$P(y)$	Line 2
6.	$\neg Q(y)$	Modus ponens RI_{14}

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SOLVED EXAMPLES

1. Find infix, prefix and suffix notation of

$$P \wedge \neg R \rightleftarrows P \vee Q$$

Solution:

Let $\quad f = P \wedge \neg R \rightleftarrows P \vee Q$.

Connectives must be considered in the following hierarchy

$$\neg, \wedge, \vee, \longrightarrow \text{ and } \rightleftarrows$$

Infix : $\quad f = P \wedge \neg R \rightleftarrows P \vee Q$

$\equiv (P \wedge \neg R \rightleftarrows P \vee Q$

$\equiv (P \wedge \neg R) \rightleftarrows (P \vee Q)$.

Prefix : Consider $f =$

$(P \wedge \neg R) \rightleftarrows (P \vee Q)$.

It is given by $\quad (P \wedge \neg R) \rightleftarrows (P \vee Q)$

$\equiv \rightleftarrows \wedge P \neg R \vee PQ$ is prefix notation of f

Suffix : It is given by

$(P \wedge \neg R) \rightleftarrows (P \vee Q)$

$\equiv PR \neg \wedge PQ \vee \rightleftarrows$ is suffix notation of f.

2. Conclude $\neg P$ from $\neg P \vee Q$, $\neg(Q \wedge \neg R)$ and $\neg R$.

Solution:

The premises are $\neg P \vee Q$, $\neg(Q \wedge \neg R)$ and $\neg R$.

(1)	$\neg R$	P
(2)	$\neg(Q \wedge \neg R)$	P
(3)	$\neg Q \vee R$	(2) De Morgans Law
(4)	$\neg Q$	(1), (3), Disjunctive syllogisn
(5)	$\neg P \vee Q$	P
(6)	$\neg P$	(4), (5), Disjunctive syllogisn

3. Find the equivalent formula for $(P \longrightarrow Q) \vee R$ contains $\uparrow$ only.

Solution:

We know, $\quad P \uparrow P = \neg(P \wedge P) = \neg P$.

We can write $\quad \neg P \equiv P \uparrow P \quad$ ... (*i*)

$P \wedge Q \equiv (P \uparrow Q) \uparrow (P \uparrow Q) \quad$ ... (*ii*)

and $\quad P \vee Q \equiv (P \uparrow P) \uparrow (Q \uparrow Q) \quad$ ... (*iii*)

Hence $(P \to Q) \vee R \equiv \neg P \vee Q \vee R$

$\equiv (\neg P \vee Q) \vee R$

$\equiv ((\neg P \vee Q) \uparrow (\neg P \vee Q)) \uparrow (R \uparrow R)$ [by equation (*iii*)]

$\equiv ((\neg P \uparrow \neg P) \uparrow (Q \uparrow Q)) \uparrow ((\neg P \uparrow \neg P) \uparrow (Q \uparrow Q)) \uparrow (R \uparrow R)$ [by equation (*iii*)]

$= ((P \uparrow (Q \uparrow Q)) \uparrow (P \uparrow (Q \uparrow Q)) \uparrow (R \uparrow R)$.

4. $\{\neg, \longrightarrow\}$ is functionally complete.

Solution:

A set of connectives is said to be functionally complete if any formula can be written as an equivalent formula containing only these connectives. Obviously $\{\neg, \vee, \wedge\}$ is functionally complete.

Since $P \wedge Q = \neg(\neg P \vee \neg Q)$, hence $\{\neg, \vee\}$ is also functionally complete.

Since $P \vee Q \equiv \neg P \to Q$, wherever $\vee$ comes we can substitute $\neg$ and $\to$ for $\vee$, hence $\{\neg, \vee\} = \{\neg, \to\}$ in turn implies, $\{\neg, \to\}$ is functionally complete.

5. Show that R can be concluded from $P \vee Q$, $P \longrightarrow R$ and $Q \longrightarrow R$.

Solution:

Take $\neg R$ as a premise. Therefore

{1}	(1) $\neg R$	P
{2}	(2) $Q \to R$	P
{1, 2}	(3) $\neg Q$	T since $Q \longrightarrow R, \neg R \to \neg Q$
{3}	(4) $P \to R$	P
{1, 3}	(5) $\neg P$	T since $P \longrightarrow R, \neg R \to \neg P$
{1, 2, 3}	(6) $\neg P \wedge \neg Q$	T since $P, Q \to P \wedge Q$
{1, 2, 3}	(7) $\neg(P \vee Q)$	T
{4}	(8) $(P \vee Q)$	P
{1, 2, 3, 4}	(9) F	

T since $P, \neg P \to F$.

Hence $\neg R$ can't be a premise, *i.e.* $\neg R \equiv F$ leads a contradiction. Hence $R \equiv T$ and hence R can be concluded.

6. Show that $(A \vee C)$ is not a valid conclusion of

$A \rightleftarrows (B \longrightarrow C)$, $(B \rightleftarrows (\neg A \vee \neg C))$, $C \rightleftarrows (A \vee \neg B)$

and B

Solution:

Consider $A \equiv C \equiv F$ and $B \equiv T$. Therefore

$A \rightleftarrows (B \longrightarrow C) \equiv F \rightleftarrows (T \longrightarrow F) \equiv F \rightleftarrows F \equiv T$,

$B \rightleftarrows (\neg A \vee \neg C) \equiv T \rightleftarrows (T \vee T) \equiv T \rightleftarrows T \equiv T$,

$C \rightleftarrows (A \vee \neg B) \equiv F \rightleftarrows (F \vee F) \equiv F \rightleftarrows F \equiv T$,

$B \equiv T$.

But $\quad A \vee C \equiv F \vee F \equiv F$

Hence above premises can't imply $A \vee C$.

(If $P \equiv T$ and $Q \equiv F$, then Q can't be a valid conclusion of P or P can't imply Q).

7. If $H_1, H_2, \ldots, H_n$ and P imply Q, then $H_1, H_2, \ldots, H_n$ imply $P \longrightarrow Q$.

Solution:

Given $\quad (H_1 \wedge H_2 \wedge \ldots \wedge H_n \wedge P) \to Q$

To prove that $\quad (H_1 \wedge H_2 \wedge \ldots \wedge H_n) \to P \to Q$.

i.e. to prove that if $\quad H_1 \wedge H_2 \wedge \ldots \wedge H_n \equiv T$

then $\quad P \to Q \equiv T$

Let $(H_1 \wedge H_2 \wedge \ldots \wedge H_n \equiv T)$ and let $(P \to Q \equiv F)$

$\Rightarrow \quad (H_1 \equiv H_2 \equiv \ldots \equiv H_n \equiv T)$ and $(P \equiv T$ and $Q \equiv F)$

$\Rightarrow \quad (H_1 \equiv H_2 \equiv \ldots \equiv H_n \equiv P \equiv T)$ and $(Q \equiv F)$

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$\Rightarrow$ $(H_1 \wedge H_2 \wedge \ldots \wedge H_n \wedge P) \rightarrow Q$, a contradiction to (1)

Hence the statement (2) is false.

i.e. $(H_1 \wedge H_2 \wedge \ldots \wedge H_n) \rightarrow (P \rightarrow Q)$.

8. Show that $P \rightarrow (Q \rightarrow R) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$.

Solution:

As $P \rightarrow Q$ is false only when P = T and Q = F.

Therefore if we want to show $P \rightarrow Q$ it is enough to show that $Q \equiv F$ implies $P \equiv F$.

Consider RHS

$F \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R) \equiv F$

$\Rightarrow (P \rightarrow Q) \equiv T$ and $(P \rightarrow R) \equiv F$

$\Rightarrow (P \rightarrow Q) \equiv T$ and $(P \equiv T$ and $R \equiv F)$

$\Rightarrow$ $T \rightarrow Q \equiv T$, $P \equiv T$ and $R \equiv F$

$\Rightarrow$ $Q \equiv T$, $P \equiv T$ and $R \equiv F$

Consider LHS$P \rightarrow (Q \rightarrow R) \equiv T \rightarrow (T \rightarrow F)$ by (1)

$\equiv T \rightarrow F \equiv F$.

Thus when RHS is F, then LHS is also F.

Hence **LHS $\equiv$ RHS.**

9. Prove substitution instance of tautology is a tautology.

Solution:

The basic structure of a tautology is not going to change by substitution of a formula.

e.g. $Q \vee \urcorner Q$ is going to become

$(\urcorner P \longrightarrow Q) \vee \urcorner(\urcorner P \longrightarrow Q)$

10. Show that truth value of the following formula is independent of its components.

$$(P \longrightarrow Q) \rightleftarrows (\urcorner P \vee Q)$$

Solution:

Method 1: Let us construct the truth table.

P	**Q**	**$(P \longrightarrow Q)$**	**$\urcorner P$**	**$(\urcorner P \vee Q)$**	**$(P \longrightarrow Q) \rightleftarrows (\urcorner P \vee Q)$**
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Hence $(P \longrightarrow Q) \rightleftarrows (\urcorner P \vee Q)$ is a tautology and independent of components.

Method 2 :

Consider $(P \longrightarrow Q) \rightleftarrows (\urcorner P \vee Q)$

$\equiv (P \longrightarrow Q) \rightleftarrows (P \longrightarrow Q)$

$\equiv T$ as $P \rightleftarrows P$ is always true.

Hence truth value of above formula is independent of its components.

11. Find equivalent formula consisting of $\wedge, \vee, \urcorner$ for $\overline{\vee}$.

Solution:

The connective exclusive OR, $\overline{\vee}$ is given by the truth table.

P	**Q**	**$P \overline{\vee} Q$**	**$P \wedge \urcorner Q$**	**$\urcorner P \wedge Q$**	**$(P \wedge \urcorner Q) \vee (\urcorner P \wedge Q)$**
T	T	F	F	F	F
T	F	T	T	F	T
F	T	T	F	T	T
F	F	F	F	F	F

$\Rightarrow$ $P \overline{\vee} Q = (P \wedge \urcorner Q) \vee (\urcorner P \wedge Q)$.

$= x\bar{y} + \bar{x}y$.

Note. $P \rightleftarrows Q$ if and only if $P \rightleftarrows Q \equiv T$. (That is P and Q have the same truth value.)

12. $\{\wedge, \vee\}$ is not functionally complete.

Solution:

Consider $(P \vee \urcorner P = T)$. This formula can't be written as combination of $\vee$ and $\wedge$ only.

13. $\{\uparrow\}$ is functionally complete.

Solution:

We know $P \uparrow P \equiv \urcorner(P \wedge P) \equiv \urcorner P$

$(P \uparrow Q) \uparrow (P \uparrow Q) \equiv \urcorner(\urcorner(P \wedge Q) \wedge \urcorner(P \wedge Q))$

$\equiv \urcorner(\urcorner(P \wedge Q)) = P \wedge Q$

and $(P \uparrow P) \uparrow (Q \uparrow Q) \equiv \urcorner(\urcorner(P \wedge \urcorner Q) = P \vee Q$.

Hence any formula consisting $\wedge$, $\vee$, and $\urcorner$ can be written as a formula consisting of $\uparrow$ only.

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14. Express $P \longrightarrow (\rceil P \longrightarrow Q)$ in terms of $\uparrow$ only.

Solution:

Consider

$$P \longrightarrow (\rceil P \longrightarrow Q) \equiv \rceil P \vee (\rceil(\rceil P) \vee Q))$$
$$\equiv \rceil P \vee (P \vee Q)$$
$$\equiv \rceil P \vee P \vee Q$$
$$\equiv T \vee Q$$
$$\equiv T$$

15. Express $P \uparrow Q$ in terms of $\downarrow$ only.

Solution: $P \uparrow Q = \rceil(P \wedge Q)$

$$= (P \wedge Q) \downarrow (P \wedge Q)$$
$$= ((P\downarrow P)\downarrow(Q\downarrow Q))\downarrow((P\downarrow P)\downarrow(Q\downarrow Q))$$

16. Find simplest form of $((P \to Q) \rightleftarrows (\rceil Q \to \rceil P)) \wedge R$

Solution:

Expression is $((P \to Q) \rightleftarrows (P \to Q)) \wedge R \equiv T \wedge R \equiv F.$

17. Obtain principal conjunctive normal form and principal disjunctive normal form of

$$P \vee (\rceil P \to (Q \vee (\rceil Q \to R)))$$

Solution:

Consider $A \equiv P \vee (\rceil P \longrightarrow (Q \vee (\rceil Q \to R)$

$\equiv P \vee (P \vee (Q \vee (Q \vee R))$ as $P \to Q \equiv \rceil P \vee Q$

$\equiv P \vee P \vee Q \vee Q \vee R$

$A \equiv (P \vee Q \vee R) = \text{PCNF}$

$\Rightarrow \quad \overline{A} \equiv$ product of remaining 7 max terms

$$= (P \vee Q \vee \overline{R}) \wedge (P \vee \overline{Q} \vee R) \wedge (P \vee \overline{Q} \vee \overline{R})$$
$$\wedge (\overline{P} \vee Q \vee R) \wedge (\overline{P} \vee Q \vee \overline{R})$$
$$\wedge (\overline{P} \vee \overline{Q} \vee R) \wedge (\overline{P} \vee \overline{Q} \vee \overline{R})$$

$$\Rightarrow \quad A = \overline{\overline{A}} \equiv (\overline{P} \wedge \overline{Q} \wedge R) \vee (\overline{P} \wedge Q \wedge \overline{R})$$
$$\vee (\overline{P} \wedge Q \wedge R) \vee (P \wedge \overline{Q} \wedge \overline{R})$$
$$\vee (\overline{P} \vee \overline{Q} \wedge R) \vee (P \wedge Q \wedge \overline{R})$$
$$\vee (P \wedge Q \wedge R)$$
$$= \text{PDNF}$$

18. Convert the following into infix notation.

$$P \rceil P \longrightarrow P \longrightarrow P \longrightarrow.$$

Solution:

This is in postfix notation. Hence let us go from left to right.

$$A \equiv P \rceil P \longrightarrow P \longrightarrow P \longrightarrow$$
$$\equiv (\rceil P)\, P \longrightarrow P \longrightarrow P \longrightarrow$$
$$\equiv (\rceil P \longrightarrow P)\, P \longrightarrow P \longrightarrow$$
$$\equiv ((\rceil P \longrightarrow P) \longrightarrow P)\, P \longrightarrow$$
$$\equiv ((\rceil P \longrightarrow P) \longrightarrow P) \longrightarrow P)$$

19. Show that C follows from H_1 and H_2.

$H_1 : P \longrightarrow (Q \longrightarrow R) \qquad H_2 : P \wedge Q \qquad C : R$

Solution:

We have to show $(H_1 \wedge H_2) \Rightarrow C$,

i.e. if $(H_1 \wedge H_2) \equiv T$, then $C \equiv T$.

Let $H_1 \equiv T$ and $H_2 \equiv T$.

$\Rightarrow P \longrightarrow (Q \longrightarrow R) \equiv T$ and $P \wedge Q \equiv T$

$\Rightarrow P \longrightarrow (Q \longrightarrow R) \equiv T$ and $P \equiv T$ and $Q \equiv T$

$\Rightarrow T \longrightarrow (T \longrightarrow R) \equiv T$

$\Rightarrow \qquad T \longrightarrow R \equiv T \Rightarrow R \equiv T.$

20. Without constructing a truth table, show that $A \wedge E$ is not a valid consequence of

$H_1 : A \rightleftarrows B, \quad H_2 : B \rightleftarrows (C \wedge D),$

$H_3 : C \rightleftarrows (A \vee E), \quad H_4 : A \vee E$

Solution:

If we want to prove Q is not valid consequence of P, then it is sufficient to prove

$P \equiv T$ and $Q \equiv F$ (hence $P \longrightarrow Q \equiv F$).

For the combinations $A \equiv F$, $B \equiv F$, $C \equiv T$, $D \equiv F$, $E \equiv T$, we get

$$H_1 : A \rightleftarrows B \equiv F \rightleftarrows F \equiv T,$$
$$H_2 : B \rightleftarrows (C \wedge D) \equiv F \rightleftarrows (T \wedge F) \equiv T.$$
$$H_3 : C \rightleftarrows (A \vee E) \equiv T \rightleftarrows (F \vee T) \equiv T$$

and $\quad H_4 : A \vee E \equiv F \vee T \equiv T.$

$\quad A \wedge E \equiv F \wedge T \equiv F.$

Hence $(H_1 \wedge H_2 \wedge H_3 \wedge H_4) \equiv T$ and $(A \wedge E) \equiv F$.

$\Rightarrow$ $(A \wedge E)$ is not valid conclusion of H_1, H_2, H_3 and H_4.

21. Show that following are inconsistent

$$P \longrightarrow Q,\ P \longrightarrow R,\ Q \longrightarrow \rceil R \text{ and } P$$

Solution:

Let the consistent,

$P \longrightarrow Q \equiv T$	$P \longrightarrow R \equiv T$	$Q \longrightarrow \rceil R \equiv T$	$P \equiv T$
(1) $Q \equiv T$	—	—	—
—	—	(2) $\rceil R \equiv T$	—
—	—	(3) $R \equiv F$	—
—	(4) $P \equiv F$	—	—

Hence we get $P \equiv F \equiv T$, a contradiction.

22. Show that the formula $[(\sim p \vee q) \Rightarrow (q \Rightarrow p)]$ is not a tautology.

Solution:

Boolean expression can be simplified as below :

$[(\sim p \vee q) \Rightarrow (q \Rightarrow p)]$

i.e. $[(\sim p \vee q) \Rightarrow (\sim q \vee p)]$

i.e. $[\sim(\sim p \vee q) \vee (\sim q \vee p)]$

i.e. $[(p \wedge \sim q) \vee \sim q \vee p]$

i.e. $[\sim p \vee q]$

Above expression is false for $p = 0$, $q = 1$.

So, it is not a tautology.

23. Let A be a tautology and B be any other formula. Prove that $(A \vee B)$ is a tautology.

Solution:

Since A is a tautology, it is true for all possible inputs. So, $A \vee B$ will also be true for all possible inputs, regardless of whether B is true or not.

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24. Show that proposition C is a logical consequence of the formula using truth tables.

$A \wedge (A \rightarrow (B \vee C)) \wedge (B \rightarrow \sim A)$

Solution:

If we want to show a formula Y is logical consequence of another formula X, we have to prove $X \rightarrow Y$. i.e we have to prove $X \rightarrow Y$ is a tautology. Hence $X \rightarrow Y \equiv T$ implies Y is logical consequence of X.

Let $X = A \wedge (A \rightarrow (B \vee C)) \wedge (B \rightarrow \sim A)$

Let us show $X \rightarrow C \equiv T$.

Consider following truth table.

A	B	C	$B \vee C$	C_1	$\sim A$	C_2	X	$X \rightarrow C$
				$A \rightarrow (B \vee C)$		$B \rightarrow \sim A$	$A \wedge C_1 \wedge C_2$	
T	T	T	T	T	F	F	F	T
T	T	F	T	T	F	F	F	T
T	F	T	T	T	F	T	T	T
T	F	F	F	F	F	T	F	T
F	T	T	T	T	T	T	F	T
F	T	F	T	T	T	T	F	T
F	F	T	T	T	T	T	F	T
F	F	F	F	T	T	T	F	T

In the above truth table, the last column shows that $X \rightarrow C \equiv T$. i.e. $X \Rightarrow C$.

Hence C is a logical consequence of $X \equiv A \wedge (A \rightarrow (B \vee C) \wedge (B \rightarrow \sim A))$.

SET THEORY

SET

S*et is any collection of distinct and distinguishable objects around us*.

By the form *distinct*, we mean that no object is repeated and by the term *distinguishable*, we mean that whether that object is in our collection or not. The objects belonging to a set are called *elements* or *members* of that set.

e.g. A is a set of stationary used by an student, i.e.

A = {Pen, Pencil, Eraser, Paper}

B is a set of Capitals of State, i.e. B = {Lucknow, Patna, Bhopal}

A set is represented by using all its elements between braces { } and by separating them from each other by commas (if there are more than one element).

Sets are denoted by capital letters of English alphabet while the elements are divided in general, but small letters.

If x is an element of a set A, we write $x \in A$ (read as *x belongs to A*).

If x is not an element of A, we write $x \notin A$ (read as *x does not belong to A*).

e.g. Let A = {4, 2, 8, 2, 6}. Elements of this collection are disinguishable but not distinct.

Hence A is not a set.

Let B = {*a, e, i, o, u*} is set of vowels in English.

Here elements of B are distinguishable as well as distinct. Hence B is a set.

FORMS OF REPRESENTATION OF A SET

(1) Set-builders form.

In this representation of set, we write between the braces { } a variable x which stands for each of the elements of the set, then we state the properties possessed by x.

We denote this property by $p(x)$ and separate x and $p(x)$ by a symbol : or (read as *such that*).

$A = \{x : p(x)\}$

$A = \{x : x$ is capital of States$\}$

or $A = \{x : x$ is a natural number and $2 < x < 11\}$

(2) Tabular form or Roaster form.

In this representation of set, elements of a set are listed one by one within braces { } and one separaed by each other by commas.

B = {Lucknow, Patna, Bhopal}

or B = (3, 4, 5, 6, 7, 8, 9, 10}

TYPES OF SETS

(1) Null set (or Empty set or Void set).

A set having no elements is called *empty set or void set.* It is denoted by ϕ or{ }

e.g. $A = \{x : x$ is an even number not divisible by $2\}$

B = {the set of immortal men}

(2) Singleton set.

A set having single element is called *singleton set.*

e.g. $A + \{x : x$ is present Prime Minister of India$\}$

B = {2}

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(3) Pair set.

A set having two elements is called *pair set.*

e.g. {1, 2}, {0, 3}, {4, 9} etc.

(4) Finite set.

A set having finite number of elements, i.e. a set where counting of elements is possible is called *finite set.*

e.g. A = {1, 2, 4, 6} is a finite set because it has four elements.

B = a null set ϕ is also a finite set because it has zero number of elements.

(5) Infinite set.

A set having infinite number of elements, i.e. a set where counting of elements is impossile, is called *infinite set.*

e.g. A = {x : x is a set of all natural numbers}

B = set of all points in a given plane.

SUBSETS

Set A is called subset of a set B if each element of set A is also an element of set B.

If A is a subset of set B, we represent it as $A \subseteq B$.

So, if $A \subseteq B \Leftrightarrow [x \in A \Rightarrow x \in B]$

e.g. Let A = {4, 5, 6} and B = {4, 5, 7, 8, 6}, then we write $A \subseteq B$.

Proper subset.

Set A is said to be a proper subset of a set B if

(*a*) every element of set A is an element of set B, and

(*b*) set B has at least one element which is not an element of set A.

This is expressed by writing $A \subset B$ or $B \subset A$ and read as A is a proper subset of B or B is a proper subset of A. If A is not a proper subset of B then we write it as $A \not\subset B$.

e.g. Let A = {4, 5, 6} and B = { 4, 5, 6, 7, 8}

Let A = {1, 2, 3} and B = {3, 2, 9}

$\therefore$ $A \not\subset B$ and $B \not\supset A$

SUPERSET

Let A = {1, 2, 3}, B = {2, 3,1}

$\Rightarrow$ $A \subseteq B$ Also $B \subseteq A$

Here $A \subseteq B$ can also be expressed equivalently by writing $B \supseteq A$, read as B is *superset of another set B*, if and only if set B contains all the elements of set A.

COMPARABILITY OF SETS

Two sets A and B are called *comparable* if either of these happens

(*i*) $A \subset B$

(*ii*) $B \subset A$

(*iii*) $A = B$

Similarly if neither of these above three exists, i.e. $A \not\subset B$, $B \not\supset A$ and $A \neq B$, then let A and B are called *incomparable.*

e.g. set A = {1, 2, 3} and B = {1, 2} are comparable.

But A = {1, 2, 3} and B = {2, 3, 6, 7} are incomparable.

UNIVERSAL SET

Any set which is superset of all the sets under consideration is called *universal set* and is either denoted by Ω or S or U.

e.g. Let A = {1, 2, 3}, B = {3, 4, 6, 9} and C = {0, 1}

We can take, S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} as universal set.

POWER SET

The set or family of all the subsets of a given set A is said to be power set of A and is expressed by P(A).

Mathematically $P(A) = \{x : x \subseteq A\}$

So, $x \in P(A)$

$\Rightarrow$ $x \subseteq A$

Note :

(*i*) $\phi \in P(A)$ and $A \in P(A)$ for all sets A.

(*ii*) Elements of P(A) are subsets of A.

e.g. If A = {1}, then $P(A) = \{\phi, \{1\}\}$

If A {1, 2}, then $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Similarly if A = {1, 2, 3}, then

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1,3\}, \{2,3\}, \{1, 2, 3\}\}$

So trends show that if A has n elements, then P(A) has 2^n elements. It is not only true for n = 1, 2, 3, but it is true of all n.

OPERATIONS ON SETS

(1) Union of sets.

The union of two sets A and B is the set of all those elements which are either in A or in B or in both. This set is denoted by $A \cup B$ and read as 'A union B'.

Symbolically, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

e.g. If A = {4, 5, 6}, and B = {2, 1, 3, 8},

then $A \cup B$ = {1, 2, 3, 4, 5, 6, 8}

(2) Intersection of sets.

Intersection of two sets A and B is the set of all the elements, which are common in A and B. This set is denoted by $A \cap B$ and read as '*A intersection of B*'.

Symbolically, $A \cap B = \{x : x \in A \text{ and } x \in B\}$

or $A \cap B = \{x : x \in A \cap x \in B\}$

e.g. If A = {1, 2, 3}, and B = {2, 1, 5, 6}, then $A \cap B$ = {1, 2}

(3) Difference of sets.

The difference of two sets A and B is the set of all those elements of A which are not elements of B. Sometimes, we call difference of sets as the relative components of B in A. It is denoted by $A \cap B$.

Symbolically, $A - B = \{x : x \in A \text{ and } x \notin B\}$

Similarly $B - A = \{x : x \in B \text{ and } x \notin A\}$

e.g. If A = {4, 5, 6, 7, 8, 9) and B = {3, 5, 2, 7}, then A – B = {4, 6, 8, 9}

Similarly, B – A = {3, 2}.

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Here, since $A - B \neq B - A$, so difference of two sets is not commutative.

(4) Disjoint sets.

Two sets A and B are called disjoint sets, if they have no element in common

Symbolically, $A \cap B = \phi$

(5) Symmetric difference of sets.

It is defined as set of elements that are member of either A or B but not both

Symbolically, $A \oplus B = (A \cup B) - (A \cap B)$

$= (A - B) \cup (B - A)$

(6) Complements of set.

Complement of a set A, also known as *absolute complement' of A* is the sets of all those elements of the universal sets which are not element of A. It is denoted by A° or A^{-1} or A' or A^c.

Symbolically, A^{-1} or $A^c = S - A$.

or $A^{-1} = \{x : x \in S \text{ and } x \in A\}$.

LAWS AND THEOREMS

(1) $A \cup A = A$, $A \cap A = A$

(2) $A \cup S = S$, $A \cap S = A$

(3) $A \cup \phi = A$, $A \cap \phi = \phi$

(4) $A - A = \phi$, $\phi - A = \phi$, $A - \phi = A$.

Theorem 1.

For any two sets A and B

(*a*) $A \cup B = B \cup A$ (*Commutative law of union*)

(*b*) $A \cap B = B \cap A$ (*Commutative law of intersection*)

Also

(*i*) $A \cup (B \cup C) = (A \cup B) \cup C$

(Associative law for union)

(*ii*) $A \cap (B \cap C) = (A \cap B) \cap C$.

(Associative law for intersection)

Theorem 2.

For any three sets A, B and C, following *disributive laws hold :*

(*i*) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(*ii*) $A \cap (B \cup C) = (A \cap B) (A \cap C)$.

DeMorgan's law.

For any two sets A and B

(*i*) $(A \cup B)' = A' \cap B'$

(*ii*) $(A \cap B)' = A' \cup B'$

For any three sets A, B and C, Demorgan's law can also be expressed as

(*i*) $A - (B \cup C) = (A - B) \cap (A - C)$

(*ii*) $A - (B \cap C) = (A - B) \cup (A - C)$

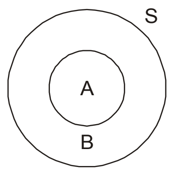
VENN DIAGRAM

It is a pictorial representation of sets.

A set is represented by circles or a closed geometrical figure inside the universal set. The universal set S is represented by a rectangular region.

Firstly we represent the set or a statement regarding sets with the help of diagram or venn diagram. The shaded area represents the set written.

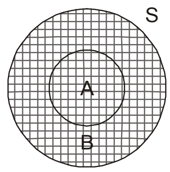
1. **Subset.**



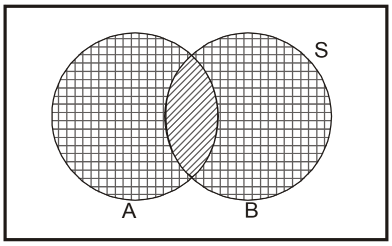
$A \subset B$.

2. **Union of sets.**

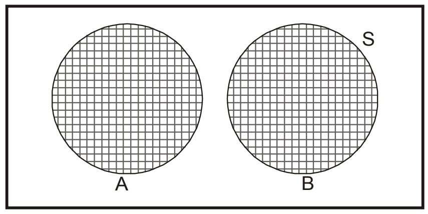
Let $A \cup B = B$. Here, whole area represented by B represents $A \cup B$.



$A \cup B$ when $A \subset B$



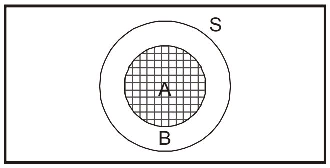
$A \cup B$ when neither $A \subset B$ nor $B \subset A$



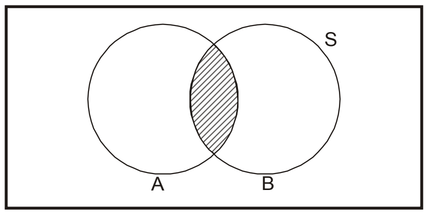
$A \cup B$ when A and B are disjoint events

3. **Intersection of sets.**

$A \cap B$ represents the common area of A and B.



$A \cap B$ when $A \subset B$ $(A \cap B = A)$



$A \cap B$ when neither $A \subset B$ or $B \subset A$

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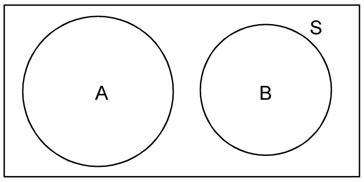
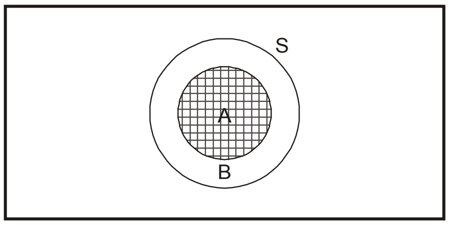


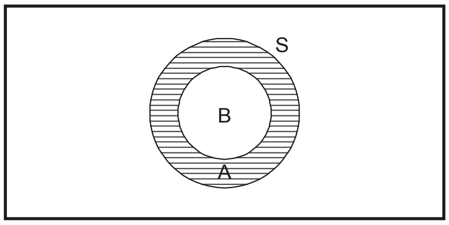
$(A \cap B = \phi)$

4. Difference of sets.

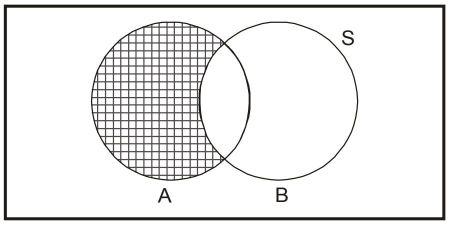
$A - B$ represents the area of A that is not in B.



$A - B$ when $A \subset B$ $(A - B = A)$

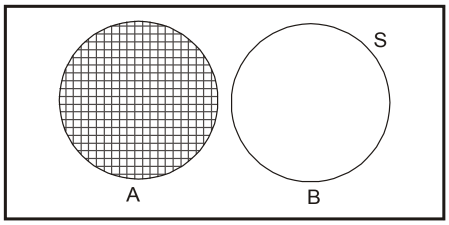


$A - B$ when $B \subset A$



$A - B$ neither

$A \subset B$ nor $B \subset A$

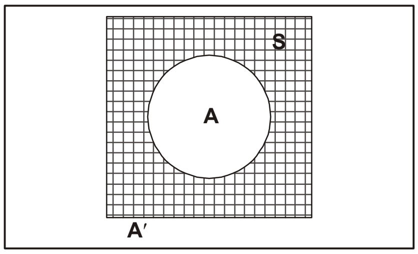


$A - B$ when A and B are

disjoint sets $(A - B = A)$

5. Complement of sets.

A' or A^c is the set of those points of universal set S which are not in A.

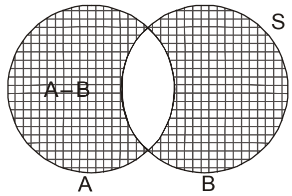


APPLICATION OF SET THEORY

The number of distinct elements of a finite set A is denoted by n (A).

Let $n(A)$, $n(B)$ and $N(A \cap B)$, where A and B are non-empty sets.

Then $\quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$



In case A and B are disjonit sets, so we have

$$A \cap B = \phi$$

$$\therefore \quad n(A \cup B) = n(A) + n(B)$$

From above Venn-diagram, following results are obtained :

(*i*) $n(A) = n(A - B) + n(A \cap B)$

(*ii*) $n(B) = n(B - A) + n(A \cap B)$

(*iii*) $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

The result, $n(A \cap B) = n(A) + n(B) - n(A \cap B)$ can be generalised as,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

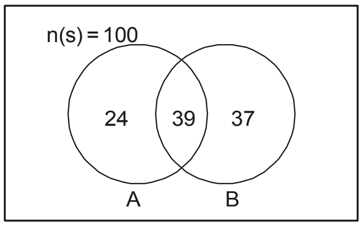
Example. If 63% of persons like oranges while 76% like apples, what can be said about the percentage of persons who like both oranges and apples ?

Solution:

Let $\quad n(S)$ = total number of persons = 100

$A = \{x : x \text{ likes oranges}\}$

$B = \{x : x \text{ likes apples}\}$



$n(A) = 63$

$n(B) = 76$

$A \cap B = \{x : x \text{ likes oranges and apples both}\}$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 63 + 76 - 100 = 39$

$\therefore \quad n(A \cap B) = 39$

Hence, 39% people like both oranges and apples.

Example. In a group of 1000 people, there are 750, who can speak Hindi and 400, who can speak Bengali. How many can speak Hindi only ? How many can speak Bengali only ? How many can speak both ?

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Solution:

Let $A = \{x : x \text{ speaks Hindi}\}$

and $B = \{x : x \text{ speaks Bengali}\}$

$\therefore$ $A - B = \{x : x$ speaks Hindi and can not speak Bengali)

$B - A = \{x : x$ speaks Bengali and can not speak Hindi)

$A \cap B = \{x : x$ speaks Hindi and Bengali both)

Given : $n(A) = 750, n(B) = 400$

$\therefore \quad n(A \cup B) = 1000$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore \quad n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 750 + 400 - 1000 = 150$

So, 150 people are speaking Hindi and Bengali both.

Again, $n(A) = n(A - B) + n(A \cap B)$

$\therefore \quad n(A - B) = n(A) - n(A \cap B)$

$= 750 - 150 = 600$

Hence, 600 people are speaking Hindi only

Finally, $n(S - A) = n(B) - n(A \cap B)$

$= 400 - 150 = \mathbf{250}$

So, 250 people are speaking Bengali only.

CARTESIAN PRODUCT OF SETS.

Ordered pair.

It is usually denoted by (x, y) is a pair of elements x and y of some sets, which is ordered in the sense that $(x, y) \neq (y, x)$ whenever $x \neq y$

Here x is called *first co-ordinate* and y is called *second co-ordinate* of the ordered pair (x, y).

e.g. ordered pairs (1, 2) and (2, 1) though consist of same elements 1 and 2, are different because they represent different points in the co-ordinate plane.

Cartesian product.

Cartesion product of two sets A and B is the set of all those pairs whose first co-ordinate is an element of A and second co-odinate is an element B.

The set is denoted by $A \times B$ and is read as '*A cross B*' or '*product set of A and B*'

i.e. $A \times B = \{(x, y) : x \in A \wedge y \in B\}$

e.g. Let $A = \{1, 2, 3\}$, and $B = \{3, 5\}$

$\therefore \quad A \times B = \{1, 2, 3\} \times \{3, 5\}$

$= \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)\}$

and, $B \times A = \{(3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3)\}$

$\therefore \quad A \times B \neq B \times A$

Similarly, we can define Cartesian product for n sets $A_1, A_2, \ldots A_n$ as

$A_1 \times A_2 \times A_3 \times \ldots. \times A_n = \{(x_1, x_2, x_3, \ldots x_n)$

$x_1 \in A \wedge x_2 \in A_2 \wedge x_3 \in A_3 \ldots \wedge x_n \in A_n\}$

The element $(x_1, x_2, \ldots x_n)$ is called as n-type of $x_1, x_2 \ldots. x_n$.

RELATIONS

Consider a set, $A = \{1, 2, 3\}$

Cartesian product set is,

$A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3),$
$(3, 1), (3, 2), (3, 3)\}$

Consider a subset from this set in such a way that first coordinate is '*less than*' the second coordinate.

If this relation is denoted by R, then R is described by

$$R = \{(x, y) \in A \times A : x < y\}$$

in which (x, y) is a member of the relation R.

Relation in a set.

A relation between two sets A and B is a subset of $A \times B$ and is denoted by R.

Thus $\quad R \subseteq A \times B$

We write xRy, if and only if $(x, y) \in R$. xRy is read as '*x is related to y*'

If A = B, then we say that *R is a relation in the set A*.

e.g. Consider the relation

$R = \{(x, y) \in N \times N : x > y$, N is a set of natural numbers.$\}$

Obviously, $R \subseteq N \times N$ and so R is relation N.

Binary relation in a set.

A binary relation R is called *defined in set* A if for any ordered pair $(x, y) \in A \times A$, it is meaningful to say that xRy is true or false.

In other words, $R = \{(x, y) \in A \times A : xRy$ is true$\}$.

i.e., a relation R in a set A is a subset of $A \times A$. Binary is two. So binary relation is a relation between two sets, which may be different or identical. For the sake of convenience a binary relation will be written as only as relation.

Domain and Range of relation.

Domain D of the relation R is defined as the set of all first elements of the ordered pairs which belong to R

i.e. $\quad D = \{x : (x, y) \in R$, for $x \in A\}$

Range E of the relation R is defined as the set of all second elements of the ordered pairs which belong to R

i.e. $\quad E = \{y : (x, y) \in R$, for $y \in B\}$

Obviously, $\quad D \subseteq A$ and $E \subseteq B$

e.g. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$.

Every subset of $A \times B$ is a relation from A to B. So, if $R = \{(2, a), (4, a), (4, c)\}$, then domain of R is the set $\{2, 4\}$ and range of R is the set $\{a, c\}$.

Total number of distinct relations from set A to set B.

Let number of elements of A and B be m and n respectively.

Then number of elements of $A \times B$ is mn.

Therefore number of elements of the power set of $A \times B$ is 2^{mn}.

Thus, $A \times B$ has 2^{mn} different subsets.

Now every subset of $A \times B$ is a relation from A to B. Hence number of different relations A to B is 2^{mn}.

TYPES OF RELATIONS IN A SET

(1) Inverse relation.

Let R be a relation from set A to set B.

Then inverse relation R^{-1} from set B to set A is defined by

$$\{(b, a) : (a, b) \in R\}$$

In other words, inverse relation R^{-1} consists of

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those ordered pairs which when reversed belong to R. Thus every relation R from set A to set B has an inverse relation R^{-1} from B to A.

e.g.Let A = {1, 2, 3}, B = {a, b} and R = {(1, a), (1, b), (3, a), (2, b)} be a relation from A to B.

The inverse relation R is

$$R^{-1} = \{(a, 1), (b, 1), (a, 3), (b, 2)\}$$

Let A = {2, 3, 4}, B = {2, 3, 4} and R = {(x, y) : $|x - y| = 1$} be a relation from A to B, i.e.

$$R = \{(3, 2), (2, 3), (4, 3), (3, 4)\}.$$

Inverse relation of R is,

$$R^{-1} = \{(3, 2), (2, 3), (4, 3), (3, 4)\}$$

It may be noted that $R = R^{-1}$.

Here every relation has an inverse relation.

If R be a relation from A to B, then $(R^{-1})^{-1} = R$.

Theorem.

If R be a relation from A toB, then domain of R is the range of R^{-1} and range of R is the domain of R^{-1}.

(2) Identity relation.

A relation R in a set A is said to be identity relation, generally denoted by I_A, if

$$I_A = \{(x, x) : x \in A\}.$$

e.g. Let A = {2, 4, 6}, then

I_A = {(2, 2), (4, 4), (6, 6)} is an identity relation in A.

(3) Universal relation.

A relation R in a set A is said to be universal relation if R is equal to A × A i.e. if R = A × A.

e.g. Let A = {1, 2, 3}, then

R = A × A = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)} is a universal relation in A.

(4) Void relation.

A relation R in a set A is said to be a void relation if R is a null set, i.e. if R = ϕ.

e.g. Let A = {2, 3, 7} and let R be defined as 'aRb if and only if a divides b', then

$$R = \phi \subset A \times A \text{ is a void relation.}$$

PROPERTIES OF RELATIONS IN A SET

(1) Reflexive relations.

Let R be a relation in a set A. Then R is called a *reflexive relation,* if $(a, a) \in R$, for all $a \in A$.

In other words, R is reflexive if every element in A is related to itself. Thus, R is reflexive if aRa holds for all $a \in A$.

As relation R in a set A is not reflexive if there is atleast one element such that $(a, a) \notin R$.

e.g. Let A = {1,2, 3, 4}. Then the relation

R_1= {(1,1), (2, 4), (3, 3), (4, 1), (4, 4)} in A is not reflexive since $2 \in A$ but $(2, 2) \notin R_1$.

(2) Anti-reflexive relations.

Let R be a relation in a set A. Then R is called an *anti-reflexive relation,* if $(a, a) \notin R$ for every $a \in A$, i.e. a/Ra for all $a \in A$.

e.g. Relation in the set of natural numbers N defined by '$x < y$' is anti-reflexive, since a is not less than a for any natural number a.

Let A = {1, 2, 3, 4}. Then relation R = {(1, 1), (2, 3), (3, 4)} is not anti-reflexive since, $(1, 1) \in R$.

(3) Symmetric relations.

Let R be a relation in a set A. Then R is said to be symmetric relation if $(a, b) \in R \Rightarrow (b, a) \in R$.

Thus R is symmetric if bRa holds whenever aRb holds.

A relation R in set A is not symmetric if there exist two distinct elements a, $b \in A$, such that aRb, but b/Ra.

e.g. Let L be the set of all straight lines in a plane. The relation R in L defined by '*x is parallel to y*' is symmetric, since if a straight line a is parallel to a straight line b, then b is also parallel to a.

$$(a, b) \in R \Rightarrow (b, a) \in R.$$

Let R be relation in the natural numbers N which is defined by '$x - y > 0$'. Then R is not symmetric since, 4 – 2 > 0 but 2 – 4 > 0.

Thus, $(4, 2) \in R$ but $(2, 4) \notin R$.

Remarks : Since $(a, b) \in R \Rightarrow (b, a) \in R^{-1}$, thus R is a symmetric relation if and only if $R = R^{-1}$.

(4) Anti-symmetric relations.

Let R be a relation in a set A, i.e.let R be a subset of A × A. Then R is said to be an anti-symmetric relation if $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$.

Thus R is anti-symmetric if $a \neq b$, then possibly $(a, b) \in R$ or possibly $(b, a) \in R$, but never both.

A relation R in a set A is not anti-symmetric if there exist elements $a, b \in A$, $a \neq b$ such that $(a, b) \in R$ and $(b, a) \notin R$.

e.g. Let A be a family of sets and let R be the relation in A defined by 'x is a subset of y'. Then R is anti-symmetric since $A \subseteq B$ and $B \subseteq A \Rightarrow A = B$.

(5) Transitive relations.

Let R be a relation in a set, i.e. let R be a subset of A × A. Then R is called *transitive relation,* if

$$(a, b) \in R \text{ and } (b, c) \in R \Rightarrow (a, c) \in R.$$

A relation R in a set A is *not transitive* if there exist elements $a, b, c \in A$, not necessarily distinct, such that $(a, b) \in R$, $(b, c) \in R$ but $(a, c) \notin R$.

e.g. Let L be the set of all straight lines in a plane and R be the relation in L defined by 'x is parallel to y'.

If a is parallel to b and b is parallel to c, then obviously a is parallel to c.

Thus $(a, b) \in R$ and $(b, c) \in R \Rightarrow (b, c) \in R$. Hence R is transitive.

Let A be set of all Indians. Let R be the relation in A defined by '*x loves y*' if a loves b and b loves c, it does not necessarily follow that a loves c, so R is not a transitive relation.

(6) Partial order.

A relation R on a Set A is said to be partial order if

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(*i*) R is reflexive, *i.e.* $(a, a) \in R \; \forall \; a \in A$

(*ii*) R is anti-symmetric, *i.e.* $\forall \; a, b \in A \; (a, b) \in R \wedge (b, a) \in R \Rightarrow a = b$

(*iii*) R is transitive, *i.e.* $\forall \; a, b, c \in A \; (a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

(7) Equivalence relations.

Let R be a relation in a set A. Then R is an equivalence relation in A if and only if

(*i*) R is reflexive, i.e. for all $a \in R$, $(a, a) \in R$.

(*ii*) R is symmetric, i.e. $(a, b) \in R \Rightarrow (b, a) \in R$, for all $a, b \in A$.

(*iii*) R is transitive, i.e. $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

e.g. Most trivial example of an equivalence relation is that of *equality* for any elements in any set

I. $a = a$, i.e. reflexive.

II. $a = b, \; \Rightarrow \; b = a$, i.e. symmetric

III. $a = b$ and $b = c \; \Rightarrow \; a = c$, i.e. transitive.

Let R be the relation in the real numbers defined by '$x \leq y$'. Then

I. $x \leq x$ i.e. $(x, x) \in R$, i.e., R is reflexive.

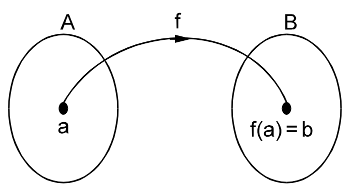
II. Let $x \leq y$ but $y \not\leq x$ i.e. $(x, y) \in R$ but not necessarily, $(y, x) \in R$, i.e. R is not symmetric. Thus R is not an equivalence relation.

Theorem.

If R be an equivalence relation in a set A, then R^{-1} is also an equivalence relation in A.

FUNCTION

Let A and B be two non-empty sets and f be a relation from A to B. If for each element $a \in A$, there exists a unique $b \in B$ (denoted by $f(a)$) such that $(a, b) \in f$, then f is called a *function (a mapping) from A to B*.



It is denoted by $f : A \to B$.

Take any element $a \in A$.

Since $f : A \to B$ is a function, there exists a unique element $b \in B$, such that $f(a) = b$.

It may be written as follows.

$$a \in A. \; f : A \to B \Rightarrow f(a) = b \in B, \text{ or}$$

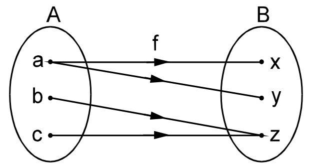
$$f : A \to B, \; a \in A \Rightarrow \exists \text{ unique } (a, b) \in f.$$

Note:

For all elements of A, images should exist in B.

All elements in B may or may not have pre-images in A

There may be one or more pre-images in A for elements in B.



$f(A) \subseteq B$ and $f \subseteq A \times B$.

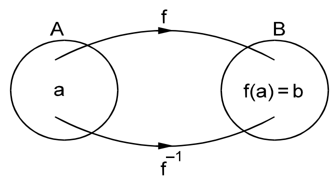
Every function from A to B is a relation from A to B but every relation from A to B need not be a function from A to B.

If an element in A has more than one image in B, f is not a function.

If $n(A) = m$, $n(B) = k$, then number of functions that can be defined from A to B is k^m.

IMAGE AND PRE-IMAGE

If $f : A \to B$ is a function and $(a, b) \in f$, then b is called *f-image of a*.



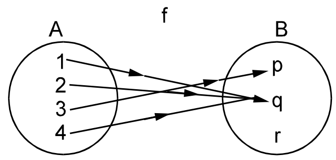
It is written as $f(a) = b$.

'a' is called a *pre-image of* 'b'.

We write this as

$$a \in f^{-1}(b) : f^{-1}(b) = \{x \in A \mid f(x) = b\}.$$

Every element of A has one and only one image in B. Every element of B may or may not have a pre-image in A. But one or more elements of A, may have the same image in B.



In above figure, there are more than one pre-images for $q \in B$.

In fact, $\quad f^{-1}(q) = \{a \in A \mid f(a) = q\} = \{1, 2, 4\}$

DOMAIN, CO-DOMAIN AND RANGE

If $f \; A \to B$ is a function, then A is called *domain*, B is called *co-domain of f*.

Range.

If $f : A \to B$ is a function, then set of all images of the elments of A is called *range of f*.

It is denoted by $\; f(A) = \{f(a)/a \in A\}$.

Range is a subset of the co-domain, i.e. $f(A) \subseteq B$.

e.g. If $A = \{1, 2, 3, 4\}$, $B = \{p, q, r\}$ and $f = \{1, q), (2, q), (3, q), (4, q)\}$, then f is a function.

Domain = A, Co-domain = B.

$$\text{Range of } f = f(A) = \{f(1), f(2), f(3), f(4)\}$$
$$= \{q, q, p, q\} = \{p, q\}$$

$\therefore \qquad f(A) \subseteq B$

There is no pre-image for the element $r \subseteq B$. Even

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then $f : A \to B$ is a function (hence for every element there may not be pre-images).

e.g. (*i*) Let A = {1, 2, 3, 4}, B = {*p, q, r*} and $g = \{(1, p), (2, q), (3, p), (4, p), (1, r)\}$.

The element $1 \in A$ has two images *p* and *r* in B.

So, $g : A \to B$ is not a function but it is only a relation.

- If N is the set of natural numbers and $f = \{(a, b) \mid a, b \in N, b = 2a\}$, then
$$f : N \to N \text{ is a function.}$$
- If $f(x) = |x|$, $\forall x \in R$, then $f : R \to R$ is a function, i.e.
$$f = \{(x, |x|)\ x \in R\}$$
$$f(x) = x \text{ if } x \geq 0, f(x) = -x \text{ if } x < 0.$$

EQUALITY OF TWO FUNCTIONS

If *f* and *g* are functions defined on the same domain A and if $f(a) = g(a)$ for every $a \in A$, then functions *f* and *g* are equal,

i.e. $f = g$

e.g. Let $f(x) = x^2 + 5$, where *x* is a real number and $g(x) = x^2 + 5$, where *x* is a complex number. Then function is not equal to the function *g,* since they have different domains although $f(x) = x^2 + 5 = g(x)$.

Let a function *g* be defined by the formula $g(x) = x^2$, where domain of *g* is the set {1, 2}.

Then $f = g$, since they both have same domain and since *f* and *g* assign same image of each element in the domain.

TYPES OF FUNCTIONS

(1) Into mappings.

A mapping $f : A \to B$ is called *into mapping* if $f(A)$ is a proper subset of B. In this case we say that *f* maps A into B.

e.g. Let $f : Z \to Z$ (set of integers) be defined by $f(x) = 2x, x \in Z$.

Then *f* is an into mapping, because $f(Z)$ (set of all even integers) is a proper subset of the codomain set Z.

(2) Onto mappings.

A mapping $f : A \to B$ is called *onto mapping* if $f(A) = B$.

In this case we say that *f* maps onto B.

e.g. Let $f : Z \to Z$ be defined by $f(x) = x + 1, x \in Z$.

Then every element *y* in the codomain set Z has a pre-image $y-1$ in the domain set Z.

Therefore, $f(z) = Z$ and *f* is an onto mapping.

(3) Injective mapping.

A mapping $f : A \to B$ is called *injective* (or one-to-one or injection) if for each pair of distinct elements of A, their *f*-images are distinct.

Thus $f : A \to B$ is injective if $x_1 \neq x_2$ in A implies $f(x_1) \neq (x_1)$ in B.

e.g. Let $f : R \to R$ be defined by $f(x) = 2x + 1, x \in R$.

Then for $x_1, x_2 \in R$ $(x_1 \neq x_2)$, we have $f(x_1) \neq f(x_2)$.

(4) Surjective mapping.

A mapping $f : A \to B$ is called *surjective* (or onto or surjection) if $f(A) = B$.

(5) Bijective mapping.

A mapping *f* $A \to B$ is called *bijective* (or bijection) if *f* is both injective and surjective.

e.g. Let $f : Z \to Z$ be defined by $f(x) = x + 2$, $x \in 1 \in Z$.

The mapping *f* is both injective and surjective. So it is bijective.

Note : If *f* is injective, each element of B has at most one *pre-image*. If *f* is surjective, each element of B has at least one *pre-image*.

(6) Many-one mapping.

A mapping $f : A \to A$ is called *many-one* if two or more distinct elements in A have the same image, i.e. if $f(x) = f(x)'$ implies $x \neq x'$.

(7) Constant mapping.

A function *f* of A into B is called *constant mapping* if same element $b \in B$ is assigned to every element in A.

In other words, $f : A \to B$ is a constant mapping if range of *f* consists of only one element.

e.g. Let $f : R \to R$ be defined by the formula $f(x) = 50$.

Then *f* is a constant function since 50 is assigned to every element.

(8) Identity mapping.

Let A be a set and the function $f : A \to A$ be defined by the formula $f(x)$.

Then *f* is called *identity function* or *identity mapping* or *identity transformation* on A.

(9) Inverse mapping.

Let *f* $A \to B$ be a one-one onto mapping.

Then mapping $f^{-1} : B \to A$ which associates to each element $b \in B$ the element $a \in A$, such that $f(a) = b$, is called *inverse mapping* of the mapping $f : A \to B$.

Hence if $f : A \to B$ be a one-one on to mapping, then $f^{-1} : B \to A$. The mapping f^{-1} is called *inverse mapping of the mapping f.*

e.g. Let A = R – {3} and B = R – {1} and function $f : A \to B$ be defined by
$$f(x) = \frac{x-2}{x-3}.$$
Then inverse function $f^{-1} : B \to A$ is given by
$$y = \frac{x-2}{x-3} \quad \text{or} \quad x = \frac{2-3y}{1-y}.$$
Hence $$f^{-1}(x) = \frac{2-3x}{1-x}.$$

SOME IMPORTANT THEOREMS

Theorem 1.

Only bijective mapping possess inverse mappings.

Theorem 2.

If $f : A \to B$ is bijective, then $f^{-1} : B \to A$ is also bijective.

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Theorem 3.

If $f : A \to B$ be bijective, then inverse mapping of f is unique.

PRODUCT OR COMPOSITIONS OF MAPPINGS

Let $f : A \to B$ and $g : B \to C$.

Then composite of functions f and g denoted by (fog) or fg is a mapping of $A \to C$ given by $(fog) : A \to C$ such that

$$(gof)(x) = gf(x) = g[f(x)] = \forall x \in A$$

It may be noted that domain of g is equal to codomain of f.

e.g. Let $f(x) = 2x + 1$ and $g(x) = 3x$ be two functions.

$$\begin{aligned}\therefore \quad (gof)(x) &= g[f(x)] = g(2x+1)\\ &= g(y) \text{ (where } y = 2x+1)\\ &= 3y \quad [\text{since } g(x) = 3x]\\ &= 3(2x+1)\\ &= 6x+3.\end{aligned}$$

Note : If $f\ A \to B$ and $g : A \to A$, then we can find both the composite mapping go f and fog.

But in general $(gof) \neq (fog)$.

$$\begin{aligned}\text{e.g. Let} \quad f(x) &= 2x - 1,\ g(x) = 5x\\ \therefore \quad (fog)(x) &= f[g(x)] = f(5x)\\ &= 2(5x) - 1\\ &= 10x - 1\\ \text{and} \quad (gof)(x) &= g[f(x)]\\ &= g(2x-1)\\ &= 5(2x-1)\\ &= 10x - 5.\\ \text{i.e.} \quad fog &\neq gof.\end{aligned}$$

Properties of Composites of mappings.

(1) If $f : A \to B$ be a bijective mapping, then

of $fof^{-1} = I_B$ and $f^{-1}\ of = I_A$

where I_A and I_B are identity mappings of the sets A.

(2) If $f : A \to B$, then

I_B of $= f$ and of $f_0\ I_A = f$

(3) If $f : A \to B$, $g : B \to C$, $h : C \to D$ be three mappings, then

$ho(gof) = (hog)\ of.$

(4) If $f : A \to B$ and $g : B \to C$ be both injective mappings, then composite mapping $gof : A \to C$ is injective.

(5) If $f : A \to B$ and $g : B \to C$ be two mappings such that $gof : A \to C$ is injective, then f is injective.

(6) If $f : A \to B$ and $g : B \to C$ be both surjective, then composite mapping $gof : A \to C$ is surjective.

(7) If $f : A \to B$ and $g : B \to C$ be two mappings such that $gof : A \to C$ is surjective, then g is surjective.

(8) If $f : A \to B$ and $g : B \to C$ be both bijective, then composite mapping $go\,f : A \to C$ is bijective.

REAL VALUED FUNCTIONS

R is the set of real numbers. If $A \subseteq R$, then a function, $f : A \to R$ is called *real valued function.*

Example. Find domain of the real valued function $f(x) = \sqrt{49 - x^2}$.

Solution:

$$\begin{aligned} \sqrt{49 - x^2} &\in R\\ \Rightarrow \quad 49 - x^2 &\geq 0\\ \Rightarrow \quad x^2 &\leq 49\\ \Rightarrow \quad |x| &\leq 7\\ \Rightarrow \quad -7 \leq\ & x \leq 7\end{aligned}$$

Thus domain of f is $[-7, 7]$

ALGEBRA

INTRODUCTION

Group is a mathematical system having an algebraic structure.

Groups are sets with an algebraic operation. Sets under an algebraic operation satisfy certain rules which we classify and put them under the name of axioms.

Notations used for some standard Sets.

N = set of all natural numbers (positive integers)
Z = set of all integers
Q = set of all rational numbers
R = set of all real numbers
C = set of all complex numbers
A^* = set of all nonzero numbers of A where $A \subseteq R$
A^+ = set of all positive numbers of A where $A \subseteq R$
A^- = set of all negative numbers of A where $A \subseteq R$.

BINARY OPERATIONS

Definition 1.

Let A be a nonempty set. Any mapping from $A \times A$ into A is called binary operation or binary composition on A.

- If f is a binary operation on a nonempty set A and $(x, y) \in A \times A$, then image $f(x, y)$ of (x, y) under f is denoted by xfy.
- The symbols +, ..., 0, * etc., is used oftenly to denoted binary operations on a nonempty set.
- If * is a binary operation on a nonempty set A, then for all $x, y \in A$, $x * y$ is a unique element of A.

Definition 2.

*Let * be a binary operation on a nonempty set A and $B \subseteq A$. Then B is said to be closed with respect to the binary operation * if $x * y \in B$ for all $x, y \in B$.*

- If * is a binary operation on a nonempty set A, then A is closed with respect to *.

  If $x, y \in A$ then $x * y$ is called *product of x and y with respect to *.*
- If A is closed with respect to *, then we also say that * *is closed on A.*

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Symbols used for some usual operations

+ = usual addition

– = usual subtraction

. = usual multiplication

÷ = usual division

e.g. + and . are binary operations on N, Z, Q, R and C.

BINARY OPERATION TABLES.

Definition 3.

*If * is a binary operation on a finite set A, then products of elements of A can be represented in a table. These tables are called Cayley binary operation tables or Cayley multiplication tables or Cayley composition tables.*

If $x, y \in A$, then $x * y$ is shown as follows :

*		y	
x		$x * y$	

Example. Write down binary operation table for usual multiplication '.' on the set A = {1, – 1}.

Solution:

$\cdot$	1	–1
1	1	–1
–1	–1	1

Example. Write down composition table for the set A = $\{1, \omega, \omega^2\}$, where ω is complex cube roots of unity, under usual multiplication.

Solution:

$\cdot$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

TYPES OF BINARY OPERATIONS.

Definition 4.

*A binary operation * on a nonempty set A is said to be associative if $(x * y) * z = x * (y * z), \ \forall x, y, z \in A$.*

e.g. + and $\cdot$ are associative binary operations on N, Z, Q, R and C.

Defintion 5.

*A binary operation * on a nonempty set A is said to be commutative if*

$$x * y = y * x, \ \forall x, y \in A.$$

e.g. + and $\cdot$ are commutative binary operations on N,Z, Q, R and C.

Example. Which of the following are binary operations on N. Also verify which binary operations are associative and commutative ?

(*i*) $a * b = a + b$

(*ii*) $a * b = a - b$

(*iii*) $a * b = a + b + 1$

(*iv*) $a * b = ab$

(*v*) $a * b = a \div b$

(*vi*) $a * b = ab + 1$

(*vii*) $a * b = a + b + ab$.

Solution:

(***i***) If a, b are natural numbers, then $a + b$ is a natural number.

Therefore * is a binary operation on N in (I).

Addition of natural numbers is both associative and commutative.

(***ii***) * is not a binary operation on N, since $3 \in N, 5 \in N$; but $3 * 5 = 3 - 5 \notin N$.

(***iii***) If a, b are natural numbers, then $a + b + 1$ is also a natural number.

Therefore * is a binary operation on N in (III).

$$a * b = a + b + 1 = b + a + 1 = b * a$$

Therefore * is commutative.

$$\begin{aligned}(a * b) * c &= (a + b + 1) * c \\ &= a + b + 1 + c + 1 \\ &= a + (b + c + 1) + 1 \\ &= a * (b + c + 1) = a * (b * c)\end{aligned}$$

Therefore * is associative.

(***iv***) If a, b are natural numbers, then ab is a natural number.

Therefore * is a binary operation on N in (IV).

Multiplication of natural numbers is both associative and commutative.

(***v***) * is not a binary operation in (V),

since, $3 \in N, 5 \in N$,

but $3 * 5 = 3 \div 5 \notin N$.

(***vi***) If a, b are natural numbers, then $ab + 1$ is a natural number

Therefore * is a binary operation on N in (VI).

$$a * b = ab + 1 = ba + 1 = b * a$$

Therefore * is commutative.

$$\begin{aligned}(a * b) * c &= (ab + 1) * c = (ab + 1)\, c + 1 \\ &= abc + c + 1 \\ a * (b * c) &= a * (bc + 1) = a(bc + 1) + 1 \\ &= abc + a + 1.\end{aligned}$$

Therefore * is not associative.

(***vii***) If a, b are natural numbers, then $a + b + ab$ is also a natural number.

Therefore * is a binary operation on N.

$$a * b = a + b + ab = b + a + ba = b * a$$

Therefore * is commutative.

$$\begin{aligned}(a * b) * c &= (a + b + ab) * c \\ &= a + b + ab + c + (a + b + ab)\, c \\ &= a + b + c + ab + bc + ac + abc \\ a * (b * c) &= a * (b + c + bc) \\ &= a + b + c + bc + a(b + c + bc) \\ &= a + b + c + ab + bc + ac + abc\end{aligned}$$

Therefore * is associative.

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ALGEBRAIC SYSTEMS OR ALGEBRAIC STRUCTURES

Definition 6.

If A is a nonempty set and B is a set of binary operations on A, then (A, B) is called an algebraic system or an algebraic structure.

*If * is a binary operation on A, then (A, *) is an algebraic system.*

e.g. (N, +), (N, .), (Z, +), (Z, .), (Q, +), (Q, .), (R, +), (R, .) are algebraic systems.

SEMIGROUPS

Definition 7.

*If S is a nonempty set and * is a binary operation on S, then algebraic system (S, *) is called a semigroup if*

$$a * (b * c) = (a * b) * c, \forall\, a, b, c \in S.$$

(*i*) If S is a nonempty set and * is an associative binary operation on S, then (S, *) is a semigroup.

(*ii*) If (S, *) is a semigroup, then element $a \cdot b$ is denoted by ab.

(*iii*) If (S, *) is a semigroup, then S is a semigroup under * or a semigroup with respect to *.

e.g. (N, +), (N, ·), (Z, +), (Z, ·), (Q, +), (Q, ·), (R, +), (R, ·), (C, +), (C, ·), are all semigroups.

Definition 8.

Let (S, *) be a semigroup.

An element $e \in S$ is called

I. *left identity of S if* $e * a = a, \ \forall a \in S$

II. *right identity of S if* $a * e = a, \ \forall a \in S$

III. *two sided identity or simple identity of S if* $a * e = e * a = a, \ \forall a \in S.$

e.g. semigroups (Z, +), (Q, +), (R, +), (C, +) contain '0' as identity.

semigroups (N, .), (Z, .), (Q, .), (R, .), (C, .) contain '1' as identity.

Theorem 1.

Let (S, *) be a semigroup. If e is a left identity and f is a right identity of S, then $e = f$.

Theorem 2.

If (S, *) is a semigroup with identity, then identity in S is unique.

Definition 9.

A semigroup with identity is called a monoid.

e.g. (Z, +) is a monoid and (N, +) is a semigroup but not a monoid.

Definition 10.

*Let (S, *) be a semigroup with identity e.*

I. *An element $a \in S$ is called left invertible if there exists an element $b \in S$ such that $b * a = e$. The element b is called left inverse of a.*

II. *An element $a \in S$ is called right invertible if there exists an element $c \in S$ such that $a * c = e$. The element c is called right inverse of a.*

III. *An element $a \in S$ is called invertible if a is both left invertible and right invertible.*

e.g. 1 is invertible in the semigroup (Z, .).

Theorem 3.

Let (S, *) be a semigroup with identity e and $a \in$ S. If b is a left inverse and c is a right inverse of a, then $b = c$.

Definition 11.

*Let (S, *) be a semigroup with identity e and $a \in S$. If a is invertible, then element which is both left inverse and right inverse of a is called inverse of a.*

Theorem 4.

Let (S, *) be a semigroup with identity e. If a is an invertible element, then inverse of a is unique.

Let (S, *) be a semigroup with identity e. If a is an invertible in S element, then inverse of a is denoted by a^{-1}.

Now, $\quad a * a^{-1} = a^{-1} * a = e.$

Theorem 5.

Let (S, *) be a semigroup with identity e. If a is an invertible element, then a^{-1} is also invertible and

$$\left(a^{-1}\right)^{-1} = a.$$

Theorem 6.

Let (S, *) be a semigroup with identity e. If a, b are two invertible elements in S, then $a * b$ is also invertible and

$$(a * b)^{-1} = b^{-1} * a^{-1}$$

Example. Show that $S\left\{\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} : a, b \in R\right\}$ is a semigroup under matrix multiplication. Verify that $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ is a left identify in S.

Solution:

$$\text{Let } \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix} \in S$$

$$\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} ac & ad \\ 0 & 0 \end{bmatrix} \in S.$$

Hence matrix multiplication is a binary operation on S. Since matrix multiplication is associative (S, .) is a semigroup.

$$\text{Let} \quad \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \in S$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$$

$\therefore \quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ is a left identity of S.

Example. If S is a nonempty set and P(S) is power set of S, then show that (P(S), $\cup$), (P(S), $\cap$) are monoids.

Solution:

Let A, B $\in$ P(S).

Now $A \subseteq S$, $B \subseteq S$ and therefore $A \cup B \subseteq S$, $A \cap B \subseteq S$.

Hence, $\quad A \cup B \in P(S), A \cap B \in P(S)$

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We know, $(A \cup B) \cup C = A \cup (B \cup C)$,

$(A \cap B) \cap C = A \cap (B \cap C)$,

$\forall$ A, B, C $\in p(S)$.

Again $A \cup \phi = \phi \cup A = A$

and $A \cap S = S \cap A = A$.

Hence (P(S), $\cup$) and (P(S), $\cap$) are monoids.

Example. If S = set of all odd integers, then show that (S, .) is a semigroup with identity (monoid)

Solution:

Now $S = \{2n + 1 : n \in z\}$.

Let $2m + 1, 2n + 1 \in S$

$(2m + 1)(2n + 1) = 2(2mn + m + n) + 1 \in S$.

Since multiplication is associative in Z, multiplication is associative in S.

Hence (S, .) is a semigroup and clearly 1 is the identity of S.

Definition 12.

*A semigroup (S, *) is called*

I. *left cancellative semigroup* if $a, b, c \in S$

$a * b = a * c$

$\Rightarrow b = c$

II. *right cancellative semigroup* if $a, b, c \in S$

$b * a = c * a$

$\Rightarrow b = c$

III. *cancellative semigroup* if it is both left cancellative and right cancellative semigroup.

- $a * b = a * c$

$\Rightarrow b = c$ is called *left cancellation law.*

- $b * a = c * a$

$\Rightarrow b = c$ is called *right cancellation law.*

e.g. (Z, +), (Z, *), (Q, +*), (Q, *.), (R, +), (R, .) are cancellative semigroups.

GROUP

A semigroup with identity and in which every element is invertible is called a *group*.

Definition 1 .

*Let G be a nonempty set and * be a binary operation on G. Then algebraic system (G, *) is called a group if*

I. $a * (b * c) = (a * b) * c, \forall a, b, c \in G$

II. $\exists e \in G \Rightarrow a * e = e * a = a, \forall a \in G$

III. $a \in G$

$\Rightarrow \exists b \in G \Rightarrow a * b = b * a = e$

- The element e in condition II of the definition of a group is called *identity element of the group.*
- The element b corresponding to a in condition III of the definition of a group is called *inverse of a* in the group.

Definition 2 .

*A group (G, *) is called an abelian group or commutative group if*

$$a * b = b * a, \forall a, b \in G.$$

e.g. (Z, +), (Q, +), (R, +), (C, +) are all abelian groups.

Example. If G = Q − {1}, if * is defined on G as $a * b = a + b - ab$, show that (G, *) is an abelian group.

Solution:

Clearly G is non-empty.

Let, $a, b \in G$

$a \in G \Rightarrow a \in Q, a \neq 1$

$b \in G \Rightarrow b \in Q, b \neq 1$

$a \in Q, b \in Q \Rightarrow a + b \in Q$,

$ab \in Q \Rightarrow a + b - ab \in Q$

$\Rightarrow a * b \in Q$

If $a * b = 1$

then $a + b - ab = 1$

$\Rightarrow a - 1 + b - ab = 0$

$\Rightarrow 1(a - 1) - b(a - 1) = 0$

$\Rightarrow (a - 1)(1 - b) = 0$

$\Rightarrow a = 1$ or $b = 1$

Contradiction : $a * b \neq 1$

$a * b \in Q, a * b \neq 1$,

$\Rightarrow a * b \in G$

Therefore* is a binary operation on G.

Let, $a, b, c \in G$.

$$a * (b * c) = a * (b + c - bc)$$
$$= a * (b + c) - bc) - a(b + c - bc)$$
$$= a + b + c - bc - ab - ac + abc$$
$$(a * b) * c = (a + b - ab) * c$$
$$= a + b - ab + c - (a + b - ab)\, c$$
$$= a + b + c - ab - ac - bc + abc$$

$\therefore a * (b * c) = (a * b) * c$

Therefore $*$ is an associative binary operation on G.

Let e be the identity element of G, then

$a * e = a, \forall a \in G$

$a * e = a$

$\Rightarrow a + e - ae = a$

$\Rightarrow e(1 - a) = 0$

$\Rightarrow e = 0$

$\therefore e = 0 \in G$ is identity element in G.

Let $a \in G$. If b is inverse of a, then

$a * b = 0$

$\Rightarrow a + b - ab = 0$

$\Rightarrow b(a - 1) = a$

$$\Rightarrow b = \frac{a}{a - 1} \in G.$$

Therefore $b = \dfrac{a}{a-1} \in G$ is inverse of a in G.

Let $a, b \in G$, then $a * b = a + b - ab$

$= b + a - ba = \boldsymbol{b * a}$

Therefore $*$ is a commutative in G.

Therefore (G, $*$) is an abelian group.

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Definition 3.

A group $(G, *)$ is said to be a nonabelian group, if $(G, *)$ is not abelian.

PROPERTIES OF A GROUP.

Theorem 1.

In a group, identity element is unique.

Theorem 2.

In a group, inverse of every element is unique.

Theorem 3.

If (G, .) be a group and $a \in G$, then $\left(a^{-1}\right)^{-1} = a$.

Theorem 4.

If (G, .) be a group and $a, b \in G$, then $(ab)^{-1} = b^{-1}a^{-1}$

Theorem 5.

If (G, .) be a group and $a_1, a_2, \ldots\ldots a_n \in G$, then $(a_1a_2 \ldots\ldots a_n)^{-1} = a_n^{-1}\, a_{n-1}^{-1} \ldots\ldots a_1^{-1}$

Theorem 6.

Cancellation laws hold in a group, i.e. If (G, .) is a group, then

I. $a, b, c \in G$, $ab = ac$
$\Rightarrow$ $b = c$

II. $a, b, c \in G$, $ba = ca$
$\Rightarrow$ $b = c$.

Definition 4.

If *(G, .) be a group and $a \in G$, $n \in Z$, then a^n is defined as follows :*

I. $a^0 = e$

II. If $n > 0$, then $a^1 = a; a^{n+1} = a^n \cdot a$

III. If $n < 0$, then $a^n = \left(a^{-n}\right)^{-1}$

Theorem 7.

Let (G, .) be a group and $a \in G$.

If $m, n \in Z$, then

I. $a^m \cdot a^n = a^{m+n} = a^n a^m$

II. $\left(a^m\right)^n = a^{mn}$

Definition 5.

Let (G, .) be a group. An element $a \in G$ is called idempotent if $a^2 = a$.

Definition 6.

A group (G, .) is called finite group if G is a finite set. The number of different elements in G is called order of the finite group (G, .). It is denoted by O(G).

Definition 7.

A group (G, .) is called infinite group if G is an infinite set. The order of an infinite group is defined to be ∞.

Example. Let G = {1, – 1, i, – i} where $i^2 = -1$.

Show that G is a finite abelian group under multiplications of complex numbers.

Solution:

The composition table for multiplication is as follows:

$\cdot$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

The multiplication of complex numbers is associative and commutative.

By observing the table it follows that 1 is the identity element in G and the inverses of $1, -1, i, -i$ are $1, -1, -i, i$ respectively.

Thus (G, .) is a *finite abelian group*.

Definition 8.

If S is a finite set containing n elements, then group of all bijections on S is called a permutation group or symmetric group. It is denoted by P_n or S_n.

Note : $o(S_n) = n!$

RESIDUE CLASSES

Definition 1.

Let n be a positive integer and $a, b \in Z$. Then a is called congruent to b modulo n if n divides $a - b$. It is denoted by $a \equiv b \pmod{n}$.

e.g. 5 divides $32 - 2 \Rightarrow 32 \equiv 2 \pmod{5}$

9 divides $-47 - 7 \Rightarrow 47 \equiv 7 \pmod{9}$

- The relation in the above definition is referred to as congruent modulo n.

Theorem 1.

The relation congruent modulo n is an equivalence relation on Z.

Definition 2.

The equivalence classes under the relation congruent modulo a positive integer n or Z are called residue classes modulo n.

- The residue class containing an integer a is denoted by $[a]$ or $\overline{a}$.
- The set of all residue classes modulo a positive integer n is denoted by Z_n.

Theorem 2 .

If n is a positive integer, then

$$Z_n = \left\{\overline{0}, \overline{1}, \overline{2}, \ldots, \overline{n-1}\right\}, \text{ and } o(Z_n) = n.$$

Theorem 3.

If $a \equiv b \pmod{n}$, $c \equiv d \pmod{n}$, then $a + c \equiv b + d \pmod{n}$, and $ac \equiv bd \pmod{n}$

Residue class Addition.

Definition 3.

The operation $\oplus$ defined on Z_n as, $\overline{a} \oplus \overline{b} = \overline{a+b}$, $\forall\ \overline{a}, \overline{b} \in Z_n$

is called *residue class addition* or addition modul n.

It is also denoted by $\oplus_n$ or $+_n$.

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- If $\overline{a}, \overline{b} \in Z_n$, then $\overline{a} \oplus \overline{b} = \overline{r}$, where r is remainder of $a + b$ when divided by n.
- The operation, addition modulo n is also defined in $\{0, 1, 2, ..., n - 1\}$ as $a \oplus b = r$, where r is remainder of $a + b$ when divided by n.

e.g. $\overline{5} \oplus_6 \overline{4} = \overline{9} = \overline{3}$; $4 \oplus_7 6 = 3$

Theorem 4.

$(Z_n, \oplus)$ is an abelian group.

Residue class Multiplication.

Definition 4.

The operation. defined on Z_n, as $\overline{a}\,\overline{b} = \overline{ab}$

$\forall\ \overline{a}, \overline{b} \in Z_n$ is called *residue class multiplication* or multiplication modulo n.

It is also denoted by $\cdot_n$ or $\cdot_n$ or $\otimes$ or $\otimes_n$ or $\times_n$.

- If $\overline{a}, \overline{b} \in Z_n$, then $\overline{a} \otimes \overline{b} = \overline{r}$

  where r is remainder of ab when divided by n.
- The operation, multiplication modulo n is also defined in $\{0, 1, 2, ..., n - 1\}$ as $a \otimes b = r$

  where r is remainder of ab when divided by n.

  e.g. $\overline{5} \cdot_6 \overline{4} = \overline{20} = \overline{2}$; $4 \cdot_7 6 = 3$

Theorem 5.

$(Z_n, \otimes)$ is a commutative semigroup with identity.

Theorem 6.

$(Z^*_n, .)$ is an abelian group iff n is prime.

PARTIAL ORDERING

A relation ~ on a set P is called partial order if it is

(*a*) reflexive $x \sim x,\ \forall\ x \in P$:

(*b*) antisymmetric : $(x \sim y) \wedge (y \sim x) \Rightarrow x = y$

(*c*) transitive : $(x \sim y) \wedge (y \sim z) \Rightarrow x \sim z$

A partial order is always denoted as $\leq$.

The set P with a partial order $\leq$ is called *partially ordered set*, denoted by $(P, \leq)$ and it is also called *poset*.

Totally ordered Poset.

A poset $(P, \leq)$ is called *totally ordered set*, if for all $x, y \in P$, $(x \leq y) \vee (y \leq x)$.

Cover of x.

Let $x \in (P, \leq)$. An element $\in (P, \leq)$ is called *cover of* x if $x \leq y$ and there exists no element $z \in P$ such that $x \leq z$ and $z \leq y$.

Hasse diagram.

It is a graph whose points are points of P, where points x and y are connected by an edge if y covers x. Any finite poset can be represented by a Hasse diagram.

e.g. Let P = {1, 2, 3, 4, 5, 6, 25, 100}.

Hasse graph is given in the figure.

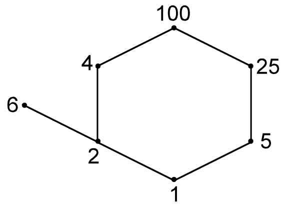


Fig. Poset

Least element.

y is least element of P if $y \leq x$ for all $x \in P$.

Greatest element.

y is greatest element of P if $x \leq y$ for all $x \in P$.

Upper bound.

Let $A \subseteq P$. An element $x \in P$ is called *upper bound of A* if $a \leq x$ for all $a \in A$.

Lower bound.

Let $A \subseteq P$. An element $y \in P$ is called *upper bound of A* if $y \leq a$ for all $a \in A$.

Least upper bound (LUB).

LUB of A is x is an upper bound and if y is any other upper bound, then $x \leq y$.

Greatest lower bound (GLB).

GLB of A is y if y is a lower bound for A and if b is any other lower bound, then $b \leq y$.

Sometimes GLB is called *infimum* and LUB is called *supremum.*

LATTICE

It is a partially ordered set $(P, \leq)$ in which any two elements $\in$ P has single GLB and single LUB.

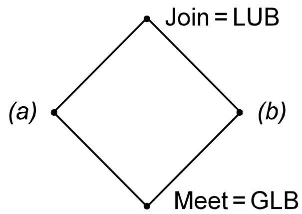
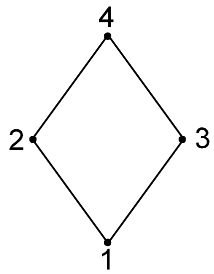


Fig. **(*a*)**

In the Fig.(*a*) GLB and LUB are shown, which are also called *meet and join* respectively.



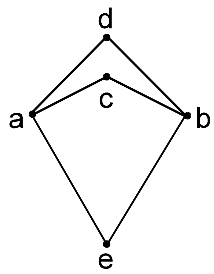


Fig.* (*b*) *Lattice Fig. **(*c*) *Non-Lattice***

The poset shown in Fig (*c*) is not a lattice since a and b have two LUBs namely c and d.

From first lattice, following relations are obtained :

$$1 \leq 2,\ 1 \leq 3,\ 2 \leq 4 \text{ and } 3 \leq 4.$$

Here 2 and 3 are not compared.

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If (P, ≤) is a lattice, then (P, ≥) is also a lattice, we define ≥ as follows

$$(x \le y) \Rightarrow y \ge x.$$

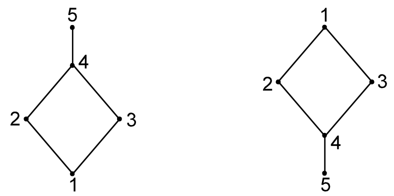
In other words,GLB and LUB are interchanged if we interchange relations ≤ and ≥.

In terms of lattices, operations of meet and join on (L, ≤) become the operations of join and meet on (L, ≤).

DUAL LATTICE

For the lattice (P, ≤) the dual is (P, ≥).

The duals are shown in the figure below. The diagram of (P, ≥) can be obtained from that of (P, ≤) by simply turning it upside down.

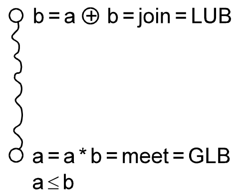


$1 \le 2 \le 4 \le 5$ $\quad 5 \ge 4 \ge 2 \ge 1$

$1 \le 3 \le 4 \le 5$ $\quad 5 \ge 4 \ge 3 \ge 1$

Let $a * b$ = meet of a and b = GLB, and $a \oplus b$ = join of a and b = LUB

Example. For $a, b \in a$ lattice (P, ≤), prove $a \le b; \Leftrightarrow a * b = a$



Solution:

Let $a \le b$.

Since $a \le a$, we have $a \le a * b$.

Since $a * b$ is lower bound for both a and b, we have

$$a * b \le a.$$

Hence $a = a * b$.

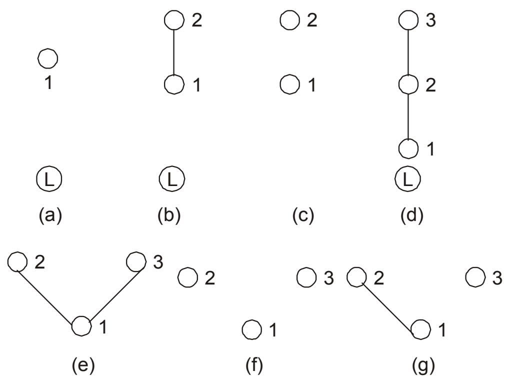
Example. Show that a lattice with less than 4 elements is a chain.

Solution:

Let (L, ≤) be a lattice.

Obviously if |L| = 1 or 2 it must be a chain.

Figure given below shows all possible combinations on three elements.



The posets shown on Figures (*c*), (*e*), (*f*) and (*g*) are not lattices as no two elements have GLB or LUB. Other posets are lattices and they are chains.

SUBLATTICE.

Let (L, *, ⊕) be a lattice and let $S \subseteq L$. The set (S, *, ⊕) is called *sublattice* iff it is closed under * and ⊕.

Sublattice is itself a lattice.

Example. Show that poset in Fig. (a) is a lattice and it is not a sublattice of the lattice in the Fig. (b).

Solution:

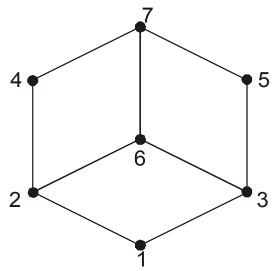


Fig. (*a*) (P_1, ≤)

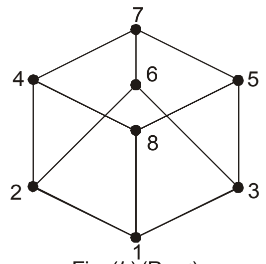


Fig. (*b*) (P_2, ≤)

In poset (P_1, ≤), any two elements have unique GLB and LUB.

Hence (P_1, ≤) is a lattice.

Since GLB (4, 5) in P_1 = 1 and LUB (4, 5) in $P_2 = 8 \notin P_1$.

Hence P_1 is not sublattice of P_2.

Closed interval.

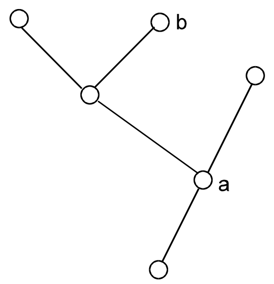


Fig. Interval [a,b]

Let $a \le b$.

Then closed interval of a and b is defined as

$$[a, b] = [x / a \le x \text{ and } x \le b\}.$$

Clearly any closed interval is a chain.

Complete lattice.

A lattice is complete if each of its non-empty subsets has a least upper bound and a greatest lower bound.

Every finite lattice is *complete*.

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BOUNDS

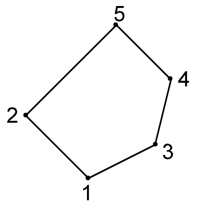
The least and greatest elements of lattice are called *bounds*, and denoted by 0 and 1.

Complement.

In a bounded lattice (L, *, ⊕, 0, 1), if $a * b = 0$ and $a \oplus b = 1$, then b is called *complement of a.*

If b is complement of a, then a is complement of b.

Consider lattice shown in the figure.



5 = (1) = greatest element

1 = (0) = least element

In that lattice, 2 has two complements 4 and 3.

0 and 1 are complement.

Complemented lattice.

A lattice (L, *, ⊕, 0, 1), is called *complemented* if every element a of L has atleast one complement b (*i.e.* $a * b = 0$ and $a \oplus b = 1$).

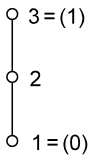


Fig. Lattice but not complemented

e.g. consider a chian of 3 elements show in the figure.

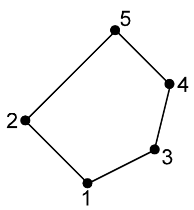
Here 2 * 3 = 2 ≠ (0) = least element

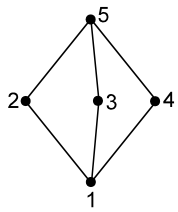
Distributive lattice.

A lattice is distributive if

$$a * (b \oplus c) = (a * b) * (a * c)$$

and $$a \oplus (b * c) = (a \oplus b) * (a \oplus c)$$





e.g.

- Every chain is a distributive lattice.
- Cancellation law can be stated as in a distributive lattice.

$$(a * b = a * c) \wedge (a \oplus b = a \oplus c) \Rightarrow b = c$$

Modular lattice.

A lattice is modular if $a \leq c$, then $a \oplus (b * c) = (a \oplus b) * c$

- We can show every distributive lattice is modular but the converse is not true.
- In a bounded distributive lattice, elements which have complements form a sublattice.

Sublattice.

Let <L, *, ⊕ > be a lattics and S ⊆ L.

Then <S, *, ⊕ >is a sublattics of <L, *, ⊕ > iff S is closed under operations * and ⊕.

Homomorphisn.

Let < L, *, ⊕ > and < S, ∧, ∨ > be two lattices.

A mapping $g : L \to S$ is called Homomorphisn if for any $a, b \in L$

$$g(a * b) = g(a) \wedge g(b)$$
$$g(a \oplus b) = g(a) \vee g(b)$$

BOOLEAN ALGEBRA

A complemented, distributive lattice is called *Boolean Algebra.*

Boolean algebra will generally be denoted by < B, *, ⊕, 0, 1> in which <B, *, ⊕ > is a lattice with two binary operations * and ⊕ and 0 and 1 are the least and greatest element of < B, ≤ > respectively.

Let < B, ∗, ⊕, 0, 1> be a Boolean algebra.

Following statements in which a, b and c denote any elements of the set B are true.

(1) $a * a = a$, and $a \oplus a = a$

(2) $a * b = b * a, a \oplus b = b \oplus a$

(3) $a * (b * c) = (a * b) * c$ and $a \oplus (b \oplus c) = (a \oplus b) \oplus c$

(4) $a * (a \oplus b) = a$ and $a \oplus (a * b) = a$

(5) $a * (b \oplus c) = (a * b) \oplus (a * c)$
and $a \oplus (b * c) = (a \oplus b) * (a \oplus c)$

(6) $(a * b) \oplus (b * c) \oplus (c * a) = (a \oplus b) * (b \oplus c) * (c \oplus a)$

(7) $(a * b = a * c) \wedge (a \oplus b = a \oplus c) \Rightarrow b = c$

If B is a bounded lattice, then

(8) $0 \leq a \leq 1$

(9) $a * 0 = 0, a \oplus 1 = 1$

(10) $a * 1 = a, a \oplus 0 = a$

If B is uniquely complemented, then

(11) $a * a' = 0, a \oplus a' = 1$

(12) $0' = 1, 1' = 0$

(13) $(a * b)' = a' \oplus b', (a \oplus b)' = a' * b'$

(14) $a ** b$ = GLB $\{a, b\}$, $a \oplus b$ = LUB $\{a, b\}$

(15) $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$

(16) $a \leq b \Leftrightarrow a * b' = 0 = b' \leq a' \Leftrightarrow a' \oplus b = 1$

Example. Prove following Boolean identities

(*i*) $a \oplus (a' * b) = a \oplus b$

(*ii*) $a * (a' \oplus b) = a * b$

(*iii*) $(a * b) \oplus (a * b') = a$

Solution:

(*i*) $a \oplus (a' * b) = (a \oplus a') * (a \oplus b) = 1 * (a \oplus b) = a \oplus b$

(*ii*) $a * (a' + b) = (a * a') \oplus (a * b)$
$= 0 \oplus (a * b)$
$= a * b$

(*iii*) LHS $= a * (b \oplus b') = a * 1 = a$

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SOLVED EXAMPLES

1. Let A = {– 2, – 1, 0, 1, 2}, B = {0, 1, 4}, and f : A → B is defined as $f(x) = x^2$ is a function. Find whether it is one-one or bijection ?

Solution: $f(x) = x^2$

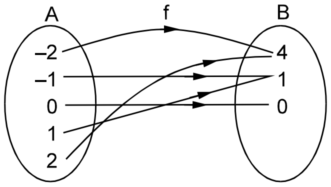
$f(-2) = 4, \quad f(-1) = 1$

$f(0) = 0, \quad f(1) = 1, \quad f(2) = 4.$

$f : A \to B$ is a function,

and $f : A \to B$ is onto (since $f(A) = B$).

$f(-1) = f(1) = 1$



hence $f : A \to B$ is not one-one, since $f: A \to B$ is not a bijection.

2. If A = {p, q, r}, B = {2, 1, 0} and f : A → B, find whether it is one-one, onto or bijection ?

(*i*) f = {(p, 2), (q, 0), (r, 2)}

(*ii*) f = {(p, 1), (q, 0), (r, 2)}

Solution:

(*i*) $(p, 2), (r, 2) \in f$

$\Rightarrow$ p and r have the same image

$\therefore$ $f : A \to B$ is not one-one.

$f(A) = \{0, 2\} \neq B$. Therefore not onto.

$\therefore$ $f : A \to B$ is not a bijection.

(*ii*) $f = \{(p, 1), (q, 0), (r, 2)\}$

Different elements in A have different f – images in B and hence f is one-one.

$f(A) = \{1, 0, 2\} = B,$

$\therefore$ $f: A \to B$ is onto

$\therefore$ $f : A \to B$ is a bijection

3. Find whether function f : [0, ∞) → R defined as $f(x) = x^2$ is one-one, onto or bijection.

Solution: $f(x) = x^2$

$f : [0, \infty) \to R$ is a function.

Let $a_1, a_2 \in [0, \infty)$ and $f(a_1) = f(a_2)$

Now $f(a_1) = f(a_2)$

$\Rightarrow$ $a_1^2 = a_2^2$

$\Rightarrow$ $a_1 = a_2$

$\therefore$ $[0, \infty) \to R$ is one-one.

There is no pre-image in $[0, \infty]$ to (-1)

$\therefore$ f is not onto.

$\therefore$ $f : [0, \infty) \to R$ is not a bijection.

4. Find whether each of the following functions is one-one or onto or one-one and onto.

(*i*) $f : R \to R$ is defined as $f(x) = \begin{cases} x, & x > 2 \\ 5x - 2 & x \leq 2 \end{cases}$

(*ii*) $f(x) = 2x - 3$

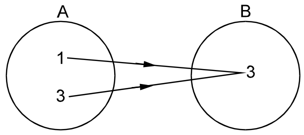
(*iii*) $f(x) = x^2$

Solution:

(*i*) $3 > 2$, then $f(3) = 3$

$1 < 2$, then $f(10) = 5\,(1) - 2 = 3$

Thus 1 and 3 have same image.Hence it is not one-one.



Let $y \in R$

If $y > 2$, take $x = y$

then $f(x) = f(y) = y.$

If $y \leq 2$, take $x = \dfrac{y+2}{5}$

Then $x \leq \dfrac{4}{5}$

$\therefore$ $f(x) = 5x - 2 = 5\left(\dfrac{y+2}{5}\right) - 2 = y.$

Thus f is onto since f is not one-one. f is not a bijection.

(*ii*) If $f : R \to R$, then $f(x) = 2x - 3 \in R$

$a_1, a_2 \in R$ $\quad f(a_1) = f(a_2)$

$\therefore$ $2a_1 - 3 = 3a_2 - 3$

$\Rightarrow$ $a_1 = a_2$

Hence f is one-one.

If $y \in R$, then $x = \dfrac{y+3}{2}$

$f(x) = 2x - 3 = 2\left(\dfrac{y+3}{2}\right) - 3 = y$

Thus f is onto. Hence f is a bijection.

(*iii*) $f(x) = x^2, \quad \forall\, x, y \in R$

$f(-1) = (-1)^2 = 1$

and $f(1) = 1^2 = 1$

Hence 1 and – 1 have same image, hence f is not one-one.

– 1 has no pre-image, hence f is not onto.

Thus f is not a bijection.

5. Find inverse functions of the following of $f(x) = 4x + 7$

Solution: $y = f(x) = 4x + 7$

$\Rightarrow$ $x = \dfrac{y-7}{4}$

$\therefore$ Inverse function $f^{-1}(y) = \dfrac{y-7}{4}$, and inverse function $f^{-1}(x) = \dfrac{x-7}{4}$

6. Find inverse function of $f(x) = \dfrac{2x+1}{3}$

Solution:

Let $y = f(x) = \dfrac{2x+1}{3}$

$\therefore$ $f^{-1}(y) = x = \dfrac{3y-1}{2}$, and $f^{-1}(x) = \dfrac{3x-1}{2}$

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GRAPH THEORY

GRAPH

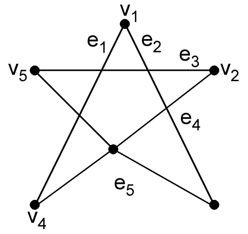
Graph is a drawing or a diagram consisting of a collection of vertices together with edges joining certain pairs of these vertices.

Mathematically, A graph G = [V(G), E(G)]

where V(G) and E(G) are two finite sets defined as

V(G) = vertex set of graph G

E(G) = edge set of graph G such that each element e of E(G) is assigned an unordered pair of vertices (u, v) called *end vertices of e*.

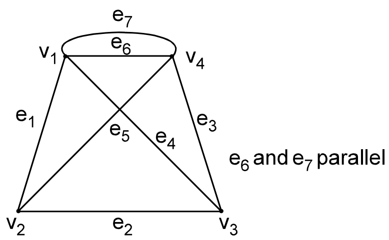


Vertices are also some times called *points or nodes.*

If e an edge with end vertices u and v, then e is said to join u and v. It is also possible to have a vertex u joined to itself by an edge, such an edge is called a *loop.*

Parallel edges.

If two (or more) edges of a graph G have the same end vertices, then these edges are called *parallel*. e.g.



Isolated vertex.

A vertex of graph G, which is not the end of any edge is called *isolated.*

Neighbours or adjacent vertex.

Two vertices which are joined by an edge are called *'adjacent'* or *'neighbours'*. The set of all such neighbours of vertex V is called *neighbourhood set of V* and is denoted by N(v).

Incidence.

An edge e of a graph G is called *incident* with the vertex v if v is an end vertex of e (or v is incident with e). Two edges e and f which are incident with a common vertex v are called *adjacent*. An edge could be incident with either 1 or 2 vertices (1 in case of loop), whereas a vertex could be incident on any finite number of edges.

MULTIGRAPH

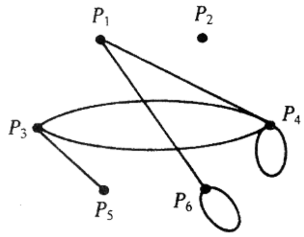
A multigraph G = G(V, E) also consists of a set V of vertices and a set E of edges except that E may contain *multiple* edges, i.e., edges connecting the same endpoints, and E may contain one or more *loops* i.e., an edge whose endpoints are the same vertex.

Diagram of a Multigraph.

Graphs (multigraphs) G = G(V, E) are pictured by diagrams in the plane as follows.

Each vertex v in V is represented by a dot (or small circle) and each edge $e = \{u, v\}$ is represented by a curve which connects its *endpoints* u and v. (we usually denote a graph, when possible, by drawing its diagram rather than explicitly listing its vertices and edges).

Example. Describe formally the multigraph shown below.



Solution.

There are six vertices, so

$$V = \{P_1, P_2, P_3, P_4, P_5, P_6\}$$

There are seven edges (of which two are multiple edges and two are loops) and thus seven pairs of vertices; hence

$E = [\{P_1, P_4\}, \{P_1, P_6\}, \{P_3, P_4\}, \{P_3, P_4\}, \{P_4, P_4\}, \{P_3, P_5\}, \{P_6, P_6\}]$

Finite multigraph.

A multigraph G = G(V, E) is *finite* if both V is finite and E is finite.

Note that a graph G with a finite number of vertices V must automatically have a finite number of edges and so must be finite.

DEGREE OF A VERTEX

Relation of adjacency and incidence in a graph G.

Let $e = \{u, v\}$ is an edge in G, i.e. u and v are endpoints of e. Then vertex u is said to be adjacent to the vertex v, and edge e is said to be incident on u and on v.

Degree and parity (even or odd) of a vertex.

Degree of a vertex v in a graph G, written deg (v), is equal to number of edges which are incident on v or, in other words, the number of edges which contain v as an endpoint. The vertex v is said to be even or odd according as deg (v) is even or odd.

Theorem : Sum of the degrees of the vertices of a graph is equal to twice the number of edges.

It follows directly from the fact that, each edge is counted twice in counting degrees of the vertices of a graph G.

First theorem of graph theory.

For any graph G with e edges and n vertices $v_1 \ldots\ldots v_n$,

$$\sum_{i=1}^{n} d(v_i) = 2e$$

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Odd or Even vertex.

Vertex of a graph is called *odd or even* depending on whether its degree is even or odd.

PATHS, CONNECTIVITY

Path and its length in a graph (multigraph) G.

A *path* α in G with origin v_0 and end v_n is an alternating sequence of vertices and edges of the form

$$v_0, e_1, v_1, e_2, v_2, \ldots, e_{n-1}, v_{n-1}, e_n, v_n$$

where each edge e_i is incident on vertices v_{i-1} and v_i.

The number of edges, n is called *length* of α. When there is no ambiguity, we denote α by its sequence of edges, $\alpha = (e_1, e_2, \ldots, e_n)$, or by its sequence of vertices $\alpha = (v_0, v_1, \ldots, v_n)$.

Simple path and trail in a graph (multigraph) G.

A path $\alpha = (v_0, v_1, \ldots, v_n)$ is simple if all the vertices are distinct. The path is a *trail* if all the edges are distinct.

Closed path and a cycle in a graph (multigraph) G.

A path $\alpha = (v_0, v_1, \ldots, v_n)$ is *closed* if $v_0 = v_n$, i.e. if origin (α) = end (α). The path α is a *cycle* if it is closed and if all vertices are distinct except $v_0 = v_n$. A cycle of length k is called k-cycle. A cycle in a graph must therefore have length three or more.

SPECIAL GRAPHS

Types of graphs.

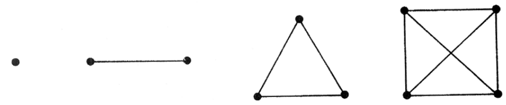
(Here the term graph does not include multigraphs.)

(1) Complete graph.

A graph G is *complete* if each vertex is connected to every other vertex. The complete graph with n vertices is denoted by K_n.

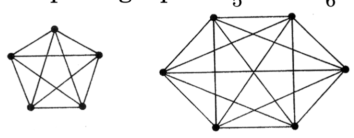
Complete graphs K_1, K_2, K_3, and K_4.

First draw the appropriate number n of vertices. Then draw an edge from each vertex to every other vertex. The required diagrams are given below.



K_1 K_2 K_3 K_4

e.g. complete graphs K_5 and K_6.



K_5 K_6

Number of edges, m in complete graph K_n.

Each pair of vertices determines an edge. Thus

$m = C(n, 2) = \dfrac{n(n-1)}{2}$ since there are $C(n, 2)$ ways of selecting two vertices out of n vertices.

Example. Find the number of edges, m in the graphs (*i*) K_8, (*ii*) K_{12}, and (*iii*) K_{15}.

Solution.

(*i*) $m = \dfrac{8 \cdot 7}{2} = 28$, (*ii*) $m = \dfrac{12 \cdot 11}{2} = 66$

(*iii*) $m = \dfrac{15 \cdot 14}{2} = 105$.

(2) Regular graph.

A graph G is *regular of degree* or *k-regular* if every vertex has degree k. In other words, a *graph is regular if every vertex has the same degree.*

Connected regular graphs of degrees 0, 1 and 2:

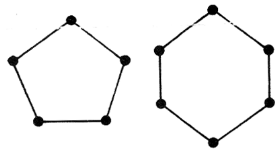
The connected 0-regular graph is the *trivial graph* with one vertex and no edges.

The connected 1-regular graph is the graph with two vertices and one edge connecting them.

The connected 2-regular graph with n vertices is the graph which consists of a single n-cycle.

Figure below shows the connected 0-regular and 1-regular graphs and some of the connected 2-regular graphs.

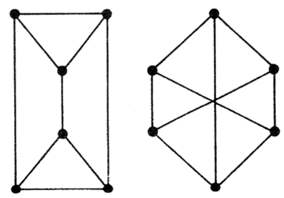
(a) 0-regular ***(b) 1-regular***



(c) 2-regular

Draw two-3 regular graphs with six vertices.

e.g.



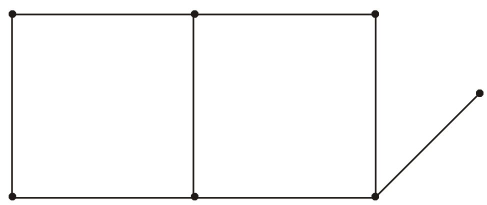
(3) Empty or Trivial graph.

A graph with no edges.

(4) Bipartite graphs.

If vertex set V of a graph G can be partitioned into two non-empty disjoint subsets X and Y in such a way that each edge of G has one end in X and one end in Y, then G is called *bipartite.*

The partition $V = X \cup Y$ is called a *bipartition of G.*



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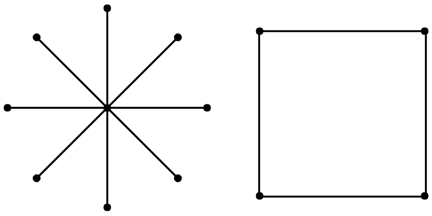
Complete bipartite graph :

If every vertex in X is joined to every vertex of Y, then it is called *complete bipartite graph* denoted by $k_{m,\,n}$, where m and n are vertices in X and Y respectively.

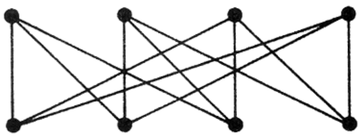
Any complete bipartite graph with a bipartition into two sets of m and n vertices is isomorphic to $k_{m,\,n}$.



Diameter of any complete bipartite graph :

The diameter of $K_{1,\,1}$ will be one since there are only two vertices and the shortest path between them is length one. All other bipartite graphs will have diameter two since any two points in either M or N will be exactly distance 2 apart. (One edge to reach the other subgroup of vertices and one to return.)

Example. Which connected graphs can be both regular and bipartite?

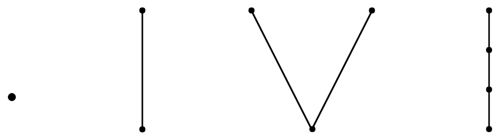


Solution.

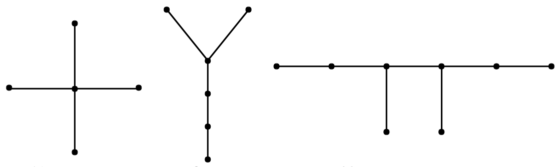
The bipartite graph $K_{m,m}$ is regular of degree m since each vertex is connected to m other vertices and hence its degree is m. Subgraphs of $K_{m,m}$ can also be regular if m disjoint edges are deleted. For example, the subgraph of $K_{4,4}$ shown above is 3-regular. We can continue to delete m disjoint edges and each time obtain a regular graph of one less degree. These graphs may be disconnected, but in any case their connected components have the desired properties.

(5) Tree graph.

A graph is called a **cycle** if it contains no cycles. Since loops are cycles of length one while a pair of parallel edges produces a cycle of length two, any acyclic graph must be simple. A graph G is called a tree if it is a connected acyclic graph. As the number of vertices increases, the drawing of tree become complicated.



1 vertex ***2 vertex*** ***3 vertex*** ***4 vertex***



5 vertex ***6 vertex*** ***7 vertex***

Fig. Trees with different vertices

Forest.

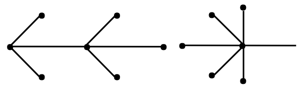


Fig. : Forest

Let G be an acyclic graph. Then any sub graph of G must also contains no cycles. The connected components of G are also acyclic and so they are trees, and therefore an acyclic graph is also called a *Forest.*

Theorem 1.

- Let u and v be distinct vertices of a Tree T, then there is precisely one path from u to v.
- Let G be a graph without any loops. If for every pair of distinct vertices u and v of G there is precisely one path from u to v, then G is a tree.

Theorem 2.

Let T be a tree with at least two vertices and let P $= u_0, u_1 \ldots\ldots u_n$ be a longest path in T. Then both u_0 and u_n have degree 1.

Corollary : Any tree T with atleast two vertices has more than one vertex of degree 1.

Theorem 3.

If T is a tree with n vertices, then it has precisely $(n-1)$ edges.

Theorem 4.

If G be an acyclic graph with n vertices and k connected components, i.e. $w(G) = k$, then G has n-k edges.

SUBGRAPHS

If G and H are two graphs will vertex sets V(H), V(G) and edge sets E(H) and E(G) respectively such that V(H) $\leq$ V(G) and E(H) $\leq$ E(G), then we call H as a subgraph of G (or G as a supergraph of H).

If V(H) $\subset$ V(G) and E(H) $\subset$ E(G), then A is a proper subgraph of G and if V(H) = V(G), then H is a spanning subgraph of G.

Vertex deleted subgraphs.

If from a graph G with vertex set V, a subset U of V is deleted and all the edges which have a vertex in U as an end point, then (G-U) is called *vertex deleted subgraph.*

Edge deleted subraphs.

If a subset F of E is deleted from G, then (G-F) denotes the subgraph of G with vertex set V and edge set E-F, then (G-F) is called *edge deleted subgraph.*

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Underlying simple graph.

By deleting from a graph G all loops and in each collection of parallel edges all edges but one a simple spanning subgraph of G is obtained, called *underlying simple graph of G.*

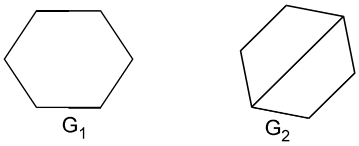


Fig. G_1 is a proper spanning subgraph of G_2.

If G is a graph with vertex set V, and U is a subset of V, then subgraph G(U) of G induced by U is defined to be the graph having vertex set U and edge set consisting of those edges of G that have both ends in U. Similarly if F is a non-empty subset of the edge set E of G, then subgraph G(F) of G induced by F is the graph whose vertex set of ends of edges in F and whose edge set is F.

PATHS AND WALKS

A walk in a graph G is a finite sequence

$$W = v_0e_1, v_1e_2, ... v_{k-1}e_kv_k$$

whose terms are alternately vertices and edges such that, for $1 \le i \le k$, the edge has ends v_{i-1} and v_i.

The vertex V_0 is called *origin* and V_k as the terminus of walk W. They need not be distinct.

Vertics v_1 ..... v_{k-1} are called *internal vertices.*

The number of edges in W are called *length of the walk*. Vertices and edges can be repeated in a walk. A trivial walk is one containing no edges and has length zero. A walk is closed or open depending on whether origin and terminus are same or different.

If edges $e_1, e_2, ... e_k$ of walk W are distinct, then such walk is called *trail*. So a trail is a walk if no edge is repeated.

If vertices $v_0, v_1, ... v_k$ of the walk $W = v_0e_1, v_1e_2, ... e_kv_k$ are distinct, then W is called *a path*. A path is a walk if no vertex is repeated. So every path is a trail and not every trail is a path.

CONNECTED GRAPHS AND CYCLES

Concatenation.

A vertex u is said to be connected to a vertex v in a graph G if there is a path in G from u to v. The converse is also true as v is also connected with u by the reverse path. If u is connected with v and v is connected with w, then u is connected with w.

This process of joining two walks which have a common end point to form a longer walk is called *concatenation (stringing together).*

Connected graphs.

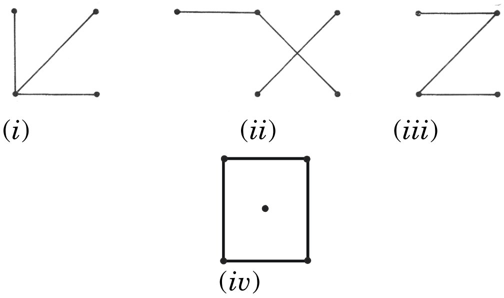
A graph G is called **connected** if every two of its vertices are connected.

Given any vertex u of a graph G, let C(u) denote the set of all vertices in G that are connected to u. Then subgraph of G induced by u is called *connected component containing u.*

Diameter of a connected graph G.

The diameter of G, written diam (G), is the maximum distance between any two of its vertices.

Example. Determine whether or not each of the graphs given below is connected.



Solution.

(*i*) Yes. There is a path between any two vertices of the graph.

(*ii*) No. Here A, B, and Y are connected, and C and X are connected, but there is no path from A, B, to either C or X.

(*iii*) Yes. There is a path between any two vertices of the graph.

(*iv*) No. There is no path from C to any other vertex of the graph.

Cycle.

A non trivial closed trail in a graph a is called a **cycle**, if its origin and internal vertices are distinct.

The closed trail $C = v_1, v_2, ... v_n, v_1$ is cycle, if C has atleast 1 edge and $v_1, v_2, ... v_n$ are n distinct vertices.

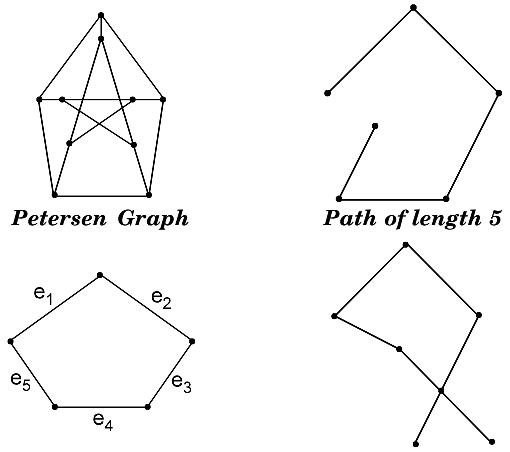
A cycle with K edges is called **K-cycle.** A K-cycle is called **odd or even** depending on whether k is odd or even. Any two cycles with same length are isomorphic. An cycle is denoted by C_n.

Some cycles are defined as

1 cycle → Loop

2 cycle → Pair of parallel edges

3 cycle → triangle



Petersen Graph ***Path of length 5***

Cycle of length 5 ***Trail of length 5***

Fig. Path, Cycle and Trails

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OPERATIONS ON GRAPHS

(1) Union.

Given two graphs G_1 and G_2. Their union will be a graph such that

$$V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$$

and $$E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$$

(2) Intersection.

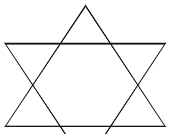
When two graphs G_1 and G_2 with at least one vertex in common, then their intersection will be a graph such that

$$V(G_1 \cap G_2) = V(G_1) \cap V(G_2)$$
$$E(G_1 \cap G_2) = E(G_1) \cap E(G_2)$$

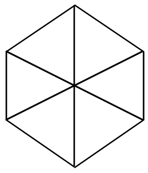
(3) Complement.

Complement $\overline{G}$ of G is defined as a simple graph with the same vertex set as G and where two vertices u and v are adjacent only when they are not adjacent in G.

e.g.



G



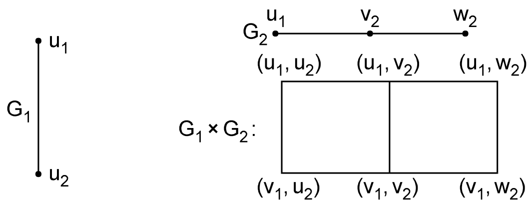
Complement of G

(4) Product of graphs.

To define product $G_1 \times G_2$ of two graphs, consider any two points $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V = V_1 \times V_2$.

Then u and v are adjacent in $G_1 \times G_2$ whenever $[u_1 = v_1$ and u_2 adj $v_2]$ or $[u_2 = v_2$ and u_1 adj $v_1]$.

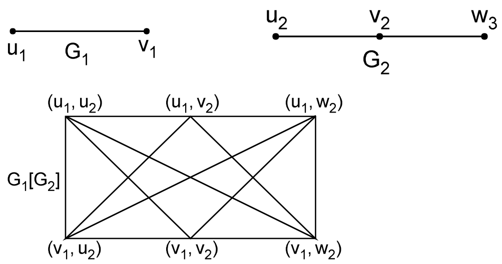
e.g.



(5) Composition.

The composition $G = G_1[G_2]$ also has $V = V_1 \neq V_2$ as its point set and $u = (u_1, u_2)$ is adjacent with $v = (v_1, v_2)$ whenever $[u_1$ adj $v_1]$ or $[u_1 = v_1$ and u_2 adj $v_2]$.

e.g.



MATRIX REPRESENTATION OF GRAPHS

A graph can be represented inside a computer by using adjacency matrix or incidence matrix of a graph.

Let G be a graph with n vertices v_1 .... v_n. The adjacency matrix of G with respect to this particular listing of n vertices is the $n \times n$ matrix

$$A(G) = (a_{ij}),$$

where, a_{ij} = number of edges joining the vertex v_i to v_j.

If G has no loops, then all entries of the main diagonal will be 0 and if G has no parallel edges, then entries of A(G) are either 0 or 1.

Given a $n \times n$ symmetric matrix $A = (a_{ij})$, in which all the entries are non-negative integers, one can associate with it a graph G whose adjacency matrix is A by letting G have n vertices and joining vertex i to vertex j by a_{ij} edges.

Theorem 1.

Let G be a graph with n vertices v_i .... v_n and let A denote adjacency matrix of G with respect to this listing of the vertices. Let k be any positive integer and let A^k denote matrix multiplication of A, k times. Then $(ij)^{\text{th}}$ element of A^k is the number of different $v_i - v_j$ walks in G of length K.

This result can be used to check whether a graph is connected or not.

Theorem 2.

Let G be a graph with n vertices v_1, ... v_n and let A denote adjacency matrix of G with respect to this listing of the vertices.

Let $B = (b_{ij})$ be the matrix.

$\therefore$ $B = A + A^2 + ... + A^{n-1}$

Then G is a connected graph if for every pair of distinct indices i and j, we have $b_{ij} \neq 0$ i.e., iff B has no zero entries of the main diagonal.

This result can also be used to check the connect-edness of a graph by using its adjacency matrix.

Example. A graph G has the following adjacency matrix. Check whether it is connected.

$$A(G) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Solution.

Since here, $n = 5$, therefore $B = A + A^2 + A^3 + A^4$

We find, A^2, A^3 and A^4 by matrix multiplication

$$A^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix},$$

$$A^3 = \begin{bmatrix} 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 2 & 1 & 0 & 0 & 3 \\ 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 3 & 3 & 0 \end{bmatrix}, A^4 = \begin{bmatrix} 2 & 1 & 0 & 0 & 3 \\ 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 5 & 4 & 0 \\ 0 & 0 & 4 & 5 & 0 \\ 3 & 3 & 0 & 0 & 6 \end{bmatrix}$$

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Now $B = A + A^2 + A^3 + A^4 = \begin{bmatrix} 3 & 1 & 3 & 1 & 4 \\ 1 & 3 & 1 & 3 & 4 \\ 3 & 1 & 7 & 5 & 4 \\ 1 & 3 & 5 & 7 & 4 \\ 4 & 4 & 4 & 4 & 8 \end{bmatrix}$

Since B has no non zero entry off the main diagonal. The graph is connected.

Incidence matrix.

Another matrix associated with graph G is the incidence matrix.

Let G has n vertices listed as v_1 ...... v_n and t edges listed as e_1 ..... e_i.

The incidence matrix of a is the $n \times t$ matrix

$$M(G) = (m_{ij}),$$

where m_{ij} is number of times the vertex V_i is incident with the edge e_j,

i.e.

$$m_{ij} \begin{cases} 0 \text{ if } v_i \text{ is not an end of } e_j \\ 1 \text{ if } v_i \text{ is an end of non loop } e_j \\ 2 \text{ if } v_i \text{ is an end of the loop } e_j \end{cases}$$

Sum of elements in the i^{th} row of MG gives degree of the vertex v_i.

BRIDGE OR CUT EDGE

On removing an edge from a graph, the number of connected components of the graph either remains unchanged or it increases by exactly 1. So, an edge e of a graph G is called *bridge or cut edge*, if the subgraph G-e has more connected components than G has.

Theorem 1.

Let e be an edge of the graph G and, as usual let G–e be the subgraph obtained by deleting e.

Then $\quad \omega(G) \le \omega(G–e) \le \omega(G) + 1$

where $\omega(a)$ is number of connected components of G.

Theorem 2.

An edge e of a graph G is a bridge if e is not a part of any cycle in G.

Theorem 3.

Let G be a graph with n vertices, and q edges and, let ω(G) denote number of connected components of G. Then G has at least n-ω(G) edges, i.e. $q \ge n$-ω(G).

Corollary : A graph with n vertices less than $(n - 1)$ edges can not be connected.

Theorem 4.

Let G be a graph with n vertices, then following three statements are equivalent.

I. G is tree

II. G is an acyclic graph witn $(n - 1)$ edges.

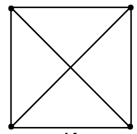
III. G is a connected graph witn $(n - 1)$ edges.

SPANNING TREES AND CONNECTOR PROBLEMS

Let G be a graph. A subgraph H of G is called a **spanning subgraph** of G if vertex set of H is same as the vertex set of G. Similarly, a spanning tree of a graph G is a spanning subgraph of G, that is a tree.

Theorem.

A graph G is connected if it has a spanning tree.



K_4

Cayley's theorem.

The complete graph K_n has n^{n-2} different spanning trees.

Spanning trees of K_4.

Since, here, $n = 4$, so, there will be $4^{4-2} = 16$ spanning trees.

A weighted graph is a graph G in which each edge e has been assigned a real number $\omega(e)$, called *weight of e*.

If H is a subgraph of a weighted graph, weight ω(H) of H is sum of the weights $\omega(e_1)$ + ... + $\omega(e_k)$, where $[e_1, ..., e_k]$ is set of edges of H.

Since many optimisation problems could be solved by using weighted graphs, hence try to find a subgraph of the weighted graph, with minimum (or maximum) weight.

Now look at a connected spanning subgraph of G which should not have a cycle, i.e. look at a spanning tree of G and finally look for a spanning tree with minimum weights, such a tree is called *minimal spanning tree or an optimal tree for G.*

Thus there are two algorithms for finding such a tree. A necessary condition for these algorithms is that no weights should be negative.

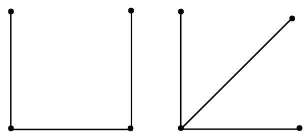


Fig. spanning trees for k_4

Kruskal's Algorithm.

In this algorithm, choose an edge of G which has smallest weight among the edges of G which are not loops. This algorithm gives an acyclic subgraph T of G and theorem given below proves that T is a minimal spanning tree of G.

Following steps are required.

Step 1 : Choose e_1, an edge of G, such that weight of e_1, $\omega(e_1)$ is as small as possible and e_1 is not a loop.

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Step 2 : If edges $e_1, e_2, ... e_i$ have been chosen, then choose an edge e_{i+1} not already chosen, such that

(*i*) induced subgraph $G[(e_1,...... e_{i+1})]$ is acyclic, and

(*ii*) $\omega(e_{i+1})$ is as small as possible.

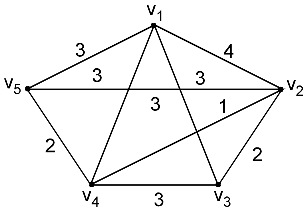
Step 3 : If G has n vertices, stop after n-1 edges have been chosen. Otherwise repeat step 2.

Theorem.

Let G be a weighted connected graph in which weights of the edges are all non-negative numbers.

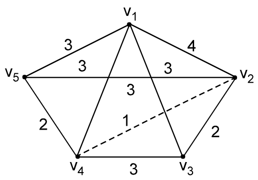
Let T be a subgraph of G obtained by Kruskal's algorithm. Then T is a minimal spanning tree of G.

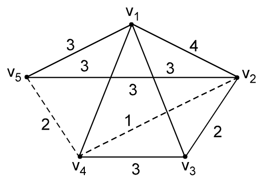
Example. Given below is a weighted graph G with 5 vertices. Apply Kruskal's Algorithm to determine a minimal spanning tree.



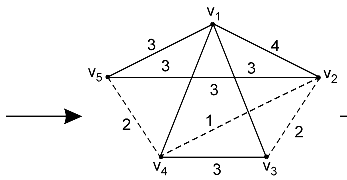
Solution.

According to step 1, choose, $e = v_2v_4$ as it has minimum weight.

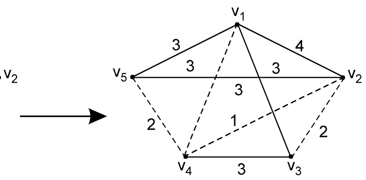




Choose $e = v_2\ v_3$

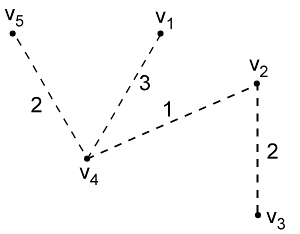


Choose $e = v_4\ v_5$



Choose $e = v_1\ v_4$

Since vertices are 5 and we have choosen 4 edges stop, minimal spanning tree of graph is



Greedy Algorithms.

Kruskal's algorithm is an example of a type of algorithm known as **Greedy**. Greedy algorithms are essentially algorithms that proceed by selecting the choice that looks best at the moment. All processes cannot be handled so simply.

Prim's Algorithm.

Another algorithm used for finding a minimal spanning tree is *Prim's algorithm.* It chooses a vertex first and chooses and edge with smallest weight incident on that vertex.

The algorithm involves following steps.

Step 1 : Choose any vertex v_1 of G.

Step 2 : Choose an edge $e_1 = v_1v_2$ of G such that $v_2 \neq v_1$ and e_1 has smallest weight among the edges of G incident with v_1.

Step 3 : If edges $e_1, e_2, ..., e_i$ have been chosen involving end points $v_1, v_2, ... v_{i+1}$.

Choose an edge $e_{i+1} = v_jv_k$ with $v_j \in \{v_i \ ...... v_{i+1}\}$ and $v_k \notin [v_1, ... v_{i+1}]$ such that e_{i+1} has smallest weight among the edges of G with precisely one end in $[v_1, ... v_{i+1}]$.

Step 4 : Stop after $n - 1$ edges have been chosen. Otherwise go to step 3.

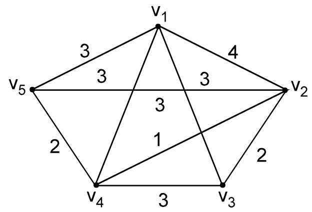
Theorem.

Let G be a weighted connected graph in which weights of the edges are all non-negative numbers.

Let T be a subgraph of G obtained by Prim's algorithm. Then T is a minimal spanning tree of G.

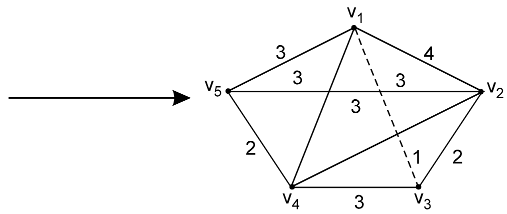
Example. Find minimal spanning tree of the weighted graph in previous example using Prim's algorithm.

Solution.



According to step 1, we choose vertex v_1.

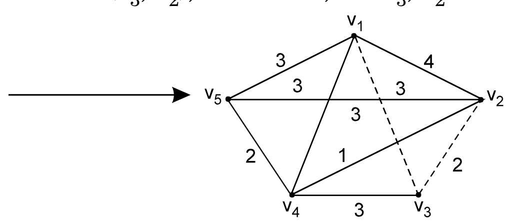
Now edge with smallest weight incident on v_1 is $e = v_1, v_3$ so we choose $e = v_1, v_3$ (or $e = v_1, v_5$).



Now, we look on to the weights,

$w(v_1, v_2) = 4,\ \ w(v_1, v_4) = 3,\ \ \ w(v_1, v_5) = 3,\ \ w(v_3, v_2) = 2,\ \ w(v_3, v_4) = 3$

Since minimum is $w(v_3, v_2)$, we choose, $e = v_3, v_2$



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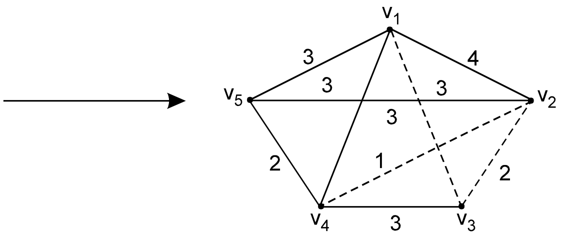
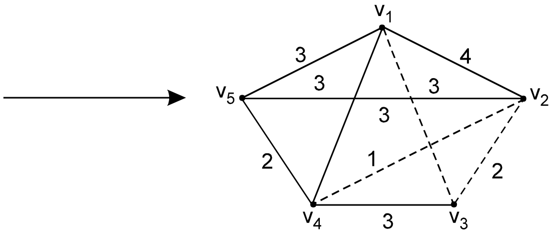
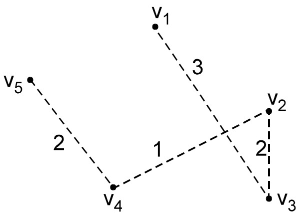
Again, $w(v_2, v_5) = 3$, $w(v_2, v_4) = 1$ and $w(v_3, v_4) = 3$.
We choose $e = v_2, v_4$



Now, $w(v_4, v_5) = 2$. We choose $e = v_4, v_5$



The minimal spanning tree is



CUT VERTICES AND CONNECTIVITY

Cut vertex is analogue of a bridge. A vertex v of a graph is called a *cut vertex of G* if

$$\omega(G - v) > \omega(G),$$

where $\omega(G)$ are number of components in graph G, i.e. a cut vertex breaks a graph into a subgraph having more connected components, then G has

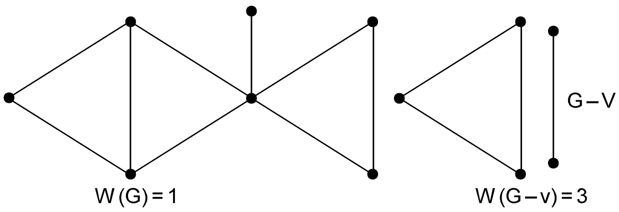


Fig. Graph after deleting cut vertex v

Theorem.

Let G be a graph with n vertices, when $n \geq 2$. Then G has at least two vertices which are not cut vertices.

If a connected graph G has a cut vertex v, then connectedness of G is vulnerable at v. Let G be a simple graph. The connectivity of G, denoted by $k(G)$, is the smallest number of vertices in G whose deletion from G leaves either a disconnected graph or k_1.

$$k(k_n) = n - 1.$$

Connected graph has

$k(G) = 1$, iff either $G = k_2$ or G has a cut vertex.

$k(G) = 0$, iff either $G = k_1$ or G is disconnected.

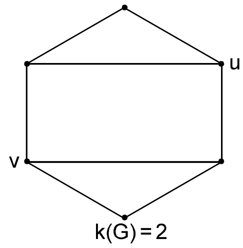


Fig. Connectivity of graph (by deleting u and v)

A simple graph G is called *n-connected* ($n \geq 1$) if

$$k(G) \geq n.$$

Let u and v be two vertices of a graph G. A collection $\{p_{(1)} ..... p_{(n)}\}$ of u-v paths is said to be *internally disjoint* if, given any distinct pair $P_{(i)}$ and $P_{(j)}$ in the collection, u and v are the only vertices $P_{(i)}$ and $P_{(j)}$ have in common.

BINARY TREES

Trees are principally used in the data representation. Here, all trees will be rooted, *i.e.* they will have a distinguished vertex r (called **root**) with the property that for every vertex $v \in V$, there is a directed r-v graph.

The trees will also be levelled, i.e. root r will constitute level 0, neighbours of r will constitute level 1 and neighbours of vertices in level 1 will have level 2 and so on. The neighbours on next level are called *children (descendent)* and previous level are called *Father (predecessor).*

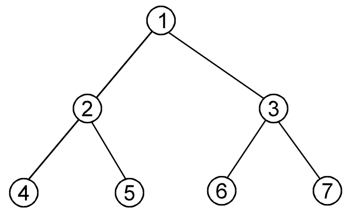
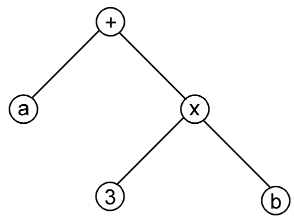


Fig. Full binary tree

A binary tree is a rooted, levelled tree in which any vertex has at most two children. If every vertex of binary tree has either two children or no children, then we say the tree is a *full binary tree*.

Use of binary tree in data representation

We have to express $a + 3b = a + 3 \times b$, then we will draw following tree :



EULER TOURS.

A trail is defined as Euler trail if it includes every edge of G. So, a trail is Euler if each edge G is in the trail exactly once.

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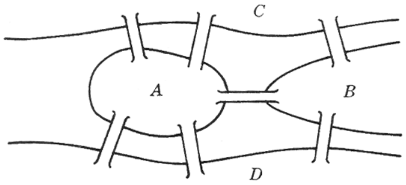
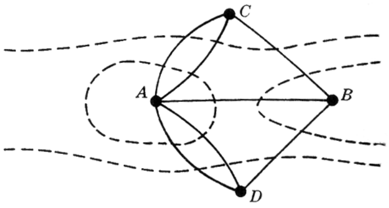
A tour of G is a closed walk of G which includes every edge of G at least once. An Euler tour of G is a tour which includes each edge of G exactly once. So, an Euler tour is just a closed Euler trail. A graph G is called *Eulerian or Euler* if it has as Euler tour.

Konisberg Bridge Problem



(*a*) *Konigsberg in 1736*



(b) Euler's graphical representation

The eighteenth-century East Prussian town of Königsberg included two islands and seven bridges as shown in above in Fig (a). Beginning anywhere and ending anywhere, can a person walk through town crossing all seven bridges but not crossing any bridge twice. The people of Königsberg wrote to the celebrated Swiss mathematician L. Euler about this question. Euler proved in 1736 that such a walk is impossible. He replaced the islands and the two sides of the river by points and the bridges by curves, thus obtaining Fig. (b). It is not difficult to see that the walk in Königsberg is possible if and only if the multigraph in Fig. (b) is traversable. But this multigraph has four odd vertices, and hence it is not traversable. Thus one cannot walk through Königsberg so that each bridge is crossed exactly once.

Theorems on Euler graph.

Theorem 1.

Let G be a graph in which degree of every vertex is at least two. Then G contains a cycle.

Theorem 2.

A connected graph G is Euler if and only if degree of every vertex is even.

Theorem 3.

A connected graph G is Euler iff G has cycles $C_{(1)}$,..... $C_{(n)}$ such that every edge of G belongs to exactly on cycle $C_{(i)}$, i.e. G is union of edge disjoint cycles.

Theorem 4.

A connected graph G has an Euler trail iff it has at most two odd vertices, i.e. it has either no vertices of odd degree or exactly two vertices of odd degree.

HAMILTONIAN GRAPHS

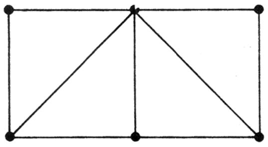
It is a graph with a closed path that includes every vertex exactly once. Such a path is a cycle and is called *Hamiltonian cycle*. An Eulerian cycle uses every edge exactly once but may repeat vertices, while a Hamiltonian cycle uses each vertex exactly once (except for the first and last) but may skip edges.

Hamiltonian path in a graph G is a path which contains every vertex of G, i.e. it contains every vertex of G once and only once. Hamiltonian cycle in a graph G is a cycle which contains every vertex of G.

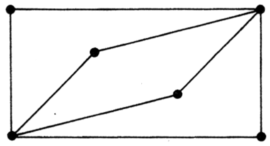
Now, A graph G is called Hamiltonian if it has a Hamiltonian cycle.

A simple graph G is called *maximal non-Hamiltonian* if it is not Hamiltonian but addition of any edge connecting two non-adjacent vertices forms a Hamiltonian graph.

A graph with six vertices which is Hamiltonian but not Eulerian.



(a) Hamiltonian and noneulerian



(b) Eulerian and nonhamiltonian

There are many possible solutions to this problem and one of these is shown above in Fig. (a). However every solution must have a cycle that includes every vertex exactly once (hamiltonian), but must not have a closed trail that uses every edge exactly once. (Eulerian). When a candidate Hamiltonian graph has been identified, one can easily determine if it is Eulerian by looking for vertices of odd degree. At least one such vertex should exist, the graph is not Eulerian.

Theorems on Hamiltonian graph.

Theorem 1 (Dirac).

If G is a simple graph with n vertices where $n \geq 3$, and the degree $d(v) \geq \frac{n}{2}$ for every vertex v of G, then G is Hamiltonian.

Theorem 2.

Let G be a simple graph with n vertices and u and v be non-adjacent vertices in G such that

$$d(u) + d(v) \geq n$$

If G + uv denote the subgraph of G obtained by joining u and v by an edge, then G is Hamiltonian iff G + uv is Hamiltonian.

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CLOSURE OF A GRAPH

Let G be a simple graph.

If there are two non-adjacent vertices u_1 and v_1 in G such that $d(u_1) + d(v_1) \geq n$ in G, join u_1 and v_1 by an edge to form the supergraph G_1. If two vertices u_2 and v_2 are there such that $d(u_2) + d(v_2) \geq n$ in G_1, join u_2 and v_2 by an edge to form the supergraph G_2.

Continue in this was to join pairs of non adjacent vertices whose degree sum is at least n. The final supergraph thus obtained is called *closure of G* and is denoted by C(G).

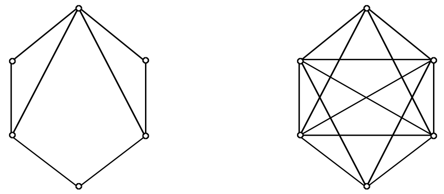


Fig. Graph and its closure C(G)

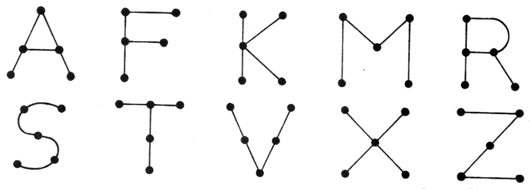
Theorem due to Bondy and Chvatal.

A simple graph is Hamiltonian iff its closure C(G) is Hamiltonian.

ISOMORPHISM OF GRAPHS

Let G(V, E) and G*(V*, E*) are graphs and $f : V \to V^*$ is a one-to-one correspondence beta sets of vertices such that $\{u, v\}$ is an edge of G if and only if $\{f(u), f(v)\}$ is an edge of G*. Then f is *isomorphism* between G and G*, and G and G* are called *isomorphic* graphs. Normally, we do not distinguish between isomorphic graphs (even though their diagrams may "look different").

Example. Figure shows ten graphs pictured as letters. Which of the ten graphs are isomorphic to M?



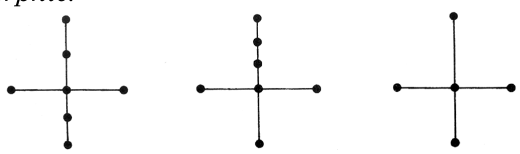
Solution.

M consists of five vertices in a single line. Thus S, V, and Z (and M itself) are isomorphic to M.

HOMEOMORPHIC GRAPHS

Given any graph G, we can get a new graph by dividing an edge of G with additional vertices. Two graphs G and G* are called *homeomorphic* if they can be obtained from isomorphic graphs by this method.

Graphs which are called homeomorphic, but not isomorphic.



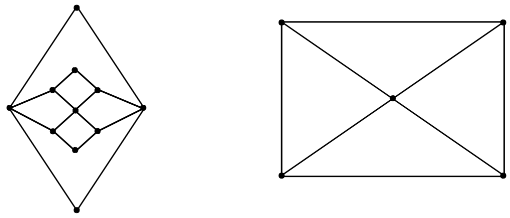
(a) ***(b)*** ***(c)***

Graphs (a) and (b) in figure are not isomorphic; but they are homeomorphic since each can be obtained from (c) by adding appropriate vertices.

PLANAR GRAPHS

It is a graph drawn in the plane in such a way that any pair of edges meet only at their end vertices.

Planar graph is a graph which is isomorphic to a plane graph, *i.e.* it can be redrawn as a plane graph.



Jordan Curve in the plane is a continuous non-interesecting curve whose origin and terminus coincide.

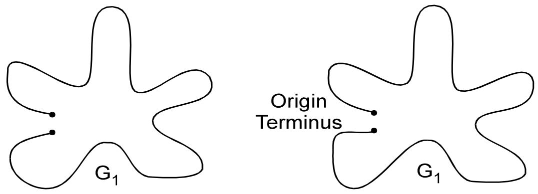


Fig. G_1 is Jordan Curve but G_2 is not

MATCHINGS

Let four applicants a_1, a_2, a_3, and a_4 are available to fill six vacant positions p_1, p_2, p_3, p_4, p_5 and p_6.

Applicant a_1 is qualified to fill position p_2 or p_5.

Applicant a_1 is qualified to fill position p_2 or p_5. Applicant a_2 can fill p_2 or p_5.

Applicant a_3 is qualified for p_1, p_2, p_3, p_4, p_5 or p_6.

Applicant a_4 can fill jobs p_2 or p_5.

This situation is represented by the graph in figure below.

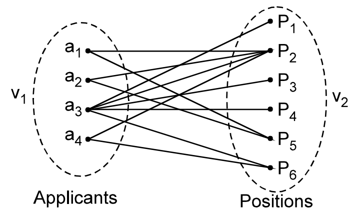


Fig. (a) Bipartite graph

The vacant positions and applicants are represented by vertices. The edges represent the qualifications of each applicant for filling different positions. The graph clearly is bipartite, the vertices falling into two sets $V_1 = \{a_1, a_2, a_3, a_4\}$ and $V_2 = \{p_1, p_2, p_3, p_4, p_5, p_6\}$.

The most likely questions one is to ask in this situation are :

Is it possible to hire all the applicants and assign each a position for which he is suitable ?

If the answer is no, what is the maximum number of positions that can be filled from the given set of applicants ?

This is a problem of matching (or assignment) of one set of vertices into another. More formally, a matching in a graph is a subset of edges in which no two edges are adjacent. A single edge in a graph is obviously a matching.

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Maximal maching.

It is a matching to which no edge in the graph can be added.

e.g. in a complete graph of three vertices (*i.e.* a triangle) any single edge is a maximal matching. The edges shown by heavy lines.

(a) *(b)*

Fig. Graph and two of its maximal matchings.

Clearly, a graph may have many different maximal matchings and of different sizes. Among these, the maximal matchings with the largest number of edges are called *largest maximal matchings.*

In Fig. *(b)*, a largest maximal matching is shown in heavy lines. The number of edges in a largest maximal matching is called *matching number of the graph.*

Theorem 1.

A complete matching of V_1 into V_2 in a bipartite graph exists if and only if every subset of r vertices in V_1 is collectively adjacent to or more vertices in V_2 for all values of r.

Problem of distinct representatives.

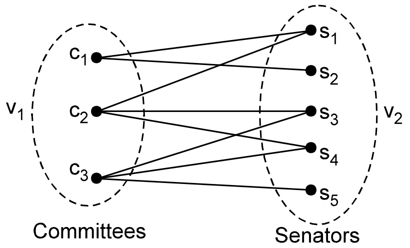

Fig. Membership of committees

Five senators s_1, s_2, s_3, s_4 and s_5 are members of three committees, c_1, c_2 and c_3. The membership is shown in the figure.

One member from each committee is to be represented in a super committee. Is it possible to send one distinct representative from each of the committees ?

This problem is one of finding a complete matching of a set V_1 into set V_2 in a bipartite graph.

Now, use Theorem 1 and check if r vertices from V_1 are collectively adjacent to at least r vertices from V_2, for all values of r.

Result is shown in the table below (ignore last column for the time being).

	V_1	V_2	$r - q$
$r = 1$	$\{c_1\}$	$\{s_1, s_2\}$	-1
	$\{c_2\}$	$\{s_1, s_3, s_4\}$	-2
	$\{c_3\}$	$\{s_3, s_4, s_5\}$	-2
$r = 2$	$\{c_1, c_2\}$	$\{s_1, s_2, s_3, s_4\}$	-2
	$\{c_2, c_3\}$	$\{s_1, s_3, s_4, s_5\}$	-2
	$\{c_3, c_1\}$	$\{s_1, s_2, s_3, s_4, s_5\}$	-3
$r = 3$	$\{c_1, c_2, c_3\}$	$\{s_1, s_2, s_3, s_4, s_5\}$	-2

Thus for this example, condition for the existence of a complete matching is satisfied as stated in Theorem 1. Hence, it is possible to from the super committee with one distinct representative from each committee.

The problem of distinct representatives just solved was a small one.

A larger problem would have become unwieldy, if there are M vertices in V_1. Theorem 1 requires that we take all $2^M - 1$ nonempty subsets of V_1 and find the number of vertices of V_2 adjacent collectively to each of these. In most cases, however, following simplified version of Theorem 1 will suffice for detection of a *complete matching in any large graph.*

Theorem 2.

In a bipartite graph, a complete matching V_1 into V_2 exists if (but not only if) there is a positive integer m for which the following condition is satisfied :

Degree of every vertex in $V_1 \geq m \geq$ degree of every vertex in V_2.

In the bipartite graph of above figure,

Degree of every vertex in $V_1 \geq 2 \geq$ degree of every vertex in V_2.

Therefore, there exists a complete matching.

In the bipartite graph, no such number is found, because

$$\text{degree of } p_2 = 4 > \text{degree of } a_1.$$

COVERINGS

In a graph G, a set g of edges is said to cover G if every vertex in G is incident on at least one edge in g. A set of edges that covers a graph G is called *edge covering*, *covering subgraph*, or *simply a covering of G.*

e.g. a graph G is trivially its own covering. *Spanning tree* in a connected graph (or a spanning forest in an unconnected graph) is another covering. Hamiltonian circuit (if it exists) in a graph is also a covering.

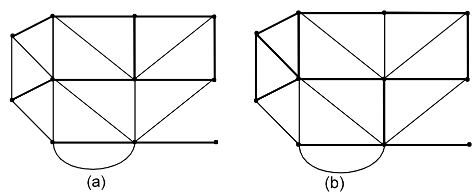

Fig. Graph and two of its minimal coverings

Here we shall investigate minimal covering-a covering from which no edge can be removed without destroying its ability to cover the graph.

- Covering exists for a graph if and only if the graph has no isolated vertex.
- Covering of an n-vertex graph will have at least $\left\lceil \frac{n}{2} \right\rceil$ edges. ($\lceil x \rceil$ denotes smallest integer not less than x.)
- Every pendant edge in a graph is included in every covering of the graph.

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- Every covering contains a minimal covering.
- If we denote remaining edges of a graph by $(G-g)$, the set of edges g is a covering if and only if, for every vertex v,

  degree of vertex in $(G-g) \leq$ (degree of vertex v in G) $-$ 1
- No minimal covering can contain a circuit, for which we can always remove an edge from a circuit without leaving any of the vertices in the circuit uncovered. Therefore a minimal covering of an n-vertex graph can contain no more than n-edges.
- A graph in general has many minimal coverings, and they may be of different sizes (i.e. consisting of different numbers of edges). The number of edges in a minimal covering of the smallest size is called *covering number of the graph.*

Theorem 3.

A covering g of a graph is minimal if and only if g contains no paths of length three or more.

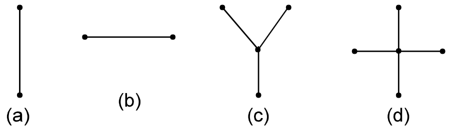


Fig. Star graphs of one, two, three, and four edges.

Let graph in the above figure represents street map of a part of a city, each of the vertices is a potential trouble spot and must be kept under the surveillance of a patrol car. How will you assign a minimum number of patrol cars to keep every vertex covered?

The answer is a smallest minimal covering. The covering shown in Fig. (a) is an answer, and it requires six patrol cars. Clearly, since there are 11 vertices and no edge can cover more than two, less than six edges cannot cover the graph.

SOLVED EXAMPLES

1. Prove by induction that expression for the number of diagonals in a polygon of n sides is

$$\left[\frac{n(n-3)}{2}\right]$$

Solution.

The result holds for $n = 4$, i.e. for quadrilateral.

Assume that it holds for $n = m$, if there are $\left[\frac{m(m-3)}{2}\right]$ diagonals of m sides polygon.

Now, increase the sides to $m + 1$, which results in addition of one corner to the polygon. To these edges diagonals from all other edges can be added except the neighbouring edges.

$\therefore$ Number of diagonals added $= (m+1) - 2$

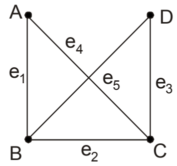
$$= m - 1$$

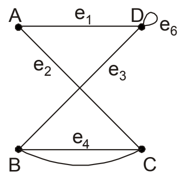
$\therefore$ Total number of diagonals

$$= \left[\frac{m(m-3)}{2}\right] + (m-1) = \frac{[((m+1)(m+1)-3)]}{3}$$

Hence the result is proved by induction.

2. Consider the multigraph G = G (V, E) shown in the figure.





(i) Find the number of vertices and edges.

(ii) Are there any multiple edges or loops ? If So, what are they ?

Solution.

(i) G contains four vertices A, B, C, D; and six edges, e_1, e_2, ..., e_6 (although the edges e_2 and e_3 across at a point, the diagram does not indicate that the intersection point is a vertex of G.)

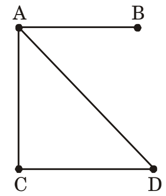
(ii) The edges e_4 and e_5 are multiple edges since they both have the same endpoints B and C. The edge e_6 is a loop.

3. Draw a diagram for each of the following graphs G = G (V, E) :

V = {A, B, C, D}, E = [{A, B}, {D, A}, {C, A}, {C, D}]

Solution.

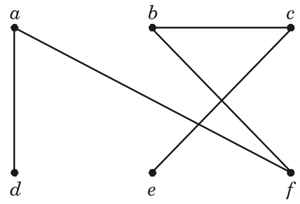
First draw vertices of the graph, and then connect the appropriate vertices to indicate the edges of the graph, as shown in the figure.



4. Draw a diagram for the graphs G = G (V, E)

V = {a, b, c, d, e, f}, E = [{a, d}, {a, f}, {b, c}, {b, f}, {c, e}]

Solution.



5. Let G = G (V, E) has five vertices. Find the maximum number of m of edges in E if

(i) G is a graph, and

(ii) G is a multigraph.

Solution.

(i) There are C (5, 2) = 10 ways of choosing two vertices from V ; hence $m = 10$.

(ii) Since multiple edges are peritted, G can have any number of edges (and loops), finite or infinite; hence no such maximum number m exists.

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COMBINATORIES

INTRODUCTION

Objects (or things) can be arranged in many ways. Let there are three objects marked a, b, c on a table. From these, two objects can be selected at a time in three different ways as ab, ac, bc. In this way selection of two objects from three objects in three ways is called *combination*.

The above selections ab, ac, bc can also be arranged as ba, ca, cb. So, we can understand that two objects can be selected from three objects and arranged in six ways. These arrangements are called *permutations*.

FUNDAMENTAL PRINCIPLE

If A is a finite set, then number of different elements in A is denoted by $n(A)$.

Here $n(A)$ is non-negative integer.

e.g. If $A = \{2, 5, 7\}$, then $n(A) = 3$

If $A = \left\{a, \frac{2}{3}, 5, 0, \frac{1}{3}\right\}$, then $n(A) = 5$

If $C = \phi$ (empty set), then $n(C) = 0$.

Fundamental principle of counting.

If a man can do a work in p different ways and another work in q different ways, then he can do the two works one after the other in pq ways.

Now treating one way of doing a work as one element, we form the two sets A and B.

We denote the set of ways of doing the first work by A and set of ways of doing the second work by B. Then

$n(A) = p$ and $n(B) = q$.

Number of ways one can do the two works in an order is

$n(A \times B) = n(A) \times n(B) = pq$.

If same principle is extended to 'm' works, then

$$n(A_1 \times A_2 \times ..... \times A_m) = n(A_1)\, n(A_2) \,.....\, n(A_m).$$

PERMUTATIONS

(1) Linear permutations.

From the elements of a finite set A (in general distinct elements) taking some or all elements and arranging them linearly is called *linear permutation*.

(2) Similar permutations.

If in a given set of things, some elements are alike, permutations formed from them are called *similar permutations*. In this case, number of permutations will change because we can not identify the change among the like things.

(3) Circular permutations.

If 'n' distinct things are arranged in a circular manner giving preference only to relative positions, then it is called *circular permutation*.

In a circular permutation, if places are changed circularly, the permutations will not change.

e.g. If $A = \{4, 5, 6\}$, then permutations obtained by arranging all elements in A in an order are:

456, 465, 546, 564, 645, 654.

- From a set A of n elements, number of permutations taken $r\ (\le n)$ at a time is denoted by ^nP_r, or $P(n, r)$.

Notation.

The product of first 'n' natural numbers is called *factorial n* and is denoted by $n!$.

We define $0! = 1$ and we observe that $1! = 1$.

e.g. (*i*) $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

(*ii*) Taking all the letters at a time from A, A, B, the possible number of permutations is 3. They are AAB, ABA, BAA.

Here we can not find difference between A's but if you find it, then number of permutations is

$$3P_3 = 3! = 6.$$

Theorem.

The number of permutations of 'n' dissimilar things taken $r\ (r \le n)$ at a time is

$$^nP_r = n\,(n-1)\,(n-2) \,.......\, (n-r+1)$$

A finite bijective mapping is a permutation.

If $S = \{a, b, c, d, e\}$ and defining a permutation f as

$$f(a) = b;\quad f(b) = d;\quad f(c) = a;\quad f(d) = e;\quad f(e) = c$$

then we denote the permutation f as $f = \begin{bmatrix} a\,b\,c\,d\,e \\ b\,d\,a\,e\,c \end{bmatrix}$

$$\begin{aligned} ^nP_n &= n(n-1)\,(n-2) \,......\, (n-n+1) \\ &= n\,(n-1)\,(n-2) \,.......\, 2 \cdot 1 \\ &= 1.2.3. \,..........\, (n-2)\,(n-1)n = n! \end{aligned}$$

i.e. number of permutations containing all objects from n distinct things is $^nP_n = n!$.

Note : $^nP_r = n(n-1)\,(n-2) \,......\, (n-r+1)$

$$= \frac{n(n-1)\,(n-2) \,.......\, (n+r+1)\,\{(n-r)!\}}{(n-r)!}$$

$$= \frac{1 \cdot 2 \cdot 3 \,..........\, (n-1) \cdot n}{(n-r)!} = \frac{n!}{(n-r)!}$$

$$\therefore \quad ^nP_r = \frac{n!}{(n-r)!}$$

Example. Find the value of P (4, 3)

Solution:

We know, $P(n, r) = n\,(n-1) \,....\, (n-r+1)$

Here $n = 4, r = 3$,

$\therefore \quad n - r + 1 = 4 - 3 + 1 = 2.$

Hence $\quad P(n, r) = P(4, 3) = 4.3.2 = 24$

Alternately : $P(n, r) = \dfrac{n!}{(n-r)!}$

$$\therefore \quad P(4, 3) = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 4 \times 3 \times 2 = 24$$

Theorem. $^nP_r = {}^{(n-1)}P_r + r \cdot {}^{(n-1)}P_{(r-1)}$

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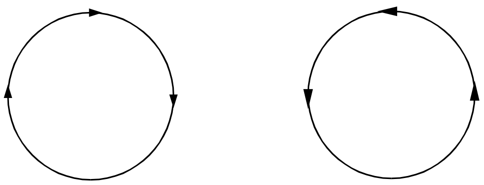
CIRCULAR PERMUTATIONS

Types of circular permutations

Circular permutations are of two types.

(*i*) Clockwise (*ii*) Anti-clockwise.

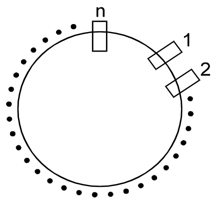


Clockwise ***Anti-clockwise***

Considering direction, the above two permutations are different, otherwise they are the same.

Number of Circular permutations.

Theorem : The number of circular permutations of n different things taken all at a time is $(n-1)!$



From the required n objects, object a can be put in any one of n places. After filling a place by a the remaining $(n-1)$ vacancies can be filled by $(n-1)$ objects in $(n-1)!$ ways. So the number of circular permutations formed with 'n' objects is $(n-1)!$.

In circular permutations, direction of arrangements of the objects is also important. The above mentioned circular permutations should be considered as total number of clock-wise and anti-clockwise permutations.

Since number of permutations are equal in both the directions and if the direction is not considered, then number of circular permutations = $\dfrac{(n-1)!}{2}$

Example. Find the number of ways that a garland is made with 18 flowers such that two specified flowers should be side by side in the garland.

Solution:

First we should treat that two specified flowers as one, hence total 17 (18 – 1) flowers can be arranged circularly in 17! ways.

Two specified flowers can be arranged in 2! = 2 ways

Thus, based on the given condition, 18 flowers can be arranged as a garland in $2 \times 17!$ ways.

But when preparing a garland, we should not consider the direction, so flowers in a garland can be arranged in $\dfrac{2 \times 17!}{2} = 17!$ ways.

Theorem.

The number of permutations of 'n' dissimilar things, taken r at a time, when the repetition of things is allowed any number of times, is n^r.

Example. Using 9 digits 1, 2, 3, 4, 5, 6, 7, 8, 9, how many 3 digits numbers can be written

(*i*) without repetition of digit?

(*ii*) using a digit any number of times?

Solution:

(***i***) Above 3 blank places are to be filled with 3 different digits without repetition of a digit. This number is equivalent to the number of permutations formed from 9 digits which are taken 3 at a time,

i.e. $^9P_3 = 9 \times 8 \times 7 = 504$ ways.

(***ii***) If a digit can be used in any number of times, then number of permutations obtained is

$^9P_3 = 9 \times 9 \times 9 = 729.$

RESTRICTED PERMUTATION.

The number of permutations of n different objects take r at time in which m particular objects are

(*i*) never included will be $P(n-m, r)$

(*ii*) always included will be $C(n-mr-m)\,P(r, r)$, where $(n \geq m, r \geq m)$

Example. There are 9 objects and 9 boxes. Out of 9, 5 objects cannot fit to be put in 3 small boxes. How many arrangements can be made, such that each object can be put in one box only?

Solution:

Since 5 objects cannot fit to be put 3 small boxes, these 5 objects can be put in the remaining 6 boxes in 6_{P_5} ways.

The remaining 4 objects can be put in 4_{P_4} ways.

∴ Total number of arrangements

$= {}^6P_5 \times {}^4P_4 = (6 \times 5 \times 4 \times 3 \times 2) \times (4 \times 3 \times 2 \times 1)$

$= 720 \times 24 =$ **17280.**

Theorem.

There are n objects. Out of which m $(< n)$ objects are identical and the remaining are dissimilar. Using all objects, the number of permutations from n objects is $\dfrac{n!}{m!}$.

In n objects, if m objects are of one type, p objects are of another type and q objects are of different type etc., then number of permutations formed from $n (= m + p + ... + q)$ objects $= \dfrac{n!}{m!\,p!.......q!}$

- In n objects, if m objects are of one type and $(n-m)$ objects are of another type, then number of permutations using all objects at a time

  $= \dfrac{n!}{m!(n-m)!}$

- In n objects, if m objects are of one type, q objects are of another type and $(n - p - q)$ objects are of different type, then using all the objects, number of permutations formed $= \dfrac{n!}{p!q!(n-p-q)!}$

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Example. Find the number of permutations formed by using all the letters of the word 'ENGINEERING' when

(*i*) *all E's come together.*

(*ii*) *all the N's and all the E's come together.*

(*iii*) all N's come together and E's do not come together.

Solution:

In the given word 'ENGINEERING', there are 11 letters out, of which there are 3 E's, 3 N's, 2 G's, 2 I's and 1 R.

(i) Number of permutations in which all the E's come together $= \dfrac{9!}{3!2!2!1!1!}$

(ii) Number of permutations that can be formed with all the N's and all the E's which come together

$$= \frac{7!}{2!2!1!1!1!}$$

(iii) Number of permutations that can be formed in which all the N's that come together and E's do not come together

$$= \frac{9!}{3!2!2!1!1!} - \frac{7!}{2!2!1!1!1!}$$

COMBINATIONS

n, r are integers. If $n \geq r$, for the set of n elements a subset of r elements is called a *combination*. Therefore a combination is an unordered selection of r elements from a set of n elements. The number of combinations of r elements selected from n elements is denoted by ^nC_r or $\binom{n}{r}$ or $C(n, r)$.

e.g. Out of 40 students in a class any 4 students can sit on a bench without considering the order in $^{40}C_4$ ways.

Theorem.

Number of combinations of n dissimilar things, taken r at a time is

$$^nC_r = \frac{n!}{r!\,(n-r)!}$$

There is no importance for the order of arrangements in combinations.

Theorem.

- $^nC_r = {}^nC_{n-r}$
- $^nC_n = \dfrac{n!}{n!\,(n-n)!} = \dfrac{n!}{n!\,0!} = 1$ (since $0! = 1$)
- $^nC_0 = \dfrac{n!}{0!(n-0)!} = \dfrac{n!}{n!} = 1$

Theorem.

If $n_{c_r} = {}^nC_s$, then either $r = s$, or $r + s = n$.

Theorem.

$^{(n-1)}C_{(r-1)} + {}^{(n-1)}C_r = {}^nC_r$

Theorem.

If n, r are positive integers with $1 \leq r \leq n$, then

- $^nC_r + {}^nC_{(r-1)} = {}^{(n+1)}C_r$
- $^{(n+1)}C_r - {}^nC_r = {}^nC_{(r-1)}$

or $^{(n+1)}C_r - {}^nC_{(r-1)} = {}^nC_r$

Corollary : If m and n are positive integers with $m \neq n$, then total $m + n$ distinct objects can be divided into two groups of m objects and n objects in $\dfrac{(m+n)!}{m!n!}$ ways.

Note : Let $2n$ objects are to be divided into 2 equal groups. Applying above corollary, there will not be any change in the division, if bags A and B are filled with the objects n and n instead of m and n objects. But interchange can be done in 2! ways. Hence number of dividing $2n$ objects n and n objects is

$$\frac{(2n)!}{2n!n!} = \frac{(2n!)}{2!\,(n!)^2}$$

- $(m + n + p)$ objects can be divided into 3 groups of m objects, n objects and p objects in $\dfrac{(m+n+p)!}{m!n!\,p!}$ ways.
- $3n$ objects can be divided into 3 equal groups in $\dfrac{(3n)!}{3!(n!)^3}$ ways.

Corollary : Let there are p objects of one type and q objects are of another type, then number of combinations selected from the total $(p + q)$ objects = $[(p + 1)(q + 1) - 1]$.

If there are p objects of one type q objects of 2nd type and so on.... and t objects of another type, then total number of combinations formed from $(p + q + ... + t)$ objects = $\{[(p + 1)(q + 1) ..... (t + 1)] - 1\}$

Here if $p = q = ... = t = 1$, and they are 'n' in number, then

total number of combinations formed

$= (1 + 1)(1 + 1) .... (1 + 1) - 1$ (n times)

$= (2, 2, ... 2) - 1 = 2^n - 1$ (n times) $= 2^n - 1$

Remember that number of subsets in a power set formed from the 'n' elements is $2^n - 1$ (except the empty set).

Example. In how many ways, a team of 3 boys and 3 girls can be selected from 6 boys and 4 girls?

Solution:

3 boys can be selected from 6 boys in

$$^6C_3 = \frac{6!}{3!3!} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20 \text{ ways}$$

3 girls can be selected from 4 girls in $^4C_3 = {}^4C_1 = 4$ ways

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Hence from fundamental principle, required number of ways of selection = 20 × 4 = 80

- *Number of straight lines drawn through 'n' points on a circle*

$$= {}^nC_2 = \frac{n(n-1)}{2\times 1} = \frac{n(n-1)}{2}$$

Example. How many straight lines can be drawn through 10 points on a circle?

Solution:

Since two points are necessary to draw a straight line, then

number of straight lines drawn through 10 points

$${}^{10}C_2 = \frac{10\times 9}{2\times 1} = \mathbf{45.}$$

Example. In an election, 4 batches of people containing 15, 20, 18, 22 have to convass in 4 districts. If 75 people themselves come forward to convass, in how many ways can they distribute among different districts ?

Solution:

Since 4 groups are distinct, number of ways of sending

75 people to 4 districts = $\dfrac{75!}{15!\,20!\,18!\,22!}$

RESTRICTED COMBINATIONS

The number of combinations of n objects taken r at a time in which m particular objects are

(*i*) never included in C $(n - m, r)$ ways,

(*ii*) always included in C $(n - m, r - m)$ ways where $(n \geq m$ and $n \geq m)$.

Example. Find the diagonals of a polygon on n sides.

Solution:

Number of straight lines formed by joining the 2 vertices from 'n' vertices of the polygon.

Number of ways of selecting 2 vertices from 'n' vertices

$$= {}^nC_2 = \frac{n(n-1)}{2\times 1}$$

Out of these straight lines, 'n' lines are the sides of polygon, therefore

$$\text{number of diagonals} = \frac{n(n-1)}{2\times 1} - n = \frac{n^2 - n - 2n}{2}$$

$$= \frac{n^2 - 3n}{2} = \frac{n(n-3)}{2}$$

Example. Find the number of triangles formed by joining the vertices of the polygon.

Solution:

Since a triangle is formed with any three diagonals of the polygon, therefore number of triangles formed with 3 vertices

$$= {}^nC_3 = \frac{n(n-1)(n-2)}{3\times 2\times 1} = \frac{n(n-1)(n-2)}{6}$$

Example. In how many ways, can a committee of 5 members be formed from 8 men and 5 women such that at least 2 women should be there in each committee ?

Solution:

If there are atleast 2 women in each 5 member committee, then the committees are

(***a***) 3 men, 2 women

Number of selections $= {}^8C_3 \times {}^5C_2 = 56 \times 10 = \mathbf{560}$

(***b***) 2 men, 3 women

Number of selections $= {}^8C_2 \times {}^5C_3 = 28 \times 10 = \mathbf{280}$

(***c***) 1 man, 4 women

Number of selections $= {}^8C_1 \times {}^5C_4 = 8 \times 5 = \mathbf{40}$

(***d***) no man, 5 women

Number of selections $= 8c_0 \times 5c_5 = 1 \times 1 = \mathbf{1}$

$\therefore$ Number of ways of selecting a 5 member committee = 560 + 280 + 40 + 1 = **881.**

Example. There are 3 questions in 1st section, 3 questions in the 2nd section and 2 questions in the 3rd section in a question paper of an exam. In how many ways can a student who appeared for the exam select to answer any 5 questions of the paper choosing at least 1 question from each section.

Solution:

Out of 8 questions, student can select 5 questions in following ways :

Number of ways of selection in 1st way

$$= {}^3C_3 \times {}^3C_1 \times {}^2C_1 = 6$$

Number of ways of selection in 2nd way

$$= {}^3C_2 \times {}^3C_2 \times {}^2C_1 = 18$$

Number of ways of selection in 3rd way

$$= {}^3C_2 \times {}^3C_1 \times {}^2C_2 = 9$$

Number of ways of selection in 4th way

$$= {}^3C_1 \times {}^3C_2 \times {}^2C_2 = 9$$

COUNTING

Basic counting rules.

There are two basic counting rules which can be used to solve many different counting problems.

(*i*) **Sum rule :** If first task can be done in n_1 ways and a second task in n_2 ways, and if these tasks are such that they cannot be done at the same time, then there are $n_1 + n_2$ ways to do either task.

(*ii*) **Product rule :** Whenever a precedure can be broken down into two tasks and then there are n_1 ways to do the first task and n_2 ways to do the next task after the first task has been done, then there are n_1n_2 ways to do the procedure.

Example. How many one-one functions are there from a set with m elements to one with n elements ?

Solution:

Whenever $m > n$, there are no one-one functions from a set with m elements to a set with n elements.

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Let us say $m \le n$. If elements in the domain are assumed to be $a_1, a_2, \ldots a_n$, then there are n ways to choose value of the function at a_1.

Since the function is one-one, there are only $(n-1)$ ways to pick up for a_2, as value of the function. In function general value can be chosen in $n-k+1$ ways for a_k. Applying product rule, there are $n(n-1)$ ..... $(n-k+1)$ one-one functions from a set with m elements to one with n elements.

Example. How many bit strings of length nine either start with 1 bit or end with the two bits 00 ?

Solution:

Precedure can be divided into two tasks.

First task is constructing a bit string of length nine beginning with a 1 bit. This can be done in

2^8 ways = 256 ways.

Second task of constructing a bit string of length nine ending with two bits 00. This can be done in

$2^7 = 128$ ways.

From this we have to remove the number of ways of choosing 1 in the first bit and corresponding 00 in the last two bits. This can be done in

2^6 ways = 64 ways.

Therefore, total number of bit strings of length nine that can be constructed, that begin with 1 and end with a 00, in

$256 + 128 - 64 = 320$ ways.

EXERCISE – I

MCQ TYPE QUESTIONS

1. $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$ is equivalent to

(*a*) $S \wedge R$ (*b*) $S \to R$

(*c*) $S \vee R$ (*d*) none of these

2. The functionally complete set is

(*a*) $\{\neg, \wedge, \vee\}$ (*b*) $\{\downarrow, \wedge\}$

(*c*) $\{\uparrow\}$ (*d*) $\{\to, \wedge\}$

3. $(P \vee Q) \wedge (P \to R) \wedge (Q \to R)$ is equivalent to

(*a*) P (*b*) Q

(*c*) R (*d*) True $\equiv$ T

4. $\neg(P \to Q)$ is equivalent to

(*a*) $P \wedge \neg Q$ (*b*) $\neg P \wedge Q$

(*c*) $\neg P \vee Q$ (*d*) $P \vee Q$

5. Logical expression $(A \wedge B) \to (C' \wedge A) \to (A \equiv 1)$ is

(*a*) contradiction

(*b*) tautology

(*c*) well-formed formula

(*d*) none of these

6. Which one of the following propositional forms is a tautology?

(*a*) $p \wedge \neg q$

(*b*) $p \Rightarrow (p \vee q)$

(*c*) $p \Rightarrow (p \wedge q)$

(*d*) $(p \vee q) \Rightarrow (p \wedge q)$

7. The translation of the sentence (given in English): '*Some cats are black but all buffaloes are black*' using the following notations :

$B(x)$: x is a black

$C(x)$: x is cat

$Bf(x)$: x is a buffalo

$(\forall x)$: for all x

$(\exists x$: for some x.

in logical symbolism is given by

(*a*) $(\exists x)\,(C(x) \wedge B(x)) \wedge (\forall x)\,(Bf(x) \to B(x))$

(*b*) $(\exists x)\,(C(x) \wedge B(x)) \wedge (\forall x)\,(Bf(x) \wedge B(x))$

(*c*) $(\exists x)\,(C(x) \to B(x)) \wedge (\forall x)\,(Bf(x) \wedge B(x))$

(*d*) $(\exists x)(C(x) \to B(x)) \wedge (\forall x)(Bf(x) \to B(x))$

8. In propositional logic, which of the following is equivalent to $p \to q$?

(*a*) $\sim p \to q$ (*b*) $\sim p \vee q$

(*c*) $\sim p \vee \sim q$ (*d*) $p \to \sim q$

9. In predicate logic, $\sim \forall P(x)$ is equivalent to

(*a*) $\exists x P\ (x)$ (*b*) $\exists x \sim P(x)$

(*c*) $\forall x \sim P(x)$ (*d*) $\sim \exists x P(x)$

10. Which of the following are tautologies ?

(*a*) $((P \vee Q) \wedge Q) \leftrightarrow Q$

(*b*) $(P \vee (P \to Q)) \to P$

(*c*) $((P \vee Q) \wedge P) \to Q$

(*d*) $((P \vee Q) \wedge \neg P) \to Q$

11. Identify the valid conclusion from the premises $P \vee Q$, $Q \to R$, $P \to M$, $\neg M$

(*a*) $P \wedge (Q \vee R)$ (*b*) $P \wedge (Q \wedge R)$

(*c*) $R \wedge (P \vee Q)$ (*d*) $Q \wedge (P \vee R)$

12. Which one of the following statements is true when exactly two of P, Q, or R are TRUE ?

(*a*) $(P \wedge \overline{Q} \wedge R) \vee (P \wedge \overline{Q} \wedge \overline{R}) \wedge (\overline{P} \wedge Q \wedge \overline{R})$

(*b*) $(P \vee Q \vee R) \wedge (P \vee \overline{Q} \vee R) \wedge (P \vee Q \vee \overline{R})$

(*c*) $(\overline{P} \wedge Q \wedge R) \vee (P \wedge \overline{Q} \wedge R) \vee (P \wedge Q \wedge \overline{R})$

(*d*) $(P \wedge \overline{Q} \wedge R) \vee (P \wedge \overline{Q} \wedge \overline{R}) \vee (\overline{P} \wedge Q \wedge \overline{R})$

13. $P \to (Q \to R)$ is equivalent to

(*a*) $(P \wedge Q) \to R$

(*b*) $(P \vee Q) \to R$

(*c*) $(P \vee Q) \to \neg R$

(*d*) $(P \wedge Q) \to \neg R$

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14. What can we correctly say about proposition P1 :

$$P1 : (p \vee \urcorner q) \wedge (q \rightarrow r) \vee (r \vee p)$$

(*a*) P1 is tautology

(*b*) P1 is a contradiction

(*c*) If p is true and q is false and r is false, then P1 is true

(*d*) If p is true and q is true and r is false, then P1 is true

15. Which of the following is a declarative statement?

(*a*) It is beautiful

(*b*) He says "*It is correct*"

(*c*) Two may not be an even integer

(*d*) I love you

16. The correct prefix formula is

(*a*) $\rightarrow \urcorner, P \vee Q \Longleftrightarrow \urcorner R \urcorner S$

(*b*) $\rightarrow P \urcorner Q \vee \Longleftrightarrow \urcorner RS$

(*c*) $\rightarrow \rightarrow PQ \rightarrow \rightarrow QR \rightarrow PR$

(*d*) $\rightarrow \urcorner PV \Longleftrightarrow QSP$

17. The negation of the statement $\exists x \forall y, P(x, y)$ is

(*a*) $\forall x \exists y, P(x, y)$

(*b*) $\forall x \exists y, \urcorner P(x, y)$

(*c*) $\exists x \exists y, \urcorner P(x, y)$

(*d*) $\forall y \forall x, \exists z \urcorner P(x, y, z)$

18. Which of the following sets are null sets ?

(*a*) {0} (*b*) {ϕ}

(*c*) { } (*d*) ϕ

19. Which of the following set (*s*) are empty ?

(*a*) $\{x : x = x\}$ (*b*) $\{x : x \neq x\}$

(*c*) $\{x : x = x^2\}$ (*d*) $\{x : x \neq x^2\}$

20. If A = $\{x, y\}$, then power set of A is

(*a*) $\{\{x\}, \{y\}\}$ (*b*) $\{\{\phi\}, \{x, y\}\}$

(*c*) $\{\phi, \{x\}, \{y\}\}$ (*d*) none of these

21. If A = $\{x, y\}$, then proper subsets of A are

(*a*) $\{x\}, \{y\}$ (*b*) $\phi, \{x, y\}$

(*c*) $\phi, \{x\}, \{y\}$ (*d*) none of these

22. If A and B are sets and $A \cup B = A \cap B$, then

(*a*) $A = \phi$ (*b*) $B = \phi$

(*c*) $A = B$ (*d*) none of these

23. If X, Y, Z are sets such that $X \subset Y$ and $Y \subset Z$, then

(*a*) $X \cup Y = Z$ (*b*) $X \cap Y = Z$

(*c*) $X \cap Y \cap Z = Z$ (*d*) none of these

24. If X and Y are two sets, then $X \cap (Y \cup X)^c$ equals

(*a*) X (*b*) Y

(*c*) ϕ (*d*) none of these

25. If P, Q, R are subsets of a set A, then

$R \times (P^c \cup Q^c)^c = (R \times P) \cap (R \times Q)$

(*a*) True

(*b*) False

26. For any two subsets X and Y of a set A, define X o Y = $(X^c \cap Y) \cup (X \cap Y^c)$. Then for any three subsets X, Y, Z of the set A is

X o (Y o Z) = (X o Y) o Z

(*a*) True

(*b*) False

27. Relation R defined on the set A = {1, 2, 3, 4}, by R = {(1, 1), (2, 2) {3, 3)} is

(*a*) reflexive (*b*) symmetric

(*c*) transitive (*d*) none of these

28. If relation R is defined on N by

R = {(a, b): a divides b; $a, b \in$ N}.

Then R is

(*a*) reflexive (*b*) symmetric

(*c*) transitive (*d*) none of these

29. Relation R defined on a set N by

R = $\{(a, b) : |a - b|$ is divisible by 5}, is

(*a*) reflexive (*b*) symmetric

(*c*) transitive (*d*) all of these

30. Relation R is defined on the set N as $\{(a, b) : a, b$ are both odd}, is

(*a*) reflexive (*b*) symmetric

(*c*) transitive (*d*) none of these

31. If R is an equivalence relation on a set A, then R^{-1} is

(*a*) reflexive (*b*) symmetric

(*c*) transitive (*d*) all of these

32. "n/m" means that n is a factor of m, then the relation 'I' is

(*a*) reflexive and symmetric

(*b*) transitive and symmetric

(*c*) reflexive, transitive and symmetric

(*d*) reflexive, transitive and not symmetric

33. The function $f : N \rightarrow N$ defined by $f(n) = 2n + 3$ is

(*a*) surjective (*b*) not surjective

(*c*) injective (*d*) none of these

34. Given : A = $\{x, y, z\}$; B = $\{u, v, w\}$

The function $f : A \rightarrow B$ defined by $f(x) = u$, $f(y) = v$, $f(z) = w$, is

(*a*) surjective (*b*) bijective

(*c*) injective (*d*) none of these

35. If $f : X \rightarrow Y$ and $a, b \subseteq X$, then $f(a \cap b)$ is equal to

(*a*) $f(a) - f(b)$

(*b*) $f(a) \cap f(b)$

(*c*) a proper subset of $f(a) \cap f(b)$

(*d*) $f(b) - f(a)$

36. If A and B are sets, then which of the following is FALSE ?

(*a*) $A - B' = A \cap B$

(*b*) $A \subset B \Rightarrow B' \subset A'$

(*c*) $A - (A - B) = A \cap B$

(*d*) None of these

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37. A = {string of 0's and 1's}.

Relation d on A is defined as xdy if x is a sub string of y (Ex. 01 d 101), Then d is

(a) symmetric
(b) anti symmetric
(c) equivalence
(d) reflexive and symmetric

38. If relation R over $\{a, b, c\}$ is given by R = $\{(a, a), (a, b), (b, a), (b, b), (c, c)\}$,

then which of the following properties does R have?

(a) Symmetry (b) Reflexivity
(c) Transitivity (d) All of these

39. If R = $\{(a, b)/a, b$ are positive integers such that $a - b$ is on ODD positive integer$\}$, then relation R is

(a) reflexive (b) symmetric
(c) transitive (d) equivalence

40. $A - (B \cup C)$ is

(a) $(A - B) \cup (A - C)$ (b) $A - B - C$
(c) $(A - B) \cap (A - C)$ (d) $A - (\overline{B \cap C})$

41. $A \cup B$ is NOT equivalent to

(a) $\left(\overline{\overline{A} \cap \overline{B}}\right)$ (b) $\left(\overline{\overline{A} - B}\right)$
(c) $\left(\overline{\overline{B} - A}\right)$ (d) $\left(\overline{A - B}\right)$

42. $A \cup B = A \cap B$ if and only if

(a) A is empty set
(b) B is empty set
(c) A and B are non-empty sets
(d) A and B are empty sets

43. If A = $\{x \mid -1 < x < 1\}$ = B, then function $f(x) = x/2$ from A to B is

(a) injective
(b) surjective
(c) both injective and surjective
(d) neither injective nor surjective

44. The domain and range are same for

(a) constant function
(b) identity function
(c) absolute value function
(d) greatest integer function

45. The function $f : Z \to Z$ given by $f(x) = x^2$ is

(a) one-one
(b) onto
(c) one-one and onto
(d) none of these

46. The number of functions

$f : (1, 2, 3, \ldots\ldots\ldots, n), \to \{1, 2, \ldots\ldots\ldots, m\}$,

which are one-to-one is

(a) m^n
(b) $m(m-1)(m-2) \ldots\ldots\ldots (m-n+1)$
(c) n^m
(d) $n(n-1)(n-2) \ldots\ldots\ldots (n-m+1)$

47. Which of the following relations may be characterised as a function defined on the set I = {1, 2, 3, 4, 5} ?

(a) $\{(x, y) \mid x, y \in I, x < y\}$
(b) $\{(x, y) \mid x, y \in I, x > y\}$
(c) $\{(x, y) \mid x, y \in I, x = y/2\}$
(d) None of these

48. Which of the following is a partition of the set

S = {1, 2, 3, 4, 5, 6} ?

(a) [{(1, 3, 5), {2, 4}, {3, 6}]
(b) [{1, 5}, {2}, {3, 6}]
(c) [{(1, 5}, {2}, {4}, {1, 5}, {3, 6}]
(d) [{1, 2, 3, 4, 5, 6}]

49. When A = {1, 2, 3, 4, 5} and r defined on A by xry if $x + 1 = y$, then r^3 is

(a) {(1, 3) (2, 4)} (b) {(1, 3) (2, 5)}
(c) {(1, 4) (2, 5)} (d) {(1, 4) (4, 5)}

50. If R = {(1, 1), (3, 1), (2, 3), (4, 2)}, then which of the following represents R^2, where R^2 is R composite R ?

(a) {(1, 1), (3, 1), (2, 3), (4, 2)}
(b) {(1, 1), (9, 1), (4, 9), (16, 4)}
(c) {(1, 3), (3, 3), (3, 4), (3, 2)}
(d) {(1, 1), (2, 1), (4, 3), (3, 1)}

51. If I is set of positive integers and relation R is defined over the set I by xRy if $x^y = y^x$, then R is

(a) reflexive but neither symmetric nor transitive
(b) reflexive and transitive but not symmetric
(c) reflexive and symmetric but not transitive
(d) reflexive, symmetric and transitive

52. 'Subset' relation on a set of sets is

(a) a partial ordering
(b) an equivalence relation
(c) transitive and symmetric only
(d) transitive and anti-symmetric only

53. The correspondence $f : N \to N$ is such that

$f(x) = y$ and for $x = p_1^{e1}, p_2^{e2}, \ldots p_k^{ek}$

where p_i are distinct primes and

integers $e_i \geq 1$, if $y = e_1 + e_2 + \ldots. + e_k$, then

(a) f is not a function
(b) f is an onto function
(c) f is an $1 - 1$ function
(d) f is neither $1 - 1$ nor onto function

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54. The power set of the set

$S = \{3, \{1, 4\}, 5\}$ is

(*a*) $\{S, 3, 1, 4, \{1, 3, 5\}, \{1, 4, 5\} \{3, 4\}, \phi\}$

(*b*) $\{S, 3, \{1, 4,\}, 5\}$

(*c*) $\{S, \{3\}, \{3, \{1, 4\}\}, \{3, 5\}, \phi\}$

(*d*) none of these

55. If $f(x)$ and $g(x)$ are defined on domains A, B respectively, then domain of $f(x) + g(x)$ is

(*a*) $A \cup B$ (*b*) $A \cap B$

(*c*) $A \Delta B$ (*d*) $A - B$

56. The function $f : R - \{2\} \to R$ defined by

$f(x) = (x^2 + 2x)/(x - 2)$ is

(*a*) one-one and onto

(*b*) one-one but not onto

(*c*) neither one-one nor onto

(*d*) not one-one but onto

57. If set $S = \{x \mid x$ is a student in your class$\}$ and relation $x \text{ R } y \leftrightarrow x$ sits in the same row as y, then which of the following statements about R is TRUE ?

(*a*) Reflexive only

(*b*) Transitive only

(*c*) Refleixve, transitvie and asymmetric

(*d*) Reflexive, transitive and symmetric

58. If set A has n elements, then number of functions that can be defined from A into A is

(*a*) n^2 (*b*) $n\,!$

(*c*) n^n (*d*) n

59. If $n \geq 2$, then number of surjections that can be defined from $\{1, 2, 3, ...n\}$ onto $\{1, 2\}$ is

(*a*) $2n$ (*b*) ${}^nP_2$

(*c*) 2^n (*d*) $2^n–2$

60. Number of bijective functions from set A to itself when A contains 106 elements, is

(*a*) 106 (*b*) 106^2

(*c*) 106 ! (*d*) 2^{106}

61. If $f : Z \to Z$ be defined as $f(x) = x^2$, $x \in Z$, then function f is

(*a*) bijection (*b*) injection

(*c*) surjection (*d*) none of these

62. A relation over the set $S = \{x, y, z\}$ is defined by : $\{(x, x), (x, y), (y, x), (x, z), (y, z), (y, y), (z, z)\}$.

What properties hold for this relation?

(*a*) Symmetric

(*b*) Reflexive

(*c*) Antisymmetric

(*d*) Irreflexive

63. A relation on the integers 0 through 4 is defined by :

$R = \{(x, y): x + y \leq 2x\}$.

Which of the properties listed below applies to this relation?

I. Tronsitivity

II. Symmetry

III. Reflexivity

(*a*) I only (*b*) III only

(*c*) both I and III (*d*) both II and III

64. The symmetric difference of sets A and B is

(*a*) $(A - B) \cup (B - A)$

(*b*) $(A \cup B) - (A \cap B)$

(*c*) $(A - B) \cap (B - A)$

(*d*) $(A \cap B) - (A \cup B)$

65. Order of the power set of a set of order n is

(*a*) n (*b*) $2n$

(*c*) n^2 (*d*) 2^n

66. If x, y are real numbers such that ordered pairs $(x + y, x - y)$ and $(2x + 3y, 3x - 2y)$ are equal, then (x, y) is equal to

(*a*) (1, 1) (*b*) (2, 3)

(*c*) (3, – 2) (*d*) (0, 0)

67. $f : R \to R$ is a function defined by $f(x) = 10\,x - 7$. If $g = f^{-1}$, then $g(x) =$

(*a*) $\dfrac{1}{10x - 7}$ (*b*) $\dfrac{1}{10x + 7}$

(*c*) $\dfrac{x + 7}{10}$ (*d*) $\dfrac{x - 7}{10}$

68. If $|A| = m$ and $|B| = n$, then number of possible relations $R : A \to B$ is

(*a*) mn (*b*) $(mn)^2$

(*c*) 2^{m+n} (*d*) 2^{mn}

69. Assume R and S are (non empty) relations in a set A. Which of the following given relations is FALSE ?

(*a*) If R and S are transitive, then $R \cup S$ is transitive

(*b*) If R and S are transitive, then $R \cap S$ is transitive

(*c*) If R and S are symmetric, then $R \cup S$ is symmetric

(*d*) If R and S are reflexive, then $R \cup S$ is reflexive

70. If R be a relation from

$A = \{1, 2, 3, 4\}$ to $B = \{1, 3, 5\}$.

i.e. $(a, b) \in R$ iff $a < b$, then RoR^{-1} is

(*a*) $\{(1, 3); (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$

(*b*) $\{(3, 1), (5, 1) (3,2), (5,2), (5,3), (5,4)\}$

(*c*) $\{(3, 3), (3, 5), (5, 3), (5, 5)\}$

(*d*) $\{(3, 3), (3, 4), (4, 5)\}$

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71. If $f : R \to R$ defined by $f(x) = x^2 + 1$, then values of $f^{-1}(17)$ and $f^{-1}(-3)$ are respectively

(*a*) $\{\phi\}, (4, -4\}$ (*b*) $\{3, -3\}, \{\phi\}$

(*c*) $\{\phi\}, \{3, -3\}$ (*d*) $\{4, -4\}, \phi$

72. If function $f : R \to R$ is given by $f(x) = x^2 + 2x - 3$ and function $g : R \to R$ is gien by $g(x) = 3x - 4$,

then $(gof)(x)$ is given by

(*a*) $9x^2 + 18x + 5$ (*b*) $3x^2 + 6x - 13$

(*c*) $x^2 + x - 7$ (*d*) $x^2 - 5x - 1$

73. With respect to the relation '*x divides y*', which of the following sets are totally ordered ?

I. {36, 3, 9}

II. {7, 77, 11}

III. (3,6, 24, 12}

IV. {1, 2, 3, ...}

(*a*) I, II and III only (*b*) II and III only

(*c*) I and III only (*d*) All of these

74. If Q be the set of non-zero rational numbers and the relation R be defined over the set Q by

xRy if $x = 1/y$, $x, y \in Q$, then

(*a*) R is an equivalence relation

(*b*) R is reflexive

(*c*) R is symmetric

(*d*) R is transitive

75. Let n(A) denotes the number of elements in set A. If $n(A) = p$ and $n(B) = q$, then how many ordered pairs (a, b) are there with $a \in A$ and $b \in B$?

(*a*) p^2 (*b*) $p \times q$

(*c*) $p + q$ (*d*) $2pq$

76. Which of the following relations are functions?

I. $N = \{(x, y)/y = x^2, x \in \{-1, 0, 1, 2, 3\}\}$

II. $P = \{(x, y)/y^2 = x, x \in \{4, 9, 16\}\}$

III. $Q = \{(x, y)/y = 4x^2 - 14, x \in \{-1, 1, 2, 3\}\}$

(*a*) I only (*b*) I and II only

(*c*) I and III only (*d*) III only

77. In a group of 72 students, 47 have background is Electronics, 59 have background in Mathematics and 42 have background in both the subjects. How many students do not have background in any of the subjects ?

(*a*) 8 (*b*) 13

(*c*) 25 (*d*) 34

78. If $[x]$ denotes integer part of the real number, then the function $f(x) = x - [x]$ is/an

(*a*) even function (*b*) odd function

(*c*) periodic function (*d*) constant

79. To have inverse for the function f, f is

(*a*) one one (*b*) onto

(*c*) one one onto (*d*) identity function

80. Which of the following set is closed under addition, subtraction and multiplication?

(*a*) $A = \{x \mid x = 2^n, n \in N\}$

(*b*) $B = \{x \mid x = 3n, n \in N\}$

(*c*) $C = \{x \mid x = 3n, n \in Z\}$

(*d*) All of these

81. The set of all Equivalence classes of a set A of cardinality C

(*a*) has the same cardinality as A

(*b*) forms a partition of A

(*c*) is of cardinality 2C

(*d*) is of cardinality C^2

82. Which of the following sets is a null set ?

I. $X = \{ x \mid x = 9, 2x = 4\}$

II. $Y = \{ x \mid x = 2x. x \neq 0\}$

III. $Z = \{ x \mid x - 8 = 4\}$

(*a*) I and II only (*b*) I, II and III

(*c*) I and III only (*d*) II and III only

83. Let R be a relation "$(x - y)$ is divisible by m", where x, y, m are integers and $m > 1$, then R is

(*a*) symmetric but not transitive

(*b*) partial order

(*c*) equivalence relation

(*d*) anti symmetric and not transitive

84. Let $s(w)$ denote the set of all the letters in w where w is an English word. Let us denote set equality, subset and union relations by $=$, $\subset$ and $\cup$ respectively.

Which of the following is NOT true?

(*a*) $s(\text{ten}) \subset s(\text{twenty})$

(*b*) $s(\text{stored}) = s(\text{sorted})$

(*c*) $s(\text{sixty}) \subset (s(\text{six}) \cup s(\text{twenty})$

(*d*) None of these

85. A relation R is defined on the set of positive integers as xRy if $2x + y \leq 5$.

The relation R is

(*a*) reflexive (*a*) symmetric

(*a*) transitive (*d*) none of these

86. Let R be an equivalence relation on the set {1, 2, 3, 4, 5, 6} given by

{(1, 1), (1, 5), (2, 2), (2, 3), (2, 6) (3, 2), (3, 3), (3, 6), (4, 4), (5, 1), (5, 5), (6, 2), (6, 3), (6, 6)}.

The partition induced by R is

(*a*) {1, 2, 3, 4, 5, 6} (*b*) {{1, 3, 5, 6}, {2, 4}}

(*c*) {{1, 5}, {2, 3, 6}, {4}} (*d*) {{1, 2, 3, 4}, {5, 6}}

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87. Let Z denote the set of all integers.

Define $f : Z \to Z$ by

$$f(x) = \begin{cases} x/2 & (x \text{ is even}) \\ 0 & (x \text{ is odd}) \end{cases},$$

then f is

(*a*) onto but not one-one

(*b*) one-one but not onto

(*c*) one-one and onto

(*d*) neither one-one nor-onto

88. If $f : A \to B$ is a bijective function, then f^{-1} of f =

(*a*) $f o f^{-1}$

(*b*) f

(*c*) f^{-1}

(*d*) I_A (Identity map of the set A)

89. The matrices $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ commute under the multiplication

(*a*) if $a = b$ (or) $\theta = n\pi$, n is an integer

(*b*) always

(*c*) never

(*d*) if $a \cos\theta \neq b \sin\theta$

90. The set of all real numbers under the usual multiplication operation is not a group since

(*a*) multiplication is not a binary operation

(*b*) multiplication is not associative

(*c*) identity element does not exist

(*d*) zero has no inverse

91. The set of integers Z with the binary operation '*' defined as $a * b = a + b + 1$ for $a, b \in Z$, is a group. The identity element of this group is

(*a*) 0 (*b*) 1

(*c*) – 1 (*d*) 12

92. The identity element in the group $G = \left\{\begin{bmatrix} x & x \\ x & x \end{bmatrix} : x \in R, x \neq 0\right\}$ with respect to matrix multiplication is

(*a*) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (*b*) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

(*c*) $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ (*d*) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

93. (Z, *) is a group with $a * b = a + b + 1$, $\forall a, b \in Z$. The inverse of a is

(*a*) 0 (*b*) – 2

(*c*) $a - 2$ (*d*) $-a - 2$

94. In the group (G, .), the value of $(a^{-1} b)^{-1}$ is

(*a*) ab^{-1} (*b*) $b^{-1}a$

(*c*) $a^{-1}b$ (*d*) ba^{-1}

95. If (G, .) is a group such that $a^2 = e$, $\forall a \in G$, then G is

(*a*) semi group (*b*) abelian group

(*c*) non-abelian group (*d*) none of these

96. If (G, .) is a group, such that $(ab)^2 = a^2b^2\ \forall a, b \in G$, then G is a/an

(*a*) commutative semi group

(*b*) abelian group

(*c*) non-abelian group

(*d*) none of these

97. If (G, .) is a group such that $(ab)^{-1} = b^{-1} a^{-1}$, $\forall a, b \in G$, then G is a/an

(*a*) commutative semi group

(*b*) abelian group

(*c*) non-abelian group

(*d*) none of these

98. Let G denoted the set of all $n \times n$ non-singular matrices with rational numbers as entries. Then under multiplication G is a/an

(*a*) subgroup

(*b*) finite abelian group

(*c*) infinite, non abelian group

(*d*) infinite, abelian

99. The set of all n^{th} roots of unity under multiplication of complex numbers form a/an

(*a*) semi group with identity

(*b*) commutative semigroups with identity

(*c*) group

(*d*) abelian group

100. If the binary operation * is defined on a set of ordered pairs of real numbers as

$(a, b) * (c, d) = (ad + bc, bd)$

and is associative, then

(1, 2) * (3, 5) * (3, 4) equals

(*a*) 74, 40) (*b*) (32, 40)

(*c*) (23, 11) (*d*) (7, 11)

101. If every element of a group G is its own inverse, then G is

(*a*) finite (*b*) infinite

(*c*) cyclic (*d*) abelian

102. A subset H of a group (G, *) is a group if

(*a*) $a, b \in H \Rightarrow a * b \in H$

(*b*) $a \in H \Rightarrow a^{-1} \in H$

(*c*) $a, b \in H \Rightarrow a * b^{-1} \in H$

(*d*) H contains the identity element.

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103. G$\{e, a, b, c\}$ is an abelian group with 'e' as identity element. The order of the other elements are

(*a*) 2, 2, 3 (*b*) 3, 3, 3

(*c*) 2, 2, 4 (*d*) 2, 3, 4

104. Which of the following statements is true?

(*a*) Every equivalence relation is a partial-ordering relation.

(*b*) Number of relations form A = $\{x, y, z\}$ to B = {1, 2}, is 64.

(*c*) Empty relation ϕ is reflexive.

(*d*) Properties of a relation being symmetric and being ant-symmetric are negative of each other.

105. Which of the following statements is false ?

(*a*) If R is reflexive, then $R \cap R^{-1} \neq \phi$.

(*b*) $R \cap R^{-1} \neq \phi \Rightarrow$ R is anti-symmetric.

(*c*) If R, R′ are equivalence relations in a set A, then $R \cap R'$ is also an equivalence relation in A.

(*d*) If R, R′ are reflexive relations in A, then R – R′ is reflexive.

106. Let A = {1, 2, 3.........}.

Define ~ by $x \sim y \Leftrightarrow x$ divides y. Then ~ is

(*a*) reflexive, but not a partial-ordering

(*b*) symmetric

(*c*) an equivalence relation

(*d*) a partial-ordering relation

107. If A = {1, 2, 3}, then relation S = {(1, 1), (2, 2) is

(*a*) symmetric only

(*b*) anti-symmetric only

(*c*) both symmetric and anti-symmetric

(*d*) an equivalence relation

108. If A = {1, 2, 3, 4}. Let ~ = {(1, 2), (1, 3), (4, 2). Then ~ is

(*a*) not anti-symmetric

(*b*) transitive

(*c*) reflexive

(*d*) symmetric

109. If R = {(1, 2),(2, 3),(3, 3)} be a relation defined on A = {1, 2, 3}, then R . R (= R^2) is

(*a*) R itself

(*b*) {(1, 2),(1, 3),(3, 3)}

(*c*) {(1, 3),(2, 3),(3, 3)}

(*d*) {(2, 1),(1, 3),(2, 3)}

110. The universal relation A × A on A is

(*a*) an equivalence relation

(*b*) anti-symmetric

(*c*) a partial ordering relation

(*d*) not symmetric and not anti-symmetric

111. Total number of different partitions of a set having four elements is

(*a*) 16 (*b*) 8

(*c*) 15 (*d*) 4

112. A partition of {1, 2, 3, 4, 5} is the family

(*a*) {(1, 2),(3, 4),(3, 5)} (*b*) {ϕ(1, 2),(3, 4),(5)}

(*c*) {(1, 2, 3),{5}} (*d*) {{1, 2,} {3, 4, 5}}

113. The less than relation, <, on reals is

(*a*) a partial ordering since it is asymmetric and reflexive.

(*b*) a partial ordering since it is anti-symmetric and reflexive.

(*c*) not a partial ordering because it is not asymmetric and not reflexive.

(*d*) not a partial ordering because it is not anti-symmetric and not reflexive.

114. Hasse diagrams are drawn for

(*a*) partially ordered sets

(*b*) lattices

(*c*) boolean Algebra

(*d*) none of these

115. A self-complemented, distributive lattice is called

(*a*) Boolean algebra

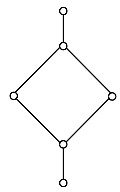
(*b*) Modular lattice

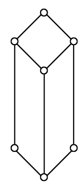
(*c*) Complete lattice

(*d*) Self dual lattice

116. Which of the following lattices are Boolean ?

(*a*)

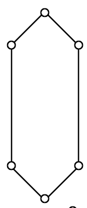
(*b*) 

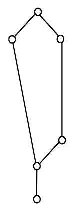
(*c*) 

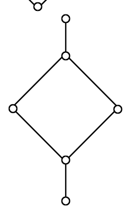
(*d*) None of these

117. Which of the following is not a "lattice" ?

(*a*)

(*b*) 

(*c*) 

(*d*) 

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118. Let D_{30} = {1, 2, 3, 4, 5, 6, 10, 15, 30} and relation I be partial ordering on D_{30}. The all lower bounds of 10 and 15 respectively are

(*a*) 1, 3 (*b*) 1, 5

(*c*) 1, 3, 5 (*d*) none of these

119. Complement of each element of D_6 in $[D_6 ; \vee, \wedge]$, is

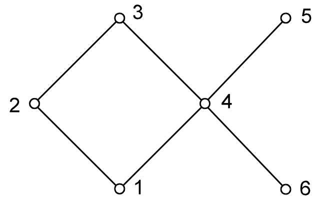
(*a*) $\overline{1} = 2, \overline{2} = 3, \overline{3} = 2, \overline{6} = 1$

(*b*) $\overline{1} = 6, \overline{2} = 3, \overline{3} = 6, \overline{6} = 1$

(*c*) $\overline{1} = 6, \overline{6} = 1, \overline{3} = 2, \overline{2} = 3$

(*d*) none of these

120. Maximal and minimal elements of the Poset are



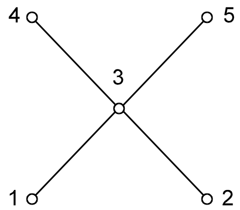
(*a*) Maximal 5, 6; Minimal 2

(*b*) Maximal 5, 6; Minimal 1

(*c*) Maximal 3, 5; Minimal 1, 6

(*d*) None of these

121. The greatest and least elements of the Poset are



(*a*) greatest 4, 5; least 1, 2

(*b*) greatest 5; least 1

(*c*) greatest None; least none

(*d*) none of these

122. Different partially ordered sets may be represented by the same Hasse diagram if they are

(*a*) same

(*b*) lattices with same order

(*c*) isomorphic

(*d*) order-isomorphic

123. The absorption law is defined as

(*a*) $a * (a * b) = b$

(*b*) $a * (a \oplus b) = b$

(*c*) $a * (a * b) = a \oplus b$

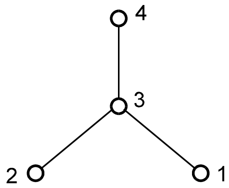
(*d*) $a * (a \oplus b) = a$

124. If lattice (C, ≤) is a complemented chain, then

(*a*) $|C| \le 1$ (*b*) $|C| \le 2$

(*c*) $|C| > 1$ (*d*) C doesn't exist

125. If set A = {1, 2, 3, 4}, then ordered pairs in the relation determined by the Hasse diagram in the figure given below, are described as



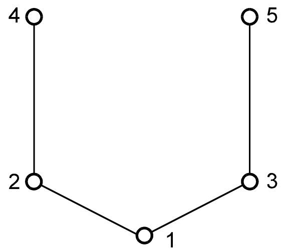
(*a*) {(1, 1), (2, 2), (3, 3), (4, 4), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}

(*b*) (1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}

(*c*) both (*a*) and (*b*)

(*d*) none of these

126. Matrix of the partial order whose Hasse diagram is given below is



(*a*) $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (*b*) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(*c*) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (*d*) None of these

127. Principle of duality is defined as

(*a*) ≤ is replaced by ≥

(*b*) LUB becomes GLB

(*c*) all properties are unaltered when ≤ is replaced by ≥

(*d*) all properties are unaltered when ≤ is replaced by ≥ other than 0 and 1 element.

128. Every finite subset of a lattice has

(*a*) a LUB and a GLB

(*b*) many LUBs and a GLB

(*c*) many LUBs and many GLBs

(*d*) either some LUBs or some GLBs

129. In any undirected graph, the sum of degrees of all the nodes

(*a*) must be even

(*b*) is twice the number of edges

(*c*) must be ODD

(*d*) need not be EVEN

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130. Number of vertices of ODD degree in a graph is
(*a*) always EVEN
(*b*) always ODD
(*c*) either EVEN or ODD
(*d*) always ZERO

131. A graph in which all nodes are of equal degree is called
(*a*) multi graph (*b*) non regular graph
(*c*) regular graph (*d*) complete graph

132. Maximum degree of any node in a simple graph with n vertices is
(*a*) $n - 1$ (*b*) n
(*c*) $\frac{n}{2}$ (*d*) $n - 2$

133. Two isomorphic graphs must have
(*a*) same number of vertices
(*b*) same number of edges
(*c*) equal number of vertices
(*d*) all of these

134. A given connected graph G is a Eular graph if and only if all vertices of G are of
(*a*) same degree (*b*) even degree
(*c*) odd degree (*d*) different degree

135. Total number of edges in a complete graph of n vertices is
(*a*) n (*b*) $\frac{n}{2}$
(*c*) $\frac{n(n-1)}{2}$ (*d*) $\frac{n(n+1)}{2}$

136. A tree with 'n' nodes has
(*a*) $\frac{n}{2}$ edges (*b*) n edges
(*c*) $n - 1$ edges (*d*) $n + 1$ edges

137. The number of circuits in a tree with 'n' nodes is
(*a*) Zero (*b*) One
(*c*) $n - 1$ (*d*) $\frac{n}{2}$

138. A graph is a tree if and only if it
(*a*) is completely connected
(*b*) is minimally connected
(*c*) contains a circuit
(*d*) is planar

139. A graph with 'n' vertices and $n - 1$ edges that is not a tree, is
(*a*) connected (*b*) disconnected
(*c*) eular (*d*) a circuit

140. A complete graph with "n" vertices is
(*a*) 2-chromatic (*b*) $\frac{n}{2}$-chromatic
(*c*) $(n-1)$-chromatic (*d*) n-chromatic

141. Number of different rooted labeled trees with n vertices is
(*a*) 2^{n-1} (*b*) 2^n
(*c*) n^{n-1} (*d*) n^n

142. T is a graph with n vertices. T is connected and has exactly $n - 1$ edges, then
(*a*) T is a tree
(*b*) T contains no cycles
(*c*) every pair of vertices in T is connected by exactly one path
(*d*) addition of a new edge will create a cycle.

143. The length of a Hamiltanian path (if exists) in a connected graph of n vertices is
(*a*) $n - 1$ (*b*) n
(*c*) $n + 1$ (*d*) $\frac{n}{2}$

144. A simple graph in which there exists an edge between every pair of vertices is called
(*a*) complete graph (*b*) eular graph
(*c*) planner graph (*d*) regular graph

145. If a graph requires k different colours for its proper colouring, then chromatic number of the graph is
(*a*) 1 (*b*) k
(*c*) $k - 1$ (*d*) $\frac{k}{2}$

146. Degree of each vertex in K_n is
(*a*) n (*b*) $n - 1$
(*c*) $n - 2$ (*d*) $2n - 1$

147. The parity of vertex D will be
(*a*) odd (*b*) even
(*c*) uncertain (*d*) none of these

148. The number of ways in which a team of eleven players can be selected from 22 players including 2 of them and excluding 4 of them is
(*a*) ${}^{16}C_{11}$ (*b*) ${}^{16}C_5$
(*c*) ${}^{16}C_9$ (*d*) ${}^{20}C_9$

149. In how many ways can 5 red and 4 white balls be drawn from a bag containing 10 red and 8 white balls ?
(*a*) ${}^8C_5 \times {}^{10}C_4$ (*b*) ${}^{10}C_5 \times {}^8C_4$
(*c*) ${}^{18}C_9$ (*d*) None of these

150. How many 10 digits numbers can be written by using the digits 1 and 2 ?
(*a*) ${}^{10}C_1 + {}^9C_2$ (*b*) 2^{10}
(*c*) ${}^{10}C_2$ (*d*) 10 !

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151. The number of words that can be formed out of the letters of the word 'COMMITTEE' is

(a) $\frac{9!}{(2!)^3}$ (b) $\frac{9!}{(2!)^2}$

(c) $\frac{9!}{2!}$ (d) 9 !

152. How many different committees of 5 can be formed from 6 men and 4 women on which exact 3 men and 2 women serve ?

(a) 6 (b) 20

(c) 60 (d) 120

153. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then number of ways in which they can be seated is

(a) $\frac{m!n!}{(m+n)!}$ (b) $\frac{m!(m+1)!}{(m-n+1)!}$

(c) $\frac{m!n!}{(m-n+1)!}$ (d) none of these

154. There are $(n + 1)$ white and $(n + 1)$ black balls each set numbered 1 to $n + 1$. The number of ways in which the balls can be arranged in a row so that adjacent balls are of different colours, is

(a) $(2n + 2)!$ (b) $(2n + 2)! \times 2$

(c) $(n + 1)! \times 2$ (d) $2\{(n + 1)!\}^2$

155. The number of ways in which 10 persons can sit around a circular table so that none of them has the same neighbours in any two arrangements, is

(a) 9 ! (b) $\frac{1}{2}(9!)$

(c) 10 ! (d) $\frac{1}{2}(10!)$

156. In the above problem, if the first digit cannot be 0, then

(a) 42100 (b) 421200

(c) 42400 (d) 421600

157. Six teachers and six students have to sit round a circular table such that there is a teacher between any two students. The number of ways in which they can sit is

(a) $6! \times 6!$ (b) $5! \times 6!$

(c) $5! \times 5!$ (d) none of these

158. In how many ways can 10 people be seated in a row so that a given pair is not next to each other?

(a) $10! - 9!$

(b) $10! - 2(9!)$

(c) $9!$

(d) none of these

159. The number of ways in which three girls and nine boys can be seated in two vans each having numbered seats with four in the back and three in the front ?

(a) 7! (b) 2 (7!)

(c) 7 (13!) (d) 12!

160. Suppose a licence plate contains two letters followed by three digits with the first digit not zero. How many different licence plates can be printed ?

(a) 608000 (b) 608200

(c) 608400 (d) 608600

NUMERICAL TYPE QUESTIONS

1. Number of subsets of a set of order three is ________
2. Number of proper subsets of a set of order three is ________
3. The number of distinct relations on a set of 3 elements is ________
4. The number of elements in the power set of the set $\{\{a, b\}, \{c\}\}$ is ________
5. The number of distinct reflexive and symmetric relations that can be defined on a 3 element set is ________
6. If P(A) be the collection of all subsets of A = $\{a, b, c\}$ and R be a relation defined as *"x is disjoint from y"* over P(A), then number of elements in R is ________
7. If set A has 3 elements and set B has 4 elements, then number of injections that can be defined from A into B is ________
8. If sets A and B have 3 and 6 elements each, then minimum number of elements in A $\cup$ B is ________
9. In a beauty contest, half the number of experts voted for Mr. A and two thirds voted for Mr. B. 10 voted for both and 6 did not vote for either. How many experts were there in all ?
10. In a classroom containing 28 people, there are 18 people who speak English, 15 people who speak Hindi and 22 people who speak Telugu, 9 people speak both English and Hindi, while 1 people speak both Hindi and Telugu. Further 13 persons speak both Telugu and English. How many people speak all three languages ?
11. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} x+2 & (x \le -1) \\ x^2 & (-1 \le x \le 1) \\ 2-x & (x \ge 1) \end{cases}.$$

Then value of $f(-1.75) + f(0.5) + f(1.5)$ is ________

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12. If * is defined on R* as $a * b = \frac{ab}{2}$, then identity element in the group (R*, *) is ________

13. The inverse of $-i$ in the multiplicative group, $\{1, -1, i, -i\}$ is ________

14. In the group G = {2, 4, 6, 8} under multiplication modulo 10, the identity element is ________

15. If a, b are positive integers, define $a * b = \alpha$ where $ab \equiv \alpha$ (modulo 7), with this * operation, then inverse of 3 in group G {1, 2, 3, 4, 5, 6} is ________

16. Let D_{30} = {1, 2, 3, 5, 6, 10, 15, 30} and relation I be a partial ordering on D_{30}. The all upper bounds of 10 and 15 respectively is ________

17. Let D_{30} = {1, 2, 3, 5, 6, 10, 15, 30} and relation I be a partial ordering on D_{30}. The lub of 10 and 15 respectively is ________

18. The minimum number of spanning trees in a connected graph with '*n*' nodes is ________

19. The number of colours required to properly colour the vertices of every planar graph is ________

20. If G and G* are isomorphic graphs, then number of connected components of G* if G has connected components, are ________

21. If v is an isolated vertex in a graph (multigraph) G, then the degree is ________

22. The degree of vertex D in G will be ________

23. The number of different permutations of the word BANANA is ________

24. The number of diagonals that can be drawn by joining the vertices of an octagon is ________

25. In an examination there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct is ________

26. Ramesh has 6 friends. In how many ways can he invite one or more of them at a dinner ? ________

27. The number n of ways that an organization consisting of twenty-six members can elect a president, treasurer, and secretary (assuming no person is elected to more than one position) is ________

28. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to ________

29. The ______ number of ways can 5 prizes be distributed among 4 boys when every boy can take one or more prizes

30. The number of different signals which can be given from 6 flags of different colours taking one or more at a time, is ________

31. The number of ways to arrange the letters of the word CHEESE are________

32. The number of words from the letters of the word 'BHARAT' in which B and H will never come together, is________

33. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is ________

34. If a five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5, without repetition, then total number of ways this can be done is ________

35. If sides AB, BC, CA of a triangle ABC have 3, 4 and 5 interior points respectively on them, then total number of triangles that can be constructed by using these points as vertices, is ________

36. A student can take one of four Mathematics sections and one of five English sections. The number n of ways he can register for the two courses, is ________

37. In a football championship, 153 matches were played. Every team played one match with each other. The number of teams participating in the champion-ship is ________

38. There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is ________

39. Three persons enter a railway compartment. If there are 5 seats vacant, _____ number of ways can they take these seats

40. Number n of license plates that can be made where each plate contains two distinct letters followed by three different digits is ________

41. The number of ways to cut a six sided convex polygon whose vertices are labeled into four triangles using diagonal lines that do not cross is ________

42. There are three identical red balls and four identical blue balls in a bag. Three balls are drawn. The number of different colour combinations is ________

43. The ______ number of five different letter words can be formed out of the word "LOGARITHMS"

44. The ______ number of ways can 5 men and 3 women be seated in a row so that two women cannot sit side by side

45. If 6 identical coins are arranged in a row, then number of arrange-ments in which 4 are heads and 2 are tails are ________

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EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. Which one of the following is NOT equivalent to $p \leftrightarrow q$?

(*a*) $(\rceil p \vee q) \vee (p \vee \rceil q)$

(*b*) $(\rceil p \vee q) \vee (q \to p)$

(*c*) $(\rceil p \wedge q) \vee (p \wedge \rceil q)$

(*d*) $(\rceil p \wedge \rceil q) \vee (p \wedge q)$

2. Consider the following two statements.

S1 : if a candidate is known to be corrupt, then he will not be elected

S2 : if a candidate is kind, he will be elected

Which one of the following statements follows from S_1 and S_2 per sound interference rules of logic?

(*a*) If a person is known to corrupt, he is kind

(*b*) If a person is not known to be corrupt, he is not kind

(*c*) If a person is kind, he is not known to be corrupt

(*d*) If a person is not kind, he is not known to be corrupt

3. Which one of the following well formed formulae is tautology ?

(*a*) $\forall x \exists y \mathrm{R}(x, y) \leftrightarrow \exists y\ \forall x \mathrm{R}(x, y)$

(*b*) $\left(\forall x\left[\exists y \mathrm{R}(x, y) \to (x, y)\right]\right) \to \forall x \exists y \mathrm{S}(x, y)$

(*c*) $\left[\left(\forall x \exists y(p(x, y) \to \mathrm{R}(x, y))\right)\right]$

$\to \left[\forall x \exists y(\neg \mathrm{P}(x, y) \mathrm{V} \mathrm{R}(x, y)\right]$

(*d*) $\forall x \forall y p(x, y) \to \forall x \forall y p(y, x)$

4. Suppose U is the power set of the set S = {1, 2, 3, 4, 5, 6}. For any T □ U, let |T| denote the number of element in T and T' denote the complement of T. For any T, R □ U, let T\R be the set of all elements in T which are not in R. Which one of the following is true ?

(*a*) $\forall \mathrm{X} \in \mathrm{U}\left(|\mathrm{X}| = |\mathrm{X}'|\right)$

(*b*) $\exists \mathrm{X} \in \mathrm{U} \exists \mathrm{Y} \in \mathrm{U}\left(|\mathrm{X}| = 5, |\mathrm{Y}| = 5 \text{ and } \mathrm{X} \cap = \mathrm{Y} = \phi\right)$

(*c*) $\forall \mathrm{X} \in \mathrm{U} \forall \mathrm{Y} \in \mathrm{U}\left(|\mathrm{X}| = 2, |\mathrm{Y}| = 3 \text{ and } \mathrm{X}/\mathrm{Y} = \phi\right)$

(*d*) $\forall \mathrm{X} \in \mathrm{U} \forall \mathrm{Y} \in \mathrm{U}\left(\mathrm{X}/\mathrm{Y} = \mathrm{Y}'/\mathrm{X}'\right)$

5. For a set A, the power set of A is denoted by 2^A. If A = {5, {6}, {7}}, which of the following options are TRUE ?

I. $\phi \in 2^A$ II. $\phi \subseteq 2^A$

III. $\{5, \{6\}\} \in 2^A$ IV. $\{5\{6\}\} \subseteq 2^A$

(*a*) I and III only

(*b*) II and III only

(*c*) I, II and III only

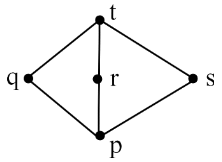
(*d*) I, II and IV only

6. If $g(x) = 1 - x$ and $h(x) = \dfrac{x}{x-1}$, then $\dfrac{g(h(x))}{h(g(x))}$ is

(*a*) $\dfrac{h(x)}{g(x)}$ (*b*) $\dfrac{-1}{x}$

(*c*) $\dfrac{g(x)}{h(x)}$ (*d*) $\dfrac{x}{(1-x)^2}$

7. Suppose L = {p, q, r, s, t} is a lattice represented by the following Hasse diagram :



For any $x, y \in$ L, not necessarily distinct, $x \vee y$ and $x \wedge y$ are join and meet of x, y respectively. Let $L^3 = \{(x, y, z) : x, y, z \in L\}$ be the set of all ordered triplets of the elements of L. Let p_r be the probability that an element $(x, y, z) \in L^3$ chosen equiprobably satisfies $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$. Then

(*a*) $p_r = 0$ (*b*) $p_r = 1$

(*c*) $0 < p_r \leq \dfrac{1}{5}$ (*d*) $\dfrac{1}{5} < p_r < 1$

8. Let R be the relation on the set of positive integers such that a aRb if and only if a and b are distinct and have a common divisor other than 1. Which one of the following statements about R is true?

(*a*) R is symmetric and reflexive but not transitive

(*b*) R is reflexive but not symmetric and not transitive

(*c*) R is transitive but not reflexive and not symmetric

(*d*) R is symmetric but not reflexive and not transitive

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9. Let R be a relation on the set of ordered pairs of positive integers such that $((p, q), (r, s)) \in R$ if and only if $p - s = q - r$. Which one of the following is true about R ?

(*a*) Both reflexive and symmetric

(*b*) Reflexive but not symmetric

(*c*) Not reflexive but symmetric

(*d*) Neither reflexive nor symmetric

2014

10. Consider the statement :

"Not all that glitters is gold"

Predicate *glitters* (x) is true if x glitters and predicate *gold* (x) is true if x is gold. Which one of the following logical formulae represents the above statement?

(*a*) $\forall x : glitters\ (x) \Rightarrow \neg\, gold\ (x)$

(*b*) $\forall x : gold\ (x) \Rightarrow glitters\ (x)$

(*c*) $\exists x : gold\ (x) \wedge \neg\, glitters\ (x)$

(*d*) $\exists x : glitters\ (x) \wedge \neg\, gold\ (x)$

11. Which one of the following propositional logic formulas is TRUE when exactly two of p, q, and r are TRUE?

(*a*) $((p \leftrightarrow q) \wedge r) \cup (p \wedge q \wedge \sim r)$

(*b*) $(\sim (p \leftrightarrow q) \wedge r) \cup (p \wedge q \wedge \sim r)$

(*c*) $((p \rightarrow q) \wedge r) \cup (p \wedge q \wedge \sim r)$

(*d*) $(\sim (p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

12. Which one of the following Boolean expressions is NOT a tautology?

(*a*) $((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c)$

(*b*) $(a \leftrightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c))$

(*c*) $(a \wedge b \wedge c) \rightarrow (c \vee a)$

(*d*) $a \rightarrow (b \rightarrow a)$

13. Consider the following statements:

P: Good mobile phones are not cheap

Q: Cheap mobile phones are not good

L: P implies Q

M: Q implies P

N: P is equivalent to Q

Which one of the following about L, M, and N is CORRECT?

(*a*) Only L is TRUE.

(*b*) Only M is TRUE.

(*c*) Only N is TRUE.

(*d*) L, M and N are TRUE.

14. The CORRECT formula for the sentence, "not all rainy days are cold" is

(*a*) $\forall d$ (Rainy $(d) \wedge \sim$ Cold (d))

(*b*) $\forall d$ ($\sim$ Rainy $(d) \rightarrow$ Cold (d))

(*c*) $\exists$d($\sim$ Rainy (d) $\rightarrow$ Cold (d))

(*d*) $\exists d$ (Rainy $(d) \wedge \sim$ Cold (d))

15. Consider the following relation on subsets of the set S of integers between 1 and 2014. For two distinct subsets U and V of S we say U < V if the minimum element in the symmetric difference of the two sets is in U.

Consider the following two statements:

S1 : There is a subset of S that is larger than every other subset

S2 : There is a subset of S that is smaller than every other subset

Which one of the following is CORRECT?

(*a*) Both S1 and S2 are true

(*b*) S1 is true and S2 is false

(*c*) S2 is true and S1 is false

(*d*) Neither S1 nor S2 is ture

16. Let X and Y be finite sets and $f : X \rightarrow Y$ be a function. Which one of the following statements is TRUE?

(*a*) For any subsets A and B of X, $|f(A \cup B)| = |f(A)| + |f(B)|$

(*b*) For any subsets A and B of X, $f(A \cap B) = f(A) \cap f(B)$

(*c*) For any subsets A and B of X, $|f(A \cap B)| = \min\{|f(A)|, |f(B)|\}$

(*d*) For any subsets S and T of Y, $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

17. Consider the set of all functions

$f : \{0,1 ..., 2014\} \rightarrow \{0,1..., 2014\}$ such that

$f(f(i)) = i$, for all $0 \leq i \leq 2014$. Consider the following statements.

P. For each such function it must be the case the for every i, $f(i) = i$.

Q. For each such function it must be the case that for some i, $f(i) = i$.

R. Each such function must be onto.

Which one of the following is CORRECT?

(*a*) P, Q and R are true

(*b*) Only Q and R are true

(*c*) Only P and Q are true

(*d*) Only R is true

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2013

18. What is the logical translation of the following statement?

"None of my friends are perfect."

(*a*) $\exists x\ (F(x) \wedge \neg P(x))$

(*b*) $\exists x\ (\neg F(x) \wedge P(x))$

(*c*) $\exists x(\neg F(x) \wedge \neg P(x))$

(*d*) $\neg \exists x\ (F(x) \wedge P(x))$

19. Which one of the following is NOT logically equivalent to $\neg\exists x\ (\forall y(\alpha) \wedge \forall z\ (\beta))$?

(*a*) $\forall x\ (\exists z\ (\neg\beta) \rightarrow \forall y\ (\alpha))$

(*b*) $\forall x\ (\forall z\ (\beta) \rightarrow \exists y\ (\neg \alpha))$

(*c*) $\forall x\ (\forall y\ (\alpha) \rightarrow \exists z\ (\neg \beta))$

(*d*) $\forall x\ (\exists y\ (\neg \alpha) \rightarrow \exists z\ (\neg \beta))$

20. A binary operation $\oplus$ on a set of integers is defined as $x \oplus y = x^2 + y^2$. Which one of the following statements is TRUE about $\oplus$?

(*a*) Commutative but not associative

(*b*) Both commutative and associative

(*c*) Associative but not commutative

(*d*) Neither commutative nor associative.

2012

21. What is the correct translation of the following statement into mathematical logic?

"Some real numbers are rational"

(*a*) $\exists x$ (real(x) $\vee$ rational (x))

(*b*) $\forall x$ (real (x) $\rightarrow$ rational (x))

(*c*) $\exists x$ (real (x) $\wedge$ rational (x))

(*d*) $\exists x$ (rational (x) $\rightarrow$ real (x))

22. How many onto (or surjective) functions are there from an n-element ($n \geq 2$) set to a 2-element set?

(*a*) 2^n (*b*) $2^n - 1$

(*c*) $2^n - 2$ (*d*) $2(2^n - 2)$

2011

23. Which one of the following options is CORRECT given three positive integers x, y and z, and a predicate

$P(x) = \neg (x = 1) \wedge \forall y\ (\exists z(x = y * z) \Rightarrow (y = x) \vee (y = 1))$

(*a*) P(x) being true means that x is a prime number

(*b*) P(x) being true means that x is a number other than 1

(*c*) P(x) is always true irrespective of the value of x

(*d*) P(x) being true means that x has exactly two factors other than 1 and x

2010

24. Suppose the predicate F(x, y, t) is used to represent the statement that person x can fool person y at time t. Which one of the statements below expresses best the meaning of the formula

$\forall x \exists y \exists t\ (\neg F\ (x, y, t))$?

(*a*) Everyone can fool some person at some time

(*b*) No one can fool everyone all the time

(*c*) Everyone cannot fool some person all the time

(*d*) No one can fool some person at some time

25. What is the possible number of reflexive relations on a set of 5 elements?

(*a*) 2^{10} (*b*) 2^{15}

(*c*) 2^{20} (*d*) 2^{25}

26. Consider the set S = {1, ω, ω^2}, where ω and ω^2 are cube roots of unity. If * denotes the multiplication operation, then structure {S, *} forms

(*a*) a group (*b*) a ring

(*c*) an integral domain (*d*) a field

27. Let G = (V, E) be a graph. Define $\xi(G) = \sum_d i_d \times d$, where i_d is number of vertices of degree d in G. If S and T are two different trees with ξ(S) = ξ(T), then

(*a*) $|S| = 2|T|$

(*b*) $|S| = |T| - 1$

(*c*) $|S| = |T|$

(*d*) $|S| = |T| + 1$

28. In a binary tree with n nodes, every node has an odd number of descendants. Every node is considered to be its own descendant. What is the number of nodes in the tree that have exactly one child?

(*a*) 0 (*b*) 1

(*c*) $(n - 1)/2$ (*d*) $n - 1$

29. The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences can not be the degree sequence of any graph?

I. 7, 6, 5, 4, 4, 3, 2, 1

II. 6, 6, 6, 6, 3, 3, 2, 2

III. 7, 6, 6, 4, 4, 3, 2, 2

IV. 8, 7, 7, 6, 4, 2, 1, 1

(*a*) I and II (*b*) III and IV

(*c*) IV only (*d*) II and IV

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2009

30. Which one of the following is the most appropriate logical formula to represent the statement:

" Gold and silver ornaments are precious"

The following notations are used :

$G(x)$: x is a gold ornament.

$S(x)$: x is a silver ornament.

$P(x)$: x is precious.

(*a*) $\forall x(P(x) \to (G(x) \wedge S(x)))$

(*b*) $\forall x((G(x) \wedge S(x)) \to P(x))$

(*c*) $\exists x((G(x) \wedge S(x)) \to P(x))$

(*d*) $\forall x((G(x) \vee S(x)) \to P(x))$

31. Consider the following well-formed formulae:

I. $\neg\forall x(P(x))$

II. $\neg\exists x(P(x))$

III. $\neg\exists x(\neg P(x))$

IV. $\exists x(\neg P(x))$

Which of the above are equivalent?

(*a*) I and III (*b*) I and IV

(*c*) II and III (*d*) II and IV

32. Which one of the following is NOT neccessarily a property of a Group?

(*a*) Commutativity

(*b*) Associativity

(*c*) Existence of inverse for every element

(*d*) Existence of identity

33. Consider the binary relation

$R = \{(x, y), (x, z), (z, x), (z, y)\}$ on the set $\{x, y, z\}$.

Which one of the following is TRUE ?

(*a*) R is symmetric but NOT antisymmetric.

(*b*) R is NOT symmetric but antisymmetric.

(*c*) R is both symmetric and antisymmetric.

(*d*) R is neither symmetric nor antisymmetric.

34. For the composition table of a cyclic group shown below

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

Which one of the following choices is correct ?

(*a*) a, b are generators (*b*) b, c are generators

(*c*) c, d are generators (*d*) d, a are generators

35. The binary operation $\square$ is defined as follows:

P	Q	P $\square$ Q
T	T	T
T	F	T
F	T	F
F	F	T

Which one of the following is equivalent to $P \vee Q$?

(*a*) $\neg Q \square \neg P$ (*b*) $P \square \neg Q$

(*c*) $\neg P \square Q$ (*d*) $\neg P \square \neg Q$

36. What is the chromatic number of an n-vertex simple connected graph which does not contain any odd length cycle ? Assumen $n \geq 2$.

(*a*) 2 (*b*) 3

(*c*) n – 1 (*d*) n

37. Which one of the following is TRUE for any simple connected undirected graph with more than 2 vertices ?

(*a*) No two vertices have the same degree.

(*b*) At least two vertices have the same degree.

(*c*) At least three vertices have the same degree.

(*d*) All vertices have the same degree.

2008

38. Let fsa and pda be two predicates such that fsa(x) means x is a finite state automaton, and pda(y) means that y is a pushdown automaton. Let equivalent be another predicate such that equivalent (a, b) means a and b are equivalent. Which of the following first order logic statements represents the following

Each finite state automaton has an equivalent pushdown automaton.

(*a*) $(\forall x\, \text{fsa}(x)) \Rightarrow (\exists y\, \text{pda}(y) \wedge \text{equivalent}(x, y))$

(*b*) $\sim \forall y\, (\exists x\, \text{fsa}(x)) \Rightarrow \text{pda}(y) \wedge \text{equivalent}(x, y))$

(*c*) $\forall x\, \exists y\, (\text{fsa}(x)) \wedge \text{pda}(y) \wedge \text{equivalent}(x, y))$

(*d*) $\forall x\, \exists y\, (\text{fsa}(y)) \wedge \text{pda}(x) \wedge \text{equivalent}(x, y))$

39. P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(*a*) Only I and II

(*b*) Only I, II, and III

(*c*) Only I, II and IV

(*d*) All of I, II, III, and IV

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40. If P, Q, R are subsets of the universal set U, then $(P \cap Q \cap R) \cup (P^c \cap Q \cap R) \cup Q^c \cup R^c$ is

(*a*) $Q^c \cup R^c$ (*b*) $P \cup Q^c \cup R^c$

(*c*) $P^c \cup Q^c \cup R^c$ (*d*) U

2007

41. Let S be a set of *n* elements. The number of ordered pairs in the largest and the smallest equivalence relations on S are

(*a*) *n* and *n* (*b*) n^2 and *n*

(*c*) n^2 and 0 (*d*) *n* and 1

42. Consider the set S = {*a, b, c, d*}.

Consider following 4 partitions $\pi_1, \pi_2, \pi_3, \pi_4$ on S :

$\pi_1 = \{\overline{abcd}\}$,

$\pi_2 = \{\overline{ab}, \overline{cd}\}$,

$\pi_3 = \{\overline{abc}, \overline{d}\}$,

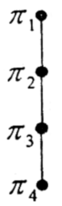
$\pi_4 = \{\overline{a}, \overline{b}, \overline{c}, \overline{d}\}$.

Let $\prec$ be partial order on the set of partitions

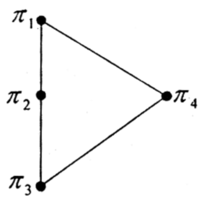
$S' = \{\pi_1, \pi_2, \pi_3, \pi_4\}$ defined as follows :

$\pi_i \prec \pi_j$ if and only if π_i refines π_j. The poset digram for $(S', \prec)$ is

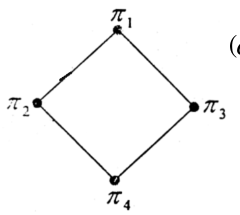
(*a*)



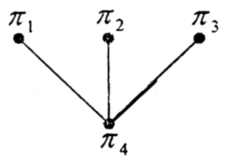
(*b*)



(*c*)



(*d*)



43. Consider set of (column) vectors defined by

$$X = \{x \in R^3 \mid x_1 + x_2 + x_3 = 0$$

where $x^T = [x_1, x_2, x_3]^T\}$.

Which of the following is **TRUE**?

(*a*) $\{[1, -1, 0]^T, [1, 0, -1]^T\}$ is a basis for the subspace X.

(*b*) $\{[1, -1, 0]^T, [1, 0, -1]^T\}$ is a linearly independent set, but it does not span X and therefore is not a basis of X

(*c*) X is not a subspace of R^3

(*d*) None of these

44. Let G be the non-planar graph with the minimum possible number of edges. Then G has

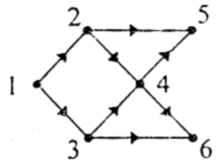
(*a*) 9 edges and 5 vertices

(*b*) 9 edges and 6 vertices

(*c*) 10 edges and 5 vertices

(*d*) 10 edges and 6 vertices

45. Consider the DAG with V = {1,2,3,4,5,6}, shown below.



Which of the following is **NOT** a topological ordering ?

(*a*) 1 2 3 4 5 6 (*b*) 1 3 2 4 5 6

(*c*) 1 3 2 4 6 5 (*d*) 3 2 4 1 6 5

46. Let *Graph*(*x*) be a predicate which denotes that *x* is a graph. Let *Connected*(*x*) be a predicate which denotes that *x* is connected. Which of the following first order logic sentences **DOES NOT** represent the statement; "*Not every graph is connected*"?

(*a*) $\neg\forall x\,(\text{Graph}(x) \Rightarrow \text{Connected}(x))$

(*b*) $\exists x\,(\text{Graph}(x) \wedge \neg\,\text{Connected}(x))$

(*c*) $\neg\forall x\,(\neg\,\text{Graph}(x)) \vee \text{Connected}(x))$

(*d*) $\forall x(\text{Graph}(x) \Rightarrow \neg\,\text{Connected}(x))$

47. Which of the following graphs has an Eulerian circuit ?

(*a*) Any *k*-regular graph where *k* is an even number

(*b*) A complete graph on 90 vertices

(*c*) The complement of a cycle on 25 vertices

(*d*) None of these

2006

48. Which one of the first order predicate calculus statements given below correctly expresses the following English statement ?

Tigers and lions attack if they are hungry or threatened.

(*a*) $\forall x\ [(\text{tiger}\ (x) \wedge \text{lion}\ (x)) \to \{(\text{hungry}\ (x) \vee \text{threatened}\ (x)) \to \text{attacks}\ (x)\}]$

(*b*) $\forall x\ [(\text{tiger}\ (x) \vee \text{lion}\ (x)) \to \{(\text{hungry}\ (x) \vee \text{threatened}\ (x)) \wedge \text{attacks}\ (x)\}]$

(*c*) $\forall x\ [(\text{tiger}\ (x) \vee \text{lion}\ (x)) \to \{(\text{attacks}\ (x) \to (\text{hungry}\ (x) \vee \text{threatened}\ (x))\}]$

(*d*) $\forall x\ [\text{tiger}\ (x) \vee \text{lion}\ (x)) \to \{(\text{hungry}\ (x) \vee \text{threatened}\ (x)) \to \text{attacks}\ (x)\}]$

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49. Consider the following propositional statements

$P_1 : ((A \wedge B) \rightarrow C)) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$

$P_2 : ((A \vee B) \rightarrow C)) \equiv ((A \rightarrow C) \vee (B \rightarrow C))$

Which one of the following is true ?

(*a*) P_1 is a tautology, but not P_2

(*b*) P_2 is a tautology, but not P_1

(*c*) P_1 and P_2 are both tautologies

(*d*) Both P_1 and P_2 are not tautologies

50. A logical binary relation ⊙, is defined as follows

A	B	A ⊙ B
True	True	True
True	False	True
False	True	False
False	False	True

Let ~ be the unary negation (NOT) operator, with higher precedence, then ⊙. Which one of the following is equivalent to $A \wedge B$?

(*a*) (~ A ⊙ B) (*b*) ~ (A ⊙ ~ B)

(*c*) ~ (~A ⊙ ~ B) (*d*) ~ (~ A ⊙ B)

51. Let, X, Y, Z be sets of sizes *x, y* and *z* respectively. Let W = X × Y and E be the set of all subsets of W. The number of functions from Z to E is

(*a*) z (*b*) $z \times 2^{xy}$

(*c*) z (*d*) 2^{xyz}

52. The set {1, 2, 3, 5, 7, 8, 9} under multiplication modulo 10 is not a group. Given below are four plausible reasons. Which one of them is false ?

(*a*) It is not closed

(*b*) 2 does not have an inverse

(*c*) 3 does not have an inverse

(*d*) 8 does not have an inverse

53. A relation R is defined on ordered pairs of integers as follows

(x, y) R (u, v) if $x < u$ and $y > v$.

Then R is

(*a*) neither a Partial Order nor an Equivalence Relation

(*b*) a Partial Order but not a Total Order

(*c*) a Total Order

(*d*) an Equivalence Relation

54. Let E, F and G be finite sets.

Let $X = (E \cap F) - (F \cap G)$ and

$Y = (E - (E \cap G)) - (E - F)$.

Which one of the following is true?

(*a*) $X \subset Y$

(*b*) $X \supset Y$

(*c*) $X = Y$

(*d*) $X - Y \neq \emptyset$ and $Y - X \neq \emptyset$

55. Let S = {1, 2, 3, ..., *m*} *m* > 3.

Let $X_1, ..., X_n$ be subsets of S each of size 3.

Define a function *f* from S to the set of natural numbers as, *f*(*i*) is the number of sets X_j that contain the element *i*, i.e. is $f(i) = |\{j | i \in X_j\}|$.

Then $\sum_{i=1}^{m} f(i)$ is

(*a*) $3m$ (*b*) $3n$

(*c*) $2m + 1$ (*d*) $2n + 1$

NUMERICAL TYPE QUESTIONS

2015

1. The cardinality of the power set of {0, 1, 2, ..., 10} is ______

2. The number of onto function (surjective function) from set X = {1, 2, 3, 4} to set Y = {a, b, c} is ______.

3. The number of 4 digit numbers having their digits in non-decreasing order (from left to right) constructed by using the digits belonging to the set {1, 2, 3} is _____.

2014

4. The base (or radix) of the number system such that the following equation holds is ___________.

$$\frac{312}{20} = 13.1$$

5. Let S denote the set of all functions $f : \{0, 1\}^4 \rightarrow \{0, 1\}$. Denote by N the number of functions from S to the set {0,1}. The value of $\log_2 \log_2 N$ is______.

6. Let G be a group with 15 elements. Let L be a subgroup of G. It is known that L ≠ G and that the size of L is at least 4. The size of L is _______.

7. If V_1 and V_2 are 4-dimensional subspaces of a 6-dimensional vector space V, then the smallest possible dimension of $V_1 \cap V_2$ is _______.

8. There are two elements *x*, *y* in a group (G, *) such that every element in the group can be written as a product of some number of *x*'s and *y*'s in some order. It is known that

$$x * x = y * y = x * y * x * y = y * x * y * x = e$$

where e is the identity element. The maximum number of elements in such a group is _________.

9. A pennant is a sequence of numbers, each number being 1 or 2. An n-pennant is a sequence of numbers with sum equal to n. For example, (1,1,2) is a 4-pennant. The set of all possible 1- pennants is {(1)}, the set of all possible 2-pennants is {(2), (1,1)}and the set of all 3-pennants is {(2,1), (1,1,1), (1,2)}. Note that the pennant (1, 2) is not the same as the pennant (2,1). The number of 10- pennants is ___________.

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ANSWERS

EXERCISE – I

MCQ Type Questions

1. (c)	**2.** (c)	**3.** (c)	**4.** (a)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (b)	**9.** (b)	**10.** (a,d)
11. (c,d)	**12.** (c)	**13.** (a)	**14.** (c)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (c, d)	**19.** (b)	**20.** (d)
21. (c)	**22.** (c)	**23.** (d)	**24.** (c)	**25.** (a)	**26.** (a)	**27.** (b,c)	**28.** (a,c)	**29.** (d)	**30.** (b,d)
31. (d)	**32.** (d)	**33.** (b,c)	**34.** (b)	**35.** (c)	**36.** (d)	**37.** (b)	**38.** (d)	**39.** (b)	**40.** (c)
41. (d)	**42.** (d)	**43.** (a)	**44.** (b)	**45.** (b)	**46.** (b)	**47.** (d)	**48.** (d)	**49.** (c)	**50.** (d)
51. (d)	**52.** (a)	**53.** (b)	**54.** (d)	**55.** (b)	**56.** (c)	**57.** (d)	**58.** (c)	**59.** (d)	**60.** (c)
61. (d)	**62.** (b)	**63.** (c)	**64.** (a,b)	**65.** (d)	**66.** (d)	**67.** (c)	**68.** (d)	**69.** (a)	**70.** (c)
71. (d)	**72.** (b)	**73.** (c)	**74.** (c)	**75.** (b)	**76.** (c)	**77.** (a)	**78.** (b)	**79.** (b)	**80.** (c)
81. (b)	**82.** (a)	**83.** (c)	**84.** (d)	**85.** (c)	**86.** (c)	**87.** (a)	**88.** (d)	**89.** (a)	**90.** (d)
91. (c)	**92.** (c)	**93.** (d)	**94.** (b)	**95.** (b)	**96.** (b)	**97.** (b)	**98.** (c)	**99.** (d)	**100.** (a)
101. (d)	**102.** (c)	**103.** (a)	**104.** (b)	**105.** (d)	**106.** (d)	**107.** (c)	**108.** (b)	**109.** (c)	**110.** (a)
111. (c)	**112.** (d)	**113.** (d)	**114.** (a, d)	**115.** (a)	**116.** (d)	**117.** (b)	**118.** (b)	**119.** (c)	**120.** (c)
121. (c)	**122.** (d)	**123.** (d)	**124.** (b)	**125.** (a)	**126.** (c)	**127.** (d)	**128.** (a)	**129.** (b)	**130.** (a)
131. (c)	**132.** (a)	**133.** (d)	**134.** (b)	**135.** (c)	**136.** (c)	**137.** (a)	**138.** (b)	**139.** (b)	**140.** (d)
141. (c)	**142.** (a)	**143.** (a)	**144.** (a)	**145.** (d)	**146.** (b)	**147.** (b)	**148.** (c)	**149.** (b)	**150.** (b)
151. (a)	**152.** (d)	**153.** (b)	**154.** (d)	**155.** (b)	**156.** (b)	**157.** (b)	**158.** (b)	**159.** (c)	**160.** (c)

Numerical Type Questions

1. 8	**2.** 6	**3.** 512	**4.** 4	**5.** 8	**6.** 11	**7.** 24
8. 6	**9.** 24	**10.** 6	**11.** 1	**12.** 2	**13.** i	**14.** 6
15. 5	**16.** 30	**17.** 30	**18.** 2	**19.** 2	**20.** 8	**21.** 0
22. 4	**23.** 60	**24.** 20	**25.** 63	**26.** 63	**27.** 15600	**28.** 7200
29. 1024	**30.** 1956	**31.** 120	**32.** 240	**33.** 69760	**34.** 216	**35.** 205
36. 20	**37.** 18	**38.** 1023	**39.** 60	**40.** 468000	**41.** 12	**42.** 30
43. 30240	**44.** 14400	**45.** 15				

EXERCISE – II

MCQ Type Questions

1. (c)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (d)	**8.** (d)	**9.** (c)	**10.** (d)
11. (b)	**12.** (b)	**13.** (d)	**14.** (d)	**15.** (a)	**16.** (d)	**17.** (b)	**18.** (d)	**19.** (a, d)	**20.** (a)
21. (c)	**22.** (c)	**23.** (a)	**24.** (b)	**25.** (c)	**26.** (a)	**27.** (c)	**28.** (a)	**29.** (d)	**30.** (d)
31. (b)	**32.** (a)	**33.** (d)	**34.** (c)	**35.** (b)	**36.** (a)	**37.** (b)	**38.** (a)	**39.** (b)	**40.** (d)
41. (b)	**42.** (c)	**43.** (b)	**44.** (b)	**45.** (d)	**46.** (d)	**47.** (a)	**48.** (d)	**49.** (d)	**50.** (d)
51. (d)	**52.** (c)	**53.** (a)	**54.** (c)	**55.** (b)					

Numerical Type Questions

1. (2048)	**2.** (36)	**3.** (15)	**4.** (5)	**5.** (16)	**6.** (5)	**7.** (2)	**8.** (4)	**9.** (89)

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EXPLANATIONS

EXERCISE – I

NUMERICAL TYPE QUESTIONS

20. The graphs G* must also have eight connected components.

21. The vertex v is isolated if it does not belong to any edge. Thus v is isolated if and only if $\deg(v) = 0$.

Consider the multigraph G where

$V(G) = \{A, B, C, D\}$ and

$E(G) = [\{A, C\}, \{A, D\}, \{B, B\}, \{B, C\}, \{C, A\}, \{C, B\}, \{D, B\}, \{D, D\}]$

27. The president can be elected in twenty-six different ways; following this, the treasurer can be elected in twenty-five different ways (since the person chosen president is not eligible to be treasurer); and following this, the secretary can be elected in twenty-four different ways.

Thus, by above principle of counting, there are $n = 26 \times 25 \times 24 = 15\,600$ different ways in which the organization can elect the officers.

36. There are four choices for Mathematics and five choices for English.

$\therefore \quad n = 4 \times 5 = 20$

40. $n = 26 \times 25 \times 10 \times 9 \times 8$

$= 468000.$

i.e. there are twenty-six choices for the first letter, but only twenty-five choices for the second letter which must be different from the first letter. Similarly, choices for the digits are 10, 9 and 8 since the digits must be distinct.

43. Total letters in the word "LOGARITHMS" is 10,

i.e. $n = 10$ and we have to form five-letter words

$r = 5$

$\therefore$ Required number of words,

$$P(10, 5) = \frac{10!}{(10-5)!} = \frac{10!}{5!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5!} = 10 \times 9 \times 8 \times 7 \times 6 = \mathbf{30240}$$

44. Since there is no restriction on men, they can sit in a row in $5! = 120$ ways.

Since women cannot sit side by side they can sit in between two men or in the first place or in the last place as shown below :

X $\boxed{\text{M}}$ X $\boxed{\text{M}}$ X $\boxed{\text{M}}$ X $\boxed{\text{M}}$ X $\boxed{\text{M}}$ X

If sign 'X' shows the women place, there are 6 vacancies for women to sit. In these 6 vacancies 3 women can sit in

$$^6P_3 = 120 \text{ ways.}$$

Hence number of ways where 5 men and 3 women can sit with the given condition

$= 120 \times 120 = \mathbf{14400.}$

45. Here we find the number of permutations from 6 objects in which 4 are of one type (heads) and 2 are of another type (tails).

$\therefore$ Required number of ways

$$= \frac{6!}{4!2!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{(4 \times 3 \times 2 \times 1)(2 \times 1)} = \mathbf{15}$$

EXERCISE – II

MCQ TYPE QUESTIONS

2. If the person is kind, he is not known to be corrupt

4. Counter example :

(A) Let $X = \{1\}$

$\Rightarrow \quad X' = S - X = \{2, 3, 4, 5, 6\}$

$\Rightarrow \quad |X| = |X'|$

(B) Since $|S| = 6$

and $|X| = 5 = |Y|$

$\therefore$ Atleast 4 elements common in X and Y

$\Rightarrow \quad X \cap Y = \phi$ is false

(C) Counter Example :

Let $X = \{1, 2\}$

and $Y = \{2, 3, 4\}$

then $\frac{X}{Y} = \{1\} = \phi$

(D) $\frac{X}{Y} = X - Y = X \cap Y'$

and $\frac{Y'}{X'} = Y' - X' = Y' \cap (X')$

$= Y' \cap X = X \cap Y'$

$\therefore \quad \frac{X}{Y} = \frac{Y'}{X'}, \cancel{\forall}\ X, Y \in U$

5. $A = \{5, \{6\}, \{7\}\}$

Power Set of A, $(2^A) = \{\phi, 5, \{6\}, \{7\}, \{5, \{6\}\}, \{5, \{7\}\}, \{\{6\}, \{7\}\}, \{5, \{6\}, \{7\}\}\}$

Options I, II and III are satisfied.

$\therefore$ Option (*c*) is correct.

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6. $g(x) = 1 - x$ and $h(x) = \dfrac{x}{x-1}$

Now $g(h(x)) = g\left(\dfrac{x}{x-1}\right) = 1 - \dfrac{x}{x-1}$

$$= \frac{x-1-x}{x-1}$$

$$= \frac{-1}{x-1}$$

and $h(g(x)) = h(1-x) = \dfrac{1-x}{1-x-1} = \dfrac{1-x}{-x}$

Now $\dfrac{g(h(x))}{h(g(x))} = \dfrac{-1}{x-1} \times \dfrac{(-x)}{1-x} = \dfrac{-x}{(x-1)^2}$

Now from option (a),

$$\frac{h(x)}{g(x)} = \frac{x}{x-1} \times \frac{1}{1-x}$$

$$= \frac{-x}{(x-1)^2}$$

$\therefore$ Option (a) is correct.

7. $\dfrac{1}{5} < P_r < 1$

8. R is not reflexive not transitive

R is symmetric

consider $a = 3$, $b = 6$, $c = 8$

The common divisor of 3 & 6 is 3

The common divisor of 6 & 8 is 2

But there is no common divisor of 3 & 8

Hence 3R6, 6R8 but not 3R8

9. Since $\quad p - q \neq q - p$

$\therefore \quad (p, q)\ R\ (p, q)$

$\Rightarrow$ R is not reflective

Let $\quad (p, q)\ R\ (r, s)$

then $\quad p - s = q - r$

$\Rightarrow \quad r - q = s - p$

$\Rightarrow \quad (r, s)\ R\ (p, q)$

$\Rightarrow$ R is symmetric

10. It means "It is false that every glitter is gold" or "some glitters are not gold".Then we can say "atleast one glitter object is not gold".

11. P = Tq = F and r = T

Option a will become false

Option c will become false

Option d is always false

12.

$a \wedge c \sim b \rightarrow (a \wedge c)(a \leftrightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c))$

a	b	c	$a \leftrightarrow c$	$\sim b$
F	F	F	T	T
F	T	T		
F	F	T	F	T
F	T	F		
F	T	F	T	F
F	T	T		F
T	T	F	F	F
T	F			
T	F	F	F	T
F	T	F		
T	F	T	T	T
T	T	T		
T	T	F	F	F
F	F	T		
T	T	T	T	F
T	F	T		

Hence $(a \leftrightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c))$ is not Tautology.

13. g : mobile is good $\quad$ c : mobile is cheap

P : Good mobile phones are not cheap

$$\cong g \rightarrow \neg c \cong (\neg g \vee \neg c)$$

$$[\because a \rightarrow b \equiv \neg a \vee b]$$

Q : Cheap mobile phones are not good

$$\cong c \rightarrow \neg g \cong (\neg c \vee \neg g)$$

$\therefore$ Both P and Q are equivalent which means P and Q imply each other.

14. Given statement is

$$\sim \forall d\left[r(d) \rightarrow c(d)\right]$$

$$\equiv \sim \forall d\left[\sim r(d) \vee c(d)\right]$$

$$\equiv \exists d\left[r(d) \wedge \sim c(d)\right]$$

Since $p \rightarrow q \equiv \sim p \vee q$ and let $r(d)$ be rainy day, $c(d)$ be cold day).

15. Let U = {1, 2, ... 1008}

V = {1009, ... 2014}

Since subset S_1 will be greater than every other subset *i.e.* V & subset S_2 will be less than every other subset *i.e.* U, So both statements will be true.

16. $f : X \rightarrow Y$ defined by $f(a) = 1, f(b) = 1, f(c) = 2$ where $X = \{a, b, c\}$ $Y = \{1, 2\}$

Let $A = \{a, c\}$, $B = \{b, c\}$ be subsets of X

then $|f(A \cup B)| = 2$; $|f(A)| = 2$; $|f(B)| = 2$

$f(A \cap B) = \{2\}$; $f(A) = \{1,2\}$; $f(B) = \{1,2\}$

$f(A) \cap f(B) = \{1,2\}$

$|f(A \cap B)| = 1$

$\therefore$ Option (a), (b), (c) are not true

Hence, Option (d) is true

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17. Let us consider a function (counter example) as

$f(0) = 1$, $f(1) = 0$, $f(2) = 3$, $f(3) = 2$,....., $f(2012) = 2013$,

$f(2013) = 2012$ and $f(2014) = 2014$

Clearly $f(f(i)) = i$ for $0 \le i \le 2014$

Here $f(i) \ne i$ for every i and $f(i) = i$ for some i

Also f is onto

Hence, only Q and R are true

18. Given : Statement

"None of my friends are perfect"

To find : Logical translation of it.

Analysis : The statement means that of all the friends that I have, none of them are prefect.

Hence, if we look at only friend and if any of them being perfect. We get logical

$$\exists\forall x(F(x) \wedge P(x))$$

This means there is an x who is friend of mine and is also perfect.

Now, we need to negate the statement since none are perfect.

$\therefore$ $\rightarrow \exists x(F(x) \wedge P(x))$

$\rightarrow$ option (D)

In english, this statement is there is no x s.t., $F(x)$ and $P(x)$ is true simultaneously.

20. $x \oplus y = x^2 + y^2$

$y \oplus x = y^2 + x^2$

$= x^2 + y^2$ (Q addition is cumulative)

$= x \oplus y$

$\therefore$ $\oplus$ is cumulative

$(x \oplus y) \oplus z = (x^2 + y^2)^2 + z^2$

$x \oplus (y \oplus z) = 2 \oplus (y^2 + z^2)^2$

$= x^2 + (y^2 + z^2)^2$

Clearly, $(x \oplus y) \oplus z$ ^{1} $x \oplus (y \oplus z)$

$\therefore$ $\oplus$ is not associative

21. Option A: There exists x which is either real or rational and can be both.

Option B: All real numbers are rational

Option C: There exists a real number which is rational.

Option D: There exists some number which is not rational or which is real.

22. Total number of functions is 2^n, out of which there will be exactly two functions where all elements map to exactly one element, so total number of onto functions is $2^n - 2$

24. Given : $F(x, y, t)$ a predicate

$\forall x \exists y \exists t (\neg F(x, y, t))$

Since $\forall x \exists y \exists t (\neg F(x, y, t))$

$\Rightarrow \forall x \neg (\forall y \forall t (F(x, y, t))$

So, we can say no one can fool everyone all the time.

25. Total number of reflexive relations in set of n elements

$= 2^{n^2 - n}$

$= 2^{5^2 - 5}$ .....(since $n = 5$)

$= 2^{20}$

26. *A group should satisfy following relations :*

(*i*) Closure

(*ii*) Inverse

(*iii*) Identity

(*iv*) Associative

in $(1, \omega, \omega^2)$

We can say

$1 * \omega = \omega$ all are elements

$\omega * \omega^2 = 1$ of group than

$\omega^2 * 1 = \omega^2$ it is closure

Hence, we think about inverse, then this should satisfy

$a * e = a$

where e is identity element means $\exists\, e \in G$ have identity element 1

means $1 * \omega = \omega$, $1 * 1 = 1$
$1 * \omega^2 = \omega^2$ identities

Hence it should be inverse

means $a * b = 3$

means b is the inverse element a, b, e 64

here $1 * 1 = 1$, $\omega * \omega^2 = 1$, $\omega^2 * \omega = 1$ } inverse

So, it also hold Associative for this

$a * (b * c) = (a * b) * c$

means $1 * (\omega * \omega^2) = (1 * \omega) * \omega^2$ } Associative

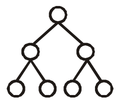
means $(1, \omega, \omega^2)$ is a group.

27. Since $\Sigma\partial(V) = 2e$

$\therefore$ $\xi(s) = \xi(T)$

$\Rightarrow$ $|S| = |T|$

28. Given that a node has odd number of descendants possible only when it is fully binary tree like.



- So in fully binary tree, every node has two child or zero child (zero child when there is one node exactly)
- So, number of nodes in tree that type having exactly one child is zero = e]

29. Havell-Hakimi algorithm can be used to check whether a given degree sequence is a graph or not.

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The algorithm is

1. remove top node of the sequence.
2. subtract "1" from as many nodes in remaining sequence as the degree of top node that was removed.
3. rearrange this sequence in non increasing order.
4. check if resulting sequence is a graph.
5. proceed again to step 1.

If the given sequence is not a graph we will see a voilation in step 4, such as presence of negative degrees in the sequence. Otherwise the algorithm will bottom out with a degree sequence consisting of only even no. of 1's and any number of 0's.

Now applying the algorithm to the degree sequence I, II, III and IV, one by one :

I. 7, 6, 5, 4, 3, 2, 1

6, 5, 4, 4, 3, 2, 1	(Step 1)
5, 4, 3, 3, 2, 1, 0	(Step 2)
5, 4, 3, 3, 2, 1, 0	(Step 3)
Sequence is a graph	(Step 4)
4, 3, 3, 2, 1, 0	(Step 1)
3, 2, 2, 1, 0, 0	(Step 2)
3, 2, 2, 1, 0, 0	(Step 3)
Sequence is a graph	(Step 4)
2, 2, 1, 0, 0	(Step 1)
1, 1, 0, 0, 0	(Step 2)
1, 1, 0, 0, 0	(Step 3)
Sequence is a graph	

Now, the algorithm ends, since the sequence has only 0's and even number of 1's.

The final sequence corresponds to following valid greah.

Similarly for sequence II.

II. 6, 6, 6, 6, 3, 3, 2, 2

6, 6, 6, 3, 3, 2, 2	(Step 1)
5, 5, 5, 2, 2, 1, 2	(Step 2)
5, 5, 5, 2, 2, 2, 1	(Step 3)
Sequence is a graph	(Step 4)
5, 5, 2, 2, 2, 1	(Step 1)
4, 4, 1, 1, 1, 1	(Step 2)
4, 4, 1, 1, 1, 1	(Step 3)
Sequence is a graph	(Step 4)
4, 1, 1, 1, 1	(Step 1)
3, 0, 0, 0, 1	(Step 2)
3, 1, 0, 0, 0	(Step 3)
Sequence is a graph	(Step 4)
1, 0, 0, 0	(Step 1)
0, – 1, – 1, 0	(Step 2)
0, 0, – 1, – 1	(Step 3)

The sequence is not a graph (Step 4), since negative degrees not possible in a valid graph. So, algorithm ends.

II is cannot be the degree sequence of any graph.

Similarly we can show that III is degree sequence of some graph and IV is not a degree sequence of any grpah.

So, correct answer is choice (d) II and IV.

30. Given : gold and silver ornament are precious.

We use

gold and silver gold $\vee$ silver

$\forall$ $((G(x) \vee S(x)))$

gold and silver $\rightarrow$ precious

$$\forall x((G(x) \vee S(x)) \rightarrow P(x)$$

Alternately

"Gold and silver ornaments are precious"

Gold ornaments are alone precious.

Silver ornaments are also alone precious.

Gold and silver ornaments are also precious

$\forall x((G(x) \vee S(x)) \rightarrow P(x))$

31. Universal qualification of $P(x)$ is denoted $\forall x\ P(x)$. The symbol $\forall$ is called *universal quantifier.*

The existential qualification of predicate $P(x)$ is statement, there exist a value of x for which $P(x)$ is true. The existential qualification of $P(x)$ is denoted by $\exists(x)\ P(x)$. The symbol $\exists$ is called *existential quantitier.*

$\therefore$ $\neg \forall x\ P(x) = \exists x(\neg P(x))$

$\neg$ sign is for not

32. Group is an algebraic system having property of closed under operation, associative, contain identity element and exist inverse of every element.

Alternately

A group (G, *) is a monoid with identity e that the additional property that for every element a ε G there exist an element a′ ε G such that $a * a' = a^i$ $a = e$. Thus group is set together with binary operation $*$.

It follows property :

(*i*) *Associativity :* $(a * b) * c = a * (b * c)$

(*ii*) *Inverse for every element :*

$$a * e = e * a \text{ for every } a \in G$$

(*iii*) *Existance of identity :* $a * a' = a' * a$

33. $R = \{(x, y), (x, z), (z, x), (z, y)\}$

If $(a, b) \in R$ then $(b, a) \notin R$ to be symmetric but $(x, y) \in R$ and $(y, x) \notin R$

So, R is NOT Symmetric

If $(a, b) \in R$ then $(b, a) \in R$ to be Antisymmetric but $(x, z) \in R$ and $(z, x) \in R$

So, R is NOT Antisymmetric

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34.

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

Hence c, d is generator

Alternately

Every element in Cyclic Group can be represented in the power of any generator.

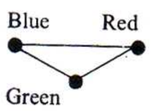
From given table

$$
\begin{aligned}
a*a &= a \\
b*b &= a \\
c*c &= c^2 = b \\
c^3 &= c*c^2 = c*b = d \\
c^4 &= c*c^3 = c*d = a \\
d*d &= d^2 = b \\
d^3 &= d*d^2 = d*b = c \\
d^4 &= d*d^3 = d*c = a
\end{aligned}
$$

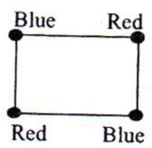
35.

P	Q	P□Q	P∨Q	P□¬Q
T	T	T	T	T
T	F	T	T	T
F	T	F	T	T
F	F	T	F	F

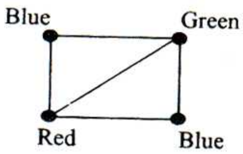
36. Chromatic number of n-vertex simple connected undirected graph which does not contain any odd length cycle will be 2(n > = 2).



(For n=3 Odd Length Cycle)

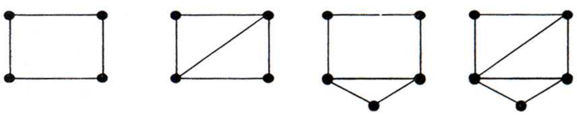


(For n=4 Even Length Cycle)



(For n=4 Odd Length Cycle)

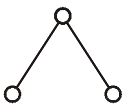
37. Atleast 2 vertices have the same degree.



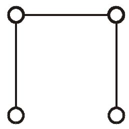
Alternately

For simple corrected graph with more than two vertices at least 2 vertices have the same degree

$n = 3$



$n = 4$



38. "For x which is an fsa, there exists a y which is a pda and which is equivalent to x."

$(\forall x \text{ fsa } (x)) \Rightarrow (\exists y \text{ pda } (y) \wedge \text{equivalent } (x, y))$ is the logical representation.

39. IT can solved by truth table.

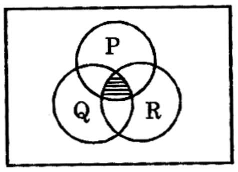
P	Q	P∨ ~ Q	(−P ∧ Q)	P ∧ Q	P ∧ −Q	~ P∧ ~ Q
T	T	T	F	T	F	F
T	F	T	F	F	T	F
F	T	F	T	F	F	F
F	F	T	F	T	F	T

Using above table, we get

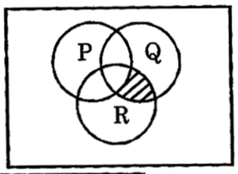
P	Q	I	II	III	IV
T	T	T	T	T	T
T	F	T	T	T	T
F	T	F	F	F	F
F	F	T	T	T	T

40.

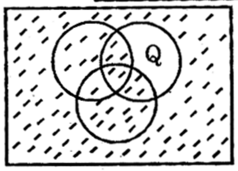
P ∩ Q ∩ R =



P^C ∩ Q ∩ R =

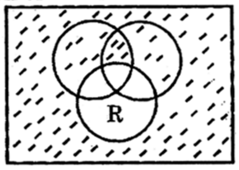


Q^C =



Union of all universal set is

R^C =



41. Smallest equivalance relation for set of n elements contains n ordered pair and largest equivalance relation contains n^2 ordered pairs.

42. Since, $\pi_i \; \alpha \; \pi_j$ only if π_i refines π_j

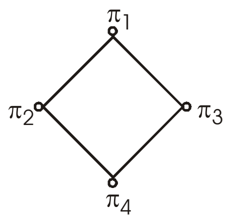
$\pi_1 = \left\{\overline{abcd}\right\}$ refines

$\pi_2 = \left\{\overline{ab}, \overline{cd}\right\}$

$\pi_3 = \left\{\overline{abc}, \overline{d}\right\}$

and these both refines

$\pi_4 = \left\{\overline{a}, \overline{b}, \overline{c}, \overline{d}\right\}$



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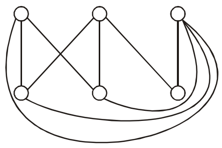
43.
$$X = \{x \in R^3 \mid x_1 + x_2 + x_3 = 0\}$$
$$X^T = [x_1, x_2, x_3]^T \text{ then}$$

$\{[1, -1, 0]^T, [1, 0, -1]^T\}$ is a linearly independent set because one cannot be obtained from another by scalar multiplication. However $(1, -1, 0)$ and $(1, 0, -1)$ do not span X, since all such combinations (x_1, x_2, x_3) such that $x_1 + x_2 + x_3 = 0$ cannot be expressed as linear combinations of $(1, -1, 0)$ $(1, 0, -1)$.

44. Kurtowsk is 3, 3 ($K_{3,3}$) is non–planer graph with minimum number of edges.

It has 9 edges and 6 vertices.



45. Here, $\text{INDEG}(v)$ = In degree of verter v.

Since,
$$\text{INDEG}(1) = 0$$
$$\text{INDEG}(2) = 1$$
$$\text{INDEG}(3) = 1$$
$$\text{INDEG}(4) = 2$$
$$\text{INDEG}(5) = 2\text{m}$$
$$\text{INDEG}(6) = 2$$

Node 1 is with zero indegree, so first node in ordering will always 1. Hence answer (d) is not a topological ordering.

46. $\forall_x$ [Graph (x) $\Rightarrow$ 7 connected (x)] represents "for every x if x is a graph then it is not connected"

47. A k – regular graph is one in which vertex is of degree k. If k is even then, such a graph will have an Eulerian circuit.

48. Given statement can be interpreted as follows :

If x is lion or tiger, if it is hungry or it is threatened, x will attack.

$$P(x) = \text{lion}(x) \vee \text{tiger}(x)$$
$$Q(x) = \text{hungry}(x) \vee \text{threatened}(x)$$
$$R(x) = \text{attacks}(x)$$
$$P(x) \rightarrow (Q(x) \rightarrow R(x))$$

Expanding, $\forall x\ [(\text{lion}(x) \vee \text{tiger}(x)) \rightarrow \{(\text{hungry}(x) \vee \text{threatened}(x)) \rightarrow \text{attacks}(x)\}]$

49.

A	B	C	A ∨ B	A ∧ B	(A ∨ B) → C	(A ∧ B) →	CA → C	B → C
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	F	F
T	F	T	T	F	T	T	T	T
T	F	F	T	F	F	T	F	T
F	T	T	T	F	T	T	T	T
F	T	F	T	F	F	T	T	F
F	F	T	F	F	T	T	T	T
F	F	F	F	F	T	T	T	T

(A → C) ∧ (B → C)	(A → C) ∨ (B → C)
T	T
F	F
T	T
F	T
T	T
F	T
T	T
T	T

It is apparent from the truth table that P_1 and P_2 are not tautologies.

50.

A	B	A⊙B	~(~A⊙B)	A∧B
T	T	T	~ F = T	T
T	F	T	~ T = F	F
F	T	F	~T = F	F
F	F	T	~T =F	F

51. Number of functions from a set A to a set B where

$$|A| = m$$

and $|B| = n$ is n^m

$$W = X \times Y$$
$$\therefore \quad |W| = xy$$
$$|E| = 2^{xy}$$
$$|Z| = z$$

Hence number of functions from Z to E

$$= (2^{xy})^z = 2^{xyz}$$

52. Binary operation multiplication modulo 10 is defined as follows

$$aob = (a \times b)\ \%\ 10$$

The identity element for the given set is 1.

For closure property, $2 \text{ o } 7 = 4 \notin$ set

$\therefore$ Set is not closed.

2 and 8 do not have an inverse, 3o7 = 1

$\therefore$ 7 is the inverse of 3.

Hence (c) is false

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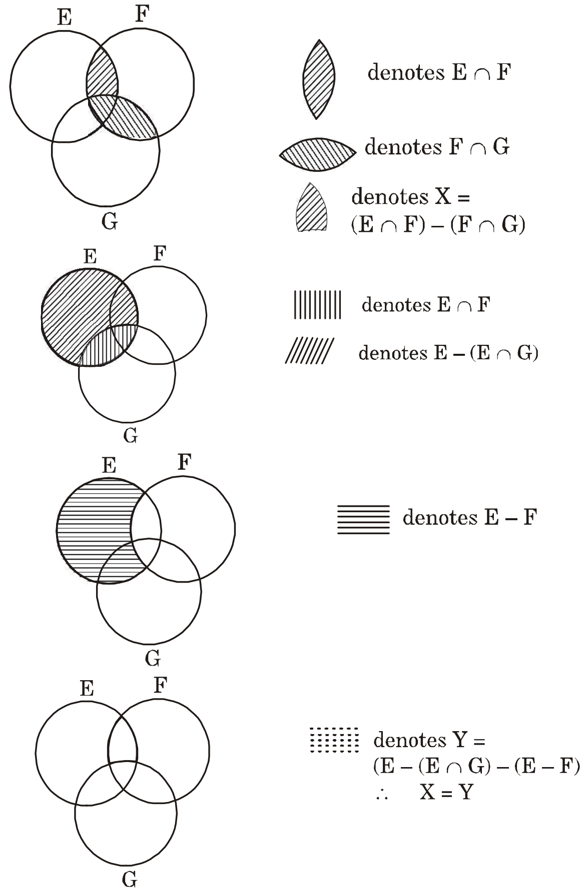
53. Reflexive property is an essential condition for Partial order, Total order as well as Equivalence relation.

Given set is not reflexive as $\forall\ x, y \in I$,

(x, y) is not related to (x, y)

54. X and Y sets can be figured out using Venn diagrams.



55. Consider an example,

$$S = \{1,2, 3, 4\}$$
$$m = 4$$
$$X_1 = \{1, 2, 3\}, X_2 = \{1, 3, 4\}\ n = 2$$
$$\sum_{i=1}^{m} f(i) = f(1) + f(2) + f(3) + f(4)$$
$$= 2 + 1 + 2 + 1 = 6 = 3 \times 2 = 3n$$

NUMERICAL TYPE QUESTIONS

1. Cordinality of power set

$\{0, 1, 2, ..., 10\} = 2^{11} = 2048$

3. 1111, 1112, 1113, 1122, 1123, 1133, 1222, 1223, 1233, 1333, 2222, 2223, 2233, 2333, 3333

Total number of 4-digit numbers are 15.

4. Let 'x' be the base or radix of the number system

$$\frac{2\times x^\circ + 1\times x + 3\times x^2}{0\times x^\circ + 2\times x} = 3 \times x^\circ + 1 \times x + 1 \times x^{-1}$$

$$\Rightarrow \quad \frac{2 + x + 3x^2}{2x} = 3 + x + \frac{1}{x}$$

$$\Rightarrow \quad \frac{3x^2 + x + 2}{2x} = \frac{3x + x^2 + 1}{x}$$

$$\Rightarrow \quad 3x^2 + x + 2 = 6x + 2x^2 + 2$$

$$\Rightarrow \quad x^2 - 5x = 0$$

$$\Rightarrow \quad x(x - 5) = 0$$

$$\Rightarrow \quad x = 0 \text{ or } x = 5$$

As base or radix of a number system cannot be zero, here $x = 5$

5. The number of functions from A to B where size of A = $|A|$ and size of B = $|B|$ is $|B|^{|A|}$

$\{0,1\}^4 = \{0,1\}\ X\{0,1\}X\{0,1\}\ X\{0,1\} = 16$

$|S| = 2^{16}$

$N = 2^{|S|}$

loglog N = log log $2^{|S|}$ = log$|S|$ = log 2^{16} = 16

6. Order of subgroup divides order of group (Lagrange's theorem).

3, 5 and 15 can be the order of subgroup. As subgroup has atleast 4 elements and it is not equal to the given group, order of subgroup can't be 3 and 15. Hence it is 5.

7. Let the basis of 6-dimensional vector space be {e1, e2, e3,e4, e5, e6}. In order for V1 $\cap$ V2 to have smallest possible dimension V1 and V2 could be, say, {e1, e2, e3,e4} and {e3, e4, e5, e6} respectively. The basis of V1 $\cap$ V2 would then be {e3, e4}. => Smallest possible dimension = 2.

8. $x \times x = e \Rightarrow x$ is its own inverse

$y \times y = e \Rightarrow y$ is its own inverse

$(x \times y)\times (x \times y) = e \Rightarrow (x\times y)$ is its own inverse

$(y \times x) \times (y \times x) = e \Rightarrow (y \times x)$ is its own inverse

also $x \times x \times e = e \times e$ can be rewritten as follows

$$x \times y \times y \times x = e \times y \times y \times e = e \quad [\because\ y \times y = e]$$

$(x \times y) \times (y \times x) = e$ shows that $(x \times y)$ and $(y \times x)$ Are each other's inverse and we already know that $(x \times y)$ and $(y \times x)$ are inverse of its own.

As per (G,*) to be group any element should have only one inverse element (unique)

This process $x \times y = y \times x$ (is one element)

So the elements of such group are 4 which are $\{x, y, e, x \times y\}$

9. No twos : 1 1 1 1 1 1 1 1 1 1 $\Rightarrow$ 1 pennant

Single two : 2 1 1 1 1 1 1 1 1 $\Rightarrow$ 9!/8!1!

= 9 pennants

Two twos : 2 2 1 1 1 1 1 1 $\Rightarrow$ 8!/6!.2! = 28

Three twos : 2 2 2 1 1 1 1 $\Rightarrow$ 7!/3!.4! = 35

Four twos : 2 2 2 2 1 1 $\Rightarrow$ 6!/4!.2! = 15

Five twos : 2 2 2 2 2 $\Rightarrow$ 1

Total = 89 pennants.

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1
CHAPTER

Digital Logic

LOGIC FUNCTIONS & MINIMIZATION

BASIC LOGICAL FUNCTIONS

While each logical element or condition must always have a logic value of either "0" or "1"; ways are required to combine different logical signals or conditions to provide a logical result.

e.g., consider the logical statement: "If I move the switch on the wall up, the light will turn on." At first glance, this seems to be a correct statement. However, looking at a few other factors, it is realized that there's more to it than this. In this example, a more complete statement would be: "If I move the switch on the wall up andthe light bulb is good *and* the power is on, the light will turn on."

If these two statements are logical expressions and use logical terminology, first statement can be reduced to:

Light = Switch

This means nothing more than that the light will follow the action of the switch, so that when the switch is up/on/true/1, the light will also be on/true/1. Conversely, if the switch is down/off/false/0, the light will also be off/false/0.

For second version of the statement, a slightly more complex expression is

Light = Switch *and* Bulb *and* Power

Normally, symbols are used rather than words to designate the *and* function that are used to combine the separate variables of Switch, Bulb, and Power in this expression. The symbol normally used is a dot, which is the same symbol used for multiplication in some mathematical expressions.

Using this symbol, three-variable expression becomes.

Light = Switch • Bulb • Power

LOGIC GATES

Logic gate is an elementary building block of a digital circuit . Most logic gates have two inputs and one output. At any given moment, every terminal is in one of the two *binary conditions* represented by different voltage levels :

low (0) or *high* (1),

Generally logic state of a terminal can, and does, change often, as the circuit processes data.

In most logic gates, the low state is approximately zero volts (0 V), while high state is approximately five volts positive (+5 V).

There are seven basic logic gates :

1. AND
2. OR
3. XOR
4. NOT
5. NAND
6. NOR
7. XNOR

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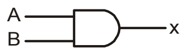
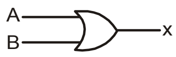
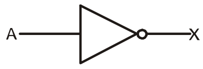
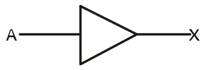
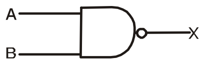
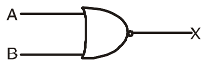
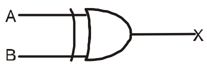
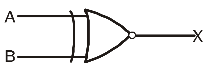
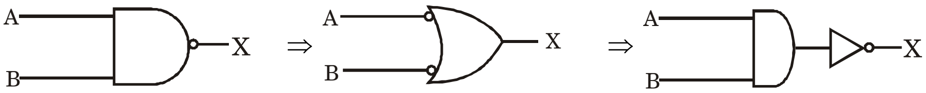
Name	*Graphical symbol*	*Algebraic function*	*Truth table*	*Named because*
AND		$X = A \cdot B = AB$	**A B X** 0 0 0 0 1 0 1 0 0 1 1 1	If 0 is called "false" and 1 is called "true," the gate acts in the same way as the logical "AND" operator.
OR		$X = A + B$	**A B X** 0 0 0 0 1 1 1 0 1 1 1 1	It behaves after the fashion of logical inclusive "OR." Output is "true" if either or both of the inputs are "true." If both inputs are "false," then the output is "false."
Inverter		$X = \overline{A}$	**A X** 0 1 1 0	It differentiate it from other types of electronic inverter devices, has only one input. It reverses the logic state.
Buffer		$X = A$	**A X** 0 0 1 1	It provides same input and output.
NAND		$X = (\overline{AB})$	**A B X** 0 0 1 0 1 1 1 0 1 1 1 0	It operates as an AND gate followed by a NOT gate. output is "false" if both inputs are "true." Otherwise, the output is "true."
NOR		$X = (\overline{A+B})$	**A B X** 0 0 1 0 1 0 1 0 0 1 1 0	It is a combination OR gate followed by an inverter. Its output is "true" if both inputs are "false." Otherwise, the output is "false."
Exclusive-OR (XOR)		$X = A \oplus B$ or $X = \overline{A}B + A\overline{B}$	**A B X** 0 0 0 0 1 1 1 0 1 1 1 0	
Exclusive NOR or equivalence		$X = A \odot B$ or $X = \overline{A}\,\overline{B} + AB$	**A B X** 0 0 1 0 1 0 1 0 0 1 1 1	It is a combination of XOR gate followed by an inverter. Its output is "true" if the inputs are the same, and"false" if the inputs are different.

Using combinations of logic gates, complex operations can be performed. In theory, there is no limit to the number of gates that can be arrayed together in a single device. But in practice, there is a limit to the number of gates that can be packed into a given physical space. Arrays of logic gates are found in digital integrated circuits (ICs). As IC technology advances, the required physical volume for each individual logic gate decreases and digital devices of the same or smaller size become capable of performing ever-more-complicated operations at ever-increasing speeds.

UNIVERSAL GATES.

Universal gates are the ones which can be used for implementing any gate like AND, OR and NOT, or any combination of these basic gates; NAND and NOR gates are universal gates; but there are some rules that need to be followed when implementing NAND or NOR based gates.

NAND Gate



$X = (A \cdot B)'$ $\qquad$ $X = A' + B' = (A \cdot B)'$

NOR Gate

$X = (A + B)'$ $\qquad$ $X = A' \cdot B' = (A + B)'$

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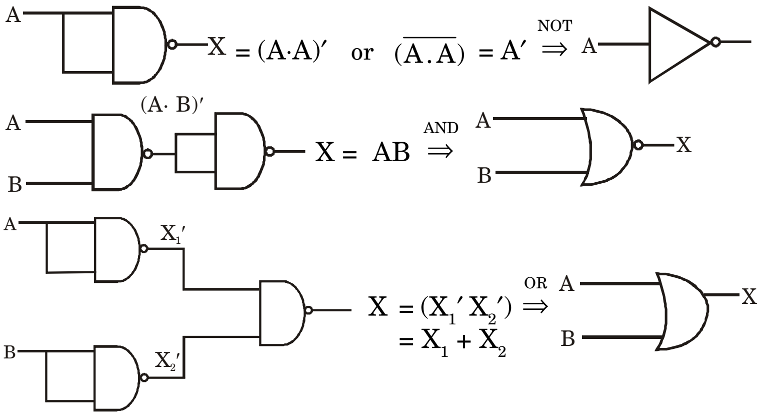
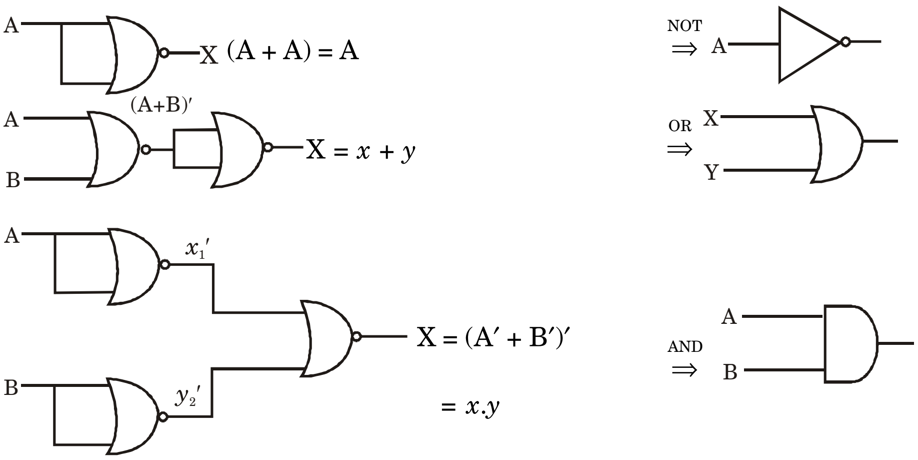
REALIZATION OF LOGIC GATE USING NAND GATES.



Realization of logic gate using NOR Gates



BOOLEAN EXPRESSIONS

Digital circuits use ON-OFF devices to implement operations of a system of logic called *two-valued* using *Boolean expression*. The statements may take the form of algebraic expressions, logic block diagrams, or truth tables, as well as circuits. Boolean expression is composed of variables and terms. The simplification of Boolean expressions can lead to more effective computer programs, algorithms and circuits.

Theorems of Boolean algebra

S. No.	Theorem	Name
1.	(*a*) A + B = B + A (*b*) A.B = B . A	Commutative law
2.	(*a*) (A + B) + C = A + (B + C) (*b*) (A. B) . C = A . (B . C)	Associative law
3.	(*a*) A. (B + C) = A. B + A . C (*b*) A + (B . C) = (A + B) . (A + C)	Distributive law
4.	(*a*) A + A = A (*b*) A . A = A	Identity law
5.	$\overline{\overline{A}}$ = A	Involution (Negation) law
6.	(*a*) A + A . B = A (*b*) A. (A + B) = A	Absorption law

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S. No.	Theorem	Name
7.	(*a*) $0 + A = A$ (*b*) $1 . A = A$ (*c*) $1 + A = 1$ (*d*) $0 . A = 0$	Boolean postulates
8.	(*a*) $\overline{A} + A = 1$ (*b*) $\overline{A} . A = 0$	
9.	(*a*) $A + \overline{A} . B = A + B$ (*b*) $A . (\overline{A} + B) = A . B$	
10.	(*a*) $\overline{A+B} = \overline{A} . \overline{B}$ (*b*) $\overline{A.B} = \overline{A} + \overline{B}$	DeMorgan's laws

FORMS OF BOOLEAN EXPRESSIONS.

(1) Sum of Product form (SOP)

e.g. $W = (\overline{X}\ \overline{Y}\ \overline{Z}) + (\overline{X}\ Y\ \overline{Z}) + (\overline{X}\ \overline{Y}\ Z)$

Each term in such an expression is called *minterm*.

(2) Product of Sum form (POS)

e.g. $S = (P + Q + R)(P + \overline{Q} + R)(\overline{P} + Q + R)$

Each term in such an expression is called *maxterm*.

Minterm and Maxterm

n variables can be combined to form 2^n minterms or maxterms.

Each minterm is obtained from an AND term of n variables with each variable being primed, if corresponding bit of binary number is 0 and unprimed, if it is 1.

Each maxterm is obtained from an OR term of n variables with each variable being unprimed, if corresponding bit is a 0 and primed if it is 1.

Maxterm is complement of its corresponding minterm and vice versa

		Minterm *term*	Maxterm *term*
x	y		
0	0	$x'y'$ (m_0)	$x + y$ (m_0)
0	1	$x'y$ (m_1)	$x + y'$ (m_1)
1	0	xy' (m_2)	$x' + y$ (m_2)
1	1	xy (m_3)	$x' + y'$ (m_3)

MINIMIZATION OF BOOLEAN EXPRESSION.

(1) Convert equation from POS form to SOP form.

(2) Remove parenthesis if any in the expression.

(3) If there are two or more identical terms, keep only one of them and drop the other.

(4) If a variable and its complements are present in a term, reduce it to 0. ($A. \overline{A} = 0$).

(5) Group two terms of which one contains a variable and other its complement, except for which both are identical. They can be reduced to a single term and in the reduced term, above variable will be absent ($C + \overline{C} = 1$)

e.g. $(A . B . \overline{C}) + (A. B. C) = A. B (\overline{C} + C) = A . B$

(6) If there are two terms which are identical except that one contains an extra variable, reduce them into a single term by dropping the larger one.

e.g. $B.C + \overline{A} . B . C = B . C (1 + \overline{A}) = B . C (1) = B . C$

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Minimal form and Prime implicant.

A boolean expression which cannot be reduced further is said to be in *minimal form.* Each term in it is called *prime implicant.*

TRUTH TABLE

It is used to help show the function of a logic gate. If truth table is not sure and guidence is required on how go about drawing them for individual gates of logic circuits, then *truth table section link* is used.

Logic Gates representation using Truth table

NOT gate	
A	$\bar{A}$
0	1
1	0

Inputs		Outputs					
A	B	AND	NAND	OR	NOR	EXOR	EXNOR
0	0	0	1	0	1	0	1
0	1	0	1	1	0	1	0
1	0	0	1	1	0	1	0
1	1	1	0	1	0	0	1

For each combination of inputs to a logical function, there is an associated minterm :

A	B	C	Minterm
0	0	0	A′ B′ C′
0	0	1	A′B′C
0	1	0	A′B C′
0	1	1	A′BC
1	0	0	AB′C′
1	0	1	AB′C
1	1	0	ABC′
1	1	1	ABC

The minterm associated with each input combination is the AND, or product, of the input variables. Minterms and maxterms are useful for deriving Boolean equations from truth tables. For each combination of inputs to a logical function, there is an associated maxterm.

A	B	C	Maxterm
0	0	0	A + B + C
0	0	1	A + B + C′
0	1	0	A + B′ + C
0	1	1	A + B′ + C′
1	0	0	A′ + B + C
1	0	1	A′ + B + C′
1	1	0	A′ + B′ + C
1	1	1	A′ + B′ + C′

The maxterm associated with each input combination is the OR, or sum of the input variables. Maxterms and minterms are useful for deriving Boolean equations from truth tables.

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A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

Techniques of deriving Boolean equations from a Truth table.

Two commonly used techniques are follows :

1. The minterms corresponding to each line in the truth tale for which the output is logic 1 are extracted and combined using OR opertors. This method results in an equation said to be in the *sum-of-products from.*

   e.g.

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
0	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

SOP expression = $A'B'C + A'BC + AB'C' + AB'C$

For functions whose output is logic 1 fewer times than it is logic 0, it is generally easier to extract a sum-of-products equation. The sum-of-products and products of sums equations complement each other and return identical resulsts. An equation in either form can be transformed into its alternative form by means of the appropriate DeMorgan transformation

2. The maxterms corresponding to each line in the truth table for which the output is logic 0 are combined using and AND operators. This method results in an equation said to be in *product-of-sums* form.

Example. *Find the logic 0 outputs and invert those inputs in the product of sum format :*

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

POS expression = $(A + B + C)\ (A + B' + C)\ (A' + B' + C)\ (A' + B' + C')$

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KARNAUGH'S MAP

Karnaugh map which is used in the simplification of Boolean expressions, is actually a truth table in another form.

It is a diagram consisting of a number of squares. Each term represents a definite min term in a Boolean function.

Total number of squares are 2^n, where n is number of variables in the Boolean expression

e.g. a Karnaugh's map for a Boolean expression with four variables will have 16 squares and three variables will have 8 squares. 1 is entered corresponding to min terms present and 0 in other squares.

If two squares either in row or column have difference in only one bit, they are said to be in *pairs*.

Pair

If two adjacent squares either in row or column contain 1'*s*, they are said to be in *pairs* and corresponding two terms reduce to a single term. The reduced term will contain common variable in the two terms and number of variables in it will be one less.

Quad

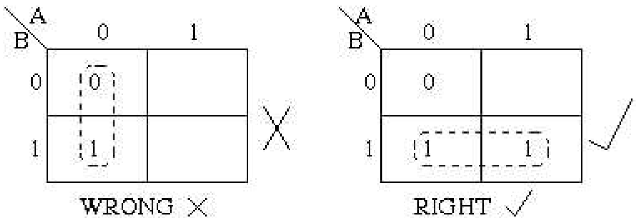
For three or more variables besides pairs, *quads* may occur. These are a group of four 1's horizontally or vertically or it may be in 4 adjacent squares of 2 rows and 2 columns. Hence 4 terms can be reduced to a single term containing only variables common to all 4 terms and will contain 2 variables less. Similarly with 4 or more variables *octets* may occur.

Rules of Karnaugh Map Simplification

Karnaugh map uses following rules for the simplification of expressions by grouping together adjacent cells containing *ones :*

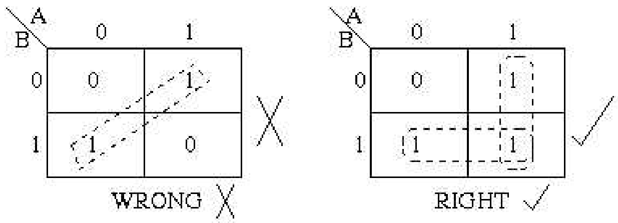
1. Zeros not allowed.

Groups may not include any cell containing a zero.



2. No diagonals.

Groups may be horizontal or vertical, but not diagonal.



3. Only power of 2 numbers of cells in each group.

Groups must contain 1, 2, 4, 8, or in general 2^n cells.

If n = 1, a group will contain two 1's, since $2^1 = 2$.

If n = 2, a group will contain four 1's since $2^2 = 4$.

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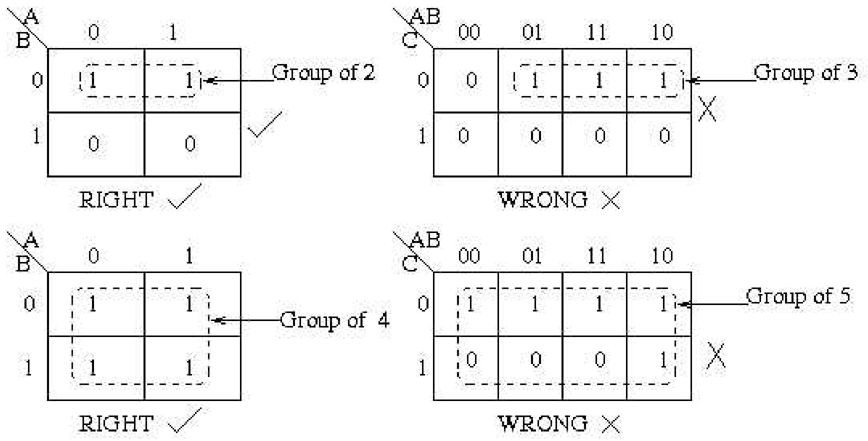
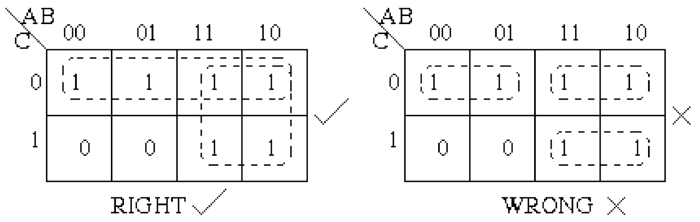
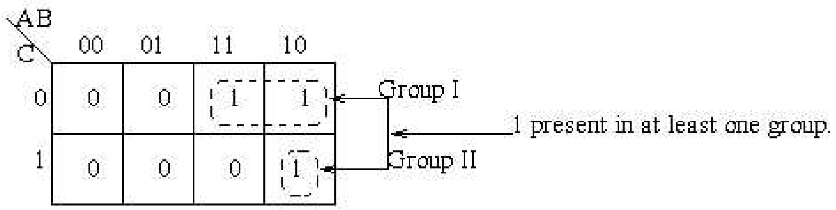


4. Each group should be as large as possible.

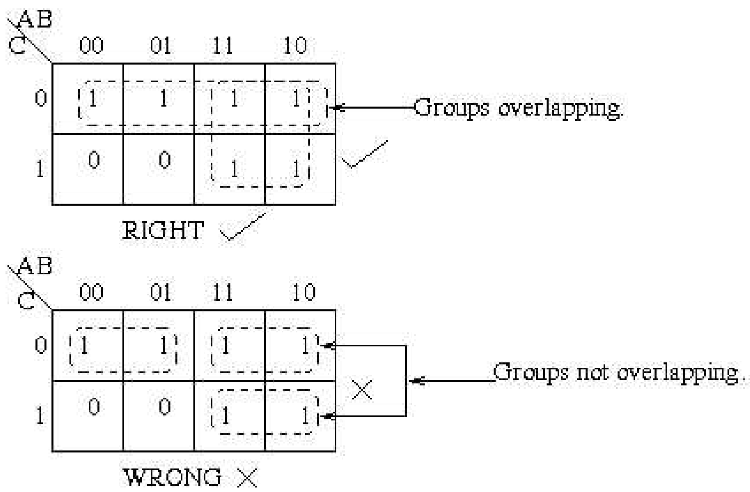


(Note that no Boolean laws broken, but not sufficiently minimal)

5. Each cell containing a *one* must be in at least one group.



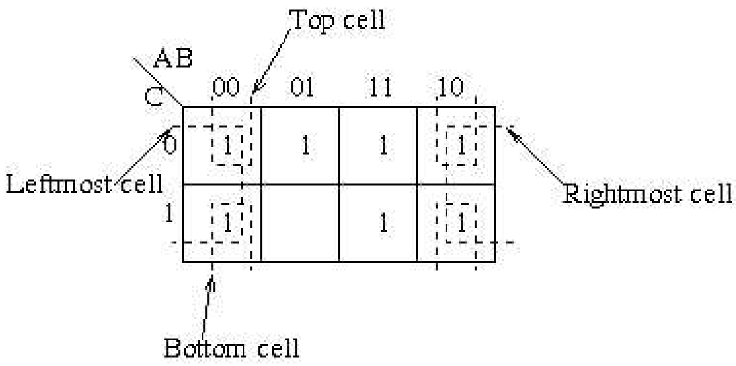
6. Groups may overlap.



7. Overlapping allowed.

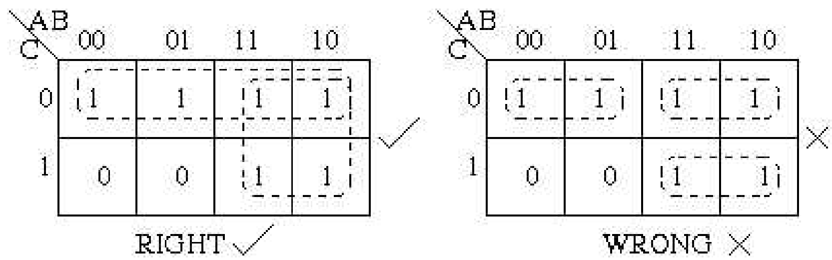
Groups may wrap around the table.

The leftmost cell in a row may be grouped with the rightmost cell and the top cell in a column may be grouped with the bottom cell.



8. Fewest number of groups possible.

There should be as few groups as possible, as long as this does not contradict any of the previous rules.



Example. Simplify $y = A + \overline{A}B$ by use of a Karnaugh map.

Solution:

A, the first term of y, spans both squares in the top row of the Karnaugh map; thus the row is filled by 1's.

The second term $\overline{A}$. B, spans only the lower right-hand square of the Karnaugh map and a 1 is so entered. 0's are entered in the balance of the map to give completed Karnaugh map.

	$\overline{B}$	B
$\overline{A}$	0	1
A	1	1

When collecting patterns of adjacent squares, subareas of the map can be used more than once.

For above Karnaugh map, all 1's are used atleast once by the two patterns consisting of upper row (A) and right column (B).

Thus $y = A + B$

DRAWING KARNAUGH'S MAP

Method for drawing Karnaugh's map is same for POS term. But max terms are entered by 0'*s* in the corresponding squares. It is checked in this case whether 0'*s* are in pairs or in quads or in octets etc.

Karnaugh map is a way of minimizing the boolean functions using diagrams which are made up of squares. By minimization we imply a function with minimum number of terms and each term with lowest number of literals.

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1. Two variables Karnaugh-map.

Truth table for 2 variables x and y :

k-map location number	x	y	Output function
0	0	0	$x'y'$
1	0	1	$x'y$
2	1	0	xy'
3	1	1	xy

x \ y	0	1
0	$x'\,y'$	$x'\,y$
1	xy'	xy

2. Three variable Karnaugh map.

Truth table for three variables x, y, z :

k-map location number	x	b	z	output function
0	0	0	0	$x'\,y'\,z'$
1	0	0	1	$x'\,y'\,z$
2	0	1	0	$x'\,y\,z'$
3	0	1	1	$x'\,y\,z$
4	1	0	0	$x\,y'\,z'$
5	1	0	1	$x\,y'\,z$
6	1	1	0	$x\,y\,z'$
7	1	1	1	xyz

x \ yz	0 0	0 1	1 1	1 0
0	$x'\,y'z'$	$x'\,y'z$	$x'\,y\,z$	$x'\,y\,z'$
1	$x\,y'z'$	$x\,y'\,z$	$x\,y\,z$	$x\,y\,z'$

3. Four variables Karnaugh map.

Truth table for four variables x, y, z, w :

K-map location	x	y	z	w	Output function
0	0	0	0	0	$x'y'z'w'$
1	0	0	0	1	$x'y'z'w$
2	0	0	1	0	$x'y'zw'$
3	0	0	1	1	$x'y'zw$
4	0	1	0	0	$x'yz'w'$
5	0	1	0	1	$x'yz'w$
6	0	1	1	0	$x'yzw'$
7	0	1	1	1	$x'yzw$
8	1	0	0	0	$xy'z'w'$
9	1	0	0	1	$xy'z'w$
10	1	0	1	0	$xy'zw'$
11	1	0	1	1	$xy'zw$
12	1	1	0	0	$xyz'w'$
13	1	1	0	1	$xyz'w$
14	1	1	1	0	$xyzw'$
15	1	1	1	1	$xyzw$

$x\,y$ \ $z\,w$	0 0	0 1	1 1	1 0
0 0	0	1	3	2
0 1	4	5	7	6
1 1	12	13	15	14
1 0	8	9	11	10

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Prime implicants.

Final Product term obtained from k-map after combining all possible adjacent square is called *prime implicants.*

Essential term.

When one minterm can only be represented by one prime implicant, then it is called *essential term*.

It helps in determining other possible simplifications for functions with multiple variables.

Decimal digit	*(BCD) 8421*	*Excess-3 5043210*	*84–2–1*	*2421*	*Binary*
0	0000	0011	0000	0000	0000
1	0001	0100	0111	0001	0001
2	0010	0101	0110	0010	0010
3	0011	0110	0101	0011	0011
4	0100	0111	0100	0100	0100
5	0101	1000	1011	1011	0101
6	0110	1001	1010	1100	0110
7	0111	1010	1001	1101	0111
8	1000	1011	1000	1110	1000
9	1001	1100	1111	1111	1001

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SOLVED EXAMPLES

1. Simplify Boolean function, $F = \bar{x}yz + \bar{x}y\bar{z} + x\bar{y}\bar{z} + x\bar{y}z$

Solution:

$x \backslash yz$	00	01	11	10
0	0	0	1	1
1	1	1	0	0

$F = \bar{x}y + x\bar{y}$

2. Simplify $f = \bar{A}\,.\,\bar{B}\,.\,C + \bar{A}\,.\,B\,.\,C$ using a Karnaugh map.

Solution:

The three variable map is constructed as follows :

$A \backslash BC$	00	01	11	10
0	0	1	1	0
1	0	0	0	0

$\mathbf{F = \bar{A}\,.\,C.}$

3. Simplify Boolean function, $F = \bar{x}yz + x\bar{y}\bar{z} + xyz + xy\bar{z}$

Solution:

$x \backslash yz$	00	01	11	10
0			1	
1	1		1	1

$\mathbf{F = yz + x\bar{z}}$

4. Simplify $F = x\,.\,yz + x\,.\,\bar{y}\,.\,z$ using a Karnaugh map.

Solution:

$x \backslash yz$	00	01	11	10
0	0	0	0	0
1	0	1	1	0

$F = xz$

5. Simplify Boolean function, $F = \bar{A}C + \bar{A}B + A\bar{B}C + BC$

Solution:

$A \backslash BC$	00	01	11	10
0		1	1	1
1		1	1	

$\mathbf{F = C + \bar{A}B}$

6. Use a Karnaugh map to simplify $F = \bar{A} + \bar{B} + A\,.\,B\,.\,\bar{C}\,.$

Solution:

$A \backslash BC$	00	01	11	10
0	1	1	1	1
1	1	1		1

$f = \bar{A} + \bar{B} + B\,.\,\bar{C}$

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7. Minimise following Boolean expression using Karnaugh's map.

$$W = \bar{x}\,\bar{y}z + \bar{x}yz + x\bar{y}\,\bar{z} + x\bar{y}z + xyz$$

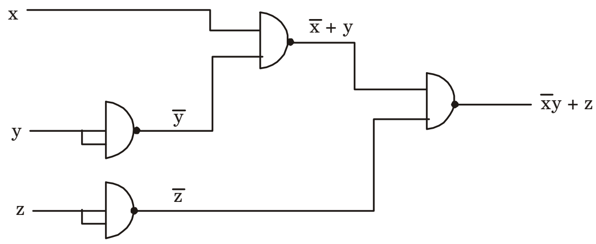
Solution:

Making K-map for the given Boolean expression

x \ yz	00	01	11	10
0	0	1	1	0
1	1	1	1	0

By the Karnaugh's map, $W = z + x\bar{y}$

Realization of equation $W = z + x\bar{y}$ using NAND gates is



8. Reduce Boolean function, $y = A.B.C.D + A\,\bar{B}.C.D + \bar{A}.B.C.D + \bar{A}.\bar{B}.C.D$ to a simpler expression using Karnaugh map.

Solution:

AB \ CD	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

$y = CD$

9. Simplify Boolean function, $F = \bar{A}\,\bar{B}\,\bar{C} + \bar{B}C\bar{D} + \bar{A}BC\bar{D} + A\bar{B}\,\bar{C}$

Solution:

AB \ CD	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

$$F = \bar{B}\,\bar{D} + \bar{B}\,\bar{C} + \bar{A}C\bar{D}$$

10. Simplify Boolean functions, $F(w, x, y, z) = \sum(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

Solution:

wx \ yz	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		1
10	1	1		

$$F(w, x, y, z) = \bar{y} + \bar{w}\,\bar{z} + x\bar{z}$$

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11. Simplify Boolean function, $f = A . B . C . D + A . C . D . + A . D.$

Solution:

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	1	1	0
10	0	1	1	0

$F(A, B, C, D) = A. D$

12. Minimize $f(W, X, Y, Z) = \sum(0, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14, 15)$

Solution:

YZ \ WX	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	1	0	1	1
10	1	0	1	1

$F(W, X, Y, Z) = X\overline{Y} + WX + \overline{X}\,\overline{Z} + \overline{X}Y$

13. Minimize $f(W, X, Y, X) = \sum(1, 3, 7, 11, 15) + \sum d\,(0, 2, 5)$

Solution:

YZ \ WX	00	01	11	10
00	X	0	0	X
01	1	X		
11	1	1	1	1
10	0	0	0	0

$= YZ + \overline{W}Z$

14. Simplify following Boolean function in sum-of-products form by means of a four-variable map.

$F = (A, B, C, D) = \sum(0, 2, 8, 9, 10, 11, 14, 15)$

Draw the logic diagram with

(*a*) AND - OR gates

(*b*) NAND gates.

Solution:

Combining squares with 1′s gives simplified functions in sum of products :

AB \ CD	00	01	11	10
00	1			1
01				
11			1	1
10	1	1	1	1

$F = \overline{B}\,\overline{D} + A\overline{B} + AC$

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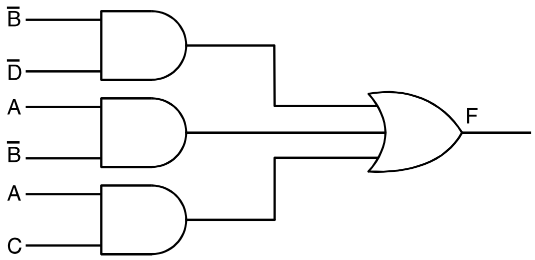
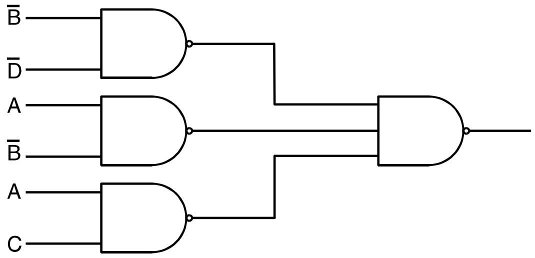
(*a*) Logic diagram with AND - OR gates.



(*b*) Logic diagram with NAND gates



15. Simplify Boolean function F together with the don't-care conditions in

(*a*) sum-of-products form and

(*b*) product-of-sums form.

$$F(w, x, y, z) = \Sigma(0, 1, 2, 3, 7, 8, 10)$$
$$d(w, x, y, z) = \Sigma(5, 6, 11, 15)$$

Solution:

wx \ yz	00	01	11	10
00	1	1	1	1
01	0	X	1	X
11	0	0	X	0
10	1	0	X	1

Fig. (a) Sum of Product

wx \ yz	00	01	11	10
00	1	1	1	1
01	0	X	1	X
11	0	0	X	0
10	1	0	X	1

Fig (b) Product-of-sum

This results in a simplified sum-of-products functions, $F = \overline{x}\,\overline{z} + \overline{w}\,z$

In (*b*), the 0'*s* are combined with any X'*s* convenient to simplify complement of the function. The complement function is simplified to :

$$\overline{F} = wz + x\overline{z}$$

Complementing again, we get a simplified product of sums function, $F = (\overline{w} + \overline{z})(\overline{x} + z)$

16. Simplify f= $\Sigma(8, 12, 13, 14) + d\Sigma(3, 7, 9, 10)$

Solution:

YZ \ WX	00	01	11	10
00	0	0	1	1
01	0	0	1	X
11	X	X	0	0
10	0	0	1	X

Here $f = W\overline{Y} + W\overline{Z}$

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17. Minimize f = $\sum(0,3,4,7,8) + d\sum(10,11,12,13,14,15)$

Solution:

AB \ CD	00	01	11	10
00	1	0	1	0
01	0	0	1	0
11	X	X	X	X
10	1	0	X	X

Here $$f = \overline{C}\,\overline{D} + CD = C . D$$

18. By use of a Karnaugh map, shows that $f_1 = f_2$ where

$$f_1 = A . B . C + A . B , \quad f_2 = A . (B + C) = A . B + A . C$$

Solution:

Construct the map where it is seen that mapping of f_1 and f_2 are identical.

A	00	01	11	10
0	0	0	0	0
1	1	1	1	1

19. Implement following function with NAND gates :

$$F(x, y, z) = \sum(0, 6)$$

x \ yz	00	01	11	10
0	1	0	0	0
1	0	0	0	1

Fig. (a) Map simplification in sum of products.

Solution:

Simplified function in sum of products from

$$F = \bar{x}\,\bar{y}\,\bar{z} + xy\bar{z}$$

The two-level NAND implementation is shown in Fig. (*b*).

Complement of the function in sum of products can be determined by combining 0's in the map.

$$\overline{F} = \bar{x}\,y + x\bar{y} + z$$

Two-level NAND gate for generating $\overline{F}$ is shown in Fig. (*c*).

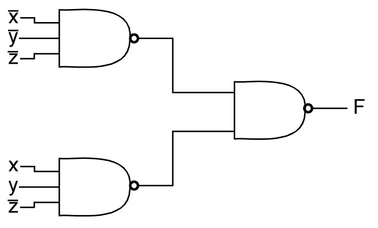


Fig. (b) $F = \bar{x}\,\bar{y}\,\bar{z} + xy\bar{z}$

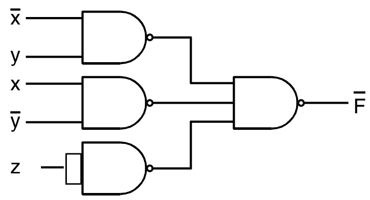
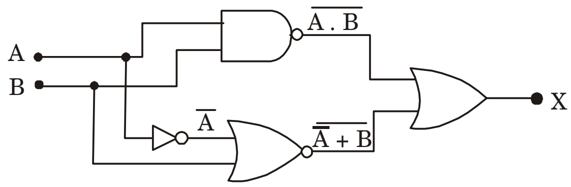


Fig. (c) $F = \bar{x}\,y + x\bar{y} + z$

20. Write an algebraic expression for x, the given logic circuit.



Solution:

With outputs of the intermediate logic gates as indicated in figure, overall output must be

$$X = \overline{A.B} + \overline{\overline{A} + B}$$

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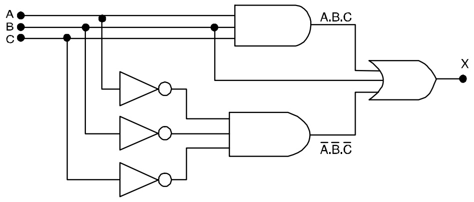
21. Draw a simplified logic circuit for the logic diagram shown in the figure.



Solution:

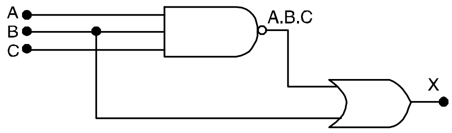
Output of logic circuit shown in the figure is

$$X = A.B.C + \bar{A}.\bar{B}.\bar{C} + B \quad \text{...}(i)$$

$$= (AC + 1)B + \bar{A}.\bar{B}.\bar{C}$$

$$= B + \bar{A}\bar{B}\bar{C} \quad \text{...}(ii)$$

Equation (*ii*) can be realized with only two logic gates as shown by figure given below.



22. For the case of three variables, show that NAND gates can be interconnected to form equivalent of an OR gate.

Solution:

Desired logic function is, $X = A + B + C$ ... (*i*)

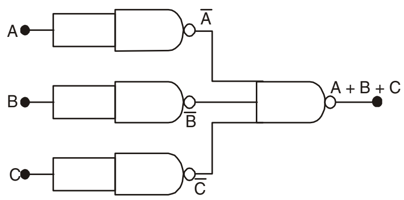
Applying negation and first of DeMorgan's laws to (*i*) we get

$$X = \overline{\overline{A+B+C}} = \overline{\bar{A}.\bar{B}.\bar{C}} \quad \text{...}(ii)$$

Now (*ii*) could be synthesized by a NAND gate if the complements of the three variables were available. But by the identity law,

$$\overline{A.A} = \bar{A} \quad \text{...}(iii)$$

An interpretation of (*iii*) is that, if A is input to both gates of a two-input NAND gate, then its output is $\bar{A}$. Thus, (*iii*) is realized by the logic circuit of figure given below.



23. Realize logic function f = A . B + $\bar{A}$. B using only NOR logic gates.

Solution:

Negation law and DeMorgan's laws give

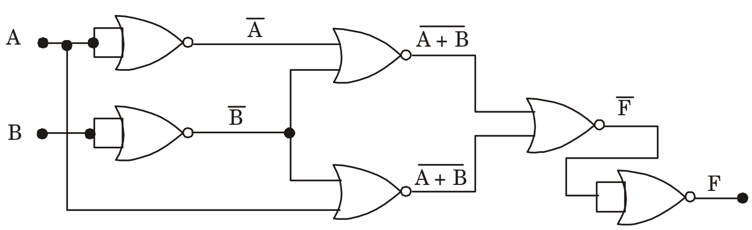
$$\bar{f} = A.B + \bar{A}.B$$

$$\bar{f} = \overline{A.B + \bar{A}.B}$$

$$\bar{f} = (\bar{A} + \bar{B}).(A + \bar{B})$$

$$\bar{f} = \overline{\overline{(\bar{A}+\bar{B}).(A+\bar{B})}}$$

$$\bar{f} = \overline{\overline{(\bar{A}+\bar{B})} + \overline{(A+\bar{B})}}$$



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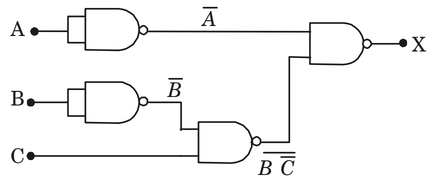
24. Use NAND gates only to construct a logic circuit with output X = A + $\overline{B}$. C.

Solution:



25. A digital system has four bits of a 4-bit word 'ABCD' as input. The output Y is equal to 1 when two adjacent bits equal to 1, or any three or all Four bits are 1.

(*a*) Draw Karnaugh map for Y and minimal expressions.

(*b*) Realise using 2-input and 3-input NAND gates only.

Solution: Truth Table :

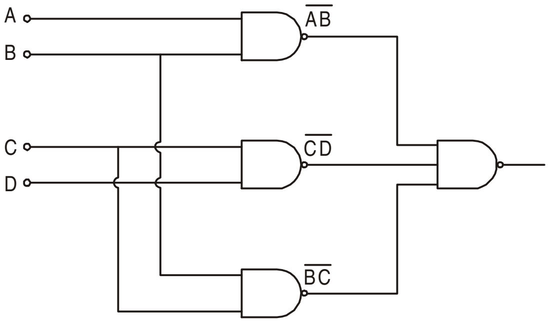
Decimal	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

(*a*) K-map

AB \ CD	00	01	11	10
00	0	0	1	0
01	0	0	1	1
11	1	1	1	1
10	0	0	1	0

Y = AB + CD + BC

(*b*) $Y = \overline{\overline{Y}} = \overline{\overline{AB+CD+BC}} = \overline{\overline{AB}\,\overline{CD}\,\overline{BC}}$



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COMBINATIONAL CIRCUITS

INTRODUCTION

A combinational circuit consists of logic gates whose output at any time are determined directly from the present combination of inputs irrespective of previous inputs.

e.g. half adder, full subtractor, code conversion, binary adder, decimal adder, magnitude-comparator, decoder, demultiplexer, encoder, multiplexer, ROM, PLA etc.

Combinational circuit consists of

(*i*) Input variables

(*ii*) Logic gates

(*iii*) Output variables.

The logic gates accept signals from the inputs and generate signals at the outputs. This process transforms binary information from given input data to the required output data.

DESIGN OF COMBINATIONAL CIRCUIT.

It starts from a specification of the problem and culminates in a logic diagram or set of Boolean equations from which the logic diagram can be obtained.

The procedure involves following steps :

1. From the specifications of the circuit, determine required number of inputs and outputs, and assign a letter symbol to each.
2. Derive the truth table that defines the required relationship between inputs and outputs.
3. Obtain the simplified Boolean functions of each outputs as function of the input variables.
4. Draw the logic diagram.
5. Verify the correctness of the design.

ARITHMETIC CIRCUITS

Half Adder

A logic circuit for the addition of two one-bit numbers is called half-adder. The addition process is reproduced in truth table.

Here, A and B are the two inputs and S (SUM) and C (CARRY) are the two outputs.

INPUTS		**OUTPUTS**	
A	**B**	**Sum**	**Carry**
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

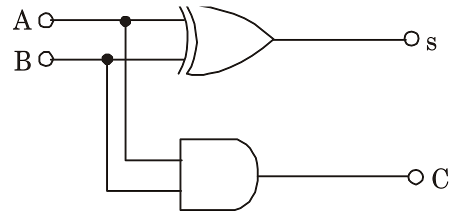

Fig. Realization of an Half-adder

From truth table, logical expressions for S and C outputs are obtained as

$$S = \overline{A}\,B + A\,\overline{B} = A \oplus B$$

$$C = AB$$

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Full Adder

A half adder has only two inputs and there is no provision to add a carry coming from lower order bits when multibit addition is performed. For this purpose, a third input terminal is added and this circuit is used to add A_n, B_n, and C_{n-1}, where A_n and B_n are nth order bit of the numbers A and B respectively and C_{n-1} is carry generated from the addition of $(n-1)$th order bits. This circuit is referred to as full-adder and its truth table is given below.

Inputs			Outputs	
A_n	B_n	C_{n-1}	S_n	C_n
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

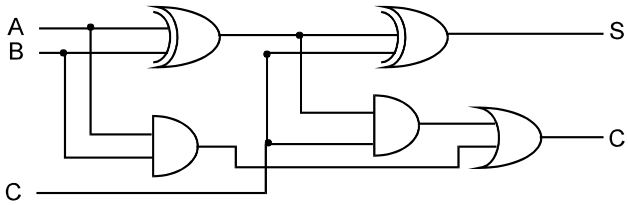


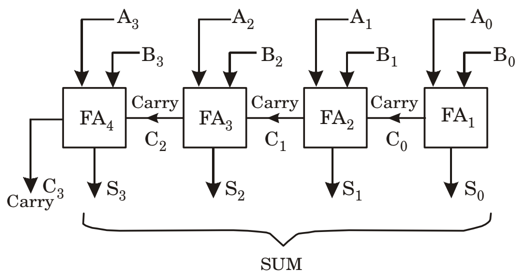
Fig. Realization of a Full adder

From the truth table, logical expressions for the outputs S_n and C_n are obtained as

$$\text{Sum} = A \oplus B \oplus C$$
$$\text{Carry} = AB + BC + AC$$

Binary Parallel Adder

It is a digital function that produces arithmetic sum of two binary numbers in parallel. It consists of full-adders connected in cascade, with the output carry from one full adder connected to the input carry of the next full-adder.



Half Subtractor

A logic circuit for the subtraction of B (subtrahend) from A (minuend) where A and B are 1-bit number is called *half-subtractor*. The subtraction process is reproduced in truth table.

Here, A and B are two inputs and D (difference) and C (borrow) are two outputs.

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INPUTS		OUTPUTS	
A	**B**	**D**	**C**
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

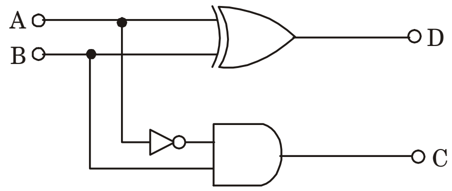

Fig. Realization of a half-subtractor

From truth table, logical expressions for D and C are obtained as

$$D = \overline{A}B + A\overline{B} = A \oplus B$$

$$C = \overline{A}B$$

Full Subtractor

A *full-subtractor* circuit is required for performing multibit subtraction where in a borrow from the previous bit position may also be there.

Full-subtractor have three inputs :

(*i*) A_n (minuend)

(*ii*) B_n (subtrahend)

(*iii*) C_{n-1} (borrow from previous stage)

Full-subtractor have two outputs :

(*i*) D_n (difference)

(*ii*) C_n (borrow).

Its truth table is given below.

INPUTS			OUTPUTS	
$\mathbf{A_n}$	$\mathbf{B_n}$	$\mathbf{C_{n-1}}$	$\mathbf{D_n}$	$\mathbf{C_n}$
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

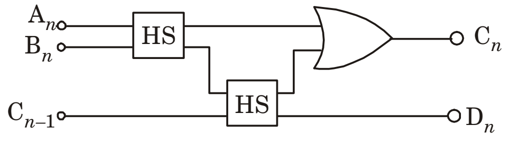

Fig. Realization of Full subtractor

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CD \ AB	00	01	11	10
00	0	1	0	1
01	0	x	x	1
11	0	x	x	0
10	0	x	x	1

Expression for X using SOP form.

$X = \overline{A}B + A\overline{C}D + A\overline{B}\,\overline{D}$

CD \ AB	00	01	11	10
00	0	0	1	1
01	0	x	x	0
11	0	x	x	1
10	0	x	x	0

X = AB + ACD + ABC

Expression for W

W = AB + ACD

= A (B + CD)

DECODERS

Decoder is a combinational logic circuit that receives coded information on n lines and feeds them to 2^n unique output lines after conversion. If n-bit decoded information has unused or don't-care combinations, the decoder output will have less than 2^n outputs.

A	B	Out 1	Out 2	Out 3	Out 4
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

Fig. Realization of 4 × 16 decoder using two 3 × 8 decoders is shown below.

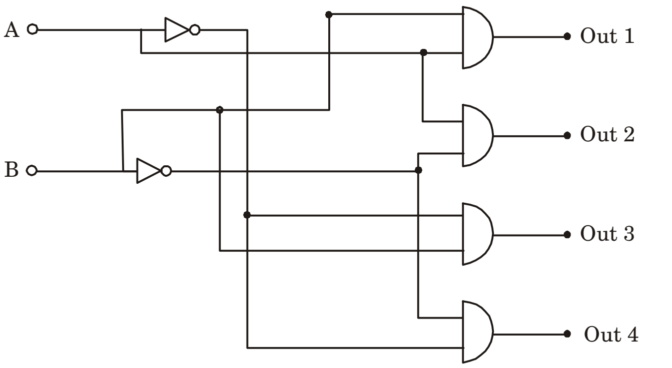


Fig. Realization of four output decoder using gates

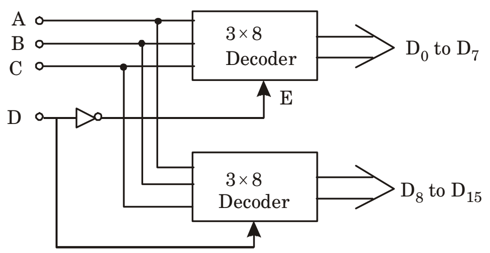


Fig. 4 × 16 decoder

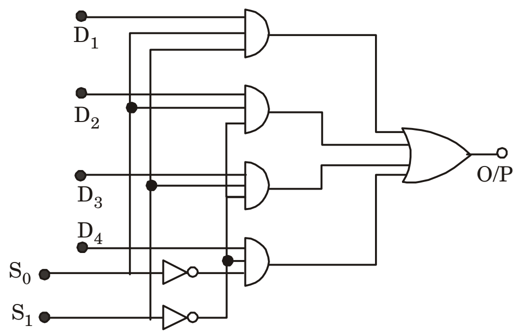
MULTIPLEXER

It is a combinational logic circuit that selects binary information from one of the multiple input lines (D_{n-1} ..... D_1, D_0) and directs it to an output line according to a received *select code* ($S = S_{n-1}$ ....$S_1 S_0$).

A block diagram of a four input multiplexer is shown in the figure below and it's truth table is given below.

Truth Table

S_0	S_1	O/P
0	0	D_4
0	1	D_3
1	0	D_2
1	1	D_1



Output = $D_4\,\overline{S_0}\,\overline{S_1} + D_3\,\overline{S_0}\,S_1 + D_2\,S_0\,\overline{S_1} + D_1\,S_0\,S_1$

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Example. Implement following function with a multiplexer, $f(A, B, C, D) = \sum(0, 1, 3, 4, 8, 9, 15)$.

Solution.

This is a 4 variable function.

Number of selection lines will be 4 – 1 = 3 and input $2^3 = 8$ lines with A as MSB and D as LSB.

	I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7
$\overline{A}$	0	1	2	3	4	5	6	7
A	8	9	10	11	12	13	14	15
	1	1	0	$\overline{A}$	$\overline{A}$	0	0	A

Minterms of the functions are encircled.

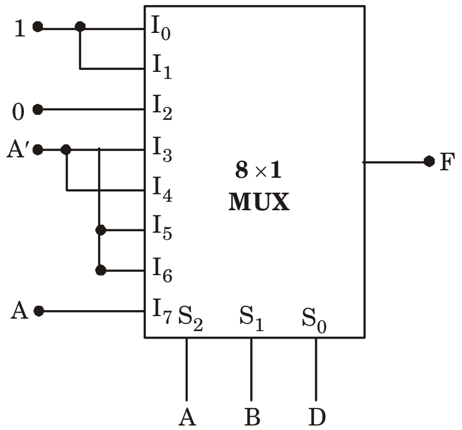
By inspection of each column

(*a*) If both minterms in a column are not circled, apply 0 to the corresponding MUX input. This will be I_2, I_5, I_6.

(*b*) If both terms are circled, apply 1 to the MUX input. This will be I_0, I_1.

(*c*) If only top row of column is circled, apply $\overline{A}$ to the corresponding MUX input. This will be I_3, I_4.

(*d*) If only bottom row of column is circled, apply A to the corresponding MUX input. This will be I_7.



It is not necessary that MSB or left most variable should be chosen as the residue. By selection of any other variable, the table will change as follows.

	I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7
$\overline{B}$	0	1	2	3	8	9	10	11
B	4	5	6	7	12	13	14	15
$\overline{C}$	0	1	4	5	8	9	12	13
C	2	3	6	7	10	11	14	15
$\overline{D}$	0	2	4	6	8	10	12	14
D	1	3	5	7	9	11	13	15

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1.11 DEMULTIPLEXER (DEMUX)

It is a combinational logic circuit that connects a single-input line (D) to one of multiple output lines (f_{n-1}, ..., f_1, f_0) according to a received n-bit select coded (S = S_{n-1} .... S_1 S_0). Demultiplexing is reverse process of multiplexing.

Figure below shows a demultiplexer, through use of 2 selection lines, four different outputs are possible.

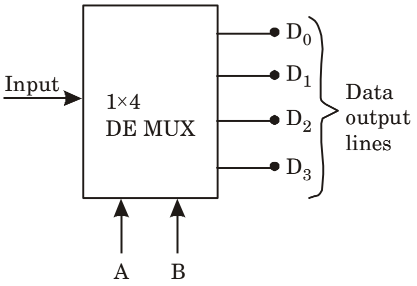

Fig. Block diagram

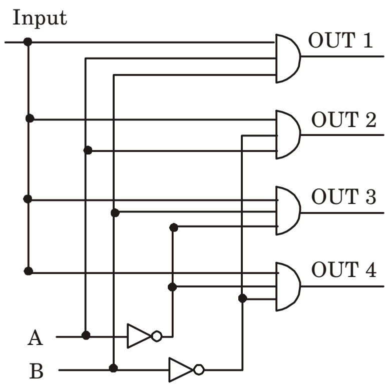

Fig. Realization of four output demultiplexer using gates

Block diagram shown in figure is associated with truth table given below.

A	B	Input	Out 1	Out 2	Out 3	Out 4
0	0	0	0	0	0	0
0	0	1	0	0	0	1
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	0	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	0	0
1	1	1	1	0	0	0

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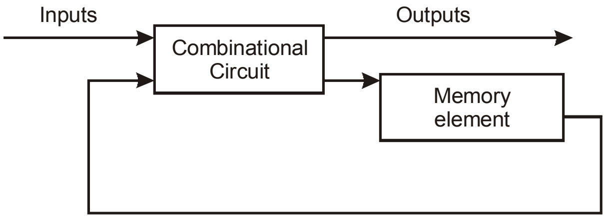
SEQUENTIAL CIRCUITS

INTRODUCTION

In the sequential circuits, outputs are implicily or explicitly delayed and connected as inputs. Such circuits have a feedback from output to the input. The output depends not only on the present state but also on the past history of the inputs, *i.e.* they have memory.

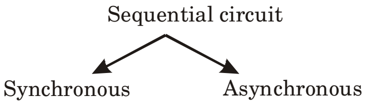
The basic element used in sequential circuits is a flip-flop that can be made using logic gates and is available in IC form also. These are basic building blocks of many sequential logic circuits such as registers, counters, static RAM etc.

In sequential circuit, logic output depends on stored levels and also the input levels, whereas combinational logic output depends on the inputs levels.



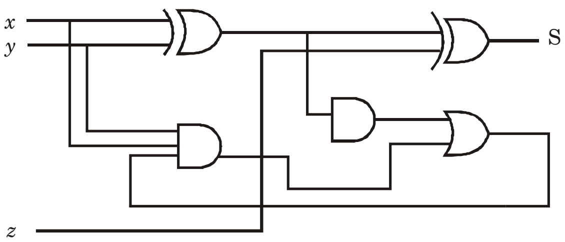
The memory elements has capability of storing binary information.

TYPES OF SEQUENTIAL CIRCUITS



1. **Asynchronous sequential circuit**

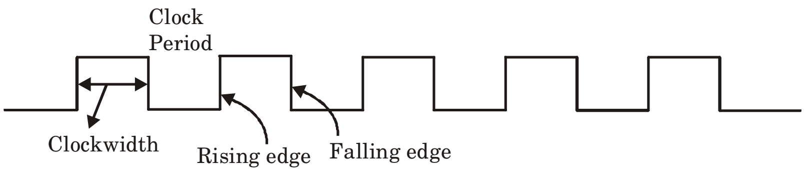
   This is a system whose outputs depend upon the order in which its input variables change and can be affected at any instant of time.



   Flip-Flops are synchronous, which means that the state changes DC current in response to a clock pulse. So it does have a clock input.

2. **Synchronous sequential circuit**

   This type of circuit changes their states and output values at discrete instants of time. All state transitions in such circuit occur only when the clock value is either 0 or 1 or happen at the rising or falling edges of the clock depending on the type of memory elements used in the circuit. These are also called *clocked sequential circuit.*



   Latches are asynchronous, which means that the output changes very soon after the input changes. It does not have a clock input.

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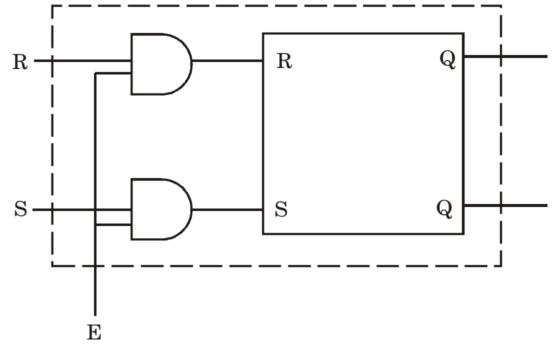
Level Triggered enables.

This is found in latches devices whose contents are affected by any change to the inputs. So, contents of a latch can be controlled by an enable input, and this devices is called *level triggered.*

e.g.



FLIP-FLOPS

Flip-Flops is another name for a bistable multivibrator. It is capable of storing 1 bit of binary data. *It has two stable states* : 'one' and 'zero'.

The output stays low or high and to change it, the circuit must be derived by an input called *trigger*. Until the trigger arrives, output voltage remains low or high indefinitely.

Uses of Flip-Flops

Flip-Flops are used for following :

(1) Memory device or data latch
(2) Relay circuit as a means of delaying information or data.
(3) Frequency divider (divide by 2 circuit)
(4) Creating registers or latches
(5) Creating counters and shift register
(6) Binary rate multipliers or dividers
(7) Flags or Semaphores
(8) Time state machines.

Preset and Clear

To get some computers started, an operator has to push a reset button. This sends a reset or CLEAR signal to all flip-flops. Also, it is necessary in some digital systems to PRESET certain flip-flops.

In a clocked flip-flop PRESET and CLEAR inputs are called *synchronous*, because they activate flip-flops independently of the clock.

TYPES OF FLIP-FLOPS.

1. **Edge triggered flip-flop**

   It responds only during brief instant the clock switches from one voltage level to another. It is enabled only during a change in value of the logic and circuit enabled during the transition from 01

   When triggering occurs on the positive going edge of the clock, it is called *positive-edge triggering.*

   When triggering occurs on trailing edge of the clock activates the gates, allowing data to be recognized, it is called *negative-edge triggering.* Circuit enabled during the transition from logic 1 to logic 0.

2. **R-S Flip-Flop (Direct coupled RS Flip-Flop)**

   This logic circuit has two stable states, which can be achieved by giving proper inputs to R and S inputs. The flip-flop will assume one of its two stable states depending upon any asymmetry in the circuit, hence both R and S high at the same time is forbidden because it is contradiction.

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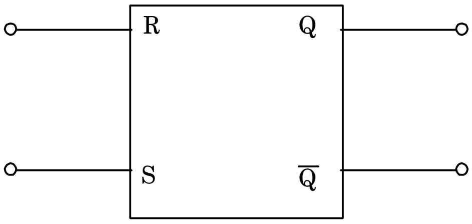


Fig. Block diagram

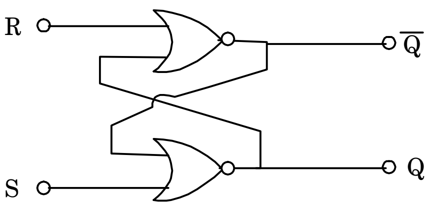


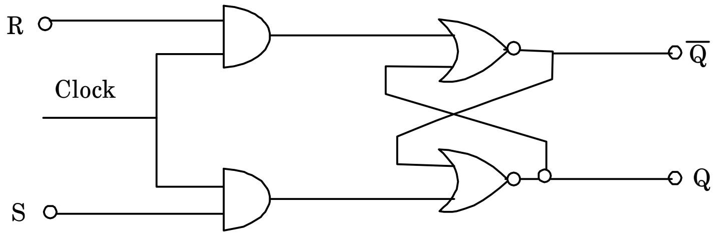
Fig. Logic diagram

INPUTS	OUTPUTS			MODE
S	**R**	**Q**	$\overline{Q}$	
0	0	Q	$\overline{Q}$	Hold
0	1	0	1	Reset
1	0	1	0	Set
1	1	0	0	Prohibited

3. Clocked R-S Flip-Flop

It is sometimes required that the time at which a flip-flop is set or reset be controlled by a timer or clock.

By adding AND gates at the input to which clock pulses are applied, the flip-flop can be made to respond to input levels appearing only during occurrence of a clock pulse.



Following table shows state to which the flip-flop will go during the next clock pulse, given present state and the present inputs. Here R = S = 1 is a prohibited state.

Characteristic table.

	Inputs		Outputs		Mode
CLK	S	R	Q_n	$\overline{Q}_n$	
0	0	0	Q	$\overline{Q}_n$	Hold
0	0	1	Q	$\overline{Q}_n$	Hold
0	1	0	Q	$\overline{Q}_n$	Hold
0	1	1	Q	$\overline{Q}_n$	Hold
1	0	0	Q	$\overline{Q}_n$	Hold
1	0	1	0	1	Reset
1	1	0	1	0	Set
1	1	1	1	1	Prohibited

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Characteristic equation.

Q \ SR	00	01	11	10
0			X	1
1	1		X	1

$$Q\,(t+1) = S + \overline{R}\,Q$$

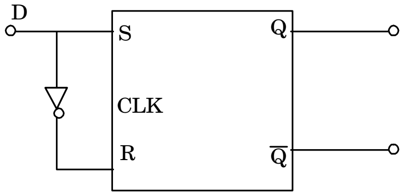
$$S\,R = 0$$

Characteristic equation specifies value of the next state as a function of present state and the inputs. The relation SR = 0 specifies that both S and R cannot be equal to 1 simultaneously.

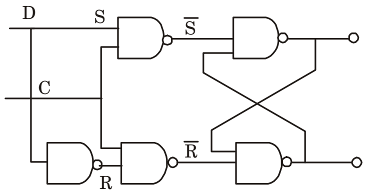
4. D flip-flop

This is a flip-flop with a delay (D) and is equal to exactly one cycle of the clock. The prohibited condition of both R and S high can be avoided by D flip-flop. In D flip flop, input to R is through an inverter from S, so that input to R is always the complement of S and is never same.

Note : Q (t+1) is always equal to D.



(a) Logic diagram



***(b) Realization by NAND* gates**

Q	D	Q(t + 1)
0	0	0
0	1	1
1	0	0
1	1	1

(c) Characteristic table

Q \ D	0	1
0	0	1
1	0	1

$$Q\,(t+1) = D$$

(d) Characteristic equation

5. J-K Flip-Flop

It is called a *universal flip-flop* because other flip-flops like D, R-S and T can be derived from it. In a R-S flip-flop, input R = S = 1 leads to an indeterminate output. The R-S flip-flop circuit may be refined so that if both inputs are high, then flip-flop switches to its complement state, *i.e.* if Q = 1, then Q (t + 1) will be 0 and vice-versa.

Q \ JK	00	01	11	10
0			1	1
1	1			1

$$Q\,(t+1) = JQ' + K'Q$$

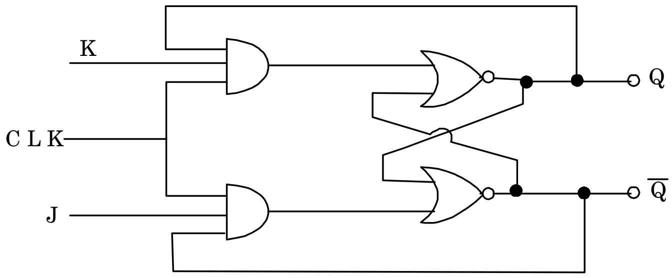


Fig. Logic diagram

6. Master-Slave Flip-Flop

It is constructed from two separate flip-flops. When CLK is 0, output of the inverter is 1. Since clock input of slave is 1, flip-flop is enabled and output Q is equal to Y. The master flip-flop is disabled because CLK = 0. When pulse becomes 1, information at the external R and S inputs is transmitted to the master flip-flop. The slave flip-flop is isolated as long as the pulse is at its level 1, because output of the inverter is 0. When pulse returns to 0, master flip-flop is isolated, which prevents external inputs from affecting it. The slave flip-flops of master slave coincide with negative edge transition of the pulse.

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The master-slave combination can be constructed for any type of flip-flop by adding a clocked R-S flip-flop with an inverted clock to form the slave.

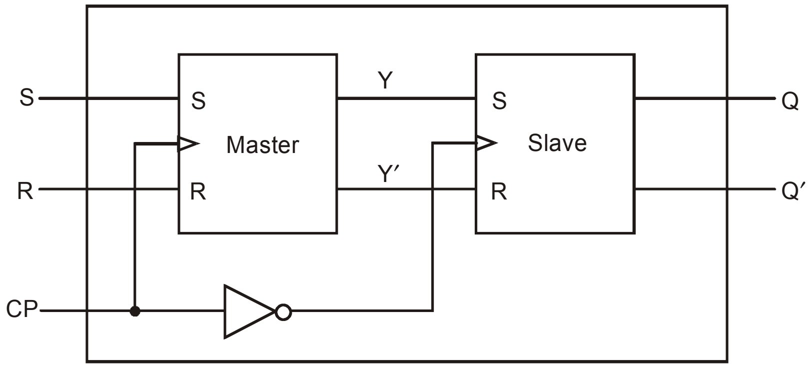


Fig. Logic diagram of master-slave flip-flop

In a master-slave flip-flop, it is possible to switch output of the flip-flop and its input information with the same clock pulse.

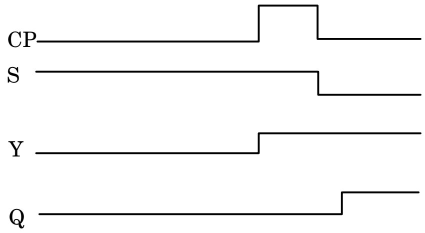
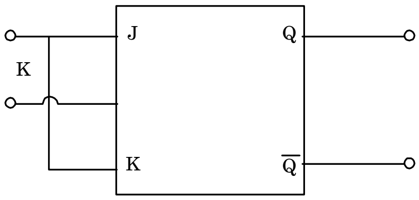


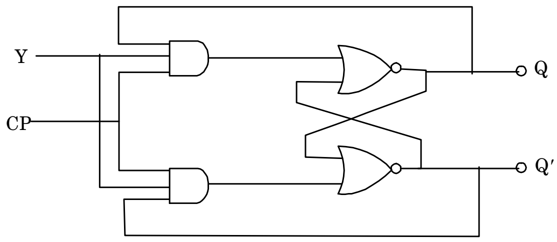
Fig. Timing relationships in a master-slave flip-flop

7. T Flip-Flop

It is a single input version of J-K flip-flop. In this, both inputs are tied together. Regardless of the present state of flip-flop, output of the flip-flop is complemented, when clock pulse occurs while input T is high.



(a) Logic diagram



(b) Realization by NOR gates

Q	T	Q(t + 1)
0	0	0
0	1	1
1	0	1
1	1	0

(c) Characteristic table

Q \ T	0	1
0	0	1
1	1	0

$$Q\,(t + 1) = T\,\overline{Q} + \overline{T}\,Q$$

(d) Characteristic equation

EXCITATION TABLE OF FLIP-FLOP.

Truth table of a flip-flop is also referred to as *characteristic table* and specifies operational characteristic of the flip-flop.

In the design of sequential circuits, situations occurs in which *present state* and *next state* of the circuit are specified, and then find input conditions that must prevail to cause desired transition of the state *e.g.* output of an S–R flip-flop before the clock pulse is $Q_n = 0$ and it is desired that output does not change when clock pulse is applied.

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From truth table (or characteristic table) of an S–R flip-flop, following conditions are obtained :

(1) $S_n = R_n = 0$ (first row)

(2) $S_n = 0$, $R_n = 1$ (third-row)

A tabulation of these conditions called *excitation table*. Table gives excitation tables of S–R, J–K, T, and D flip-flops. This is derived from characteristic table of the flip-flop.

Table. Excitation table of flip-flops

Present State	*Next State*	*S–R* S_n	*FF* R_n	*J–K* J_n	*FF* K_n	*T-FF* T_n	*D-FF* D_n
0	0	0	×	0	×	0	0
0	1	1	0	1	×	1	1
1	0	0	1	×	1	1	0
1	1	×	0	×	0	0	1

State Table

The state table representation of a sequential circuit consists of three sections labelled :

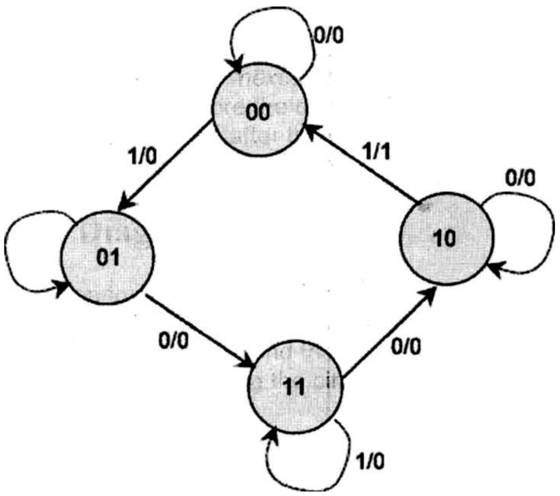
(1) Present state : It designates the state of flip-flops before the occurrence of a clock pulse.

(2) Next state : It shows the states of flip-flops after the clock pulse

(3) Output section : It lists values of the output variables during the present state.

STATE DIAGRAM.

In addition to graphical symbols, tables or equations, flip-flops can also be represented graphically by a state diagram. In this diagram, a state is represented by a circle, and the transition between states is indicated by directed lines (or arcs) connecting the circles. An example of a state diagram is shown in figure below.



The binary number inside each circle identifies the state the circle represents. The directed lines are labelled with two binary numbers separated by a slash (/). The input value that causes the state transition is labelled first. The number after the slash symbol / gives the value of the output.

e.g. Consider a sequential circuit shown below. It has one input X, one output Z and two state variables Q1Q2 (thus having four possible present states 00, 01, 10, 11).

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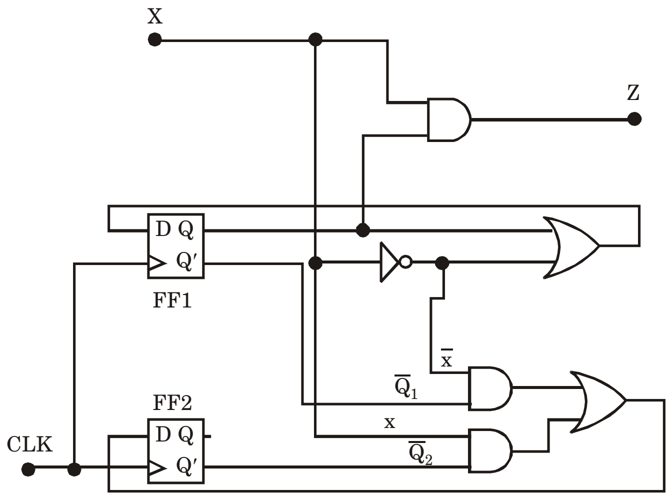


Behaviour of the circuit is determined by the following Boolean expressions :

$$Z = X.Q_1$$
$$D1 = x' + Q_2$$
$$D2 = x.\overline{Q}_2 + \overline{x} + \overline{Q}_1$$

These equations can be used to form the state table.

Let present state (i.e. Q_1Q_2) = 00, and input x = 0.

Under these conditions, we get Z = 0, D1 = 1, and D2 = 1.

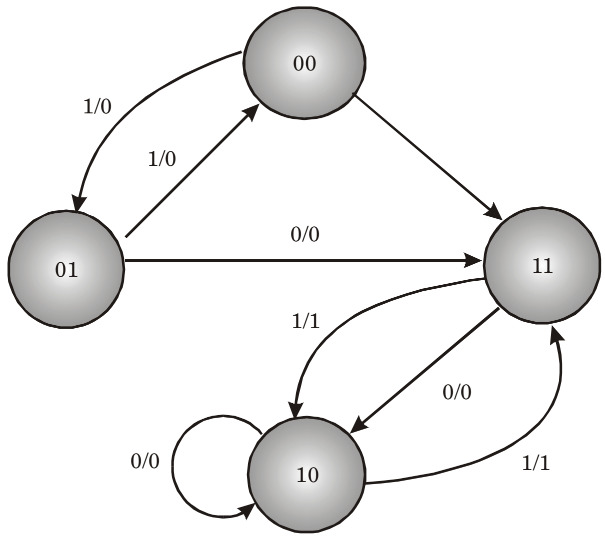
Thus next state of the circuit D1D2 = 11, and this will be the present state after thclock pulse has been applied.

Output of the circuit corresponding to the present state Q_1Q_2 = 00 and X = 1 is Z = 0.

This data is entered into the state table as shown in the table below.

Present State	**Next State**		**Output**	
$\mathbf{Q_1Q_2}$	***x* = 0**	***x* = 1**	***x* = 0**	***x* = 1**
0 0	1 1	1 1	0	0
0 1	1 1	0 0	0	1
1 0	1 0	1 1	0	1
1 1	1 0	1 0	0	1

State diagram for the sequential circuit shown in the above figure is shown below :



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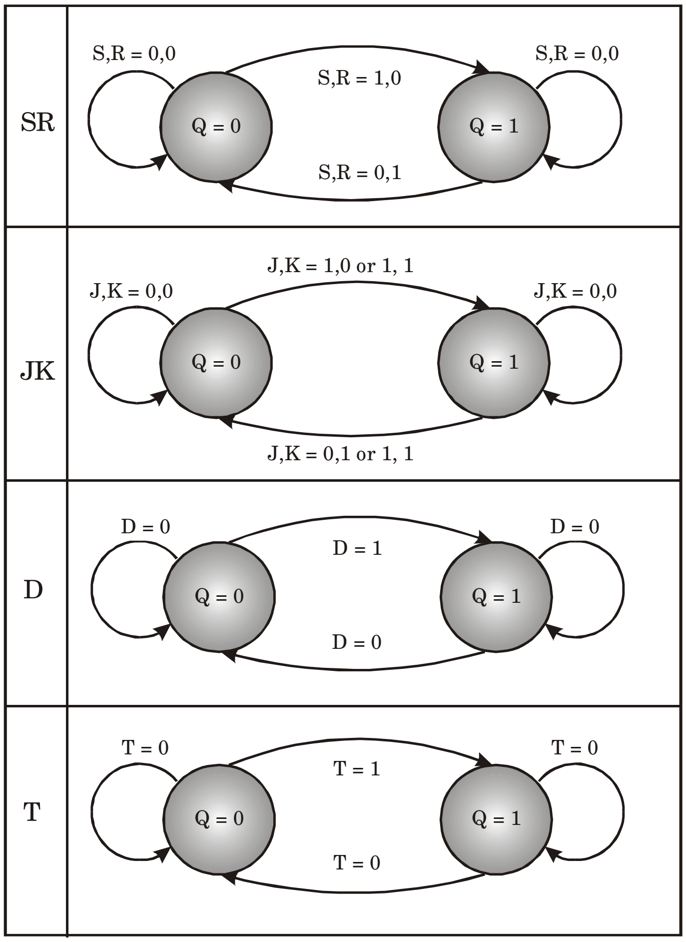
State diagrams of some Flip-Flops



OPERATING CHARACTERISTICS OF FLIP-FLOPS.

The operating characteristics apply to all flip-flops regardless of the particular form of the circuit. These are typically found in data sheets for integrated circuits. These specify the performance, operating requirements, and operating limitations of the circuit.

1. **Propagation Delay time**

   It is the interval of time required after an input signal has been applied for the resulting output change to occur.

2. **Set-Up time**

   It is the minimum interval required for the logic levels to be maintained constantly on the inputs (J and K, or S and R, or D) prior to the triggering edge of the clock pulse in order for the levels to be reliably clocked into the flip-flop.

3. **Hold time**

   It is the minimum interval required for the logic levels to remain on the inputs after the triggering edge of the clock pulse in order for the levels to be reliably clocked into the flip-flop.

4. **Maximum clock frequency**

   It is the highest rate that a flip-flop can be reliably triggered.

5. **Power dissipation**

   It is the total power consumption of the device.

6. **Pulse widths**

   These are the minimum pulse widths specified by the manufacturer for the CLOCK, SET and CLEAR inputs.

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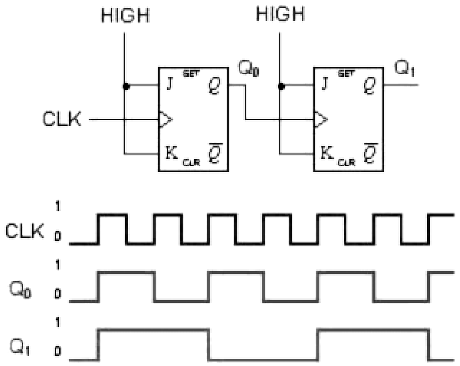
APPLICATIONS OF FLIP-FLOPS.

1. **Frequency division**

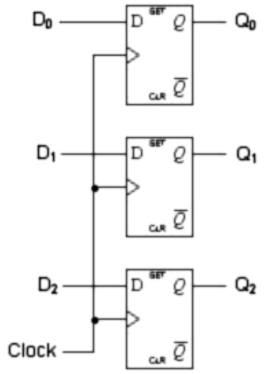
   When a pulse waveform is applied to the clock input of a J-K flip-flop that is connected to toggle, the Q output is a square wave with half the frequency of the clock input. If more flip-flops are connected together as shown in the figure below, further division of the clock frequency can be achieved.



   Q output of the second flip-flop is one-fourth the frequency of the original clock input. This is because frequency of the clock is divided by 2 by the first flip-flop, then divided by 2 again by the second flip-flop. If more flip-flops are connected this way, the frequency division would be 2 to the power n, where n is number of flip-flops.

2. **Parallel data storage**

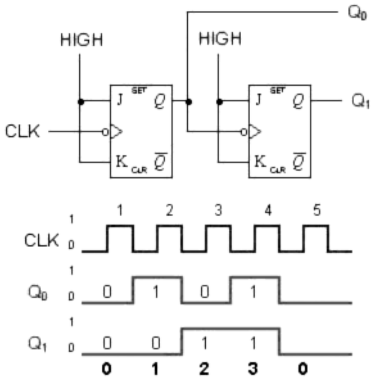
   In digital systems, data are normally stored in groups of bits that represent numbers, codes, or other information. So, it is common to take several bits of data on parallel lines and store them simultaneously in a group of flip-flops. This operation is shown in the figure below.



   Each of the three parallel data lines is connected to the D input of a flip-flop. Since all the clock inputs are connected to the same clock, data on the D inputs are stored simultaneously by the flip-flops on the positive edge of the clock.

3. **Counting**

   Another very important application of flip-flops is in digital counters.



   A counter that counts from 0 to 3 is shown in the timing diagram. The two-bit binary sequence repeats every four clock pulses. When it counts to 3, it recycles back to 0 to begin the sequence again.

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COUNTERS

Classification of Counters

1. **Asynchronous counter.**
   When flip-flops are connected serially and output of preceding flip-flop clocks the succeeding flip-flop, it is asynchronous counter as the change of states occurs one after the other.

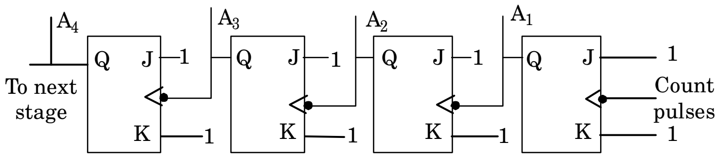


Fig. 4-bit binary ripple counter(asynchronus)

2. **Synchronous counter.**
   In the synchrounous counter, all flip-flops are clocked simultaneously with the clocks inputs connected in parallel to the clock pulse. This increases speed of operation of the counters. The pulse to be counted are applied at the clock input terminal.

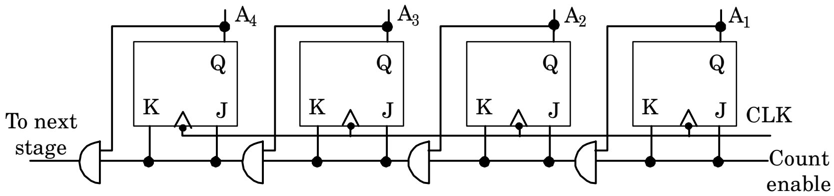
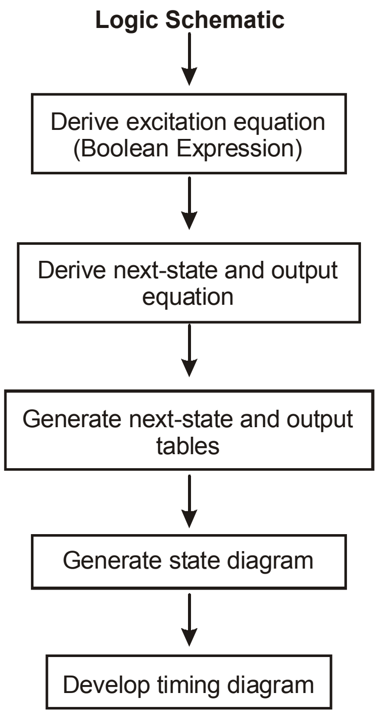


Fig. 4-bit synchronous binary counter

ANALYSIS OF SEQUENTIAL CIRCUITS

The behaviour of a sequential circuit is determined from the inputs, outputs and the states of its flip-flops. Both the output and the next state are a function of the inputs and the present state.

Method of sequential circuit Analysis



Start with the logic schematic from which excitation equations for each flip-flop input can be derived. Then, to obtain next-state equations, insert excitation equations into the characteristic equations. The output equations can be derived from the schematic, and once output and next-state equations are obtained, the next-state and output tables as well as state diagrams can be generated. When this stag is reached, use either the table or the state diagram to develop a timing diagram which can be verified through simulation.

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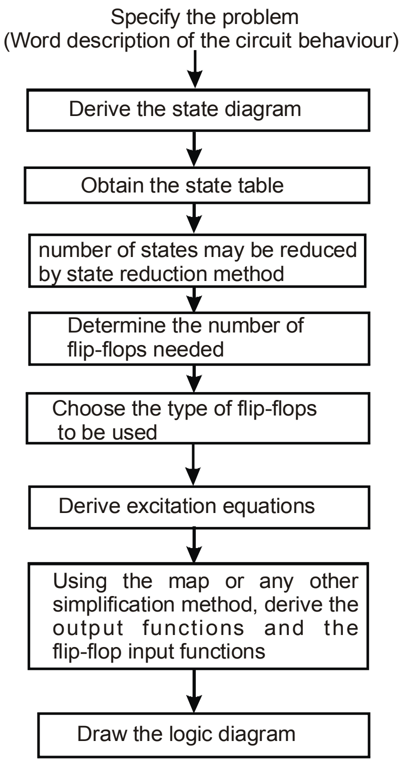
DESIGN OF SYNCHRONOUS SEQUENTIAL CIRCUITS

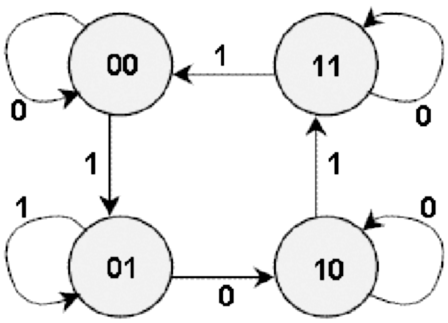
The design of a synchronous sequential circuit starts from a set of specifications and culminates in a logic diagram or a list of Boolean functions from which a logic diagram can be obtained. In contrast to a combinational logic, which is fully specified by a truth table, a sequential circuit requires a state table for its specification.

First step in the design of sequential circuits is to obtain a state table or an equivalence representation, such as a state diagram.

A synchronous sequential circuit is made up of flip-flops and combinational gates. The design of the circuit consists of choosing the flip-flops and then finding the combinational structure which, together with the flip-flops, produces a circuit that fulfils the required specifications. The number of flip-flops is determined from the number of states needed in the circuit.



Example. Design a synchronous sequential circuit whose state diagram is given below. The type of flip-flop to be use is J-K.



State diagram

Solution:

From the state diagram, generate the state table. Note that there is no output section for this circuit. Two flip-flops are needed to represent four states and are designated q_0, q_1, q_2, q_3,. The input variable is labelled x.

Present State	Next State	
Q_0 Q_1	x = 0	x = 1
q_0 0 0	0 0	0 1
q_1 0 1	1 0	0 1
q_2 1 0	1 0	1 1
q_3 1 1	1 1	0 0

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State table.

Now derive the excitation table and the combinational structure. The table is now arranged in a different form, where the present state and input variables are arranged in the form of a truth table.

Excitation table for JK flip-flop

Output Transitions	Flip-flop inputs
Q → Q(next)	J K
0 → 0	0 X
0 → 1	1 X
1 → 0	X 1
1 → 1	X 0

Excitation table of the circuit

Present State	Next State	Input	Flip-flop Inputs	
Q_0 Q_1	Q_0 Q_1	x	J_0K_0	J_1K_1
0 0	0 0	0	0 X	0 X
0 0	0 1	1	0 X	1 X
0 1	1 0	0	1 X	X 1
0 1	0 1	1	0 X	X 0
1 0	1 0	0	X 0	0 X
1 0	1 1	1	X 0	1 X
1 1	1 1	0	X 0	X 0
1 1	0 0	1	X 1	X 1

The simplified Boolean functions for the combinational circuit can now be derived.

Input variables are : Q_0, Q_1, and x;

Output variables are : J_0, K_0, J_1 and K_1.

The information from the truth table is plotted on the Karnaugh maps as shown below :

QoQ1 \ x	0	1
00	0	0
01	1	0
11	X	X
10	X	X

J0 map

QoQ1 \ x	0	1
00	X	X
01	X	X
11	0	1
10	0	0

K0 map

QoQ1 \ x	0	1
00	0	1
01	0	X
11	0	X
10	0	1

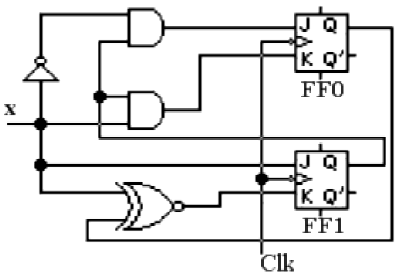
J1 map

QoQ1 \ x	0	1
00	X	X
01	1	0
11	0	1
10	X	X

K1 map

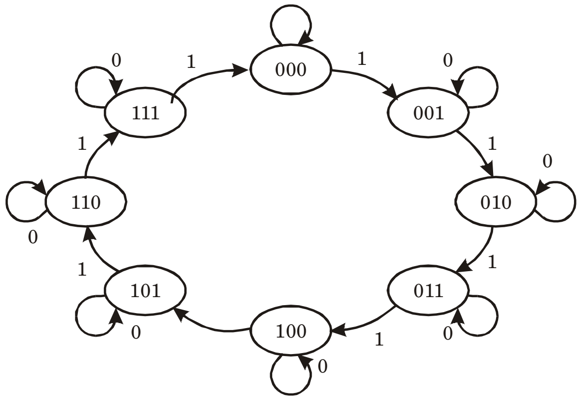
Note : The symbol ⊙ is exclusive-NOR.

Logic diagram of the sequential circuit is shown below :



COUNTER DESIGN USING FSM

3-bit counter design



Present State			Next State					
$Q_2(t)$	$Q_1(t)$	$Q_0(t)$	$Q_2(t+1)$	$Q_1(t+1)$	$Q_0(t+1)$	T_2	T_1	T_0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	0	1	1
1	0	1	0	1

$D_1 = Q_2Q'_0 + Q_2Q'_1 + Q'_2Q_1Q_0$

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	1	1	0
1	1	0	0	1

$D_1 = Q'_1Q_0 + Q_0Q'_0$

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	1	1	1
1	0	0	0	0

$D_0 = Q'_0$

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	0	0	0
1	0	1	1	0

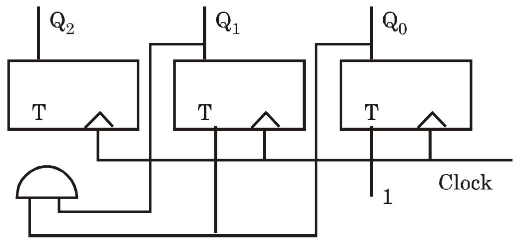
$T_2 = Q_1Q_0$

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	0	0	0
1	1	1	1	1

$T_1 = Q_0$

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	1	1	1
1	1	1	1	1

$T_0 = 1$



Example. Design an 3-bit non-ripple up/down counter using FSM. The incoming pulses are sent to the clock input of all FF's so that they are avtivated whenever a new pulse (a logic 1) comes. The FSM has $2^3 = 8$ states (000 through 111) and one input I.

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When I = 0, FSM counts down otherwise it counts up.

Present State			Input	Next State			Signal for T-FF		
$Q_2(t)$	$Q_1(t)$	$Q_1(t)$	I	$Q_2(t+1)$	$Q_1(t+1)$	$Q_0(t+1)$	T_2	T_1	T_0
0	0	0	0	1	1	1	1	1	1
0	0	0	1	0	0	1	0	0	1
0	0	1	0	0	0	0	0	0	1
0	0	1	1	0	1	0	0	1	1
0	1	0	0	0	0	1	0	1	1
0	1	0	1	0	1	1	0	0	1
0	1	1	0	0	1	0	0	0	1
0	1	1	1	1	0	0	1	1	1
1	0	0	0	0	1	1	1	1	1
1	0	0	1	1	0	1	0	0	1
1	0	1	0	1	0	0	0	0	1
1	0	1	1	1	1	0	0	1	1
1	1	0	0	1	0	1	0	1	1
1	1	0	1	1	1	1	0	0	1
1	1	1	0	1	1	0	0	0	1
1	1	1	1	0	0	0	1	1	1

Directly from this table, we get

$$T_0 = 1,\ T_1 = IQ_0 + I'Q'_0,\ T_2 = IQ_1Q_2 + I'Q'1Q'_0$$

which indicate that input to a T-FF is

AND of all Q outputs lower bits when counting up, or

AND of all Q′ output of the lower bits when counting down.

If other types of flipflops are used, the logic is not as simple.

CLOCKED SEQUENTIAL CIRCUIT DESIGN

A sequential circuit uses clock signal input for the memory elements which are flip-flops. All flip-flops are clocked simultaneously. The output and next state of a clocked sequential circuit depends upon external inputs and present state of the circuit.

There are two commonly used models of such circuits :

(*i*) *Mealy model*

(*ii*) *Moore model.*

Next state and output for these models are expressed as follows :

For Mealy model,

$$\text{Next state} = F_1 \text{(present state, inputs)}$$
$$\text{Outputs} = F_2 \text{(present state, inputs)}$$

For Moore model,

$$\text{Next state} = F_1 \text{(present state, inputs)}$$
$$\text{Outputs} = F_2 \text{(Present state)}$$

For Moore model, signal path from external inputs to output logic block is not present.

For these circuits, sequence of inputs, present and next states, and output can be represented by a *state table* or a *state diagram.*

Consider a clocked sequential circuit represented by the state table given in the table. Figure given below represents its state diagram. The state table gives information about present state of the circuit, next state of circuit for all the combinations of the external inputs (X is an external input) and outputs for all the combinations of the inputs. The column for output is necessary only if outputs are taken from logic gates. In case outputs are taken directly from the flip-flops, then output column is not required and this information is available in the present state column.

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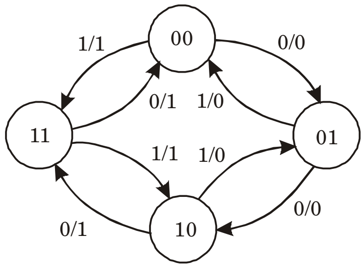

Fig. State diagram for state table

The state diagram represents information of state table in graphical form. In this diagram, a state is represented by a circle and directed lines connecting states indicate the transition between states when inputs are applied and the circuit is clocked. The input and output conditions are labelled with two binary numbers separated by the symbol'/' along with the directed line. The first number indicates input value that causes state transition, and the second number (after the symbol /) gives value of the output.

Table : State table of a sequential circuit

Present state		Next state				Output	
A	B	X = 0		X = 1		X = 0	X = 1
		A	B	A	B		
0	0	0	1	1	1	0	1
0	1	1	0	0	0	0	0
1	0	1	1	0	1	1	0
1	1	0	0	1	0	1	1

In case an input condition does not cause a change of state, the fact is indicated by a directed line terminating on the same circle from which it originated.

MOD-3 Counters

A mod-3 counter (non binary counter) requires *two* flip-flops.

The counter is to cycle through its three states in the manner indicated in the table.

Since thus, there is a counter state S = 0 corresponding to Q_1Q_0 = 00.

The next input cycle is to advance the counter to its second state S = 1, in which Q_1Q_0 = 01.

The cycle following is to advance the counter to S = 2, in which Q_1Q_0 = 10.

Finally, one additional input cycle should reset the counter back to S = 0.

A ripple counter is not possible in the present case. In a ripple counter one of the flip-flops would receive the clock input from gate output of the other flip-flop. When S goes from S = 2 to S = 0, Q_1 changes state and hence requires a clocking transition. However, Q_0 does not change state and hence cannot provide such a clocking transition. The situation is not improved if we interchange states of Q_1 and Q_0. Now for designing assume synchronous operation with external clocking signal applied to both flip-flops. Also assume that we have JK flip-flops, and we must still determine how to arrange for the J and K inputs always to be at such logic levels that the two flip-flops make the proper transition at the proper time.

With this end in view, it is convenient to present the truth table of a JK flip-flop in the form of Table, which specifies required logic at J and K if a flip-flop is to go from a state Q_n (before clocking transition) to a state Q_{n+1} (after clocking transition). The entry X means "don't care".

Referring to tables, we can now fill in the K maps for J_0, K_0, J_1, and K_1 for the two flip-flops FF0 and FF1 which form our mod-3 counter. The result is given in the figure below. To obtain this figure, when count is 0, outputs of both flip-flops are Q_1Q_0 = 00, next state of the counter is Q_1Q_0 = 01. Q_1 must remain at Q_1 = 0, so that we require that J_1 = 0 and K_1 = X while Q_0 must go to 1, so that we require J_0 = 1 and K_0 = X. The case Q_1Q_0 = 11 never arises, so that corresponding K-map boxes are all marked X. The K- maps of above figure when put in simplest form yield.

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$$J_0 = \overline{Q}_1 \quad K_0 = 1 \quad \text{and} \quad J_1 = Q_0 \quad K_1 = 1$$

Counter state S	Q_1	Q_0
0	0	0
1	0	1
2	1	0
0	0	0
•	•	•
•	•	•

$Q_n \longrightarrow$	Q_{n+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

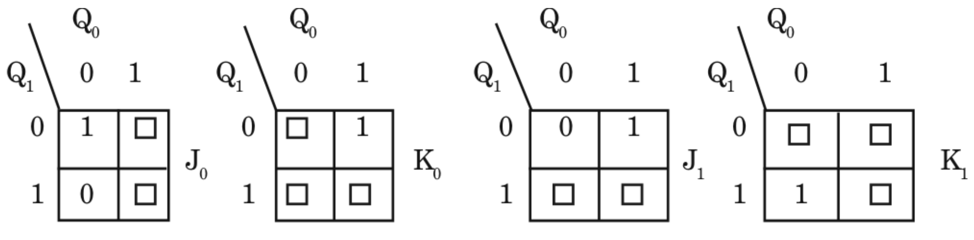

Fig. K maps for a mod-3 counter

The resulting counter circuit shown in Fig. *(a)* below. Specification of the logic values for Q_1 and Q_0 corresponding to each counter state, as in table is called *state assignment*. The waveform chart for the counter, which can be deduced from the circuit or simply read from the state assignment table, is shown in Fig. *(b)* below.

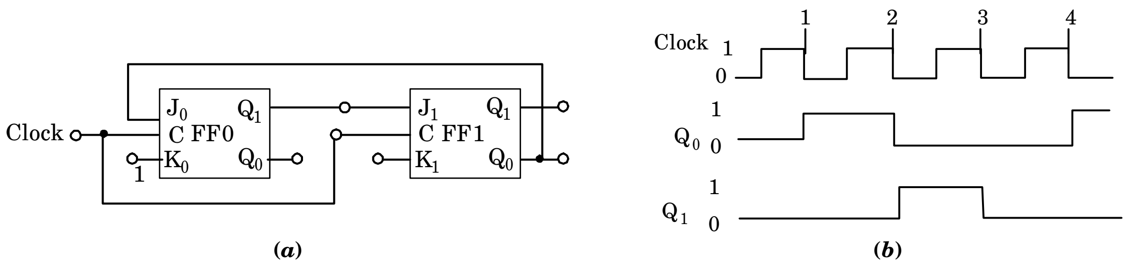

Fig. mod-3 counter.

MOD-5 Counters

Consider a mod-5 counter to operate in accordance with the state assignment of table. Three flip-flops are now required.

Counter state S	Q_2	Q_1	Q_0
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
0	0	0	0

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Consider that the count is S = 2, so that $Q_2Q_1Q_0 = 010$.

At the next clock pulse Q_0 will become 1 while Q_2 and Q_1 will not change state.

Thus, we set $J_0 = 1$, $K_0 = X$, $J_1 = X$, $K_1 = 0$, $J_2 = 0$, and $K_2 = X$.

The other portions of the K maps are completed in similar fashion.

SHIFT REGISTERS

These are a type of sequential logic circuit, mainly for storage of digital data. They are a group of flip-flops connected in a chain, so that output from one flip-flop becomes input of the next flip-flop. Most of the registers possess no characteristic internal sequence of states. All the flip-flops are driven by a common clock, and all are *set or reset* simultaneously.

It can be used in four different configurations depending upon the way in which data is entered into and taken out of it. A special form of counter - the shift register counter, is also introduced.

CONFIGURATIONS.

(1) Serial input, serial output (SISO).

The data can be moved in and out of the register, one bit at a time.

(2) Parallel input serial output (parallel to serial converter, PISO).

The data can be loaded simultaneously. But data can be removed from the register one bit at a time by clock pulse.

(3) Serial-input, parallel output (serial-to-parallel converter, SIPO).

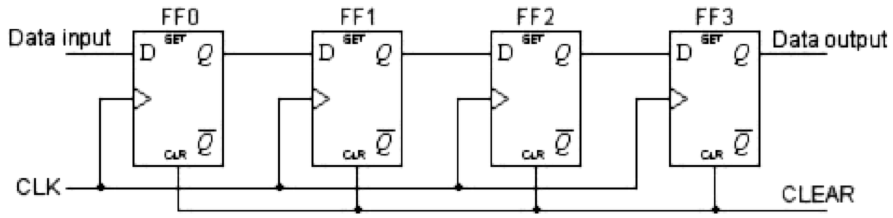
Data is loaded serially one bit at a time but the data stored can be read simultaneously.

(4) Parallel input, parallel output (PIPO).

The data can be loaded into the stages simultaneously and can also be taken out or read simultaneously.

1. Serial In - Serial Out Shift Registers

A basic four-bit shift register can be constructed using four D flip-flops, as shown below.



Operation of the circuit :

The register is first cleared, forcing all four outputs to zero. The input data is then applied sequentially to the D input of the first flip-flop on the left (FF0). During each clock pulse, one bit is transmitted from left to right. Assume a data word to be 1001. The least significant bit of the data has to be shifted through the register from FF0 to FF3.

	FF0	FF1	FF2	FF3
CLEAR	0	0	0	0

In order to get the data out of the register, they must be shifted out serially. This can be done destructively or non-destructively. For destructive readout, the original data is lost and at the end of the read cycle, all flip-flops are reset to zero.

	FF0	FF1	FF2	FF3	
0000	1	0	0	1	0000

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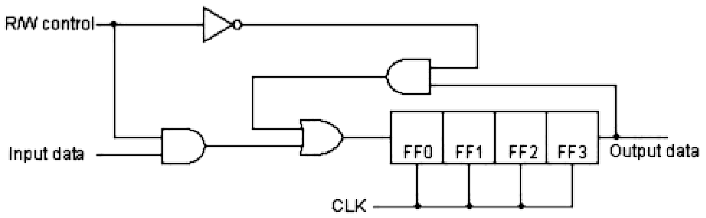
To avoid loss of data, an arrangement for a non-destructive reading can be done by adding two AND gates, an OR gate and an inverter to the system. The construction of this circuit is shown below.



The data is loaded to the register when control line is HIGH (ie WRITE). The data can be shifted out of the register when the control line is LOW (ie READ). This is shown below.

WRITE	FF0	FF1	FF2	FF3
1001	0	0	0	0

2. Serial In - Parallel Out Shift Registers

For this type of register, data bits are entered serially same as serial in parallel out shift register but the difference is the way in which the data bits are taken out of the register. Once the data are stored, each bit appears on its respective output line, and all bits are available simultaneously. A construction of a four-bit serial in - parallel out register is shown below.

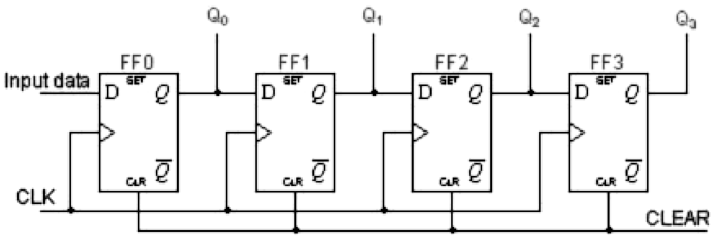
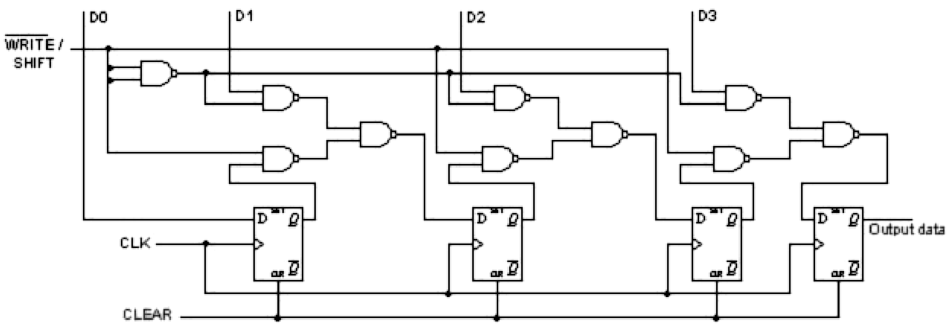


Figure given below shows, how the four-bit binary number 1001 is shifted to the Q outputs of the register.

CLEAR	Q0	Q1	Q2	Q3
1001	0	0	0	0

3. Parallel In - Serial Out Shift Registers

A four-bit parallel in - serial out shift register is shown below. The circuit uses D flip-flops and NAND gates for entering data (ie writing) to the register.



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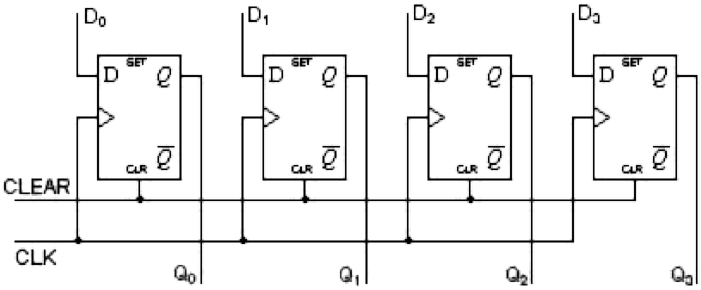
D0, D1, D2 and D3 are the parallel inputs, where D0 is the most significant bit and D3 is the least significant bit. To write data in, the mode control line is taken to LOW and the data is clocked in. The data can be shifted when the mode control line is HIGH as SHIFT is active high. The register performs right shift operation on the application of a clock pulse, as shown in the animation below.

	Q0	Q1	Q2	Q3
CLEAR	0	0	0	0

4. Parallel In - Parallel Out Shift Registers

For parallel in - parallel out shift registers, all data bits appear on the parallel outputs immediately following the simultaneous entry of the data bits. The following circuit is a four-bit parallel in - parallel out shift register constructed by D flip-flops.

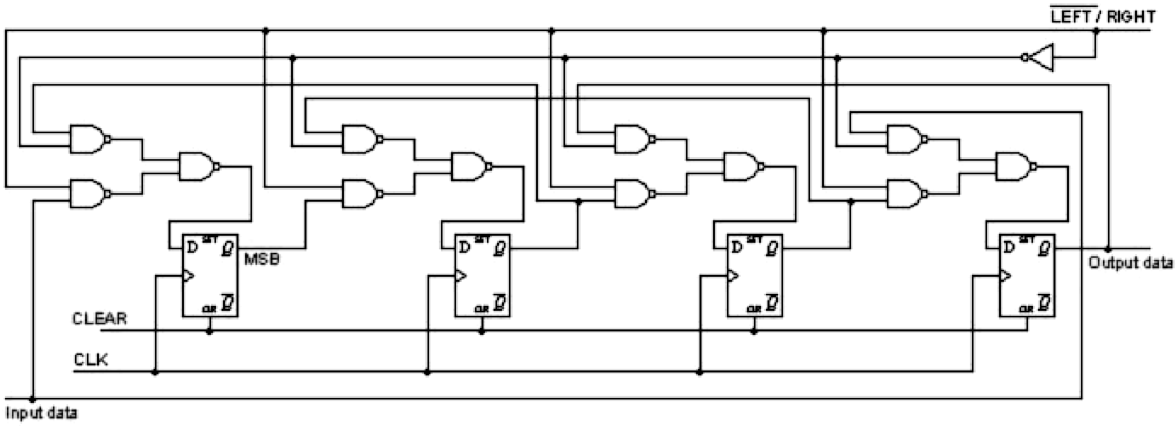


The D's are the parallel inputs and the Q's are the parallel outputs. Once the register is clocked, all the data at the D inputs appear at the corresponding Q outputs simultaneously.

5. Bidirectional Shift Registers

The registers discussed so far involved only right shift operations. Each right shift operation has the effect of successively dividing the binary number by two. If the operation is reversed (left shift), this has the effect of multiplying the number by two. With suitable gating arrangement a serial shift register can perform both operations.

A bidirectional, or reversible, shift register is one in which the data can be shift either left or right. A four-bit bidirectional shift register using D flip-flops is shown below.



Here a set of NAND gates are configured as OR gates to select data inputs from the right or left adjacent bistables, as selected by the LEFT/RIGHT control line.

The animation below performs right shift four times, then left shift four times. Note that order of the four output bits are not the same as order of the original four input bits but they are actually reversed.

RIGHT	FF0	FF1	FF2	FF3
11111001	0	0	0	0

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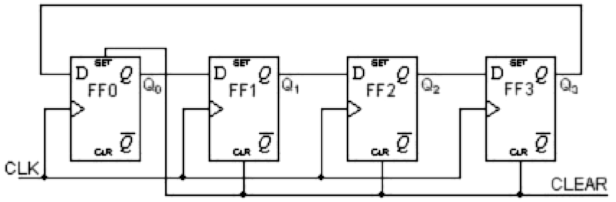
SHIFT REGISTER COUNTERS.

These are basically shift registers with the serial outputs connected back to the serial inputs in order to produce particular sequences. These registers are classified as counters because they exhibit a specified sequence of states.

Common types of shift register counters

1. **Ring Counter.**

   It is basically a circulating shift register in which the output of the most significant stage is fed back to the input of the least significant stage. The following is a 4-bit ring counter constructed from D flip-flops. The output of each stage is shifted into the next stage on the positive edge of a clock pulse. If the CLEAR signal is high, all the flip-flops except the first one FF0 are reset to 0. FF0 is preset to 1 instead.

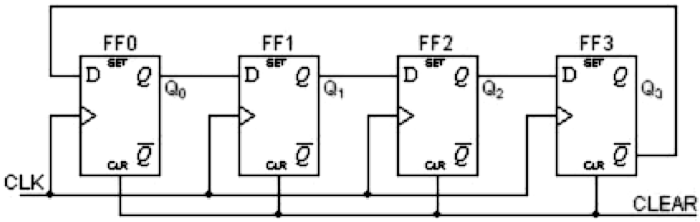


Clock Pulse	Q3	Q2	Q1	Q0
0	0	0	0	1
1	0	0	1	0
2	0	1	0	0
3	1	0	0	0

CLEAR	FF0	FF1	FF2	FF3
	0	0	0	0

Since the count sequence has 4 distinct states, the counter can be considered as a mod-4 counter. Only 4 of the maximum 16 states are used, making ring counters very inefficient in terms of state usage. But the major advantage of a ring counter over a binary counter is that it is self-decoding. No extra decoding circuit is needed to determine what state the counter is in.

2. **Johnson Counter** .



Clock Pulse	Q3	Q2	Q1	Q0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	1
3	0	1	1	1
4	1	1	1	1
5	1	1	1	0
6	1	1	0	0
7	1	0	0	0

Johnson counters are a variation of standard ring counters, with the inverted output of the last stage fed back to the input of the first stage. They are also known as twisted ring counters. An n-stage Johnson counter yields a count sequence of length 2n, so it may be considered to be a mod-2n counter. The circuit above shows a 4-bit Johnson counter. The state sequence for the counter is given in the table as well as the animation on the left.

CLEAR	FF0	FF1	FF2	FF3
	0	0	0	0

Again, the apparent disadvantage of this counter is that the maximum available states are not fully utilized. Only eight of the sixteen states are being used.

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Both Ring and the Johnson counter must initially be forced into a valid state in the count sequence because they operate on a subset of the available number of states, otherwise, the ideal sequence will not be followed.

APPLICATIONS OF SHIFT REGISTERS.

1. **To produce Time delay**

   The serial in -serial out shift register can be used as a time delay device. The amount of delay can be controlled by:

   (*i*) number of stages in the register, and

   (*ii*) clock frequency

2. **To simplify Combinational logic**

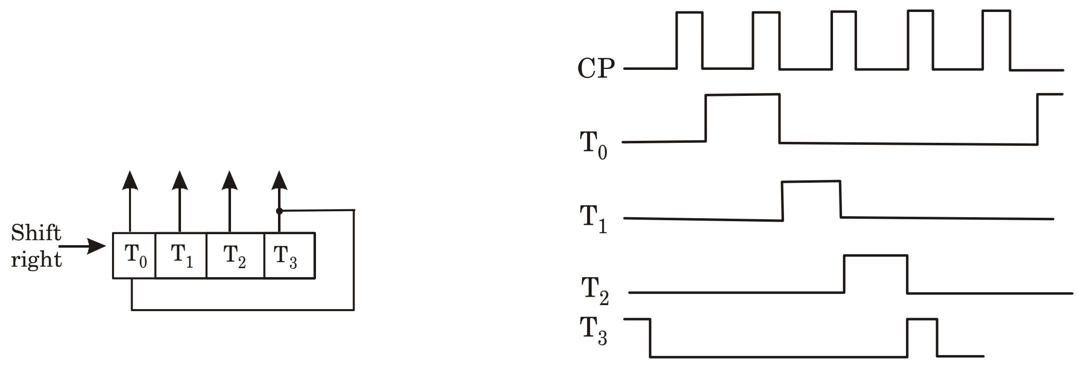
   The ring counter technique can be effectively utilized to implement synchronous sequential circuits. A major problem in the realization of sequential circuits is the assignment of binary codes to the internal states of the circuit in order to reduce the complexity of circuits required. By assigning one flip-flop to one internal state, it is possible to simplify the combinational logic required to realize the complete sequential circuit. When the circuit is in a particular state, the flip-flop corresponding to that state is set to HIGH and all other flip-flops remain LOW.

3. **To convert Serial data to Parallel data**

   A computer or microprocessor-based system commonly requires incoming data to be in parallel format. But frequently, these systems must communicate with external devices that send or receive serial data. So, serial-to-parallel conversion is required. As shown in the previous sections, a serial in -parallel out register can achieve this.

RING COUNTER

It is a circular shift register with only one flip-flop being set at any particular time; all others are cleared. The single bit is shifted from one flip-flop to the other to produce sequence of timing signals. Fig. *(a)* shows a 4-bit shift register connected as a ring counter. Initial value of the register is 1000, which produces the variable T_0. The single bit is shifted right with every clock pulse and cirulates back from T_3 to T_0. Each flip-flop is in the 1 state once evey four clock pulses and produces one of the four timing signals shown in Fig. *(b)*. Each output becomes a 1 after the negative-edge transition of a clock pulse and remains 1 during the next clock pulse.



(a) ***(b)***

Fig. Generation of timing signals

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SOLVED EXAMPLES

1. Draw the circuit diagram of mod-6 counter using J-K flip-flops.

Solution:

(*i*) Number of flip-flops = $2^{N-1} < 6 < 2^N$

where N is number of flip-flops.

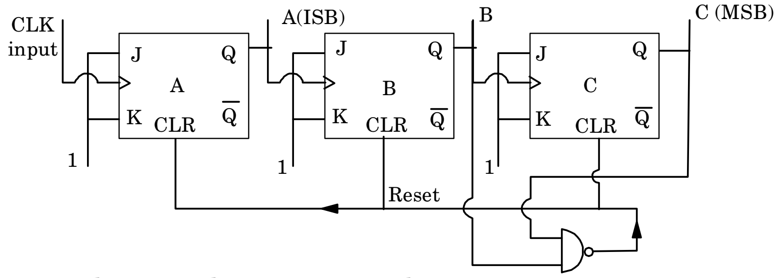
We should choose higher power of N, so N = 3.

(*ii*) N – 1 = 5 or 101 (binary), 5 is the maximum decimal number.

(*iii*) Modulo 6 = 110

(*iv*) Connect all flip-flops as ripple counter.

(*v*) Assuming flip-flop to have only a clear line, connect all flip-flop outputs for which Q = 1 at the binary count *n* as inputs to a NAND gate. Connect NAND gate output to the RESET input of the counter.

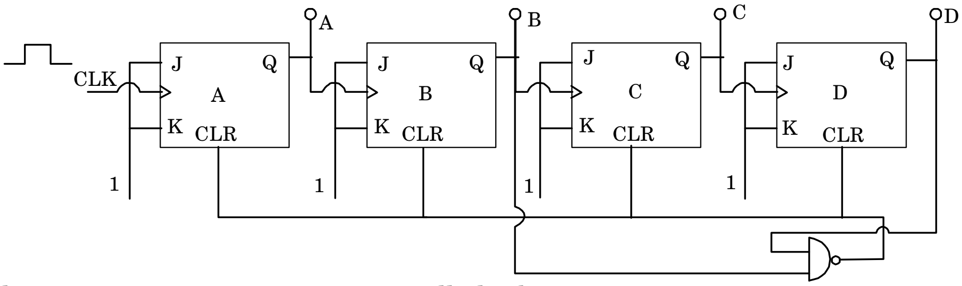


2. Usingmod-6 counter realise a mod-12 counter with a square wave output.

Solution:

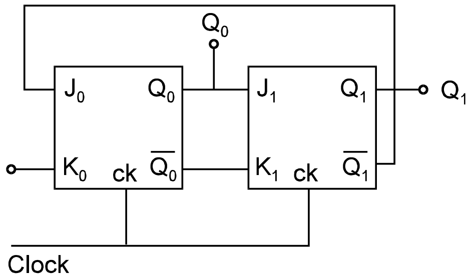
Modulo - 12 counter (Binary 1100)

Binary of decimal number 11 is 1011. Only Q_D, Q_C has to toggle to 0.



As clock input is a square wave, output will also be square.

3. If figure assume initially $Q_0 = Q_1 = 0$, find logic states of Q_0 and Q_1 immediately after 777th clock pulse.



Solution:

$$Q_0Q_1 = 00 \rightarrow 11 \rightarrow 00 \rightarrow 11$$

$$\text{Number of states occupied} = \frac{777}{4} = 194 \quad \text{Quotient 1.}$$

$\therefore$ Logic after 777 clock pulse is $Q_0Q_1 = 00$.

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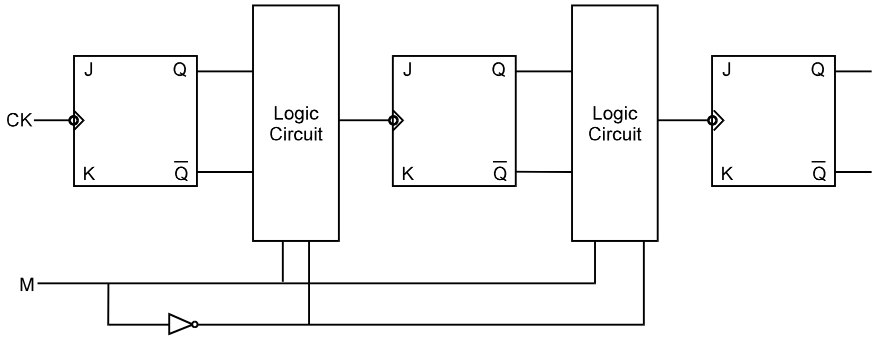
4. The circuit shown below uses TTL FFs, the FFs are triggered at negative transition of the clock. It is desired that when M = 1, the circuit should function as an up-counter (in 8421 BCD) and when M = 0, as a down counter. Design combinational circuit interposed between flip-flops, so that the circuit works as desired.



Solution:

This ins an 3-bit up/down synchrouous counter.

Clock pulse	Q_C	Q_B	Q_A	*Down*
0	0	0	0	
1	0	0	1	
2	0	1	0	
3	0	1	1	
4	1	0	0	
5	1	0	1	
6	1	1	0	
7	1	1	1	

UP

(1) FF Q_A toggles on each clock pulse in both UP and DOWN sequence.

Hence $J_A = K_A = 1.$

(2) For UP sequence, Q_B changes state if $Q_A = 1$. For the DOWN sequence, Q_B changes state if $Q_A = 0$.

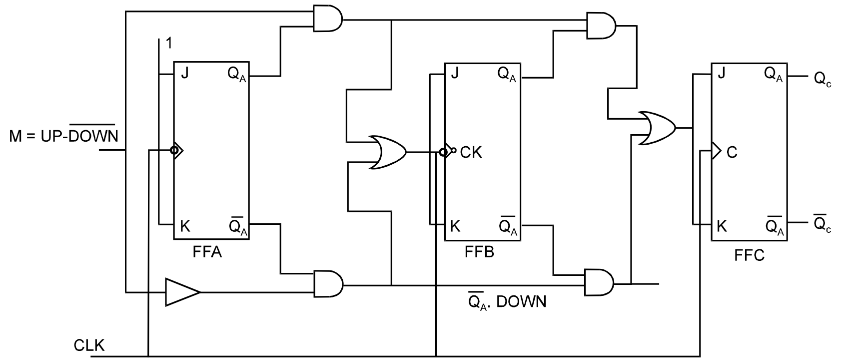
Thus $J_B = K_B = Q_A \cdot \text{UP} + \overline{Q}_A \cdot \text{DOWN}$

(3) For UP sequence, Q_C changes state when both $Q_A = Q_B = 1$.

For DOWN sequence, Q_C changes state when both $Q_A = Q_B = 0$.

Hence, $J_C = K_C = Q_B \cdot Q_A \cdot \text{UP} + \overline{Q}_B \cdot \overline{Q}_A \cdot \text{DOWN}$

Now, implementation done is shown below.



(a) Three bit UP/DOWN synchronous counter

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Q (t)	Q (t + 1)	
0	0	0
0	1	1
1	0	1
1	1	0

(b) Characteristic table

Q \ T	0	1
0		1
1	1	

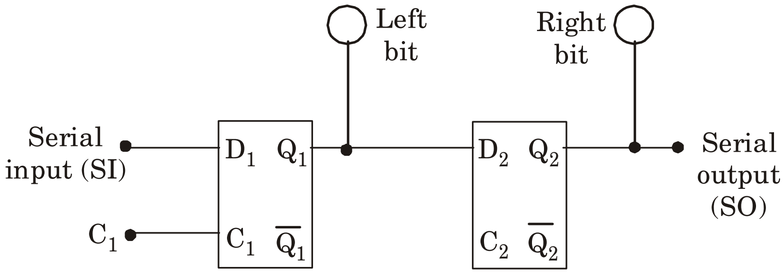
$$Q(t+1) = T\overline{Q} + \overline{T}Q$$

(c) Characteristic equation

5. Using D flip-flops, design a 2-bit shift register that moves incoming data from left to right.

Solution:

Figure below shows 2-bit shift register. Each data bit is first entered as output of the left-hand M/S D flip-flops. Upon low transition of the clock, data bit is stably available at output Q_1 which is also connected as input of the right-hand M/S D flip-flop. Over next clock pulse, the new arriving data bit at D_1 and previous data bit at D_2 are set in the respective master flip-flops. As the clock pulse makes its transition to low logic, data bits are set in the respective slave flip-flops; the arriving data bit is set as output Q_1 while previous data bit is set as the output Q_2. The shifting process continues as long as pulses are transmitted to the flip-flops.



6. Convert modulo-4 ripple counter to a synchronous counter (a counter for which all bits are set at the same point in time by action of the system clock).

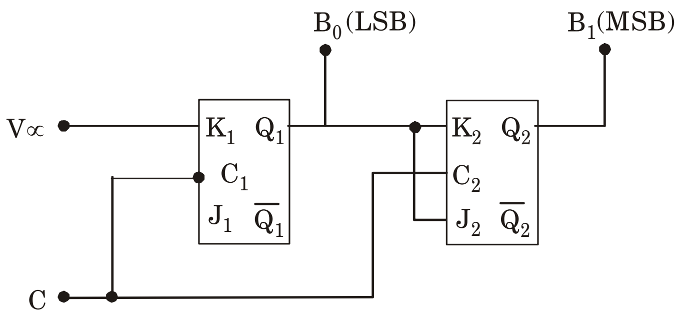
Solution:

For this 2-bit counter, modification is relatively simple in that the output Q_1 becomes input to J_2 K_2.

Output $B_1 = Q_2$ is complemented each time that $Q_1 = B_0$ is 1 and is unchanged each time that $Q_1 = B_0$ is 0.

Outputs for the first four clock cycles are tabulated as follows :

B_1	B_0	C
0	0	0
0	1	↓
1	0	↓
1	1	↧
0	0	↓



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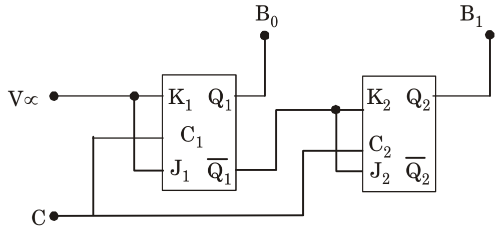
7. Design a modulo-4 up-down counter.

Solution:

In figure shown below, complementary output of the left-hand flip-flop is connected, so that

$$J_2 = K_2 = \overline{Q_1}.$$

B_1	B_0	C
1	1	0
1	0	↓
0	1	↓
0	0	↓
1	1	↓



Assume counter has been preset, so that $Q_1 = B_0 = Q_2 = B_1 = 1$.

Since $J_1 = K_1 = V_{CC}$, Q_1 will toggle each cycle of the clock.

For first clock cycle, $\overline{Q_1} = K_2 = J_2 = 0$; thus Q_2 is unchanged.

For second clock cycle $\overline{Q_1} = J_2 = K_2 = 1$; thus Q_2 is toggled.

For third clock cycle, $\overline{Q_1} = J_2 = K_2 = 0$ and Q_2 is unchanged. Q_2 is complemented over the fourth clock cycle since $\overline{Q_1} = 1$. The down counter outputs are tabulated for the first four clock cycles.

8. Design a 2-bit up-down counter, a counter capable of incrementing from 0 to M-1 and reset, or decrementing from M-1 to 0 and reset based upon a direction command.

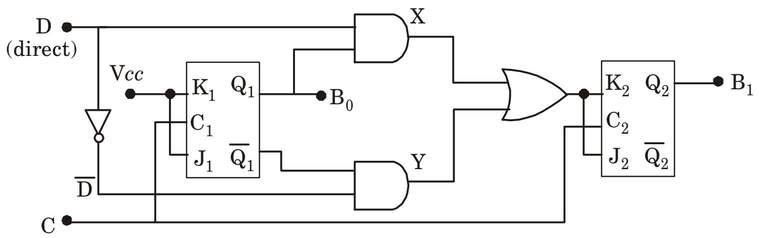
Solution:

We require a circuit to pass Q_1 to successive flip-flop for up count and to pass $\overline{Q_1}$ to the successive flip-flop for down count.

For circuit of figure below, P. 7. D = 1 for up count and D = 0 for down count.

If D = 1 ($\overline{D} = 0$), then $X = D.\ Q_1 = Q_1$ and $Y = \overline{D}.\ \overline{Q_1} = 0$;

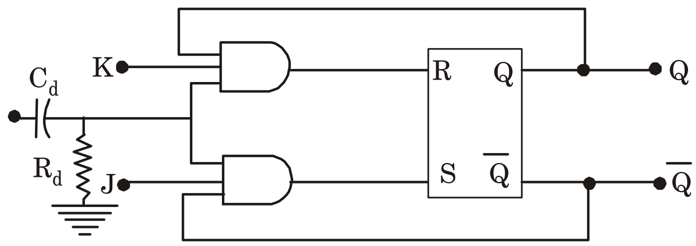
For D = 0 (D = 1), $X = D.\ Q_1 = 0$ and $Y = D.\ \overline{D} = \overline{Q_1}$, or circuit functions as the down counter.



9. Use an RS flip-flop, AND gates, and an RC differentiator network to design a JK flip-flop that is triggered on the leading edge of the clock pulse.

Solution:

A JK flip-flop can be built up from an RS flip-flop by ANDing Q and $\overline{Q}$ output with K and J inputs, respectively. The clock pulse leading edge can be differentiated by a RC network to form positive spike to set Q and $\overline{Q}$ according to the switching pattern of J and K at the time of arrival of the clock pulse leading edge. The resulting logic circuit is shown in the figure.



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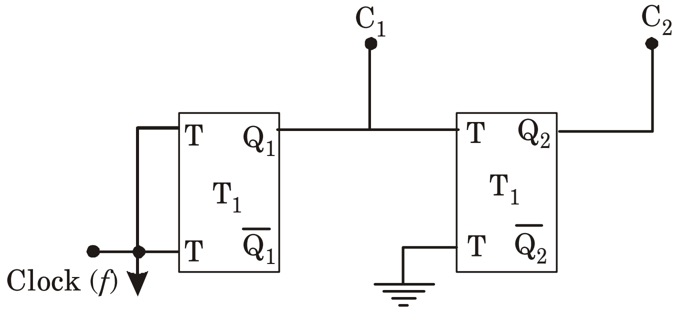
10. Use two T flip-flops to design a circuit that receives the system clock at frequency f and generates two new lower frequency clock signals (C_1 and C_2) with characteristics $f_1 = f/2$ and $f_2 = f/4$.



Solution:

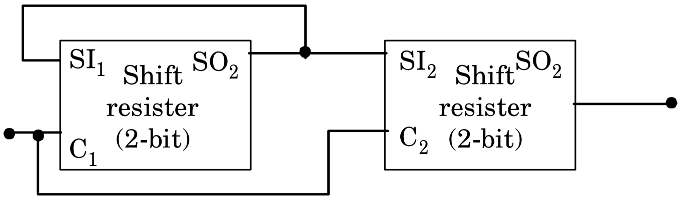
Use two cascade-connected T flip-flops as shown in above figure.

Input of the left-hand flip-flop is connected to clock input C.

Output $Q_1 = C_1$ will toggle every other clock pulse; thus its frequency = $\frac{f}{2}$.

Output $Q_2 = C_2$ toggles every other cycle of C_1; thus its freuqency = $\frac{(f/2)}{2}$ = 4.

11. Design a 2-bit shift register system that transfers data from shift register 1 to shift register 2, but retains the data in shift register 1.



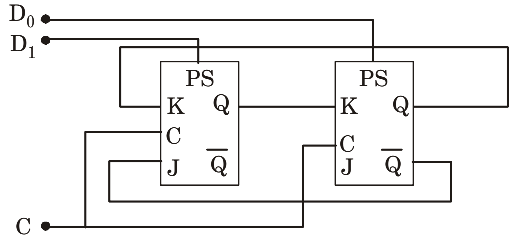
Solution:

The shift register loses information in the right-hand bit after each clock cycle. However, if right-hand bit were fed back to the input or left-hand bit as the serial input, the data is retained. This principle (called *recirculating*) is used in the proposed circuit of figure to retain data of shift register 1 while simultaneously transferring the data to shift register 2. For this set of 2-bit registers, two clock cycles are necessary to implement complete data transfer. The system clock is ANDed with a shift control (not illustrated) that is logic high for the duration of two clock cycles to effect the data transfer.

12. Using JK flip-flop, design a 2-bit shift register that is initially parallel loaded with $D = D_1D_0$.

Solution:

M/S JK flip-flops with preset (PS) feature are selected. The data can then be loaded through the PS terminals. The resulting shift register is shown in the figure.



13. Design clocked sequential circuit whose state diagram is given below. The type of flip-flop to be used is D.

Solution:

Total number of state is 4, therefore 2 flip-flops are used (A, B)

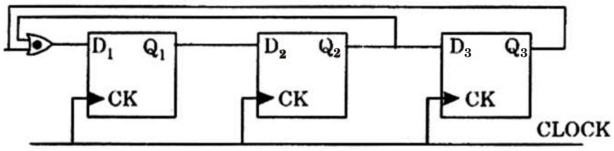
Q_t	Q_{t+1}	D
0	0	0
0	1	1
1	0	0
1	1	1

Fig. Excitation table for D flipflop

Present state A	Present state B	Input X	Next State A	Next State B	D_A	D_B	Y
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0
0	1	0	0	0	0	0	0
0	1	1	1	0	1	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	1	1	0
1	1	0	0	0	0	0	0
1	1	1	1	1	1s	1	1

Fig. State Table.

14. Consider synchronous sequential circuit in figure.

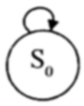


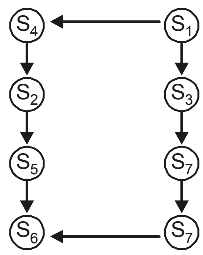
(*a*) Draw a state diagram which is implemented by the circuit. Use following names for the states corresponding to the values of flip-flops as given below.

Q_1	Q_2	Q_3	State
0	0	0	S_0
0	0	1	S_1
–	–	–	–
–	–	–	–
–	–	–	–
1	1	1	S_7

(*b*) Given that initial of the circuit is S_4, identify the set of states which are not reachable.

Solution:

(*a*) 



(*b*) $\{S_0\}$

NUMBER SYSTEMS

INTRODUCTION

The rightmost digit is multiplied by 10^0, the next digit to the left is multiplied by 10^1, and so on. Each digit to the left has a multiplier that is 10 times the previous digit.

- To multiply a number by 10 simply shift it to the left by one digit, and fill in the rightmost digit with a 0 (moving the decimal place one to the right). To divide a number by 10, simply shift the number to the right by one digit (moving the decimal place one to the left).
- To see how many digits a number needs, simply take the logarithm (base 10) of the absolute value of the number, and add 1 to it. The integer part of the result is the number of digits.

  e.g. $\log_{10}(33) + 1 = 2.5.$

  The integer part of that is 2, so 2 digits are needed.
- With n digits, 10^n unique numbers (from 0 to 10^n-1) can be represented.

  If n = 3, 1000 (=10^3) numbers can be represented 0-999.
- Negative numbers are handled easily by simply putting a minus sign (-) in front of the number. However, this does lead to the somewhat awkward situation where 0 = -0. Avoid this situation with binary representations, but with a little bit of effort.

  Representing fractions is a simple extension of this idea.

  $$25.43_{10} = 2*10 + 5*1 + 4*0.1 + 3*0.01 = 2*10^1 + 5*10^0 + 4*10^{-1} + 3*10^{-2}$$
- If there are m digits to the right of the decimal point, smallest number that can be represented is 10^{-m}.

  e.g. if m = 4, the smallest number that can be represented is $0.0001=10^{-4}$.

Binary Representation of Positive integers

Binary representations of positive can be understood in the same way as their decimal counterparts.

e.g. $86_{10} = 1*64 + 0*32 + 1*16 + 0*8 + 1*4 + 1*2 + 0*1$

or $86_{10} = 1*2^6 + 0*2^5 + 1*2^4 + 0*2^3 + 1*2^2 + 1*2^1 + 0*2^0$

or $86_{10} = 1010110_2$

The subscript 2 denotes a binary number. Each digit in a binary number is called a *bit*. The number 1010110 is represented by 7 bits. Any number can be broken down this way, by finding all of the powers of 2 that add up to the number in question (in this case 2^6, 2^4, 2^2 and 2^1). This is exactly analagous to the decimal deconstruction of the number 125 that was done earlier. Likewise similar set of observations can be made.

- To multiply a number by 2, simply shift it to the left by one digit, and fill in the rightmost digit with a 0. To divide a number by 2, simply shift the number to the right by one digit.
- To see how many digits a number needs, simply take the logarithm (base 2) of the number, and add 1 to it. The integer part of the result is the number of digits.

  e.g. $\log_2(86) + 1 = 7.426.$

  The integer part of that is 7, so 7 digits are needed.
- With n digits, 2^n unique numbers (from 0 to 2^n-1) can be represented.

  If n = 8, 256 (=2^8) numbers can be represented 0-255.

DECIMAL NUMBER SYSTEM.

There are ten digits 0 to 9. The base of the decimal number system is 10.

Value of the nth digit of the number from the right side = nth digit $\times 10^{n-1}$

$= n$th digit $\times$ (base)$^{n-1}$.

e.g.
$$4598 = 4000 + 500 + 90 + 8$$
$$= 4 \times 10^3 + 5 \times 10^2 + 9 \times 10^1 + 8 \times 10^0$$

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BINARY NUMBER SYSTEM.

It is cumbersome for humans to deal with writing, reading and remembering individual bits, because it takes many of them to represent even fairly small numbers.

A number of different ways have been developed for handling of binary data easier for us. The most common is hexadecimal.

In the binary number system there are only two digits 0 and 1. The binary digits are called *bits*. The base of the binary number system is 2. Two is written as 10 and three as 11. Again for four we have to write 100. In this way we proceed further.

Value of the nth bit of the number from right side = nth bit × 2^{n-1}

= nth bit × $(\text{base})^{n-1}$.

e.g. 1011 (binary number) = $1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 8 + 0 + 2 + 1 = 11$ (decimal number)

Hexadecimal Number System

In hexadecimal notation, 4 bits (a nibble) are represented by a single digit. There is obviously a problem with this since 4 bits gives 16 possible combinations, and there are only 10 unique decimal digits, 0 to 9. This is solved by using the first 6 letters (A..F) of the alphabet as numbers.

$$
\begin{aligned}
\text{1A5D (hex)} &= 1 \times 16^3 + A \times 16^2 + 5 \times 16^1 + D \times 16^0 \\
&= 4096 + 10 \times 256 + 80 + 13 \times 1 \\
&= 4096 + 2560 + 80 + 13 \\
&= 6749 \text{ (decimal)}
\end{aligned}
$$

There are some significant advantages to using hexadecimal when dealing with electronic representations of numbers (if people had 16 fingers, we wouldn't be saddled with the awkward decimal system). Using hexadecimal makes it very easy to convert back and forth from binary because each hexadecimal digit corresponds to exactly 4 bits (log_2(16) = 4) and each byte is two hexadecimal digit. In contrast, a decimal digit corresponds to $\log_2(10) = 3.322$ bits and a byte is 2.408 decimal digits. Clearly hexadecimal is better suited to the task of representing binary numbers than is decimal.

e.g. number $CA3_{16} = 1100\ 1010\ 0011_2$ ($1100_2 = C_{16}$, $1010_2 = A_{16}$, $0011_2 = 3_{16}$).

It is convenient to write the binary number with spaces after every fourth bit to make it easier to read.

Converting back and forth to decimal is more difficult, but can be done in the same way as before.

$$3235_{10} = C_{16}{*}256 + A_{16}{*}16 + 3_{16}{*}1 = C_{16}{*}16^2 + A_{16}{*}16^1 + 3_{16}{*}16^0$$

$$3235_{10} = 12{*}256 + 10{*}16 + 3{*}1 = 12{*}16^2 + 10{*}16^1 + 3{*}16^0$$

Octal Number System

It is often convenient to handle groups of bits, rather than individually. The most common grouping is 8 bits, which forms a byte. A single byte can represent 256 (2^8) numbers. Memory capacity is usually referred to in bytes. Two bytes is usually called a *word*, or *short word* (though word-length depends on the application). A two-byte word is also the size that is usually used to represent integers in programming languages. A long word is usually twice as long as a word. A less common unit is the nibble which is 4 bits, or half of a byte.

The base of the octal number system is 8. The digits are 0 to 7 same as those for the decimal number system. The number 8 of the decimal number system is represented by 10, 9 by 11 and so on.

Obviously this can be represented by exactly 3 bits. Two octal digits can represent numbers up to 64, and three octal digits up to 512. A byte requires 2.667 octal digits. Octal used to be quiete common, it was the primary way of doing low level I/O on some old DEC computers. It is much less common today but is still used occasionally

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e.g. 645 (octal) $= 6 \times 8^2 + 4 \times 8^1 + 5 \times 8^0$

$= 6 \times 64 + 4 \times 8^1 + 5 \times 1$

$= 384 + 32 + 5$

$= 421$ (decimal)

5273 (octal) $= 5 \times 8^3 + 2 \times 8^2 + 7 \times 8^1 + 3 \times 8^0$

$= 5 \times 512 + 2 \times 64 + 7 \times 8 + 3 \times 1$

$= 2747$ (decimal)

NUMBERS WITH DIFFERENT BASES.

DECIMAL (base 10)	BINARY (base 2)	OCTAL (base 8)	HEXADECIMAL (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

CONVERSION OF NUMBER SYSTEMS

1. Conversion of Decimal number to Binary number

In a binary number, weights of bits from the right side are 1, 2, 4, 8, 16 and so on. This concept can be utilised for the conversion of a decimal number to a binary number. A procedure is to be developed to examine which multiples of 2 are present in the decimal number.

e.g. 12 (decimal number) has one 8 and one 4. It has no 2 and 1. In other words it has zero 2 and zero 1. Its binary equivalent is 1100. Based on this concept for decimal to binary conversion, the decimal number is divided by 2 successively. At each stage the quotient and remainder are noted down. The quotient of one stage is divided by 2 at the next stage. The procedure is repeated till the quotient becomes zero. If remainder of the 1st stage is R_1, remainder of the 2nd stage R_2, and so on, the binary number is given by

$$\text{Binary number} = R_i\, R_{i-1}\, R_{i-2} \ldots\ldots\, R_3\, R_2\, R_1$$

where, R_f = remainder of ith stage.

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e.g. Binary equivalent of the decimal number 39.

	Quotient	Remainder	Remark
Divide 39 by 2.	19	1 (LSB)	There is one 1 and nineteen 2s.
Divide 19 by 2. *(This is equivalent to division by 4)*	9	1	There is one 2 and nine 4s.
Divide 9 by 2. *(It is equivelent to division by 8)*	4	1	There is one 4 and four 8s.
Divide 4 by 2. *(It is equivelent to division by 16)*	2	0	There is zero 8 and two 16s.
Divide 2 by 2. *(It is equivalent to division by 32).*	1	0	There is zero 16 and one 32
Divide 1 by 2. *(It is equivalent to division by 64)*	0	1(MSB)	There is one 32 and zero 64.

The binary number is 100111.

The last remainder is MSB and the 1st remainder LSB.

2. Conversion of a Decimal number to Octal number

For conversion of a decimal number to an octal number, the technique of division by 8 is used.

e.g., Equivalent octal number of 461 (decimal).

8	461	Remainder
8	57	5 (least significant digit)
8	7	1
	0	7 (most significant digit)

Hence octal number is 715.

Equivalent octal number of the decimal number 2747.

	8 2747	Remainder
8	343	3 (LSD)
8	42	7
8	5	2
	0	5 (MSD)

Hence octal number is 5273

3. Conversion of a Decimal number to Hexadecimal number

For the conversion of a decimal number to a hexadecimal number, the technique of division by 16 can be used.

e.g. Hexadecimal number of the decimal number 10767.

16	10767	Remainder
16	672	15 = F
16	42	0
16	2	10 = A
	0	2

Hence hexadecimal number is 2A0F.

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4. Conversion of Binary number to Octal number

The base of an octal number is 8. As $8 = 2^3$, for binary to octal conversion groups of 3 bits are formed in the binary number from the right. After forming the groups, each 3-bit binary group is replaced by its octal equivalent.

e.g. Octal equivalent of the binary number 1101110.

$$\begin{aligned}(1101110)_2 &= (1)\,(101)\,(110)\\ &= (001)\,(101)\,(110)\\ &= (1)\,(5)\,(6)\\ &= (156)_8\\ &= 156 \text{ (octal)}.\end{aligned}$$

5. Conversion of an Octal number to Binary number

For the conversion of an octal number to a binary number each digit of the given octal number is converted to its 3-bit binary equivalent.

e.g. Conversion 527 (octal) to binary.

$$\begin{aligned}527 \text{ (octal)} &= \underset{5}{(101)} \quad \underset{2}{(010)} \quad \underset{7}{(111)}\\ &= 101010111 \text{ (binary)}.\end{aligned}$$

6. Conversion of a Binary number to Hexadecimal number

The base of a hexadecimal number is 16. As $16 = 2^4$, for binary to hexadecimal conversion groups of four bits are made in the binary number from the least significant bit (from the right). After forming the group, each 4-bit binary group is replaced by its hexadecimal equivalent.

e.g. Conversion of binary number 1101011010 to its equivalent hexadecimal number.

Making groups of 4 bits from the right in the given binary number, we have the groups

$$(1101011010)_2 = (11)\,(0101)\,(1010).$$

Here group of the most significant binary bits contains only 2 binary bits. This can be extended to 4 binary bits by putting zeros in MSB positions.

If MSBs are extended by zeros, then number is not affected. Thus given number is grouped.

$$\begin{aligned}(1101011010)_2 &= (0011)\,(0101)\,(1010)\\ &= (3)\,(5)\,(A)\\ &= (35A)_{16} = 35A \text{ (hex)}.\end{aligned}$$

7. Conversion of a Hexadecimal number to Binary number

For the conversion of a hexadecimal number to binary number, each digit of the given hexadecimal number is converted to its 4-bit binary equivalent.

e.g. Conversion of hexadecimal number 4F2D to its binary equivalent.

$$\begin{aligned}(4F2D)_{16} &= \underset{4}{(0100)} \quad \underset{F}{(1111)} \quad \underset{2}{(0010)} \quad \underset{D}{(1101)}\\ &= (0100111100101101)_2\end{aligned}$$

Hexadecimal numbers are used with microprocessors and microcomputers for the convenience of the programmer. But microprocessor accepts the binary equivalent of a hexadecimal number. There is a provision for such machine language conversion in the microprocessor-based system.

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8. **Conversion of Octal number to Hexadecimal number and Vice versa**

The conversion of a hexadecimal number to octal number or *vice-versa* can be done through the binary method.

e.g. Conversion of 5A (hexadecimal) to octal.

$$5A\,(\text{hex}) = \underset{5}{(0101)}\ \underset{A}{(1010)}$$
$$= 01011010\,(\text{binary})$$
$$01011010\,(\text{binary}) = (01)\,(011)\,(010) = (001)\,(011)\,(010)$$
$$= (1)\,(3)\,(2) = 132\,(\text{octal})$$

Conversion of octal number 456 to hexadecimal.

$$456\,(\text{octal}) = \underset{4}{(100)}\ \underset{5}{(101)}\ \underset{6}{(110)}$$
$$= 100101110\,(\text{binary}).$$

Now, equivalent binary number is divided into groups of 4 bits to get the equivalent hexadecimal number.

$$100101110\,(\text{binary}) = (1)\,(0010)\,(1110)$$
$$= (0001)\,(0010)\,(1110)$$
$$= (1)\,(2)\,(E)$$
$$= 12\,E\,(\text{hex}).$$

COMPARISON OF NUMBER SYSTEMS

Advantages of Hexadecimal number system over Octal number system

1. For long computer word length, binary representation of hexadecimal number is shorter.
2. For the word length divisible by 4 but not by 3, hexadecimal system is more convenient. Octal number system require the extension of zero bits ahead of the most significant bit for such a case.

Advantages of Octal number system over Hexadecimal number system

1. In the hexadecimal number system, letters are used to represent decimal numbers 10 to 15. The numbers using letters resemble with words or names. Thus, use of letters in hexadecimal numbers creates confusion. There is no such confusion in the octal number system as octal number system uses only digits 1 to 7.
2. The conversion of octal numbers to decimal numbers is simpler as an octal number contains only digits. The letters used in hexadecimal numbers have to be converted into equivalent decimal numbers during the conversion process.

CONVERSION OF REAL NUMBER

In the decimal number system, digits after the decimal point have weights as follows :

$$0.538 = 0.5 + 0.03 + 0.008$$
$$= 5 \times \frac{1}{10} + 3 \times \frac{1}{100} + 8 \times \frac{1}{1000}$$
$$= 5 \times 10^{-1} + 3 \times 10^{-2} + 8 \times 10^{-3}.$$

In the binary number system, weights of binary bits after the binary point are as follows :

$$0.1011 = 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}$$
$$= 1 \times \frac{1}{2} + 0 \times \frac{1}{4} + 1 \times \frac{1}{8} + 1 \times \frac{1}{16}$$
$$= 0.5 + 0 + 0.125 + 0.0625$$
$$= 0.6875\,(\text{decimal}).$$

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Example. Convert the binary real number 1011.011 to a decimal real number.

Solution. Binary real number consists of two parts–an integer and a fracion. Decimal equivalent are written for both integer and the fraction and they are added to give decimal real nmber.

$$1011.011 = (1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0) + (0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{3})$$
$$= (8 + 0 + 2 + 1) + \left(0 \times \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right)$$
$$= 11 + (0 + 0.25 + 0.125)$$
$$= 11.375 \text{ (decimal real number).}$$

SIGNED BINARY INTEGERS

A minus sign (-) is not used to represent negative numbers. Binary numbers are represented with only two symbols, 0 and 1. There are a few ways to represent negative binary numbers. The simplest of these methods is called 1's *complement*, where sign of a binary number is changed by simply toggling each bit (0's become 1's and vice-versa). This has some difficulties, like zero can be represented in two different ways (for an eight bit number these would be 0000 0000 and 1111 1111). Method used is called *two's complement notation* which avoids the pitfalls of one's complement, but which is a bit more complicated.

To represent an n bit signed binary number, the leftmost bit, has a special significance. The difference between a signed and an unsigned number is given in the table below for an 8 bit number.

Value of bits in signed and unsigned binary numbers

	Bit 7	Bit 6	Bit 5	Bit 4	Bit 3	Bit 2	Bit 1	Bit 0
Unsigned	$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$
Signed	$-(2^7) = -128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

Binary	Unsigned	Signed
0010 0011	35	35
1010 0011	163	–93
1111 1111	255	–1
1000 0000	128	–128

If Bit 7 is not set, the representation of signed and unsigned numbers is the same. However, if Bit 7 is set, the number is always negative. For this reason Bit 7 is sometimes called *sign bit*.

Signed numbers are added same as unsigned numbers, but the only difference is in the way they are interpreted. This is important for designers of arithmetic circuitry because it means that numbers can be added by the same circuitry regardless of whether or not they are signed.

To form a two's complement number that is negative, simply take the corresponding positive number, invert all the bits, and add 1.

e.g. Forming the number negative 35 as a two's complement integer:

$$35_{10} = 0010\ 0011_2$$
$$\text{invert} \rightarrow 1101\ 1100_2$$
$$\text{add } 1 \rightarrow 1101\ 1101_2$$

So 1101 1101 is our two's complement representation of -35. Check this by adding up contributions from the individual bits

$$1101\ 1101_2 = -128 + 64 + 0 + 16 + 8 + 4 + 0 + 1 = -35.$$

Since 0000 0011 represents the number 3, hence 1111 1101 represents the number –3.

Note that a number can be extended from 4 bits to 8 bits by simply repeating the leftmost bit 4 times.

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Consider the following examples

Decimal	4 bit	8 bit
3	0011	0000 0011
–3	1101	1111 1101
7	0111	0000 0111
–5	1011	1111 1011

Consider the last case which uses the number –5. As a 4 bit number, this is represented as

$$1011 = -8 + 2 + 1 = -5$$

8 bit number is, $1111\ 1011 = -128 + 64 + 32 + 16 + 8 + 2 + 1 = -5.$

In the second case, sum of the contributions from the leftmost 5 bits (–128 + 64 + 32 + 16 + 8 = –8) is same as the contribution from the leftmost bit in the 4 bit representation (–8)

This process is called *sign-extension*, and can be applied whenever a number is to be represented by a larger number of bits.

Most processors even have two separate instructions for shifting numbers to the right :

1. **LSR (Logical Shift Right).**

   It simply shifts the bits to the right and usually fills a zero in as the lefmost bit.

2. **ASR (Arithmetic Shift Right).**

   It shifts all of the bits to the right, while keeping the leftmost bit unchanged. With ASR 1010 (–6) becomes 1101 (–3).

There is only one instruction for a left shift (since LSL is equivalent to ASL).

BINARY FRACTIONS.

Positive binary fractions

The representation of unsigned binary fractions proceeds exactly the same as decimal fractions.

e.g. $0.625_{10} = 1*0.5 + 0*0.25 + 1*0.125 = 1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 0.101_2$

Each place to the right of the decimal point represents a negative power of 2, just as for decimals they represent a negative power of 10. Similarly, if there are m bits to the right of a decimal, the precision of the number is 2^{-m} (versus 10^{-m} for decimal). Though it is possible to represent numbers greater than one by having digits to the left of the decimal place we will restrict ourselves to numbers less than one. These are commonly used by Digital Signal Processors.The largest number that can be represented by such a representation is $1\text{-}2^{-m}$, the smallest number is 2^{-m}.

For a fraction with 15 bits of resolution this gives a range of approximately 0.99997 to 3.05E-5.

This representationis easily extended to represent all positive numbers by having digits to the left of the decimal point represent the integer part, and the digits to the right representing fractional part.

$$6.625_{10} = 110.101_2$$

Signed binary fractions

These are formed much like signed integers. Working with a single digit to the left of the decimal point, and this will represent the number -1 (= $-(2^0)$). The rest of the representation of the fraction remains unchanged. Therefore this leftmost bit represents a sign bit just as with two's complement integers. If this bit is set, the number is negative, otherwise the number is positive. The largest positive number that can be represented is still $1\text{-}2^{-m}$ but the largest negative number is -1. The resolution is still $1\text{-}2^{-m}$.

There is a terminology for naming the resolution of signed fractions. If there are m bits to the right of the decimal point, the number is said to be in Q*m* format. For a 16 bit number (15 bits to the right of the decimal point) this results in Q15 notation.

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Signed binary fractions are easily extended to include all numbers by representing the number to the left of the decimal point as a 2's complement integer, and the number to the right of the decimal point as a positive fraction.

Thus $$-6.625_{10} = (-7 + 0.375)_{10} = 1001.011_2$$

As with two's complement integers, the leftmost digit can be repeated any number of times without affecting the value of the number.

CONVERSION OF DECIMAL FRACTION TO BINARY FRACTION.

The conversion of a decimal fraction to a binary fraction is obtained using the technique of successive multiplication by 2. The integer part is noted down after each multiplication and the new fracion is used for further multiplication.

Examples : *Convert decimal fraction 0.375 to its binary equivalent.*

Fraction	***Fraction × 2***	***Remainder fraction***	***Integer***
0.375	0.750	0.75	0 (MSB)
0.75	1.50	0.50	1
0.50	1.00	0.00	1 (LSB)

0.375 (decimal) = 0.11 (binary).

Examples : *Converst 10.7 (decimal) to its binary equivalent.*

For the above decimal real number the binary equivalents are obtained for the integer and the fraction separately.

First 10 (decimal) is converted to its equivalent binary number.

2	10	Remainder
2	5	0 (LSB)
2	2	1
2	1	0
	0	1 (MSB)

10 (decimal) = 1010 (binary)

Now, 0.7 (decimal) is converted to its equivalent binary fraction.

$$\begin{array}{r} 0.7 \\ \hline \times 2 \\ 1.4 \\ \hline \times\ 2 \\ 0.8 \\ \hline \times\ 2 \\ \hline 1.6 \\ \hline \times\ 2 \\ \hline 1.2 \end{array}$$

0.7 (decimal) = $(0.1011)_2$

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Examples : *Convert decimal number 25 to its binary equivalent.*

Remainder can be written in a simpler form as shown below :

2	25	Remainder
2	12	1(LSB)
2	6	0
2	3	0
2	1	1
	0	1 (MSB)

Binary number = 11001

BINARY ADDITION AND SUBTRACTION

Binary Addition

When 1 is added to 1, the sum is 10 (binary) = 2 (decimal)

Rules :

A	*B*	*A + B*
0	0	0
0	1	1
1	0	1
1	1	10

e.g.

1 1 0 1	(13 decimal)
+ 0 0 0 1	(+ 1 decimal)
1110	(14 decimal)

and

0 0 1 1	(3 decimal)
+ 0 1 1 1	(7 decimal)
1 0 1 0	(10 decimal)

Binary Subtraction

Rules :

A	*B*	*A–B (Difference)*	*Borrowed*
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

COMPLEMENTS

Complements are used in digital computers simplifying the subtraction operation and for logical manipulations.

Types of Complements

There are two types of complements for each base-r system :

(*i*) *r's* Complement.

Given a positive N in base *r* with an integer part of *n* digits.

Then *r's* complement of N is defined as $r^n - N$ for $N \neq 0$ and 0 for N = 0.

Examples :

10's complement of $(52520)_{10}$ is

$$10^5 - 52520 = 47480.$$

2's complement of $(101100)_2$ is

$$(2^6)_{10} - (101100)_2 = (1000000 - 101100)_2 = 010100.$$

2's complement of $(0.0110)_2$ is

$$(1 - 0.0110)_2 = 0.1010.$$

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(*ii*) (*r* – 1)'*s* Complement.

Given a positive number N in base *r* with an integer of *n* digits and a fraction part of *m* digits.

Then $(r - 1)$'*s* complement of N is defined as $r^n - r^{-m} - N$.

Examples :

- 9'*s* complement of $(52520)_{10}$ is

$$(10^5 - 1 - 52520) = 99999 - 52520 = 47479.$$

No fraction part, so $10^{-m} = 10^0 = 1$.

- 9'*s* complement of $(0.3267)_{10}$ is

$$(1 - 10^{-4} - 0.3267) = 0.9999 - 0.3267 = 0.6732.$$

- 1'*s* complement of $(101100)_2$ is

$$(2^6 - 1) - (101100) = (111111 - 101100)_2 = 010011.$$

- 1'*s* complement of $(0.0110)_2$ is

$$(1 - 2^{-4})_{10} - (0.0110)_2 = (0.1111 - 0.0110)_2 = 0.1001.$$

Note : When value of then base is subsituted, the two types receive the names 2'*s* and 1'*s* complement for binary numbers, or 10'*s* and 9'*s* complement for decimal numbers.

9's, 10's, 1's and 2's Compliments

9'*s* Complement.

To obtain 9'*s* complement of a decimal number each digit of the number is subtracted from 9.

Examples :

9'*s* complement of 45 is $(99 - 45) = 54$,

9'*s* complement of 523 is $(999 - 523) = 476$.

10'*s* Complement.

10'*s* complement of a decimal number = its 9'*s* complement + 1.

10'*s* complement of 45 is $54 + 1 = 55$

10'*s* complement of 534 is $476 + 1 = 477$.

Now add the number and units 10'*s* complement and examine the sum.

Examples :

```
   45  (decimal number)
 + 55  (its 10's complement)
 ---------------------------
   00
```

Carry = 1

In this case the decimal number is of two digits. If we consider the sum of the number and its 10'*s* complement only upto two digits, it is zero. In other words we are neglecting the carry of the last stage.

e.g.

```
   5 2 3  (decimal number)
 + 4 7 7  (its 10's complement)
 ------------------------------
   000
```

Carry = 1

Decimal number 523 is of three digits. If sum of the number and its 10'*s* complement only upto three digits is considered, it is zero. Thus 10'*s* complement gives negative value of the number.

10'*s* complement of a decimal number = – decimal number

1'*s* Complement.

In the binary number system, it is similar to the 9'*s* complement in the decimal system. 1'*s* complement of a binary number is obtained by subtracting each bit of the number from 1.

1'*s* complement of 01 is 10. The 1'*s* complement of 111 is 000. Thus the 1'*s* complement of a binary number can be taken by changing the bit 1 to 0 and 0 to 1.

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Examples :

1's complement of 100110 = 011001.

1's complement of 0000 is 1111.

1's complement of 11111 is 00000.

2's complement.

In the binary number system, it is similar to the 10's complement in the decimal number system.

2's complement of binary number = its 1's complement + 1.

Examples :

2's complement of 10011 = 01100 + 1 = 01101

2's complement of 111 = 000 + 1 = 001

2's complement of 0000 = 1111 + 1 = 0000

Examining the sum when a binary number is added to its 2's complement :

Let binary number = 1 0 0 1

Its 1's complement = 0 1 1 0

Its 2's complement = 0 1 1 0 + 1

= 0 1 1 1.

Number + its 2's complement = 1 0 0 1

+ 0 1 1 1

0 0 0 0

Carry

Last carry is lost if processor is of 4 bits or sum is considered only upto 4 bits.

For 8-bit processor, the number and 2's complement will be written in 8 bits as given below :

Number = 00001001

1's complement = 11110110

2's complement = 11110110 + 1

= 11110111

Number = 00001001

+ 2's complement = + 11110111

00000000

Carry

Examples :

The last carry will be lost, if 8 bit sum is considered.

- + 4 (decimal) = 00000100(binary)

  – 4 = Its 2's complement = 11111011 + 1

  = 11111100

  + 4 = 00000100

  – 4 = 11111100

  = 00000000

- Add + 5 and – 7

  7 = 00000111

  – 7 = 11111000 + 1

  = 11111001

  + 5 = 00000101

  – 7 = 11111001

  –2 = 11111110

Check : 2 = 00000010

– 2 = 11111101 + 1

= 11111110

Thus 2's complement of a binary number represents its negative.

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Subtraction with *r's* Complements

Subtraction of two positive numbers (M – N), both of base r, is done as follows :

(*i*) Add minuend M to the *r's* complement of the subtrahend N.

(*ii*) Inspect the result obtained in step 1 for an end carry :

(*a*) If an end carry occurs, discard it.

(*b*) If an end carry does not occur, take *r's* complement of the number obtained in step 1 and place a negative sign in front.

Examples :

- Using 10's complement, subtract 72532 – 3250

	M = 72532	72532
	N = 03250	
10's complement of	N = 96750	+96750
	end carry → 1	69282
		69282

- Subtract $(3250 - 72532)_{10}$

	M = 03250	03250
	N = 72532	
10's complement of	N = 27486	+27468
	no carry	30718

Hence 69282 = – (10's complement of **30718**)

Subtraction with (r-1)'s Complement

Subtraction of M–N, both positive numbers in base r is done as follows :

(*i*) Add minuend M to the (*r* – 1)'s complement of the subtrahend N.

(*ii*) Inspect the result obtained in step 1 for an end carry.

(*a*) If an end carry occurs, add 1 to the least significant digit (end-around carry).

(*b*) If an end carry does not occur, add 1 to the (*r* – 1)'s complement of the number obtained in step 1 and place a negative sign in front.

Example : Use 1's complement to perform M – N with given binary number.

(*a*)	M = 1010100		1010100
	N = 1000100		
1's complement of	N = 0111011		+ 0111011
	end around carry	1__	0001111
			+ 1
			0010000

(*b*) M = 1000100	1000100
N = 1010100	
1's complement of N = 0101011	0101011
no carry__	1101111

Hence 10000 = – (1's complement of 1101111)

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Binary codes for Decimal digits

It require a minimum of four bit. Numerous codes can be obtained by arranging four or more bits in ten distinct possible combinations.

Few possibilities are shown in the table below :

Decimal digit	*(BCD) 8421*	*Excess-3 5043210*	*84–2–1*	*2421*	*Binary*
0	0000	0011	0000	0000	0100001
1	0001	0100	0111	0001	0100010
2	0010	0101	0110	0010	0100100
3	0011	0110	0101	0011	0101000
4	0100	0111	0100	0100	0110000
5	0101	1000	1011	1011	1000001
6	0110	1001	1010	1100	1000010
7	0111	1010	1001	1101	1000100
8	1000	1011	1000	1110	1001000
9	1001	1100	1111	1111	1010000

Example. Apply (r – 1)'s complement subtraction to find the difference $(010)_2 - (111)_2$.

Solution.

The 1'*s* complement of the subtrahend is

$$\begin{array}{r} 111 \\ -111 \\ (000)_2 \end{array}$$

Performing the addition,

$$\begin{array}{r} 010 \\ +000 \\ (010)_2 \end{array}$$

Since no end-carry resulted in the addition, the answer is formed as the negative at the 1'*s* complement of $(010)_2$, or

$$\begin{array}{r} 111 \\ -010 \\ -(101)_2 \end{array}$$

Example. Use r's complement subtraction to find the difference $(111)_2 - (010)_2$.

Solution.

Forming the 2'*s* complement of the subtrahend,

$$\begin{array}{r} 111 \\ -010 \\ 101 \\ +1 \\ (110)_2 \end{array}$$

The resulting addition gives an end-carry which is discarded to give the answer,

$$\begin{array}{r} 111 \\ +110 \\ (1101)_2 \end{array}$$

Hence $$(111)_2 - (010)_2 = (101)_2$$

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FLOATING-POINT REPRESENTATION

DECIMAL CASES.

- 3.141592653589 ...
- 2.71828 ...
- (N_A)
  6.023×10^{-32}
- ($\hbar$)

In programming, a floating point number -123.45×10^{-6} is expressed as –123.45E –6.

In general, a floating-point number can be written as

$$\pm \; M \times B^E$$

where, M = fraction mantissa or significand.
E = exponent.
B = base. In decimal case, B = 10

BINARY CASES.

Example : A 32-bit word is used in MIPS computer to represent a floating-point number:

S	E	M
1 bit	8 bits	23 bits

$$(-1)S \times M \times 2^E$$

It represent :

(1) implied base is 2 (not explicitly shown in the representation).
(2) exponent can be represented in signed 2's complement (but also see biased notation later).
(3) implied decimal point is between the exponent field E and the significand field M.
(4) more bits in field E mean larger range of values representable.
(5) More bits in field M mean higher precision.
(6) zero is represented by all bits equal to 0: $0000 \ldots 000_2$

Normalization

To efficiently use the bits available for the significand, it is shifted to the left until all leading 0's disappear (as they make no contribution to the precision). The value can be kept unchanged by adjusting the exponent accordingly.

Moreover, as MSB of the significand is always 1, it does not need to be shown explicitly. The significand could be further shifted to the left by 1 bit to gain one more bit for precision. The first bit 1 before the decimal point is implicit. The actual value represented is

$$(-1)S \times (1. + M) \times 2^E$$

However, to avoid possible confusion in the following, the default normalization does not assume this implicit 1 unless otherwise specified.

Zero is represented by all 0's and is not (and cannot be) normalized.

e.g. *A binary number x = 0.0001101001101 can be represented in 14-bit floating-point form in the following ways (1 sign bit, a 4-bit exponent field and a 9-bit significand field) :*

- $x = 0.0001101001101 \times 2^0$ $\boxed{0}$ $\boxed{0000}$ $\boxed{000110100}$
- $x = 0.001101001101 \times 2^{-1}$ $\boxed{0}$ $\boxed{1111}$ $\boxed{001101001}$
- $x = 0.01101001101 \times 2^{-2}$ $\boxed{0}$ $\boxed{1110}$ $\boxed{011010011}$
- $x = 0.1101001101 \times 2^{-3}$ $\boxed{0}$ $\boxed{1101}$ $\boxed{110100110}$
- $x = 1.101001101 \times 2^{-4}$ $\boxed{0}$ $\boxed{1100}$ $\boxed{101001101}$ with an implied 1.0 :

  By normalization, highest precision can be achieved.

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BIASED NOTATION FOR EXPONENT.

To simplify the hardware for comparing two exponents, we may want to avoid 2's complement representation for the exponent. This can be done by simply adding 1 (a bias) at the MSB of the exponent field and the resulting representation is called *biased notation*.

Consider a 5-bit exponent field (range of exponents : $-2^4 \sim 2^4 - 1$) :

Decimal Expoennts	Signed -2's Complement	Biased Notation (Excess-16)	Decimal Value of Biased Notation
15	01111	11111	31
14	01110	11110	30
...	...	...	...
1	00001	10001	17
0	00000	10000	16 (bias)
–1	11111	01111	15
...	...	...	...
–15	10001	00001	1
–16	10000	00000	0

The bias depends on number of bits in the exponent field. If there are e bits in this field, then

$$\text{Bias} = 2^{c-1},$$

which lifts the representation by half of the range to get rid of the negative parts represented by 2's complement. The range of actual exponents represented is still the same.

With the biased exponent, the value represented by the notation is:

$$(-1)^{S} \times (1. \times M) \times 2^{E-Bias}$$

Floating-Point Notation of IEEE 754

The IEEE 754 floating-point standard uses 32 bits to represent a floating-point number, including 1 sign bit, 8 exponent bits and 23 bits for the significand. As the implied base is 2, an implied 1 is used, i.e., the significand has effectively 24 bits including 1 implied bit to the left of the decimal point not explicitly represented in the notation.

When working with any type of digital electronics in which numbers are being represented, it is important to understand different ways numbers are represented in these systems. Numbers are represented by two voltage levels which can represent a one or a zero. The number system based on ones and zeroes is called *bi*nary *system*.

To represent the positive integer one hundred and twenty-five as a decimal number, we can write (with the postivie sign implied)

$$125_{10} = 1*100 + 2*10 + 5*1 = 1*10^2 + 2*10^1 + 5*10^0$$

The subscript 10 denotes the number as a base 10 (decimal) number.

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SOLVED EXAMPLES

1. Convert the decimal number $(52)_{10}$ to an equivalent binary number.

Solution.

division	*integer quotient*	*remainder*
52/2	26	0
26/2	13	0
13/2	6	1
6/2	3	0
3/2	1	1
1/2	0	1

The binary number is formed from the remainder terms as $(110100)_2$.

2. Convert the decimal fraction $(0.4375)_{10}$ to a binary fraction.

Solution.

multiplication	*integer*	*fraction portion*
0.4375×2	0	0.8750
0.8750×2	1	0.7500
0.7500×2	1	0.5000
0.5000×2	1	0.0000

The converted binary fraction is formed from the integer portion as $(0.0111)_2$.

3. Convert the decimal number $(10.75)_{10}$ to its binary equivalent.

Solution.

The integer portion and fraction portion are converted separately and combined to form the complete binary number.

division	*integer quotient*	*remainder*
10/2	5	0
5/2	2	1
2/2	1	0
1/2	0	1

Thus, $(10)_{10} = (1010)_2$.

Converting the fraction portion

multiplication	*integer portion*	*fraction portion*
0.75×2	1	0.500
0.500×2	1	0.000

Hence, $(0.75)_{10} = (0.11)_2$.

The complete conversion is

$$(10.75)_{10} = (1010)_2 + (0.11)_2 = (1010.11)_2$$

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4. Convert decimal 41 to binary

Solution.

	Integer	remainder	
2	41		
2	20	1	
2	10	0	
2	5	0	
2	2	1	
2	1	0	
	0	1	101001

5. Convert $(0.6875)_{10}$ to binary.

Solution.

integer	fraction	Coefficient
$0.6875 \times 2 = 1$	$+ 0.3750$	$a_{-1} = 1$
$0.3750 \times 2 = 0$	$+ 0.7500$	$a_{-2} = 0$
$0.7500 \times 2 = 1$	$+ 0.5000$	$a_{-3} = 1$
$0.5000 \times 2 = 1$	$+ 0.000$	$a_{-4} = 1$

$(0.6875)_{10} = (0.1011)_2$

6. From the octal equivalent of $(100)_{10}$.

Solution.

division	***integer quotient***	***remainder***
100/8	12	4
12/8	1	4
1/8	0	1

The octal number is formed from the remainder terms as $(144)_8$.

7. Find the octal equivalent of $(100.250)_{10}$.

Solution.

multiplication	***integer portion***	***fraction portion***
0.250×8	2	0.000

Hence, $(0.250)_{10} = (0.2)_8$,

The complete conversion is

$$(0.250)_{10} = (144)_8 \, (0.2)_8$$
$$= (144.2)_8$$

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EXERCISE – I

MCQ TYPE QUESTIONS

1. Analog systems are different from digital systems because they
(*a*) use transistors
(*b*) handle information in analog form
(*c*) handle information in digital form
(*d*) are slow

2. Digital circuits easier to design than analog circuits because
(*a*) they do not control electricity precisely over a wide range.
(*b*) they are made in the form of ICs.
(*c*) all elements of digital circuits are from the same family.
(*d*) they are smaller in size.

3. A digital gate can respond to an input signal in
(*a*) about a second
(*b*) about a hundredth of a second
(*c*) a few billionth of a second
(*d*) a few millisecond

4. Analog methods are not used for handling extremely precise information because
(*a*) precise information always involves numbers which are inherently digital
(*b*) there are limits to how closely an analog signal can reproduce the information
(*c*) analog information never need to be precise
(*d*) they are very expensive

5. The operation which is commutative but not associative is
(*a*) AND (*b*) OR
(*c*) EX-OR (*d*) NAND

6. The functional capacity for LSI devices is
(*a*) 1 to 11 gates
(*b*) 12 to 99 gates
(*c*) 100 to 10,000 gates
(*d*) more than 10,000 gates

7. The functional capacity for VLSI devices is
(*a*) 1 to 11 gates
(*b*) 12 to 99 gates
(*c*) 100 to 10,000 gates
(*d*) more than 10,000 gates

8. SMD on a data sheet for an IC means
(*a*) Single Memory Device
(*b*) Serial Mode Data
(*c*) Synchronous Mode Device
(*d*) Surface Mount Device

9. Which of the following logic expression is incorrect?
(*a*) $1 \oplus 0 = 1$ (*b*) $1 \oplus 1 \oplus 0 = 1$
(*c*) $1 \oplus 1 \oplus 1 = 1$ (*d*) $1 \oplus 1 = 0$

10. Which of the following respectively represent commutative law, Associative law and Distributive law ?
I. $A \cdot (B \cdot C) = (A \cdot B) \cdot C$
II. $A \cdot (B + C) = A \cdot B + A \cdot C$
III. $A + B = B + A$
Codes :
(*a*) I, III and II
(*b*) II, I and III
(*c*) III, II and I
(*d*) III, I and II

11. Which of the following boolean algebra expression is incorrect ?
(*a*) $\overline{ABC} + BC + AC = C$
(*b*) $(A + B)\,\overline{[\overline{A}\,(\overline{B} + \overline{C})]} + \overline{B}\,\overline{C} + \overline{A}\,\overline{C} = 1$
(*c*) $AB + \overline{A}C + BC = AB + AC$
(*d*) $AB + A\overline{C} = B$

12. Which of the following boolean algebra expression is incorrect ?
(*a*) $AB + A(B + C) + B(B + C) = B + AC$
(*b*) $[A\overline{B}\,(C + B\overline{D}) + \overline{A}\,\overline{B}]\,C = B\overline{C}$
(*c*) $\overline{A\overline{B}\,(C + \overline{D})} + \overline{A}\,B + \overline{C}D$
(*d*) $(A + C)\,(ABC + ACD) = ABC + ACD$

13. Which of the following boolean algebra expression is incorrect ?
(*a*) $AB + \overline{A}\,\overline{B}\,C + A = A + BC$
(*b*) $[A\overline{B} + \overline{A}B\overline{C} + \overline{A}BCD + \overline{A}B\overline{C}\,\overline{D}E = \overline{A}\,B$
(*c*) $AB + (\overline{A} + \overline{B})\,C + AB = AB + A\overline{C} + \overline{B}C$
(*d*) $(A + \overline{A})\,(AB + AB\overline{C}) = AB$

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14. Simplified form of the boolean expression (X + Y + XY) (X + Z) is

(*a*) X + Y + Z (*b*) XY + YZ
(*c*) X + YZ (*d*) XZ + Y

15. The simplified form of the boolean expression $(X+\overline{Y}+Z)(Z+\overline{Y}+\overline{Z})(X+Y+Z)$ is

(*a*) $\overline{X}Y+\overline{Z}$ (*b*) $X+\overline{Y}Z$
(*c*) X (*d*) $XY+\overline{Z}$

16. Which of the following expression remove hazard from : $xy + z\overline{x}$?

(*a*) $xy + z\overline{x}$ (*b*) $xy + z\overline{x} + wyz$
(*c*) $xy + z\overline{x} + yz$ (*d*) $xy + z\overline{x} + wz$

17. Consider the boolean expression :

$$\overline{x}\,y\,\overline{z} + \overline{x}\,\overline{y}\,z + x(y+z).$$

The equivalent product of sums form is

(*a*) $\overline{x}\,y\,\overline{z} + \overline{x}\,\overline{y}\,z + xy + xz$

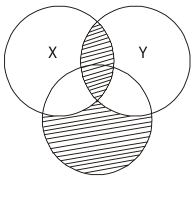
(*b*) $(x+\overline{y}+z)(x+y+\overline{z})(x+y+z)$

(*c*) $(\overline{y}+\overline{z})(\overline{x}+y+z)$

(*d*) $(y+z)(x\overline{y}+z)$

18. In the following Karnaugh map, corresponding switching function in its minimal form is

YZ \ WX	00	01	11	10
00	1	1	1	
01				
11		1	1	
10	1		1	1



(*a*) $F(w, x, y, z) = x'z'$

(*b*) $F(w, x, y, z) = (x' + z')(w + z')(x + y + z)$

(*c*) $F(w, x, y, z) = xz + w'z + x'y'z'$

(*d*) $(w, x, y, z) = x'z' + wz'wz' + xyz$

19. Which of the following boolean expressions is not logically equivalent to all of the rest ?

(*a*) $ab + (cd)' + cd + bd'$

(*b*) $a(b + c) + cd$

(*c*) $ab + ac + (c \oplus d)'$

(*d*) $bd' + c'd' + ab + cd$

20. The boolean expression $\overline{A}BE + BCDE + B\overline{C}\,\overline{D}E + \overline{B}\,\overline{C}D\overline{E}$ can be simplified to $BE + \overline{B}D\overline{E}$, if the don't care conditions are

(*a*) $ABCDE + A\overline{B}CD\overline{E}$

(*b*) $ABCDE + A\overline{B}CD\overline{E} + ABC\overline{D}E$

(*c*) $AB\overline{C}DE + A\overline{B}CD\overline{E} + ABC\overline{D}E$

(*d*) none of these

21. In the following Karnaugh map, which one of the following represents minimal Sum-Of-Products of the map ?

yz \ wx	00	01	11	10
00	0	X	0	X
01	X	1	X	1
11	0	X	1	0
10	0	1	X	0

(*a*) $xy + y'z$ (*b*) $wx'y' + xy + xz$
(*c*) $w'x + y'z$ (*d*) $xz + y$

22. The minimal cover for the maximal compatibility classes {*ae, acd, ad, bd*} is

(*a*) *ae, acd, ad* (*b*) *acd, ad, bd*
(*c*) *ae, acd, bd* (*d*) *ae, ad, bd*

23. What is the form of the boolean expressions $AB + \overline{B}\,\overline{C} = Y$?

(*a*) Product-of-sums (*b*) Sum-of-products
(*c*) Karnaugh map (*d*) Matrix

24. Consider a function that is defined by the following truth table :

A	B	C	*f*(A, B, C)
0	0	0	1
1	0	0	1
0	1	0	1
1	1	0	1
0	0	1	1
1	0	1	0
0	1	1	X
1	1	1	X

Which of the following statements about the minimal sum-of-products and minimal product-of-sums implementations is correct ?

Note that *x* represents don't care term.

(*a*) They are logically equivalent because the don't cares are used in the same way.

(*b*) They are logically not equivalent because the don't cares are used in different ways.

(*c*) They are logically not equivalent by definition

(*d*) They are logically equivalent by definition.

25. Consider following switching function :

$f(w, x, y, z) = w'x' + w'xy' + wx'z' + wxy$

For this function, which of the following is list of essential prime implicants ?

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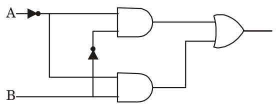
(*a*) $w'\,x'$, $w'\,y'$, $x'\,y'$, wxy, wyz

(*b*) wxy, wyz'

(*c*) $w'\,x'$, $w'\,y'$, $x'\,z'$

(*d*) $w'\,x'$, $w'\,y'$, $x'\,z$, wxy'

26. What is the Boolean expression for the circuit given below ?



(*a*) F(A, B) = A′ B + A′ B′

(*b*) F(A, B) = A ⊕ B

(*c*) F(A, B) = A′ + B′

(*d*) F(A, B) = A′ B′

27. The number of Boolean functions that can be defined for n Boolean variables over k-valued Boolean algebra are

(*a*) 2^{2^n} (*b*) k^{k^n}

(*c*) nk (*d*) n

28. Which of the following statements is true ?

(*a*) (A + B) (A + C) = AC + BC

(*b*) (A + B) (A + C) = AB + C

(*c*) (A + B) (A + C) = A + BC

(*d*) (A + B) (A + C) = AC + B

29. A switching function is symmetric with respect to a set of literals if and only if the function remains unchanged after

(*a*) two of these literals are interchanged

(*b*) any permutation of the literals

(*c*) all the literals are changed in anticlockwise order

(*d*) all the literals are changed in clockwise order

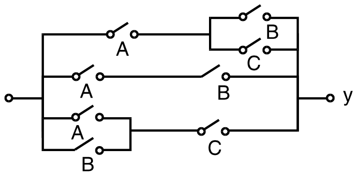
30. Match List-I with List-II and select the correct answer using the codes given below the lists :

List-I	List-II
A. A ⊕ B = 0	**1.** A ≠ B
B. $\overline{A+B} = 0$	**2.** A = B
C. $\overline{A} \cdot B = 0$	**3.** A = 1 & B = 1
D. A ⊕ B = 1	**4.** A = 1 & B = 0

Codes :

	A	B	C	D
(*a*)	1	2	4	3
(*b*)	2	3	4	1
(*c*)	1	3	2	4
(*d*)	2	4	1	3

31. The minimum Boolean expression for the following circuit is



(*a*) AB + AC + BC

(*b*) A + BC

(*c*) A + B

(*d*) A + B + C

32. For a binary half-subtractor having two inputs A and B, the correct set of logical expression for the outputs D (= A minimum B) and X (= borrow) are

(*a*) $D = AB + \overline{A}B$, $X = \overline{A}B$

(*b*) $D = \overline{A}B + A\overline{B}$, $X = A\overline{B}$

(*c*) $D = \overline{A}B + A\overline{B}$, $X = \overline{A}B$

(*d*) $D = AB + \overline{A}\,\overline{B}$, $X = A\overline{B}$

33. A graphical display of the fundamental products in a truth-table is known as

(*a*) mapping (*b*) graphing

(*c*) T-map (*d*) kanangh-map

34. The time required for a gate or inverter to change its state is called

(*a*) rise time (*b*) decay time

(*c*) propagation time (*d*) charging time

35. The time required for a pulse to change from 10 to 90 percent of its maximum value is called

(*a*) rise time (*b*) decay time

(*c*) propagation time (*d*) operating speed

36. The maximum frequency at which digital data can be applied to gate is called

(*a*) operating speed

(*b*) propagation speed

(*c*) binary level transaction period

(*d*) charging time

37. The time required for a pulse to decrease from 90 to 10 per cent of its maximum value is called

(*a*) rise time

(*b*) decay time

(*c*) binary level transition period

(*d*) propagation delay

38. Which of the following statements is wrong ?

(*a*) Propagation delay is the time required for a gate to change its state.

(*b*) Noise immunity is the amount of noise which can be applied to the input of a gate without causing the gate to change state.

(*c*) Fan-in of a gate is always equal to fan-out of the same gate.

(*d*) Operating speed is the maximum frequency at which digital data can be applied to a gate.

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39. A small dot or circle printed on top of an IC indicates

(*a*) V_{CC} (*b*) Pin 4

(*c*) Pin 14 (*d*) Pin 1

40. In which of the following gates, the output is 1, if and only if at least one input is 1 ?

(*a*) NOR (*b*) AND

(*c*) OR (*d*) NAND

41. In which of the following gates, the output is 0 if and only if at least one input is 1 ?

(*a*) OR (*b*) AND

(*c*) NOR (*d*) NAND

42. The NAND can function as a NOT gate if

$\overline{A.B} = \overline{A} + \overline{B}$

(*a*) inputs are connected together

(*b*) inputs are left open

(*c*) one input is set to 0

(*d*) one input is set to 1

43. What is the minimum number of two-input NAND gates used to perform the function of two input OR gate ?

(*a*) one (*b*) two

(*c*) three (*d*) four

44. Which of the following gates are added to the inputs of the OR gate to convert it to the NAND gate ?

(*a*) NOT (*b*) AND

(*c*) OR (*d*) XOR

45. What logic function is produced by adding an inverter to the output of an AND gate ?

(*a*) NAND (*b*) NOR

(*c*) XOR (*d*) OR

46. Which of the following gates is known as coincidence detector ?

(*a*) AND gate (*b*) OR gate

(*c*) NOT gate (*d*) NAND gate

47. An OR gate can be imagined as

(*a*) switches connected in series

(*b*) switches connected in parallel

(*c*) MOS transistors connected in series

(*d*) none of these

48. Which of the following gates would output 1 when one input is 1 and other input is 0 ?

(*a*) OR gate

(*b*) AND gate

(*c*) NAND gate

(*d*) both (*a*) and (*c*)

49. The output of NOR gate is

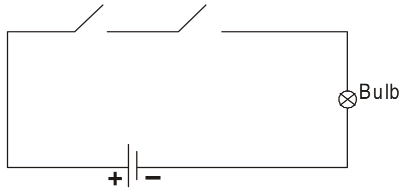
(*a*) high if all of its inputs are high

(*b*) low if all of its inputs are low

(*c*) high if all of its inputs are low

(*d*) high if only of its inputs is low

50. What logic gate is represented by the circuit shown below ?



(*a*) NAND (*b*) NOR

(*c*) AND (*d*) EQUIVALENCE

51. A toggle operation cannot be performed using a single

(*a*) NOR gate (*b*) AND gate

(*c*) NAND gate (*d*) XOR gate

52. If $A \oplus B = C$, then

(*a*) $A \oplus C = B$ (*b*) $B \oplus C = A$

(*c*) $A \oplus B \oplus C = 0$ (*d*) all of these

53. Which combination of gates does not allow the implementation of an arbitrary boolean function?

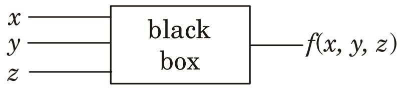
(*a*) OR gates and AND gates only

(*b*) OR gates and exclusive – OR gate only

(*c*) OR gates and NOT gates only

(*d*) NAND gates only

54. The black box in the following figure consists of a minimum complexity circuit that uses only AND, OR and NOT gates :

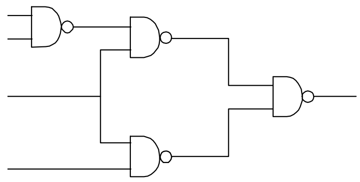


The function $f(x, y, z) = 1$ whenever x, y are different and 0 otherwise. In addition the 3 inputs x, y, z are never all the same value. Which of the following equation lead to the correct design for the minimum complexity circuit ?

(*a*) $x'y + xy'$ (*b*) $x + y'z$

(*c*) $x'y'z' + xy'z$ (*d*) $xy + y'z + z'$

55. The logic expression for the output of the circuit shown in the figure is



(*a*) $\overline{A}\,\overline{B}\,C + CD$ (*b*) $\overline{A}\,\overline{B}\,\overline{C} + CD$

(*c*) $A\,B\,C + \overline{C}\,\overline{D}$ (*d*) $\overline{A}\,\overline{B} + \overline{C}\,\overline{D}$

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56. Which table shows the logical state of a digital circuit output for every possible combination of logical states in the inputs ?

(*a*) Function table (*b*) Truth table
(*c*) Routing table (*d*) ASCII table

57. Which table shows the electrical state of a digital circuit's output for every possible combination of electrical states in the inputs ?

(*a*) Function table (*b*) Truth table
(*c*) Routing table (*d*) ASCII table

58. How many truth tables can be made from one function table ?

(*a*) One (*b*) Two
(*c*) Three (*d*) Any numbers

59. A positive AND gate is also a negative

(*a*) NAND gate (*b*) NOR gate
(*c*) AND gate (*d*) OR gate

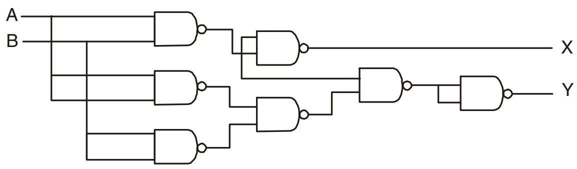
60. How many full adders are required to construct an m-bit parallel adder ?

(*a*) $m/2$ (*b*) $m - 1$
(*c*) m (*d*) $m + 1$

61. Which of the following adders can add three or more numbers at a time ?

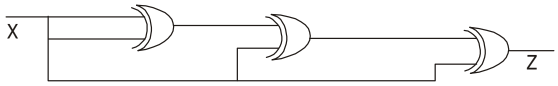
(*a*) Parallel adder
(*b*) Carry-look-ahead adder
(*c*) Carry-save-adder
(*d*) Full adder

62. What logic function is performed by the circuit shown below ?



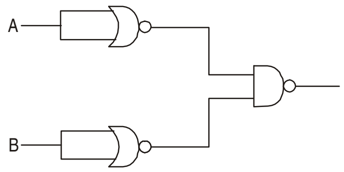
(*a*) ring counter (*b*) ripple counter
(*c*) full adder (*d*) half adder

63. Output of the following circuit is



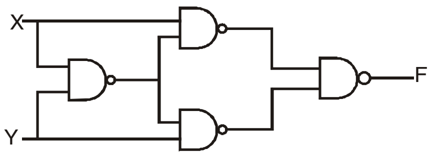
(*a*) 0 (*b*) 1
(*c*) x (*d*) x'

64. In the circuit shown below, which logic function does this circuit generate ?



(*a*) OR (*b*) NOR
(*c*) NAND (*d*) EX-OR

65. Which of the following logic gates is equivalent to the circuit shown below ?



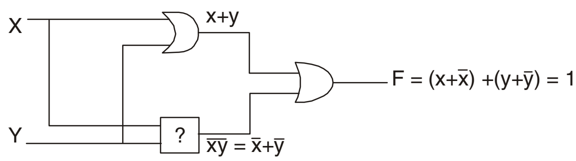
(*a*) OR (*b*) AND
(*c*) NOT (*d*) EX-OR

66. A full-adder is a logic circuit which can add two single order bits plus a carry in from a previous adder. Its incomplete truth table is given in the table below. The missing entry in the outputs for SUM and CARRY out are

Input			Outputs	
A	B	C_{in}	Sum	$Carry_{out}$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	?	?

(*a*) 0 0 (*b*) 0 1
(*c*) 1 0 (*d*) 1 1

67. Consider the following circuit :



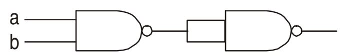
To make it a tautology, the ? marked box should be

(*a*) OR gate (*b*) AND gate
(*c*) NAND gate (*d*) EX-OR gate

68. The number of full and half-adders required to add 16-bit numbers is

(*a*) 8 half-adders, 8 full-adders
(*b*) 1 half-adder, 5 full-adders
(*c*) 16 half-adders, 0 full-adders
(*d*) 4 half-adders, 12 full-adders

69. The combinational circuit given below is implemented with two NAND gates. To which of the following individual gates is its equivalent ?



(*a*) NOT (*b*) OR
(*c*) AND (*d*) XOR

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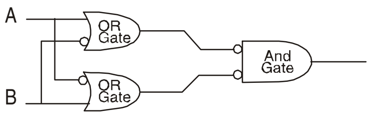
70. What is the Boolean expression for the following circuit?



(*a*) F(A, B) = (A + B′) · (B + A′)

(*b*) F(A, B) = 1 (Tautology)

(*c*) F(A, B) = O (inconsistency)

(*d*) F(A, B) = A ⊕ B (A exclusive OR' ed with B)

71. Which of the following circuit can be used as parallel to serial converter ?

(*a*) Multiplexer (*b*) Demultiplexer

(*c*) Decoder (*d*) Digital counter

72. The function of a multiplexer is

(*a*) to decode information

(*b*) to select 1 out of N input data sources and to transmit it to single channel

(*c*) to transit data on N lines

(*d*) to perform serial to parallel conversion

73. Which of the following sets of component (s) is/are sufficient to implement any arbitrary Boolean function ?

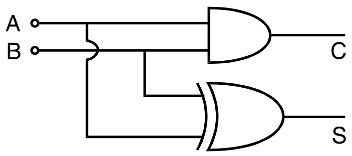
(*a*) XOR gates, NOT gates

(*b*) 2 to 1 multiplexors

(*c*) AND gates, XOR gates

(*d*) Three-input gates that output (A . B) + C for the inputs A . B. and C.

74. For the circuit shown for AB = 00, AB = 01; C, S values respectively are



(*a*) 0, 0 and 0, 1 (*b*) 0, 0 and 1, 0

(*c*) 0, 1 and 0, 0 (*d*) 1, 0 and 0, 0

75. An AND circuit

(*a*) is a memory circuit

(*b*) gives an output when all input signals are present simultaneously

(*c*) is a – *ve* OR gate

(*d*) is a linear circuit

76. A comparison between serial and parallel adder reveals that serial order

(*a*) is slower

(*b*) is faster

(*c*) operates at the same speed as parallel adder

(*d*) is more complicated

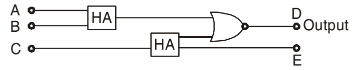
77. The circuit given below is a

(*a*) full adder

(*b*) full subtractor

(*c*) parity checker

(*d*) none of these



78. Which of the following expressions is not equivalent to $\overline{x}$?

(*a*) *x* NAND *x* (*b*) *x* NOR *x*

(*c*) *x* NAND 1 (*d*) *x* NOR 1

79. The full adder adds the Kth bits of two numbers to the

(*a*) difference of the previous bits

(*b*) sum of all previous bits

(*c*) carry from $(K - 1)^{th}$ bit

(*d*) sum of previous bit

80. A demultiplexer is used to

(*a*) route the data from single input to one of many outputs

(*b*) select data from several inputs and route it to single output

(*c*) perform serial to parallel conversion

(*d*) all of these

81. Parallel adders are

(*a*) combinational logic circuits

(*b*) sequential logic circuits

(*c*) both (*a*) and (*b*)

(*d*) none of these

82. In which of the following adder circuits, the carry look ripple delay is eliminated ?

(*a*) Half adder

(*b*) Full adder

(*c*) Parallel adder

(*d*) Carry-look-ahead adder

83. A combinational logic circuit which generates a particular binary word or number is

(*a*) decoder

(*b*) multiplexer

(*c*) encoder

(*d*) demultiplexer

84. Which one of the following set of gates are best suited for parity checking and pacity generation.

(*a*) AND, OR, NOT gates

(*b*) EX-NOR or EX-OR gates

(*c*) NAND gates

(*d*) NOR gates

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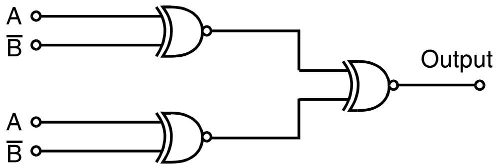
85. The output of the circuit shown in the figure is equal to



(*a*) 0 (*b*) 1

(*c*) $\overline{A}B + A\overline{B}$ (*d*) $(\overline{A * B}) * (\overline{A * B})$

86. In the following question, match each of the items A, B and C on the left with an approximation item on the right

A. Shift register can be used	**1.** for code conversion
B. A multiplexer can be used	**2.** to generate memory slip to select
C. A decoder can be used	**3.** for parallel to serial conversion
	4. as many to-one switch
	5. for analog to digital conversion

Codes :

	A	B	C
(*a*)	1	2	3
(*b*)	3	4	1
(*c*)	5	4	2
(*d*)	1	3	5

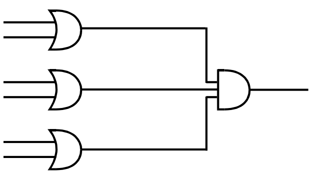
87. Which one of the following will give the sum of full adders as output ?

(*a*) Three point majority circuit
(*b*) Three bit parity checker
(*c*) Three bit comparator
(*d*) Three bit counter

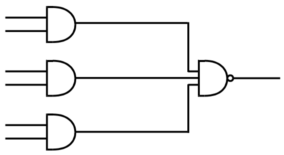
88. A combinational circuit is one in which the output depends on the

(*a*) input combination at the time
(*b*) input combination and the previous output
(*c*) input combination at that time and the previous input combination
(*d*) present output and the previous output

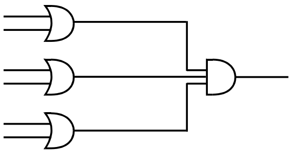
89. The circuit shown in the figure is equivalent to



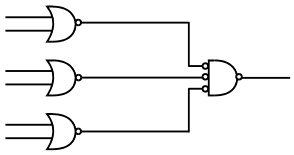
(*a*)



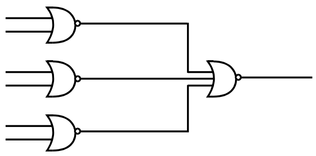
(*b*)



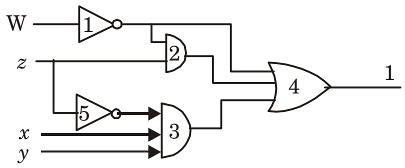
(*c*)



(*d*)



90. Consider following gate network



Which one of the following gates is redundant ?

(*a*) Gate No. 1 (*b*) Gate No. 2
(*c*) Gate No. 3 (*d*) Gate No. 4

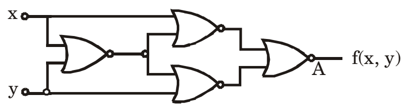
91. The digital multiplexer is basically a combination logic circuit to perform the operation

(*a*) AND – AND (*b*) OR – OR
(*c*) AND – OR (*d*) OR – AND

92. Adders

(*a*) adds 2 bits
(*b*) is called so because a full adder involves two half-adders
(*c*) needs two input and generates two output
(*d*) all of these

93. Identify the logic function performed by the circuit shown in the given figure



(*a*) exclusive OR (*b*) exclusive NOR
(*c*) NAND (*d*) NOR

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94. The inverter, OR-gate and AND gate are called deeision-making elements because they can recognize some input ____while disregarding others. A gate recognize a word when its output is ______.

(*a*) words, high (*b*) bytes, low
(*c*) bytes, high (*d*) character, low

95. A full- adder adds ______ bits and produce a sum and a

(*a*) 8, SUBTRACTION
(*b*) 3, CARRY
(*c*) 16, DIVIDE
(*d*) All of these

96. A toggle operation cannot be performed using a single

(*a*) NOR gate (*b*) AND gate
(*c*) NAND gate (*d*) XOR gate

97. The EXCLUSIVE NOR gate is equivalent to which gate followed by an inverter ?

(*a*) OR (*b*) AND
(*c*) NAND (*d*) XOR

98. Odd parity of word can be conveniently tested by

(*a*) OR gate (*b*) AND gate
(*c*) NOR gate (*d*) XOR gate

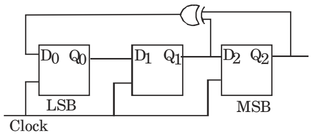
99. When the maximum clock rate is quoted for a logic family, then it applies to a

(*a*) shift register (*b*) flip-flop
(*c*) counter (*d*) single logic gate

100. Consider the circuit given below with initial state $Q_0 = 1$, Q_1 $Q_2 = 0$. The state of the circuit is given by the value $4Q_2 + 2Q_1 + Q_0$.

The correct state sequence of the circuit is

(*a*) 1, 3, 4, 6, 7, 5, 2
(*b*) 1, 2, 5, 3, 7, 6, 4
(*c*) 1, 2, 7, 3, 5, 6, 4
(*d*) 1, 6, 5, 7, 2, 3, 4



101. The clock signals are used in sequential logic circuits to

(*a*) tell the time of the day
(*b*) tell how much time has elapsed since the system was turned on
(*c*) carry serial data signals
(*d*) synchronize events in various parts of system

102. When an inverter is placed between both inputs of an SR flip-flop, then resulting flip-flop is

(*a*) JK flip-flop
(*b*) D flip-flop
(*c*) T flip-flop
(*d*) Master slave JK flip-flop

103. The functional difference between SR flip-flop and JK flip-flop is that JK flip-flop

(*a*) is faster than SR flip-flop
(*b*) has a feed back path
(*c*) accepts both inputs 1
(*d*) does not require external clock

104. The master slave JK flip-flop is effectively a combination of

(*a*) a SR flip-flop and a T flip-flop
(*b*) an SR flip-flop and a D flip-flop
(*c*) a T flip-flop and a D flip-flop
(*d*) two T flip-flops

105. The number of flip-flops required in a module N counter is

(*a*) $\log_2 (N) + 1$ (*b*) $\lfloor \log_2 (N) \rfloor$
(*c*) $\lceil \log_2 (N) \rceil$ (*d*) $\log_2 (N - 1)$

106. What is the maximum counting speed of a 4-bit binary counter which is composed of flip-flops with a propagation delay of 25 ns ?

(*a*) 1 MHz (*b*) 10 MHz
(*c*) 100 MHz (*d*) 4 MHz

107. A shift register can be used for

(*a*) parallel to serial conversion
(*b*) serial to parallel conversion
(*c*) digital delay line
(*d*) all of these

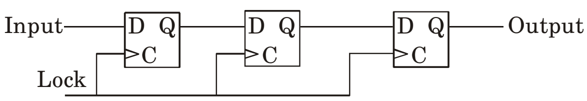
108. 74LS138 chip functions as

(*a*) decoder/demultiplexer
(*b*) encoder
(*c*) multiplexer
(*d*) memory element

109. The output of a sequential circuit depends on

(*a*) present inputs only
(*b*) past inputs only
(*c*) both present and past inputs
(*d*) present outputs only

110. The circuit shown in the figure below is a



(*a*) full adder (*b*) half adder
(*c*) shift register (*d*) 2-bit multiplexer

111. Consider an RS flip-flops with both inputs set to 0. If a momentary '1' is applied at the input S, then the output

(*a*) Q will flip from 0 to 1
(*b*) Q will flip from 0 to 1 and then back to 0
(*c*) Q will flip from 1 to 0
(*d*) Q will flip from 0 to 1 and then back to 0

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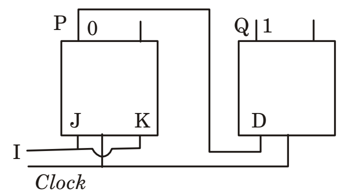
112. Flip-flop outputs are always
(*a*) complimentary
(*b*) the same
(*c*) independent of each other
(*d*) same as inputs

113. In a ripple counter using edge triggered JK filp-flops, the pulse input is applied to the
(*a*) clock input of all flip-flops
(*b*) clock input of one flip-flops
(*c*) J and K inputs of all flip-flops
(*d*) J and K inputs of one flip-flop.

114. In a positive edge triggered JK flip-flop, a low J and low K produces
(*a*) no change
(*b*) low state
(*c*) high state
(*d*) toggle

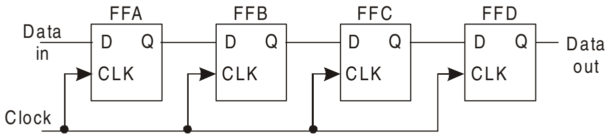
115. The following arrangement of master-slave flip flops has the initial state of P, Q as 0, 1 respectively.



After three clock cycles, the output state P, Q respectively are
(*a*) 1,0 (*b*) 1,1
(*c*) 0,0 (*d*) 0,1

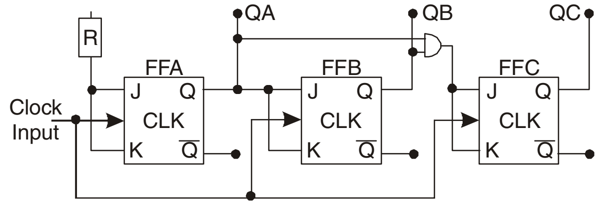
116. A JK flip-flop has its J input connected to logic level 1 and its input to the Q output. A clock pulse is fed to its clock input. The flip-flop will now
(*a*) change its state at each clock pulse
(*b*) go to state 1 and stay there
(*c*) go to state 0 and stay there
(*d*) retain its previous state

117. Consider the logic circuit shown below :



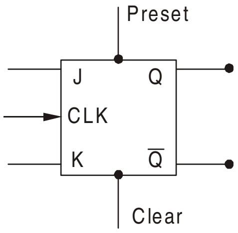
The logic circuit operates as a
(*a*) 4-bit asynchronous counter
(*b*) 4-bit synchronous counter
(*c*) BCD counter
(*d*) Serial-In Serial-Out shift register

118. The logic circuit shown below is a 3-bit



(*a*) shift register
(*b*) asychronous binary up counter
(*c*) asynchronous binary down-counter
(*d*) synchronous binary up counter

119. The inputs of the J-K flip-flop, shown below are :
PRESET = CLEAR = 1; J = K = 0



If a single clock pulse is applied, then device will
(*a*) toggle (*b*) set
(*c*) reset (*d*) not change states

120. Popular application of flip-flop are
(*a*) counters (*b*) shift registers
(*c*) transfer register (*d*) all of these

121. Register is a
(*a*) set of capacitor used to register input instructions in a digital computer
(*b*) set to paper tapes and cards put in a file
(*c*) temporary storage unit within the CPU having dedicated or general purpose use
(*d*) part of the auxiliary memory

122. For which of the following flip-flops, the output is clearly defined for all combinations of two inputs ?
(*a*) P type flip-flop (*b*) R-S flip-flop
(*c*) J-K flip-flop (*d*) T flip-flop

123. It is difficult to design asynhronous sequential circuit because
(*a*) external clock is to be provided
(*b*) it is more complex
(*c*) memory required is enormous
(*d*) generally they involve stability problem

124. The main difference between JK and RS flip-flop is that
(*a*) JK flip does not need a clock pulse
(*b*) there is a feedback in JK flip-flop
(*c*) JK flip-flop accepts both inputs as 1
(*d*) JK flip-flop is acronym of Junction cathode multivibrator

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125. If a clock with time period 'T' is used with n stage shift register, then output of final stage will be delayed by

(*a*) nT sec (*b*) $(n-1)$T sec

(*c*) n/T sec (*d*) $(2n-1)$T sec

126. Which of the following unit will choose to transform decimal number to binary code ?

(*a*) Encoder (*b*) Decoder

(*c*) Multiplexer (*d*) Demultiplexer

127. The ring counter is analogous to

(*a*) toggle switch (*b*) latch

(*c*) stepping switch (*d*) S-R flip-flop

128. If the input J is connected through K input of J-K, then flip-flop will behave as a

(*a*) D type flip-flop (*b*) T type flip-flop

(*c*) S-R flip-flop (*d*) Toggle switch

129. The clear data and present input of the JK flip-flop are known as

(*a*) synchronous inputs

(*b*) directed inputs

(*c*) either (*a*) or (*b*)

(*d*) indirect inputs

130. Which of the following flip-flop is free from race-around problem ?

(*a*) Q flip-flop

(*b*) T flip-flop

(*c*) SR flip-flop

(*d*) Master-slave JK flip-flop

131. In a digital counter circuit feedback loop is introduced to

(*a*) improve distortion

(*b*) improve stability

(*c*) reduce the number of input pulses to reset the counter

(*d*) synchronous input and output pulses

132. The flip-flops which operate in synchronism with external clock pulses are known as

(*a*) synchronous flip-flop

(*b*) asynchronous flip-flop

(*c*) either of the above

(*d*) none of these

133. A stable multivibrator are used as

(*a*) comparator circuit

(*b*) squaring circuit

(*c*) frequency to voltage converter

(*d*) voltage to frequency converter

134. When a large number of analog signals are to be converted an analog multiplexer is used. In this case most suitable A.D. converter will be

(*a*) up down counter type

(*b*) dual stop type

(*c*) forward counter type

(*d*) successive approximation type

135. The astable multivibrator has

(*a*) two quasi stable states

(*b*) two stable states

(*c*) one stable and one quasi-stable state

(*d*) none of these

136. A J-K flip-flop has its J-input connected to logic level 1 and its input to the Q output pulse is fed to its clock input the flip-flop will now

(*a*) change its state at each clock pulse

(*b*) go to state 1 and stay there

(*c*) go to state 0 and stay there

(*d*) retain its previous state

137. How many illegitimate states has synchronous mod-6 counter ?

(*a*) 3 (*b*) 2

(*c*) 1 (*d*) 0

138. A pulse train can be delayed by a finite number of clock periods using

(*a*) a serial-in serial-out shift register

(*b*) a serial-in parallel-out shift register

(*c*) a parallel-in serial-out shift register

(*d*) a parallel-in parallel-out shift register

139. A 2 bit binary multiplier can be implemented using

(*a*) 2 input ANDs only

(*b*) 2 input XORs and 4 input AND gates only

(*c*) 2 input NORs and one XNOR gate

(*d*) XOR gates and shift registers

140. If in a shift resistor Q_0 is fed back to input the resulting counter is

(*a*) Twisted ring with N : 1 scale

(*b*) Ring counter with N : 1 scale

(*c*) Twisted ring with 2N : 1 scale

(*d*) Ring counter with 2 N : 1 scale

141. A ring counter same as

(*a*) up-down counter

(*b*) parallel-counter

(*c*) shift register

(*d*) none of these

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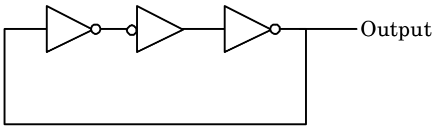
142. The dynamic hazard problem occurs in
(*a*) combinational circuit alone
(*b*) sequential circuit only
(*c*) both (*a*) and (*b*)
(*d*) none of these

143. Which of the following conditions must be met to avoid race around problem ?
(*a*) $\Delta t < t_p < T$ (*b*) $T > \Delta t > t_p$
(*c*) $2 t_p < \Delta t < T$ (*d*) none of these

144. A n-stage ripple counter will count upto
(*a*) 2^n (*b*) $2^n - 1$
(*c*) n (*d*) 2^{n-1}

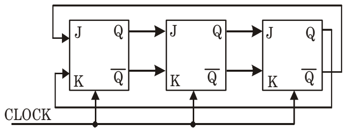
145. The circuit shown in the figure given below



(*a*) is an oscillating circuit and its output is a square wave
(*b*) is one whose output remains stable in '1' state
(*c*) is one having output semains stable '0' state
(*d*) having a single pulse of 3 times propagation delay

146. A mod-2 counter followed by a mod-5 counter is
(*a*) same as a mode-5 counter followed by a mod-2 counter
(*b*) a decade counter
(*c*) a mod-7 counter
(*d*) none of these

147. For the initial state of 000, the function performed by the arrangement of the J-K flip-flop in the given figure is



(*a*) Shift Register (*b*) Mod-3 Counter
(*c*) Mod-6 Counter (*d*) Mod-2 Counter

148. Match List I with List II and select the correct answer form the codes given below the list.

List I	List II
A. A shift register can be used	1. for code conversion
B. A multiplexer can be used	2. to generate memory chip select
C. A decoder can be used	3. for parallel to serial conversion
	4. as many to one switch
	5. for analog to digital conversion

Codes:

	A	B	C
(*a*)	3	1	2
(*b*)	4	3	2
(*c*)	3	4	2
(*d*)	2	3	4

149. Sign extension is a step in
(*a*) floating point multiplication
(*b*) signed 16 bit integer addition
(*c*) arithmetic left shift
(*d*) converting a signed integer from one size to another

150. The 2's complement representation of $(-539)_{10}$ in hexadecimal is
(*a*) ABE (*b*) DBC
(*c*) DE5 (*d*) 9E7

151. Consider excess-3 code that is used to represent integers 0 through 9 as shown below:

Number	Code (abcd)
0	1100
1	0010
2	1010
3	0110
4	1110
5	0001
6	1001
7	0101
8	1101
9	0011

Which of the following expressions is the correct one for an invalid code ?
(*a*) $b' \cdot c' \cdot d' + c \cdot d$
(*b*) $b' \cdot c' \cdot d' + a \cdot c \cdot d$
(*c*) $b' \cdot c' \cdot d' + b \cdot c \cdot d + a \cdot c \cdot d + b' \cdot c' \cdot d'$
(*d*) $b' \cdot c' \cdot d' + b \cdot c \cdot d + a \cdot c \cdot d$

152. A logic circuit which is used to change a BCD number into an equivalent decimal number is
(*a*) decoder
(*b*) encoder
(*c*) multiplexer
(*d*) code converter

153. The weight which makes the complement operation easier in BCD form is
(*a*) 8-4-2-1 (*b*) Excess-3
(*c*) 2-4-2-1 (*d*) 3-2-1-0

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154. In 2's complement representation, a certain negative number – N is 1011. The representation of + N is

(*a*) 0100 (*b*) 0101

(*c*) 0110 (*d*) 0011

155. The expression for sum of A, B in the half adder is given by

(*a*) AB (*b*) A + B

(*c*) $A \oplus B$ (*d*) none of these

156. Booth's coding in 8 bits for the decimal number – 57 is

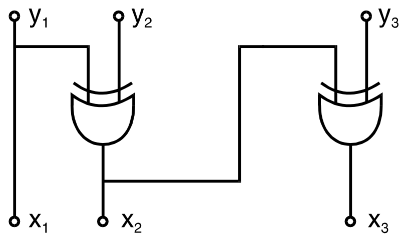
(*a*) 0 – 100 + 1000

(*b*) 0 – 100 + 100 – 1

(*c*) 0 – 1 + 100 – 10 + 1

(*d*) 0 0 – 10 + 100 – 1

157. The logic circuit given below converts a binary code y_1, y_2, y_3 into



(*a*) Excess-3 code (*b*) Gray code

(*c*) BCD code (*d*) Hamming code

158. In 2's complement addition, overflow

(*a*) is flagged whenever there is carry from sign bit addition

(*b*) cannot occur when a positive value is added to a negative value

(*c*) is flagged when the carries from sign bit and previous bit match

(*d*) none of these

159. A circuit produces 1's complement of the input word one application is binary subtraction. It is called

(*a*) logic gate

(*b*) register

(*c*) multiplexes

(*d*) controlled inverter

160. Decimal number 5 in level parity self correcting code is

(*a*) 10000 (*b*) 01011

(*c*) 01100 (*d*) 00101

161. Which of the following 4 bit combination are invalid in the BCD code ?

(*a*) 0010 (*b*) 0101

(*c*) 1000 (*d*) 1010

162. Which of the following is true about BCD ?

(*a*) It is a 8–4–2–1 weighted code

(*b*) $(1234567)_{10}$ needs 4 bytes in BCD representation

(*c*) Conversion to and from the decimal system can be done easily

(*d*) All of these

163. Binary coded decimal (BCD) numbers express each digit as a

(*a*) byte (*b*) nibble

(*c*) bit (*d*) all of these

164. The binary equivalent of the octal Numbers 13.54 is

(*a*) 1011.1011 (*b*) 1101.1110

(*c*) 1001.1110 (*d*) all of these

165. The binary equivalent of the hexadecimal number 7BD is

(*a*) 01111 0111101 (*b*) 111010111101

(*c*) 1011101111101 (*d*) all of these

166. The range of the numbers which can be stored in an eight bit register is

(*a*) – 128 to + 127

(*b*) – 128 to + 128

(*c*) – 999999 + + 999999

(*d*) none of these

167. The most commonly used character codes for transmission is

(*a*) EBCDIC (*b*) ASCII

(*c*) both (*d*) none of these

168. The decimal number 80 can be represented in BCD code as

(*a*) 1000 0001 (*b*) 0101 0000

(*c*) 0010 0000 (*d*) 10000000

169. The range for 13-bits complement numbers, expressed in decimal, is from

(*a*) – 127 to + 128 (*b*) – 127 to + 127

(*c*) – 128 to + 128 (*d*) – 128 to +127

170. In binary system decimal 0.875 is represented by

(*a*) 0.001 (*b*) 0.0101

(*c*) 0.011 (*d*) 0.111

171. Which code is a weighted code

(*a*) Gray (*b*) Excess-3

(*c*) Shift counter (*d*) 5211

172. Two's complement can be obtained from one's complement by

(*a*) adding 1

(*b*) subtracting 1

(*c*) putting 1 as the leading bit

(*d*) none of these

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173. When an old number is converted into the binary number, the least significant digit (LSD) is

(*a*) 0 (*b*) 1
(*c*) 0 or 1 (*d*) none of these

174. Even parity is being used for error checking computer data. If data word to be transmitted is 0101 0111, the value of the parity will be

(*a*) 0 (*b*) 1
(*c*) 1 or 0 (*d*) 01

175. Number 84 in BCD is

(*a*) 1000 0100 (*b*) 0100 0100
(*c*) 1000 101 (*d*) 1000 1100

176. What is the hexadecimal equivalent of binary number 1010 1111?

(*a*) AF (*b*) 9E
(*c*) 8C (*d*) None of these

177. What is the binary equivalent of the hexadecimal number B3?

(*a*) 1011 0001 (*b*) 1101 0011
(*c*) 1011 0011 (*d*) 1001 0101

178. The ASII code is

(*a*) numeric code
(*b*) same as BCD code
(*c*) an alphanumeric code
(*d*) seldom used

179. Which one of the following is the result of dividing $0011\ 1000_2$ by 1000_2?

(*a*) $(1100)_2$ (*b*) $(1110)_2$
(*c*) $(0011)_2$ (*d*) $(0111)_2$

180. The ASCII

(*a*) is a subset of 8-bit EBCDIC
(*b*) is used only in Western Countries
(*c*) is version II of the ASC standard
(*d*) has 128 character, including 32 control characters

181. A 12 bit binary number has an accuracy equivalent to the decimal fraction ?

(*a*) 1/1024 (*b*) 1/2048
(*c*) 1/4096 (*d*) 1/6400

182. (r-1)'s complement of the (r-1)'s comple-ment of an integer number is

(*a*) origianl number
(*b*) (r-2)'s complement of the number
(*c*) r's compliment
(*d*) none of these

183. The result of the addition $37_{16} + 19_{16}$ is

(*a*) 56_{16} (*b*) 50_{16}
(*c*) 59_{16} (*d*) none of these

184. The result of the addition $A0_{16} + 6B_{16}$ is

(*a*) $10\ B_{16}$ (*b*) $16\ B_{16}$
(*c*) AOB_{16} (*d*) $A6B_{16}$

185. The result of the subtraction $FD_{16} - 88_{16}$ is

(*a*) 75_{16} (*b*) 65_{16}
(*c*) $5E_{16}$ (*d*) 10_{16}

186. The **1**'s complements of the binary number 1101101 is

(*a*) 0000010 (*b*) 0010010
(*c*) 0010011 (*d*) 1101110

187. The 2's complements of binary number 010111.1100 is

(*a*) 101001.1100 (*b*) 101000.0100
(*c*) 010111.0011 (*d*) 101000.0011

188. The sign magnitude representation of binary number + 1101.011 is

(*a*) 01101.011 (*b*) 11101.011
(*c*) 00010.100 (*d*) 10010.100

189. The sign magnitude representation of binary - 1101.011 is

(*a*) 01101.011 (*b*) 10010.100
(*c*) 11101.011 (*d*) 10010.101

190. The excess 3 code is also called

(*a*) cyclic reundancy code
(*b*) weighted code
(*c*) self complimenting code
(*d*) algebraic code

191. Which of the following is an unweighted code?

(*a*) 8421 code (*b*) excess 3 code
(*c*) 2421 code (*d*) 5211 code

192. What is the 2's complement representation of – 5/8?

(*a*) 0.1010 (*b*) 1.0010
(*c*) –0.1101 (*d*) 0.0110

193. What is the equivalent of $(10110011100011110000)_2$ in base 32?

(*a*) 11 9 23 31 (*b*) 22 14 7 16
(*c*) 11 9 7 16 (*d*) 11 14 23 16

194. If negative numbers are stored in 2's complement form, then range of numbers that can be stored in 8 bits, is

(*a*) – 127 to + 127
(*b*) – 127 to + 128
(*c*) – 128 to + 127
(*d*) – 128 to + 128

195. What is the binary equivalent of Gray code 11100?

(*a*) 01011 (*b*) 10101
(*c*) 00111 (*d*) 10111

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196. The hexadecimal number A 23 F is represented in binary is

(*a*) 1010 0010 0011 1110

(*b*) 1010 0100 0011 1111

(*c*) 1010 0010 0011 1111

(*d*) 1111 0011 0010 1010

197. Let $a_n\ a_{n-1}$ ...., $a_1\ a_0$ be the binary representation of an integer b. Then b is divisible by 3 if

(*a*) number 1's of is divisible by 3

(*b*) number 0's of is divisible by 3

(*c*) number 1's of is divisible by 6

(*a*) difference of alternate sum, i.e. $(a_0 + a_2 + ...) - (a_1 + a_3 + ...)$ is divisible by 3

198. Which of the following binary numbers is the same as its 2's complement?

(*a*) 1010 (*b*) 0101

(*c*) 1000 (*d*) 1001

199. Which of the following binary equals its **1**'s complement ?

(*a*) 1010

(*b*) 1001

(*c*) No such number exists

(*d*) None of these

200. Which of the following is the binary representation of the hexadecimal number 3B7F?

(*a*) 0100 1001 1110 1101

(*b*) 0110 0011 1011 1100

(*c*) 0011 1011 0111 1111

(*d*) 0010 0100 0000 1010

201. Each computer can represent a limited number of real numbers exactly in any floating-point representation. The Alpha computer represents floating-point numbers in normalized, base 2 format shown below :

S	Maintssa		Exponent	
0	1	7	8	1 1

The exponent is biased (excess- 8). What is the approximate relative error of representing the decimal value +0.4 on the Alpha Computer?

(*a*) 0.5% (*b*) 2.2%

(*c*) 3.8% (*d*) 5.1%

202. A computer with a 32-bit word size uses 2's complement to represent numbers. The range of integers that can be represented by this computer is

(*a*) -2^{32} to 2^{32} (*b*) -2^{32} to 2^{31}

(*c*) -2^{31} to $2^{31}-1$ (*d*) -2^{32} to 2^{31}

NUMERICAL TYPE QUESTIONS

1. The number of canonical expressions that can be developed over a 3-valued boolean algebra is_____

2. If set $x = \{a, b, c, d\}$, then number of binary operations that can be defined on x is _____

3. How many boolean functions of three variables $f(x, y, z)$ have the property that

$$f(x, y, z) = (f(x, y, z))\ ?$$

4. The minimum number of NAND gates required to implement the Boolean function. $A + A\overline{B} + A\overline{B}C$ is equal to _____

5. How many lines the truth table for a four-input NOR gate would contain to cover all possible input combinations ?

6. The largest number of data inputs which a data selector with two control inputs can have is _______

7. The number of 2-input multiplexers required to construct a 2^{10} input multiplexer, is ______

8. The minimum number of 2–input NAND gates requird to implement the function $F = (x' + y')(z + w)$ is ________

9. A one-to-four line demultiplexer is to be implemented using a memory. The number of bits each word has is _______

10. If a logic gates has four inputs, then total number of possible input combinations is _____

11. Assuming that only the X and Y logic inputs are available and their complements $\overline{X}$ and $\overline{Y}$ are not available, what is the minimum number of two-input NAND gates require to implement $X \oplus Y$?

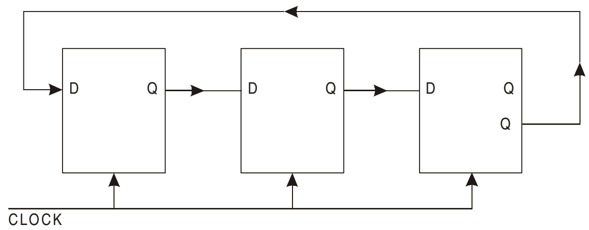
12. A sequential circuit outputs a ONE when an even number (> 0) of one's are input; otherwise the output is ZERO. The minimum number of states required is _________

13. The ________ number of bits are required to encode all twenty six letters, ten symbols, and ten numerals.

14. The ________ number of flip-flop are needed to divide the input frequency by 64

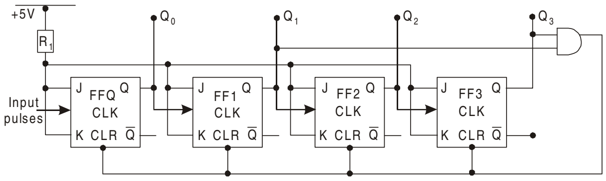
15. The number of flip-flops required in a decade counter is _______

16. Consider the counter shown below :



The modulo (number of pressing states) of this counter is ______

17. Consider the figure given below :



Initially all flip-flops are cleared. How many clock pulses have to be applied to the system before the output from FF3 becomes a HIGH level ?

18. A 2 MHz signal is applied to the input of a J-K flip-flop which is operating in the '*toggle*' mode. The frequency of the signal at the output will be ______ MHz

19. The number of clock pulses needed to shift one byte of data from input to the output of a 4-bit shift register is ______

20. The _______ number of minimum are needed for a 3-bit up-counter using standard binary and using T flip-flops Assume unlimited fan-in.

21. To build a mod-19 counter the number of flip-flop required is ______

22. With the use of an electronic counter six capsules are to be filled in bottles automatically. In such a counter _______number of flip-flops are required

23. A decimal number has 25 digits. The number of bits needed for its equivalent binary representation is, approximately ______

24. The octal equivalent of 111 010 is ______

25. The decimal equivalent of binary number 0.0111 is _______

26. The result of truncating the decimal number – 22.56 is _______

27. The _______ different number a 6-bit binary word can represent

28. The ______ number of fractional digits 2^{-48} will have

29. The decimal equivalent of hexa decimal number 566 is ________

30. The octal equivalent of decimal 324.987 is_____

31. The 10's complement of $(715)_8$ is_______

32. The 9's complements of 381 is _______

33. The sign magnitude representative of decimal number –637 is _______

34. If 12A $7C_{16} = X_8$, then value of x is______

35. The ______ number of 1's are present in the binary representation of $15 \times 256 + 5 \times 16 + 3$

36. Consider a number system with radix 20. The excess code that is necessary or its equivalent binary coded representation is ___________

37. If $(yx\ 567)_8 + (12yx\ 5)_8 = (x\ 71\ yx)_8$, then values of x and y are ________

38. A decimal number has 30 digits. Approximately, _______ number of digits would the binary representation have

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. Consider a 4-bit Johnson counter with an initial value of 0000. The counting sequence of this counter is

(*a*) 0, 1, 3, 7, 15, 14, 12, 8, 0

(*b*) 0, 1, 3, 5, 7, 9, 11, 13, 15, 0

(*c*) 0, 2, 4, 6, 8, 10, 12, 14, 0

(*d*) 0, 8, 12, 14, 15, 7, 3, 1, 0

2. The binary operator ≠ is defined by the following truth table.

p	q	p ≠ q
0	0	0
0	1	1
1	0	1
1	1	0

Which one of the following is true about the binary operator ≠ ?

(*a*) Both commutative and associative

(*b*) Commutative but not associative

(*c*) Not commutative but associative

(*d*) Neither commutative nor associative

3. Consider the operations

f(X, Y, Z) = X′ Y Z + X Y′ + Y′Z′ and

g(X, Y, Z) = X′ YZ + X′Y Z′ + XY.

Which one of the following is correct ?

(*a*) Both {*f*} and {*g*} are functionally complete

(*b*) Only {*f*} is functionally complete

(*c*) Only {*g*} is functionally complete

(*d*) Neither {*f*} nor {*g*} is functionally complete

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4. Consider a CSMA/CD network that transmits data at a rate of 100 Mbps (10^8 bits second) over a 1 km(kilometer) cable with no repeaters. If the minimum frame size required for this network is 1250 bytes, what is the signal speed (km/sec) in the cable ?

(*a*) 8000 (*b*) 10000

(*c*) 16000 (*d*) 20000

5. Given the function F = P′ + QR, where F is a function in three Boolean variables P, Q and R and P′ = !P, consider the following statements

(S1)F $= \Sigma(4,5,6)$

(S2)F $= \Sigma(0,1,2,3,7)$

(S3)F $= \Pi(4,5,6)$

(S4)F $= \Pi(0,1,2,37)$

Which of the following is true ?

(*a*) (S1)-False, (S2)-True, (S3)-True, (S4)-False

(*b*) (S1)-True, (S2)-False, (S3)-False, (S4)-True

(*c*) (S1)-False, (S2)-False, (S3)-True, (S4) True

(*d*) (S1)-False-True, (S2) True, (S3)-False, (S4)-False

2014

6. Consider the following Boolean expression for F:

F(P, Q, R, S) = $PQ + \bar{P}QR + \bar{P}Q\bar{R}S$

The minimal sum-of products form of F is

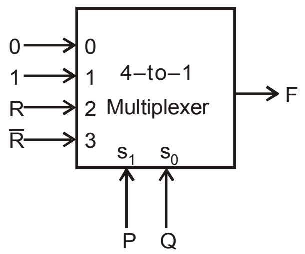
(*a*) PQ + QR + QS

(*b*) P + Q + R + S

(*c*) $\bar{P}+\bar{Q}+\bar{R}+\bar{S}$

(*d*) $\bar{P}R+\bar{P}\bar{R}S+P$

7. Consider the 4-to-1 multiplexer with two select lines S_1 and S_0 given below.



The minimal sum of-products form of the Boolean expression for the output F of the multiplexer is

(*a*) $\bar{P}Q+Q\bar{R}+P\bar{Q}R$

(*b*) $\bar{P}Q+\bar{P}Q\bar{R}+PQ\bar{R}+P\bar{Q}R$

(*c*) $\bar{P}QR+\bar{P}Q\bar{R}+Q\bar{R}+P\bar{Q}R$

(*d*) $PQ\bar{R}$

8. The dual of a Boolean function F($x_1, x_2, \ldots, x_n$, +, · , ′), written as F^D, is the same expression as that of F with + and · swapped. F is said to be self-dual if F = F^D. The number of self-dual functions with n Boolean variables is

(*a*) 2^n (*b*) 2^{n-1}

(*c*) 2^{2^n} (*d*) $2^{2^{n-1}}$

9. Let $k = 2^n$. A circuit is built by giving the output of an n-bit binary counter as input to an n-to-2^n bit decoder. This circuit is equivalent to a

(*a*) k-bit binary up counter

(*b*) k-bit binary down counter

(*c*) k-bit ring counter

(*d*) k-bit Johnson counter

10. Consider the following minterm expression for F.

$$F(P,Q,R,S) = \sum 0, 2, 5, 7, 8, 10, 13, 15$$

The minterms 2, 7, 8 and 13 are 'do not care' terms. The minimal sum of-products form for F is

(*a*) $Q\bar{S}+\bar{Q}S$

(*b*) $\bar{Q}\bar{S}+QS$

(*c*) $\bar{Q}\bar{R}\bar{S}+\bar{Q}R\bar{S}+Q\bar{R}S+QRS$

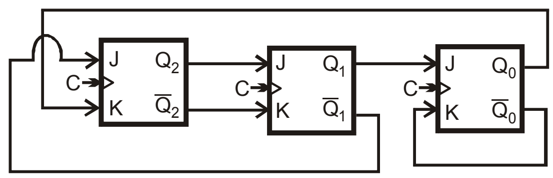
(*d*) $\bar{P}\bar{Q}\bar{S}+\bar{P}QS+PQS+P\bar{Q}\bar{S}$

11. What is the optimized version of the relation with respect to algebric equation $\pi_{A1}(\pi_{A2}(\sigma_{F1}(\sigma_{F2}(r))))$, where A1, A2 are sets of attributes in r with A1 $\subset$ A2 and F1, F2 are Boolean expressions based on the attributes in r?

(*a*) $\pi_{A1}(\sigma_{(F1 \wedge F2)}(r))$ (*b*) $\pi_{A1}(\sigma_{(F1 \vee F2)}(r))$

(*c*) $\pi_{A2}(\sigma_{(F1 \wedge F2)}(r))$ (*d*) $\pi_{A2}(\sigma_{(F1 \vee F2)}(r))$

12.



The above synchronous sequential circuit built using JK flip-flops is initialized with $Q_2Q_1Q_0 = 000$. The state sequence for this circuit for the next 3 clock cycles is

(*a*) 001, 010, 011

(*b*) 111, 110, 101

(*c*) 100, 110, 111

(*d*) 100, 011, 001

13. Let $\oplus$ denote the Exclusive OR (XOR) operation. Let '1' and '0' denote the binary constants. Consider the following Boolean expression for F over two variables P and Q.

$$F(P, Q) = ((1 \oplus P) \oplus (P \oplus Q)) \oplus ((P \oplus Q) \oplus (Q \oplus 0))$$

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The equivalent expression for F is

(*a*) P + Q

(*b*) $\overline{P+Q}$

(*c*) $P \oplus Q$

(*d*) $\overline{P \oplus Q}$

2013

14. The smallest integer that can be represented by an 8-bit number in 2's complement form is

(*a*) –256 (*b*) –128

(*c*) –127 (*d*) 0

15. In the following truth table, V = 1 if and only if the input is valid.

Inputs				Outputs		
D_0	D_1	D_2	D_3	X_0	X_1	V
0	0	0	0	x	x	0
1	0	0	0	0	0	1
x	1	0	0	0	1	1
x	x	1	0	1	0	1
x	x	x	1	1	1	1

What function does the truth table represent?

(*a*) Priority encoder

(*b*) Decoder

(*c*) Multiplexer

(*d*) Demultiplexer.

16. Which one of the following expressions does NOT represent exclusive NOR of x and y?

(*a*) $xy + x'y'$ (*b*) $x \oplus y'$

(*c*) $x' \oplus y$ (*d*) $x' \oplus y'$

17. A RAM chip has a capacity of 1024 words of 8 bits each (1K × 8). The number of 2 × 4 decoders with enable line needed to construct a 16K × 16 RAM from 1K × 8 RAM is

(*a*) 4 (*b*) 5

(*c*) 6 (*d*) 7

2012

18. The decimal value 0.5 in IEEE single precision floating point representation has

(*a*) fraction bits of 000...000 and exponent value of 0

(*b*) fraction bits of 000...000 and exponent value of –1.

(*c*) fraction bits of 100...000 and exponent value of 0

(*d*) no exact representation.

19. The truth table

X	Y	f(X,Y)
0	0	0
0	1	0
1	0	1
1	1	1

represents the Boolean function

(*a*) X (*b*) X + Y

(*c*) $X \oplus Y$ (*d*) Y

20. Suppose a circular queue of capacity $(n - 1)$ elements is implemented with an array of n elements. Assume that the insertion and deletion operations are carried out using REAR and FRONT as array index variables, respectively. Initially, REAR = FRONT = 0. The conditions to detect *queue full and queue empty* are

(*a*) *full:* (REAR + 1) mod n = = FRONT
empty: REAR = = FRONT

(*b*) *full*: (REAR + 1) mod n = = FRONT
empty: (FRONT + 1) mod n = = REAR

(*c*) *full*: REAR = = FRONT
empty: (REAR + 1) mod n = = FRONT

(*d*) *full*: (FRONT + 1) mod n = = REAR
empty: REAR = = FRONT

21. What is the minimal form of the Karnaugh map shown below? Assume that X denotes a don't care term.

cd \ ab	00	01	11	10
00	1	X	X	1
01	X			1
11				
10	1			X

(*a*) $\bar{b}\bar{d}$

(*b*) $\bar{b}\bar{d} + \bar{b}\bar{c}$

(*c*) $\bar{b}\bar{d} + a\bar{b}\bar{c}d$

(*d*) $\bar{b}\bar{d} + \bar{b}\bar{c} + \bar{c}\bar{d}$

2011

22. The minimum number of D flip-flops needed design and a mod-258 counter is

(*a*) 9 (*b*) 8

(*c*) 512 (*d*) 258

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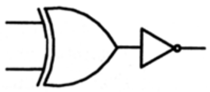
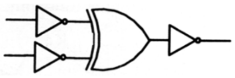
23. The simplified SOP (Sum of Product) form of the Boolean expression

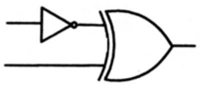
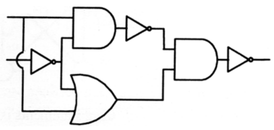
$(P+\overline{Q}+\overline{R}).(P+\overline{Q}+R).(P+Q+\overline{R})$ is

(*a*) $(\overline{P}.Q+\overline{R})$ (*b*) $(P+\overline{Q}.\overline{R})$

(*c*) $(\overline{P}.Q+R)$ (*d*) $(P.Q+R)$

24. Which one of the following circuits is NOT equivalent to a 2-input XNOR (exclusive NOR) gate?

(*a*)  (*b*) 

(*c*)  (*d*) 

2010

25. The minterm expansion of

$f(P, Q, R) = PQ + Q\overline{R} + P\overline{R}$ is

(*a*) $m_2 + m_4 + m_6 + m_7$

(*b*) $m_0 + m_1 + m_3 + m_5$

(*c*) $m_0 + m_1 + m_6 + m_7$

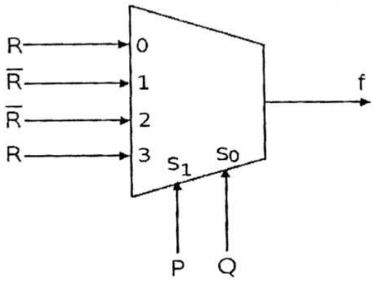
(*d*) $m_2 + m_3 + m_4 + m_5$

26. P is a 16-bit signed integer. The 2's complement representation of P is $(F87B)_{16}$. The 2's complement representation of 8*P is

(*a*) $(C3D8)_{16}$ (*b*) $(187B)_{16}$

(*c*) $(F878)_{16}$ (*d*) $(987B)_{16}$

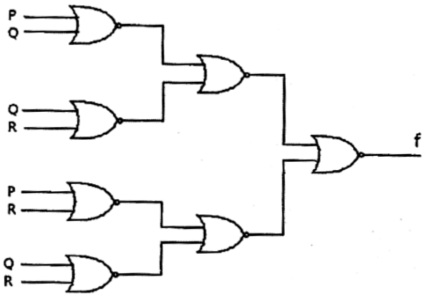
27. The Boolean expression for the output *f* of the multiplexer shown below is



(*a*) $\overline{P \oplus Q \oplus R}$ (*b*) $P \oplus Q \oplus R$

(*c*) $P + Q + R$ (*d*) $\overline{P + Q + R}$

28. What is the boolean expression for the output *f* of the combinational logic circuit of NOR gates given below?



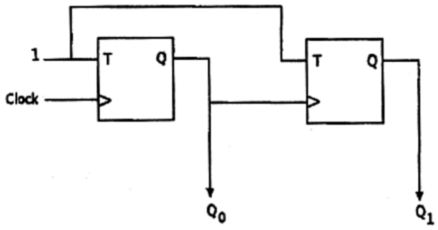
(*a*) $\overline{Q + R}$

(*b*) $\overline{P + Q}$

(*c*) $\overline{P + R}$

(*d*) $\overline{P + Q + R}$

29. In the sequential circuit shown below, if the initial value of the output Q_1Q_0 is 00, what are the next four values of Q_1Q_0?



(*a*) 11, 10, 01, 00 (*b*) 10, 11, 01, 00

(*c*) 10, 00, 01, 11 (*d*) 11, 10, 00, 01

2009

30. $(1217)_8$ is equivalent to

(*a*) $(1217)_{16}$ (*b*) $(028F)_{16}$

(*c*) $(2297)_{10}$ (*d*) $(0B17)_{16}$

31. What is the minimum number of gates required to implement the Boolean function (AB + C) if we have to use only 2-input NOR gates ?

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

2008

32. In the IEEE floating point representation the hexadecimal value 0x00000000 corresponds to the

(*a*) normalized value 2^{-127}

(*b*) normalized value 2^{-126}

(*c*) normalized value + 0

(*d*) special value + 0

33. In the Karnaugh map shown below, X denotes a don't care term. What is the minimal form of the function represented by the Karnaugh map?

(*a*) $\overline{b}.\overline{d} + \overline{a}.\overline{d}$ (*b*) $\overline{a}.\overline{b} + \overline{b}.\overline{d} + \overline{a}.b.\overline{d}$

(*c*) $\overline{b}.\overline{d} + \overline{a}.b.\overline{d}$ (*d*) $\overline{a}.\overline{b} + \overline{b}.\overline{d} + \overline{a}.\overline{d}$

cd \ ab	00	01	11	10
00	1	1		1
01	X			
11	X			
10	1	1		X

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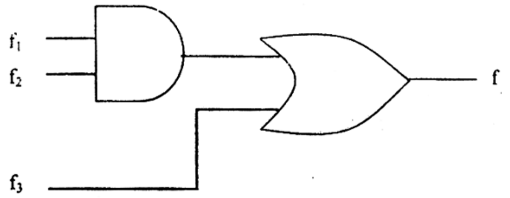
34. Let r denote number system radix. The only value(s) of r that satisfy the equation

$\sqrt{121_r} = 11_r$ is/are

(*a*) decimal 10 (*b*) decimal 11

(*c*) decimal 10 and 11 (*d*) any value > 2

35. Given f_1, f_3 and f in canonical sum of products form (in decimal) for the circuit



$f_1 = \Sigma$ m (4, 5, 6, 7, 8)

$f_3 = \Sigma$ m (1, 6, 15)

f = Σ m (1, 6, 8, 15)

Then f_2 is

(*a*) Σ m (4, 6) (*b*) Σ m (4, 8)

(*c*) Σ m (6, 8) (*d*) Σ m (4, 6, 8)

36. If P, Q, R are Boolean variables, then

$$\left(P+\overline{Q}\right)\left(P.\overline{Q}+P.R\right)\left(\overline{P}.\overline{R}+\overline{Q}\right)$$

simplifies to

(*a*) $P.\overline{Q}$ (*b*) $P.\overline{R}$

(*c*) $P.\overline{Q}+R$ (*d*) $P.\overline{R}+Q$

2007

37. What is the maximum number of different Boolean functions involving *n* Boolean variables ?

(*a*) n^2 (*b*) 2^n

(*c*) 2^{2^n} (*d*) 2^{n^2}

38. How many 3-to-8 line decoders with an enable input are needed to construct a 6-to-64 line decoder without using any other logic gates ?

(*a*) 7 (*b*) 8

(*c*) 9 (*d*) 10

39. Consider the following Boolean function of four variables

$$f(w, x, y, z) = \sum(1, 3, 4, 6, 9, 11, 12, 14)$$

The function is

(*a*) independent of one variable

(*b*) independent of two variables

(*c*) independent of three variables

(*d*) dependent on all the variables

40. Let $f(w, x, y, z) = \sum(0,4,5,7,8,9,13,15)$.

Which of the following expressions are **NOT** equivalent to *f* ?

(*a*) $x'y'z' + w'xy' + wy'z + xz$

(*b*) $w'y'z' + wx'y' + xz$

(*c*) $w'y'z' + wx'y' + xyz + xy'z$

(*d*) $x'y'z' + wx'y' + w'y$

41. Define the connective * for the Boolean variable *X* and Y as :

$$X^* Y = XY + X'Y'$$

Let $Z = X * Z$.

Consider the following expression P, Q and R.

$P : X = Y * Z Q : \quad Y = X * Z$

$R : X * Y * Z = 1$

Which of the following is **TURE ?**

(*a*) Only *P* and *Q* are valid

(*b*) Only *Q* and *R* are valid

(*c*) Only *P* and *R* are valid

(*d*) All *P*, *Q*, *R* are valid

42. Suppose only one multiplexer and one inverter are allowed to be used to implement any Boolean function of *n* variables. What is the minimum size of the multiplexer needed?

(*a*) 2^n line to 1 line

(*b*) 2^{n+1} line to 1 line

(*c*) 2^{n-1} line to 1 line

(*d*) 2^{n-2} line to 1 line

43. In a look-ahead carry generator, the carry generate function G_i and the carry propagate function P_i for inputs, A_i and B_i are given by :

$$P_i = A_i \oplus B_i \text{ and } G_i = A_i B_i$$

The expressions for the sum bit S_i and the carry bit C_{i+1} of the look-ahead carry adder are given by:

$$S_i = P_i \oplus C_i \text{ and } C_{i+1} = G_i + P_i C_i,$$

where C_0 is input carry.

Consider a two-level logic implementation of the look-ahead carry generator. Assume that all P_i and G_i are available for the carry generator circuit and that the AND and OR gates can have any number of inputs. The number of AND gates and OR gates needed to implement the look-ahead carry generator for a 4-bit adder with S_3, S_2, S_1, S_0 and C_4 as its outputs are respectively

(*a*) 6, 3 (*b*) 10, 4

(*c*) 6, 4 (*d*) 10, 5

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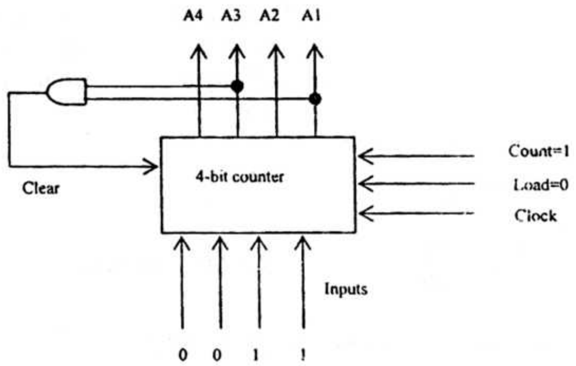
44. The control signal functions of a 4-bit binary counter are given below (where X is "don't care"):

Clear	Clock	Load	Count	Function
1	X	X	X	Clear to 0
0	X	0	0	No change
0		1	X	Load input
0		0	1	Count next

The counter is connected as follows :



Assume that the counter and gate delays are negligible. If the counter starts at 0, then it cycles through the following sequence

(*a*) 0, 3, 4

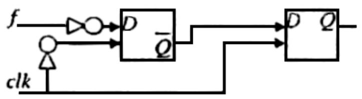
(*b*) 0, 3, 4, 5

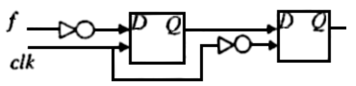
(*c*) 0, 1, 2, 3, 4

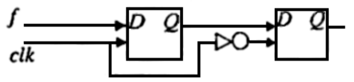
(*d*) 0, 1, 2, 3, 4, 5

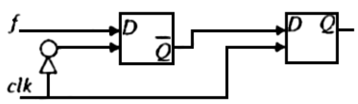
2006

45. You are given a free running clock with a duty cycle of 50% and a digital waveform f which changes only at the negative edge of the clock. Which one of the following circuits (using clocked D flip-flops) will delay the phase of f by 180° ?

(*a*) 

(*b*) 

(*c*) 

(*d*) 

46. Given two three bit numbers $a_2a_1a_0$ and $b_2b_1b_0$ and c, the carry in, the function that represents the carry generate function when these two numbers are added is

(*a*) $a_2b_2 + a_2a_1b_1 + a_2a_1a_0b_0 + a_2a_0b_1b_0 + a_1b_2b_1 + a_1a_0b_2b_0 + a_0b_2b_1b_0$

(*b*) $a_2b_2 + a_2b_1b_0 + a_2a_1b_1b_0 + a_1a_0b_2b_1 + a_1a_0b_2 + a_1a_0b_2b_0 + a_2a_0b_1b_0$

(*c*) $a_2 + b_2 + (a_2 \oplus b_2)\,(a_1 + b_1 + (a_1 \oplus b_1)\,(a_0 + b_0))$

(*d*) $a_2b_2 + \overline{a_2}\, a_1b_1 + \overline{a_2\, a_1}\, a_0b_0 + \overline{a_2}\, a_0\, \overline{b_1}\, b_0 + a_1\, \overline{b_2}\, b_1 + \overline{a_1}\, a_0\, \overline{b_2}\, b_0 + a_0\,\overline{b_2\, b_1}\, b_0$

47. Consider a boolean function $f(w, x, y, z)$. Suppose that exactly one of its inputs is allowed to change at a time.

If function happens to be true for two input vectors $i_1 = < w_1, x_1, y_1, z_1 >$ and $i_2 = < w_2, x_2, y_2, z_2 >$, we would like the function to remain true as the input changes from i_1 to i_2 (i_1 and i_2 differ in exactly one bit position), without becoming false momentarily.

Let $f(w, x, y, z) = \sum(5, 7, 11, 12, 13, 15)$.

Which of the following cube covers of f will ensure that the required property is satisfied ?

(*a*) $\bar{w}xz,\ wx\bar{y},\ x\bar{y}z,\ xyz,\ wyz$

(*b*) $wxy,\ \bar{w}xz,\ wyz$

(*c*) $wx\overline{yz},\ xz,\ w\bar{x}yz$

(*d*) $wzy,\ wyz,\ wxz,\ \bar{w}xz,\ x\bar{y}z,\ xyz$

48. We consider the addition of two 2's complement numbers $b_{n-1}\, b_{n-2} \ldots. b_0$ and $a_{n-1}\, a_{n-2} \ldots.. a_0$. A binary adder for adding unsigned binary numbers is used to add the two numbers. The sum is denoted by $c_{n-1}\, c_{n-2} \ldots.\, c_0$ and the carry-out by c_{out}.

Which one of the following options correctly identifies the overflow condition ?

(*a*) $c_{out}\,\overline{\left(a_{n-1} \oplus b_{n-1}\right)}$

(*b*) $a_{n-1}\, b_{n-1}\, \overline{c_{n-1}} + \overline{a_{n-1}\, b_{n-1}}\, c_{n-1}$

(*c*) $c_{out} \oplus c_{n-1}$

(*d*) $a_{n-1} \oplus b_{n-1} \oplus c_{n-1}$

49. Consider numbers represented in 4-bit gray code. Let $h_3h_2h_1h_0$ be the gray code representation of a number n and let $g_3\, g_2\, g_1\, g_0$ be the gray code of $(n + 1)$ (modulo 16) value of the number. Which one of the following functions is correct ?

(*a*) $g_0\,(h_3h_2h_1h_0) = \sum(1, 2, 3, 6, 10, 13, 14, 15)$

(*b*) $g_1(h_3h_2h_1h_0) = \sum(4, 9, 10, 11, 12, 13, 14, 15)$

(*c*) $g_2\,(h_3\, h_2\, h_1\, h_0) = \sum(2, 4, 5, 6, 7, 12, 13, 15)$

(*d*) $g_3\,(h_3h_2h_1h_0) = \sum(0, 1, 6, 7, 10, 11, 12, 13)$

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NUMERICAL TYPE QUESTIONS

2015

1. The minimum number of JK flip-flops required to construct a synchronous counter with the count sequence (0, 0, 1, 1, 2, 2, 3, 3, 0, 0....) is ________.
2. The number of min-terms after minimizing the following Boolean expression is _______.

   $[D' + AB' + A'C + AC'D + A'C'D]'$
3. A half adder is implemented with XOR and AND gates. A full adder is implemented with two half adders and one OR gate. The propagation delay of an XOR gate is twice that of an AND/OR gate. The propagation delay of an AND/OR gate is 1.2 microseconds. A 4-bit ripple-carry binary adder is implemented by using four full adders. The total propagation time of this 4-bit binary adder in microseconds is ____________.
4. Suppose X_i for i = 1, 2, 3 are independent and identically distributed random variables whose probability mass functions are $Pr[X_i = 0] = Pr[X_i = 1] = 1/2$ for i =1, 2, 3. Define another random variable $Y = X_1X_2 \oplus X_3$, where $\oplus$ denotes XOR. Then $Pr[Y = 0 \mid X_3 = 0]$ = _____ .
5. The total number of prime implicates of the function

   $f(w, x, y, z) = \Sigma(0, 2, 4, 5, 6, 10)$ is _____ .
6. Consider the equation $(43)_x = (y3)_8$ where x and y are unknown. The number of possible solution is _______ .

2014

7. Consider the equation $(123)_5 = (x8)_y$ with x and y as unknown. The number of possible solutions is _______ .

ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*b*)	**2.** (*a*)	**3.** (*c*)	**4.** (*b*)	**5.** (*d*)	**6.** (*c*)	**7.** (*d*)	**8.** (*d*)	**9.** (*b*)	**10.** (*d*)
11. (*d*)	**12.** (*b*)	**13.** (*c*)	**14.** (*c*)	**15.** (*b*)	**16.** (*c*)	**17.** (*d*)	**18.** (*d*)	**19.** (*a*)	**20.** (*c*)
21. (*a*)	**22.** (*d*)	**23.** (*b*)	**24.** (*a*)	**25.** (*c*)	**26.** (*a*)	**27.** (*b*)	**28.** (*c*)	**29.** (*b*)	**30.** (*b*)
31. (*a*)	**32.** (*c*)	**33.** (*d*)	**34.** (*c*)	**35.** (*a*)	**36.** (*a*)	**37.** (*b*)	**38.** (*c*)	**39.** (*d*)	**40.** (*c*)
41. (*c*)	**42.** (*a*)	**43.** (*c*)	**44.** (*a*)	**45.** (*a*)	**46.** (*a*)	**47.** (*b*)	**48.** (*d*)	**49.** (*c*)	**50.** (*c*)
51. (*b*)	**52.** (*b*)	**53.** (*a*)	**54.** (*a*)	**55.** (*a*)	**56.** (*b*)	**57.** (*a*)	**58.** (*b*)	**59.** (*b*)	**60.** (*c*)
61. (*c*)	**62.** (*d*)	**63.** (*a*)	**64.** (*a*)	**65.** (*d*)	**66.** (*d*)	**67.** (*c*)	**68.** (*b*)	**69.** (*c*)	**70.** (*c*)
71. (*a*)	**72.** (*b*)	**73.** (*c*)	**74.** (*a*)	**75.** (*b*)	**76.** (*a*)	**77.** (*a*)	**78.** (*d*)	**79.** (*c*)	**80.** (*a*)
81. (*a*)	**82.** (*c*)	**83.** (*b*)	**84.** (*b*)	**85.** (*b*)	**86.** (*b*)	**87.** (*d*)	**88.** (*a*)	**89.** (*d*)	**90.** (*b*)
91. (*c*)	**92.** (*d*)	**93.** (*b*)	**94.** (*a*)	**95.** (*b*)	**96.** (*b*)	**97.** (*d*)	**98.** (*d*)	**99.** (*b*)	**100.** (*d*)
101. (*d*)	**102.** (*b*)	**103.** (*c*)	**104.** (*a*)	**105.** (*c*)	**106.** (*b*)	**107.** (*d*)	**108.** (*a*)	**109.** (*c*)	**110.** (*c*)
111. (*a*)	**112.** (*a*)	**113.** (*d*)	**114.** (*a*)	**115.** (*a*)	**116.** (*d*)	**117.** (*a*)	**118.** (*d*)	**119.** (*d*)	**120.** (*d*)
121. (*c*)	**122.** (*c*)	**123.** (*d*)	**124.** (*c*)	**125.** (*b*)	**126.** (*a*)	**127.** (*c*)	**128.** (*a*)	**129.** (*c*)	**130.** (*d*)
131. (*c*)	**132.** (*a*)	**133.** (*a*)	**134.** (*d*)	**135.** (*a*)	**136.** (*a*)	**137.** (*a*)	**138.** (*d*)	**139.** (*b*)	**140.** (*c*)
141. (*c*)	**142.** (*c*)	**143.** (*b*)	**144.** (*a*)	**145.** (*a*)	**146.** (*a*)	**147.** (*c*)	**148.** (*c*)	**149.** (*a*)	**150.** (*c*)
151. (*c*)	**152.** (*a*)	**153.** (*c*)	**154.** (*b*)	**155.** (*c*)	**156.** (*b*)	**157.** (*b*)	**158.** (*d*)	**159.** (*b*)	**160.** (*b*)
161. (*d*)	**162.** (*d*)	**163.** (*b*)	**164.** (*a*)	**165.** (*a*)	**166.** (*a*)	**167.** (*d*)	**168.** (*d*)	**169.** (*b*)	**170.** (*d*)
171. (*d*)	**172.** (*a*)	**173.** (*b*)	**174.** (*b*)	**175.** (*a*)	**176.** (*a*)	**177.** (*c*)	**178.** (*c*)	**179.** (*d*)	**180.** (*d*)
181. (*c*)	**182.** (*a*)	**183.** (*b*)	**184.** (*a*)	**185.** (*a*)	**186.** (*b*)	**187.** (*b*)	**188.** (*a*)	**189.** (*c*)	**190.** (*c*)
191. (*b*)	**192.** (*d*)	**193.** (*b*)	**194.** (*c*)	**195.** (*d*)	**196.** (*c*)	**197.** (*d*)	**198.** (*c*)	**199.** (*c*)	**200.** (*c*)
201. (*b*)	**202.** (*c*)								

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Numerical Type Questions

1. 64	**2.** 2^{16}	**3.** 6	**4.** 0	**5.** 16	**6.** 4	**7.** 1023
8. 4	**9.** 1	**10.** 16	**11.** 5	**12.** 2	**13.** 6	**14.** 6
15. 4	**16.** 6	**17.** 4	**18.** 8	**19.** 16	**20.** 2	**21.** 5
22. 6	**23.** 75	**24.** 72	**25.** 0.4375	**26.** –22	**27.** 64	**28.** 48
29. 1382	**30.** 504.771	**31.** 539	**32.** 618	**33.** 9687	**34.** 224174	**35.** 8
36. 4	**37.** 4,3	**38.** 90				

EXERCISE – II

MCQ Type Questions

1. (*d*)	**2.** (*a*)	**3.** (*b*)	**4.** (*d*)	**5.** (*a*)	**6.** (*a*)	**7.** (*a*)	**8.** (*d*)	**9.** (*c*)	**10.** (*b*)
11. (*a*)	**12.** (*c*)	**13.** (*d*)	**14.** (*b*)	**15.** (*a*)	**16.** (*d*)	**17.** (*a*)	**18.** (*b*)	**19.** (*a*)	**20.** (*a*)
21. (*b*)	**22.** (*a*)	**23.** (*b*)	**24.** (*d*)	**25.** (*a*)	**26.** (*a*)	**27.** (*b*)	**28.** (*a*)	**29.** (*a*)	**30.** (*b*)
31. (*d*)	**32.** (*d*)	**33.** (*a*)	**34.** (*d*)	**35.** (*c*)	**36.** (*a*)	**37.** (*c*)	**38.** (*b*)	**39.** (*b*)	**40.** (*d*)
41. (*d*)	**42.** (*c*)	**43.** (*c*)	**44.** (*d*)	**45.** (*b*)	**46.** (*a*)	**47.** (*a*)	**48.** (*c*)	**49.** (*b*)	

Numerical Type Questions

1. (3) **2.** (1) **3.** (19.2) **4.** (0.75) **5.** (3) **6.** (5) **7.** (3 to 3)

EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

31.

$$\begin{aligned} y &= A(B + C) + AB + (A + B)C \\ &= AB + AC + AB + AC + BC \\ &= AB + AC + BC \end{aligned}$$

32. *Truth-table for half-subtractor :*

A	B	X	D
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

Drawing K-maps.

B \ A	0	1
0	0	0
1	1	0

$f = \bar{A}B$

B \ A	0	1
0	0	1
1	1	0

$f = A\bar{B} + \bar{A}B$

68. The one half-adder can add the least significant bit of the two numbers.

Full adders are required to add the remaining 15 bits as they all involve adding carries.

73. XOR gate can be used to implement NOT by doing $A \oplus 1$.

Using AND gate and the NOT realization of XOR, a NAND gate can be obtained, which is a universal gate.

Hence (*c*) is correct.

The 2 to 1 multipliexer gives the function

AX + BX′, where X is selector input.

It can be used to implement a NOT by making A = 0 and B = 1, thus X′ is obtained.

An AND gate can be obtained by putting B = 0 and thus getting A.X.

Using NO and AND, a NAND can be obtained, which is a universal gate. So (*b*) is wrong.

We also can't NOT gate can't be implemented A. B. + C. So, (*d*) is wrong.

90. Before **Gate No. 4** being removed

Output, F $= xy\bar{z} + \bar{w}z + \bar{w}$

After **Gate No. 4** being removed,

Output, F $= xy\bar{z} + z = xy\bar{z} + z$

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156. 0 – 100 + 100 – 1 evaluates to 57 in Booth1's coding as below

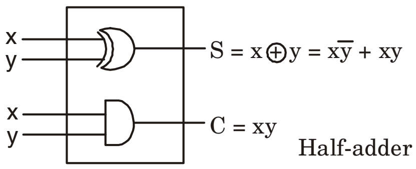
$$(0*2^7) - (1*2^6) + (0*2^5) + (0*2^4) + (1*2^3) + (0*2^2) + (0*2^1) - (1*2^0)$$

$= -64 + 8 - 1 = \mathbf{-57}$

157. In Gray code successive code word differs by the complementation of a single bit and this logic has been complemented in the given circuit.

158. Any digital circuit can be realized by a circuit consisting only NOT and AND gates ($\urcorner, \wedge$). Hence it is enough to show that both the gates can be realized by Half-adder.

Let x and y be two inputs and S and C be two outputs of a Half-adder.



Then $S = x \oplus y = x\overline{y} + \overline{x}y$

and $C = xy$.

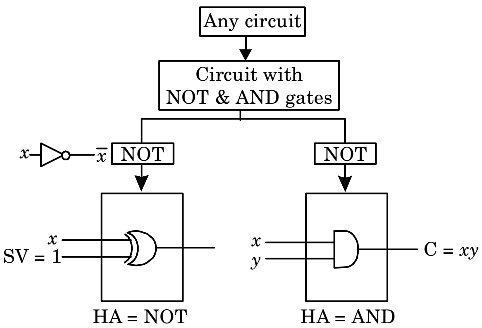
Consider $x \oplus 1 = x.\ \overline{1} + \overline{x}\ .\ 1$

$= x\ .\ 0 + \overline{x}$

$= \overline{x}\ .$

$\therefore \quad \overline{x} = x \oplus 1.$

Hence NOT gate can be realized by a half-adder with x and 1 as inputs. AND gate can be realised by the output $C = xy$ of a half-adder.



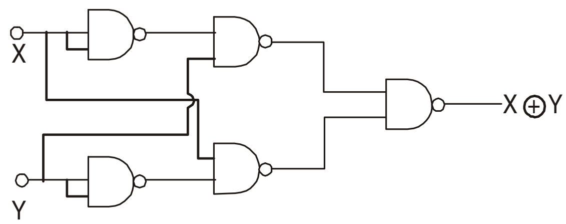
Hence any circuit (digital) can be realized by only Half address adders.

NUMERICAL TYPE QUESTIONS

4.
$$y = A + A\overline{B} + A\overline{B}C$$
$$= A + A\overline{B}\,(1 + C)$$
$$= A + A\overline{B} = A\,(1 + \overline{B})$$
$$= \mathbf{A}$$

Hence there is no need of any NAND gate.

11.



EXERCISE – II

MCQ TYPE QUESTIONS

1. The counting sequence for given 4-bit Johnson Counter with initial value 0000 is

0, 8, 12, 14, 15, 7, 3, 1, 0

2. The operator $\neq$ is both cumutative and associative as $p \neq q = q \neq p$ and $p \neq (q \neq r) = (p \neq q) \neq r$

3. Both {f} and {g} are functionally complete

4. Given L = 1250 Bytes

B = 100 mbps

d = 1 km

V = ?

In CSMA/CD, $L = 2 \times \dfrac{d}{v} \times B$

$\Rightarrow \quad V = \dfrac{2dB}{L}$

$= \dfrac{2 \times 10^3 \times 10^8}{10^4}$

$\Rightarrow \quad V = 20{,}000$ km/sec

6. $PQ + \overline{P}QR + \overline{P}Q\overline{R}S$

$= PQ + \overline{P}Q\,(R + \overline{R}S)$

$= PQ + \overline{P}Q\left(\left(R + \overline{R}\right)(R{+}S)\right)$

$\left[\because A + BC = (A + B)\,(A + C)\right]$

$= PQ + \overline{P}Q(R + S)$

$\left[\because \ R + \overline{R} = 1\right]$

$= Q\left(P{+}\overline{P}(R + S)\right)$

$= Q\left(\left(P + \overline{P}\right)(P + R + S)\right)$

$\left[\because \ A + BC = (A + B)(A + C)\right]$

$= Q\,(P + R + S) \qquad \because\left[P + \overline{P} = 1\right]$

$= PQ + QR + QS$

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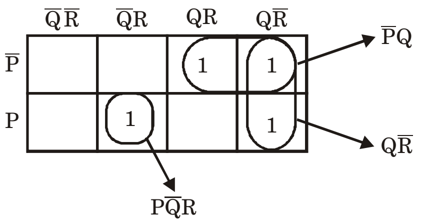
7.



$$\overline{P}\overline{Q}.O + \overline{P}Q.1 + P\overline{Q}.R + PQ\overline{R}$$

$$= \overline{P}Q + P\overline{Q}.R + PQ\overline{R}$$

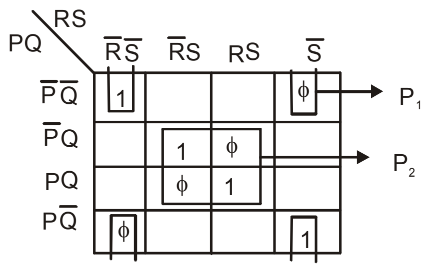
Hence the minimized expression is

$$\overline{P}Q + Q\overline{R} + P\overline{Q}R$$

8. According to the passage, kaliningrad is a part of Russia despite it not being contiguous with the rest of Russia.

9. Among given option, option (*d*) *i.e.* the number of people with malarial fever has increased this year do not contradic the conclusion.

10.



The K-map for the function F is as follows :

$P_1 = \overline{Q}\overline{S}$ and $P_2 = QS$

$\therefore$ F (P, Q, R, S) = $P_1 + P_2 = \overline{Q}\overline{S} + QS$

11. π is used to select a subset of attributes and σ_p is used to select subset of tuples matching the predicate P.

$$\Pi_{A_1}\left(\Pi_{A_2}\left(\sigma_{F_1}\left(\sigma_{F_2}(r)\right)\right)\right)$$

$$= \Pi_{A_1}\left(\Pi_{A_2}\left(\sigma(F_1 \wedge F_2)(r)\right)\right)$$

and as $A_1 \subset A_2$, so final relation will be displaying values for attributes present in set A.

$$\Pi_{A_1}\left(\Pi_{A_2}\left(\sigma(F_1 \wedge F_2)(r)\right)\right) = \Pi_{A_1}\left(\sigma(F_1 \wedge F_2)(r)\right)$$

12.

P.S.			FFinputs						N.S.		
Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0	Q_2^+	Q_1^+	Q_0^+
			$(\overline{Q}_1)$	(Q_0)	(Q_2)	$(\overline{Q}_2)$	(Q_1)	$(\overline{Q}_0)$			
0	0	0	1	0	0	1	0	1	1	0	0
1	0	0	1	0	1	0	0	1	1	1	0
1	1	0	0	0	1	0	1	1	1	1	1

13. $F(P,Q) = \left((1 \oplus P) \oplus (P \oplus Q)\right) \oplus \left((P \oplus Q) \oplus (Q \oplus 0)\right)$

$$= \left(\overline{P} \oplus \left(P\overline{Q} + \overline{P}Q\right)\right) \oplus \left(\left(P\overline{Q} + \overline{P}Q\right) \oplus Q\right)$$

$$= \left[\overline{P}\left(PQ + \overline{P}\overline{Q}\right) + P\left(P\overline{Q} + \overline{P}Q\right)\right]$$

$$\oplus \left[\left(PQ + \overline{P}\overline{Q}\right)Q + \left(P\overline{Q} + \overline{P}Q\right)\overline{Q}\right]$$

$$= \left(\overline{P}\overline{Q} + P\overline{Q}\right) \oplus \left(PQ + P\overline{Q}\right) = \overline{Q} \oplus P$$

$$= PQ + \overline{P}\overline{Q} = \overline{P \oplus Q}$$

14. **Option (*a*) :** 256 cannot be represented by 8-bit 2′s complement no.

or 2′S complement representation, the general formula for n-bits is

$(-2^{n-1}$ to $2^{n-1} - 1)$

for $n = 8$, the max negative number is $-2^{8-1} = -2^7 = -128$

Hence answer is option (*b*).

15. In following truth table, $v = 1$ if and only if the input is valid

Inputs				Outputs		
D_0	D_1	D_2	D_3	X_0	X_1	V
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

What function does the truth table represent?

Ans. A

It is a 4 × 2 priority encoder, where highest priority bits (inputs) are to the right and "X" indicates an irrelevant value.

i.e. Any input value there yields the same output since it is superseded by higher-priority input. And the output V indicates if the input is valid.

16. Exclusive NOR of x & y $\overline{x \oplus y}$

Truth table:

x	y	$2 \oplus y$	$\overline{x \oplus y}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

Option (A):

x	y	xy	$x'y'$	$xy + x'y'$	Same as $\overline{x \oplus y}$
0	0	0	1	1	
0	1	0	0	0	
1	0	0	0	0	
1	1	1	1	1	

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Option (B):

x	y	y'	$x \oplus y'$
0	0	1	1
0	1	0	0
1	0	1	0
1	1	0	1

$\equiv \overline{x \oplus y}$

Option (C):

x	y	x'	$x' \oplus y$
0	0	1	1
0	1	1	0
1	0	0	0
1	1	0	1

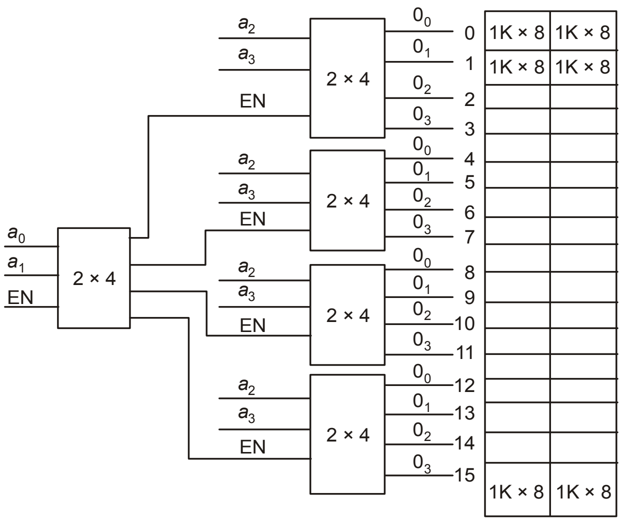
$\equiv \overline{x \oplus y}$

Option (D):

x	y	x'	y'	$x' \oplus y'$
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

This is not equivalent to $\overline{x \oplus y}$ So, the answer is (d)

17. 16 K × 16 RAM



$$16\,k \times 16 = 2^4 + 2^{10} + 2^4 \text{ bit } = 2^{18} \text{ bits}$$

$$1K \times 8 = 2^{10} + 2^3 \text{ bits} = 2^{13} \text{ bits}$$

so no of 1K × 8 RAM chips needed = $\frac{2^{18}}{2^{13}} 2^5 = 32$

18. $(0.5)_{10} = (1.0)_2 \times 2^{-1}$

So, exponent = – 1 and fraction is 000 - - - 000

19.
$$XY' + XY = X(Y'+Y)$$
$$= X.1 = X$$

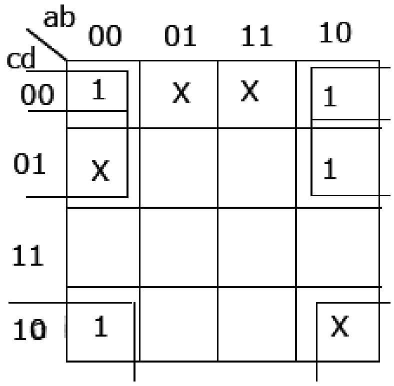
20. The ***counter example*** for the condition *full*: REAR = FRONT is Initially when the Queue is empty REAR = FRONT = 0 by which the above *full* condition is satisfied which is false

The ***counter example*** for the condition *full*: (FRONT + 1) mod n = REAR is Initially when the Queue is empty REAR = FRONT = 0 and let n = 3, so after inserting one element REAR = 1 and FRONT = 0, at this point the condition *full* above is satisfied, but still there is place for one more element in Queue, so this condition is also false

The ***counter example*** for the condition *empty* : (REAR + 1)mod n = FRONT is Initially when the Queue is empty REAR = FRONT = 0 and let n = 2, so after inserting one element REAR =1 and FRONT = 0, at this point the condition empty above is satisfied, but the queue of capacity n – 1 is full here

The ***counter example*** for the condition *empty* : (FRONT+l)mod n = REAR is Initially when the Queue is empty REAR = FRONT = 0 and let n = 2, so after inserting one element REAR = 1 and FRONT = 0, at this point the condition *empty* above is satisfied, but the queue of capacity n – 1 is full here

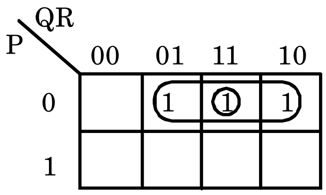
21.



22. $2^n \geq 258$

$\Rightarrow$ n = 9

23.



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$f = (P + \overline{R})(\overline{P} + Q)$

$= P + QR$

Alternate method

$(P + \overline{Q} + \overline{R}).(P + \overline{Q} + R).(P + Q + \overline{R})$

$$= \overline{\overline{\left(P+\overline{Q}+\overline{R}\right).\left(P+\overline{Q}+R\right).\left(P+Q+\overline{R}\right)}}$$

$$= \overline{\overline{P}QR + \overline{P}Q\overline{R} + \overline{P}\overline{Q}R}$$

$$= \overline{\overline{P}Q\left(R+\overline{R}\right) + \overline{P}\overline{Q}R}$$

$$= \overline{\overline{P}Q + \overline{P}\overline{Q}R}$$

$$= \overline{\overline{P}\left(Q+\overline{Q}R\right)}$$

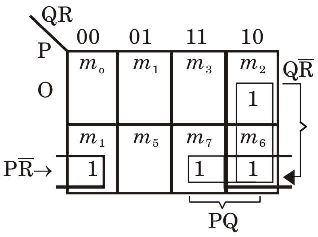
$$= \overline{\overline{P}\left(Q+R\right)}$$

$$= P + \overline{Q}\,\overline{R}$$

24. All options except option '*d*' gives EX-NOR gates

25. Given : $f(P, Q, R) = PQ + Q\overline{R} + P\overline{R}$

By K-map



These are min terms means sum of products in k-map

Then answer is $\mathbf{PQ + Q\overline{R} + P\overline{R}}$

So answer is $\boldsymbol{m_2 + m_4 + m_6 + m_7}$ because all other answer conflict the requirment.

26. $(F87B)_{16} = \underbrace{1111}_{F}\,\underbrace{1000}_{8}\,\underbrace{0111}_{7}\,\underbrace{1011}_{B}$

But it is 2's complement of P

So for getting original numbers, take 2's complement of $(F87B)_{16}$

It is $\quad P = 0000\ 01111\ 0000101$

$= (1925)_{10}$

Then $\quad P * 8 = 1925 \times 8$

$= 15400$

$(15400)_{10} = 0011110000101000$

Now, 2's complement of this 15400 in binary

$= \underbrace{1100}_{C}\,\underbrace{0011}_{3}\,\underbrace{1101}_{D}\,\underbrace{1000}_{8}$

$= (C3D8)_{16}$

27. Make the table

$P(S_1)$	$Q(S_0)$	Z	
0	0	R	0
0	1	$\overline{R}$	1
1	0	$\overline{R}$	2
1	1	R	3

as given in figure

Add these all for result because output is the mixing of these signals.

$\therefore\ \overline{P}\overline{Q}R + \overline{P}Q\overline{R} + P\overline{Q}\overline{R} + PQR$

$= \overline{P}\left(\overline{Q}R + Q\overline{R}\right) + P\left(Q\overline{R} + QR\right)$

$= \overline{P}(Q \oplus R) + P(Q \Theta R)$

$= P \oplus Q \oplus R$

Alternately

P and Q are lines out of which only one will enable at a time.

P	Q	F
0	0	R
0	1	$\overline{R}$
1	0	$\overline{R}$
1	1	R

$\therefore\ P'Q'R + P'QR' + PQ'R' + PQR$

$= P'(Q'R + QR') + P(Q'R' + QR)$

$= P'(Q \oplus R) + P(Q \odot R)$

$= P'(Q \oplus R) + P(Q \oplus R)'$

$= P'Z + PZ'$ ....[take $(Q \oplus R) = Z$]

$= P \oplus Z$

$= P \oplus Q \oplus R$

28. Output

$$= \overline{\left(\overline{(P+Q)} + \overline{(Q+R)}\right)} + \overline{\left(\overline{(P+R)} + \overline{(Q+R)}\right)}$$

$$= \left(\overline{P+Q} + \overline{Q+R}\right)\cdot\left(\overline{(P+R)} + \overline{(Q+R)}\right)$$

$$\left(\because \left(\overline{A+B}\right) = \overline{A}\ \overline{B}\right)$$

$$= \left(\overline{P}\overline{Q} + \overline{Q}\overline{R}\right)\left(\overline{P}\overline{R} + \overline{Q}\overline{R}\right)$$

$$= \overline{P}\overline{Q}\overline{P}\overline{R} + \overline{P}\overline{O}\overline{Q}\overline{R} + \overline{P}\overline{Q}\overline{R}\overline{R} + \overline{Q}\overline{R}\overline{Q}\overline{R}$$

$$= \overline{P}\overline{Q}\overline{R} + \overline{Q}\overline{P}\overline{R} + \overline{P}\overline{Q}\overline{R} + \overline{Q}\overline{R}$$

$$= \overline{P}\overline{Q}\overline{R} + \overline{Q}\overline{R}$$

$$= \overline{Q}\overline{R}(\overline{P}+1) \quad .....\left[\text{since}(\overline{P}+1) = 1\right]$$

$$= \overline{(Q+R)}$$

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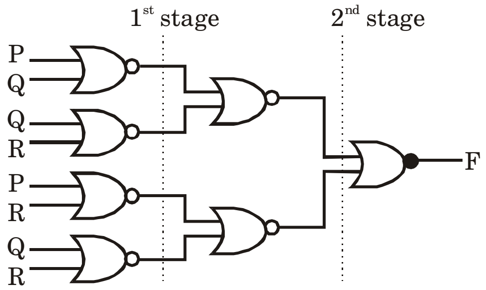
Alternately



At first stage

- $\overline{(P+Q)}$
- $\overline{(Q+R)}$
- $\overline{(P+R)}$
- $\overline{(Q+R)}$

At second stage

- $\overline{\overline{(P+Q)}+\overline{(Q+R)}}$
- $\overline{\overline{(P+R)}+\overline{(Q+R)}}$

$$F = \overline{\overline{\overline{(P+Q)}+\overline{(Q+R)}}+\overline{\overline{(P+R)}+\overline{(Q+R)}}}$$

$$= \overline{\overline{\overline{\overline{(P+Q)}+\overline{(Q+R)}}}}\cdot\overline{\overline{\overline{\overline{(P+R)}+\overline{(Q+R)}}}}$$

$$= \left(\overline{(P+Q)}+\overline{(Q+R)}\right)\cdot\left(\overline{(P+R)}+\overline{(Q+R)}\right)$$

$$= (P'Q'+Q'R')\cdot(P'R'+Q'R')$$

$$= \underline{P'Q'R'+P'Q'R'+P'Q'R'+P'Q'R'}+Q'R'$$

$$= P'Q'R' + Q'R'$$

$$= Q'R'\,(P'+1)$$

$$= Q'\,R'$$

$$= \overline{Q+R}$$

29. Initially Q_1 Q_0

0 0

Here T flip-flop

If T = 0 the next state is present state

If T = 1 next state is compliment of present

Since T = 1 initially, so outputs is compliment

Next second Q_1 remain 1 since Q_0 become 0 when TIP is 1 (compliment of present), so 2n second clock not present remains in same state.

So sequence is like 11, 10, 01, 00

30. $(1217)_8 = 001\ 010\ 001\ 111$

$0010\ 1000\ 1111 = (028F)_{16}$

Alternately

Given digit in octal 1217.

Convert them in binary $\overline{001}\ \overline{010}\,\overline{001}\,\overline{111}$

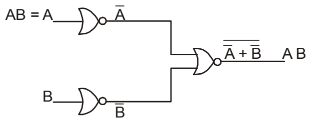
Now convert binary to hexa

$$\begin{array}{ccc} \overline{0010} & \overline{1000} & \overline{1111} \\ 2 & 8 & F \end{array}$$

$$= (028F)_{16}$$

31. For implementing (AB + C), first we implement AB.

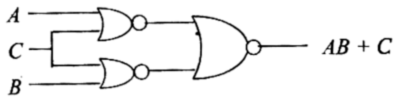
For AND operation 3 NOR GATE required



Now for (AB + C)

Hence five NOR gates required.

Alternately



32. Hexadecimal value 0x00000000 in binary

$\underline{0000\ 0000}$ $\underline{0000\ 0000}$ $\underline{0000\ 0000}$ $\underline{0000\ 0000}$

$\underline{0000\ 0000}$

Hence it is normalized to value to.

33. K-Map is

cd \ ab	00	01	11	10
00	1	1		1
01	x			
11	x			
10	1	1		x

a′d′ + b′d′

34. $\sqrt{121_r} = (11)_r$

So we till take base *r* any value that is greater than 2.

35. f_1 and f_2 are connected with and gates, so when $f_2 = \Sigma m(6, 8)$ then f is true for Σm (1, 6, 8, 15)

36. $(P + Q')\,(PQ' + PR)\,(P'R' + Q')$

$= (PQ' + PR + PQ' + PRQ')\,(P'R' + Q')$

$= (PQ' + PR + PRQ')\,(P'R' + Q')$

$= (PQ' + PR\,(1 + Q'))\,(P'R' + Q')$

$= (PQ' + PR)\,(P'R' + Q')$

$= PQ' + PRQ'$

$= PQ'\,(1 + R)$

$= PQ'$

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37. For n binary variables, we can obtain 2^n distinct minterm or maxterms.

Since function can be either 1 or 0 for each minterm or maxterm, therefore we can form 2^{2^n} function from n variables.

38. Eight 3 – to– 8 line decoders will be needed to construct a 6 – to – 64 line decoder without any other logic gates.

39. Karnaugh map of given function will be as follows:

wx \ yz	00	01	11	10
00		1	1	
01	1			1
11	1			1
10		1	1	

Simplified form of f will be

$$f = x'z + xz'$$

Therefore, it is independent of two variables.

40. Each of the functions P, Q and R includes same minterms which f includes, where S doesn't includes same minterms as f.

$$P = \Sigma (0, 4, 5, 7, 8, 9, 13, 15)$$
$$= Q = R = f$$

whereas $S = \Sigma (0, 2, 3, 5, 7, 8, 9) \neq f$

41. Truth table for X, Y and Z

X	Y	Z = X * Y	Y * Z	X * Z	X * Y * Z
0	0	1	0	0	1
0	1	0	0	1	1
1	0	0	1	0	1
1	1	1	1	1	1

From above truth table we can say that all P, Q and R are valid.

42. Multiplexes of size 2^{n-1} to 1 will be needed.

43. A 4-bit adder with S_3, S_2, S_1, S_0 and C_4 as its output require 2-1 bit adder and a single one bit adder require 5 AND gates and 2 OR gates in two level logic. Hence required gate are 10 AND gate and 4 OR gates.

44. Counter will start with 0 for Load = 0, Count = 1 and clock it will go to 1, then 2, then 3, then 4 and at last at 5. As soon as it will reach to 5, the clear will become one (1) which will clear the counter to zero.

45. Clock with duty cycle 50%

In choice (c), there are two D-Flip Flop if left most is activated or clock value 1 the clock value of right most 1 D flip-flop is 0, so it delay the phase f by 180°.

46.
$$\begin{array}{ccc} a_2 & a_1 & a_0 \\ b_2 & b_1 & b_0 \end{array}$$

Carry will be generated in the following cases

$$a_2 = 1, b_2 = 1$$
$$a_2 \oplus b_2 = 1$$

and $a_1 = b_1 = 1$

$$a_2 \oplus b_2 = a_1 \oplus b_1 = 1$$

and $a_0 = b_0 = 1$

Considering options :

In option (b), second term is invalid

as if $a_2 = 1, b_2 = 0, b_1 = 1, b_0 =1, c \neq 1$

In option (c), a_2 is invalid term

In option (d), $a_2\, a_1, b_1$, is invalid as, if

$$a_2 = 0, a_1 = b_1 = 1, b_2 = 0, c^1 \neq = 1$$

Hence only option (a) encompasses possibilities.

47.

wx \ yz	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	1	1	1	0
10	0	0	1	0

The cube covers of (a) contain following cells.

$\overline{w}\,xz$	0101, 0111
$wx\,\overline{y}$	1101, 1100
$x\,\overline{y}\,z$	0101,1101
xyz	0111, 1111
wyz	1011,1111

Atleast one member of a pair should satisfy the condition of differing by one bit by at least one member of each of other pairs. This is true regarding the above cubes.

48.
$$B = b_{n-1}\, b_{n-2} \ldots b_0$$
$$A = a_{n-1}\, a_{n-2} \ldots a_0$$
$$S = c_{n-1}\, c_{n-2} \ldots c_0$$

Over flow condition,

$$V = b'_{n-1}\, a'_{n-1}\, c_{n-2} + b_{n-1}\, a_{n-1}\, c'_{n-2}$$

c_{n-1} is c_{out}, so

$$V = c_{out} \oplus c_{n-1}$$

Hence over flow condition is

$$c_{out} \oplus c_{n-1}$$

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49. $h_3h_2h_1h_0$ is gray code representation of number n, $g_3g_2g_1g_0$ is gray code representation of $(n+1)$ (modulo 16) $(n+1)$ is 5 bit in decimal it range is 0 to 31.

So, $g_1(h_3h_2h_1h_0) = \Sigma(4, 9, 10, 11, 12, 13, 14, 15)$ is correct function for $(n+1)$ mod 16.

NUMERICAL TYPE QUESTIONS

1. The minimum number of flip-flop required is 3.

2.

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	$\otimes$	1
10	1	1	1	1

Ans = 1, ABCD

3. Propagation delay for AND/OR Gate

$= 1 \cdot 2$ s

Propagation delay for XOR Gate

$= 2 \times 1.2$

$= 2.4$ s

Propagation time for a half adder

$= 1.2 + 2.4$

$= 3.6$ s

Propagation time for full adder

$= 2 \times 3.6 + 1.2$

$= 7.2 + 1.2$

$= 8.4$ s

Total 4-adders are used in 4-bit binary adder

Hence $8.4 \times 4 =$ **33.6 s**

4. Given $X_3 = 0$

$\therefore \quad Y = X_1X_2 \oplus X_3 = 0$

gives $\quad X_1X_2 = 0$

$\Rightarrow \quad X_1 = 1, \; X_2 = 0$

or $\quad X_1 = 0, X_2 = 1$

or $\quad X_1 = 0, X_2 = 0$

$$\therefore \quad \Pr\left[Y = 0 \middle| X_3 = 0\right] = \frac{\Pr\left[(Y=0)\cap(X_3=0)\right]}{\Pr(X_3=0)}$$

$$= \frac{\Pr[X_1 = 1, X_2 = 0, X_3 = 0] + \Pr[X_1 = 0, X_2 = 1, X_3 = 0] + \Pr[X_1 = 0, X_2 = 0, X_3 = 0]}{\Pr[X_3 = 0]}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}}{\frac{1}{2}}$$

($\because X_1, X_2, X_3$ are independent)

$$= \frac{3}{4} = 0.75$$

5.

wx \ yz	00	01	11	10
00	*1	0	0	1
01	1	1*	0	1*
11	0	0	0	0
10	0	0	0	1*

There are three groups, and all the containing esential-1, so all are prime implicants as well as essential prime implicants also.

6.

$$(43)_x = (y3)_8$$

$\Rightarrow \quad 3 + 4x = 3 + 8y$

$\Rightarrow \quad 4x = 8y$

$\Rightarrow \quad x = 2y$

$\Rightarrow \quad x \geq 5$

and $\quad y \leq 7$

$\therefore$ 5 solutions are possible which are (14, 7), (12, 6), (10, 5), (8, 4) and (6, 3)

7. Here it is given that x is real

Now $\quad |x^2 - 2x + 3| = 11$

$\Rightarrow \quad x^2 - 2x + 3 = 11$ or $x^2 - 2x + 3 = -11$

For $\quad x^2 - 2x + 3 = 11 \Rightarrow x^2 - 2x - 8 = 0$

$b^2 - 4ac = 4 - 4(1)(-8) = 36 > 0$

Hence roots are real.

and for $\quad x^2 - 2x + 14 = 0$

$b^2 - 4ac = 4 - 4(1)(14) < 0$

Hence roots are imaginary

$\therefore$ The value of x is the root of $x^2 - 2x - 8 = 0$

Now $\quad x = \dfrac{+2 \pm \sqrt{4 - 4(1)(-8)}}{2}$

$= \dfrac{2 \pm \sqrt{36}}{2} = \dfrac{2 \pm 6}{2}$

$\therefore \quad x = 4, -2.$

Now for $\quad x = 4,$

$|-x^3 + x^2 - x| = |-64 + 16 - 4| = 52.$

and for $\; x = -2$

$|-x^3 + x^2 - x| = |+8 + 4 + 2| = 14$

Hence possible values of $|-x^3 + x^2 - x| = 14, 52$

■■

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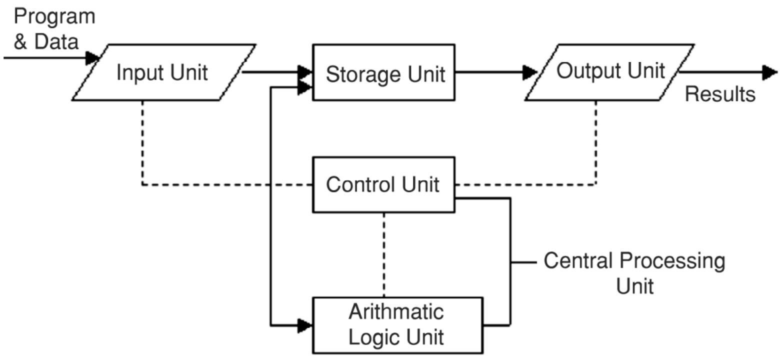
2

CHAPTER

Computer Organization and Architecture

MACHINE INSTRUCTIONS & ADDRESSING MODES

BASIC COMPUTER OPERATIONS



A computer as shown above performs basically five major operations or functions irrespective of their size and make :

1. **Input.**

   This is the process of entering data and programs in to the computer system. Computer is an electronic machine like any other machine which takes as inputs raw data and performs some processing giving out processed data. Therefore, input unit takes data to the computer in an organized manner for processing.

2. **Storage.**

   The process of saving data and instructions permanently is called storage. Data has to be fed into the system before the actual processing starts, because processing speed of Central Processing Unit (CPU) is so fast that the data has to be provided to CPU with the same speed. Therefore data is first stored in the storage unit for faster access and processing. This storage unit or the primary storage of the computer system is designed to do the above functionality. It provides space for storing data and instructions.

   Storage unit performs following major functions :

   (*i*) All data and instructions are stored here before and after processing.

   (*ii*) Intermediate results of processing are also stored here.

3. **Processing.**

   Performing operations like arithmetic and logical operations is called *processing*. The Central Processing Unit (CPU) takes data and instructions from the storage unit and makes all sorts of calculations based on the instructions given and the type of data provided. It is then sent back to the storage unit.

4. **Output.**

   This is the process of producing results from the data for getting useful information. Similarly output produced by the computer after processing must also be kept somewhere inside the computer before being given in human readable form. Again the output is also stored inside the computer for further processing.

5. **Control.**

   Controlling of all operations like input, processing and output are performed by control unit. It takes care of step by step processing of all operations in side the computer.

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MACHINE INSTRUCTIONS

The instruction sets can be differentiated by

(*i*) Operand storage in the CPU
(*ii*) Number of explicit operands per instruction
(*iii*) Operand location
(*iv*) Operations
(*v*) Type and size of operands

Note : Type of internal storage in the CPU is the most basic differentiation.

Machine Type

(*i*) **Stack** (operands are implicitly on top of the stack)
(*ii*) **Accumulator** (one operand is implicitly the accumulator)
(*iii*) **Set of registers** (all operands are explicit either registers or memory locations)

All machines use a general purpose architecture because registers are :

(*i*) faster than memory
(*ii*) easier for a compiler to use
(*iii*) used more effectively

Advantages and Disadvantages of Each class of Machine

Machine Type	Advantages	Disadvantages
Stack	Simple model of expression evaluation. Good code density.	A stack can't be randomly accessed. It makes it difficult to generate efficient code.
Accumulator	Minimizes internal state of machine. Short instructions	Since accumulator is only temporary storage, memory traffic is highest.
Register	Most general model for code generation	All operands must be named, leading to longer instructions.

Classification of General Purpose Register Machines

There are two major instruction set characteristics that divide General Purpose Register (GPR) architectures which concern

(*i*) whether an ALU instruction has two or three operands
    ADD R3, R1, R2 or ADD R1, R2
    R3 <- R1 + R2 or R1 <- R1 + R2
(*ii*) how many of the operands may be memory addressed in ALU instruction
    (*a*) Register- Register (Load/Store)
        ADD R3, R1, R2 (R3 <- R1 + R2)
    (*b*) Register - Memory
        ADD R1, A(R1 <- R1 + A)
    (*c*) Memory - Memory
        ADD C, A, B (C <- A + B)

INSTRUCTIONS

There are three types of instructions :

1. One-byte instructions.

(*i*) Data transfer instructions that copy the contents from one register (or memory) into another register (or memory) are 1-byte instructions.
e.g. MOV

(*ii*) Arithmetic/Logic instructions without the ending letter I are 1 byte instructions.
e.g. ADD, SUB, ORA, ANA, INR, DCR

2. Two-byte instructions.

(*i*) In a two-byte instruction, the 1^{st} byte of the instruction is its opcode and the 2^{nd} byte is either data or address.
e.g. MVI B, 05 Move 05 to register B.
IN 01 Read data at port B.

(*ii*) These instructions are generally represented by three letters with the ending letter I.
e.g. MVI, ADI, ANI, ORI (Exception : LXI instructions)

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3. Three-byte instructions

Instructions that load 16 bits data or refer to 16-bit memory address are 3-byte instruction.

e.g. LXI H, 2400H	Load H-L pair with 2400 H.
LDA 2500 H	Get the content of the memory 2500 H into accumulator.

ADDRESSING MODES

These are the ways how architectures specify address of an object they want to access.

e.g., in GPR machines, an addressing mode can specify a constant, a register or a location in memory.

Computers use addressing mode techniques for accommodating one or both of the following provisions :

(*i*) To give programming versatility to the user by providing such facilities as pointers to memory, counters for loop control, indexing of data and program relocation.

(*ii*) To reduce number of bits in the addressing field of the instruction.

TYPES OF ADDRESSING MODES.

1. ***Implied mode.***
In this mode, the operands are specified implicitly in the definition of the instructions. All register reference instructions that use an accumulator are implied mode instructions. Zero-address instructions in a stack organized computer are implied mode instructions.

2. ***Immediate mode.***
In this mode, the operand is specified in the instructions itself. Immediate mode instructions are useful for initializing registers to a constant value. Immediate mode instruction does not have an address field.

3. ***Register mode.***
In this mode, the operands are in registers that reside within the CPU. The particular register is selected from a register field in the instructions. A n-bit field can specify any one of 2^n registers.

4. ***Register indirect mode.***
In this mode, a register specified in instructions contain the address of the operand rather than operand itself. The advantages of a register indirect mode instruction is that the address field of the instruction uses fewer bit to select a register than would have been required to specify a memory directly.

5. ***Autoincrement or autodecrement mode.***
This is similar to the register indirect mode except that the register is incremented or decremented after (or before) its value is used to access memory.

6. ***Direct address mode.***
In this mode the effective address is equal to the address part of the instructions. The operand resides in the memory and its address is given directly by the address field of the instructions.

7. ***Indirect address mode.***
In this mode, address field of the instructions gives the address where the effective address is stored in memory. Control fetches the instructions from memory and uses its address part to access memory again to read the effective address.

8. ***Relative address mode.***
In this mode the content of the program counter is added to the address part of the instructions in order to get effective address. This effective address has position in memory is relative to the address of the next instructions.

9. ***Indexed addressing mode.***
In this mode, the content of an index register is added to the address part of the instructions to obtain the effective address. The address field of the instructions defines the beginning address of a data array in memory.

10. ***Base register addressing mode.***
In this mode, the content of a base register is added to the address part of the instructions to obtain the effective address. A base register is assumed to hold a base address and the address field of the instructions give a displacement relative to this base address. The base register addressing mode is used in computers to facilitate the relocation of programs in memory.

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11. ***Immediate and Displacement addressing modes.***

These dominate addressing mode usage. The major question for displacement-style addressing mode is that of the **range of displacement** used. Choosing the displacement field size is important because it directly affects instruction length. According to measurements taken on the data access on a GPR architecture using SPEC benchmarks displacement values are widely distributed.

Another important instruction set measurement is the **range of values for immediates** . Small immediate values are used most heavily. However, large immediates are sometimes used, most likely in address calculations.

Common Addressing Modes *(names may differ among architectures)*

Addressing modes	Example Instruction	Meaning	When used
Register	Add R4, R3	R4 <- R4 + R3	When a value is in a register
Immediate	Add R4, #3	R4 <- R4 + 3	For constants
Displacement	Add R4, 100(R1)	R4 <- R4 + M[100+R1]	Accessing local variables
Register deffered	Add R4, (R1)	R4 <- R4 + M[R1]	Accessing using a pointer or a computed address
Indexed	Add R3, (R1+R2)	R3 <- R3 + M[R1+R2]	Useful in array addressing: R1 - base of array R2 - index amount
Direct	Add R1, (1001)	R1 <- R1 + M[1001]	Useful in accessing static data
Memory deferred	Add R1, @(R3)	R1 <- R1 + M[M[R3]]	If R3 is the address of a pointer *p*, then mode yields **p*
Auto- increment	Add R1, (R2)+	R1 <- R1 +M[R2] R2 <- R2 + *d*	Useful for stepping through arrays in a loop. R2 - start of array *d* - size of an element
Auto-decrement	Add R1,-(R2)	R2 <- R2-*d* R1 <- R1 + M[R2]	Same as autoincrement. Both can also be used to implement a stack as push and pop
Scaled	Add R1, 100(R2)[R3]	R1<-R1+M[100+R2+R3*d]	Used to index arrays. May be applied to any base addressing mode in some machines.

Notation :

<- - assignment

M - name for memory:

M[R1] refers to contents of memory location whose address is given by the contents of R1

ADDRESSING MODES IN BRANCH ADDRESSING

The content of IP (Instruction Pointer) denotes the address of instruction currently being executed. To fetch next instruction this is changed. This is replaced by a value called *Effective Branch Addess* (EBA).

There are four modes :

Mode 1 : Intrasegment Direct.

EBA is sum of offset value and content of IP.

EBA = (IP) + offset value

Mode 2 : Intrasegment Indirect.

This is equivalent to indirect addressing mode, *i.e.* EA is calculated as usual and then EBA is nothing but content of EA.

EBA = content of the address shown by EA.

Then IP is replaced by EBA.

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Following steps show how to calculate EBA :

Step 1 : From instruction and addressing mode find EA.

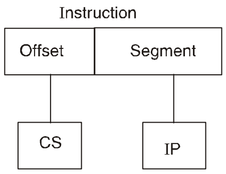
Step 2 : EA may be a register address or an address in main memory.

Step 3 : Find content of EA's register or main memory.

Step 4 : EBA is the content of EA.

Step 5 : IP is replaced by EBA.

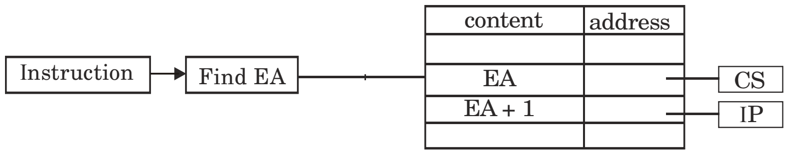
Mode 3 : Address sent by microprocessor to memory is the sum of Code Segment (CS) register and Instruction Pointer (IP). If we want to switch over to other segment's, we have to change CS. In this intersegment direct mode, one part of instruction is stored in IP and another part is stored in CS.



Intersegment Direct Mode

Mode 4 : Intersegment Indirect.

In intersegment indirect, EBA is a content of EA. Here contents of CS and IP are continuous two words in memory located by EA.



Intersegment Indirection Addressing

Encoding of Addressing Modes

Addressing modes of operands are encoded depends on

(*i*) range of addressing modes

(*ii*) degree of independence between opcodes and modes

For **small** number of addressing modes or *opcode/addressing mode combinations,* the addressing mode can be encoded in **opcode.**

For a **larger** number of combinations, typically a separate **address specifier** is needed for each operand.

Architect has to balance following competing forces when encoding instruction set :

(*i*) The desire to have **as many** registers and addressing modes **as possible**.

(*ii*) The **impact of the size** of the register and addressing mode fields on the average instruction size and hence on the average program size.

(*iii*) A desire to have instructions encode into lengths that are **easy to handle** in the implementation (multiples of bytes, fixed-length) with possible sacrificing in average code size.

DLX ARCHITECTURE.

It should have following qualities :

(*i*) Simple load/store instruction set

(*ii*) Design for pipelining efficiency

(*iii*) Easily decoded instruction set

(*iv*) Efficiency as a compiler target

Registers for DLX

1. **32-bit general purpose registers (GPRs),** named R0, R1, ..., R31.
   The value of R0 is always 0.
2. **Thirty-two floating-point registers (FPRs).**
   These are which can be used as
   (*i*) 32 single precision (32-bit) registers or
   (*ii*) even-odd pairs holding double-precision values. Thus, the 64-bit FPRs are named F0,F2,...,F30
3. **Few special registers.**
   These can be transferred to and from the integer registers.

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Data types for DLX

1. *For integer data*
   (*i*) 8-bit Bytes
   (*ii*) 16-bit half words
   (*iii*) 32-bit words
2. *For floating point*
   (*i*) 32-bit single precision
   (*ii*) 64-bit double precision

DLX operations work on 32-bit integers and 32- bit or 64-bit floating point. Bytes and half words are loaded into registers with either zeros or the sign bit replicated to fill the 32 bits of the registers.

Memory

(*i*) Byte addressable
(*ii*) Big Endian mode
(*iii*) 32-bit address
(*iv*) Two addressing modes (Immediate and Displacement).
Register deferred and absolute addressing with 16-bit field are accomplished using R0.
(*v*) Memory references are load/store between memory and GPRs or FPRs
(*vi*) Access to GPRs can be to a byte, to a halfword, or to a word
(*vii*) All memory accesses must be aligned
(*viii*) There are instructions for moving between a FPR and a GPR

Instructions

(*i*) Instruction layout for DLX
(*ii*) Complete list of instructions in DLX
(*iii*) 32 bits(fixed)
(*iv*) Must be aligned

OPERATIONS.

There are four classes of instructions :

1. **Load/Store**
   Any of the GPRs or FPRs may be loaded and stored except that loading R0 has no effect.
2. **ALU Operations**
   All ALU instructions are register-register instructions.
   The operations are :
   - Add
   - Subtract
   - AND
   - OR
   - XOR
   - Shifts

   Compare instructions compare two registers (=,!=,<,>,=<,=>).
   If the condition is true, these instructions place a 1 in the destination register, otherwise they place a 0.
3. **Branches/Jumps**
   All branches are conditional. The branch condition is specified by the instruction, which may test the register source for zero or nonzero.
4. **Floating-Point Operations**
   The operations are :
   - Add
   - Subtract
   - Multiply
   - Divide

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DLX INSTRUCTION SET

Instruction type/opcode	Instruction meaning
Data transfers	**Move data between registers and memory, or between the integer and FP or special register; only memory address mode is 16-bit displacement + contents of a GPR**
LB, LBU, SB	Load byte, load byte unsigned, store byte
LH, LHU, SH	Load halfword, load halfword unsigned, store halfword
LW, SW	Load word, store word (to/from integer registers)
LF, LD, SF, SD	Load SP float, load DP float, store SP float, store DP float (SP - single precision, DP - double precision)
MOVI2S, MOVS2I	Move from/to GPR to/from a special register
MOVF, MOVD	Copy one floating-point register or a DP pair to another register or pair
MOVFP2I, MOVI2FP	Move 32 bits from/to FP tegister to/from integer registers
Arithmetic / Logical	**Operations on integer or logical data in GPRs; signed arithmetics trap on overflow**
ADD, ADDI, ADDU, ADDUI	Add, add immediate (all immediates are 16-bits); signed and unsigned
SUB, SUBI, SUBU, SUBUI	Subtract, subtract immediate; signed and unsigned
MULT, MULTU, DIV, DIVU	Multiply and divide, signed and unsigned; operands must be floating-point registers; all operations take and yield 32-bit values
AND, ANDI	And, and immediate
OR, ORI, XOP, XOPI	Or, or immediate, exclusive or, exclusive or immediate
LHI	Load high immediate - loads upper half of register with immediate
SLL, SRL, SRA, SLLI, SRLI, SRAI	Shifts: both immediate(S__I) and variable form(S__); shifts are shift left logical, right logical, right arithmetic
S__, S__I	Set conditional: "__"may be LT, GT, LE, GE, EQ, NE
Control	**Conditional branches and jumps; PC-relative or through register**
BEQZ, BNEZ	Branch GPR equal/not equal to zero; 16-bit offset from PC
BFPT, BFPF	Test comparison bit in the FP status register and branch; 16-bit offset from PC
J, JR	Jumps: 26-bit offset from PC(J) or target in register(JR)
JAL, JALR	Jump and link: save PC+4 to R31, target is PC-relative(JAL) ot a register(JALR)
TRAP	Transfer to operating system at a vectored address
RFE	Return to user code from an exception; restore user code
Floating point	**Floating-point operations on DP and SP formats**
ADDD, ADDF	Add DP, SP numbers
SUBD, SUBF	Subtract DP, SP numbers
MULTD, MULTF	Multiply DP, SP floating point
DIVD, DIVF	Divide DP, SP floating point
CVTF2D, CVTF2I, CVTD2F, CVTD2I, CVTI2F, CVTI2D	Convert instructions: CVTx2y converts from type x to type y, where x and y are one of I(Integer), D(Double precision), or F(Single precision). Both operands are in the FP registers.

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IMPLEMENTATION OF DLX.

Every DLX instruction can be implemented in at most following five clock cycles :

1. Instruction fetch cycle (IF)

IR <- MEM[PC]

NPC <- PC +4

Operation:

(*i*) Send out the PC and fetch the instruction from memory into the instruction register (IR)

(*ii*) Increment the PC by 4 to address the next sequential instruction

(*iii*) IR is used to hold the instruction that will be needed on subsequent clock cycles

(*iv*) NPC is used to hold the next sequential PC (program counter)

2. Instruction decode/register fetch (ID)

A <- Regs[$IR_{6..10}$]

B <- Regs[$IR_{11..15}$]

Imm <- $((IR_{16})^{16}\#\#IR_{16..31})$

Operation :

(*i*) Decode the instruction and access the register file to read the registers.

(*ii*) Output of the general-purpose registers are read into two temporary registers (A and B) for use in later clock cycles.

(*iii*) Lower 16 bits of the IR are also sign-extended and stored into the temporary register IMM, for use in the next cycle.

(*iv*) Decoding is done in parallel with reading registers, which is possible because these fields are at a fixed location in the *DLX instruction format.* This technique is called ***fixed-field decoding.***

3. Execution/Effective address cycle (EX)

ALU operates on the operand prepared in the prior cycle, performing one of the following four functions depending on DLX instruction type :

(*i*) Memory reference.

ALUOutput <- A +Imm

Operation : ALU adds the operands to form effective address and places the result into the register ALUOutput

(*ii*) Register-Register ALU instruction.

ALUOutput <- A *op* B

Operation : ALU performs the operation specified by the opcode on the value in register A and on the value in register B. The result is placed in the register ALUOutput.

(*iii*) Register- Immediate ALU instruction.

ALUOutput <- A *op* Imm

Operation : ALU performs the operation specified by the opcode on the value in register A and on the value in register Imm. The result is placed in the register ALUOutput.

(*iv*) Branch.

ALUOutput <- NPC + Imm

Cond <- (A op 0)

Operation :

(*a*) The ALU adds the NPC to the sign-extended immediate value in Imm to compute the address of the branch target.

(*b*) Register A, which has been read in the prior cycle, is checked to determine whether the branch is taken.

(*c*) Comparison operation *op* is the relational operator determined by the branch opcode e.g. *op* is "==" for the instruction BEQZ.

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4. Memory access/branch completion cycle (MEM)

The only DLX instructions active in this cycle are loads, stores, and branches.

(*i*) ***Memory reference.***

LMD <- Mem[ALUOutput] or Mem[ALUOutput] <- B

Operation:

(*a*) Access memory if needed

(*b*) If instruction is load , data returns from memory and is placed in the LMD (load memory data) register

(*c*) If instruction is store, data from the B register is written into memory.

(*d*) In either case the address used is the one computed during the prior cycle and stored in the register ALUOutput

(*ii*)**Branch.**

if (cond) PC <- ALUOutput

else PC <- NPC

Operation :

(*a*) If instruction branches, the PC is replaced with branch destination address in the register ALUOutput

(*b*) Otherwise, PC is replaced with the incremented PC in the register NPC

5. Write-back cycle (WB)

(*i*) **Register-Register ALU instruction.**

Regs[$IR_{16..20}$] <- ALUOutput

(*ii*) **Register-Immediate ALU instruction.**

Regs[$IR_{11..15}$] <- ALUOutput

(*iii*) **Load instruction.**

Regs[$IR_{11..15}$] <- LMD

Operation :

(*a*) Write result into the register file, whether it comes from the memory(LMD) or from ALU (ALUOutput)

(*b*) Register destination field is in one of two positions depending on the opcode

COMPUTER CONFIGURATION

Components of the computer

1. Central Processing Unit (CPU)
2. Computer Memory (RAM and ROM)
3. Data bus
4. Ports
5. Motherboard
6. Hard disk
7. Output Devices
8. Input Devices

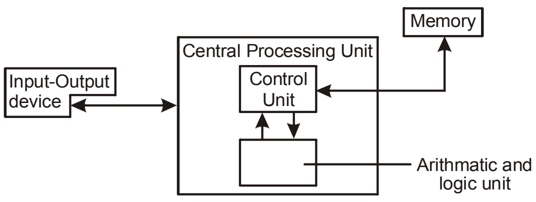
All these components are inter-connected for the personal computer to work.

FUNCTIONAL UNITS.

The computer allocates the task between its various functional units. The computer system is divided into following three separate units for its operation :

1. Central Processing Unit (CPU)

ALU and the CU of a computer system are jointly called *Central Processing Unit* (CPU). It can be called as brain of any computer system. It is just like brain that takes all major decisions, makes all sorts of calculations and directs different parts of the computer functions by activating and controlling the operations.



Block Diagram of CPU

2. Arithmetic Logical Unit (ALU)

After entering data through the input device, it is stored in the primary storage unit. The actual processing of the data and instruction are performed by Arithmetic Logical Unit.

Major operations performed by the ALU are :

addition, subtraction, multiplication, division, logic and comparison.

Data is transferred to ALU from storage unit when required. After processing the output is returned back to storage unit for further processing or getting stored.

3. Control Unit (CU)

It acts like the supervisor seeing that things are done in proper fashion. The control unit determines the sequence in which computer programs and instructions are executed. Processing of programs are stored in the main memory, interpretation of the instructions and issuing of signals for other units of the computer to execute them. It also acts as a switch board operator when several users access the computer simultaneously. Thereby it coordinates activities of computer's peripheral equipment as they perform input and output. Therefore it is the manager of all operations.

ARITHMETIC LOGIC UNIT (ALU).

ALU is the circuit which does all arithmetic and logic operations. It is combinational circuit.

It consists of two parts :

1. Arithmetic circuit
2. Logic circuit

1. Arithmetic circuit

In arithmetic circuit, basic operations are :

(*i*) Addition

(*ii*) Subtraction

(*iii*) Increment

(*iv*) Decrement

(*v*) Shift

Basic Arithmetic Circuits

(*i*) ***Binary Adder :***

It is a digital circuit that generates arithmetic sum of two binary numbers of any length.

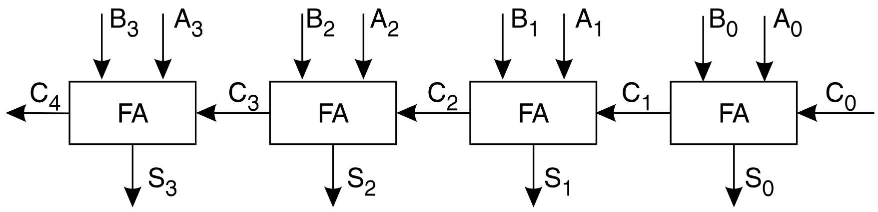


Fig. Four bit binary adder

(*ii*) ***Adder Subtractor :***

When M = 0, it works as adder

Then $B \oplus 0 = B$

When M = 1, it works as subtractor

Then $B \oplus 1 = \overline{B}$ and C_0 is 1.

Therefore, $A + \overline{B} + 1 = (A - B)$

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For signed numbers, result is A – B. Provided there is no overflow.

For unsigned numbers result is A – B if A ≥ B

or 2's complement (B – A) if A < B

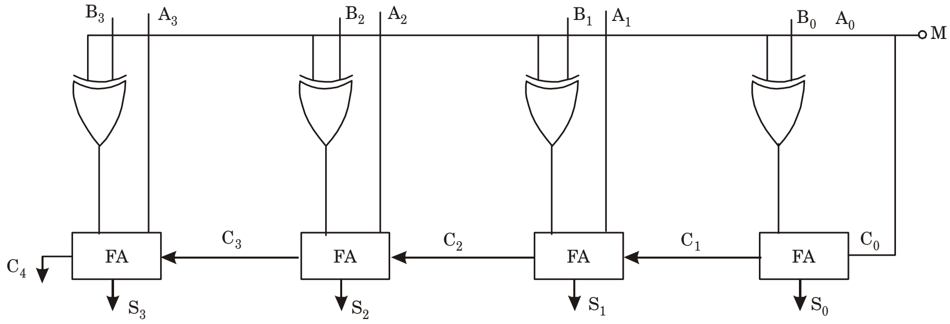

Fig. Four bit adder subtraction

(*iii*) *Binary Incrementer :*

Any binary number can be incremented by 1 with binary counters. When count enable is active, clock pulse transition increments the contents of registers by 1.

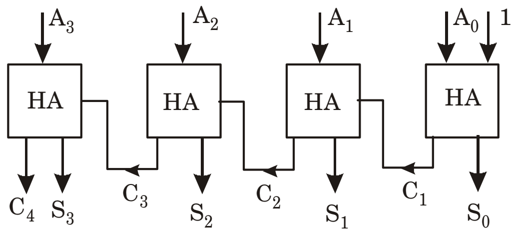

Fig. Four bit binary incrementer

Some Arithmetic operations :

Symbolic designation	Description
$R_3 \leftarrow R_1 + R_2$	Contents of R_1 plus R_2 transfered to R_3
$R_3 \rightarrow R_1 - R_2$	Contents of R_1 minus R_2 transfered to R_3
$R_1 \leftarrow \overline{R}_1$	1's complement of contents of R_1
$R_1 \leftarrow \overline{R}_1 + 1$	2's complement of contents of R_1 (negate)
$R_3 \leftarrow R_1 + \overline{R}_2 + 1$	R_1 plus 2's complement of R_2 (subtract)
$R_1 \leftarrow R_1 + 1$	Increment R_1 by 1
$R_1 \leftarrow R_1 - 1$	Decrement R_1 by 1

2. LOGIC CIRCUITS.

Four basic logic gates (AND, OR, XOR and NOT) can realize rest logic gates.

Output	Operations
$A \cap B$	AND
$A \cup B$	OR
$A \oplus B$	XOR
$F = \overline{A}$	Complement

Applications of logical operations :

(*i*) Change bit values

(*ii*) Delete a group of bits

(*iii*) Insert new bit values into a Register.

e.g. Register A = processor register.

Register B = Logic operand extracted from memory and placed in B.

CPU CONTROL UNIT

INTRODUCTION

The computer reads the instruction from memory and place it in the control register. Then control section interprets the binary code of the instruction and execute it as a sequence of micro-operations. Therefore control unit is responsible for initiating some micro-operations in CPU.

INSTRUCTIONS

1. Micro-instruction

The content of word in ROM at a given address specifies a microinstruction.

2. Micro-program

A sequence of microinstructions constitutes a microprogram.

TYPES OF CONTROL UNIT

According to internal hardware, control units are of following types :

1. *Hard-wired Control unit*

When control unit is made up of only hardware.

(*i*) Hardwired control unit with Flip-flop.

Control unit attains several states and for each state some output signals are activated. For each state separate flip-flops are used in control unit. For any output signal, some flip-flops of control unit are set to 1 and some set to 0. But in this design, more number of flip-flops are required.

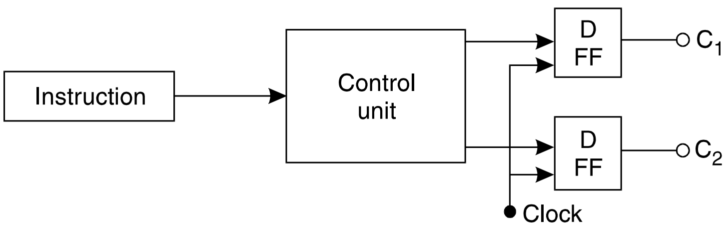

Fig. Hard-wired control unit flip-flop

(*ii*) Hard-wired control unit with Registers.

In this control unit design, a register is used to select one of the output signals. An n-bit register can represent 2^n states (2^n combinations of n-bits)

2. *Programmed Control unit*

When control unit has inbuilt memory (ROM) in which programmes are permanently stored and ROM is responsible for outputting the control signals.

Micro-programmed Control unit.

In this, memory consists of control variables that initiate micro-operations. Memory element for any unit is usually ROM. The control sequence is permanent which needs no alteration. Memory element stores micro-instructions which specify operations for the components in control unit. Control unit also consists of a special circuit to select the next address as specified by the micro-instruction.

MEMORY SYSTEM

There are two types of computer memory :

1. Primary memory

It is accessible directly by the processing unit

e.g., RAM

As soon as the computer is switched off, contents of the primary memory is lost. Data can be stored and retrieved much faster with primary memory compared to secondary memory.

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Primary memory in the computer is in the form of IC's (Integrated Circuits). These circuits are called *Random Access Memory (RAM)*. Each of RAM's locations stores one *byte* of information. (One *byte* is equal to *8 bits*). A bit is an acronym for *binary digit*, which stands for one binary piece of information. This can be either 0 or 1.

Primary or internal storage section is made up of several small storage locations (ICs) called *cells*. Each of these cells can store a fixed number of bits called *word length***.**

Capacity of Primary memory.

As each cell of memory contains one character or 1 byte of data, so capacity is defined in terms of *byte or words*.

e.g., 64 kilobyte (KB) memory is capable of storing $64 \times 1024 = 65536$ bytes. (1 kilobyte is 1024 bytes).

A memory size ranges from few kilobytes in small systems to several thousand kilobytes in large mainframe and super computer. In personal computer, we find memory capacity in the range of 64 KB, 4 MB, 8 MB and even 16 MB (MB = Million bytes).

2. **Secondary memory**

   It is located outside the computer.

Note : Primary memory is more expensive than secondary memory and because of this, size of primary memory is less than that of secondary memory.

SOME RELATED TERMS.

1. **Random Access Memory (RAM)**

   Primary storage is called *random access memory* (RAM) because it is possible to randomly select and use any location of the memory directly store and retrieve data. It takes same time to any address of the memory as the first address. It is also called *read / write memory*. The storage of data and instructions inside the primary storage is temporary. It disappears from RAM as soon as power to the computer is switched off. The memories, which loose their content on failure of power supply, are called *volatile memories*. So RAM is volatile memory.

2. **Read Only Memory (ROM)**

   There is another memory in computer, called *Read Only Memory* (ROM). It is the ICs inside the PC that form the ROM. The storage of program and data in the ROM is permanent. ROM stores some standard processing programs supplied by the manufacturers to operate the personal computer. ROM can only be read by the CPU but it cannot be changed. The basic input/output program is stored in the ROM that examines and initializes various equipment attached to the PC when switch is made ON. The memories, which do not loose their content on failure of power supply, are called *non-volatile* memories. ROM is non-volatile memory.

3. **Programmable Read Only Memory (PROM)**

   There is another type of primary memory in computer, called PROM. It is not possible to modify or erase programs stored in ROM, but it is possible to store program in PROM chip. Once the programmes are written, it cannot be changed and remain intact even if power is switched off. Therefore programs or instructions written in PROM or ROM cannot be erased or changed.

4. **Erasable Programmable Read Only Memory (EPROM)**

   It overcome the problem of PROM and ROM. EPROM chip can be programmed time and again by erasing the information stored earlier in it. Information stored in EPROM exposing the chip for some time ultraviolet light and it erases chip is reprogrammed using a special programming facility. When the EPROM is in use, information can only be read.

5. **Cache Memory**

   The speed of CPU is extremely high compared to the access time of main memory. Therefore performance of CPU decreases due to the slow speed of main memory. To decrease mismatch in operating speed, a small memory chip is attached between CPU and Main memory whose access time is very close to the

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processing speed of CPU. It is called *cache memory*. Cache memories are accessed much faster than conventional RAM. It is used to store programs or data currently being executed or temporary data frequently used by the CPU. So each memory makes main memory to be faster and larger than it really is. It is also very expensive to have bigger size of cache memory and its size is normally kept small.

6. **Registers**

CPU processes data and instructions with high speed, there is also movement of data between various units of computer. It is necessary to transfer the processed data with high speed. So the computer uses a number of special memory units called *registers*. They are not part of the main memory but they store data or information temporarily and pass it on as directed by the control unit.

MEMORY ARCHITECTURE AND INTERFACE

The first step when designing a high bandwidth interconnect scheme between processor and DRAM in an IRAM system is to provide the proper memory architecture and interface. Higher bandwidth was achieved by improvements in the interface (EDO, SDRAM, RDRAM). Since bandwidth is limited by the small number of external pins, there was no real motivation for major architectural enhancements. IRAM and embedded DRAM remove this limitation and open the way for new DRAM organizations, which can also be optimized for the specific type of logic core on the die.

Section Organization

The memory subsystem in an IRAM is divided into blocks called *memory sections*. It is defined as minimum memory partition that can be potentially addressed independently. Therefore, multiple sections enable parallelism of accesses. Each section consists of a section controller and a number of memory banks, connected through the section bus. The controller provides the interface between banks and the processor. Its structure and functionality can depend on the specific processor architecture and it does not affect organization of the rest of the section.

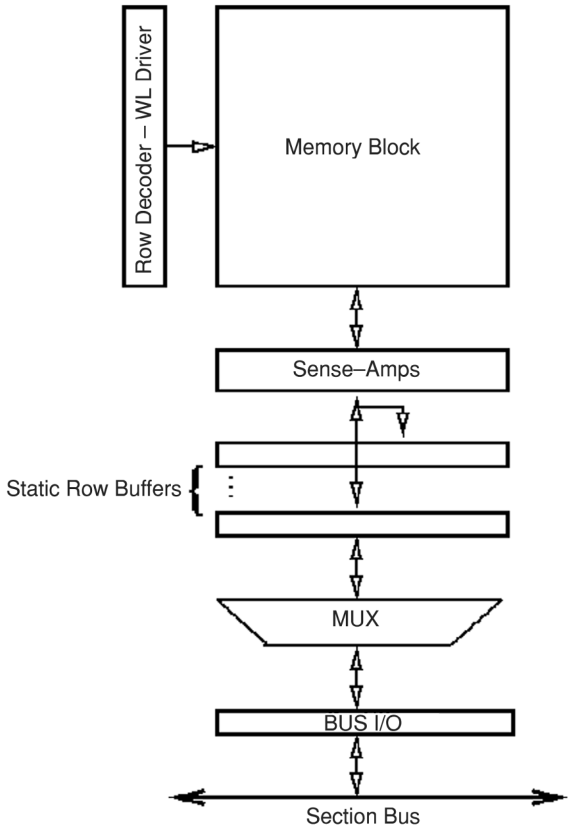

Fig. Block diagram of the memory bank

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Basic components :

(*i*) Memory block

(*ii*) Row decoder

(*iii*) Sense amplifiers.

Each row access delivers to the sense amplifiers a large number of bits, which are a multiple of the processor word. Here assume that word length is 64 bits, ignoring fact that a single processor address may describe a larger amount of data under certain schemes. These bits can be stored in one of the static row buffers (rows of static latches). Using buffers at the outputs of the sense-amplifiers had been proposed in the past in order to decouple the next read access from reading out to the output the data of the previous one. Here it is proposed to use one or potentially more registers to provide both decoupling and caching. Multiple rows from a single bank can be concurrently open if they are stored in separate row buffers. This reduces number of expensive RAS accesses to the memory block. The exact number of buffers is an architectural decision that depends both on performance and area overhead, but in any case there is atleast one. Words stored in a row buffer can be updated or read through the section bus, one per clock cycle. The section bus must be as wide as a word. There is no need for a large column decoder in the bank, since operations are either performed on a word or a whole row. Only it is required to be able to specify the specific row buffer or word within a row buffer involved in an operation. This information, along with the rest of the control for a memory operation, must be brought through the section bus, either in encoded or decoded form.

Floor plan of Momory bank.

Actual floorplan of a memory bank is different from what the block diagram implies due to following reasons :

(*i*) It has to be constructed from the small rectangular DRAM arrays, which usually have a capacity of 128 to 256Kbits because of the restrictions to the maximum width of bit-lines and word-lines in a DRAM array

(*ii*) Aspect ratio of a DRAM cell is 2 : 1 and width of a sense amplifier is four times wider than a memory cell, sense-amplifier and I/O circuits have to be placed at both sides of the memory array.

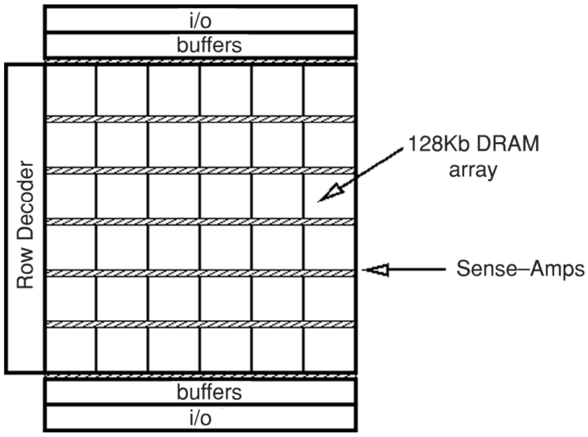

Fig. Floorplan of a memory bank.

Above figure present floorplan of the memory bank as a two dimensional organization of 128Kbit arrays. A linear organization would fail to amortize area cost of the decoder or buffers and I/O circuits over a decent amount of memory. In any case, only a single memory row (word-line) is activated per access to a bank. Row buffers are placed at the top and bottom and not in between arrays to increase associativity, in other words the number of rows that can use a specific set of buffers. Transfer from internal sense-amplifiers to the buffers can be made at no cost by using segmented (or hierarchical) bit-lines, which would enable reductions in the number of sense-amplifier rows as well. If segmented bit lines are available, they can be constructed on top of the arrays, at the cost of an extra layer of metal.

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DESIGN PARAMETERS.

Memory architecture has a number of design parameters that can be adjusted according to the requirements of a specific implementation.

(*i*) Number of Memory sections

It determines available access parallelism. This number has no effect on memory size but it can increase significantly area of the memory crossbar, if it is decided to address each one independently. The increased number of sections also puts high demands on the address generation unit on the processor side as well, due to TLB lookups, conflict resolution etc.

(*ii*) Size of a Memory bank

Larger bank sizes are attractive since area cost of bank periphery is amortized over a large memory area and there are less receivers/drivers on the section bus. Specifically, increased bank height amortizes cost of row buffers and I/O circuits, while increase bank length amortizes cost of the decoder. In terms of energy, it is better to have many small banks. On each RAS access, it is required to activate a whole bank, thus smaller the better.

Width of the bank has a second effect as well. By increasing it, width of row buffers also increases. While initially increasing block size is good in terms of performance due to prefetching, after a certain point we end up prefetching data that will never be used, which is unacceptable in terms of energy.

(*iii*) Number of Row buffers per bank

Increasing number of buffers comes with an area penalty due to

(*a*) area they take up

(*b*) additional complexity of multiplexing at the bank's interface.

Additional cache size does improve performance (up to the point where compulsory misses dominate), but it may not be worth it to sacrifice much of area for it, as it could be used for on-chip DRAM.

To define appropriate interface, architectures and corresponding interfaces of several high-performance DRAMs are examined. An interface similar to that of RDRAM was not preferred since most of architectural decisions associated with it were based on characteristics of off-chip interconnect. RDRAM uses narrow busses and achieves high-bandwidth at the cost of higher latency. Still, in architecture the section bus is significantly wider, so high-bandwidth can be achieved without hurting latency. In addition, packet-based interface used in RDRAM would add extra latency and complexity without offering any advantages. For these reasons, interface was based on interfaces defined for Synchronous DRAM.

Memory Interface Commands

Command	Arguments	Operation
Precharge	#bank	Precharge bit-lines
Read2B	#bank, #address, #buffer	Read memory row to buffer <- data restoring is optional
Write2B	#bank, #address, #buffer	Write back buffer to memory row
Read2S	#bank, #buffer, #word	Read word from buffer to section bus
Write2S	#bank, #buffer, #word	Write word from section bus to buffer
Refresh	#bank, #row	Refresh memory row

Above table presents sixcommands of the proposed interface for the IRAM memory architecture. These commands are transfered to memory banks through the section bus.

Read and write RAS accesses just transfer rows (multiple words) from memory rows to a row buffer and vice versa. Read accesses have optional restoring. Restoring memory data that will be overwritten soon is energy-inefficient. From the energy perspective, it is better to make all the updates on a specific row in a row buffer and write it back only once at the end. Commands Read2S and Write2S transfer one of the words in the data buffer to the section bus and vice versa.

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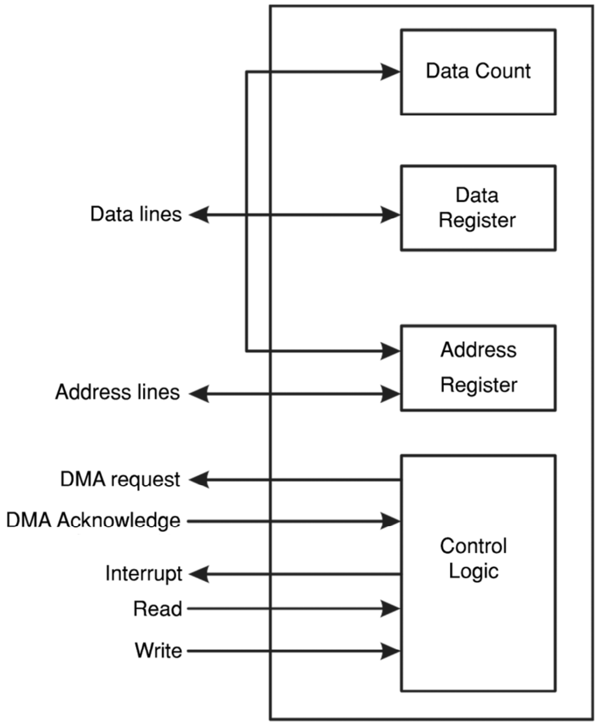
I/O INTERFACE

DIRECT MEMORY ACCESS (DMA)



Device wishing to perform DMA asserts the processors bus request signal. Processor completes the current bus cycle and then asserts the bus grant signal to the device. Then device asserts the bus grant ack signal.

The processor senses change in the state of bus grant ACK signal and starts listening to the data and address bus for DMA activity.

The DMA device performs transfer from the source to destination address. During these transfers, processor monitors addresses on the bus and checks if any location modified during DMA operations is cached in the processor.

If processor detects a cached address on the bus, it can take one of the two actions :

(*i*) Processor invalidates the internal cache entry for the address involved in DMA write operation

(*ii*) Processor updates the internal cache when a DMA write is detected

Once DMA operations have been completed, the device releases the bus by asserting the bus release signal. Processor acknowledges the bus release and resumes its bus cycles from the point it left off.

INTERRUPT ACKNOWLEDGE CYCLE.

Interrupt handling in a scenario where hardware does not support identifying the device that initiated the interrupt. In such cases, the possible interrupting devices need to be polled in software.

- A device asserts the interrupt signal at a hardwired interrupt level.
- The processor registers the interrupt and waits to finish the current instruction execution. Once the current instruction execution is completed, the processor initiates the interrupt handling by saving the current register contents on the stack.
- Then processor switches to supervisor mode and initiates an interrupt acknowledge cycle.
- No device responds to the interrupt acknowledge cycle, so the processor fetches the vector corresponding to the interrupt level.
- The address found at the vector is the address of the interrupt service routine (ISR).
- The ISR polls all the devices to find the device that caused the interrupt. This is accomplished by checking the interrupt status registers on the devices that could have triggered the interrupt.
- Once the device is located, control is transferred to the handler specific to the interrupting device.
- After the device specific ISR routine has performed its job, ISR executes the “return from interrupt” instruction, which results in restoring the processor state. The processor is restored back to user mode.

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SYNCHRONIZATION.

Synchronization requirements for DMA and Interrupts

Many times software designers have to work with data structures that are shared with interrupts or DMA devices. This requires performing atomic updates to the shared critical regions.

Synchronization with Interrupts

When a data structure is shared with an ISR, disabling the interrupt to execute critical region updates is a good technique. Disabling of interrupts should be restricted to only the code that updates the critical region. Keeping the interrupts disabled for a long time will increase the interrupt latency.

Another option is to make use of the fact that interrupts are processed at instruction boundaries. A single instruction that performs read as well as write could be used to perform an atomic transaction.

Synchronization with DMA

Sharing data structures with a DMA device is tricky. The processor can initiate a DMA operation at a bus cycle boundary. This means that a new DMA operation can be started in the middle of an instruction execution. Best mechanism to perform critical region updates is to use the read-modify-write bus cycle. With this instruction, atomic updates can be made to critical regions as the read and write are glued together in a special bus cycle.

Another option is to disable DMA operation.

Precautions

Extreme caution should be used when employing these techniques :

(*i*) Some processors also support disabling DMA operations by using locked bus cycles. The processor could execute lock instruction to disable external bus grants. When critical region updates have been completed, unlock instruction is used to allow bus grants.

(*ii*) Another mechanism to prevent DMA might be to temporarily disable the device that will perform DMA. e.g., if DMA operations are being performed by an Ethernet controller, disabling Ethernet controller will make sure no DMA operations are started when a critical region update is being made.

PIPELINING

It is a technique of decomposing a sequential process into suboperations, with each sub-process being executed in a special dedicated segment that operates concurrently with all others. It is the way to speed up the machine by building a hardware out of several functional units and then pipeline them. About 30% of instructions are jump instruction which affect the functioning of pipeline.

$$\text{Average instruction time} = (1 - P_j)(1) + P_j [P_t (1 + b) + (1 - P_t)(1)]$$

$$\text{Execution efficiency} = \frac{1}{1 + bP_jP_t}$$

where, P_j = probability that instruction is a jump.

P_t = probability that the jump is taken.

E_b = jump penalty (loss of cycles due to jump).

e.g. If $b = 4$, $P_j = 0.3$, $P_t = 0.65$, then machine runs at less than 60 percent of its potential speed.

Basic Performance Issues in Pipelining

Pipelining increases the CPU instruction throughput - the number of instructions completed per unit of time. But it does not reduce execution time of an individual instruction. In fact, it usually slightly increases execution time of each instruction due to overhead in the pipeline control.

The increase in instruction throughput means that a program runs faster and has lower total execution time.

Limitations on Practical depth of a pipeline

It arise from following :

(*i*) Pipeline latency.

Execution time of each instruction does not decrease puts limitations on pipeline depth;

(*ii*) Imbalance among pipeline stages.

It reduces performance since the clock can run no faster than the time needed for the slowest pipeline stage;

(*iii*) Pipeline overhead.

It arises from the combination of *pipeline register delay* (setup time plus propagation delay) and *clock skew.*

Once the clock cycle is as small as sum of the clock skew and latch overhead, no further pipelining is useful, since there is no time left in the cycle for useful work.

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PIPELINE HAZARDS.

There are situations that prevent next instruction in the instruction stream from being executing during its designated clock cycle, called *hazards*.

Classes of Hazard

There are three classes of hazards :

1. **Structural Hazards.**
   These arise from resource conflicts when hardware cannot support all possible combinations of instructions in simultaneous overlapped execution.
2. **Data Hazards.**
   These arise when an instruction depends on the result of a previous instruction in a way that is exposed by overlapping of instructions in the pipeline.
3. **Control Hazards.**
   These arise from the pipelining of branches and other instructions that change the PC.

Hazards in pipelines can make it necessary to stall the pipeline.

Processor can stall on following events :

(*i*) C*ache miss.*
It stalls all the instructions on pipeline both before and after the instruction causing the miss.

(*ii*) H*azard in pipeline.*
Eliminating a hazard often requires that some instructions in the pipeline to be allowed to proceed while others are delayed. When the instruction is stalled, all the instructions issued *later* than the stalled instruction are also stalled. Instructions issued *earlier* than the stalled instruction must continue, since otherwise the hazard will never clear.

A hazard causes pipeline bubbles to be inserted. *Following table shows how the stalls are actually implemented. As a result, no new instructions are fetched during clock cycle*

CONDITIONS FOR STALLS REQUIREMENT.

Consider following sequence of instructions :

		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
LW	**R1**, 0(R1)	IF	ID	EX	**MEM**	**WB**			
SUB	R4, **R1**, R5		IF	ID	**EX**	MEM	WB		
AND	R6, **R1** R7			IF	ID	**EX**	MEM	WB	
OR	R8, **R1**, R9				IF	**ID**	EX	MEM	WB

LW instruction **does not** have the data until the end of clock cycle 4 (**MEM**) , while **SUB** instruction needs to have the data by the beginning of that clock cycle (**EX**).

For **AND** instruction, the result can be forwarded immediately to the ALU (**EX**) from the MEM/WB register(**MEM**).

OR instruction has no problem, since it receives value through the register file (**ID**). In clock cycle no. 5, WB of the LW instruction occurs "*early*" in first half of the cycle and register read of the OR instruction occurs "*late*" in the second half of the cycle.

For **SUB** instruction, the forwarded result would arrive too late - at the end of a clock cycle, when needed at the beginning.

The load instruction has a delay or latency that cannot be eliminated by forwarding alone. Instead, we need to add hardware, called ***a pipeline interlock***, to preserve the correct execution pattern. In general, a pipeline interlock ***detects*** a hazard and ***stalls*** the pipeline until the hazard is cleared.

Pipeline with a Stall and Legal forwarding.

		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	9
LW	**R1**, 0(R1)	IF	ID	EX	**MEM**	WB				
SUB	R4, **R1**, R5		IF	ID	***stall***	**EX**	MEM	WB		
AND	R6, **R1** R7			IF	***stall***	ID	EX	MEM	WB	
OR	R8, **R1**, R9				***stall***	IF	ID	EX	MEM	WB

The only necessary forwarding is done for R1 from **MEM** to **EX**.

There is no need to forward R1 for **AND** instruction because now it is getting value through the register file in ID (as **OR** above).

There are techniques to reduce number of stalls even in this case.

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CACHE AND MAIN MEMORY

INTRODUCTION

Memory Hierarchy

In principle, for a simple single processor machine, the memory architecture is quite simple. Memory is connected to the memory address lines and the memory data lines so that whenever an address is presented to the memory, data corresponding to that address appears on the data lines. This is adequate for processors which can address a relatively small address space.

Cache Memory

Cache is a small amount of high-speed memory, usually with a memory cycle time comparable to the time required by the CPU to fetch one instruction. The cache is usually filled from main memory when instructions or data are fetched into the CPU. Often main memory will supply a wider data word to the cache than the CPU requires, to fill the cache more rapidly. The amount of information which is replaces at one time in the cache is called *line size* for the cache. This is normally width of the data bus between cache memory and the main memory. A wide line size for the cache means that several instruction or data words are loaded into the cache at one time, providing a type of prefetching for instructions or data.

Since cache is small, effectiveness of the cache relies on following properties of most programs :

(*i*) **Spatial locality :** Most programs are highly sequential; the next instruction usually comes from the next memory location.

Data is usually structured, and data in these structures normally are stored in contiguous memory locations.

(*ii*) **Short loops :** These are a common program structure, especially for the innermost sets of nested loops. This means that the same small set of instructions is used over and over.

Generally, several operations are performed on the same data values, or variables.

Associative Memory

When a cache is used, there must be some way in which memory controller determines whether the value currently being addressed in memory is available from the cache. There are several ways that this can be accomplished. One possibility is to store both the address and the value from main memory in the cache, with the address stored in a type of memory called *associative memory* or, *content addressable memory*.

This memory has the property that when a value is presented to the memory, *address* of value is returned if the value is stored in the memory, otherwise an indication that the value is not in the associative memory is returned. *All* of the comparisons are done simultaneously, so the search is performed very quickly. This type of memory is very expensive, because each memory location must have both a comparator and a storage element. A cache memory can be implemented with a block of associative memory, together with a block of "*ordinary*" memory. The associative memory would hold *address* of the data stored in the cache, and ordinary memory would contain the data at that address. Such a cache memory might be configured as shown below.

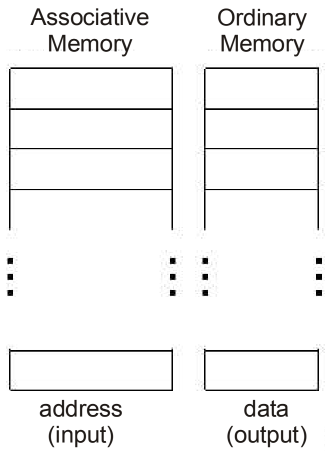

Fig. Cache implemented with associative memory

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If address is not found in the associative memory, then value is obtained from main memory.

Cache memories normally allow one of the following two things to happen when data is written into a memory location for which there is a value stored in cache :

(*i*) Write through cache.

Both the cache and main memory are updated at the same time. This may slow down the execution of instructions which write data to memory, because of the relatively longer write time to main memory. Buffering memory writes can help speed up memory writes if they are relatively infrequent, however.

(*ii*) Write back cache.

Only cache is updated directly by the CPU; cache memory controller marks the value so that it can be written back into memory when word is removed from the cache. This method is used because a memory location may often be altered several times while it is still in cache without having to write the value into main memory. This method is often implemented using an "*altered*" bit in the cache. *ALTERED* bit is set whenever a cache value is written into by the processor. Only if *altered* bit is set is it necessary to write the value back into main memory (*i.e.*, only values which have been altered must be written back into main memory). The value should be written back immediately before the value is replaced in the cache.

Cache Miss

Cache miss on an instruction fetch requires that the processor '*stall*' or wait until the instruction is available from main memory.

A *cache miss* on a *data read* may be less serious; instructions can, continue execution until the data to be fetched is actually required. In practice, data is used almost immediately after it is fetched.

Cache miss on a data word write may be even less serious; if the write is buffered, the processor can continue until the write buffer is full.

If miss rate for reads in a cache memory are known, number of read-stall cycles can be calculated as

$$\text{Read-stall cycles} = \text{Reads} \times \text{Read miss rate} \times \text{Read miss penalty}$$

For writes, the expression is similar, except that effect of the write buffer must be added in

$$\text{Write-stall cycles} = (\text{Writes} \times \text{Write miss rate} \times \text{Write miss penalty}) + \text{Write buffer stalls}$$

In many cache memory systems, penalties are the same for a cache read or write. Here we can use a single miss rate, and miss penalty :

$$\text{Memory-stall cycles} = \text{Memory access} \times \text{Cache miss rate} \times \text{Cache miss penalty}$$

SECONDARY STORAGE

In this type of memory, the cost per bit of storage is low. The operating speed is slower than that of the primary storage. Huge volume of data are stored here on permanent basis and transferred to the primary storage as and when required.

SECONDARY STORAGE DEVICES

1. Magnetic Tape

These are used for large computers like mainframe computers where large volume of data is stored for a longer time. In PC tapes can be used in the form of cassettes. The cost of storing data in tapes is inexpensive. Tapes consist of magnetic materials that store data permanently. It can be 12.5 mm to 25 mm wide plastic film-type and 500 meter to 1200 meter long which is coated with magnetic material. The deck is connected to the central processor and information is fed into or read from the tape through the processor. It similar to cassette tape recorder.

Advantages of Magnetic tape.

(*i*) **Compact:** A 10-inch diameter reel of tape is 2400 feet long and is able to hold 800, 1600 or 6250 characters in each inch of its length. The maximum capacity of such tape is 180 million characters. Thus data are stored much more compactly on tape.

(*ii*) **Economical:** Cost of storing characters is very less as compared to other storage devices.

(*iii*) **Fast:** Copying of data is easier and fast.

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(*iv*) **Long term Storage and Re-usability:** Magnetic tapes can be used for long term storage and a tape can be used repeatedly with out loss of data.

2. Magnetic Disk

Gramophone record is circular like a disk and coated with magnetic material. Magnetic disks used in computer are made on the same principle. It rotates with very high speed inside the computer drive. Data is stored on both the surface of the disk. Magnetic disks are most popular for *direct access* storage device. Each disk consists of a number of invisible *concentric circles* called *tracks*. Information is recorded on tracks of a disk surface in the form of tiny magnetic spots. The presence of a magnetic spot represents *one bit* and its absence represents zero bit. The information stored in a disk can be read many times without affecting the stored data. So the reading operation is non-destructive. But to write a new data, the existing data is erased from the disk and new data is recorded.

3. Floppy Disk

It is similar to magnetic disk discussed above. They are 5.25 inch or 3.5 inch in diameter. They come in single or double density and recorded on one or both surface of the diskette. The capacity of a 5.25-inch floppy is 1.2 mega bytes whereas for 3.5 inch floppy it is 1.44 mega bytes. It is cheaper than any other storage devices and is portable. The floppy is a low cost device particularly suitable for personal computer system.

4. Optical Disk

With every new application and software there is greater demand for memory capacity. It is the necessity to store large volume of data that has led to the development of optical disk storage medium.

Types of Optical disks.

Optical disks divided into following categories :

(***i***) **Compact Disk/ Read Only Memory (CD-ROM) :**

These are made of reflective metals. CD-ROM is written during the process of manufacturing by high power *laser beam.*

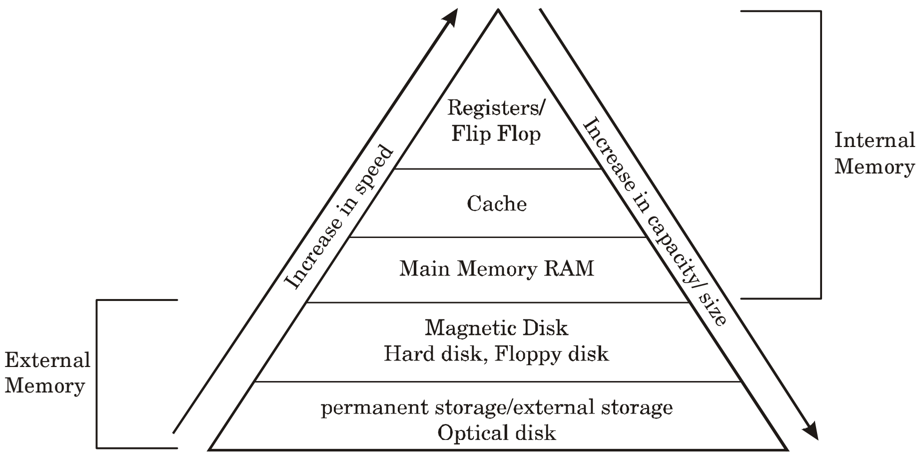
(***ii***) **Write Once, Read Many (WORM) :**

It allows the user to write data permanently on to the disk. Once the data is written, it can never be erased without physically damaging the disk.

(***iii***) **Erasable Optical Disk:**

These are optical disks where data can be written, erased and re-written. This also applies a laser beam to write and re-write the data.

MEMORY HIERARCHY



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SOLVED EXAMPLES

1. When a cache is 10 times faster than main memory, and cache can be used 90% of the time, how much speed we gain by using the cache ?

Solution.

Let M = Main memory access time

C = Cache memory access time = M/10 (given)

$$\text{Total access time using cache} = 0.9 + 0.1\text{ M} = 0.9\,\frac{\text{M}}{10} + 0.1\text{M} = 0.19\text{ M}$$

$$\therefore \qquad \text{Speed up} = \frac{\text{M}}{0.19\text{M}} \simeq 5.3$$

2. A machine has CPI (cycles per instruction) 2.0 when all memory accesses hit in the cache. The only data accesses are loads and stores, and these total 40% of the instructions. If miss penalty is 25 clock cycles and miss rate is 2%, how much faster would the machine be if all the instructions were cache hits ?

Solution.

Performance for machine that always hits,

$$\begin{aligned}\text{CPU}_{\text{execution time}} &= (\text{CPU clock cycles} + \text{Memory clock cycles}) \times \text{clock cycles} * \text{IC}\\ &= (\text{IC} * \text{CPI} + 0) * \text{clock cycle} \qquad \text{... [where IC} \equiv \text{instruction cycle]}\\ &= \text{IC} \times 2.0 \times \text{clock cycle}\end{aligned}$$

Now for machine with the real cache,

$$\begin{aligned}\text{Memory clock cycles} &= \text{IC} * \text{Memory reference per instruction} * \text{miss rate} * \text{Miss penalty}\\ &= \text{IC} * (1 + 0.4) * 0.2 * 25\\ &= \text{IC} * 0.7\end{aligned}$$

where middle term (1 to 0.4) represents one instruction access and 0.4 data accesses per instruction.

Hence total performance is

$$(\text{CPU execution time})_{\text{Cache}} = (\text{IC} * 2.0 + \text{IC} * 0.7) * \text{clock cycle} = 2.7 \times \text{IC} \times \text{clock cycle}$$

$$\therefore \qquad \frac{(\text{CPU execution time})_{\text{Cache}}}{\text{CPU execution time}} = \frac{2.7 \times \text{IC} \times \text{clock cycle}}{2.0 \times \text{IC} \times \text{clock cycle}} = 1.35$$

Hence machine with no cache misses is 1.35 times faster.

3. For a machine, assume the cache miss penalty is 50 clock cycles, and all instructions normally take 2.0 clock cycles. Assume the miss rate is 2%, and there is an average of 1.33 memory references per instruction.

What is the impact on performance when behaviour of the cache is included ?

Solution.

$$\text{CPU time} = \text{IC} \times \left(\text{CPI}_{\text{execution}} + \frac{\text{Memory clock cycle}}{\text{Instruction}}\right) \times \text{clock cycle time}$$

$\therefore$ Performance including cache misses,

$$\begin{aligned}(\text{CPU time})_{\text{with cache}} &= \text{IC} \times \{2.0 + (1.33 \times 2\% \times 50\} \times \text{clock cycle time}\\ &= \text{IC} \times 3.33 \times \text{clock cycle time}\end{aligned}$$

The clock cycle time and instruction count (TC) are the same, with or without a cache, so CPU time increases with CPI from 2.0 for a "*perfect cache*" to 3.33 with a cache that can miss. Hence, including memory hierarchy in the CPI, calculations stretches the CPU time by a factor of 1.67. Without any memory hierarchy at all, the CPI would increase to (2.0 + 50 × 1.33) or 68.5.

4. Consider the unpipelined machine with 10 ns clock cycles. It uses four cycles for ALU operations and branches whereas five cycles for memory operations. Assume that the relative frequencies of these operations are 40%, 20% and 40%, respectively. Let due to clock skew and setup pipelining, the machine adds 1ns of overhead to the clock. How much speed in the instruction execution rate will we gain from a pipeline ?

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Solution.

$$\begin{aligned}\text{Average instruction execution time} &= \text{Clock cycle} \times \text{Average CPI}\\ &= 10 \text{ ns} \times [(40\% + 20\%) \times 4 + 40\% \times 5]\\ &= 10 \text{ ns} \times 4.4 = 44 \text{ ns}\end{aligned}$$

In the pipelined implementation, clock must run at the speed of slowest stage plus overhead, which will be 10 + 1 or 11 ns; this is the average instruction execution time.

Thus speed up from pipelining,

$$(\text{Speed up})_{\text{pipelinging}} = \frac{\text{Average instruction time unpiplined}}{\text{Average instruction time pipelined}} = \frac{44 \text{ ns}}{11 \text{ ns}} = \textbf{4 times}$$

5. Assume that the time required for the five functional units, which operate in each of the five cycles are :

10 ns, 8 ns, 10 ns, 10 ns and 7 ns.

Assume that pipelining adds 1 ns of overhead. Find the speed up versus single cycle data path.

Solution.

Since unpipelined machine executes all instructions in a single clock cycle, its average time per instruction is simply the clock cycle time. The clock cycle time is equal to sum of the times for each step in the execution.

$$\text{Average instruction exection time} = 10 + 8 + 10 + 10 + 7 = 45 \text{ ns}$$

Clock cycle time on the pipelined machine must be the largest time for any stage in the pipeline (10ns) plus overhead of 1 ns, for a total of 11ns.

Since CPI is 1, this yields an average instruction execution time of 11ns.

$$\therefore \quad \text{Speed from pipelining} = \frac{\text{Average instruction time unpiplined}}{\text{Average instruction time pipelined}} = \frac{45 \text{ ns}}{11 \text{ ns}} = \textbf{4.1 times.}$$

6. An asynchronous serial communication controller which uses a start stop scheme for controlling the serial I/O of a system, is programmed for a string of length seven bits, one parity bit (odd parity) and one stop bit. The transmission rate is 1200 bits/second.

(*i*) What is the complete bit stream that is transmitted for the string '0110101' ?

(*ii*) How many strings can be transmitted per second ?

Solution.

(i) 1011010101 ***(ii)*** 120

7. Consider a CRT display that has a next mode display format of 80 × 25 characters with a 9 × 12 character cell. What is the video buffer RAM for the display to be used in monochrome (1 bit per pixel) graphics mode ?

Solution.

$$\begin{aligned}\text{Number of bits required} &= \text{Number of characters} \times \text{cell size}\\ &= 80 \times 25 \times 9 \times 12 = 216000 \text{ bits}\\ \therefore \quad \text{Size of RAM} &= 27000 \text{ bytes}\end{aligned}$$

8. Write an ALP for shifting a 16-bit number left by 2-bits.

Solution.

The number is stored in the memory locations 2501 and 2502 H. The result to be stored in the memory locations 2503 and 2504 H.

Add	Machine codes	Memories	Operands
2000	2A, 01, 25	LHLD	2501 H
2003	29	DAD	A
2004	29	DAD	A
2005	22, 03, 25	SHLD	2503 H
2008	76	HLT	

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DATA

2501–96, LSBs of the number.

2502–`15, MSBs of the number

Result

2503–58, LSBs of the result

2504–56, MSBs of the result.

9. A bus-organized CPU similar to figure given below has 16 registers with 32 bits in each, an ALU and a destination decoder.

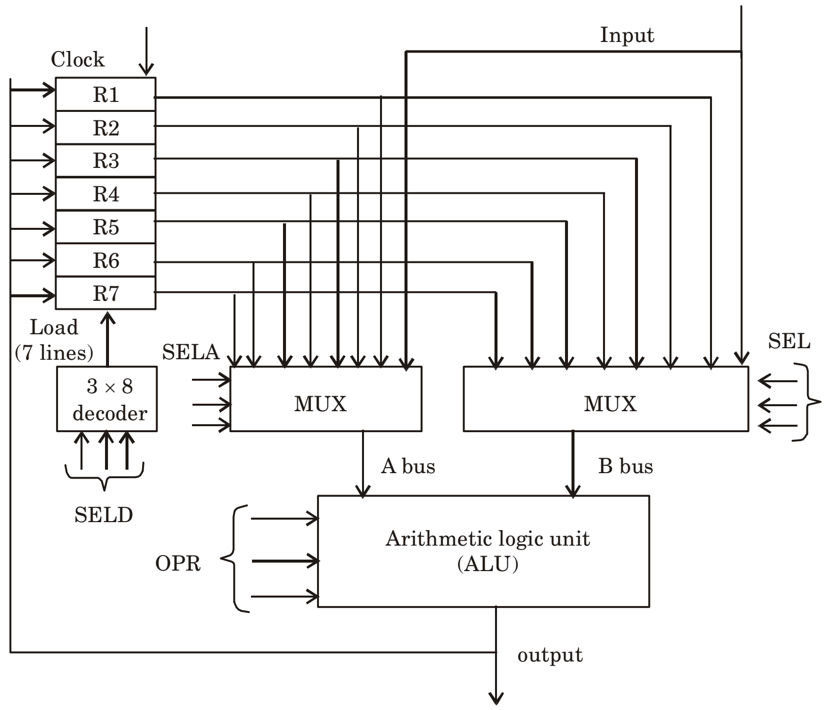
(*a*) How many multiplexers are there in the A bus, and what is the size of each multiplexer ?

(*b*) How many selection inputs are needed for MUX A and MUX B ?

(*c*) How many inputs and outputs are there in the decoder ?

(*d*) How many inputs and outputs are there in the ALU for data including input and output ?

(*e*) Formulate a control word for the system assuming that the ALU has 35 operations.



(*a*) Block diagram

3	3	3	3
SEL A	SEL B	SEL D	OPR

(*b*) Control word

Fig. Register set with common ALU.

Solution.

(*a*) 32 multiplexers, each of size 16×1.

(*b*) 4 inputs each to select one of 16 registers.

(*c*) 4-to-16 line decoder.

(*d*) 32 + 32 + 1 = 65 data input lines.

(*e*) 32 + 1 = 33 data output lines.

4	4	4	6	= 18 bits
SEL A	SEL B	SEL D	OPR	

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10. Consider the multiplication of two 40 × 40 matrices using a vector processor.

(*a*) How many product terms are there in each inner product, and how many inner products must be evaluated ?

(*b*) How many multiply add operations are needed to calculate the product matrix ?

Solution.

(*a*) There are 40 product terms in each inner product.

$40^2 = 1600$ inner product must be evaluated, one for each element of the product matrix.

(*b*) $40^3 = 64{,}000$.

11. Consider a computer with four floating point pipeline processors. Let each processor uses a cycle time of 40 ns. How long will it take to perform 400 floating-point operations ? Is there a difference if the same 400 operations are carried out using a single pipeline processor with a cycle time of 10 ns ?

Solution.

Divide 400 operations into each of the four processors,

$$\text{processing time} = \frac{400}{4} \times 40 = 4000 \text{ n sec}$$

Using a single pipeline, processing time = 400 × 10 = 4000 n sec

12. The instruction format of a CPU is

OP CODE	MODE	Reg R

← One memory word →

Mode and Reg R together specify the operand, Reg R specifies a CPU register and Mode specifies an addressing mode. In particular, Mode = 2 specifies that 'the register Reg R contains the address of the operand, after fetching the operand, the contents of Reg R are incremented by 1'.

An instruction at memory location 2000 specifies Mode = 2 and Reg R refers to program counter (PC).

(*i*) What is the address of the operand ?

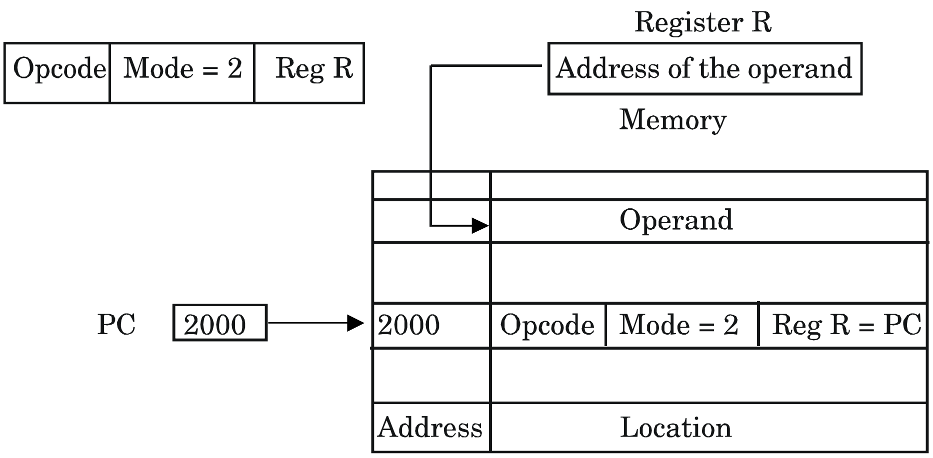
(*ii*) Assuming that this is a non-jump instruction, what are the contents of PC after the execution of this instruction ?

Solution.

(*i*) Address of the operand is the content of PC.

(*ii*) 2001

Method.



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EXERCISE – I

MCQ TYPE QUESTIONS

1. A low memory can be connected to microprocessor by using

(*a*) INTER (*b*) $\overline{\text{RESET IN}}$
(*c*) HOLD (*d*) READY

2. To put the microprocessor in the wait state

(*a*) lower the HOLD input
(*b*) lower the READY input
(*c*) raise the HOLD input
(*d*) raise the READY input

3. A basic instruction that can be interpreted by a computer generally has

(*a*) an operand and an address
(*b*) a decoder and an accumulator
(*c*) sequence register and decoder
(*d*) an address and decoder

4. In a microprocessor system with memory mapped I/O

(*a*) devices have 8-bit addresses
(*b*) devices are accessed using IN and OUT instructions
(*c*) there can be a maximum of 256 input devices and 256 output devices
(*d*) arithmetic and logic operations can be directly performed with the I/O data.

5. The addressing mode used in an instruction of the form ADD X, Y, is

(*a*) absolute (*b*) immediate
(*c*) indirect (*d*) index

6. Which of the following is responsible for coordinating various operatings using timing signals ?

(*a*) ALU (*b*) control unit
(*c*) memory unit (*d*) I/O unit

7. The register which keeps track of the execution of a program and which contains the memory address of the instruction currently being executed is called

(*a*) index register
(*b*) memory address register
(*c*) program counter
(*d*) instruction register

8. The register which holds the address of the location to or from which data are to be transferred is called

(*a*) index register
(*b*) instruction register
(*c*) memory address register
(*d*) memory data register

9. The register which contains the data to be written into or read out of the addressed location is called

(*a*) memory address register
(*b*) memory data register
(*c*) program computer
(*d*) index register

10. Which of the following is used as storage locations both in the ALU and the control section of a computer ?

(*a*) Accumulator (*b*) Register
(*c*) Adder (*d*) Decoder

11. The register used as a working area in CPU is

(*a*) program counter (*b*) instruction register
(*c*) instruction decoder (*d*) accumulator

12. Which of the following information holds the information before going to the decoder ?

(*a*) control register (*b*) data register
(*c*) accumulator (*d*) address register

13. The device which is used to connect a peripheral to bus is called

(*a*) control register
(*b*) interface
(*c*) communication protocol
(*d*) none of these

14. Which of the following is a set of general purpose internal registers ?

(*a*) stack (*b*) scratch pad
(*c*) address register (*d*) status register

15. Which of the following register is used in the control unit of the CPU to indicate the next instruction which is to be executed ?

(*a*) Accumulator
(*b*) Index register
(*c*) Instruction decoder
(*d*) Program counter

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16. An interrupt can be temporarily ignored by the counter is called
(*a*) vectored interrupt
(*b*) non-maskable interrupt
(*c*) maskable interrupt
(*d*) low priority interrupt

17. The ability to temporarily halt the CPU and use this time to send information on buses is called
(*a*) direct memory access
(*b*) vectoring the interrupt
(*c*) polling
(*d*) cycle stealing

18. In a microprocessor system, the RST instruction will cause an interrupt
(*a*) only if an interrupt service routine is not being executed
(*b*) only if a bit in the interrupt mask is made 0
(*c*) only if interrupts have been enabled by an EI instruction
(*d*) none of these

19. An instruction used to set the carry flag in a computer can be classified as
(*a*) data transfer (*b*) process control
(*c*) logical (*d*) program control

20. In a generic microprocessor instruction cycle time is
(*a*) shorter than machine cycle time
(*b*) larger than machine cycle time
(*c*) exactly double the machine cycle time
(*d*) exactly the same as the machine cycle time

21. The stack pointer in the microprocessor is a
(*a*) 16 bit register that point to stack memory locations
(*b*) 16 bit accumulator
(*c*) memory location in the stack
(*d*) flag register used for the stack

22. Micro program is
(*a*) the name of source program in micro computers
(*b*) the set of instructions indicating the primitive operations in a system
(*c*) primitive form of macros used in assembly language programming
(*d*) program of very small size

23. In a multiprocessor configuration, two co-processors are connected to the host 8086 processor. The two co-processor instruction sets
(*a*) must be the same
(*b*) may overlap
(*c*) must be disjoint
(*d*) must be the same as that of the host

24. A certain processor supports only the immediate and the direct addressing modes. Which of the following programming language features cannot be implemented on this processor ?
(*a*) Pointers (*b*) Arrays
(*c*) Records (*d*) All of these

25. The 8085 microprocessor responds to the presence of an interrupt
(*a*) as soon as the TRAP pin becomes 'high'
(*b*) by checking the TRAP pin for 'high' status at the end of each instruction fetch
(*c*) by checking the TRAP pin for 'high' status at the end of the execution of each instruction
(*d*) by Checking the TRAP pin for 'high' status at regular intervals

26. Match List I with List II and select the correct answer form the codes given below the list.

List I	List II
A. Indirect addressing	1. Loops
B. Immediate addressing	2. Pointers
C. Autodecrement addressing	3. Constants

Codes:

	A	B	C
(*a*)	3	2	2
(*b*)	1	3	2
(*c*)	2	3	1
(*d*)	3	1	2

27. Which of the following need not necessarily be saved on a context switch between processes ?
(*a*) General purpose registers
(*b*) Translation lookaside buffer
(*c*) Program counter
(*d*) All of these

28. If a processor does not have any stack pointer register, then
(*a*) it cannot have subroutine call instruction
(*b*) it can have subroutine call instruction, but no nested subroutine calls
(*c*) nested subroutine calls are possible, but interrupts are not
(*d*) all sequences of subroutine calls and also interrupts are possible

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29. CPU has two modes–privileged and non-privileged. In order to change the mode from privileged to non-privileged

(*a*) a hardware interrupt is needed

(*b*) a software interrupt is needed

(*c*) a privileged instruction (which does not generate an interrupt) is needed

(*d*) a non-privileged instruction (which does not generate an interrpt) is needed

30. In the absolute addressing mode

(*a*) operand is inside the instruction

(*b*) address of the operand is inside the instruction

(*c*) register containing the address of the operand is specified inside theinstruction

(*d*) location of the operand is implicit.

31. Which of the following is not typically found in the status register of a micro processor ?

(*a*) Overflow (*b*) Zero result

(*c*) Negative result (*d*) None of the above

32. The function of program counter (PC) holds

(*a*) temporary

(*b*) address for memory

(*c*) memory operand

(*d*) address for instruction

33. When a subroutine is called, then address of the instruction following the CAL instruction is stored in/on the.

(*a*) Stack pointer (*b*) Accumulator

(*c*) Program counter (*d*) Stack

34. The Program Counter (PC)

(*a*) is a register

(*b*) during exacution of the current instruction, its content changes

(*c*) both (*a*) and (*b*)

(*d*) none of these

35. The TRAP interrupt mechanism of the 8085 microposser executs

(*a*) an RST by hardware

(*b*) the instruction supplied by external device through the INTA signal

(*c*) an instruction from memory location 20 H

(*d*) a NOP

36. Pseudo-instructions are

(*a*) assembler directives

(*b*) instructions in any program that have no corresponding machine code instruction

(*c*) instruction in any program whose presence or absence will not change the output for any input

(*d*) none of these

37. The number of instructions needed to add 'n' numbers and store the result in memory using only one address instructions is

(*a*) n (*b*) $n + 1$

(*c*) $n - 1$ (*d*) independent of n

38. The addressing mode used in the instruction PUSH B is

(*a*) direct (*b*) register

(*c*) register indirect (*d*) immediate

39. The process of fetching and exacuting instructions, one at a time, in order of increasing addreses is called

(*a*) instruction execution

(*b*) straight line sequencing

(*c*) instruction fetch

(*d*) random sequencing

40. The CPU of a computer takes instruction from the memory and executes them. This process is called

(*a*) load cycle

(*b*) time sequence

(*c*) fetch-execute cycle

(*d*) clock cycle

41. In a microcomputer, WAIT states are used to

(*a*) make the processor wait during a DMA operation

(*b*) make the processor wait during a power interrupt processing

(*c*) make the processor wait during a power shutdown

(*d*) interface slow peripherals to the processor

42. When a program is being executed in an 8085 microprocessor, its Program Counter contains

(*a*) number of instructions in the current program that have already been executed

(*b*) the total number of instructions in the program being executed

(*c*) memory address of the instruction that is being currently executed

(*d*) memory address of the instruction that is to be executed next

43. When the HLT instruction of a 8085 micro-processor is executed, the microprocessor

(*a*) is disconnected from the system bus till the reset is pressed

(*b*) halts execution of the program and returns to monitor

(*c*) enters into a halt state and the buses are tri-stated

(*d*) reloads the program form the locations 0024 and 0025 H.

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44. Match the following in the context of an 8085 microprocessor.

	List I		List II
A.	DAA	1.	Program control instruction
B.	LXI	2.	Data movement instruction
C.	RST	3.	Interrupt instruction
D.	JMP	4.	Arithmetic instruction

Codes :

	A	B	C	D
(*a*)	1	2	3	4
(*b*)	4	2	3	1
(*c*)	3	2	1	4
(*d*)	2	3	4	1

45. Serial input data of 8085 can be loaded into bit 7 of the accumulator by
(*a*) executing a RIM instruction
(*b*) execution RST 1
(*c*) using TRAP
(*d*) none of these

46. Which of the following interrupts are unmaskable interrupts ?
(*a*) RST 5.5 (*b*) RST 7.5
(*c*) TRAP (*d*) INTR*1*

47. The memory address range to which RAM will respond
(*a*) 0000 H to 1 FFF H
(*b*) 0000 H to 5FFF H
(*c*) 4000 H to 5FFF H
(*d*) 3000 H to FFFF H

48. The address range to which I/O chip will respond is
(*a*) 0000 H to FFFF H
(*b*) 0000 H to 5FFF H
(*c*) 4000 H to 5FFFF H
(*d*) 3000 H to FFFF H

49. A stack is a/an
(*a*) 8-bit register in the microprocessor
(*b*) 16-bit register in the microprocessor
(*c*) set of memory locations in R/W memory reserved for storing information temporarily during he execution of a program
(*d*) 16-bit memory address stored in the h program counter

50. A device employing INTA line for device interrupt puts the CALL instruction on the data bus while
(*a*) $\overline{\text{INTA}}$ is active
(*b*) HOLD is active
(*c*) READY is active
(*d*) none of these

51. Which is the most appropriate match for the items in List I with the items in the List II ?

	List I		List II
A.	Indirect Addressing	1.	Array implementation
B.	Indexed Addressing	2.	Writing relocatable code
C.	Base Register Addressing	3.	Passing array as parameter

Codes :

	A	B	C
(*a*)	3	1	2
(*b*)	2	3	1
(*c*)	3	2	1
(*d*)	1	3	2

52. To put the microprocessor in the wait state
(*a*) lower the HOLD input
(*b*) lower the READY input
(*c*) raise the HOLD input
(*d*) raise the READY input

53. Both the arithmetic logic unit (ALU) and control section of CPU employ special purpose storage locations called
(*a*) decoders (*b*) buffers
(*c*) multiplexer (*d*) registers

54. A basic instruction that can be interpreted by a computer generally has
(*a*) an operand and an address
(*b*) a decoder and an accumulator
(*c*) sequence register and decoder
(*d*) an address and decoder

55. The differences between PLA and RON is
(*a*) PLA is combinational ROM is sequential
(*b*) PLA economises on the number of minterms
(*c*) PLA has fixed AND array, ROM has fixed OR array.
(*d*) none of these

56. The control unit of computer
(*a*) performs ALU operations on the data
(*b*) controls the operation of the output devices
(*c*) is a device for manually operating the computer
(*d*) directs the other unit of computers

57. The ALU of a computer normally contains a number of high speed storage elements called
(*a*) semi conductor memory
(*b*) registers
(*c*) hard disk
(*d*) magnetic disks

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58. The register which keeps track of the execution of a program and which contains the memory address of the instruction currently being executed is called
(*a*) index register
(*b*) memory address register
(*c*) program counter
(*d*) instruction register

59. The register which holds the address of the location to or from which data are to be transferred is called
(*a*) index register
(*b*) instruction register
(*c*) memory address register
(*d*) memory data register

60. The register which contains the data to be written into or read out of the addressed location is called
(*a*) memory address register
(*b*) memory data register
(*c*) program computer
(*d*) index register

61. The unit of a computer system which executes program, communicates with and often controls the operation of other subsystems of the computer is the
(*a*) CPU (*b*) control unit
(*c*) flo unit (*d*) peripheral unit

62. The ability of a medium sized computer system to increase in data processing capability by addition of such devices as mass storage device, I/O devices etc is called
(*a*) computer expandability
(*b*) computer mobility
(*c*) computer enahancement
(*d*) computer upward capability

63. The technique which repeatedly uses the same block of internal storage during different stage of problem is called
(*a*) overlay (*b*) overlapping
(*c*) swapping (*d*) reuse

64. The register used as a working area in CPU is
(*a*) program counter
(*b*) instruction register
(*c*) instruction decoder
(*d*) accumulator

65. Which of the following information holds the information before going to the decoder ?
(*a*) control register (*b*) data register
(*c*) accumulator (*d*) address register

66. Which of the following unit is used to supervise each instruction in the CPU ?
(*a*) Control logic unit (*b*) Accumulator
(*c*) ALU (*d*) Control register

67. The bus which is used to transfer data from main memory to peripheral device is
(*a*) data bus (*b*) input bus
(*c*) DMA bus (*d*) output bus

68. The device which is used to connect a peripheral to bus is called
(*a*) control register
(*b*) interface
(*c*) communication protocol
(*d*) none of these

69. The bus connected between the CPU and main memory that permits transfer of information between main memory and the CPU is called
(*a*) DMA bus (*b*) memory bus
(*c*) address bus (*d*) control bus

70. Which of the following register is used in the control unit of the CPU to indicate the next instruction which is to be executed ?
(*a*) Accumulator
(*b*) Index register
(*c*) Instruction decoder
(*d*) Program counter

71. An interrupt can be temporarily ignored by the counter is called
(*a*) vectored interrupt
(*b*) non-maskable interrupt
(*c*) maskable interrupt
(*d*) low priority interrupt

72. The ability to temporarily halt the CPU and use this time to send information on buses is called
(*a*) direct memory access
(*b*) vectoring the interrupt
(*c*) polling
(*d*) cycle stealing

73. Microprogramming is a technique for
(*a*) writing small programs effectively
(*b*) programming output/input routines
(*c*) programming the microprocessors
(*d*) programming the control steps of a computer

74. A device that works in conjunction with a computer but not as part of it is called
(*a*) microprocessor
(*b*) peripheral device
(*c*) hardware
(*d*) memory

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75. A system of letters, numbers symbols adopted by computer manufacturer as an abbreviated form of instruction sets is called

(*a*) mesh (*b*) monitor
(*c*) modern (*d*) mnemonic

76. When a subroutine is called, then address of the instruction following the CAL instruction is stored in/on the.

(*a*) Stack pointer (*b*) Accumulator
(*c*) Program counter (*d*) Stack

77. In 8085 microprocessor, the value of the most significant bit of the result following the execution of any arithmetic or Boolean instruction is stored in the

(*a*) carry status flag
(*b*) auxiliary carry status flag
(*c*) sign status flag
(*d*) zero status flag

78. PLA

(*a*) produces sum of prducts as the output
(*b*) is dedicated for a particular operation.
(*c*) is general
(*d*) both (*a*) and (*b*)

79. The sequence of events that happen during a typical fetch operation is

(*a*) PC $\rightarrow$ MAR $\rightarrow$ Memory $\rightarrow$ MDR $\rightarrow$ IR
(*b*) PC $\rightarrow$ Memory $\rightarrow$ MDR $\rightarrow$ IR
(*c*) PC $\rightarrow$ Memory $\rightarrow$ IR
(*d*) PC $\rightarrow$ MAR $\rightarrow$ Memory $\rightarrow$ IR

80. Which of the following is not a form of memory ?

(*a*) Instruction cache
(*b*) Instruction register
(*c*) Instruction opcode
(*d*) Translation lookaside buffer

81. Which memory is difficult to interface with processor ?

(*a*) Static memory (*b*) Dynamic memory
(*c*) ROM (*d*) RAM

82. Desirable characteristic(s) of a memory system is (are)

(*a*) speed and reliability
(*b*) low power consumption
(*c*) durability and compactness
(*d*) all of these

83. The minimum time delay required between initiation of two successive memory operations is called

(*a*) memory cycle time
(*b*) memory access time
(*c*) transmission time
(*d*) skip time

84. For a memory system, the cycle time is

(*a*) same as the access time
(*b*) longer than the access time
(*c*) shorter than the access time
(*d*) submultiple of the access time

85. Which of the following statements is wrong ?

(*a*) RAM is a type of volatile
(*b*) Magnetic tape is non-volatile
(*c*) Magnetic core and semiconductor memories are used as mass memory medium
(*d*) An EPROM can be programmed, erased and reprogrammed by the user with an EPROM programming instrument

86. The memory which is ultraviolet light erasable and electrically programmable is

(*a*) ROM (*b*) PROM
(*c*) RAM (*d*) EPROM

87. Which memory is non-volatile and may be written only once ?

(*a*) RAM (*b*) EE-ROM
(*c*) EPROM (*d*) PROM

88. The refreshing rate of dynamic RAMs is approximately once in

(*a*) two micro seconds
(*b*) two milli seconds
(*c*) fifty milli seconds
(*d*) two seconds

89. In comparison with static RAM memory, the dynamic RAM memory has

(*a*) lower bit density and higher power consumption
(*b*) higher bit density and higher power consumption
(*c*) lower bit density and lower power consumption
(*d*) higher bit density and lower power consumption

90. Disadvantage of dynamic RAM over static RAM is

(*a*) higher power consumption
(*b*) variable speed
(*c*) need to refresh the capacitor charge every once in two milliseconds
(*d*) higher bit density

91. EEPROM is

(*a*) electrically erasable (*b*) easily erasable
(*c*) non-erasable (*d*) effective erasable

92. The access time of magnetic bubble memory is approximately

(*a*) 30 nano seconds
(*b*) 30 micro seconds
(*c*) 30 milli seconds
(*d*) 0.3 seconds

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93. Serial access memories are useful in applications where
- (*a*) data consists of numbers
- (*b*) short access time is required
- (*c*) each stored word is processed differently
- (*d*) data naturally needs to flow in and out in serial form

94. What is the main advantage of magnetic core memory over semiconductor RAM memory ?
- (*a*) more compact and smaller
- (*b*) more economical
- (*c*) a bit does not have to be written after reading
- (*d*) non-volatile

95. Fastest type of memory from the following list is
- (*a*) tape
- (*b*) semiconductor memory
- (*c*) disk
- (*d*) bubble memory

96. The use of hardware in Memory management is through segment relocation and protection is
- (*a*) to perform address translation to reduce size of the memory
- (*b*) to perform address translation to reduce execution time overhead
- (*c*) both (*a*) and (*b*)
- (*d*) none of these

97. Memory refreshing may be done
- (*a*) by the CPU that contains a special regress counter, only
- (*b*) by an external refresh controller, only
- (*c*) either by the CPU or by an external refresh controller
- (*d*) none of these

98. Memory consisting of electronic circuits attached into silicon chip is known as
- (*a*) magnetic core memory
- (*b*) emiconductor memory
- (*c*) thin film memory
- (*d*) MOS memory

99. Choose the correct statement(s) from the following?
- (*a*) PROM contains a programmable AND array and a fixed OR array
- (*b*) PLA contains a fixed AND array and a programmable OR array
- (*c*) PROM contains a fixed AND array and programmable OR array
- (*d*) None of these

100. Which of the following is not true of primary storage?
- (*a*) It represents the decimal number through string of binary digits.
- (*b*) It stores operating system programs.
- (*c*) It stores data while they are being processed by CPU.
- (*d*) It stores the bulk of data used by computer application.

101. A dynamic RAM consists of
- (*a*) 6 transistors
- (*b*) 2 transistors and 2 capacitors
- (*c*) 1 transistor and 1 capacitor
- (*d*) 2 capacitors only

102. Semiconductor memory is
- (*a*) somewhat slower than magnetic core memory.
- (*b*) a volatile memory
- (*c*) somewhat longer than magnetic core memory
- (*d*) all of these

103. Which of the following is the internal memory of the system (computer) ?
- (*a*) CPU register (*b*) Cache
- (*c*) Main memory (*d*) All of these

104. The bus connected between the CPU and main memory that permits transfer of information between main memory and the CPU is called
- (*a*) DMA bus (*b*) memory bus
- (*c*) address bus (*d*) control bus

105. A software program stored in a ROM that can not be changed easily is called
- (*a*) hardware (*b*) linker
- (*c*) editor (*d*) firmware

106. A device that works in conjunction with a computer but not as part of it is called
- (*a*) microprocessor
- (*b*) peripheral device
- (*c*) hardware
- (*d*) memory

107. Match List I with List II and select the correct answer form the codes given below the list.

List I	List II
A. An 8-bit wide 5 word sequential memory will have	1. 8 Fixed 'AND' gates and 4 programmable OR' gate
B. A 256 × 4 EPROM has	2. Eight 4 bit shift registers
	3. 4 words of 32-bits each
	4. 8 address pins and 4 data pins output

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Codes:

	A	B
(*a*)	1	2
(*b*)	3	2
(*c*)	2	3
(*d*)	4	2

108. An advantage of memory interfacing is that
(*a*) a large memory is obtained
(*b*) effective speed of the memory is increased
(*c*) the cost of the memory is reduced
(*d*) a non-volatile memory is obtained

109. In a virtual memory system the address space specified by the address lines of the CPU must be .... than the physical memory size and .... than the secondary storage size.
(*a*) smaller, smaller
(*b*) smaller, larger
(*c*) larger, smaller
(*d*) larger, larger

110. List I shows some operating system abstractions and List II hardware components Match List I with List II and select the correct answer form the codes given below the list.

List I	List II
A. Thread	1. Interrupt
B. Virtual address space	2. Memory
C. File system	3. CPU
D. Signal	4. Disk

Codes.

	A	B	C	D
(*a*)	2	4	3	1
(*b*)	1	2	3	4
(*c*)	3	2	4	1
(*d*)	4	1	2	3

111. Which of the following is/are advantages of virtual memory ?
(*a*) Faster access to memory on an average.
(*b*) Processes can be given protected address spaces.
(*c*) both (*a*) and (*b*)
(*d*) Programms larger than the physical memory size can be run.

112. Which of the following need extra hardware for DRAM refreshing ?
(*a*) 8085
(*b*) Motoral-68000
(*c*) both (*a*) and (*b*)
(*d*) none of these

113. Choose the correct statement(s) from the following.
(*a*) PROM contains a programmable AND array and a fixed OR array
(*b*) PLA contains a fixed AND array and a programmable OR array
(*c*) PROM contains a fixed AND array and a programmable OR array
(*d*) PLA contains a programmable AND array and a programmable OR array.

114. Which memory is difficult to interface with processor ?
(*a*) Static memory (*b*) Dynamic memory
(*c*) ROM (*d*) RAM

115. Desirable characteristic(s) of a memory system is (are)
(*a*) speed and reliability
(*b*) low power consumption
(*c*) durability and compactness
(*d*) all of these

116. The minimum time delay required between initiation of two successive memory operations is called
(*a*) memory cycle time
(*b*) memory access time
(*c*) transmission time
(*d*) skip time

117. Non-volatility is an important advantage of
(*a*) CCD's (*b*) RAM
(*c*) Magnetic bubbles (*d*) PROM

118. Which memory is non-volatile and may be written only once ?
(*a*) RAM (*b*) EE-ROM
(*c*) EPROM (*d*) PROM

119. Which of the following memories must be refreshed many times per second ?
(*a*) Static RAM (*b*) Dynamic RAM
(*c*) EPROM (*d*) ROM

120. Memory refreshing may be done
(*a*) by the CPU that contains a special regress counter, only
(*b*) by an external refresh controller, only
(*c*) either by the CPU or by an external refresh controller
(*d*) none of these

121. A charge coupled device has
(*a*) low cost per bit
(*b*) high cost per bit
(*c*) low density
(*d*) none of these

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122. Semiconductor memory is

(*a*) somewhat slower than magnetic core memory

(*b*) a volatile memory

(*c*) somewhat larger than the magnetic core memory

(*d*) none of these

123. Memory consisting of electronic circuits attached into silicon chip is known as

(*a*) magnetic core memory

(*b*) semiconductor memory

(*c*) thin film memory

(*d*) MOS memory

124. Commonly used auxiliary storage device are

(*a*) magnetic tape, magnetic drum and magnetic disk.

(*b*) magnetic core, magnetic tape and magnetic drum.

(*c*) magnetic disk, semiconductor memory and magnetic drum.

(*d*) magnetic tape, monolithic storage and magnetic disk.

125. Choose the correct statement(s) from the following ?

(*a*) PROM contains a programmable AND array and a fixed OR array

(*b*) PLA contains a fixed AND array and a programmable OR array

(*c*) PROM contains a fixed AND array and programmable OR array

(*d*) None of these

126. Each cell of a statio Random Access memory contains

(*a*) 6 MOS transistors

(*b*) 4 MOS transistors and 2 capacitors

(*c*) Two 2-input NORs and One X-NOR gate

(*d*) XOR gates and shift registers

127. Semiconductor memory is

(*a*) somewhat slower than magnetic core memory.

(*b*) a volatile memory

(*c*) somewhat longer than magnetic core memory

(*d*) All of these

128. The input unit of a computer

(*a*) feeds the data in CPU

(*b*) retrieves the data from CPU

(*c*) directs all other units

(*d*) all of these

129. A hardware unit which is used to monitor computer processing is

(*a*) console (*b*) dot matrix printer

(*c*) mouse (*d*) ROM

130. Which of the following is the internal memory of the system (computer) ?

(*a*) CPU register (*b*) Cache

(*c*) Main memory (*d*) All of these

131. Which of the following memory is capable of operating at electronic speed ?

(*a*) Semi conductor memory

(*b*) Magnetic disks

(*c*) Magnetic drums

(*d*) Magnetic tapes

132. A software program stored in a ROM that can not be changed easily is called

(*a*) hardware (*b*) linker

(*c*) editor (*d*) firmware

133. Read cycle must be followed by

(*a*) Random access cycle

(*b*) write cycle

(*c*) ROM cycle

(*d*) any of these

134. Scratch-pad memory is a ...... memory

(*a*) Last-in First-out

(*b*) First-in First-out

(*c*) Local permanent memory

(*d*) Local temporary memory

135. The larger the RAM of a computer, the faster is its speed, since it eliminates

(*a*) need for ROM

(*b*) need for external memory

(*c*) frequent disk I/Os

(*d*) need for a data-wide path

136. What is the byte capacity of a drum which is 5 inch high, 10 inch diameter, and which has 60 tracks per inch bit density of 800 bits per inch ?

(*a*) 942000 bytes

(*b*) 471000 bytes

(*c*) 188400 bytes

(*d*) 16384 bytes

137. The idea of cache memory is based on

(*a*) the property of locality of reference

(*b*) the heuristic 90-10 rule

(*c*) the fact that only a small portion of a program is referenced relatively frequently

(*d*) all of these

138. Property of locality of reference may fail, if a program has

(*a*) many conditional jumps

(*b*) many unconditional jumps

(*c*) many operands

(*d*) all of these

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139. What is the correct sequence of time delays that happen during a data transfer from a disk to memory?

(*a*) seek time, access time, transfer time
(*b*) seek time, latency time, transfer time
(*c*) latency time, seek time, transfer time
(*d*) latency time, access time, transfer time

140. If each address space represents one byte of storage space, how many address lines are needed to access RAM chips arranged in a 4 × 6 array, where each chip is 8K × 4 bits ?

(*a*) 13 (*b*) 15
(*c*) 16 (*d*) 17

141. Four memory clips of 16 × 4 size have their address bases connected together. The system will be of size

(*a*) 64 × 64 (*b*) 16 × 16
(*c*) 32 × 8 (*d*) 256 × 1

142. Each cell of a statio Random Access memory contains

(*a*) 6 MOS transistors
(*b*) 4 MOS transistors and 2 capacitors
(*c*) Two 2-input NORs and One X-NOR gate
(*d*) XOR gates and shift registers

143. The ability of a medium sized computer system to increase in data processing capability by addition of such devices as mass storage device, I/O devices etc is called

(*a*) computer expandability
(*b*) computer mobility
(*c*) computer enahancement
(*d*) computer upward capability

144. The bus which is use1d to transfer data from main memory to peripheral device is

(*a*) data bus (*b*) input bus
(*c*) DMA bus (*d*) output bus

145. How many addresses are required for 25 × 40 video RAM ?

(*a*) 1020 (*b*) 1920
(*c*) 1000 (*d*) 2000

146. Arrange the following configurations for CPU in decreasing order of operating speeds:

Hard wired control, vertical micro-programming, horizontal micro-programming.

(*a*) Hard wired control, vertical micro-programming, horizontal micro-programming
(*b*) Hard wired control, horizontal micro-programming, vertical micro-programming
(*c*) Horizontal micro-programming, vertical micro-programming, hard wired control.
(*d*) Vertical micro-programming, horizontal micro-programming, hard wired control.

147. The main difference (a) between a CISC and a RISC processor is/are that a RISC processor typically

(*a*) has fewer instructions and addressing modes
(*b*) has more registers
(*c*) is easier to implement using hard-wired control logic
(*d*) all of these

148. Comparing the time T1 taken for a single instruction on a pipelined CPU with time T2 taken on a non-pipelined but idential CPU, we can say that

(*a*) T1 = T2
(*b*) T1 > T2
(*c*) T1 < T2
(*d*) T1 is T2 plus time taken for one instruction fetch cycle

149. Performance of a pipelined processor suffers if

(*a*) the pipeline stages have different delays
(*b*) consecutive instructions are dependent on each other
(*c*) the pipeline stages share hardware resources
(*d*) all of these

150. A micro-programmed control unit

(*a*) is faster than a hard-wired control unit
(*b*) facilitates easy implementation of new instructions
(*c*) is usefull when very small programs are to be run
(*d*) usually refers to the control unit of a microprocessor

151. Which of the following are typical characteristics of a RISC machine ?

(*a*) Highly pipelined
(*b*) Multiple register sets
(*c*) Both (*a*) and (*b*)
(*d*) None of these

152. Each cell of a station Random Access memory contains

(*a*) 6 MOS transistors
(*b*) 4 MOS transistors and 2 capacitors
(*c*) Two 2-input NORs and One X-NOR gate
(*d*) XOR gates and shift registers.

153. Which of the following statements is true ?

(*a*) ROM is a Read / Write memory
(*b*) PC points to the last instruction that was executed
(*c*) Stack works on the principle of LIFO
(*d*) All instructions affect the flags.

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154. In a 8085 microprocessor system with memory mapped I / O

(*a*) I / O devices have 8-bit addresses

(*b*) I / O devices are accessed using IN and OUT instructions

(*c*) there can be a maximum of 256 input devices and 256 output devices

(*d*) arithmetic and logic operations can be directly performed with the I / O data.

155. In a microcomputer, wait states are used to

(*a*) make the processor wait during a DMA operation

(*b*) make the processor wait during an interrupt processing

(*c*) make the processor wait during a power shutdown

(*d*) interface slow peripherals to the processor

156. The access time of magnetic bubble memory is approximately

(*a*) 30 nano seconds (*b*) 30 micro seconds

(*c*) 30 milli seconds (*d*) 0.3 seconds

157. When we move from the outermost track to the innermost track in a magnetic disk, then density (bits per linear inch)

(*a*) increases

(*b*) decreases

(*c*) remains the same

(*d*) either remains constant or decreases

158. The use of hardware in Memory management is through segment relocation and protection is

(*a*) to perform address translation to reduce size of the memory

(*b*) to perform address translation to reduce execution time overhead

(*c*) both (*a*) and (*b*)

(*d*) none of these

159. The parallel operation is preferred because

(*a*) circuitry is simple

(*b*) it requires less memory

(*c*) it is faster than series operation

(*d*) all of these

160. The process of entering data into a storage location

(*a*) causes variation in its address number

(*b*) adds to the contents of the location

(*c*) is called a readout operation

(*d*) is destructive of previous contents

161. In comparison to the internal (main) memory, tape or disk memory is

(*a*) slower and more expensive

(*b*) slower and less expensive

(*c*) faster and more expensive

(*d*) faster and less expensive

162. The number of records contained within a block of data on magnetic tape is defined by the

(*a*) block definition

(*b*) record contain clause

(*c*) blocking factor

(*d*) record per block factor

163. In magnetic disks, data is organized on the plotter in a concentric sets or rings called

(*a*) sector (*b*) track

(*c*) head (*d*) block

164. The number of bits that are typically stored on each track of a magnetic disk is usually

(*a*) the same

(*b*) different

(*c*) depend on the program to be stored

(*d*) fifty

165. When we move from the outermost track to the innermost track in a magnetic disk, then density (bits per linear inch)

(*a*) increases

(*b*) decreases

(*c*) remains the same

(*d*) either remains constant or decreases

166. Commonly used auxiliary storage device are

(*a*) magnetic tape, magnetic drum and magnetic disk

(*b*) magnetic core, magnetic tape and magnetic drum

(*c*) magnetic disk, semiconductor memory and magnetic drum

(*d*) magnetic tape, monolithic storage and magnetic disk

167. Which of the following memory is capable of operating at electronic speed ?

(*a*) Semi conductor memory

(*b*) Magnetic disks

(*c*) Magnetic drums

(*d*) Magnetic tapes

168. The ALU of a computer normally contains a number of high speed storage elements called

(*a*) semi conductor memory

(*b*) registers

(*c*) hard disk

(*d*) magnetic disks

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169. The process of entering data into a storage location
(*a*) causes variation in its address number
(*b*) adds to the contents of the location
(*c*) is called a readout operation
(*d*) is destructive of previous contents

170. Which access method is used for obtaining a record from a cassette tape ?
(*a*) direct (*b*) sequential
(*c*) random (*d*) parallel

171. Serial access memories are useful in applications where
(*a*) data consists of numbers
(*b*) short access time is required
(*c*) each stored word is processed differently
(*d*) data naturally needs to flow in and out in serial form

172. In comparison to the internal (main) memory, tape or disk memory is
(*a*) slower and more expensive
(*b*) slower and less expensive
(*c*) faster and more expensive
(*d*) faster and less expensive

173. Fastest type of memory from the following list is
(*a*) tape
(*b*) semiconductor memory
(*c*) disk
(*d*) bubble memory

174. An advantage of blocking a tape is that
(*a*) the additional processing time is consumed
(*b*) the direct file method can be emulated
(*c*) the tapes contain less data and longer tapes
(*d*) less tape is used to store the same amount of data

175. The number of records contained within a block of data on magnetic tape is defined by the
(*a*) block definition
(*b*) record contain clause
(*c*) blocking factor
(*d*) record per block factor

176. Diskette and hard disks are
(*a*) direct access devices
(*b*) sequential access devices
(*c*) slower than magnetic tape
(*d*) used only in mainframe computers

177. In magnetic disks, data is organized on the plotter in a concentric sets or rings called
(*a*) sector (*b*) track
(*c*) head (*d*) block

178. The number of bits that are typically stored on each track of a magnetic disk is usually
(*a*) the same
(*b*) different
(*c*) depend on the program to be stored
(*d*) fifty

179. Which of the following is not true of primary storage?
(*a*) It represents the decimal number through string of binary digits.
(*b*) It stores operating system programs.
(*c*) It stores data while they are being processed by CPU.
(*d*) It stores the bulk of data used by computer application.

180. Which of the following storage devices can be carried around ?
(*a*) floppy disks (*b*) main memory
(*c*) registers (*d*) core memory

181. Which of the following coded entries are used to control access to computers ?
(*a*) Code words (*b*) Pass memory
(*c*) Binary pass (*d*) ASC11 codes

182. The hardware in which data may be stored for a computer system is called
(*a*) register (*b*) memory
(*c*) chip (*d*) peripheral

183. Which out of the following is not an alternative name for primary memory?
(*a*) Main memory (*b*) Primary storage
(*c*) Internal storage (*d*) Mass storage

184. In modern computers bipolar semiconductor chips are often used in the arithmetic logic unit. What material is used for the slower and less expensive primary storage section?
(*a*) Gallium arsenide (GaAs)
(*b*) Metal oxide semiconductor
(*c*) Silicon
(*d*) Germanium

185. Which type of memory chips are likely to be used in the primary storage of the future general of computers?
(*a*) Selenium chips
(*b*) Optical chips
(*c*) Bio chips
(*d*) Gallium arsenide chips

186. Which of the following chips can be reprogrammed with special electric pulses ?
(*a*) EPROM (*b*) PROM
(*c*) ROM (*d*) EEPROM

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187. If a computer has a 1024 K memory, then what does the letter K stand for ?
(*a*) kilometre (*b*) Thousand
(*c*) 1024 (*d*) Core

188. In a computer, 16 bits are used to specify addresses in a ram, the number of addresses will be
(*a*) 256×256 (*b*) 65,536
(*c*) 64K (*d*) any of these

189. What was the amount of memory required by the earliest operating system called dos 1.0?
(*a*) 4K (*b*) 8K
(*c*) 16K (*d*) 32K

190. The storage device which is used to compensate for the difference in rates of flow of data from one device to another is called
(*a*) cache (*b*) concentrator
(*c*) buffer (*d*) I/O device

191. As a secondary storage medium, what is the most important advantage of a video disk?
(*a*) Compactness
(*b*) Potential capacity
(*c*) Durability
(*d*) Cost effectiveness

192. Secondary storage device which uses a delivery grooveless surface and is encoded by a laser beam in the form of microscopic pits is called
(*a*) laser disk
(*b*) compact disk
(*c*) photo disk
(*d*) video disk

193. What is the size of optical compact disk which is used for recording high quality music?
(*a*) 4.7 inch (*b*) 3½ inch
(*c*) 5½ inch (*d*) 8 inch

194. Which is the most popular medium for direct access secondary storage of a computer?
(*a*) Magnetic tape
(*b*) Magnetic disk
(*c*) RAM
(*d*) ROM

195. The data on the tracks on a magnetic disks are written as
(*a*) up or down
(*b*) tiny magnetic spots
(*c*) 0 or 1
(*d*) high or low voltage

196. Which part of the diskette never be touched?
(*a*) Hub (*b*) Ole in the centre
(*c*) Oval slot (*d*) Corner

197. The two types of main memory are
(*a*) primary and secondary
(*b*) random and sequential
(*c*) ROM and RAM
(*d*) central and peripherals

198. Memory is
(*a*) a device that performs a sequence of operations specified by instructions in memory
(*b*) the device where information is stored
(*c*) a sequence of instructions
(*d*) typically characterized by interactive processing and time-slicing of the CPU's time to allow quick response to each user.

199. Virtual memory
(*a*) is a method of memory allocation by which the program is subdivided into equal portions, 8 pages and core is subdivided into equal portions
(*b*) consists of those addresses that may be generated by a processor during execution of a computation
(*c*) is a method of allocating processor time
(*d*) allows multiple programs to reside in separate areas of core at a time

200. A ROM is used to store the table for multiplication of two 8-bit unsigned integers. The size of ROM required is
(*a*) $256K \times 16$ (*b*) $64\ K \times 8$
(*c*) $4K \times 16$ (*d*) $64\ K \times 16$

201. The process of entering data into a storage location
(*a*) causes variation in its address number
(*b*) adds to the contents of the location
(*c*) is called readout operatio
(*d*) is destructive of previous contents

202. The desirable characteristic(s) of a memory system is(are)
(*a*) speed and reliability
(*b*) low power consumption
(*c*) durability and compactness
(*d*) all of these

203. Time elapsed between initiation of a memory operation and the completion of that operation is called
(*a*) memory cycle time
(*b*) memory access time
(*c*) transfer time
(*d*) skip time

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204. The minimum time delay required between the initiation of two successive memory operations is called

(*a*) memory cycle time
(*b*) memory access time
(*c*) transmission time
(*d*) waiting time

205. For a memory system, the cycle time is

(*a*) same as the access time
(*b*) longer than the access time
(*c*) shorter than the access time
(*d*) submultiple of the access time

206. The number of records contained within a block of data on magnetic tape is defined by

(*a*) block definition
(*b*) record contain clause
(*c*) blocking factor
(*d*) record per block factor

207. The idea of cache memory is based on

(*a*) the property of locality of reference
(*b*) the heuristic 90-10 rule
(*c*) the fact that only a small portion of a program is referenced relatively frequently
(*d*) all of these

208. If each address space represents one byte of storage space, how many address lines are needed to access RAM chips arranged in a 4 × 6 array, where each chip is 8K × 4 bits?

(*a*) 13 (*b*) 15
(*c*) 16 (*d*) 17

209. Consider a high speed 40ns memory cache with a successful hit ratio of 80%. The regular memory has an access time of 100 ns. What is the average effective time for CPU to access memory?

(*a*) 52 ns (*b*) 60 ns
(*c*) 70 ns (*d*) 80 ns

210. Consider a disk with the following characteristics:

Track size : 10,000 bytes
Rotational latency : 10 ms/ revolution
Block size : 1,000 bytes

What is the maximum transfer rate per track measured in bits per second as is conventional for this disk unit?

(*a*) 400 Mbps (*b*) 8 Mbps
(*c*) 6,400 Mbps (*d*) 4,250 Mbps

NUMERICAL TYPE QUESTIONS

1. Number of machine cycles required for RET instruction in 8085 microprocessor is _______

2. Microprocessor 8085 is the enhanced version of ________ with essentially the same construction set.

3. If we use 3 bits in the instruction word to indicate if an index register is to be used and if necessary, which one is to be used, then number of index registers to be used in the machine will be______

4. The capacity of program counter (PC) is ______ bits

5. The programmable interval timer is_______

6. The storage capacity of a Hollerith card which is organized into nibbles is ________

7. The ________ number of addresses are required for 25 × 40 video RAM

8. Number of machine cycles required for RET instruction in 8085 microprocessor is ______

9. The average access time for a drum rotating at 4000 revolutions per minute is ________ ms

10. The ______ number of input lines are needed to construct 1024 bit coincident core plane

11. The ______ number of RAM chips of size (256K × 1 bit) are required to build 1M Byte memory

12. The _______ number of types of storage loops exist in magnetic bubble memory

13. The ______ number of wires are threaded through the cores in a coincident-current core memory

14. Transfer of information from main storage is typically n times faster than the transfer from auxiliary storage, where n is about ________

15. The ______ number of bits can be stored in the 8K capital

16. The seek time of a disk is 30 ms. It rotates at the rate of 30 rotations/ second. The capacity of each track is 300 words. The access time is (approximately) _______ ms

17. The _________ number of RAM chips of size (256 K × 1 bit) are required to build 1M Byte memory

18. In a memory system, four 256 × 8 PROM chips are used to make total memory of size 1024 × 4. The number of address bus lines is ________

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19. The hit ratio of a cache if a system performs memory access at 30 nano seconds with the cache and 150 nano seconds without it is__________ Assume that the each uses 20 nano sec memory, choose the closest approximate.

20. A 16 × 8 organization of memory cells, can store upto __________ bits

21. A memory organization that can hold upto 1024 bits and has a minimum of 20 address lines can be organized into ___________

22. In a 4m-bit chip organization has a total of 19 external connections then is has ____ address if 8 data lines are there.

23. The gray code of a given binary number 1100 is

24. The number of 256 * 4 RAM chips required to construct 2 kB cache is __________

25. Convert decimal value $(888)_{10}$ to base 5.

26. What is the word size in bits of a 8086 processor?

27. What is the (r – 1)'s complement of 345 in octal number system?

28. What is the octal equivalent of given binary number ? $(011001)_2$

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. A file is organized so that the ordering of data records is the same as or close to the ordering of data entries in some index. Then that index is called

(*a*) Dense (*b*) Sparse

(*c*) Clustered (*d*) Unclustered

2. A positive edge-triggered D flip-flop is connected to a positive edge-triggered JK flip-flop as follows. The Q output of the D flip-flop is connected to both the J and K inputs of the JK flip-flop, while the Q output of the JK flip-flop is connected to the input of the D flip-flop is connected to both the J and K inputs of the JK flip-flop, while the Q output of the JK flip-flop is connected to the input of the D flip-flop. Initially, the output of the D flip-flop is set to logic one and the output of the JK flip-flop is cleared. Which one of the following is the bit sequence (including the initial state) generated at the Q output of the JK flip-flop when the flip-flops are connected to a free running common clock ? Assume that J = K = 1 is the toggle mode and J = K = 0 is the state-holding mode of the JK flip-flop. Both the flip-flops have non-zero propagation delays.

(*a*) 0110110 ..... (*b*) 0100100 .....

(*c*) 011101110 ..... (*d*) 011001100 .....

3. Consider a processor with byte-addressable memory. Assume that all registers, including Program Counter (PC) and Program Status Word (PSW), are of size 2 bytes. A stack in the main memory is implemented from memory location $(0100)_{16}$ and it grows upward. The stack pointer (SP) points to the top element of the stack. The current value of SP is $(016E)_{16}$. The CALL instruction is of two words, the first word is the op-code and the second word is the starting address of the subroutine. (one word = 2bytes). The CALL instruction is implemented as follows:

□□ Store the current Vale of PC in the Stack

□□ Store the value of PSW register in the stack

□ Load the starting address of the subroutine in PC

The content of PC just before the fetch of a CALL instruction is $(5FA0)_{16}$. After execution of the CALL instruction, the value of the stack pointer is

(*a*) $(016A)_{16}$ (*b*) $(016C)_{16}$

(*c*) $(0170)_{16}$ (*d*) $(0172)_{16}$

4. Consider a machine with byte addressable main memory of 20^{20} bytes, block size of 16 bytes and a direct mapped cache having 2^{12} cache lines. Let the address of two consecutive bytes in main memory be $(E201F)_{16}$ and $(E2020)_{16}$. What are the tag and cache line address (in hex) for main memory address $(E201F)_{16}$?

(*a*) E, 201 (*b*) F, 201

(*c*) E, E20 (*d*) 2, 01F

5. Consider the following code sequence having five instructions I_1 to I_5. Each of these instructions has the following format.

OP Ri, Rj, Rk

Where operation OP is performed on contents of registers Rj and Rk and the results is stored in register Ri.

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I_1 : ADD R1, R2, R_3

I_2 : MUL R7, R1, R3

I_3 : SUB R4, R1, R5

I_4 : ADD R3, R 2, R 4

I_5 : MUL R7,R8, R9

Consider the following three statements.

S1 : There is an anti-dependence between instructions I2 and I5

S2 : There is an anti-dependence between instructions I2 and I4

S3 : Within an instruction pipeline an anti-dependence always creates on or more stalls

Which one of above statements is/are correct ?

(*a*) Only S1 is true
(*b*) Only S2 is true
(*c*) Only S1 and S3 are true
(*d*) Only S2 and S3 are true

2014

6. An access sequence of cache block addresses is of length N and contains *n* unique block addresses. The number of unique block addresses between two consecutive accesses to the same block address is bounded above by K. What is the miss ratio if the access sequence is passed through a cache of associativity A ≥ *k* exercising least-recently-used replacement policy?

(*a*) *n*/N (*b*) 1/N
(*c*) 1/A (*d*) *k*/*n*

7. In designing a computer's cache system, the cache block (or cache line) size is an important parameter. Which one of the following statements is correct in this context?

(*a*) A smaller block size implies better spatial locality
(*b*) A smaller block size implies a smaller cache tag and hence lower cache tag overhead
(*c*) A smaller block size implies a larger cache tag and hence lower cache hit time
(*d*) A smaller block size incurs a lower cache miss penalty

8. If the associativity of a processor cache is doubled while keeping the capacity and block size unchanged, which one of the following is guaranteed to be NOT affected?

(*a*) Width of tag comparator
(*b*) Width of set index decoder
(*c*) Width of way selection multiplexor
(*d*) Width of processor to main memory data bus

9. The value of a *float* type variable is represented using the single-precision 32-bit floating point format of IEEE-754 standard that uses 1 bit for sign, 8 bits for biased exponent and 23 bits for mantissa. A *float* type variable X is assigned the decimal value of –14.25. The representation of X in hexadecimal notation is

(*a*) C1640000H (*b*) 416C0000H
(*c*) 41640000H (*d*) C16C0000H

10. Consider the following processors (ns stands for nanoseconds).

Assume that the pipeline registers have zero latency.

P1: Four-stage pipeline with stage latencies 1 ns, 2 ns, 2 ns, 1 ns.

P2: Four-stage pipeline with stage latencies 1 ns, 1.5ns, 1.5 ns, 1.5 ns.

P3: Five-stage pipeline with stage latencies 0.5ns, 1 ns, 1 ns, 0.6 ns, 1 ns.

P4: Five-stage pipeline with stage latencies 0.5ns, 0.5 ns, 1 ns, 1 ns, 1.1 ns.

Which processor has the highest peak clock frequency?

(*a*) P1 (*b*) P2
(*c*) P3 (*d*) P4

11. A bit-stuffing based framing protocol uses an 8-bit delimiter pattern of 01111110. If the output bit-string after stuffing is 01111100101, then the input bit-string is

(*a*) 0111110100 (*b*) 0111110101
(*c*) 0111111101 (*d*) 0111111111

2013

12. In a *k*-way set associative cache, the cache is divided into *v* sets, each of which consists of *k* lines. The lines of a set are placed in sequence one after another. The lines in set *s* are sequenced before the lines in set (*s* + l). The main memory blocks are numbered 0 onwards. The main memory block numbered *j* must be mapped to any one of the cache lines from

(*a*) $(j \bmod v) * k$ to $(j \bmod v) * k + (k - 1)$
(*b*) $(j \bmod v)$ to $(j \bmod v) + (k - 1)$
(*c*) $(j \bmod k)$ to $(j \bmod k) + (v - 1)$
(*d*) $(j \bmod k) * v$ to $(j \bmod k) * v + (v - 1)$

13. Consider the following sequence of micro-operations.

MBR ← PC

MAR ← X

PC ← Y

Memory ← MBR

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Which one of the following is a possible operation performed by this sequence?

(*a*) Instruction fetch

(*b*) Operand fetch

(*c*) Conditional branch

(*d*) Initiation of interrupt service.

14. Consider a hard disk with 16 recording surfaces (0-15) having 16384 cylinders (0-16383) and each cylinder contains 64 sectors (0-63). Data storage capacity in each sector is 512 bytes. Data are organized cylinder-wise and the addressing format is <cylinder no., surface no., sector no.>. A file of size 42797 KB is stored in the disk and the starting disk location of the file is <1200, 9, 40>. What is the cylinder number of the last sector of the file, if it is stored in a contiguous manner?

(*a*) 1281 (*b*) 1282

(*c*) 1283 (*d*) 1284

15. Consider an instruction pipeline with five stages without any branch prediction: Fetch Instruction (FI), Decode Instruction (DI), Fetch Operand (FO), Execute Instruction (EI) and Write Operand (WO). The stage delays for FI, DI, FO, EI and WO are 5 ns, 7 ns, 10 ns, 8 ns and 6 ns, respectively. There are intermediate storage buffers after each stage and the delay of each buffer is 1 ns. A program consisting of 12 instructions I_1, I_2, I_3, ..., I_{12} is executed in this pipelined processor. Instruction I_4 is the only branch instruction and its branch target is I_9. If the branch is taken during the execution of this program, the time (in ns) needed to complete the program is

(*a*) 132 (*b*) 165

(*c*) 176 (*d*) 328

2012

16. Register renaming is done in pipelined processors

(*a*) as an alternative to register allocation at compile time

(*b*) for efficient access to function parameters and local variables

(*c*) to handle certain kinds of hazards

(*d*) as part of address translation

17. The amount of ROM needed to implement a 4 bit multiplier is

(*a*) 64 bits (*b*) 128 bits

(*c*) 1 Kbits (*d*) 2 Kbits

18. A file system with 300 GByte disk uses a file descriptor with 8 direct block addresses, 1 indirect block address and 1 doubly indirect block address. The size of each disk block is 128 Bytes and the size of each disk block address is 8 Bytes. The maximum possible file size in this file system is

(*a*) 3 KBytes

(*b*) 35 KBytes

(*c*) 280 KBytes

(*d*) dependent on the size of the disk

19. Fetch_And_Add (X, *i*) is an atomic Read-Modify-Write instruction that reads the value of memory location X, increments it by the value i, and returns the old value of X. It is used in the pseudocode shown below to implement a busy-wait lock, L is an unsigned integer shared variable initialized to 0. The value of 0 corresponds to lock being available,while any non-zero value corresponds to the lock being not available.

```
AcquireLock (L) {
while (Fetch_And_Add (L, 1))
    L = 1;
}
ReleaseLock (L){
    L = 0;
}
```

This implementation

(*a*) fails as L can overflow

(*b*) fails as L can take on a non-zero value when the lock is actually available

(*c*) works correctly but may starve some processes

(*d*) works correctly without starvation

2011

20. Consider a hypothetical processor with an instruction of type LW R1, 20 (R2), which during execution reads a 32-bit word from memory and stores it in a 32-bit register R1. The effective address of the memory location is obtained by the addition of a constant 20 and the contents of register R2. Which of the following best reflects the addressing mode implemented by this instruction for the operand in memory?

(*a*) Immediate Addressing

(*b*) Register Addressing

(*c*) Register Indirect Scaled Addressing

(*d*) Base Indexed Addressing

21. A computer handles several interrupt sources of which of the following are relevant for this question.

- Interrupt from CPU temperature sensor (raises interrupt if CPU temperature is too high)
- Interrupt from Mouse (raises interrupt if the mouse is moved or a button is pressed)

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- Interrupt from Keyboard (raises interrupt when a key is pressed or released)
- Interrupt from Hard Disk (raises interrupt when a disk read is completed)

Which one of these will be handled at the **HIGHEST** priority?

(*a*) Interrupt from Hard Disk
(*b*) Interrupt from Mouse
(*c*) Interrupt from Keyboard
(*d*) Interrupt from CPU temperature sensor

22. An application loads 100 libraries at startup. Loading each library requres exactly one disk access. The seek time of the disk to a random location is given as 10 ms. Rotational speed of disk is 6000 rpm. If all 100 libraries are loaded from random locations on the disk, how long does it take to load all libraries? (The time to transfer data from the disk block once the head has been positioned at the start of the block may be neglected).

(*a*) 0.50 s (*b*) 1.50 s
(*c*) 1.25 s (*d*) 1.00 s

23. An 8 KB direct mapped write-back cache is organized as multiple blocks, each of size 32-bytes.The processor generates 32-bit addresses. The cache controller maintains the tag information for each cache block comprising of the following.

1 Valid bit

1 Modified bit

As many bits as the minimum needed to identify the memory block mapped in the cache?

What is the total size of memory needed at the cache controller to store meta-data (tags) for the cache?

(*a*) 4864 bits (*b*) 6144 bits
(*c*) 6656 bits (*d*) 5376 bits

2010

24. A main memory unit with a capacity of 4 megabytes is built using 1M × 1-bit DRAM chips. Each DRAM chip has 1K rows of cells with 1K cells in each row. The time taken for a single refresh operation is 100 nanoseconds. The time required to perform one refresh operation on all the cells in the memory unit is

(*a*) 100 nanoseconds
(*b*) $100*2^{10}$ nanoseconds
(*c*) $100*2^{20}$ nanoseconds
(*d*) $3200*2^{20}$ nanoseonds

25. A 5-stage pipelined processor has Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO) and Write Operand (WO) stages. The IF, ID, OF and WO stages take 1 clock cycle each for any instruction. The PO stage takes 1 clock cycle for ADD and SUB instructions, 3 clock cycles for MUL instruction, and 6 clock cycles for DIV instruction respectively. Operand forwarding is used in the pipeline. What is the number of clock cycles needed to execute the following sequence of instructions?

Instruction	**Meaning of instruction**
I_0: MUL R_2, R_0 R_1	$R_2 \leftarrow R_0{*}R_1$
I_1: DIV R_5, R_3, R_4	$R_5 \leftarrow R_3/R_4$
I_2: ADD R_2, R_5, R_1	$R_2 \leftarrow R_5 + R_2$
I_3: SUB R_5, R_2, R_6	$R_5 \leftarrow R_2 - R_6$

(*a*) 13
(*b*) 15
(*c*) 17
(*d*) 19

26. The progam below uses six temporary variables a, b, c, d, e, f.

```
a = 1
b = 10
c = 20
d = a + b
e = c + d
f = c + e
b = c + e
e = b + f
d = 5 + e
return d + f
```

Assuming that all operations take their operands from registers, what is the minimum number of registers needed to execute this program without spilling?

(*a*) 2 (*b*) 3
(*c*) 5 (*d*) 6

2009

27. How many 32 × 1 RAM chips are needed to provide a memory capacity of 256 k-bytes ?

(*a*) 8 (*b*) 32
(*c*) 64 (*d*) 128

28. A CPU generally handles an interrupt by executing an interrupt service routine

(*a*) as soon as an interrupt is raised.
(*b*) by checking the interrupt register at the end of fetch cycle.
(*c*) by checking the interrupt register after finishing the executing of the current instruction.
(*d*) by checking the interrupt register at fixed time intervals.

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29. Consider a 4 stage pipeline processor. The number of cycles needed by the four instructions I_1, I_2, I_3, I_4 in stages S_1, S_2, S_3, S_4 is shown below:

	S_1	S_2	S_3	S_4
I_1	2	1	1	1
I_2	1	3	2	2
I_3	2	1	1	3
I_4	1	2	2	2

What is the number of cycles needed to execute the following loop ?

for (i = 1 to 2) $\{I_1; I_2; I_3; I_4\}$

(*a*) 16 (*b*) 23
(*c*) 28 (*d*) 30

30. Consider a 4-way set associative cache (initially empty) with total 16 cache blocks. The main memory consists of 256 blocks and the request for memory blocks is in the following order:

0, 255, 1, 4, 3, 8, 133, 159, 216, 129, 63, 8, 48, 32, 73, 92, 155.

Which one of the following memory block will NOT be in cache if LRU replacement policy is used ?

(*a*) 3 (*b*) 8
(*c*) 129 (*d*) 216

2008

31. For a magnetic disk with concentric circular tracks, the seek latency is not linearly proportional to the seek distance due to

(*a*) non-uniform distribution of requests
(*b*) arm starting and stopping inertia
(*c*) higher capacity of tracks on the periphery of the platter
(*d*) use of unfair arm scheduling policies

32. Which of the following is/are true of the auto-increment addressing mode?

I. It is useful in creating self-relocating code
II. If it is included in an Instruction Set Architecture, then an additional ALU is required for effective address calculation
III. The amount of increment depends on the size of the data item accessed

(*a*) I only (*b*) II only
(*c*) III only (*d*) II and III only

33. Which of the following must be true for the RFE (Return from Exception) instruction on a general purpose processor?

I. It must be a trap instruction
II. It must be a privileged instruction
III. An exception cannot be allowed to occur during execution of an RFE instruction

(*a*) I only (*b*) II only
(*c*) I and II only (*d*) I, II and III only

34. For inclusion to hold between two cache levels L1 and L2 in a multi-level cache hierarchy, which of the following are necessary?

I. L1 must be a write-through cache
II. L2 must be a write-through cache
III. The associativity of L2 must be greater than that of L1
IV. The L2 cache must be at least as large as the L1 cache

(*a*) IV only (*b*) I and IV only
(*c*) I, III and IV only (*d*) I, II, III and IV

35. Which of the following are NOT true in a pipelined processor?

I. Bypassing can handle all RAW hazards.
II. Register renaming can eliminate all register carried WAR hazards.
III. Control hazard penalties can be eliminated by dynamic branch prediction.

(*a*) I and II only (*b*) I and III only
(*c*) II and III only (*d*) I, II and III

36. The use of multiple register windows with overlap causes a reduction in the number of memory accesses for

I. function locals and parameters
II. register saves and restores
III. instruction fetches

(*a*) I only (*b*) II only
(*c*) III only (*d*) I, II and III

37. In an instruction execution pipeline, the earliest that the data TLB (Translation Lookaside Buffer) can be accessed is

(*a*) before effective address calculation has started
(*b*) during efective address calculation
(*c*) after effective address calculation has completed
(*d*) after data cache lookup has completed

2007

38. Consider a 4-way set associative cache consisting of 128 lines with a line size of 64 words. The CPU generates a 20-bit address of a word in main memory. The number of bits in the TAG, LINE and WORD fields are respectively

(*a*) 9, 6, 5 (*b*) 7, 7, 6
(*c*) 7, 5, 8 (*d*) 9, 5, 6

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39. Consider a disk pack with 16 surfaces, 128 tracks per surface and 256 sectors per track. 512 bytes of data are stored in a bit serial manner in a sector. The capacity of the disk pack and the number of bits required to specify a particular sector in the disk are respectively

(*a*) 256 Mbyte, 19 bits (*b*) 256 Mbyte, 28 bits
(*c*) 512 Mbyte, 20 bits (*d*) 64 Gbyte, 28 bits

40. Consider a pipelined processor with the following four stages :

IF : Instruction Fetch
ID : Instruction Decode and Operand Fetch
EX : Execute
WB : Write Back

The IF, ID and WB stages take one clock cycle each to complete the operation. The number of clock cycles for the EX stage depends on the instruction. The ADD and SUB instructions need 1 clock cycle and the MUL instruction need 3 clock cycles in the EX stage. Operand forwarding is used in the pipelined processor. What is the number of clock cycles taken to complete the following sequence of instructions ?

ADD	R2,	R1,	R0	R2¬R1 + R0
MUL	R4,	R3,	R2	R4¬R3 * R2
SUB	R6,	R5,	R4	R6¬R5 – R4

(*a*) 7 (*b*) 8
(*c*) 10 (*d*) 14

2006

41. A CPU has a cache with block size 64 bytes. The main memory has k banks, each bank being c bytes wide. Consecutive c-byte chunks are mapped on consecutive banks with warp-around. All the k banks can be accessed in parallel, but two accesses to the same bank must be serialized. A cache block access may involve multiple iterations of parallel bank accesses depending on the amount of data obtained by accessing all the k banks in parallel. Each iteration requires decoding the bank numbers to be accessed in parallel and this takes $k/2$ *ns*. The latency of one bank access is 80 *ns*. If $c = 2$ and $k = 24$, then latency of retrieving a cache block starting at address zero from main memory is

(*a*) 92 ns (*b*) 104 ns
(*c*) 172 ns (*d*) 184 ns

42. A CPU has five-stage pipeline and runs at 1 GHz frequency. Instruction fetch happens in the first stage of the pipeline. A conditional branch instruction computes the target address and evaluates the condition in the third stage of the pipeline. The processor stops fetching new instructions following a conditional branch until the branch outcome is known. A program executes 10^9 instructions out of which 20% are conditional branches. If each instruction takes one cycle to complete on average, then total execution time of the program is

(*a*) 1.0 second (*b*) 1.2 seconds
(*c*) 1.4 seconds (*d*) 1.6 seconds

43. Consider a new instruction named branch-on-bit-set (mnemonic bbs). The instruction "bbs reg, pos, label" jumps to label if bit in position pos of register operand reg is one. A register is 32 bits wide and the bits are numbered 0 to 31, bit in position 0 being the least significant. Consider the following emulation of this instruction on a processor that does not have bbs implemented.

temp ← reg and mask

Branch to label if temp is non-zero

The variable temp is a temporary register. For correct emulation, the variable mask must be generated by

(*a*) *mask* ← *0x1 < < pos*
(*b*) *mask* ← 0*x ffffffff >> pos*
(*c*) *mask* ← *pos*
(*d*) *mask* ← *0xf*

NUMERICAL TYPE QUESTIONS

2015

1. Consider a system with byte–addressable memory, 32-bit logical addresses, 4 kilobyte page size and page table entries of 4 bytes each. The size of the page table in the system in megabytes is ________.

2. Consider a disk pack with a seek time of 4 milliseconds and rotational speed of 10000 rotations per minute (RPM). It has 60 sectors per track and each sector can store 512 bytes of data. Consider a file stored in the disk. The file contains 2000 sectors. Assume that every sector access necessitates a seek, and the average rotational latency for accessing each sector is half of the time for one complete rotation. The total time (in milliseconds) needed to read the entire file is _______.

3. Consider a non-pipelined processor with a clock rate of 2.5 gigahertz and average cycles per instruction of four. The same processor is upgraded to a pipelined processor with five stages; but due to the internal pipelined delay, the clock speed is reduced to 2 gigahertz. Assume that there are no stalls in the pipeline. The speed up achieved in this pipelined processor is _____.

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4. Assume that for a certain processor, a read request takes 50 nanoseconds on a cache miss and 5 nanoseconds on a cache hit. Suppose while running a program, it was observed that 80% of the processors read requests result in a cache hit. The average and access time in nanoseconds is _______.

5. A computer system implements a 40-bit virtual address, page size of 8 kilobytes, and a 128-entry translation look-aside buffer (TLB) organized into 32 sets each having four ways. Assume that the TLB tag does not store any process id. The minimum length of the TLB tag in bits is _______.

6. Consider a typical disk that rotates at 15000 rotations per minute (RPM) and has a transfer rate of 50×10^6 bytes/sec. if the average seek time of the disk is twice the average rotational delay and the controller's transfer time is 10 times the disk transfer time, the average time (in milliseconds) to read or write a 512-byte sector of the disk is _____________.

7. A computer system implements 8 kilobyte pages and a +32-bit physical address space. Each page table entry contains a valid bit, a dirty bit, three permission bits, and the translation. If the maximum size of the page table of a process is 24 megabytes, the length of the virtual address supported by the system is _________ bits.

8. Consider the sequence of machine instruction given below:

MUL R5, R0, R1

DIV R6, R2, R3

ADD R7, R5, R6

SUB R8, R7, R4

In the above sequence, R0 to R8 are general purpose registers. In the instructions shown. The first register stores the result of the operation performed on the second and the third registers.

This sequence of instructions is to be executed in a pipelined instruction processor with the following 4 stages

(1) Instruction Fetch and Decode (IF),

(2) Operand Fetch (OF),

(3) Perform Operation (PO) and

(4) Write back the result (WB).

The IF, OF and WB stages take 1 clock cycle each for any instruction. The PO stage takes 1 clock cycle for ADD or SUB instruction, 3 clock cycles for MUL instruction and 5 clock cycles for DIV instruction. The pipelined processor uses operand forwarding from the PO stage to the OF stage. The number of clock cycles taken for the execution of the above sequence of instructions is __________

2014

9. A machine has a 32-bit architecture, with 1-word long instructions. It has 64 registers, each of which is 32 bits long. It needs to support 45 instructions, which have an immediate operand in addition to two register operands. Assuming that the immediate operand is an unsigned integer, the maximum value of the immediate operand is ____________.

10. Suppose a disk has 201 cylinders, numbered from 0 to 200. At some time the disk arm is at cylinder 100, and there is a queue of disk access requests for cylinders 30, 85, 90, 100, 105, 110, 135 and 145. If Shortest-Seek Time First (SSTF) is being used for scheduling the disk access, the request for cylinder 90 is serviced after servicing ____________ number of requests.

11. Consider a 6-stage instruction pipeline, where all stages are perfectly balanced. Assume that there is no cycle-time overhead of pipelining. When an application is executing on this 6-stage pipeline, the speedup achieved with respect to non-pipelined execution if 25% of the instructions incur 2 pipeline stall cycles is ______________.

12. Consider two processors P_1 and P_2 executing the same instruction set. Assume that under identical conditions, for the same input, a program running on P_2 takes 25% less time but incurs 20% more CPI (clock cycles per instruction) as compared to the program running on P_1. If the clock frequency of P_1 is 1GHz, then the clock frequency of P_2 (in GHz) is _________.

13. A 4-way set-associative cache memory unit with a capacity of 16 KB is built using a block size of 8 words. The word length is 32 bits. The size of the physical address space is 4 GB. The number of bits for the TAG field is _____.

14. A FAT (file allocation table) based file system is being used and the total overhead of each entry in the FAT is 4 bytes in size. Given a 100×10^6 bytes disk on which the file system is stored and data block size is 10^3 bytes, the maximum size of a file that can be stored on this disk in units of 10^6 bytes is ____________.

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15. Consider a main memory system that consists of 8 memory modules attached to the system bus, which is one word wide. When a write request is made, the bus is occupied for 100 nanoseconds (ns) by the data, address, and control signals. During the same 100 ns, and for 500 ns thereafter, the addressed memory module executes one cycle accepting and storing the data. The (internal) operation of different memory modules may overlap in time, but only one request can be on the bus at any time. The maximum number of stores (of one word each) that can be initiated in 1 millisecond is ___________

16. Consider a paging hardware with a TLB. Assume that the entire page table and all the pages are in the physical memory. It takes 10 milliseconds to search the TLB and 80 milliseconds to access the physical memory. If the TLB hit ratio is 0.6, the effective memory access time (in milliseconds) is ___________.

17. An instruction pipeline has five stages, namely, instruction fetch (IF), instruction decode and register fetch (ID/RF), instruction execution (EX), memory access (MEM), and register write back (WB) with stage latencies 1 ns, 2.2 ns, 2 ns, 1 and 0.75 ns, respectively (ns stands for nanoseconds). To gain in terms of frequency, the designers have decided to split the ID/RF stage into three stages (ID, RF1, RF2) each of latency 2.2/3 ns. Also, the EX stage is split into two stages (EX1, EX2) each of latency 1 ns. The new design has a total of eight pipeline stages. A program has 20% branch instructions which execute in the EX stage and produce the next instruction pointer at the end of the EX stage in the old design and at the end of the EX2 stage in the new design. The IF stage stalls after fetching a branch instruction until the next instruction pointer is computed. All instructions other than the branch instruction have an average CPI of one in both the designs. The execution times of this program on the old and the new design are P and Q nanoseconds, respectively. The value of P/Q is___________.

18. The memoy access time is 1 nanosecond for a read operation with a hit in cache, 5 nanoseconds for a real operation with a miss in cache, 2 nanoseconds for a write operation with a hit in cache and 10 nanoseconds for a write operation with a miss in cache. Execution of a sequence of instructions involves 100 instruction fetch operations, 60 memory operand read operations and 40 memory operand write operations. The cache hit ratio is 0.9. The average memory access time (in nanoseconds) in executing the sequence of instructions is _________.

ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*d*)	**2.** (*b*)	**3.** (*a*)	**4.** (*d*)	**5.** (*a*)	**6.** (*b*)	**7.** (*c*)	**8.** (*c*)	**9.** (*b*)	**10.** (*b*)
11. (*d*)	**12.** (*b*)	**13.** (*b*)	**14.** (*b*)	**15.** (*d*)	**16.** (*b*)	**17.** (*d*)	**18.** (*c*)	**19.** (*b*)	**20.** (*d*)
21. (*a*)	**22.** (*b*)	**23.** (*d*)	**24.** (*d*)	**25.** (*c*)	**26.** (*c*)	**27.** (*b*)	**28.** (*d*)	**29.** (*d*)	**30.** (*d*)
31. (*d*)	**32.** (*d*)	**33.** (*c*)	**34.** (*c*)	**35.** (*a*)	**36.** (*a*)	**37.** (*b*)	**38.** (*c*)	**39.** (*b*)	**40.** (*c*)
41. (*d*)	**42.** (*d*)	**43.** (*c*)	**44.** (*b*)	**45.** (*a*)	**46.** (*c*)	**47.** (*c*)	**48.** (*b*)	**49.** (*c*)	**50.** (*a*)
51. (*a*)	**52.** (*b*)	**53.** (*d*)	**54.** (*a*)	**55.** (*b*)	**56.** (*d*)	**57.** (*b*)	**58.** (*c*)	**59.** (*c*)	**60.** (*b*)
61. (*a*)	**62.** (*a*)	**63.** (*a*)	**64.** (*d*)	**65.** (*b*)	**66.** (*a*)	**67.** (*c*)	**68.** (*b*)	**69.** (*b*)	**70.** (*d*)
71. (*c*)	**72.** (*d*)	**73.** (*d*)	**74.** (*b*)	**75.** (*d*)	**76.** (*c*)	**77.** (*c*)	**78.** (*d*)	**79.** (*a*)	**80.** (*c*)
81. (*b*)	**82.** (*d*)	**83.** (*b*)	**84.** (*a*)	**85.** (*d*)	**86.** (*d*)	**87.** (*d*)	**88.** (*b*)	**89.** (*a*)	**90.** (*c*)
91. (*a*)	**92.** (*b*)	**93.** (*b*)	**94.** (*d*)	**95.** (*b*)	**96.** (*c*)	**97.** (*b*)	**98.** (*b*)	**99.** (*c*)	**100.** (*d*)
101. (*c*)	**102.** (*b*)	**103.** (*d*)	**104.** (*b*)	**105.** (*d*)	**106.** (*b*)	**107.** (*d*)	**108.** (*b*)	**109.** (*c*)	**110.** (*c*)
111. (*c*)	**112.** (*c*)	**113.** (*c*)	**114.** (*b*)	**115.** (*d*)	**116.** (*b*)	**117.** (*d*)	**118.** (*d*)	**119.** (*b*)	**120.** (*b*)
121. (*a*)	**122.** (*b*)	**123.** (*b*)	**124.** (*a*)	**125.** (*c*)	**126.** (*a*)	**127.** (*b*)	**128.** (*a*)	**129.** (*a*)	**130.** (*d*)
131. (*a*)	**132.** (*d*)	**133.** (*b*)	**134.** (*d*)	**135.** (*c*)	**136.** (*d*)	**137.** (*a*)	**138.** (*c*)	**139.** (*d*)	**140.** (*d*)
141. (*b*)	**142.** (*a*)	**143.** (*a*)	**144.** (*c*)	**145.** (*c*)	**146.** (*b*)	**147.** (*d*)	**148.** (*a*)	**149.** (*d*)	**150.** (*b*)

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151. (*c*)	**152.** (*a*)	**153.** (*c*)	**154.** (*d*)	**155.** (*d*)	**156.** (*b*)	**157.** (*a*)	**158.** (*c*)	**159.** (*c*)	**160.** (*c*)
161. (*c*)	**162.** (*c*)	**163.** (*c*)	**164.** (*d*)	**165.** (*a*)	**166.** (*a*)	**167.** (*a*)	**168.** (*b*)	**169.** (*c*)	**170.** (*a*)
171. (*b*)	**172.** (*c*)	**173.** (*b*)	**174.** (*d*)	**175.** (*c*)	**176.** (*a*)	**177.** (*c*)	**178.** (*a*)	**179.** (*d*)	**180.** (*a*)
181. (*b*)	**182.** (*b*)	**183.** (*d*)	**184.** (*b*)	**185.** (*c*)	**186.** (*d*)	**187.** (*c*)	**188.** (*d*)	**189.** (*b*)	**190.** (*c*)
191. (*b*)	**192.** (*d*)	**193.** (*a*)	**194.** (*b*)	**195.** (*d*)	**196.** (*c*)	**197.** (*c*)	**198.** (*b*)	**199.** (*b*)	**200.** (*d*)
201. (*d*)	**202.** (*b*)	**203.** (*b*)	**204.** (*a*)	**205.** (*b*)	**206.** (*c*)	**207.** (*d*)	**208.** (*d*)	**209.** (*a*)	**210.** (*b*)

Numerical Type Questions

1. 3	**2.** 8080	**3.** 3	**4.** 12	**5.** 8253	**6.** 240	**7.** 1020	**8.** 3
9. 7.5	**10.** 32	**11.** 32	**12.** 3	**13.** 2	**14.** 10	**15.** 8192	**16.** 47
17. 32	**18.** 10	**19.** 92%	**20.** 128	**21.** 20	**22.** 19	**23.** 1101	**24.** 16
25. $(12023)_5$	**26.** 16 bits	**27.** 432	**28.** 31				

EXERCISE – II

MCQ Type Questions

1. (*c*)	**2.** (*a*)	**3.** (*d*)	**4.** (*a*)	**5.** (*b*)	**6.** (*a*)	**7.** (*d*)	**8.** (*d*)	**9.** (*a*)	**10.** (*c*)
11. (*b*)	**12.** (*b*)	**13.** (*d*)	**14.** (*c*)	**15.** (*c*)	**16.** (*c*)	**17.** (*d*)	**18.** (*b*)	**19.** (*b*)	**20.** (*d*)
21. (*d*)	**22.** (*b*)	**23.** (*d*)	**24.** (*d*)	**25.** (*b*)	**26.** (*b*)	**27.** (*c*)	**28.** (*c*)	**29.** (*b*)	**30.** (*d*)
31. (*b*)	**32.** (*c*)	**33.** (*d*)	**34.** (*b*)	**35.** (*a*)	**36.** (*b*)	**37.** (*b*)	**38.** (*b*)	**39.** (*a*)	**40.** (*b*)
41. (*d*)	**42.** (*b*)	**43.** (*a*)							

Numerical Type Questions

1. (4)	**2.** (14020)	**3.** (3.2)	**4.** (14)	**5.** (22)
6. (6.1-6.2)	**7.** (36)	**8.** (13)	**9.** (16383)	**10.** (3)
11. (4)	**12.** (1.6)	**13.** (20 to 20)	**14.** (99.55 to 99.65)	**15.** (10000 to 10000)
16. (122)	**17.** (1.54)	**18.** (1.68)		

EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. Low memory can be connected to microprocessor by using ready.

2. Lower the READY input the micro processor in the wait state.

3. The basic instruction that can be interpreted by a computer generally has an operand and an address.

4. The microprocessor system with memory mapped I/O arithmetic and logic operations can be directly performed with the I/O data.

5. The addressing mode used in an instruction of the form ADD, X, Y is absolute.

6. Control unit is responsible for coordinating various operating using timing signals.

7. Program counter is the track of the executing of a program and which contains the memory address of the instruction currently being executed.

8. Memory address register holds the address of the location to or from which data are to be transferred.

9. Memory data register addressed location.

10. The used as storage location both in the ALU and the control section of a computer register.

11. The register used as a working area in CPU is accumulator.

12. Data register holds information before going to the decoder.

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13. Interface device is used to connect a peripheral to bus.
14. Scratch pad is a set of general purpose internal registers.
15. Program counter to indicate the next instruction which is to be executed.
16. Non-maskable interrupt can be temporarily ignored by the counter.
17. The temporarily half the CPU and use this time to send information on buses is called cycle stealing.
18. Only if interrupts have been enabled by an EI instruction.
19. Process control instruction used to set the carry flag in a computer can be classified.
20. Exactly the same as the machine cycle time generic microprocessor instruction cycle time.
21. 16 bit register that point to stack memory location in the microprocessor.
22. Micro program is the set of instructions indicating the primitive operations in a system.
23. The two-co-processor instruction sets must be the same as that of the host.
24. Pointers require indirect addressing mode.

    Arrays and records required indexing modes.

    Recursive procedures with local variable required.
25. 8085 microprocessor responds to the presence of in interrupt by checking the TRAP pin for 'high' status at the end of the execution of each instruction.
26. Indirect addressing $\rightarrow$ pointers, Immediate Addressing $\rightarrow$ Constants, Auto decrement addressing $\rightarrow$ Loops.
27. Translation look a side buffer saved on a context switch between processes.
28. The stock pointer register then all sequences of subroutine calls and also interrupts are possible.
29. CPU has two nodes a non-privileged instruction (which does not generate an interrpt) is needed frame privileged to non-privileged.
30. The absolute addressing mode location of the operand is implicit.
31. There is no any typically found in the status register of a micro processor.
32. The function of program counter (PC) holds address for instruction.
33. Program counter the address of the instruction following the CAL instruction is stored in/on the.
34. The program counter (PC) is a register, and during execution of the current instruction its content changes.
35. TRAP interrupt mechanism of the 8085 microprocesser executs an RST by hardware.
36. Pseudo-instructions are assembler directives.
37. A typical one address instruction uses that address to specify one operand.

    The ..... by default, will be in the accumulator.

    So, to add n given numbers, $a_1, a_2, ..., a_n$ ..... a_1 to accumulator.

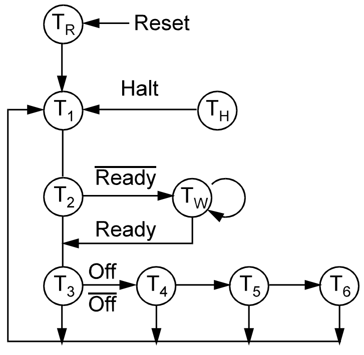
    Next the instruction ADD a_2 – adds the content of a_2 to the ..... and leaves the sum there.

    Continuing this way, n instruction needed to add n numbers and place the ..... accumulator.

    Finally, to store the result in memory 1, more instruction is needed. ..... instructions are needed.
38. The addressing mode used in the instruction BUSH BILLS register indirect.
39. The fatching and executing instructions one at a time, in order of increasing address is called straight line sennuncing.
40. The CPU of a computer takes instruction from the memory and executes them fetch-execute cycle.
41. Here to accommodated long access times, the microprocessor 8085 has a state called *WAIT state*. When microprocessor generates an address in state T_1 external control logic that monitors this state may ask the microprocessor to wait for a certain period of time which are integrals of check periods. To achieve this, external control logic makes the READY input t_0 state T_2 logic 0.

    When input to T_2 is logic 0, the microprocessor enters state T_w (L_{WAIT}) instead of T_3.

    When READY logic becomes 1, the machine cycle goes to state T_3.



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So during WAIT state, all external signals from the microprocessor are held in the same state as they were at the end of state T_2.

By this method, duration of the address is stetched and access times greater than that of microprocessor accommodated.

42. The 8085 microprocessor is the memory address of the instruction that is to be executed next.

43. The HLT instruction of the enters into a halt state and the buses are trislated.

44. DAA → Arithmetic instruction

LXI → Data movement instruction

RST → Interrupt instruction

JMP → Program control instruction.

45. Serial input data of 8085 can be loaded into bit of the accumulator by executing a RIM instruction.

46. The inturrepts are unmaskable interrupts TRAP?

47. Memory address range to RAM will respond 4000 H to 5FFFH is available.

48. I/O chip will respond is 0000H to 5FFFH.

49. A stack is a set of memory locations in R/W memory reserved for storing in formation temporarily during the execution of a program.

50. $\overline{\text{INTA}}$ is active instruction on the databus.

52. Put the microprocessor in the wait state lower the READY input.

53. ALU and section CPU employ special purpose storage locations called registers.

54. The interrepted by a computer generally has an operand and an address.

55. PLA economises on the number of minters.

56. The control unit of computer directs the other unit of computers.

57. ALU of a computer normally contains a number of high speed storage elements called registers.

58. Currently being executed is called program counter.

59. Register which holds the address of the location to or from which data are to be transferred is called memory address register.

60. The register which contains the data to be written into or read out of the addressed location is called memory data register.

61. Communicates with and of ten controls the operation of other subsystems of the computer is the CPU.

62. Ability of a medium sized computer system to increase in data processing capability by addition of such devices as mass storage device computer expand ability.

63. The internal storage during different stage of problem is called overlay.

64. The register used as a working area in CPU is accumulator.

65. The information holds the before going to the decoder data register.

66. The supervise each instruction in the CPU control logic unit.

67. The Bus is used to transfer data from main memory to peripheral device is DMA bus.

68. The device is used to connect a peripheral to bus is called interface.

69. The transfer main memory and the CPU is called memory bus.

70. An interrupt can be temporarily ignored by the counter is called maskable interrupt.

72. The temporarily halt the CPU and use this time to send information on buses is called cycle staling.

73. The microprograming is a technique for programing the control steps of a computer.

74. The works in conjunction with a computer but not as part of it is called peripheral device.

75. Computer manufactures as an abbreviated form of instruction sets is called mnemonic.

76. CAL instruction is stored in/on the program counter.

77. Boolean instruction is stored in the sign status flag.

78. PLA is the produces sum of products as the output and is dedicated for a particular operation.

79. The typical fetch operation is

PC → MAR → Memory → MDR → IR

80. The following is not a form of memory instruction opcode.

81. Dynamic memory is difficult to interface with processor.

82. Desirable charactristic (s) of a memory system is (are) speed and reliability, low power consumption, durability and compactness.

83. Minimum time delay required between in of two successive memory operations is called memory access time.

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84. For a memory system, the cycle time is same as the access time.

85. An EPROM can be programmed, erased and reprogrammed by the user with an EPROM programming instrument.

86. The EPROM memory is ultraviolet light erasable and electrically programable.

87. PROM memory is non-volatile and may be written only once.

88. Refreshing rate of dynamic RAMs is approximately once in two milli seconds.

89. The dynamic RAM memory has a lower bit density and higher power consumption.

90. Disadvantage of dynamic RAM over static RAM is need to refresh the capacitor charge every once in two milli seconds.

91. EEPROM is electrically erasable.

92. The access time of magnetic bubble memory is approximately 30 micro seconds.

93. Serial access memories are useful in applications short access time is required.

94. The main advantage of magnetic core memory over semiconductor Ram memory non-volatile.

95. Semiconductor memory is the fastest memory.

96. Memory management is through segment relations and protection is to perform address translation to reduce size of the memory, to perform address translation to reduce execution time overhead.

97. Memory refreshing may be done by an external refresh controller, only.

98. Memory consisting of electronic circuits attached into silicon chip is known as semi conducter memory.

99. PROM contains a fixed AND array and programmable array.

100. It stores the bulk of data used by computer application.

101. A dynamic Ram consists of 1 transistor and 1 capacitor.

102. Semiconductor memory is a volatile memory.

103. The internal memory of the system (computer) CPU register, cache, main memory all of these.

104. The transfer information between main memory and the CPU is called memory bus.

105. A software program stored in a Rom that can not be changed easily is called firmware.

106. Peripheral device.

107. 1. An 8-bit wide 5 words sequential memory will have – 8 address pins and 4 data pins output.

2. A256X4EPROM → Eight 4 bit shift registers.

108. The main advantage of memory interfacing is that effective speed of the memory is increased.

109. The address lines of the CPU must be **larger** than the physical memory size and **smaller** than the secondary storage size.

110. Threads refer to the execution of different processes simultaneously in the CPU.

Virtual address space is implemented in memory. *File-system* in the way in which data in a disk is origanized.

A signal is acted upon using the mechanism of interrupt.

111. *Virtual memory* is in fact stored in the secondary storage, which increases memory access time.

Processes can be given larger protected address spaces in virtual memory than is possible by just segmenting real memory.

A linker always assigns addresses independent of where the program is loaded in physical memory, so it is not a particular advantage of virtual memory.

Programs larger than physical memory size can be run as pages of the prgram can be swapped from virtual memory to physical memory as and when required.

112. The extra hardware for DRAM refreshing 8085 motoral-68000.

113. PROM contains a fixed AND array and a programmable or array are incorrect statement.

114. Dynamic memory is to difficult to interface with processor.

115. The characteristic of a memory system is (are) speed and reliability, low power consumption durability and compactness.

116. The minimum time delay required between initiation of two successive memory operations is called memory access time.

117. PROM is the non-volatility is an important advantage.

118. PROM is non-volatile and may be written only once.

119. Dynamic RAM is the memory must be refreshed many times per second.

120. Memory refreshing may be done by an external refresh controller only.

121. Low cast per bit charge coupled deviec.

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122. Semiconductor memory is a volatile memory.

123. Memory consisting of electronic circuits attached into silicon chip is known as semiconductor memory.

124. The commonly used auxiliary storage device are magnetic tape, magnetic drum and magnetic disk.

125. PROM contains a fixed AND array and programmable or array.

126. Each cell of a Static Random Access memory contains 6 mos transistors.

127. Semiconductor memory is a volatile memory.

128. Input unit of a computer feeds the data in CPU.

129. A hardware unit is used to monitor computer processing is console.

130. The internal memory of the system (computer) CPU register, Cache, main memory all of these.

131. The memory is capable of operating at electronic speed is semi conductor memory.

132. The program stored in a ROM that can not be changed easily is called firmware.

133. Read cycle must be followed by write cycle.

134. Scratch-pad memory is a local temporary memory.

135. Frequent disk I/Os.

136. 16384 bytes.

137. The cache memory is based on the properly of locality of reference.

138. Properly of locality of reference may fail, if a program has many operands.

139. Latency time, access time, transfer time.

140. 17

141. 16×16

142. Each cell of a Static Random Access memory contains 6 mos transistors.

143. The mass storage device, I/O devices is called computer expandability.

144. Bus use id to transfer data frbm main memory to peripheral device is DMA bus.

145. $25 \times 40 = 1000$/video

146. Hardwired control is the fastest. Since horizontal microprogramming involves lesser number of instructions, it is faster than vertical microprogramming.

147. CISC and RISC process or typically has fewer instruction and addressing modes, has more registers and easier to implement using hard-wired control logic.

148. $T_1 = T_2$;

149. Performance of a pipelined processor suffers if the pipeline stages have different delays, consecutive instruction are dependent on each other, and the pipeline stages share hardware resources.

150. A micro-programed control unit is facililated easy implementation of new instructions.

151. The typical RISC machine highly pipelined and multiple register sets.

152. Since RAM is not enough to store data on disk, hence disk I/O is needed which is wasteful of time.

153. *Memory mapped I / O*

1. I / O have 16 bit addresses
2. Devices are accessed like registers
3. Mass devices = 2^{16} = 256
4. Arithmetic / Logic operations can be done.

154. 8085 microprocessor system with memory mapped I/O arithmetic and logic operations can be directly performed with the I/O data.

155. A microprocessor, wait states are used to interface slow peripherals to the processer.

156. The access time of magnetic bubble memory is approximation 30 micro seconds.

157. increases

158. Memory management is through segment relocation and protection is to perform address translation to reduce size of memory and to perform address translation to reduce execution time overhead.

162. The number of records contained withing ablack of data on magnetic tape is defined by the blocking factor.

163. In magnetic disks, data is organized on the plotter in a concentric sets or rings called head.

164. Fifty

165. Move the outer most track to the inner most track in a magnetic disk then density increases.

166. The commonly used auxiliary storage device are magnetic tape, magnetic drum and magnetic disk.

167. Semi conductor memory is capable of operating at electronic speed.

168. The ALU of a computer normally contains a number of high speed storage elements called registers.

169. The process of entering data into a storage location is called a read out operation.

170. Direct access method is used for obtaining a record from a cassettle tape.

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171. Serial access memories are useful in application short access time is required.

172. In comparison to the internal main memory tape or disk memory is faster and more expensive.

173. Fastest type of memory from there list is semiconductor memory.

174. An advantage of blocking a tape is that less tape is used to store the same amount of data.

175. The number of records contained within a block of data on magnetic tape is defined by the blocking factor.

176. Diskestte and hard disks are direct access devices.

177. Data is organized on the plotter in a concentric sets or rings called head.

178. The number of bits that are typically stored on each track of a magnetic disk is usually the same.

179. It stores the bulk of data used by computer application.

180. Floppy disks is the storage devicess carry around.

181. Pass memory

182. The hardware in data may be stored for a computer system is called memory.

184. Metal oxide semiconductor.

185. Biochips

186. EEPROM

187. 1024

188. The number of addresses will be any of these.

189. 8 k

190. Difference in rates of flow of data from one device to another is called buffer.

191. Potential capacity

192. Video disk

193. 4.7 inch

194. Medium for direct access secondary storage of a computer magnetic disk.

195. The data on the tracks on a magnetic disks are written as high or low voltage.

196. Oval slot part of the diskette never be touched.

197. The two types of main memory are ROM and RAM.

198. Memory is the device when information is stored.

199. Virtual memory consists of those addresses that may be generated by a processor during execution of a computation.

200. 6416×16

201. The process of entering data into a storage location is destructive of previous contents.

202. low power consumption.

203. Time elapsed between initiation of a memory operation and the competion of that operation is called memory access time.

204. The minimum time delay required operations is called memory cycle time.

205. The memory system cycle time is longer than the access time.

206. A block of data on magnetic tape is defined by blocking factor.

207. The idea of cache memory is based on the properly of locality of reference and the heuristic 90-10 rule and the fact that only a small portion of a program is refer end relatively frequently.

208. 17

209. 52 ns

210. 8 mbps

NUMERICAL TYPE QUESTIONS

1. Number of machine cycles required for RET instruction in 8085 microprocessor is 3.

2. Microprocessor 8085 is the enhanced version of **8080** with essentially the same construction set.

3. If we use 3 bits in the instruction word to indicate if an index register is to 3.

4. The capacity of program counter (PC) is 12 bits.

5. The programmable interval timer is 8253.

6. The organized into nibbles 240.

7. Many addresses are required for $25 \times 40 = 1000$ video RAM.

8. Machine cycles required for RET instruction in 8085 microprocessor is 3.

9. 7.5 milli seconds.

10. Core plane is 32.

11. 82

12. 3

13. The wire threads cores in a coincident current core memory 2.

14. 10

15. 8k capital is 8192.

18. 10

19. 92%

20. It can be store upto 128 bits as each cell can hold one bit of data.

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21. All the others require less than 20 address bits.

22. To have 8 data lines and 19 external connections it has to have a address lines.

23. The gray code of 1001 is 1101.

24. 16 RAM chips of size 256 * 4 are need to constant a CACHE of size 2 kB.

25. Divide the given number 888 by 5 and write all the remainder terms from the bottom to up finally we will get (12023) to the base 5.

26. 8086 is a 16 bit processor.

27. (r – 1)'s complement of 345 in octal number system is 432.

28. The octal equivalent of 011001 = 31

EXERCISE – II

MCQ TYPE QUESTIONS

1. Clustered: The ordering of data record is same as or close to the ordering of data entries in same index.

2. 0110110 . . .

4.

4	12	4	$\Rightarrow$	E	201	F
TAG	cache block offset	word offset		TAG	cache block offset	word offset

7. When a cache block size is smaller, it could accommodate more number of blocks, it improves the hit ratio for cache, so the miss penalty for cache will be lowered.

8. When associativity is doubled, then the set offset will be effected, accordingly, the number of bits used for TAG comparator be effected. Width of set index decoder also will be effected when set offset is changed. Width of wag selection multiplexer wil be effected when the block offset is changed. With of processor to main memory data bus is guaranteed to be NOT effected.

9.

S	E	M
1 bit	8 bits	23 bits

$(14.25)_{10} = (1110.01)_2$

$1110.01 \times 2^0 = 1.11001 \times 2^3$

S : 1

E : $(3 + 127)_{10} = (10000010)_2$

M : 11001000 --------

Value stored :

$\underbrace{1\,1\,0\,0}_{[C}\underbrace{0\,0\,0\,1}_{1}\underbrace{0\,1\,1\,0}_{6}\underbrace{0\,1\,0\,0}_{4}\;\underset{0\,0\,0\,0]_H}{0\,0\,-\,-\,-}$

10. Clock period (CP) = max stage delay + overhead

So CP_{p1} = Max (1,2,2,1) = 2ns

CP_{p2} = Max (1, 1.5, 1.5, 1.5) = 1.5ns

CP_{p3} = Max (0.5, 1,1,0.6,1) = 1ns

CP_{p4} = Max (0.5,0.5,1,1,1.1) =1.1ns

As frequency $\alpha \frac{1}{C.P}$, so least clock period will give the highest peak clock frequency.

So, $f_{p3} = \frac{1}{1ns} = 1$ GHz

11. Given 8 – bit delimiter pattern of 01111110.

Output Bit string after stuffing is 011111$\underline{0}$0101

$\Downarrow$

Stuffed Bit

Now, Input String is 0111110101

12. This question can be solved by verifying the options suppose that the cache blocks are arranged as follows:

0	3	6	9	12
1	4	7	10	13
2	5	8	11	14

Here V = 3 and K = 5

then on verifying for any cache block only option(d) gives correct answer.

For example: Suppose we need to find for block 9 then

Option (a): (9 mod 3) ∗ 5 to (9 mod 3) ∗ 5 + 4 = 0 to 4

Option (b): (9 mod 3) to (9 mod 3) + 4 = 0 to 4

Option (c): (9 mod 5) to (9 mod 5) + 2 = 4 to 6

Option (d): (9 mod 5) ∗ 3 to (9 mod 5) ∗ 3 + 2 = 12 to 14

Now check for option (A), (B) & (C)

Check for block 11

Option (A): (11 mod 3) ∗ 5 to (11 mod 3) ∗ 5 + 4 =10 to 14

Option (B): (11 mod 3) to (11 mod 3) + 4   2 to 6

Option (C): (11 mod 5) to (11 mod 5) + 2   6 to 8

Clearly option (A) gives correct answer

13. Given: A sequence of micro operations

MBR ← PC

MAR ← X

PC ← Y

Memory ← MBR

To find: What these micro operation represents?

Analysis: On converting these operations to normal english

1. First micro operation stores the value of PC into memory Base register (MBR)
2. Second micro operation stores value of X into Memory Address register (MAR)
3. Third micro operation stores value of Y into PC
4. Fourth micro operation stores value of MBR to memory.

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So before execution of these instructions PC holds the value of next instruction to be executed. We first stores the value of PC to MBR and then through MBR to memory i.e. we are saving the value of PC in memory and then load PC with a new value. This can be done only in two types of operations conditional branch and interrupt service.

As we are not checking here for any conditions so it is interrupt service.

14. Since starting on disk starts from <1200, 9, 40> so no of sectors left on 9th surface is 24

so, on 9th surface total storage of "12288" B is possible Now, a part from 9th surface, on cylinder no. 1200 only 6 surface is left.

To storage possible on these 6 surface are

$= 6 * \underset{\text{No. of sectors on each surface}}{\underset{\downarrow}{2^6}} * 2^9 \rightarrow$ storage on each sector.

$= 196608\text{ B}$

So total on cylinder no. 1200, storage possible

$\Rightarrow 196608 + 12288 = 208896\text{ B}$

So Since file size is 42797 kB and out of which 208896 B are stored on cylinder no. 1200. So we are left only with only 43615232 B.

Since in 1 cylinder, Storage Possible is

$$= 2^4 * 2^6 * 2^9\text{ B} = 524288\text{ B}$$

So we need about $= \dfrac{43615232}{524288\text{ B}}\text{B}$

$$= 83.189 \text{ more cylinders.}$$

Hence we'll need the $\boxed{1284^{th}\text{ Cylinder}}$ to completely store the file. C_{02} after 1283rd cylinder we will left with data which will need .189

15. Given: Five stage instruction pipeline Delays for FI, DI, FO, EI and wo are 5, 7, 10, 8, 6 ns resp.

To find: Time needed to execute 12 instruction prog.

Analysis: Since the max. time taken by any stage is 10 ns and additional 1 ns is required for delay of buffer. Therefore total time for an instruction to pass from one stage to another is 11ns Now instructions are executed as follows:

$$\xrightarrow[\text{execution with time}]{I_1, I_2, I_3, I_4, I_9, I_{10}, I_{11}, I_{12}}$$

Now when I_4 is in its executioin stage we detect the branch and when I_4 is in WO stage we fetch I_9 so time for execution of instructions from I_1 to I_4 is $= 11 * 5 + (4 - 1) * 11 = 88$ ns.

and time for execution of instructions from I_9 to I_{12} is

$= 11 * 5 + (4 - 1) * 11 = 88$ ns

But we have 11ns when fetching I_9. i.e. I_9 requires only 44 ns additional instead of 55 ns because time for fetching I_9 can be overlap with WO of I_4

Hence total time is = 88 + 88 – 11 = 165 ns

16. Register renaming is done to eliminate WAR/WAW hazards.

17. For a 4 bit multiplier there are $2^4 \times 2^4 = 2^8 = 256$ combinations.

Output will contain 8 bits.

So the amount of ROM needed is $2^8 \times 8$bits = 2 Kbits.

18. Each block size = 128 Bytes

Disk block address = 8 Bytes

Each disk can contain $= \dfrac{128}{8} = 16$ addresses

Size due to 8 direct block addresses: 8×128

Size due to 1 indirect block address: 16×128

Size due to 1 doubly indirect block address: $16 \times 16 \times 128$

Size due to 1 doubly indirect block address: $16 \times 16 \times 128$

So, maximum possible file size:

$$= 8 \times 128 + 16 \times 128 + 16 \times 16 \times 128$$
$$= 1024 + 2048 + 32768$$
$$= 35840 \text{ Bytes} = 35\text{k Bytes}$$

19.
```
1.  Acquire lock (L) {
2.  While (Fetch_And_Add(L, 1))
3.                    L = 1.
        }
4.  Release Lock(L) {
5.                    L = 0;
6.      }
```

Let P and Q be two concurrent processes in the system currently executing as follows

P executes 1,2,3 then Q executes 1 and 2 then P executes 4,5,6 then L=0 now

Q executes 3 by which L will be set to 1 and thereafter no process can set

L to zero, by which all the processes could starve.

20. Here 20 will act as base and content of R_2 will be index

22. 6000 rotations ________ 60 sec

1 rotation _________ 10 ms

$\therefore$ Rotational latency = 5 ms

Time for one disk access = 15 ms

Time to load all libraries = 15×100

= 1500 ms = 1.5 sec

23.

19 + 2	8	5
↓ TAG	↓ block offset	↓ Byte offset $=2^5$

$$= \frac{2^{13}}{2^5} = 2^8$$

Required answer = $256 \times (19 + 2) = 5376$ bits

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24. Main memory unit has capacity = 4 MB

Number of DRAM chips = $\dfrac{4\,\text{MB}}{1\text{M} \times 1\,\text{bit}} = 32$

1 DRAM chip has 1 K (2^{10}) rows and 1 K(2^{10}) cells

In each row = $32 \times 2^{10} \times 2^{10}$

$= 32 \times 2^{20}$

Time taken in one refresh op^n = 100 ns

Time required to perform one refresh operation on all the cells in MM unit

$= 32 \times 2^{20} \times 100$

$= 3200 \times 2^{20}$ ns

25. Clocks

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
20	2F	2D	OF	PO	PO	PO	WO									
21		2F	2D	OF	Stall	Stall	PO	P[O	PO	PO	PO	PO	WO			
22			2F	2D	OF	-	-	-	-	-	-	-	PO	WO		
23				2F	2D	OF	-	-	-	-	-	-	-	PO	WO	

So total 15 cycles required.

Alternately

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

I_0 IF ID OF PO PO PO WO

I_1 IF ID OF .... PO PO PO PO PO POWO

I_2 IF ID OF ... POWO

I_3 IF ID OF .... PO WO

So that 15 cycles required.

26. $a = 1$

$b = 10$ Let a is in Accumulator than for b and c we want

$c = 20$ Register than **2 registers** want till now 1 for b and 1 for c

$d = a + b$ Now $d = a + b$ can be stored in Accumulator because we don't want a again

$e = c + d$. Now for ***f*** we wan't **next, one register** because we wan't ***c* & *e*** again

$f = c + e$

$b = c + e$. Now at $b = c + e$ we can store value in register b already, so we don't want another

$e = b + f$. Now at $e = b + f$ we can store it in 0 register of ***b*** because we don't want another

$d = 5 + e$ Now at $d = 5 + e$ we can store it in d because we don't want e again

than we can say only 3 registers we want.

27. Number of chips required = $\dfrac{256 * 1024 * 8}{32 * 1024 * 1} = 64$

28. A CPU generally handles an interrupt by executing ISR by checking the interrupt register after finishing execution of the current instruction.

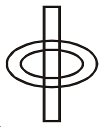
29.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
S_1	I_1		I_2	I_3		I_4	I_1		I_2	I_3		I_4											
S_2			I_1	I_2			I_3	I_4		I_1	I_2			I_3	I_4								
S_3				I_1			I_2		I_3	I_4		I_1		I_2		I_3	I_4						
S_4					I_1				I_2		I_3			I_4		I_1	I_2		I_3			I_4	

30.

Set 0	~~0~~ 48
	~~4~~ 32
	8
	~~216~~ 92
Set 1	1
	133
	129
	73
Set 2	
Set 3	~~255~~ 155
	3
	159
	63

31.



co-centric circular tracks

The seek latency is not linearly proportional to the seak distance due to arm starting and stoping inertia.

32. Auto increment add mode is basically used for incrementing a data. It depends on size of data.

33. For any exception, while executing RFE, no exception is alllowed (no interrupts). It is a priveleged instruction, as soon as exception occurs RFE is called (executed).

34. For inclusion to hole between two cache levels cache hierarchy

L1 must be write through cache.

and L2 cache must be at least as large as L1 cache.

35. In pipelined processor bypassing can handle all RAW hazards and register remaining can eliminate all register carried WAR hazard.

36. The use of multiple windows with overlap causes a reduction in the number of memory accesses for functions local and parameters, register saves and restores and instruction fetches.

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37. Translations Lookaside Buffer can be accessed during effective address calculations.

38. For identifying each of 128 lines uniquely,

number of bits required = 7 bits.

Since each line is of size 64 words, so number of bits needed to identify each word uniquely = 6 bits.

Now, since there are 20 bits in address generated by CPU

$\therefore$ Bits needed for tag = 20 – (7 + 6) = 7 bits.

39. Size of disk

= Number of surfaces × number of Tracks per surface × number of Sectors per track × capacity of each sector

= 16 × 128 × 256 × 512 bytes

= 256 Mega bytes (MB)

Since, total number of sector in disk

= 16 × 128 × 256 = 512 K = 2^{19} Sector

therefore, it requires 19 bits to specify a particular sector.

40. For each of the three instruction four stages will be there. Since ,it is a pipelined processor, so one instruction may be fetched while other is being decoded or executed or written back action performed in each clock cycle may be represented as

Clock Cycle	ADD	MUL	SUB
1	IF		
2	ID	IF	
3	EX	ID	IF
4	WB	EX	ID
5		EX	
6		EX	
7		WB	EX
8			WB

Only, two instructions cannot be executed simulteneously.

41. The main memory consists of 24 banks, each of 2 bytes. Since parallel accesses to all banks are possible, only two parallel accesses of all banks are needed to traverse the whole data.

For one parallel access,

Total time = Decoding Time + Latency Time

$$= \frac{24}{2} + 80 = 92 \text{ ns}$$

Hence, for 2 such accesses,

time = 2 × 92 = 184 ns

42. For 80% of 10^9 instructions, a single cycle is sufficient as there is no conditional branch.

For the rest 20%, an extra cycle will be needed making the number of cycles required to 2.

$$\text{One cycle takes} = \frac{1}{1\text{G}} = 10^{-9} \text{ second}$$

$\therefore$ Total time

$$= 10^{-9} \left(\frac{80}{100} \times 10^9 \times 1 + \frac{20}{100} \times 10^9 \times 2\right)$$

$$= \frac{4}{5} + \frac{2}{5} = 1.2 \text{ seconds}$$

43. Since only the bit with position *pos* is the deciding factor in jumping to the label, we have to set all other bits to 0 in temp. The mask register required for this purpose just has to have 1 in position *pos*. This can be achieved through *pos* number of left shifts over 1.

NUMERICAL TYPE QUESTIONS

1. Logical Address = 32 bits

Page size = 4 kilobyte

Page table entry = 4 byte each

No. of pages = 1 M

Page Table size = 4B × 1M = 4 MB.

2. Given seek time = 4 ms and 10000 rotation per minute i.e. $\frac{60\,\text{sec}}{10000}$ = 1 rotation

$\Rightarrow$ 1 rotation = 6 ms rotation latency

$$= \frac{1}{2} \times 6\,\text{ms} = 3 \text{ ms}$$

No. of sector = 600 per track,

$$1 \text{ sector} = \frac{6\,\text{ms}}{600} = 0.01 \text{ ms}$$

2000 sector = 2000 × 0.01 = 20 ms.

Total time required = (2000 × (4 + 3)) + 20

= 14020 ms

3. 3.2

4. Average access time can be calculated as

0.8 × 5 + 0.2 × 50 = 14ns

5. ← 40 bits →

22	5	13
TAG	SET OFFSET	WORD OFFSET

Then TAG bits are 22

6. 15000 rotation in 1 minute

$$1 \text{ rotation} = \frac{1}{15000} \text{ or } \frac{60\,\text{sec}}{1500} = 4 \text{ ms.}$$

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$$\text{rotational delay} = \frac{1}{2} \times 4 = 2 \text{ ms}$$

$$\text{Seek time} = 2 \times 2 = 4 \text{ ms}$$

$$\text{transfer time} = 0.1 \text{ ms.}$$

$$\text{Average time} = 4 + 2 + 0.1 = 6.1\text{ms}$$

7. Page size = 8 KB

PAS = 32 bits

$$\text{Number of frames} = \frac{2^{32}}{2^{13}} = 2^{19} \text{ frames}$$

One entry require 24 bits

Page Table size = $n \times e$

$$24 \times 2^{20} \times 3 = n \times 24$$

$$n = 2^{23} \text{ pages}$$

Then length of virtual Address is

$$23 + 13 = 36 \text{ bits}$$

8.

1	2	3	4	5	6	7	8	9	10	11	12	13
			IF	–	–	OF					PO	WB
		IF	–	–	OF	–				PO	WB	
	IF	OF	–	–	PO	PO	PO	PO	PO	WB		
IF	OF	PO	PO	PO	WB							

Total clock cycle require = 13

9. 1 Word = 32 bits

Each instruction has 32 bits

To support 45 instructions, opcode must contain 6-bits

Register operand1 requires 6 bits, since the total registers are 64.

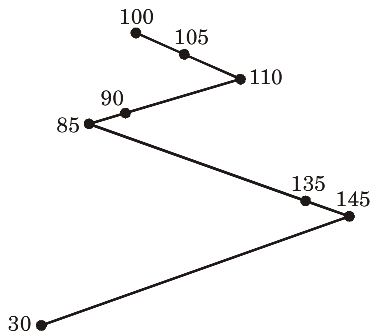
Register operand 2 also requires 6 bits

6	6	6	14
opcode	Reg opd 1	Reg opd 2	immediate opnd

← 32 bits →

14-bits are left over for immediate Operand Using 14-bits, we can give maximum 16383, Since 2^{14} =16384 from 0 to 16383.

10.



Request for cylinder is served after serving 3 requests (100, 105 and 110)

11. For 6 stages, non-pipelining takes 6 cycles

There were 2 stall cycles for pipelining for 25% of the instructions

$$\text{So pipe line time} = \left(1 + \frac{25}{100} 2\right) = \frac{3}{2} = 1.5$$

$$\text{Speed up} = \frac{\text{Non–pipeline time}}{\text{Pipeline time}} = \frac{6}{1.5} = 4$$

12. 1 cycle time for $p_1 = \frac{10^9}{1\text{GH}} = 1\text{n.s}$

Assume p_1 takes 5 cycles for a program then p_2 takes 20% more, means, 6 cycles.

p_2 Takes 25% less time, means, if p_1 takes 5 n.s, then p_2 takes 3.75 n.s.

Assume p_2 clock frequency is x GHz.

p_2 taken 6 cycles, so $\frac{6 \times 10^9}{x\text{GH}} = 3.75, x = 1.6$

13. Physical address size = 32 bits

Cache size = 16k bytes = 2^{14} Bytes

block size = 8 words = 8 × 4 Byte = 32 Bytes

(where each word = b Bytes)

$$\text{No. of blocks} = \frac{2^{14}}{2^5} = 2^9$$

← 32 bits →

TAG	SET OFFSET	BYTE OFFSET
20	7	5

block offset = 9^{bits}

$$\text{No. of sets} = \frac{2^9}{4} = 2^7$$

set offset = 7 bits

Byte offset 8 × 4 Bytes = 32 Byte = 2^5 = 5 bits

TAG = 32 – (7 + 5) = 20 bits.

14. The maximum size of file that can be stored on this disk will be 99.55 to 99.65.

15. Each write request, the bus is occupied for 100 n.s

Storing of data requires 100 n.s.

In 100 n.s. – 1 store

$$\frac{100}{10^6} \text{ n.s.} = 1\text{store}$$

$$1 \text{ m.s.} = \frac{10^6}{100} \text{ stores} = 10000 \text{ stores.}$$

16. $T_{ave} = H_1 \times (T_{TLB} + T_M) + (1 - H_1) \times (T_{TLB} + 2 \times T_M)$

T_{TLB} = time to search in TLB = 10 ms

T_M = time to access physical memory = 30 ms

H_1 = TLB hit ratio = 0.6

$T_{ave} = 0.6 \times (10 + 80) + (1 - 0.6)\,(10 + 2 \times 80)$

$T_{ave} = 0.6 \times 90\text{ms} + 0.4(170)$ ms

$T_{ave} = 54\text{ms} + 68 \text{ ms} = 122\text{ms}$

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17.

	No. of stage	Stall cycle	Stall Frequency	Clock period	Average access time
Old design	5	2	20%	2.2ns	P
New design	8	5	20%	1 ns	Q

$$P = \left[80\%(1\text{clock}) + 20\%\left(\underset{\text{Completion}}{1} + \underset{\text{stall clock}}{2}\right)\right] \times T_{c-p}$$

P = (.8 + .6) × 2.2ns = 3.08ns

$$Q = \left[80\%(1\text{clock}) + 20\%\left(\underset{\text{Completion}}{1} + \underset{\text{stall clock}}{5}\right)\right] \times T_{c-p}$$

P =(.8 +.12) × 1ns = 2ns

So the value of $\frac{P}{Q} = \frac{3.08\text{ns}}{2\text{ ns}} = 1.54$

18. Total instruction

$$= \frac{100\text{ instruction}}{\text{fetch operation}} + \begin{matrix}60\text{memory}\\ \text{operand read}\\ \text{operation}\end{matrix} + \frac{40\text{memory}}{\text{operand write op}}$$

= 200 instructions (operations)

Time taken for fetching 100 instructions (equivalent to read) = 90 * 1ns + 10*5ns =140ns

Memory operand Read operations

= 90% (60)* 1ns + 10% (60)× 5ns

= 54ns + 30 ns = 84 ms

Memory operand write operation time = 90%(40)* 2ns + 10% (40)*10ns

= 72ns + 40 ns = 112 ns

Total time taken for executing 200 instructions

= 140 + 84 + 112 = 336ns

∴ Average memory access time = $\frac{336\text{ns}}{200}$ = 1.68 ns

■■

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3 CHAPTER Data Structures and Algorithms

PROGRAMMING IN 'C'

AN OVERVIEW OF C

C character set.

Alphabets:	A, B, ... Y, Z
	a, *b*, ... *y*, *z*
Digits:	0, 1, 2 9
Some special symbols :	+, –, /, !, =, <, &, :, etc,

Keywords.

C has 32 keywords. These keywords cannot be used as variable name.

Some keywords are :

do, double, int, sizeof, while

Data types.

C have four variables data types :

char, int, float, double.

Structures.

Array provides the facility of grouping homogeneous data item into a single object while structures group together heterogeneous data items into a single object.

To store some information, a structure can be defined as follows :

```
Struct information
{
char name [30] ;
int age ;
float marks ;
char class [20];
}
```

Unions.

It is syntactically the same as a structure. In a structure, each member has a separate memory location whereas in a union all members use the same memory location. e.g.

```
union list
{
int x ;
float y ;
char z ;
} your list ;
```

Enumerated data type.

It provides the facility to specify the possible values a variable can have, by meaningful symbolic names.

Format :

```
enum tag
{
enumerated constant - 1,
enumerated constant - 2,
................
................
enumerated constant - n
} ;
```

where tag is name of an enumerated data type.

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Typedef.

The C language provides a facility to use alternate data type names, called '*typedef*'. *e.g*.

```
# include <stdio.h>
main ()
{
typedef unsigned char ABC ;
ABC a ;
a = 254 ;
printf ("%d", a) ;
}
output
254
```

The above program shows that the alternate name 'ABC' can be used for data type unsigned char.

LOOPS

The versatility of the computer lies in its ability to perform a set of instructions repeatedly. This involves repeating some portion of the program either a specified number of times or until a particular condition is being satisfied. This repetitive operation is done through a loop control structure.

Methods to repeat a part of a program

(1) Using a *while* statement

(2) Using a *for* statement

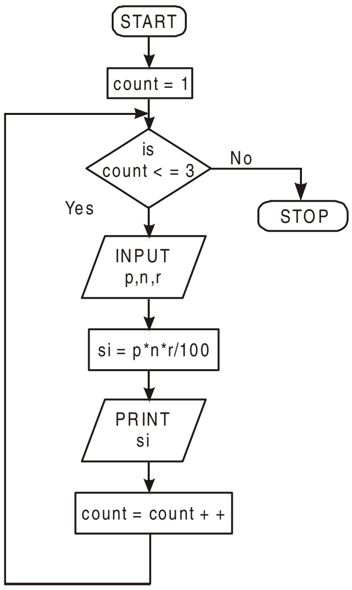
(3) Using a *do-while* statement

(1) WHILE LOOP.

It is ideally suited for programming when something is required to be done at fixed number of times.

Consider a simple example which uses a *while* loops.

The flowchart shown would help to understand operation of the *while* loop.




```
/* Calculation of simple interest for 3 sets for p, n, and r */
main ()
{
      int p, n, count ;
      float r, si ;
      count = 1 ;
      while (count < = 3)
      {
            printf ("\nEnter values of p, n and r") ;
            scanf ("%d %d %f ", &p, &n, &r) ;
            si = p * n * r / 100 ;
            printf ("Simple interest = Rs. %f ", si) ;
            count = count + 1;
      }
}
```

Some sample runs.

Enter values of p, n and r 1000 5 13.5

Simple interest = Rs. 675.000000

Enter values of p, n and r 2000 5 13.5

Simple interest = Rs. 1350.000000

Enter values of p, n and r 3500 5 3.5

Simple interest = Rs. 612.500000

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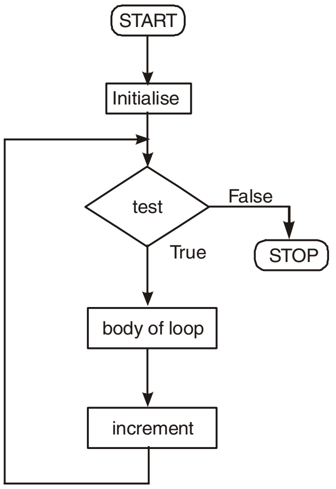
The program executes all statements after the *while* 3 times. The logic for calculating the simple interest is written within a pair of braces immediately after the *while* keyword. These statements form '*body*' of the *while* loop. The parentheses after the while contains a condition. So long as this condition remains true, all statements within the body of the *while* loop keep getting executed repeatedly. To begin with the variable, *count* is initialised to 1 and every time the simple interest logic is executed the value of count is incremented by one. The variable count is many a time called either a *loop counter* or an *index variable*.

Finally, we are using the'\n' in the *printf* **()** to ensure that every time after printing out the value of simple interest, the cursor should go to the next line, so that message 'Enter values of *p*, *n*, *r*' appears on a new line. '\n' is called *newline* and is an escape sequence. The operation of *while* loop is shown in the figure.



General form of while loop :

```
initialize loop counter ;
while (test loop counter using a condition)
{
    do this ;
    and this ;
}
```

Note:

(*i*) The statements within the *while* loop would keep on getting executed till the condition being tested remains true. When the condition becomes false, the control passes to the first statement that follows the body of the *while* loop.

(*ii*) The condition being tested may use relational or logical operators as shown below :

```
while (i <= 10)
while (i < 10 && j <= 15)
while (j > 10 && (b < 15 || c < 20))
```

(*iii*) The statements within the loop may be a single line or a block of statements. In the first case, the parentheses are optional, e.g.

```
while (i <= 10)
    i = i + 1 ;
```

is same as

```
while (i <= 10)
{
    i = i + 1;
}
```

As a rule, *while* must test a condition that will eventually become false, otherwise the loop would be executed forever, indefinitely.

```
main ()
{
    int i = 1;
    while (i <= 10)
    printf ("%d\n", i) ;
}
```

This is an indefinite loop, since i remains equal to 1 forever. The correct form would be as given below

```
main ()
{
    int = 1 ;
    while (i <= 10)
    {
        printf ("%d\n", i) ;
        i = i + 1;
    }
}
```

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Instead of incrementing a loop counter, we can even decrement it and will manage to get the body of the loop executed repeatedly. This is shown below :

```
main ()
{
      int i = 5 ;
      while (i >= 1)
      {
            printf ("\nMake the computer literate !") ;
            i = i – 1 ;
      }
}
```

It is not necessary that a loop counter must only be an int. It can even be a float :

```
main ()
{
      float a = 10.0 ;
      while (a <= 10.5)
      {
            printf ("\n Raindrops on roses ...") ;
            printf ("... and whiskers on kittens") ;
            a = a + 0.1 ;
      }
}
```

Even floating point loop counters can be decremented. Once again increment and the decrement could be of any value, not necessarily 1.

(2) FOR LOOP.

It allows to specify three things about a loop in a single line :

(*i*) Setting a loop counter to an initial value.

(*ii*) Testing the loop counter to determine whether its value has reached the number of repetitions desired.

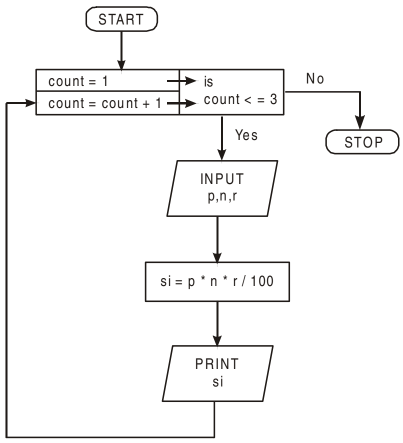
(*iii*) Increasing the value of loop counter each time the program segment (within the loop) has been executed.

General form of ***for*** *statement :*

```
for (initialize counter ; test counter ; increment counter)
{
      do this ;
      and this ;
      and this ;
}
```

We write the simple interest program using *for*. Compare this program with the one which we wrote using *while*. The flow chart is also given below.

```
/* Calculation of simple interest for 3 sets p, n and r*/
main ()
{
      int p, n, count ;
      float r, si ;
      for (count = 1; count < = 3; count = count + 1)
      {
            printf ("Enter values of p, n, and r") ;
            scanf ("%d %d %f", &p, &n, &r) ;
            si = p * n * r / 100;
            printf ("simple interest = Rs. %f\n", Si) ;
      }
}
```



If this program is compared with the one written using *while,* it can be seen that three steps are required for the loop construct have now been incorporated in the *for* statement :

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(*i*) Initialisation

(*ii*) Testing

(*iii*) Incrementation

Execution of ***for*** *statement :*

(*i*) When *for* statement is executed for the first time, the value of *count* is set to an initial value 1.

(*ii*) Now the condition *count* < = *3* is tested. Since *count* is 1, the condition is satisfied and body of the loop is executed for the first time.

(*iii*) Upon reaching the closing brace of *for*, computer sends the control back to the *for* statement, where the value of *count* gets incremented by 1.

(*iv*) Again the test is performed to check whether the new value of *count* exceeds 3.

(*v*) If the value of *count* is still within the range 1 to 3, the statements within the braces of *for* are executed again.

(*vi*) The body of the *for* loop continues to get executed till *count* doesn't exceeds the final value 3.

(*vii*) When *count* reaches the value 4, the control exits from the loop and is transferred to the statement (if any) immediately after the body for *for.*

Nesting of loops.

The way *if* statements can be nested, similarly *whiles* and *fors* can also be nested.

Consider the program given below :

```
/* Demonstration of nested loops */
main ()
{
    int r, c, sum ;
    for (r = 1; r < = 3; r ++)   /* outer loop */
    {
        for (c = 1; c <= 2; c ++)   /* inner loop */
        {
                sum = r + c;
                printf ("r = %d c = %d sum = %d\n", r, c, sum);
        }
    }
}
```

When we run this program, following output is obtained :

```
r = 1 c = 1 sum = 2
r = 1 c = 2 sum = 3
r = 2 c = 1 sum = 3
r = 2 c = 2 sum = 4
r = 3 c = 1 sum = 4
r = 3 c = 2 sum = 5
```

Here, for each value of *r*, the inner loop is cycled through twice, with the variable *c* taking values from 1 to 2.

The inner loop terminates when the value of *c* exceeds 2.

Outer loop terminates when the value of *r* exceeds 3.

The body of outer *for* loop is indented, and the body of the inner *for* loop is further indented. These multiple indentations make the program easier to understand.

Instead of using two statements, one to calculate *sum* and another to print it out, we can compact this into one single state ment by saying :

```
printf ("r = %d c = %d sum = %d\n", r, c, r + c);
```

The way *for* loops have been nested here, similarly two *while* loops can also be nested.

Not only this, a *for* loop can occur within a *while* loop, or a *while* within a *for*.

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Multiple initialisations in the *for* loop.

The initialisations expression of the *for* loop can contain more than one statement separated by a comma.

e.g. for ($i = 1, j = 2; j < = 10; j$ ++)

Multiple statements can also be used in the incrementation expression of *for* loop,

i.e. we can increment (or decrement) two or more variables at the same time. However only one expression is allowed in the set expression. This expression may contain several conditions linked together using logical operators.

Use of multiple statements in the initialisations expression also demonstrates why semicolons are used to separate the three expressions in *for* loop. If commas had been used, they could not also have been used to separate multiple statements in the initialisation expression, without confusing the compiler.

Odd Loop.

Loops that we have used so far executed the statements within them a finite number of times. However, in real life programming one comes across a situation when it is not known before hand how many times the statements in the loop are to be executed. This situation can be programmed as shown below :

```
/* Execution of a loop an unknown number of lines */
main ()
{
    char another = 'y'
    int num ;
    while (another == 'y')
    {
        printf ("Enter a number") ;
        scanf ("%d", &num) ;
        printf ("square of %d is %d", num, num * num) ;
        printf ("\nWant to enter another number y/n'") ;
        scanf ("%c", &another) ;
    }
}
```

And here is the sample output

```
enter a number 5
square of 5 is 25
Want to enter another number y/n y
Enter a number 7
square of 7 is 49
want to enter another number y/n n
```

In this program, the *while* loop would keep getting executed till the user continues to answer *y*. The moment he answer *n*, the loop terminates, since the condition *(another == 'y')* fails.

***break* STATEMENT.**

We often come across situations where we want to jump out of a loop instantly, without waiting to get back to the conditional test. The keywords *break* allows us to do this. When the keyword *break* is encountered inside any C loop, control automatically passes to the first statement after the loop. A *break* is usually associated with an *if*.

e.g. *Write a program to determine whether a number is prime or not. A prime number is one which is divisible only by 1 or itself.*

All we have to do test whether a number is prime or not, is to divide is successively by all numbers from 2 to one less than itself. If remainder of any of these division is zero, the number is not a prime.

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Following program implements this logic :

```
main ()
{
    int num, i ;
    printf ("Enter a number") ;
    scanf ("%d", &num) ;
    i = 2 ;
    while (i <= num – 1)
    {
        if (num % i == 0)
        {
            printf ("Not a prime number") ;
            break ;
        }
        i ++ ;
    }
    if (i == num)
        printf ("Prime number") ;
}
```

In this program, the moment *num % i* turns out to be zero, the message "Not a prime number" is printed and the control breaks out of the *while* loop.

The program require the *if* statement after the *while* loop because there are two ways the control could have reached outside the *while* loop :

(*i*) It jumped out because the number proved to be not a prime.

(*ii*) The loop came to an end because the value of **i** became equal to *num*.

In the second case it means that there was no number between 2 and *num – 1* that could exactly divide *num*, i.e., *num* is indeed a prime. If this is true, the program should print out the message "*prime number*".

The keyword break, breaks the control only from the while in which it is placed.

Consider following program :

```
main ()
{
    int i = 1, j = 1;
    while (i ++ <= 100)
    {
        while (j ++ <= 200)
        {
            if (j == 150)
                break ;
            else
                printf ("%d %d\n", i, j)
        }
    }
}
```

In this program when *j* equals 150, *break* takes the control outside the inner *while* only, since it is placed inside the inner *while.*

***continue* STATEMENT.**

In some programming situations we want to take the control to the beginning of the loop, bypassing the statements inside the loop which have not yet been executed. The keywords *continue* allows to do this. When the keyword *continue* is encountered inside any C loop, control automatically passes to the beginning of the loop.

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A *continue* is usually associated with an *if*.

Consider following program :

```
main ()
{
    in i, j ;
    for (i = 1; i < = 2; i ++)
    {
        for (j = 1; j < = 2; j ++)
        {
            if (i == j)
                continue
            printf ("\n %d %d\n", i, j) ;
        }
    }
}
```

Output of the above program would be

12

21

When the value of i equals that of j, the *continue* statement take the control of the *for* inner Loop by passing rest of the statements pending execution in the *for* inner loop .

(3) *DO WHILE* LOOP.

```
do
{
    this :
    and this ;
    and this ;
    and this ;
} while (this condition is true) ;
```

There is a minor difference between the working of *while* and *do while* loops. The difference is the place where the condition is tested. The *while* tests the condition before executing any of the statements within the *while* loop, while the *do while* tests the condition after having executed the statements within the loop.

This mean that *do while* would execute its statements at least once, even if the condition fails for the first time itself. But *while* will not execute its statements if the condition fails for the first time.

This difference is explained by the following program :

```
main ()
{
    while (4 < 1)
        printf ("Hello there \n") ;
}
```

Here, since the condition fails for the first time itself, the *printf ()* will not get executed at all.

Now write the same program using a *do while* loop.

```
main ()
{
    do
    {
        printf ("Hello there \n");
    } while (4 < 1) ;
}
```

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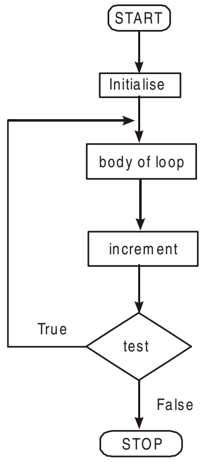

In this program, the *printf ()* would be executed once, since first the body of the loop is executed and then the condition is tested.

Apart from this peculiarity of the *do while*, the *while* and *do-while* behave exactly identically.

do while loops are rarely used in C programs, since there are comparatively fewer occasions when we want to execute a loop at least once no matter what.

break and *continue* are used with *do while* just as they would be in a *while* or a loop *for* loop.A *break* takes us out of the *do while* bypassing the conditional test. A *continue* sends us straight to the test of the end of the loop.

Given figure clarify the execution of ***do while*** *loop.*

ENUMERATED DATA TYPE

It gives an opportunity to invent our own data type and define what value the variable of this data type can take. This also help in making the program more readable, which can be an advantage when a program gets complicated or when more than one programmer would be working on it. Using enumerated data type can also help in reducing programming errors.

One could invent a data type called mar_status which can have four possible values :

single, married, divorced or widowed.

e.g. Married for instance has the same relationship to the variable *mar_status* as the number 15 has with an integer variable.

Format of the *enum* definition is similar to that of a structure. Example stated above can be implemented as shown below.

```
enum mar_status
{
    single, married, divorced, widowed
};
enum mar_status person 1, person 2 ;
```

This declaration has two parts :

(*i*) First part declares the data type and specifies its possible values. These values are called *enumerators*.

(*ii*) Second part declares variables for this data type.

Uses of enumerated data type.

Enumerated variables are used to clarify the operation of a program.

e.g. if we need to use employee departments in a payroll, program, it makes the listing easier to read if we use values like Assembly, Manufacturing, Accounts rather than the integer values 0, 1, 2 etc.

To dissect the program, first defined the data type *enum emp_dept* and specified four possible values :

assembly, manufacturing, accounts and stores.

Then define a variable *department* of the type *enum emp_dep* in a structure. The structure *employee* has three other elements containing employees information.

The program first assigns values to the variables in the structure. The statement,

```
e.department = manufacturing ;
```

assigns the value manufacturing to *e.department* variable. This is much more informative to anyone reading the program than statement like,

```
e.department = 1.
```

The next part of the program shows an important weakness of using *enum* variables ... there is no way to use the enumerated values directly in input/output functions like *printf ()* and *scanf ()*.

The *printf ()* function is not smart enough to perform the translation; the department is printed out as 1 and not manufacturing. We can write a function to print the correct enumerated values, using a *switch* statement, but that would reduce the clarity of the program.

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Renaming data types of with typedef.

There is one more technique which in some situations can help to clarify the source code of a C program. This technique is to make use of the *typedef* declaration. Its purpose is to redefine the name of an existing variable type. *e.g.* consider following statement in which the type *unsigned long int* is redefined to be of the type

TWOWORDS :

```
typedef unsigned long int TWOWORDS
```

Now we can declare variables of the type *unsigned long int* by writing.

```
TWOWORDS var1, var2 ;
```

instead of

```
unsigned long int var1, var2 ;
```

Thus, *typedef* provides a short and meaningful way to call a data type. Usually, uppercase letters are used to make it clear that we are dealing with a renamed data type.

While increase in readability is probably not great in this example, it can be significant when the name of a particular data type is long and unwieldy, as it often is with structure declarations.

By reducing length and apparent complexity of data type, *typedef* can help to clarify source listing and save time and energy spent in understanding a program.

Type casting.

Sometimes it is required to force the compiler to explicity convert the value of an expression to a particular data type.

Consider the following example :

```
main ()
{
    float a ;
    int x = 6, y = 4;
    a = x/y ;
    printf ("\nValue of a = %f", a) ;
}
```

The output is

Value of a = 1.000000

The answer turns out to be 1.000000 and not 1.5. This is because, 6 and 4 are both integers and hence $6/4$ yields an integer, 1. This 1 when stored in a is converted to 1.000000. But if we don't want the quotient to be truncated, then make either x or y as *float.* If other requirements of the program does not permit to do this, then use type casting.

The expression *(float)* causes the variable x to be converted from type *int* to type *float* before being used in the division operation.

The value of a doesn't get permanently changed as a result of type casting. Rather it is the value of the expression that undergoes type conversion whenever the cast appears.

BIT FIELDS

If in a program a variable is to take only two values 1 and 0, we really need only a single bit to store it. Similarly, if a variable is to take values from 0 to 3, then two bits are sufficient to store these values. If a variable is to take values from 0 through 7, then three bits will be enough, and so on.

For one thing, there aren't any one bit or two bit or three bit data types available in C. However, when there are several variables whose maximum values are small enough to pack into a single memory location, we can use 'bit fields' to store several values in a single integer.

Consider following example to demonstrate working of bit fields :

Let we want to store following data about an employees. Each person can :

(*a*) be male or female

(*b*) be single, married, divorced or widowed

(*c*) have one of the eight different hobbies

(*d*) can choose from any of the fifteen different schemes proposed by three company to pursue his/her hobby.

This means we need one bit to store gender, two to store marital status, three for hobby, and four for scheme.

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We need ten bits altogether, which means we can pack all this information into a single integer, since an integer is 16 bits long.

To do this using bit fields, we declare the following structure :

```
struct employee
{
    unsigned gender : 1 ;
    unsigned mar_stat : 2 ;
    unsigned hobby : 3 ;
    unsigned scheme : 4 ;
}
```

The colon in the above declaration tells the compiler that we are talking about bit fields and the number after it tells how many bits to allot for the field.

Once we have established a bit field, we can reference it just like any other structure element, as shown in the program given below :

```
# define Male 0 ;
# define female 1 ;
# define single 0 ;
# define married 1 ;
# define divorced 2 ;
# define widowed 3 ;
main ()
{
    struct employee
    {
        unsigned gender : 1 ;
        unsigned mar_stat : 2 ;
        unsigned hobby : 3 ;
        unsigned scheme : 4 ;
    }
    struct employee e;
    e.gender = male;
    e.mar_status = Divorced ;
    e.hobby = 5 ;
    e.scheme = 9 ;
    printf ("\nGender = %d", e.gender) ;
    printf ("\nMarital status = %d", e.mar_status) ;
}
```

The output is

```
Gender = 0
Marital status = 2
```

POINTERS TO FUNCTION

Every type of variable with the exception of register, has an address. Variables of the type *char, int, float* etc... can be referenced through their addresses by using pointers. Pointers can also point to C function. C functions have addresses. If function's address is known, then we can point to it, which provides another way to invoke it.

This can be done as follows:.

```
main ()
{
    int display () ;
    printf ("\nAddress of function display is %u", display) ;
    display () ; /* usual way of invoking a function */
}
display ()
```

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```
{
      puts ("\nLong live viruses !!") ;
}
```

Output of the program would be :

```
Address of function display is 1125
Long live viruses !!
```

To obtain address of a function, mention name of the function, as has been done in the *printf ()* statement above. This is similar to mentioning the name of the array to get its base address.

Consider following program to see that how using the address of a function we can manage to invoke it :

```
/* Invoking a function using a pointer to a function*/
main ()
{
      int display ();
      int (*func_ptr) () ;
      func_ptr = display ; /* assign address of function */
      printf ("\nAddress of function display is %u", func_ptr) ;
      (*func_ptr) () ; /* invokes the function display () */
}
int display ()
{
      puts ("\nLong live viruses!!") ;
}
```

Output of the program would be :

```
Address of function display is 1125
Long live viruses !!
```

In *main ( )* the function *display ( )* is declared as a function returning as *int*.

int (*func_ptr) () ;

We are declaring something, like *display ( )*, will return an *int*.

If we glance down a few lines in our program, we see the statement

func_ptr = display ;

So we know that *func_ptr* is being assigned the address of *display ()*. Therefore, *func_ptr* must be a pointer to the function *display ()*. Thus all that the declaration.

int (*func_ptr) ( ) ;

i.e. that *func_ptr* is a pointer to a function which returns an *in***t**. And to invoke the function, we are just required to write the statement.

(*func_ptr) () ;

Pointers to functions are certainly awkward and offputting.

Two possible gain of using this esoteric feature of C :

(*a*) In writing memory resident programs

(*b*) In writing viruses, or vaccines to remove the viruses

FUNCTIONS RETURNING POINTERS

The way functions return as *int,* a *float,* a *double* or any other data type, it can even return a pointer. However, to make a function return a pointer it has to be explicity mentioned in the calling function as well as in the function definition.

This program just indicates how an integer pointer can be returned from a function. This concept can be put to use while handling strings.

Here, we send the base addresses of *source* and *target* strings to *copy ()*. In the *copy ()* function, the while loop copies the characters in the source strings into the target string. Since during copying *t* is continuously incremented, before entering into the loop, initial value of *t* is safely stored in the character pointer *r*. Once copying is over, this character pointer *r* is returned to main ().

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ARRAYS

ARRAYS

The simplest data structures in Java are the primitive types `byte`, `short`, `int`, `long`, `float`, `double`, `char` and `boolean`. More correctly, we should refer to the corresponding

- `Byte`
- `Integer`
- `Float`

wrapper classes etc. Each instance of these is capable of storing a data item of the corresponding type, with public methods for manipulating that item.

The expression "data structure", however, is usually used to refer to more complex ways of storing and manipulating data, such as arrays, stacks, queues etc. We begin by discussing the simplest, but one of the most useful data structures, namely the **array**.

Array Variables

An array variable `a`, say, is declared by

```
int a [] ;
```

This declares `a` to be a variable capable of storing a reference to an array of integers.

Initialisation of Array Variables

Before using the variable `a`, it must be properly initialised, either by assignment to another properly initialised variable of the same type, or directly as in

```
a = new int[20];
```

This instructs the operating system to find consecutive storage for 20 integers and place a reference to that block of memory in the variable `a`. This is sometimes known as *allocating* the array. Note that the operations of declaration and allocation can be done together as in

```
int a[] = new int[20];
```

Either way, elements of the array can now be accessed by index as in

```
a[2] = 173;
```

which assigns the integer `173` to the array element `2`. Elements can also be accessed using an index *variable* as in `a[i]`. The variable `i` must be of type `byte`, `short` or `int`, but `int` is the most common.

Array Bounds

Arrays in Java are always **zero-based**. This means that the first element is `a[0]`, the second is `a[1]` and so on. The previous assignment to `a[2]` refers to the *third* element of the array. Correspondingly the last element of the array is `a[19]`. Attempts to access `a[20]` or beyond will lead to an `ArrayIndexOutOfBoundsException` at runtime, as will negative values.

Initialisation of Array Elements

When Java arrays are created by allocation using `new`, their elements are automatically initialised to `0` for numeric types, to the null character for `char` arrays, to `false` for `boolean` arrays and to the `null` reference for object arrays. With the possible exception of `null` for object arrays, it is generally better not to rely on implicit initialisation. Explicit initialisation makes code more secure and more intelligible. This can be done by

```
for (int i = 0; i < 20; i++) {
    a[i] = 0;
}
```

for example.

Initialisation of Both Together

Arrays can be created and initialised in one operation using a list of values, as in

```
int a[] = {24, 65, 34, 96, 12};
```

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This first allocates storage to the variable a for 5 integers, and secondly makes the corresponding assignments to the elements of the array that has been created. The length of the array is determined by the compiler at compile time in the obvious way. Note that new is not used explicitly when arrays are created by lists. Note also that initialiser lists can only be used when an array is declared. Thus

```
int a [] ;
a = {24, 65, 34, 96, 12};
```

would be illegal.

Length

When an array is created its size is held in a public constant which can accessed by the expression a.length. This is useful for ensuring that the array index stays within bounds. For example

```
for (int i = 0; i < a.length; i++) {
    a[i] = 0;
}
```

initialises all elements to 0 in a safe way.

Assignment

Arrays are treated as Objects in Java and, as such, obey the same rules for equality and assignment. We recall these here in the context of arrays.

An array variable is a *reference* variable. It stores information about where to locate the array elements. As a result, all array variables require the same amount of storage, namely one machine word, irrespective of the size of the arrays or the nature of their elements. For example, when an array variable is initialised by

```
int a[] = new int[20];
```

storage is obtained for 20 integers and the address of that block of memory is placed in the variable a. If we now declare another array variable

```
int b[];
```

and make the assignment

```
b = a;
```

the variable b now holds the same address as a. But there is still only one array; there are now two ways of addressing it. Any change to b[i] will change a[i] identically, and conversely, because they refer to one and the same memory location. For that reason, array variable assignment should be used with caution.

If you want to create a copy of an array, you have to work harder, for example:

```
b = new int[a.length];
for (int i = 0; i < a.length; i++) {
    b[i] = a[i];
}
```

There are now two distinct arrays, of the same length and with the same contents. Change to either will leave the other unaffected.

Equality

Equality behaves in a similar way. The expression

```
a == b;
```

is true if and only if the array variables a and b store the same address. More commonly you might wish to check whether a and b are of the same length and have the same contents. In that case, you need to write your own simple method.

Arrays as Parameters

Array variables can be used as parameters to methods. When an array is passed in this way, a copy is made of the actual parameter. Since this is only a reference to the array, only the reference is copied, which makes for efficient passing of arrays to methods. The method now has access to the elements of the array, via the reference, which means that they can be changed as the method chooses. The reference itself, however, will not be changed outside the method.

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Arrays of Objects

So far we have only considered arrays of elements of the various primitive types. All elements of such an array must be of the same type. But we can also have arrays of objects of any class. For example

```
String[] words = new String[20];
```

creates a String array with 20 elements. More generally we can have Object arrays. Thus

```
Object[] heap = new Object[20]
```

declares `heap` to be an array of 20 Objects.

It is important to remember that arrays must be initialised if they are to hold data that is different from the default initialisation. For arrays of Objects, the default initialisation is the `null` reference. Thus, with the declaration of `words`, we have not created any strings. Elements of the `words` array must be initialised, for example, by

```
words[0] = new String("");
```

which assigns to the array element `words[0]` a reference to the empty string.

An alternative way of initialising string arrays is by an initialiser list, for example

```
String[] words = {"the", "cat", "sat", "on", "the", "mat"};
```

which initialises `words` to be an array of length six containing the specified strings.

Multi-Dimensional Arrays

The elements of an array can themselves be arrays. This is a special case of having arrays of Objects. But in this case the original array is said to be a **multidimensional** array. The following example declares and creates a rectangular integer array with 10 rows and 20 columns:

```
int a[][] = new int[10][20];
```

It can be initialised by code such as

```
for (int i = 0; i < a.length; i++) {
    for (int j = 0; j < a[i].length; j++) {
        a[i][j] = 0;
    }
}
```

The elements are accessed as `a[i][j]`. This is a consistent notation since `a[i][j]` is element `j` of the array `a[i]`. This element is said to be in row `i` and column `j`. Note that `a.length` has the value 10 and each `a[i].length` has the value 20.

Two-dimensional arrays can, of course, be square e.g.

```
int a[][] = new int[10][10];
```

They can also have different numbers of columns in each row. For example, the following code declares a (lower) triangular array in which row `a[i]` has `i+1` elements.

```
int a[][] = new int[10][];
for (int i = 0; i < a.length; i++) {
    a[i] = new int[i+1];
}
```

Such arrays can be useful for representing symmetric matrices, `a[i][j] == a[j][i]`, where it is only necessary to operate on, and store, one copy of the off-diagonal elements. The following code initialises such an array to the unit matrix, with `1`'s on the diagonal and `0`'s elsewhere:

```
for (int i = 0; i < a.length; i++) {
    for (int j = 0; j < i; j++) {
        a[i][j] = 0;
    }
    a[i][j] = 1;
}
```

The result is

```
1
0 1
0 0 1
```

if `a.length == 3`.

Multi-dimensional arrays can also be created and initialised using initialiser lists as in

```
int a[][] = {{54, 64}, {98, 12, 67}};
```

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Array Access and Memory Allocation

One of the principal reasons arrays are used so widely is that their elements can be accessed in *constant time*. This means that the time taken to access `a[i]` is the same for each index `i`. This is because the address of `a[i]` can be determined arithmetically by adding a suitable offset to the address of the head of the array. The reason is that space for the contents of an array is allocated as a contiguous block of memory.

This feature, however, is also one of the limitations of arrays. Once the space is allocated, it cannot be extended. There is no guarantee that the next block of memory will be free at some time later in the execution of the program. In this sense arrays are static data structures.

On the other hand, arrays can be allocated dynamically in Java. This contrasts with a number of programming languages in which all array sizes must be known at compile time. This is not true of Java. The following, for example, is valid:

```
int n = 20;
int a[] = new int[n];
```

Although `n` was initialised here at declaration, it could equally have been set in any way during program execution, e.g. as a consequence of file or keyboard input. It is still true, however, that once space has been allocated, it cannot be extended.

In order to overcome this limitation, Java provides a `Vector` class in the `java.util` package which effectively provides arrays that can grow or shrink dynamically. In reality, however, once the array is full, another larger array is created and the contents of the first are copied into the second; but this is opaque to the user. We shall need a facility like this when dealing with stacks and queues. But it will be clearer to write the simple code necessary explicitly when it is needed.

LINKED LISTS

INTRODUCTION

Linked list are special list of some data elements linked to one another for implementing a list in a memory is to use a structure which is self-referential structure.

```
struct node
{
   int data;
   struct node ;
       * link;
}
```

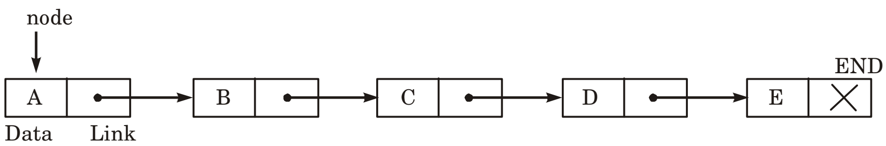

Fig. element linked stack

The use of links require more storage but provide greater flexibility. There is no need to keep an explicit counter of the number of nodes in the list. Overflow is not a problem until the computer get memory exhausted.

Queues can also be implemented as linked lists shown in the figure below. Two pointers, rear and front are used to point the queue. An empty queue is denoted by front = 0. Deletion are made at rear.

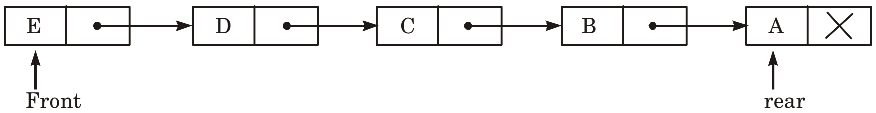

Fig. Linked queue with five elements

The continuous storage is generally preferable when structures are individually very small. Linked list provides superior when the structures are large.

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DOUBLE LINKED LIST

A double linked list in which each node contains two links :

(*i*) One to the next node in the list

(*ii*) One to the preceding node.

It is not possible to move either direction through the list while keeping only one pointer. With a doubly linked list, traversals in either direction, insertions and deletions from arbitrary position in the list can be programmed without difficulty. The cost of a doubly linked list is the extra space required in each node for a second link.

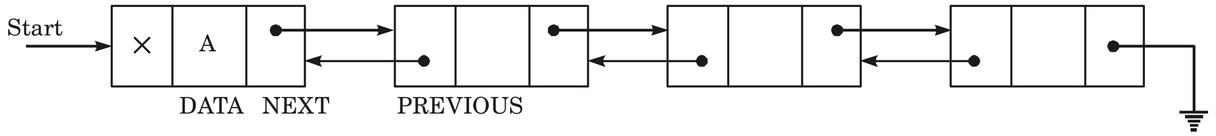

Fig. Double linked list

LINKED LISTS USING DYNAMIC VARIABLES

We have the capability of dynamically allocating and freeing a variable and these dynamic variables can be used to implement linked lists.

A linked list consists of a set of nodes, each of which has two fields:

(*i*) Information field

(*ii*) Pointer to the next node in the list.

In addition, an external pointer points to the first node in the list. A pointer variables is used to implement list pointers. Thus, we define the type of a pointer and a node by

```
struct node {
    int info ;
    struct node *next;
};
typedef struct node *NODEPTR;
```

A node of this type is identical to the nodes of the array implementation except that the *next* field is a pointer (containing address of the next node in the list) rather than an integer (containing the index within an array where next node in the list is kept).

```
NODEPTR getnode()
{
    NODEPTR p;
    p = (NODEPTR*) malloc(sizeof(struct node));
    return(p);
}
freenode(p)
NODEPTR p;
{
    free(p);
}
```

The programmer need not be concerned with managing available storage. There is no longer a need for the pointer *avail* (pointing to the first available node), since the system governs the allocating and freeing of nodes and the system keeps track of the first available node. There is no test in *getnode* to determine whether overflow has occurred. Because such a condition will be detected during the execution of the *malloc* function and is system dependent.

The procedures *insafter* (*p,x*) and *delafter* (*p,px*) are presented below using the dynamic implementation of a linked list. Assume that *list* is a pointer variable that points to the first node of a list (if any) and equals *NULL* in the case of an empty list.

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```
insafter(p, x)
NODEPTR p;
int x;
{
    NODEPTR q;
    if (p == NULL) {
       printf("void insertion\n");
       exit(1);
    q = getnode();
    q ->info = x;
    q -> next = p -> next;
    p -> next = q;
    /* end insafter */
delafter(p, px)
NODEPTR p;
int *px;
{
    NODEPTR q;
    if ((p == NULL) || (p -> next == NULL)) {
       printf("void deletion\n");
       exit(1);
    }
    q = p -> next;
    *px = q -> info;
    p -> next = q -> next;
    freenode(q);
}   /* end delafter */
```

DYNAMIC AND ARRAY IMPLEMENTATION OF LINKED LIST

The major disadvantage of dynamic implementation is that it may be more time-consuming to call upon the system to allocate and free storage than to manipulate a programmer-managed available list. Its major advantage is that a set of nodes is not reserved in advance for use by a particular group of lists.

Let a program uses two types of lists:

(*i*) Lists of integers (*ii*) Lists of characters.

Under the array representation, two arrays of fixed size would immediately be allocated. If one group of lists overflows its array, the program cannot continue. Under the dynamic representation, two node types are defined at the outset, but no storage is allocated for variable until needed. As nodes are needed, the system is called upon to provide them. Any storage not used for one type of node may be used for another. Thus as long as sufficient storage is available for the nodes actually present in the list, no overflow occurs.

Another advantage of the dynamic implementation is that a reference to *p does not involve the address computation that is necessary in computing the address of *node*[p]. To compute the address of *node*[p], contents of p must be added to the base address to the array *node*, whereas address of *p is given by the contents of p directly.

ALGORITHM TO REVERSE DIRECTION OF ALL LINKS OF A SINGLY LINKED LIST

For this we are require to traverse the list and copy its each node into another list in the start.

Algorithm → Reverse (FIRST).

Given a link list having FIRST, a pointer to the first node. This function will make a copy of the list in reverse order. A typical node in the given list consists of INFO and LINK fields; the new list is to contain nodes whose information and pointer fields are denoted by FIELD and PTR, respectively. The address of first node in old list is stored in FIRST and that of newly created list is stored in BEGIN. NEW, SAVE, and PTR are pointer variables AVAIL is a procedure to allocate memory.

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1. [Empty list ?]
   If FIRST = NULL
   then Return (NULL)
2. [Copy first node]
   If AVAIL = NULL
   then Write ('OVERFLOW')
   Return (0)
   else NEW ← AVAIL
   AVAIL ← LINK (AVAIL)
   FIELD (NEW) ← INFO (FIRST)
   BEGIN ← NULL
   PTR (BEGIN) = NULL
3. [Initialize traversal]
   SAVE ← FIRST
4. [Move to next node if not at end of list]
   Repeate through step 6 while LINK (SAVE) ≠ NULL
5. [Update save pointer]
   SAVE ← LINK (SAVE)
6. [Copy node]
   If AVAIL = NULL
   then Write ('OVERFLOW')
   Return (0)
   else NEW - AVAIL
   FIELD (NEW) ← INFO (SAVE)
   PTR (NEW) ← BEGIN
   BEGIN = NEW
7. [Return the starting address of newly created list]
   Return (BEGIN)

STORING ORDERED TABLE AS LINKED LIST

Assume that an ordered table is stored as a linked list with two external pointers :

(1) TABLE : It always points to the node containing the record with the smallest key.

(2) OTHER : It is initially equal to TABLE, but is reset each time a search is performed to point to the record that is retrieved.

If a search is unsuccessful, OTHER is reset to TABLE, write an algorithm called 'search' which implements this method and returns a pointer to a retrieved record or a nil pointer if the search is unsuccessful.

Algorithm : ***SEARCH*** (TABLE, OTHER, DATA):

This algorithm searches the DATA in the linked list. As the linked is **ordered** and TABLE pointer points to the record of shortest key, it is also equal to start of the linked list. OTHER is either equal to TABLE or points to some other record searched previously.

1. [Initialization]
   Set START = TABLE
2. [Check for the OTHER pointer and search area]
   If TABLE = OTHER Then
   Repeat step 3 while START ≠ NULL OR
   START.KEY < DATA

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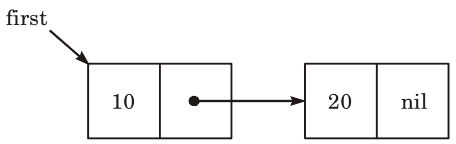
```
    else
        If OTHER.KEY < DATA Then
            Set START = OTHER
        Repeat step 3 while START ≠ NULL OR
                START.KEY < DATA
    else
        Repeat step 3 while START ≠ OTHER OR
            START.KEY < DATA
        End If
    End If
3.  [Perform search]
    If START.KEY = DATA Then
        OTHER = START
        Return OTHER
        End
    End If
    Set START = START.NEXT
4.  [Unsuccessful search]
    Set OTHER = TABLE
    Return NULL
    End
```

CREATING A LINKED LIST WITH TWO NODES OF TYPE LIST NODE

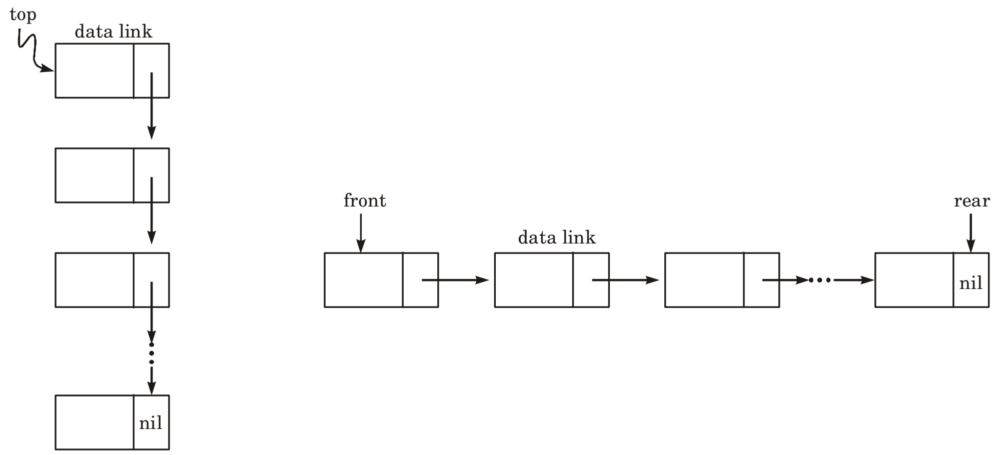
The data field of the first node is set to 10 and that of the second to 20. First is a pointer to the first node.

The resulting list structure is :




```
procedure create2(var first : pointer);
var second: pointer;
begin
    new(first);
    new(second);
    first↑.link : = second; {link first node to second}
    second↑.link : = nil; {last node}
    first↑.data : = 10; {set data of first node}
    second↑.data : = 20; {set data of second node}
end; {of create2}
```

ALGORITHM TO ADD AND DELETE TO A LINKED STACK AND LINK QUEUE



(a) Linked stack ***(b) Linked queue***

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Node and pointer are as defined below :

```
type    pointer = ↑node;
    node = record
        data: integer;
        link: pointer;
    end;
```

The following global arrays of type *pointer* are used :

top[i] = node at top of i-th stack, $1 \le i \le n$

front[i] = node at front of i-th queue, $1 \le i \le m$

rear[i] = last node in i-th queue, $1 \le i \le m$.

The initial conditions are :

top[i] = **nil**, $1 \le i \le n$

front[i] = **nil**, $1 \le i \le m$

and the boundary conditions are :

top[i] = nil iff the i-th stack is empty

front[i] = **nil** iff the i-th queue is empty

```
procedure addstack(i,y : integer);
{add y to the i-th stack, 1 ≤ i ≤ n}
var x : pointer;
begin
    new(x); {get a node}
    x↑.data : = {sets its data field}
    x↑.data : = top[i]; {attach to top of i-th stack}
    top[i] : = x; {update stack pointer}
end; {of addstack}
procedure deletestack(i : integer; var y  : integer);
{delete the top node from stack i and set y to be its data field, 1 ≤ i ≤ n}
var x: pointer;
begin
    if top[i] = nil then stackempty;
    x : = top[i];
    y : = x↑.data; {data field to top node}
    top[i] : = x↑.link; {remove top node}
    dispose(x); {free the node}
end; {of deletestack}
```

Program : Delete from a linked stack

```
procedure addqueue(i,y : integer);
{add y to queue i, 1 ≤ i ≤ m}
var x: pointer;
begin
    new(x);
    x↑.data : = y; x↑.link : = nil;
    if front[i] = nil
    then front[i] : = x {empty queue}
    else rear[i] ↑.link : = x;
    rear[i] : = x;
end; {of addqueue}
```

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Program : Add to a linked queue

procedure *deletequeue*(*i* : **integer**; **var** *y* : **integer**);

{delete the first node in queue *i* and set *y* to its *data* field, $1 \le i \le m$}

var *x*: *pointer*;

begin

```
    if front[i] = nil then queueempty;
    x : = front[i];
    front[i] : = x↑.link; {delete first node]
    y : = x↑.data;
    dispose(x); {free the node}
end; {of deletequeue}
```

PRECONDITION OF PERFORMING BINARY SEARCH IN AN ARRAY

For performing binary search in an array, the array elements should be sorted and size of the array should be known. If elements should be sorted, size of the array should be known. If elements are not arranged , then only sequential search can be preformed on it.

Binary search cannot be performed on a linked list.

A binary search cannot be performed on linked list as linked list does not meet the precondition of binary search, *i.e.* it is not sorted and number of elements are not known in the case of linked list as it is dynamic in nature.

CHAIN.

A singly linked list in which the last node has a nil link is called a chain.

(*i*) Procedure to erase a chain.

```
Procedure erase(var t : polypointer);
{free all the nodes in a chain t}
var x: polypointer;
begin
    while t < > nil do
    begin
        x : = t↑.link;
        dispose(t);
        t : = x;
    end;
end; {of erase}
```

(*ii*) Procedure to erase a circular list.

```
procedure cerase(t : polypointer);
{erase the circular list t}
var x : polypointer;
begin
    if t < > nil
    then begin
```

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```
                x : = t↑.link;s {second node}
                t↑.link : = av; {first node linked to av}
            av : = x; {second node of t becomes front of av list}
        end;
    end; {of cerase}
```

(iii) Procedure to invert a chain.

```
    Procedure invert(var x : pointer);
    {a chain pointed at by x is inverted so that if x = (a_1 . . . ., a_n)
    then after execution   x = (a_n . . . ., a_1)}
    var p,q,r : pointer;
    begin
        p : = x; q : = nil; {q trails p}
        while p < > nil do
    begin
            r : = q; q : = p; {r trails q}
            p : = p↑.link; {p moves to next node}
            q↑.link : = r; {link q to preceding node}
        end;
        x : = q;
    end; {of invert}
```

(iv) Procedure to concatenate two chains x and y.

```
    procedure concatenate(x,y : pointer; var z : pointer);
    {x = (a_1 . . . . , a_m ) and y = (b_1 . . . . , b_n ), m,n ≥ 0
    produces the new chain z = (a_1, . . . . , a_m ,b_1 , . . . . , b_n)}
    var p : pointer;
    begin
        if x = nil
        then z : = y
        else  begin
            z : = x;
            if y < > nil
            then   begin {find last node in x}
            p : = x;
            while p↑.link < > nil do p : = p↑.link;
            p↑.link : = y; {link last of x to first of y}
          end;
        end;
    end; {of concentenate}
```

(v) Procedure to find the length of a circular list.

```
    function length(a : pointer) : integer;
    {find the length of the circular list a}
    var x; pointer;
    begin
        length : = 0;
        if a < > nil
        then begin
          x : = a;
```

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```
      repeat
         length : = length + 1;
         x : = x↑.link;
      until x = a;
   end;
end; {of length}
```

STORING A POLYNOMIAL IN A LINKED LIST

Consider a polynomial of three variables x, y and z.

Let each node of the linked list have five values :

(1) Power of x

(2) Power of y

(3) Power of z

(4) Coefficient of the considered term

(5) Link to the next element (term).

power of x	power of y	power of z	coefficient	link

Fig. Node of linked list of a polynomial

Example. Create a linked list for the following polynomial.

$$f(x,y,z) = x^7y^2 - 100yz^6 + 6xyz + 20x + 40$$

Solution.

The linked list is shown in the figure below. For each product term we have a node.

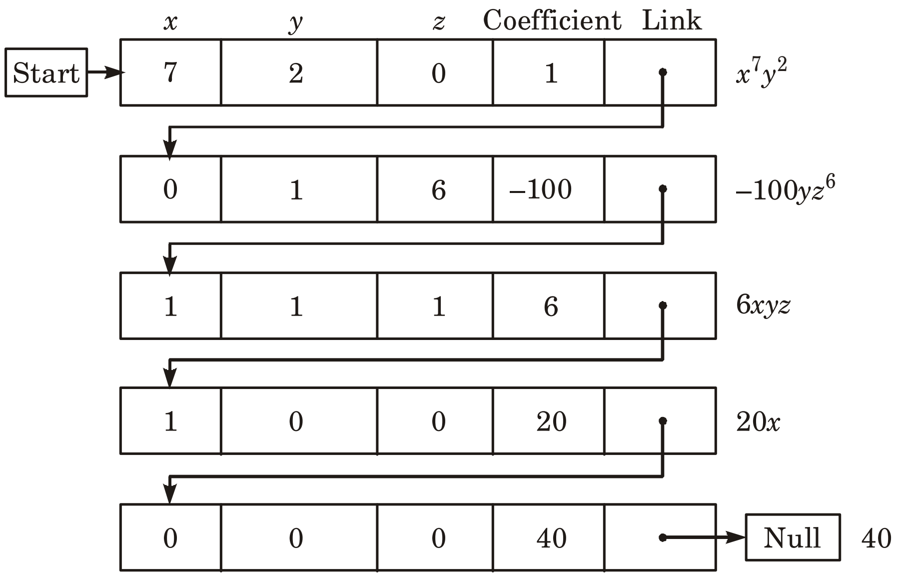

Fig. Linked list for $x^7y^2 - 100yz^6 + 6xyz + 20x + 40$

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SOLVED EXAMPLES

1. Give type definition for a link list which

(*i*) consists of node that have a data field of type integer and a link field.

(*ii*) define the type nodes to consist of three data fields and two link fields while nodes of type nodes will consists of one data field and one link field. The link field of nodes of type nodes must point to nodes of type nodes.

Solution.

(*i*)
```
type pointer  = ↑ listnode;
     listnode = record
                  data: integer;
                  link: pointer;
                end;
```

(*ii*) *The type definition*
```
type ptra  = ↑nodea;
     ptrb  = ↑nodeb;
     nodea = record
               data1: integer;
               data2: char;
               data3: real;
               linka: ptra;
               linkb: ptrb;
             end;
     nodeb = record
               data: integer;
               link: ptrb;
             end;
```

2. Let First be a pointer to a linked list. First - nil if the list is empty. Let x be a pointer to some arbitrary node in the list. Write a procedure to insert a node with data field 50 following the node pointed at by x.

Solution.
```
procedure insert(var first : pointer; x : pointer);
var t: pointer;
begin
      new(t); {get a node}
      t↑.data : = 50; {set its data field}
      if first = nil
      then begin {insert into empty list}
            first : = t;
            t↑.link : = nil;
      end
      else  begin {insert after x}
         t↑.link : = x↑.link;
         x↑.link : = r;
      end;
   end; {of insert}
```

3. Let first be a pointer to a linked list. First = nil if the list is empty. Let x be a pointer to some arbitrary node in the first and let y points to the node (if any) that precedes x and let y = nil if x = First. Write a procedure that deletes node x from the list.

Solution.
```
procedure delete(x,y : pointer; var first : pointer);
begin
if y = nil   then first : = first↑.link
      else y↑.link : = x↑.link;
dispose(x); {return the node}
end; {of delete}
```

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4. Explain how algorithm works using M = 126743 and N = 16.

Solution.

In the function Linked List (*M, N*), a while loop is executed in which last digit is separated from *M* (126743), *i.e.* 3 and is converted in the base notation of *N, i.e.* 16 and then is inserted for each digit of *M*. The process when follows for *M* = 126743 and *N* = 16, will make a link list as shown below :

1	16		→	2	16		→	6	16		→	7	16		→	4	16		→	3	16	×

Step I. Initially *M* > 0, hence *r* become *M*% 10, *i.e.* 3 and then converted in base 16 and becomes 3 again. And then inserted in list and list becomes :

3	16	×

first

Step II. In second step *r* becomes 4, and then list becomes as :

4	16		→	3	16	×

first

Step III. In third step *r* becomes 7, and then list becomes :

7	16		→	4	16		→	3	16	×

first

Step IV. In fourth step *r* becomes 6, and list becomes:

6	16		→	7	16		→	4	16		→	3	16	×

first

Step V. In this step *r* becomes 2, and list becomes :

2	16		→	6	16		→	7	16		→	4	16		→	3	16	×

first

Step VI. In this last step list becomes :

1	16		→	2	16		→	6	16		→	7	16		→	4	16		→	3	16	×

first

5. Of the three variations of linked lists (circular, with header and trailer nodes, and doubly linked) which would be most appropriate for each of the following applications and why ?

(*i*) Search a list for a key and return the keys of the two elements that come before it and the keys of the two element's that come after it.

(*ii*) A text file contains integer elements, one per line, sorted from smallest to largest. You must read the values from the file and create a sorted linked list containing the values.

(*iii*) A list that is short and frequently becomes empty. You want a list that is optimum for inserting an element into the empty list and deleting the last element from the list.

Solution.

(*i*) Doubly linked preferred for this case because to find the item linear search from beginning is to be done. After finding the item. We need to find keys of two elements before and after. In doubly linked list it is easy to go in both directions. Hence two keys before can be found easily.

(*ii*) In this case list with header and trailer nodes are preferred. Since list is sorted one. Header node point to beginning and trailer node point to the end.

(*iii*) In this case list with header and trailer nodes are preferred. It will be always easy to delete the last item if there is a trailer node.

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STACK & QUEUE

STACKS

Stacks. Stack is an ordered list in which all insertions and deletions are made at one end. This end is called the *top*.

If elements are E, D, C, B, A (in order), then first element to be removed from the stack is A.

Stack are also called LIFO (Last in First Out) lists.

The simplest way to implement a stack is to use a one dimensional array of n elements.

Var stack = array [1.n] of items top : : 0 .. n,

Program 1. *Push an item onto the stack*

```
Procedure add (item : items);
{add item to the global stack;
top is the current top of stack
and n is its maximum size}
begin
        if top = n then stack full;
        else
        top : = top + 1;
        stack [top] : = item;
end;
```

Program 2. *Pop an item from the stack*

```
        Procedure delete (var item : items);
        { remove top element from the stack and put it in item}
begin
        if top = 0, then stack empty;
        else
        item : = stack [top];
        top : = top – 1;
end;
```

Program 3. Add to a queue

```
        Procedure addq (item : items);
        {add item to the queue q}
begin
        if rear = n then queue full;
        else
        rear : = rear + 1;
        q [rear] : = item;
end;
```

Program 4. Delete from a queue

```
        Procedure deleteq (var item : items);
        {delete from the front of a queue q and put into item}
begin
        if front = rear then queue empty;
        else
        item : = q [front];
        front : = front + 1;
end;
```

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REPRESENTING STACKS IN C

A stack is an ordered collection of items, and C already contains a data type that is an ordered collection of items ; the array.

Whenever a problem solution calls for the use of a stack, it is tempting to begin a program by declaring a variable *stack* as an array. However, a stack and an array are two entirely different things. The number of elements in an array is fixed and is assigned by the declaration for the array. In general, the user cannot change this number. Fundamentally stack, is a dynamic object whose size is constantly changing as items are popped and pushed.

However, although an array cannot be a stack, it can be the home of a stack, i.e. an array can be declared large enough for maximum size of the stack. During the course of program execution, stack can grow and shrink within the space reserved for it. One end of the array is fixed bottom of the stack, while top of the stack constantly shifts as items are popped and pushed. Thus, another field is needed that, at each point during program execution, keeps track of the current position of the top of the stack.

A stack in C may therefore be declared as a structure containing two objects; an array to hold the elements of the stack, and an integer to indicate the position of the current stack top within the array. This may be done for a stack of integers by the declarations.

```
# define STACKSIZE 100
struct stack {
    int top;
    int items[STACKSIZE];
};
```

Once this has been done, an actual stack *s* may be declared by

```
    struct stack s;
```

IMPLEMENTING OPERATION AND PUSH OPERATION

The possibility of underflow must be considered in implementing *pop* operation, since the user may inadvertently attempt to pop an element from an empty stack. Such an attempt is illegal and should be avoided. However, if such an attempt is made, the user should be informed of the underflow condition.

Actions performed by a function pop :

(*i*) If stack is empty, print a warning message and halt execution.

(*ii*) Remove top element from the stack.

(*iii*) Return this element to the calling program.

Assume that the stack consists of integers, so that pop operation can be implemented as a function. This would also be the case if stack consisted of some other type of simple variable. However, if a stack consists of a more complex structure (e.g. a structure or a union), the *pop* operation would either be implemented as returning a pointer to a data element of the proper type (rather than the data element itself), or the operation would be implemented with the popped value as a parameter (in which case address of the parameter would be passed rather than the parameter, so that *pop* function could modify the actual argument).

```
pop (ps)
if (top = = -1)
printf("stack underflow");
     else
{
Printf("popped element is % d\n", stack array [top])
     top = top-1;
}
top points to last pushed item of stack.
# define max 5
```

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```
if (top = = (Max -1))
Printf ("stack overflow");
else
     {
        top = top +1;
        Stack array [to] =  push item;
     }
top always points to last added
item of stack
```

INFIX, POSTFIX, AND PREFIX

Consider sum of *A* and *B*. We apply the ***operator*** "+" to the ***operands*** *A* and *B* and write the sum as *A* + *B*. This particular representation is called *infix*.

There are two alternate notations for expressing sum of *A* and *B* using the symbols *A, B* and +.

+ AB prefix

AB + postfix

"pre-", "post-" and "in-" refer to relative position of the operator with respect to the two operands.

In *prefix notation*, operator precedes two operands.

In *postfix notation,* operator follows two operands.

In *infix notation,* operator is between two operands.

Prefix and postfix notations are not really as awkward to use as they might at first appear.

e.g. a C function to return sum of the two arguments *A* and *B* is invoked by *add(A, B).* The operator *add* precedes the operands *A* and *B*.

Examples.

(i)

Infix	***Postfix***
A + B	AB +
A + B – C	ABC – +
(A + B) * (C – D)	AB + CD – *
[(A $ B) * C – D] + [(E / F) / (G + H)]	AB $ C * D – EF / GH + / +
((A + B) * C – (D – E)) $ (F + G)	AB + C * DE – – FG + $
A – B/(C * D $ E)	ABCDE $ * / –

(ii)

Infix	**Prefix**
A + B	+ AB
A + B – C	– + ABC
(A + B) * (C – D)	* + AB – CD
[(A $ B) * C – D] + [(E / F) / (G + H)]	+ – * $ABCD / / EF + GH
((A + B) * C – (D – E)) $ (F + G)	$ – * + ABC – DE – FG
A – B/(C * D $ E)	– A / B * C $ DE

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DEQUEUE

The term dequeue means double-ended queue. It is a queue in which insertion and deletion is possible from both ends.

Deque is an ordered set of items from which, items may be deleted at either end and into which items may be inserted at either end, but not in the middle.

Operations.

(1) remvleft – Remove left node from the queue.

(2) remvright – Remove right node from the queue.

(3) insrtleft – Insert a new node in the left side of queue.

(4) insrtright – Insert a new node in the right side of queue.

Applications.

Define an input-restricted dequeue for which only the operations remvleft, remvright & insrtleft are valid, and an output-restricted dequeue as a dequeue for which only the operations remvleft, insrtleft & insrtright are valid, each of the queue can be used to represent both a stack and a queue.

While remove operation, proper care should be taken to determine whether the other end is reached or not.

ALGORITHMS

(1) Algorithm to Convert Infix Including (,), +, –, *, and / to Postfix.

POST FIX (E)

This algorithm converts the infix expression E to post fix. The last character of E is assumed to be an ∞ which will also be the last character of the post fix.

NEXT-TOKEN is a function which returns the next character in the expression E.

ISP (X) and ICP (X) are functions which reflect in-stack priority and in-comming priority.

```
(i)  STACK (1) = '–∞' ; TOP = 1
     [initialize stack]
(ii) Repeat
         x = NEXT-TOKEN (E)
         Case
            :    x  = '–∞' while (top > 1)
                    Print ((STACK(TOP));
                    TOP = TOP - 1;
                    end while
                    print ('∞');
                    return;
            :    x  is an operand
                    Print (x);
            :    x  =')' : while (STACK (TOP) ≠ '(' )
                    Print (STACK (TOP));
                    TOP = TOP - 1;
                    end while
                    TOP = TOP - 1;
            :    else : while ISP (STACK (TOP)) > = ICP (x)
                    Print ((STACK (TOP));
                    TOP = TOP - 1;
                    end while
                    Call INSERT (x, STACK, N, TOP);
                              [insert x in STACK]
     end of Repeat
```

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(2) Algorithm to Evaluate Postfix Expression.

```
Procedure eval (e : expression);
{evaluate the postfix expression e. It is assumed that the last character in e is '#'. A procedure
nexttoken is used to extract from e the next token (a token is either an operator, operand, or '#' ).
A one dimensional array stack [1..n] is used as a stack.}
var x : token;
      begin
      top = 0; {initialize stack}
      x : = nexttoken (e);
      while x < > '#' do
      begin
            if x is an operand
            then add (x) {add to stack}
            else begin {operator}
                        remove the correct number of operands for operator x from stack;
                        perform the operation x and store the result (if any) onto the stack;
               end;
            x : = nexttoken (e);
      end; {or while}
end; {or eval}
```

(3) Algorithm to Convert Expression in Infix Form into Postfix Form.

```
Procedure postfix (e : expression);
{output the postfix form of the infix expression e. nexttoken and stack are as in procedure eval.
It is assumed that the last token in e is '#'. '–#' is used at the bottom of the stack and isp
('–#') = – 1 . //
var x, y : token;
begin
      stack [1] : = '–#'; top : = 1; {initialize stack}
      x : = nexttoken(e);
      while x < > '#' do
      begin
            if x is an operand
            then write (x)
            else if x = ')'
                  then begin {unstack until '('}
                        while stack[top] < > '(' do
                        begin delete(y); write(y); end;
                        delete(y); {delete '('}
                     end
                  else begin
                        while isp (stack[top]) > = icp (x) do
                        begin delete(y); write(y); end;
                        add(x);
                     end
            x : = nexttoken(e);
      end; {or while}
      {end of expression; empty stack}
```

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while *top* > 1 **do**
begin *delete*(*y*); **write**(*y*); **end**;
written('#');
end; {of *postfix*}

Algorithm to Find Largest Integer Element Among Contents of the Stack.

FINDLARGEST (*S, N,* TOP)

Given algorithm returns the largest element among the content of stack *S* having maximum elements *N*. TOP denotes top element of the stack. It return NULL if stack is empty.

t is a temporary variable.

1. [Empty stack ?]
   If TOP ≤ 0
   return NULL;
2. [Initialization]
   max = S [TOP]
   t = TOP – 1;
3. [Repeat step 3 while t > 0]
   If max < S[t]
   max = S[t];
   t = t – 1;
4. [Return the element]
   return max;
5. [Finish]
   End

ADD & DELETE OPERATIONS TO MULTIPLE STACK.

procedure *add* (*i* : **integer**; *x* : *items*);
{add *x* to the *i*-th *stack*}
begin
if *t*[*i*] = *b* [*i*+1] **then** *stackfull* (*i*);
t[*i*] : = *t*[*i*] + 1;
v[*t*[*i*]] : = *x*; {add to *i*-th stack}
end; {of *add*}
procedure *delete* (*i* : **integer**; **var** *x* : *items*);
{*delete* topmost item of stack *i*}
begin
if *t*[*i*] = *b*[*i*] **then** *stackempty* (*i*);
x : = *v*[*t*[*i*]];
t[*i*] : = *t*[*i*] – 1;
end; {of *delete*}

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TWO STACKS CONTAINING SAME TYPE OF ENTRIES

Sometimes a program requires two stacks containing the same type of entries. If two stacks are stored in separate arrays, then one stack might overflow while there was considerable unused space in the other. A neat way to avoid this problem is to put all the space in one array and let one stack grow from one end of the array and the other at the other end grow in the opposite direction, towards the first stack. In this way, if one stack turns out to be large and the other small, then they will still both fit, and there will be no overflow until all the space is actually used. Declare a new data structure DOUBLE_STACK that includes the array and the two indices TOP A and TOP B, and write routines CREATE_DOUBLE_STACK, PUSH A, PUSH B, POP A and POP B to handle two stacks within one double-stack.

PUSH A (STACK, TOP A, TOP B, N, ITEM) :

This procedure will insert ITEM into the first stack maintained in array called STACK.

```
1.  [OVERFLOW ?]
        IF TOP A + 1 = TOP B OR TOP A = N Then
              Write 'OVERFLOW'
                  Return
        End If
        If TOP A = NULL AND TOP B = 1 Then
              Write 'OVERFLOW'
        Return
        End If
2.  [Intert the Item]
        If TOP A = NULL Then
           Set TOP A = 1
        else
           Set TOP A = TOP A + 1
        End If
           Set STACK [TOP A] = ITEM
        Return
3.  [Finish]
        End.
```

PUSH B (STACK, TOP A, TOP B, N, ITEM) :

This algorithm will insert ITEM into the second stack maintained in array called STACK.

```
1.  [OVERFLOW ?]
        If TOP B – 1 = TOP A OR TOP B = 1 Then
            Write 'OVERFLOW'
            Return
        End If
        If TOP B = NULL AND TOP A = N Then
            Write 'OVERFLOW'
            Return
        End If
```

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2. [Insert the ITEM]
    If TOP B = NULL Then
        Set TOP B = N
    else
        Set TOP B = TOP B – 1
    End If
    Set STACK [TOP B] = ITEM
    Return
3. [Finish]
    end.

POPA (STACK, TOP A, TOP B) :

This algorithm with delete ITEM from the first stack maintained in array called STACK.

1. [UNDERFLOW?]
    If TOP A = NULL Then
        Write 'UNDERFLOW'
    End If
2. [Delete the item]
    Set ITEM = STACK [TOP A]
    If TOP A = 1 Then
        Set TOP A = NULL
    else
        Set TOP A = TOP A – 1
    End If
    Return ITEM.
3. [Finish]
    End.

POPB (STACK, TOP A, TOP B) :

This procedure will delete ITEM from the second stack maintained in array called STACK.

1. [Underflow ?]
    If TOP B = NULL Then
        Write 'UNDERFLOW'
    Return
    End If
2. [Delete the Item]
    Set ITEM = STACK [TOPB]
    If TOP B = N Then
        Set TOP B = NULL
    else
        Set TOP B = TOP B + 1
    End If
    Return ITEM
3. [Finish]
    End.

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SOLVED EXAMPLES

1. A square matrix is said to be symmetric if a (i , j) = a (j, i) for i and j. Such a symmetric square matrix may be stored in an array of half the size of the matrix in terms of number of elements;

(*i*) Derive the formula for index of one dimensional array in terms of indices of symmetric square matrix for above storage scheme.

(*ii*) Write an algorithm to print the sum of each column of the above matrix stored as one dimensional array.

Solution.

(*i*) Since $a\ [i, j] = a\ [j, i]$, we need only store those elements which lie on or below the diagonal.

Let B is one dimensional array to store a symmetric matrix, then

$$B\ \{1\} = a_{11},\quad B\ [2] = a_{21},\quad B[3] = a_{22},\quad B\ [4] = a_{31}, \ldots$$

Observe first that B will contain only $\quad 1 + 2 + 3 + 4 + n = \frac{1}{2} n\ (n + 1)$

Elements, which is about half as many as a two dimensional $n*n$ array.

We require index of B (which is L) in terms of a_{ij}

$$B\ [L] = a_{ij}$$

L represents number of elements in the list up to and including a_{ij}.

Now there are $1 + 2 + 3 + \ldots + (i - 1) = \frac{i(i-1)}{1}$ elements in the row above a_{ij}, and there are j elements in the row i upto and including a_{ij}.

Accordingly, $\quad L = \frac{i(i-1)}{2} + j$ yields index that access the value a_{ij} for linear array B.

(*ii*) *ALGORITHM to find the sum and print it for each column stored as a symmetric matrix.*

SUMCOL (B, N)

Given a one dimensional Array B which has storage representation of a square matrix ($N * N$) and N, the dimension of a square matrix.

Following procedure finds out and prints the sum of each column of a square symmetric matrix which is stored in one dimensional array B.

I and J are temporary variables.

1. [Initialize]
   Set Sum ← 0
2. Repeat for J = 1 to N
3. Repeat for I = 1 to N

   set Sum ← Sum + $B\left[\frac{I(I-1)}{2} + J\right]$

   [end of step 3 loop]
   [Print the Sum]
   write′ sum of col′ J′ is′ - Sum
   set Sum ← 0
   [end of step 2 loop]
4. Return

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2. Give complete specifications of a data structure to represent STACK.

Solution.

```
structure  STACK (item)
declare    CREATE( ) → stack
           ADD (item, stack) → stack
           DELETE(stack) → stack
           TOP(stack) → item
           ISEMTS (stack) → boolean;
for all    S ∈ stack, i ∈ item let
           ISEMTS (CREATE)  ::= true
           ISEMTS (ADD(i,S)) ::= false
           DELETE(CREATE)  ::= error
           DELETE (ADD(i,S)) ::= S
           TOP(CREATE)      ::= error
           TOP(ADD(i,S))    ::= i
end
end        STACK
```

3. Write design procedures for ADD and DELETE operations on a stack.

Solution.

Procedure 1:

Push an item onto the stack

```
Procedure add (item : items);
{add item to the global stack stack ;
top is the current top of stack
and n is its maximum size}
begin
    if top = n then stackfull ;
    else
    top : = top + 1;
    stack [ top ] : = item ;
end ;
```

Procedure 2 :

pop an item from the stack

```
Procedure delete (var item : items) ;
{remove top element from the stack and put it in item}
begin
    if top = 0 then stack empty ;
    else
    item : = stack [top] ;
    top : = top – 1 ;
end ;
```

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4. Show procedures for ADD and DELETE operations to a queue.

Solution.

Procedure 1 :

Add to a queue

Procedure addq (item : items) ;

{add item to the queue q}

begin

    if rear = n then queue full ;
    else
    rear : = rear + 1 ;
    q [rear] : item ;

end ;

Procedure 2 :

Delete from a queue

Procedure deleteq (var item : items) ;

{delete from the front of a queue q and put into item}

begin

    if front = rear then queue empty ;
    else
    item : = q [front];
    front : = front + 1 ;

end ;

5. Describe the algorithm to convert an infix expression to postfix expression.

Solution.

1. [Initialize opstk with NULL]
   set Opstk ← NULL
2. [Accept Input]
   Repeat steps 3 through 4 while not end of Input
3. [Get the next input symbol]
   set symb ← next input symbol
4. [Process it]
   if symb is an operand
       then
         add symb to postfix string
       else
         while ((! empty (opstk) and (precedence of stacktop > symb))
           set topsymb ← pop (opstk)
               add topsymb to postfix string
       end if
       push (opstk, symb)
5. [Output any remaining operators]
       while (! empty (opstk))
       set topsymb ← pop (opstk)
         add topsymb to postfix string
6. Return

6. Convert following infix expressions into postfix form.

A + B * C

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Solution.

Contents of *symb,* the postfix string, and *opstk* are shown after scanning each symbol. *opstk* is shown with its top to the right.

	symb	*postfix string*	*opstk*
1	*A*	*A*	
2	+	*A*	+
3	*B*	*AB*	+
4	*	*AB*	+ *
5	*C*	*ABC*	+ *
6		*ABC* *	+
7		*ABC* * +	

Lines 1, 3 and 5 correspond to the scanning of an operand; therefore symbol (*symb*) is immediately placed on the postfix string.

In line 2, an operator is scanned and stack is found to be empty, and therefore operator is placed on the stack.

In line 4, precedence of the new symbol (*) is greater than the precedence of the symbol on the top of the stack (+); therefore new symbol is pushed onto the stack.

In lines 6 and 7, the input string is empty, and therefore stack popped and its contents are placed on the postfix string.

7. Convert following infix expression into postfix form.

((A – (B + C)) * D) $ (E + F)

Solution.

symb	*postfix string*	*opstk*
(		(
(		((
A	*A*	((
–	*A*	((–
(	*A*	((–(
B	*AB*	((–(
+	*AB*	((–(+
C	*ABC*	((–(+
)	*ABC* +	((–
)	*ABC* +–	(
*	*ABC* +–	(*
D	*ABC* +– *D*	(*
)	*ABC* +– *D* *	
$	*ABC* +– *D* *	$
(	*ABC* +– *D* *	$(
E	*ABC* +– *D* * *E*	$(
+	*ABC* +– *D* * *E*	$ (+
F	*ABC* +– *D* * *EF*	$ (+
)	*ABC* +– *D* * *F* +	$
	ABC +– *D* * *EF* + $	

8. Two stacks of positive integers are needed, one containing elements with values less than or equal to 1000s and other containing elements with values larger than 1000. The total number of elements in the small-value stack and the large-value stack combined are not more than 200 at any time, but we cannot predict how many are in each stack. Both stacks are to be implemented in one array such that there will be no overflow until all the space is actually used.

(*i*) Write the definitions for such a double-stack structure

(*ii*) Implement the PUSH operation; it should store the new item into the correct stack according to its value.

Solution.

(*i*) *Stacks can be defined as*

STACK [TOS1]

and

STACK [TOS2]

where TOS1 and TOS2 indicate top of stack 1 and top of stack 2 respectively.

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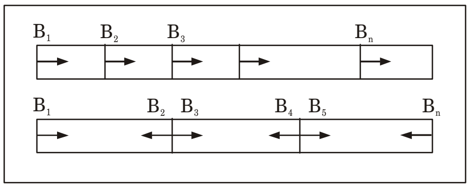
(*ii*) PUSH (STACK, ITEM)

This procedure pushes an item onto a stack. We assume TOS1 and TOS2 are global variables and hence not included in the parameter list

1. If TOS1 = TOS 2 – 1 then
   print : OVERFLOW; Return;
2. If ITEM < = 1000
   TOS1 = TOS1 + 1
   STACK [TOS1] = ITEM
   Return;

ELSE
TOS2 = TOS2 – 1
STACK [TOS2] = ITEM
RETURN;

9. Multiple stacks may be implemented using one array. Two alternative strategies for such an implementation are shown below :



where B_1 , B_2 , ... B_n denote bottoms of respective stack and arrows indicate direction of growth.
Out of the above two alternatives which one would you prefer and why ? Which alternative will need more time to declare stack full ? It may be noted that stack is declared full when all the stacks are full.

Solution.

Given a condition, a stack is declared full when all the stacks are full, the second method may decrease the number of times overflow will occur, because we partition the stack into pairs and reverse $2\,\frac{N}{K}$ (where K is number of stacks and N is continuous memory allocation) cells for each pair of stack.

Thus second method will require more time to be declare stack full.

Thus in order to minimize the overflow condition while using stack, which exclusively depends upon the given algorithm, the second method is more efficient and preferable.

QUEUE

A queue can be implemented by an array of n elements and treating it as if it was circular.

The array is declared as q [0 ... n–1].

Two variables used are :

(1) Front

(2) Rear

When front = rear, the queue is empty.

Initially we have, front = rear = 1.

Front always points to one position counter clockwise from the first element in the queue. To add an element into the queue, rear is moved one position clockwise. This is done by using the modulo operator rear : = (rear + 1) mod n.

Also front is moved one position clockwise each time a deletion is made by

front = (front + 1) mod n

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C IMPLEMENTATION OF QUEUES

A queue is represented in C by using an array to hold the elements of queue and to use two variables; *front* and *rear*, to hold the positions within the array of first and last elements of the queue.

A queue of integers may be declared and initialized by

```
# define  MAXQUEUE 100
struct queue {
    int items[MAXQUEUE];
    int front,rear;
};
struct queue q;
q.front = q.rear = MAXQUEUE-1;
```

q.front and *q.rear* are initialized to the last index of the array, rather than to –1 or 0, because last element of the array immediately precedes the first one within the queue under this representation. Since *q.rear* equals *q.front,* the queue is initially empty.

Empty function may be code as

```
empty(pq)
struct queue *pq;
{
return ((pq->front == pq->rear)? TRUE : FALSE);
}   /* end empty /*
```

Once this function exists, a test for the empty queue is implemented by the statement

```
if (empty(&q))
/* queue is empty */
else
/* queue is not empty */
```

The operation *remove*(*q*) may be code as

```
remove(pq)
struct queue *pq;
{
if  (empty(pq))    {
    printf("queue underflow");
    exit(1);
}   /*  end if */
if  (pq->front == MAXQUEUE-1)
pq->front = 0;
else
(pq->front)++;
return (pq->items[pq->front]);
} /* end remove */
insert (pq, x)
struct queue *pq;
int x;
{
/*  make room for new element */
```

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```
if  (pq->rear == MAXQUE-1)
    pq->rear = 0;
else
    (pq->rear) --;
/*  check for overflow */
if  (pq->rear == pq->front)  {
    printf("queue overflow");
    exit (1);
}   /* end if*/
pq->items[pq->rear] = x;
return;
}   /* end insert */
```

PRIORITY QUEUE

Both stack and the queue are data structures whose elements are ordered based on the sequence in which they have been inserted. The *pop* operation retrieves the last element inserted and *remove* operation retrieves the first element inserted. If there is an intrinsic order among the elements themselves (e.g. numeric order or alphabetic order), it is ignored in the stack or queue operations.

Priority queue is a data structure in which the intrinsic ordering of the elements determine the results of its basic operations.

Types.

There are two types of priority queues.

(1) Ascending priority queue.

(2) Descending priority queue.

COMPLETE SPECIFICATION OF DATA STRUCTURE REPRESENTING A QUEUE

structure *QUEUE* (*item*)

1. **declare** $CREATEQ(\,) \rightarrow queue$
2. $ADDQ(item, queue) \rightarrow queue$
3. $DELETEQ(queue) \rightarrow queue$
4. $FRONT(queue) \rightarrow item$
5. $ISEMTQ(queue) \rightarrow boolean;$
6. **for all** $Q \in queue, i\ e\ item$ **let**
7. $ISEMTQ(CREATEQ)$ :: = **true**
8. $ISEMTQ(ADD(i,Q))$ :: = **false**
9. $DELETEQ(CREATEQ)$ :: = *error*
10. $DELETEQ(ADDQ(i,Q))$ :: =
11. **if** $ISEMTQ(Q)$ **then** $CREATEQ$
12. **else** $ADDQ(i, DELETEQ(Q))$
13. $FRONT(CREATEQ)$ :: = *error*
14. $FRONT(ADDQ(i,Q))$ :: =
15. **if** $ISEMTQ(Q)$ **then** i **else** $FRONT(Q)$
16. **end**
17. **end** *QUEUE*

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ALGORITHMS FOR ADDITIONS AND DELETIONS OF ELEMENTS FROM A DEQUE.

Algorithms.

Consider a deque DQ having external pointers Start, end pointing to the start and end of the DQ respectively. Initially both are set to NULL. This algorithm also uses AVAIL and FREE for allocation and deallocation of memory to a new node. The node of a DQ has Ptr that points to next node and Info that hold the data. Fresh and Temp are used as temporary node object.

Insert_Front (DATA) :

This algorithm will insert a new node in front of DQ.

```
1.  [Initialize]
        Set Fresh = AVAIL (Node)
        Set Fresh (Info) = Data
2.  [Insert the node]
        If Start = NULL Then
            Set Start = Fresh
            Set End = Fresh
            Set End (Ptr) = Start
        else
            Set Fresh (Ptr) = Start
            Set Start = Fresh
        End If
3.  [Finish]
        End.
```

Insert_Rear (DATA) :

This algorithm will insert new node in rear of DQ.

```
1.  [Initialize]
        Set Fresh = AVAIL (Node)
        Set Fresh (Info) = Data
2.  [Insert the node]
        If End = Null Then
            Set Start = Fresh
            Set End = Fresh
            Set End (Ptr) = NULL
        else
            Set End (Ptr) = Fresh
            Set Fresh (Ptr) = NULL
            Set End = Fresh
        End If
3.  [Finish]
        End.
```

Delete_Front :

This algorithm will delete a node from the front of a DQ.

```
1.  [Empty DQ ?]
        If Start = NULL Then
            Write 'EMPTY DQ'
            Return
        End If
2.  [Delete the node]
        If Start = End Then
                      [Single node in DQ]
            FREE (Start)
            Set Start = NULL
            Set End = NULL
        else
            Set Fresh = Start
            Set Start = Start (Ptr)
            FREE (Fresh)
        End If
3.  [Finish]
        End.
```

Delete_Rear :

This algorithm will delete a node from the rear of a DQ.

```
1.  [Empty DQ]
        If End = NULL Then
            Write 'EMPTY DQ'
            Return
        End If
2.  [Delete the node]
        If start = End Then
                           [Single node in EQ]
            FREE (End)
            Set Start = NULL
            Set End = NULL
        else
            Set Fresh = Start
            Set Temp = start (Ptr)
            Repeat while Temp (Ptr) ≠ NULL
                Set Fresh = Temp
                Set Temp = Temp (Ptr)
            End Repeat
            FREE (Fresh)
            Set End = Temp
        End If
3.  [Finish]
    End
```

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IMPLEMENTING QUEUE WITH THE USE OF AN ARRAY

Let a [1 ... n] be an array of n elements.

Let f and r be variables denoting front and rear of the queue of elements continuously stored in a [1 ... n] and let $f \leq r$, i.e. queue elements are a [f], a[f + 1], ... a[r] and if we want to add an element it must be placed in a[r + 1] and if we want to delete/read an element, the element a[f] must be used.

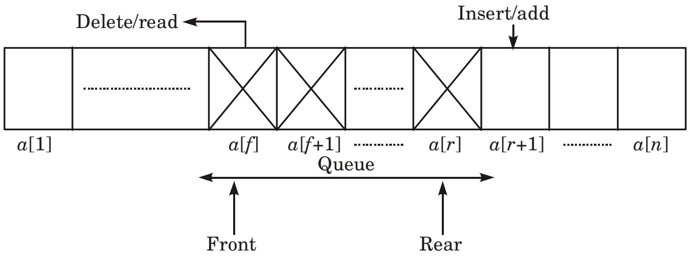

Fig. (a) Queue in an array

- $f = 1$ and $r = n$
  Queue is full. Insertion is not possible.
- $f > 1$ and $r = n$
  Queue is not full, we can move all elements to the left and reduce f and r.
- $f = r = n + 1$
  Queue is empty.
  Let $r = n$. Deletion is not possible. When an insertion is done make $f = n$.
- $f = 0$
  Queue is empty.

We can insert or delete an element in above queue.

Algorithm INSERT (x)

If $r \rightarrow n$, then (display "QUEUE IS FULL"; EXIT)

  else {$a[r + 1] = x$; $r \leftarrow r + 1$;
     }

Algorithm DELETE

If $f = 0$, then {display "QUEUE IS EMPTY"; EXIT}

  else {$x \leftarrow a[f]$;
     $f \leftarrow f - 1$;
     }

When $f = 4$ and $r = n$, we cannot insert an element. But a[1], a[2] and a[3] are empty. If we allow to fill up a[1], a[2] and a[3] after $r = n$, then wastage of space is reduced. This type of queue is called *circular queue*.

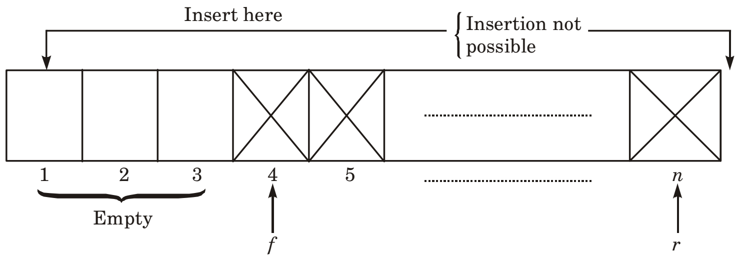

Fig. (b) Circular queue

Fig. *(c)* below shows pictorial representation of a circular queue.

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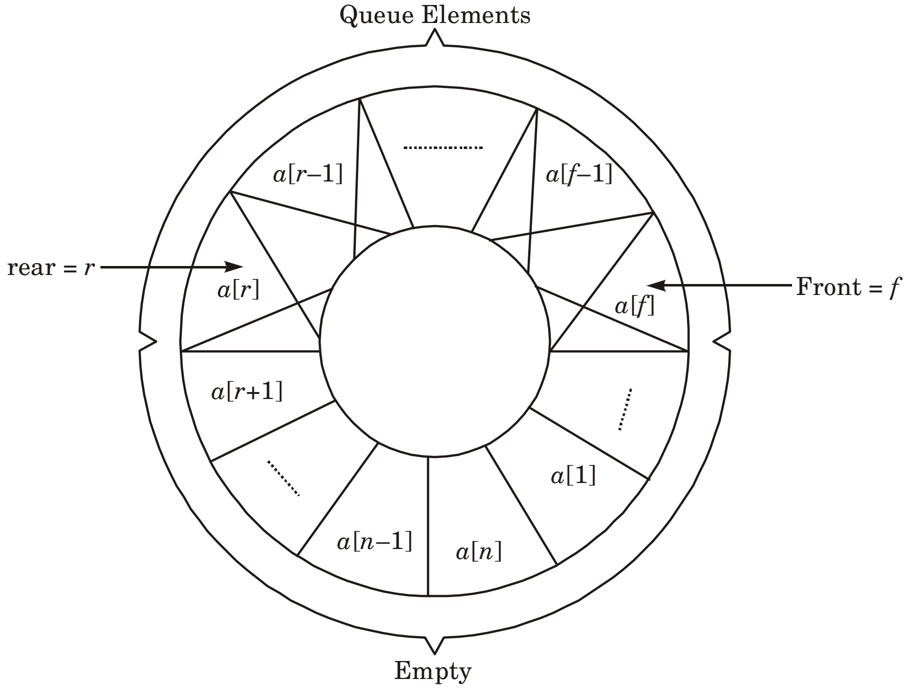

Fig. (c) Cirular Queue

In the Fig. (c), queue elements are $a[f]$, $a[f + 1]$, ... $a[r - 1]$ $a[r]$. This is similar to the array shown in Fig. *(b)*.

ALGORITHM TO IMPLEMENT ADDITION AND DELETION FROM A CIRCULAR QUEUE

Circular queue.

CQINSERT (F, R, Q, N, Y)

Given pointers to the front and rear of a circular queue are F and R. A vector (Array) Q consisting of N (max. array size) elements and an element is passed in function that will be inserted at the rear of queue.

1. [Overflow]
   If (F = 1 and R = N) OR (R + 1 = F)
   then Write ('OVERFLOW')
   Return
2. [Reset rear pointer]
   If (R = N) OR (R = NULL)
   then set R ← 1
   set Q[R] ← Y
   else
   set R ← R + 1
   set Q [R] ← Y
3. [Is F properly set]
   If F = NULL
   then F ← 1
4. Return

CQDELETE (F, R, Q, N)

Given F and R pointers to the front and rear of a circular queue respectively and a vector Q consisting of N elements. This function deletes & return the first element of the queue. Y is temporary variable.

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TREES, BINARY SEARCH TREES, BINARY HEAPS

TREES

Tree is defined as a set of vertices such that

(*i*) There is a sequence of edges from any vertex to any other.

(*ii*) There are no circuits, that is, no paths starting from a vertex and returning to the same vertex.

OR

A tree is defined as a finite set of one or more nodes such that

(*i*) There is a specially designated node called a *root.*

(*ii*) The remaining nodes are partitioned into $n \geq 0$ disjoint sets T_1 ..... T_n, where each of these sets is a tree. T_1, T_2, ... T_n are called *sub trees of the root.*

Consider tree shown in the figure below. The tree has 13 nodes. The root node is A.

The number of subtrees of a node is called its *degree*. The degree of node A is 3, of C is 1 and of J is 0.

Nodes with degree equal to zero are called *leaf nodes or terminal nodes*. The set of leaf nodes in the figure is {K, L, F, G, M, I, J}.

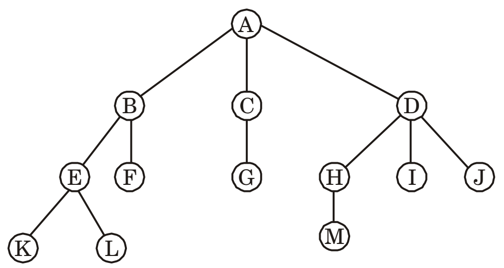
The roots of subtrees of nodes X are the children of X. The children of A are B, C and D. The parent of D is A. Children of the same parent are called *Siblings*. H, I and J are siblings.

The degree of a tree is maximum degree of nodes in the tree. The tree in the figure has a degree of 3.

The ancestors of a node are all the nodes along the path from root to that node. Ancestors of L are A, B and E.

The level of a node is defined by taking the root to be at level one. If a node is at level P, its children are at level P + 1. The height or the depth of a tree is the maximum level of any node in that tree.

A forest is a set of $n \geq 0$ disjoint trees. If root node is removed from the tree of figure below, a forest of 3 disjoint trees is created.



Ordered tree.

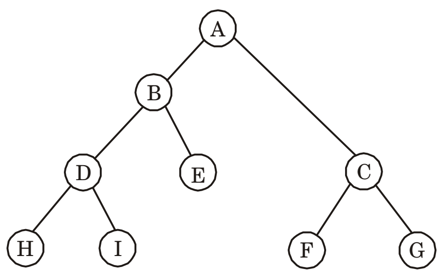
An ordered tree is a rooted tree in which children of each vertex are assigned an order.

2-tree.

A tree in which every vertex has either 0 or 2 children.

Binary tree.

A binary tree is either empty, or it consists of a root together with two binary tree called *left subtree* and *right subtree* of the root.



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Lemma 1

The maximum number of nodes on level i of a binary tree is 2^{i-1}. Also maximum number of nodes in a binary tree of depth k is

$$2^k - 1 \; k \geq 0$$

A binary tree of depth k which has exactly $2^k - 1$ nodes is called a *full binary tree of depth k*.

e.g. tree is a full binary tree.

Lemma 2

For any non empty binary tree T, if n_0 is number of terminal nodes and n_2, the number of nodes with degree 2, then

$$n_0 = n_2 + 1.$$

A binary tree with n nodes and depth k is complete if its nodes correspond to the nodes which are numbered from 1 to n in a full binary tree of depth k. (Nodes are numbered starting from nodes at level one, then going to those at level 2 and so on). Leaf nodes in a complete binary tree appear on at most two adjacent levels.

The nodes of complete binary tree may be stored in a one dimensional array tree. With the nodes numbered i being stored in tree[i].

Lemma 3

If a complete binary tree with n nodes (*i.e.* depth = $\lfloor \log_2 n \rfloor + 1$) is represented sequentially as in the figure, then for any node with index i, $1 \leq i \leq n$.

(i) Parent (i) is at [i/2] if $i \neq 1$.
When $i = 1$, i is the root and has no parent.

(ii) l child (i) is at $2i$ if $2i < n$.
If $2i > n$, then i has no left child.

(iii) r child (i) is at $2i + 1$ if $2i + 1 \leq n$.
If $2i + 1 > n$, then i has no right child.

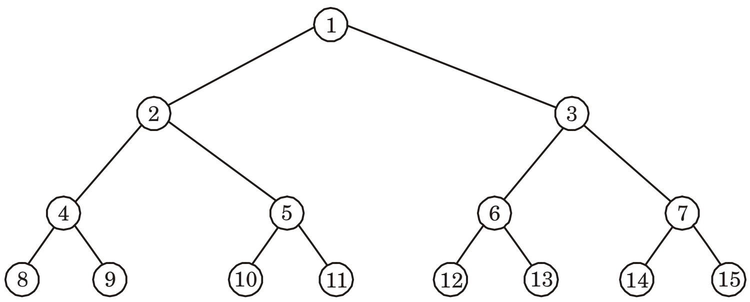

Fig. Full binary tree of depth 4.

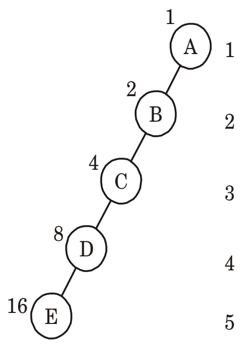

Fig. (a) Skewed binary tree

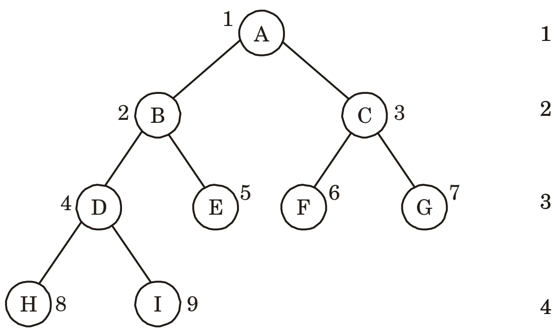

Fig. (b) Complete binary tree

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Fig. (c) and Fig. (d) shows sequential representations of the binary trees of Fig. (a) and Fig. (b) respectively.

[1]	A
[2]	B
[3]	–
[4]	C
[5]	–
[6]	–
[7]	–
[8]	D
[9]	–
[10]	–
[11]	–
[12]	–
[13]	–
[14]	–
[15]	–
[16]	E

Fig. (c)

A
B
C
D
E
F
G
H
I

Fig. (d)

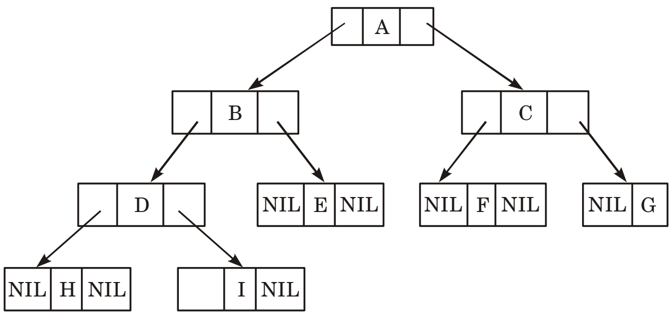

Fig. Linked representation of binary trees

If T is a tree with N vertices, then it has $m = n - 1$ edges.

When $n = 1$, T is a tree with single vertex with no edge.

Hence $m = 0 = 1 - 1 = n - 1$. Hence above problem is true for $n = 1$.

Let T_1 be a tree with $n - 1$ vertices.

Hence m_1 = number of edges of tree

$T_1 = (n - 1) - 1 = n - 2$

Let us construct a tree T from T_1 by adding a vertex u to T_1.

Clearly $$n_T = n_{T1} + 1 = n - 1 + 1 = n.$$

Case 1 : *Only one edge is added from u to T_1* [Fig. (a) below]

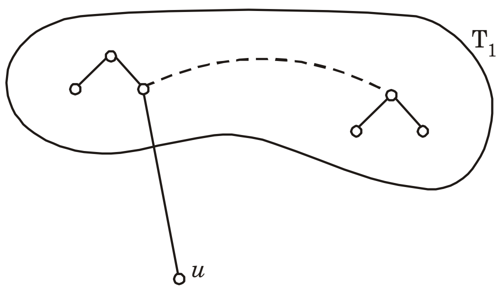

Fig. (a) T from T_1

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Clearly $m_T = m_{T1} + 1 = n - 2 + 1 = n - 1$

Hence the problem is true for T.

Case 2 : *Let two or more edges joined from u to T_1. Let u be joined to v and w of T_1.* [Fig. (*b*) below].

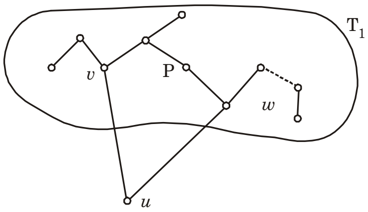


Fig.(b) uvPwu is a cycle

Since v and $w \in T_1$ and T_1 is a tree (connected acyclic graph) v and w are connected by a path P in T_1. Hence uvPwu forms a cycle in T, a contradiction.

Hence if we want to add a vertex to a tree and construct a new tree then we can add only one edge.

Thus $m = n - 1$ will true for all values of n.

If T is a spanning tree of G and the edge e ∈ E(G) and e ∉ E(T), Then T + e contains a unique cycle.

Let $e = (u, v) \in$ E(G) and $\notin$ E(T).

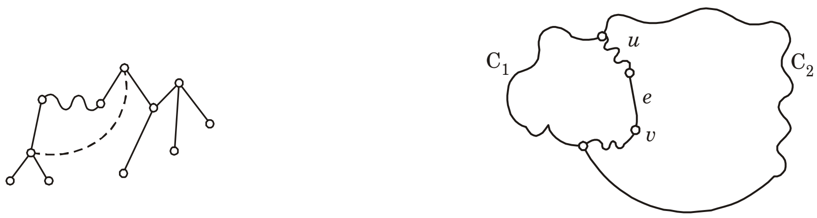
Let $u, v, \in$ V(T), then there is a unique path P connecting u and v in T.

Clearly P $\cup$ e is a cycle.

To prove uniqueness let C_1 and C_2 be the two cycles in T + e.

Since e generates C_1 and C_2, $e \in C_1$ and $e \in C_2$.

Clearly $C_1 \cup C_2 - e$ contains a cycle in T, a contradiction to T is a tree.



(a) T + e has a cycle ***(b) $C_1 \cup C_2 - e$ has a cycle in T***

Fig. (T + e) and cycle

Hence T + e contains a unique cycle. These unique cycles are called *fundamental cycles.*

Example. Is there any tour traversing all edges of G shown in the figure below ?

Solution.

The tour

1 2 3 4 5 6 7 8

travels through all edges of G. It is shown in figure below. Here no edges is repeated.

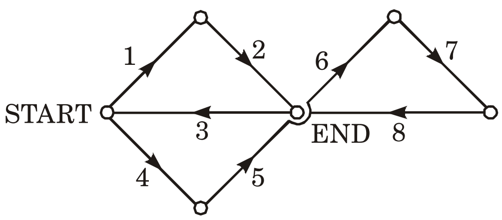


Fig. Tour of G traversing all edges

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SETS

Sets are constructed out of a finite universe of elements (say n elements). These sets may be empty or may contain any subsets of elements of U. Trees may be used to represent sets.

Assumptions.

(*i*) Elements of sets are numbers 1, 2, 3, ... n. These might be indices into a symbol table where the actual names of elements are stored.

(*ii*) Sets being represented are pairwise disjoint, i.e. if S_i and S_j, $i \neq j$ are two sets, then there is no element which is in both S_i and S_j.

Operations which may be performed on these sets are :

(a) Disjoint set union.

(b) To find the set containing an element i.

Consider sets

$$S_1 = \{1, 7, 8\ 9\}$$
$$S_2 = \{2, 5, 10\}$$
$$S_3 = \{3, 4, 1\}$$

Then tree representation will look like shown in the figure below.

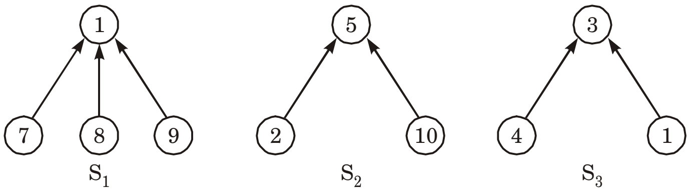
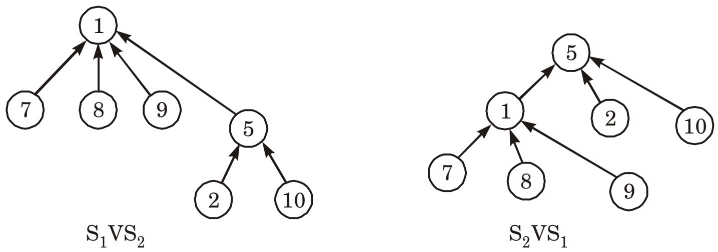
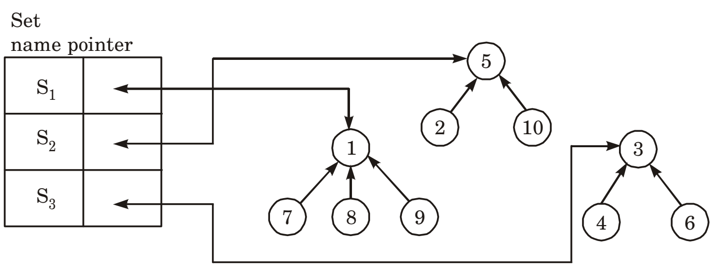


Fig. Set representation

The union of sets may be represented as



To find the union of two sets of parent field of the root of S_1 to the root of S_2. A pointer to the root of the tree representing the set is also maintained.



In addition each root has a pointer to the set name. So to determine which set a particular element resides in the parent links to the root of the tree are followed and the pointer to the set name is obtained.

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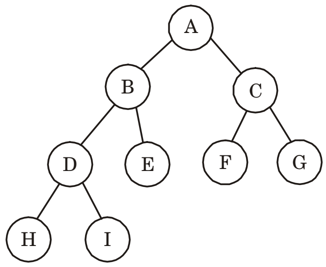
BINARY TREE TRAVERSAL

One of the important operations performed on trees is traversal. A full traversal produces a linear order for the information stored in a tree. A convention that left will be traversed before right is adopted. *Then there are 3 possible ways to traverse a tree given in figure :*



(*i*) Preorder traversal (VLR). [V-root, L-left mode, R-right more]

The node is visited before the subtrees.

e.g. for linked representation of binary trees, preorder traversal is A B D H I E C F G.

(*ii*) Inorder traversal (LVR).

The node is visited between subtrees.

e.g. for linked representation of binary trees, inorder traversal is H D I B E A F C G.

(*iii*) Post-order traversal (LRV).

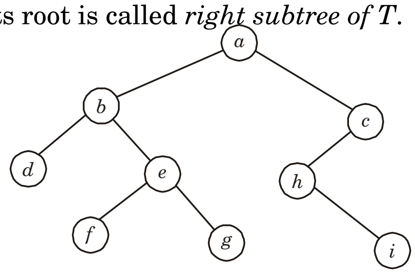
The node is visited after both of the subtrees.

e.g. for linked representation of binary trees, postorder traversal is H I D E B F G C A.

Note : A node can be an operation or an operand. Processing a node is execution of the corresponding operation and traversing is travelling through the node without considering the node.

Left subtree and right subtree.

Let T be a binary tree with root a and sons b and c. Then subtree having b as its root is called *left subtree of a* and the subtree of T having c as its root is called *right subtree of T*.



BREADTH FIRST SEARCH

Starting at vertex v and marking it as visited, breadth first search differs from depth first search in that all unvisited vertices adjacent to v are visited next. Then unvisited vertices adjacent to these vertices are visited and so on.

A breadth first search beginning at vertex v_1 of the graph is shown in the figure below would first visit v_1 and then v_2.

Next vertices v_4, v_5, v_6 and v_7 will be visited and finally v_8.

Algorithm BFS gives the details

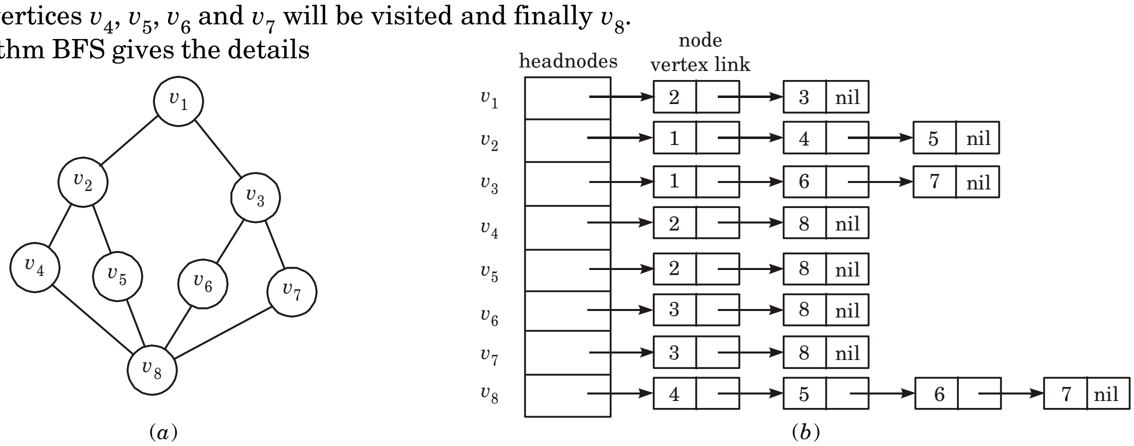


Fig. Graph G and its adjacency lists

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(1) Procedure for depth first search for an undirected graph G (V, E).

```
procedure dfs(v : integer);
{Given an undirected graph G = (V, E) with n vertices and an
array visited[n] initially set to false, this algorithm visits
all vertices reachable from v. visited is global.]
var w : integer;
begin
  visited[v] : = true;
  for each vertex w adjacent to v do
          if not visited[w] then dfs(w);
end; {of dfs}
```

(2) Procedure for breadth first search for a graph G.

```
procedure bfs(v : integer);
{A breadth first search of G is carried out beginning at vertex
v. All vertices visited are marked as visited[i] := true. The
graph G and array visited are global and visited is initialized
to false, initializequeue, addequeue, emptyqueue and deletequeue
are procedures/functions to handle queue operations]
var w : integer,
q : queue;
begin
  visited[v] : = true;
    initializequeue(q);     {q is a queue}
    addequeue(q,v);         {add vertex to queue}
    while not emptyqueue(q) do
    begin
        deletequeue(q,v);{remove from queue vertex v}
        for all vertices w adjacent to v do
            if not visited[w]
            then
            begin
                addqueue(q, w);
                visited[w] : = true;
            end; {of if and for}
    end; {of while}
end; {of bfs}
```

(3) A doubly linked list can be used to implement a binary tree.

Let each element have two pointers, left pointer and right pointer. Let left pointer point left son and right pointer point right son.

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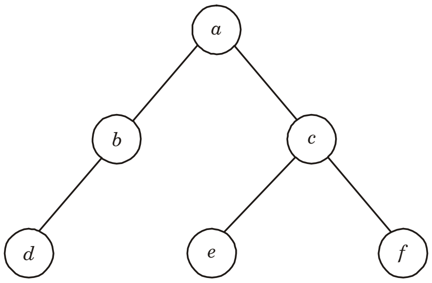
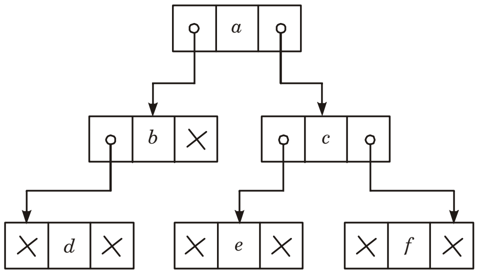
e.g. Fig. *(a)* below shows a binary tree and Fig. *(b)* shows a doubly linked list representation.



(a) T



(b) Doubly linked list of T

(4) A data structure to encode a binary search tree.

Data structure to encode a binary search tree.

LPTR	DATA	RPTR

'C' declaration of binary tree

```
typedef struct node {
        int DATA ;
struct node * LPTR, *RPTR ;
} Btree ;
```

Algorithm to traverse a binary search tree in preorder :

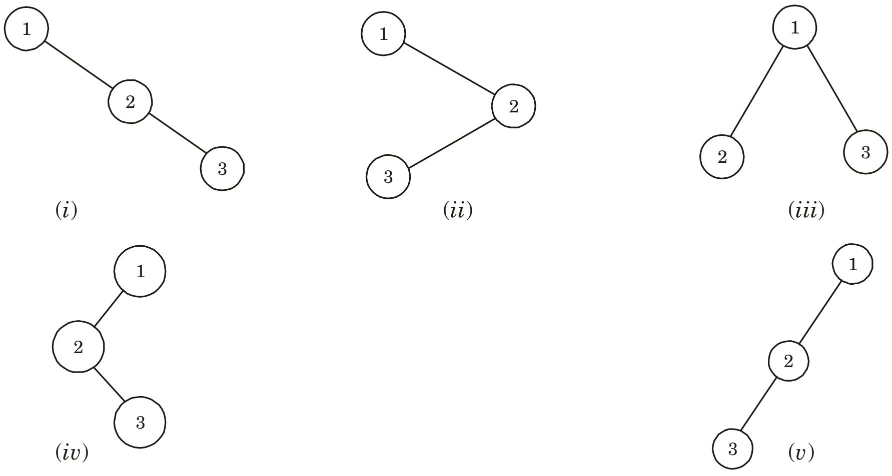
Preorder (ROOT).

Given algorithm take the root ROOT of the binary search tree and recursively traverse the tree in preorder.

1. [Check for existence of tree]
   If ROOT = NULL
   return
2. [Traverse the left subtree]
   Preorder (ROOT $\rightarrow$ LPTR)
3. [Traverse the right subtree]
   Preorder (ROOT $\rightarrow$ RPTR)
4. [Finish]
   end.

(5) All binary search trees containing the three elements 1, 2, 3.

All possible binary tree containing three elements 1, 2, and 3, are shown in the figure below.



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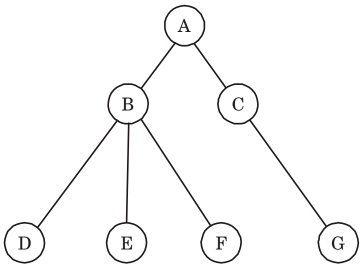
Example. Convert the following tree into a binary tree



Solution.

On down, there is/are son(s)

On right, there is/are siblings

$\therefore$ A has son B and C

B has sibling C

B has son D, E, F

D has sibling E and F

C has son G

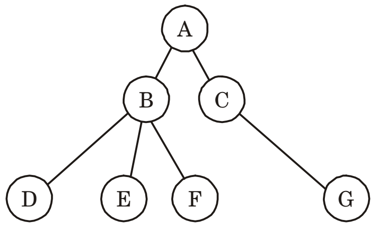


Fig. (a) Original tree

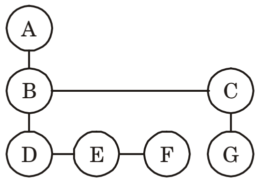


Fig. (b)

in Figure (b) which also shows in Figure (a)

while converting now,

son goes to left and sibling goes to right for the node

$\therefore$ A has son B, goes left

and B has son D, goes left and

C as sibling goes right

and so on.

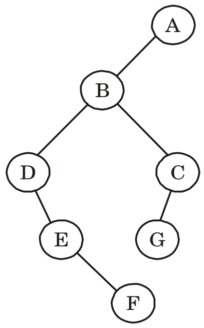


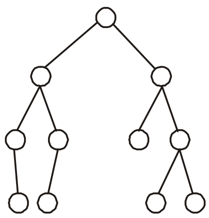
Fig. (c)

Example. Number the nodes of the following binary tree to traverse it.

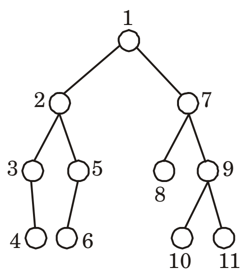
(a) Preorder

(b) Inorder

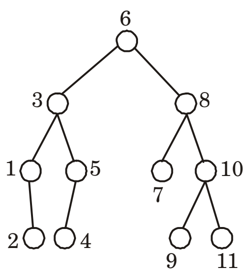
(c) Postorder



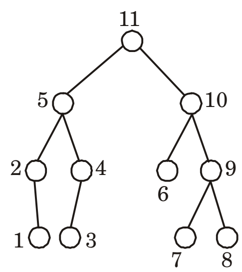
Solution.



(a) Preorder



(b) Inorder



(c) Postorder

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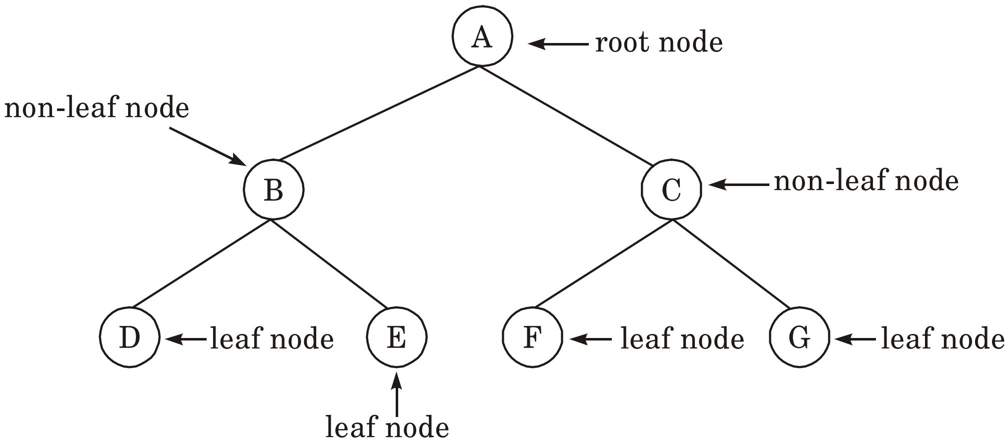
Example. A full binary tree is a binary tree where every node has 0 or 2 children but never just 1. Draw a full binary tree having 2 non-leaf nodes.

Solution.

Full binary tree having two non-leaf nodes :



ALGORITHM CHECKING BINARY SEARCH TREE

A binary search tree is a tree which gives the ascending order for preorder traversal. Thus if preorder traversal of a given binary tree is stored in an array and resultant array is sorted in ascending order, then given binary tree is the *binary search tree.*

Following is the algorithm that does the same. The algorithm takes the pointer to root of a binary tree having N elements. A is the array of N elements of store result.

Check_Binary_Tree (ROOT)

```
1.  [Initialize]
        I = 1
2.  [Traverse Tree]
        If ROOT ≠ NULL Then
        A[I] = ROOT (Info)
        Set I = I + 1
        Check_Binary_Tree (ROOT (Left))
        Check_Binary_Tree (ROOT (Right))
        End if
3.  [Array sorted ?]
        For I = 1 to N – 1 step 1
            If A[I] > A[I+1] Then
                Write 'NOT A BINARY SEARCH TREE'
                Return
            End If
        End For
        Write 'IT IS A BINARY SEARCH TREE'
4.  [Finish]
        End
```

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PREORDER, INORDER AND POSTORDER TRAVERSALS FOR BINARY TREE

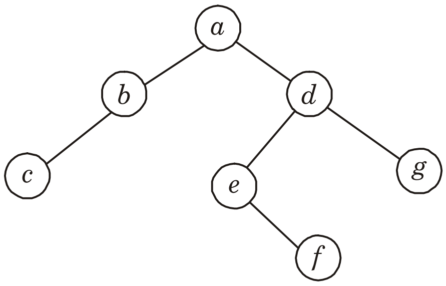


Fig. T

Preorder.

1. Process the root node.
2. Traverse the left subtree in preorder.
3. Traverse the right subtree in preorder.

a b c d e f g

Inorder.

1. Traverse left subtree in order.
2. Process the root node.
3. Traverse the right subtree in inorder.

c b a e f d g

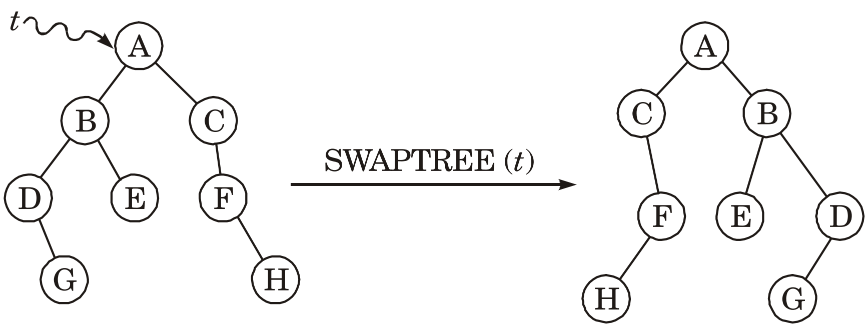
Postorder.

1. Traverse the left subtree in postorder.
2. Traverse the right subtree in postorder.
3. Process the root node.

c b f e g d a

ALGORITHM SWAPTREE (T)

It takes a binary tree pointed to by t and swaps the left and right children of every node.



Given binary tree whose root node address is given by pointer value T and a node structure (NODE) having information (INFO) part and two pointers as left-child (LPTR) and right-child (RPTR). This algorithm generates the swaptree.

1. [Null pointer]

   If I = NULL

   then return (NULL)

2. [Create a new node]

   NEW $\Leftarrow$ NODE

3. [Copy information field]

   DATA(NEW) $\leftarrow$ DATA(T)

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4. [Swap structural link and process again]

   LPTR (NEW) ← SWAPTREE (RPTR (T))

   RPTR (NEW) ← SWAPTREE (LPTR (T))

5. [Return Address of new node]

   Return (NEW)

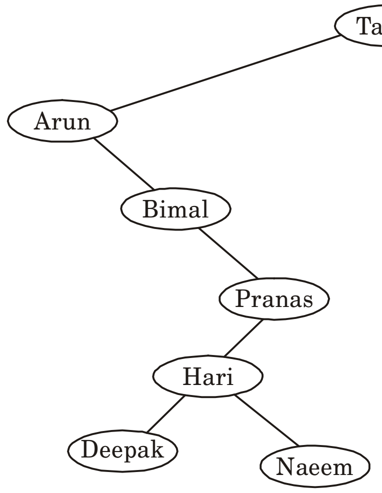
In this algorithm, a new tree is created in which every left link of old tree goes to right link and every right link of old tree goes to left link in a new tree. Hence the desired result is obtained.

Example. Create a binary search tree for the following word sequence :
Tarun; Arun; Bimal; Pranab; Hari; Naeem; Deepak,

Solution.

First node is root

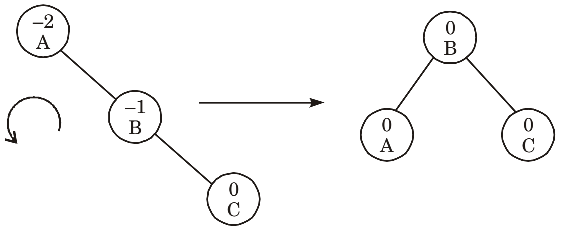
If coming node is smaller then root, it moves towards the left from root else towards right. If root has no son there, it becomes son for root else the same procedure applies to the son of root assuming it to be a new root.

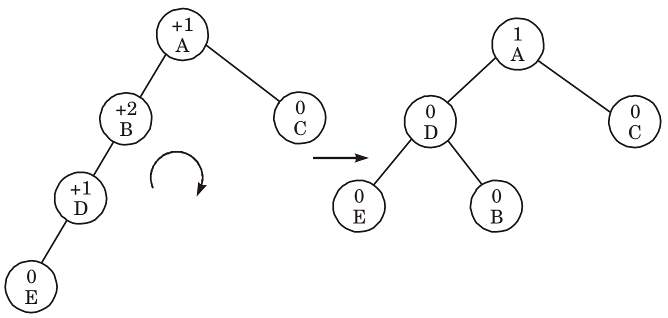


TYPES OF ROTATION

There are four types of rotations that can be used to restore AVL property in a binary search tree.

Balancing factors are written within the circles.





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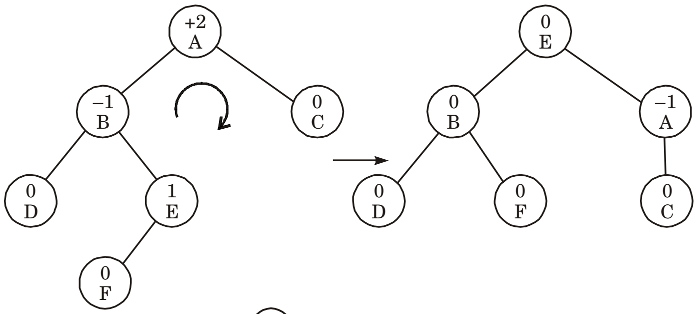

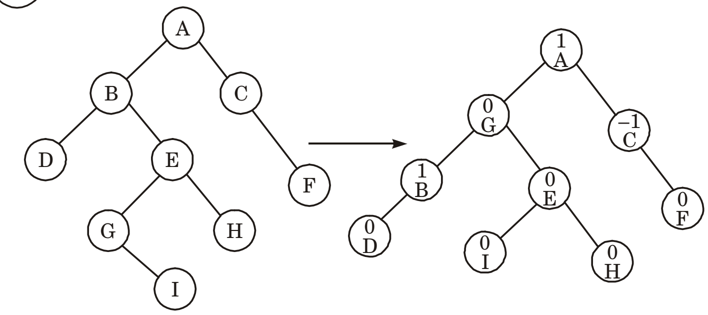

CONNECTED COMPONENTS

If G is an undirected graph, then one can determine whether or not it is connected by simply making a call to either DFS or BFS and then determining if there is any unvisited vertex. The time to do this is $O(n^2)$ if adjacency matrices are used and $O(e)$ if adjacency lists are used.

Now consider a problem to determine all the connected components of a graph. These may be obtained by making repeated calls to either DFS(v) or BFS(v), with v a vertex not yet visited. This lead to algorithm *comp* which determines all the connected components of G. The algorithm uses DFS. BFS may be used instead if desired. The computing time is not affected.

If G is represented by its adjacency lists, then total time taken by DFS is $O(e)$. The output can be completed in time $O(e)$ if DFS keeps a list of all newly visited vertices. Since **for** loops take $O(n)$ time, total time to generate all the connected components is $O(n + e)$.

By definition of a connected component, there is a path between every pair of vertices in the component and there is no path in G from vertex v to w if v and w are in two different components. Hence, if A is adjacency matrix of an undirected graph (i.e. A is symmetric) then its transitive closure A^+ may be determined in $O(n^2)$ time by first determining the connected components.

$A^+ [i, j] = 1$ iff there is a path from vertex i to j.

For every pair of distinct vertices in the same component $A^+ [i, j] = 1$.

On the diagonal $A^+ [i, j] = 1$ iff the component containing i has at least 2 vertices.

SPANNING TREES

When the graph G is connected, a depth first or breadth first search starting at any vertex visits all the vertices in G. In this case, edges of G are partitioned into two sets :

(*i*) T (for tree edges), where T is the set of edges used or traversed during the search.

(*ii*) B (for back edges), when B the set of remaining edges.

The set T may be determined by inserting the statement $T \leftarrow T \cup \{(v,w)\}$ in the **then** clauses of *dfs* and *bfs*. The edges in T form a tree which includes all the vertices of G.

Any tree consisting solely of edges in G and including all vertices in G is called a *spanning tree.*

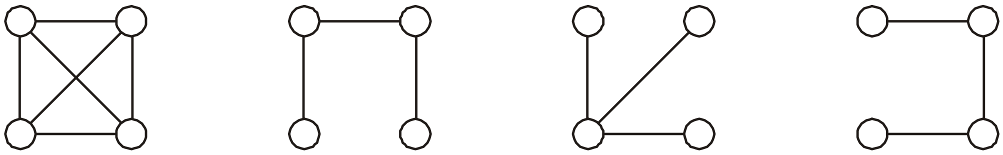

Fig. Complete graph and three of its spanning trees

Figure above shows a graph and some of its spanning trees.

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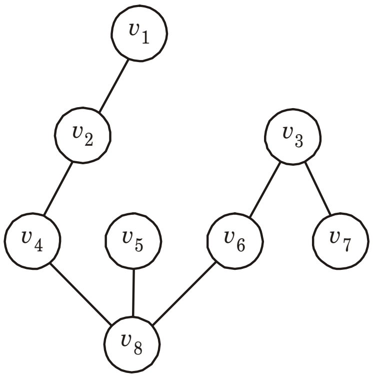
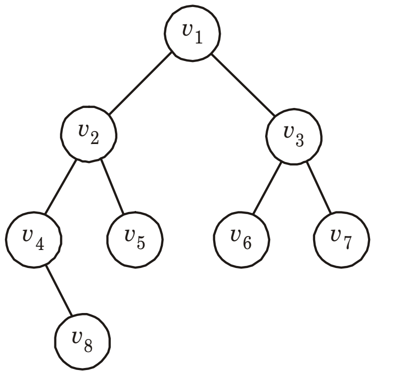
When either *DFS* of *BFS* are used, the edges of *T* form a spanning tree. The spanning tree resulting from a call to DFS is called *depth first spanning tree.*

When BFS is used, the resulting spanning tree is called *breadth first spanning tree.*



(a)* DFS *Spanning tree



(b)* BFS *Spanning tree

Fig. DFS and BFS spanning trees

Figure above shows the spanning trees resulting from a depth first and breadth first search starting at vertex v_1. If any of the edges (v,w) in B (the set of back edges) is introduced into the spanning tree T, then a cycle is formed.

This cycle consists of the edge (v,w) and all the edges on the path from w to v in T. If edge (8, 7) is introduce into the DFS spanning tree of above Fig. (a), then resulting cycle is 8, 7, 3, 6, 8.

Spanning tree of G.

Subgraph of a graph G has vertices and edges of G and no other vertices or edges. Therefore, a subgraph of G is formed from G by removing some vertices and edges of G. Then a tree T is said to be a spanning tree of G if the tree T is Subgraph of G and contains all vertices of G.

Some spanning and non spanning tree of graph G are shown below.

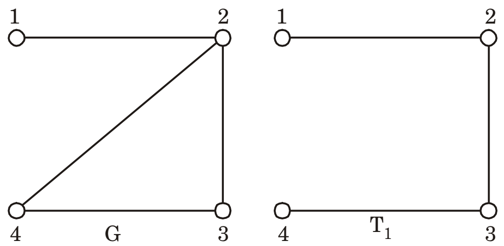


Fig. Spanning trees of G – T1 T2

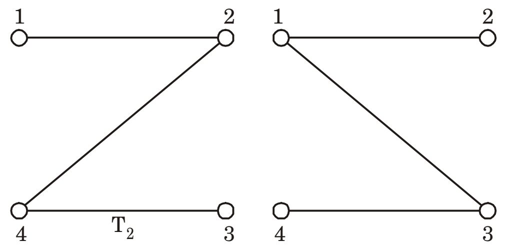


Fig. Non-Spanning tree because of edge (1, 3)

Algorithm to get a spanning tree of an undirected graph (if one exists).

SPANNING gives a structure as shown in the Fig. (*a*) below, this algorithm calculates the minimum spanning tree.

CHECK is a vector of size equal to number of nodes.

HEAD, defines outside the algorithm, points to the beginning of the linked list of edges.

We assume the FLAG field has been set to zero when structure shown in Fig. (*b*) was created.

1. [Check off nodes in the first edge]

CHECK [START (HEAD)] ← 1

CHECK [TERMIN (HEAD)] ← 1

FLAG (HEAD) ← 1

2. [Set loop and repeat untill all edges have been examined]

P ← HEAD

Repeat While $P \neq$ NULL

If FLAG (P) = 0 (those edges not alredy in tree)

then If (CHECK [START (P)] = 1 and (CHECK (TERMIN (P))] = 1

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then FLAG (P) ← (there is a path between these two nodes already)
else If (CHECK [START (P))] ≠ 0) and (CHECK [TERMIN (P)] ≠ 0)
then CHECK [START (P) ← 1 (add the edge to the tree)
CHECK [TERMIN (P)] ← 1
FLAG (P) ← 1
P ← HEAD (restart search)
P ← LINK (P) (examine next edge)

3. [Finished]
Exit.

REACH	NODEND	DATA	DIST	LISTPTR

DESTIN	EDGEPTR

Fig. (a) Edge structure

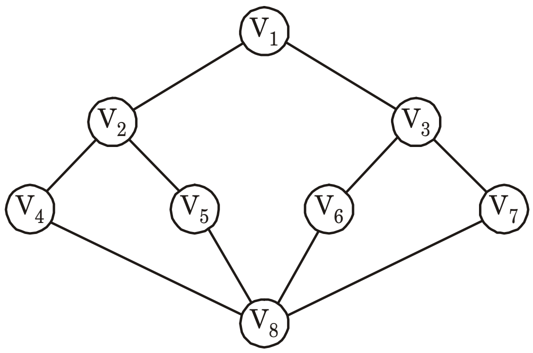
START	TERMIN	WEIGHT	FLAG	LINK

Fig. (b) Node structure

Spanning tree of a graph.

A spanning tree of a graph is an undirected tree consists of only those edges necessary to connect all the nodes in the original graph. A spanning tree has the property that for any pair of node, there exist only one path between them and for inserting any edge to a tree forms a unique cycle. Those edges left out of the spanning tree that were present in the original graph connectes path together in the tree.

Breadth first spanning tree of given graph.



Reach	No.	DATA	DIST.	*ptr*
T	1	V_1	0	→
T	2	V_2	1	→
T	3	V_3	1	→
T	4	V_4	2	→
T	5	V_5	2	→
T	6	V_6	2	→
T	7	V_7	2	→
T	8	V_8	3	→

INDEX

Distinct	*ptr*		
2	3		
4	5	1	
6	7	1	
2	8		
2	8		
3	8		
3	8		
4	5	6	7

LINKS

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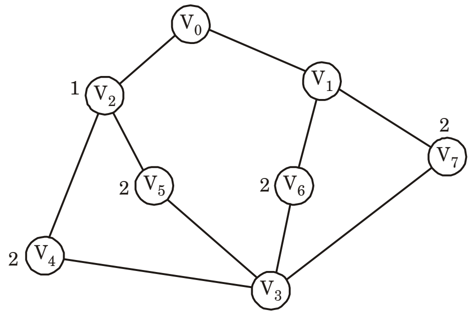


∴ BFS → V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8.

Procedure :

1. Initialize first node DIST as ZERO (0)

2. While all nodes are not processed

Delete current node from queue (of setp 1 for first time) and mark adjacent node dist as dist + 1 and if it is unvisited node, label it and add it to queue.

Note : LINK shows adjacent nodes for each node.

WALK AND TOUR

Sequence of vertices and edges in which consecutive edges are incident is called *walk.*

Tour is a walk in which any edge is not repeated.

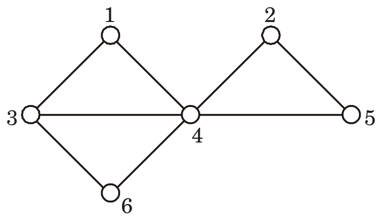


Fig. Walk, tour and path

Walk – 1 4 6 4 5 2 5 (edge (4, 6) is repeated)

Tour – 1 4 6 3 4 5 2 (No edge is repeated but the vertex 4 is repeated)

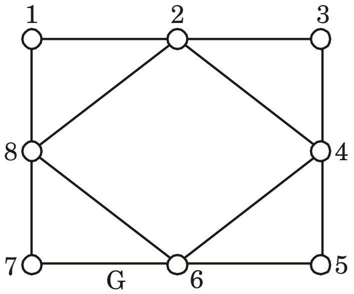
Path – 6 4 2 5

Euler tour.

When traversing is done through all edges of G and comes back to the same starting node, then this tour is called *Euler tour*.

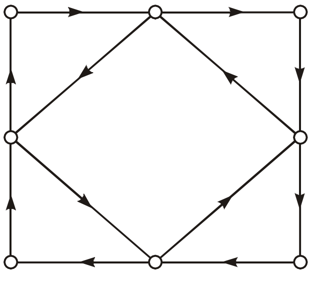
Any graph G has Eulerian tour if and only if all vertices of G are of even degree.

Example. Find the Eulerion tour of graph G shown below.



Eulerian graph

Solution.



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Since each vertex of G is of degree 2 or 4 the graph G is Eulerian.

The Eulerian tour 1 – 2 – 3 – 4 – 5 – 6 – 7 – 8 – 6 – 4 – 2 – 8 – 1 traverses all edges and return to 1.

SHORTEST PATH

When every edge of a directed path G has a non negative weight attached, then a path from one vertex v to another w such that sum of the weights on the path is as small as possible; such a path is called *shortest path.*

The weights may represent costs, time or some quantity other than distance.

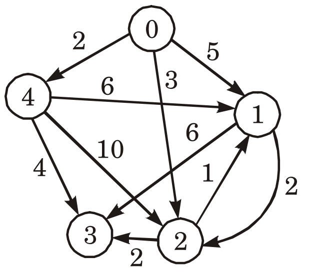

Fig. Directed graph with weights

TREE BALANCING

In a completely balanced tree, left and right subtrees of any node would have the same height. An *AVL* tree is a binary search tree in which heights of the left and right subtrees of the root differ by at most 1 and in which left and right subtrees are again AVL trees.

With each node of an AVL tree is associated a balanced factor that is left high, equal or right high according, respectively, as the left tree has height greater than, equal to, or less than that of the right subtree.

B - TREES

B - tree of order m is an m-way tree in which

(*i*) all leaves are on the same level.

(*ii*) all internal nodes except the root have at most m (nonempty) children and at least [m/2] (nonempty) children.

(*iii*) the number of keys in each internal node is one less than the number of its children, and these keys partition the keys in the children, in the fashion of a search tree.

(*iv*) the root has at most m children, but may have as few as 2 if it is not a leaf, or none of the tree consists of the root alone.

With this tree we will minimize file accesses.

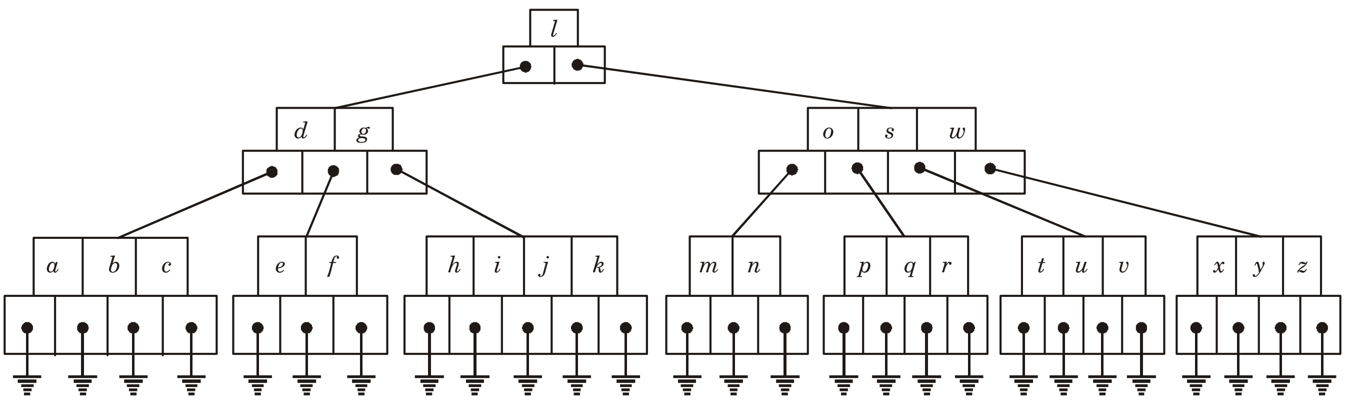

Fig. B - tree of order 5

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BINARY HEAP

A binary heap is a *heap data structure* created using a *binary tree*. It can be seen as a binary tree with two additional constraints:

- **The shape property:** the tree is a *complete binary tree;* that is, all levels of the tree, except possibly the last one (deepest) are fully filled, and, if the last level of the tree is not complete, the nodes of that level are filled from left to right.
- **The heap property:** each node is greater than or equal to each of its children according to a comparison *predicate* defined for the data structure.

Heaps with a mathematical "greater than or equal to" comparison function are called *max-heaps*; those with a mathematical "less than or equal to" comparison function are called *min-heaps.* Min-heaps are often used to implement *priority queues*.

Since the ordering of siblings in a heap is not specified by the heap property, a single node's two children can be freely interchanged unless doing so violates the shape property.

The binary heap is a special case of the *d-ary heap* in which d = 2.

It is possible to modify the heap structure to allow extraction of both the smallest and largest element in $O(\log n)$ time. To do this, the rows alternate between min heap and max heap. The algorithms are roughly the same, but, in each step, one must consider the alternating rows with alternating comparisons. The performance is roughly the same as a normal single direction heap. This idea can be generalised to a min-max-median heap.

SOLVED EXAMPLES

1. What is a heap ? How can a heap be used to represent a priority queue ? Discuss how to perform the operations of item insertion and removal in heaps used to represent priority queues.

Solution.

Heap H is a complete binary tree with say '*n*' elements and have following property.

The value at any node N is greater than or equal to value at each of the children of N.

A priority queue is a collection of elements such that each element has been assigned a priority and such that the order in which elements are deleted and processed comes from the following rules :

(1) An element of higher priority is processed before any element of lower priority.

(2) Two elements with same priority are processed according to the order in which they were added to the queue.

Priority queues can be easily implemented using heaps. Since a node will have higher value than other left and right children nodes represent the process with higher priority. Always highest priority one will be removed for processing and this will be root node. Thus to remove highest priority node from heap, following algorithm can be used :

HEAP-REMOVE-MAX (A)

1. If heap-size [A] < 1
2. then error "Heap Under flow" exit
3. max = A [1]
4. A [1] = A [heap size [A]]
5. heap size [A] = heap size [A] – 1
6. Make A [1] new heap
7. return max

HEAP-INSERT (A, Key)

1. Heap size [A] = heap size [A] + 1
2. i = heapsize [A]
3. while i > 1 and a [PARENT (*i*)] < key
4. do A[i] = A [PARENT (*i*)]
5. i = PARENT (*i*)
6. A [i] = key

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ALGORITHMS

INTRODUCTION

Programming involves various activities from the stage of conceiving the problem upto the stage of creating a model to solve the problem. The formal representation of this model as a sequence of instructions is called an *algorithm* and coded algorithm in a specific computer language is called a *program*.

Algorithm is an interpretable, finite set of instructions for dealing with contigencies and accompanying task that has recognizable end-points, end-state or result for inputs given. It is a tool for solving a well-specified computational problem. An algorithm is said to be correct if for every correct input, it halts with the correct output. Algorithms often have steps that repeat or require decisions until the task is completed. Different algorithms can be writtern for the same task, i.e., using different set of instructions, take more or less time, space or effort.

Characteristics of an Algorithm

Every algorithm should have following five characteristic features :

1. Input
2. Output
3. Definiteness
4. Effectiveness
5. Termination

ANALYZING ALGORITHM

Analyzing an algorithm means the resources that an algorithm needs. Resources can be memory requirements, communication bandwidth, logic gates computation time etc.

Phases of Analysis.

Analysis consists of two phases :

(1) Priori analysis.

The bounds of algorithms computing time are obtained by formulating a function based on theory. It is independent of programming languages and machines structures. The stress is laid on the frequency of execution of statements.

(2) Posteriori analysis.

The actual amount of space and time taken by the algorithms are recorded during execution. It is dependent on the programming languages used and machine structures.

Let us analyze time taken by the insertion sort. The time taken by the insertion sort depends upon size of input. More is the number of input, more is the time taken.

The priori analysis of an algorithm is independent of factors which depend on machine structure and programming languages. Stress is laid on the frequency of execution of statements. The O-notation (read as big oh notation) is a mathematic notation used in the priori analysis.

ASYMPTOTIC NOTATION (θ, π, θ)

1. Θ-Notation (Same order)

This notation bounds a function to within constant factors.

We say $f(n) = \Theta(g(n))$ if there exist positive constants n_0, c_1 and c_2 such that to the right of n_0, the value of $f(n)$ always lies between $c_1 g(n)$ and $c_2 g(n)$ inclusive.

In the set notation, we write as follows:

$\Theta(g(n)) = \{f(n)$: there exist positive constants c_1, c_1, and n_0 such that $c_1 g(n) \leq \text{f(n)} \leq c_2 g(n)$ for all n, $n \leq n_0$

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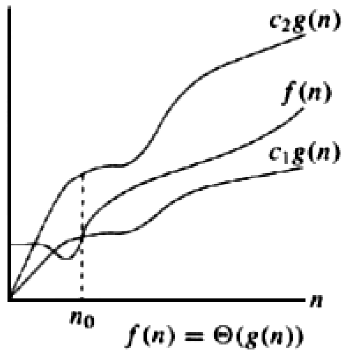
We say that is $g(n)$ an *asymptotically tight bound for* $f(n)$.



$f(n) = \Theta(g(n))$

Graphically, for all values of n to the right of n_0, the value of $f(n)$ lies at or above $c_1 g(n)$ and at or below $c_2 g(n)$. In other words, for all $n = n_0$, the function $f(n)$ is equal to $g(n)$ to within a constant factor. We say that $g(n)$ is an *asymptotically tight bound for* $f(n)$.

2. **O-Notation** (Upper Bound)

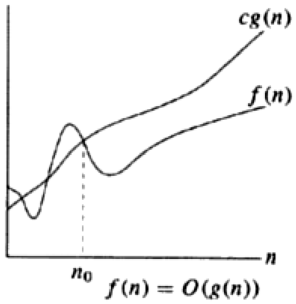
This notation gives an upper bound for a function to within a constant factor.

We write $f(n) = O(g(n))$ if there are positive constants n_0 and c such that to the right of n_0, the value of $f(n)$ always lies on or below $c\,g(n)$.

In the set notation, we write as follows:

For a given function $g(n)$, the set of functions $O(g(n)) = \{f(n)$: there exist positive constants c and n_0 such that $0 \le f(n) \le c * g(n)$ for all n_1 $n \ge n_0$

We say that the function $g(n)$ is an asymptotic upper bound for the function $f(n)$. We use O-notation to give an upper bound on a function, to within a constant factor.



$f(n) = O(g(n))$

Graphically, for all values of n to the right of n_0, the value of the function $f(n)$ is on or below g(n). We write $f(n) = O(g(n))$ to indicate that a function $f(n)$ is a member of the set $O(g(n))$ i.e.

e.g., $2n^2 = O(n^3)$, with $c = 1$ and $n_0 = 2$.

Equivalently, we may also define f is of order g as follows:

If $f(n)$ and $g(n)$ are functions defined on the positive integers, then $f(n)$ is $O(g(n))$ if and only if there is a $c > 0$ and an $n_0 > 0$ such that

$$| f(n) | \le | g(n) | \text{ for all } n \ge n_0$$

3. **Ω-Notation** (Lower Bound)

This notation gives a lower bound for a function to within a constant factor. We write $f(n) = \Omega(g(n))$ if there are positive constants n_0 and c such that to the right of n_0, the value of $f(n)$ always lies on or above $c\,g(n)$.

In the set notation, we write as follows:

For a given function $g(n)$, the set of functions

$$\Omega(g(n)) = \{f(n) : \text{there exist positive constants c and } n_0 \text{ such that } 0 = c\,g(n) = f(n) \text{ for all } n = n_0\}$$

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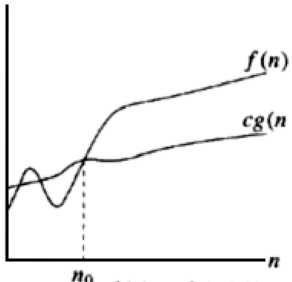
We say that the function $g(n)$ is an asymptotic lower bound for the function $f(n)$.



$f(n) = \Omega(g(n))$

The intuition behind Ω-notation is shown above.
e.g., $\sqrt{n}$ = (lg n), with $c = 1$ and $n_0 = 16$.

Properties of sets O(*f*(*n*)) and Θ(*f*(*n*))

The set of functions Θ(f(n)) is an equivalence class.

(1) Let $g(n)$ and $h(n)$ be any two functions contained in $\Theta(f(n))$. Use the definition of big-Θ to demonstrate thatthe elements of this set are reflexive, transitive and symmetric under the relation big-Θ.

(2) Big-Θ also form an equivalence class

(3) The functions $f(n)$ and $T(n)$ that respectively count the number of compares in selection sort and the number of moves in Towers of Hanoi are $\Theta(n^2)$ and $\Theta(2^n)$ respectively.

(4) The fibonacci sequence $f(n) = f(n-1) + f(n-2)$ is **not** $\Theta(2^n)$.

We know, $f(n) = O(2^n)$.

We must now find constant c_1 and N_0' such that

$$f(n) > c_1 2^n \text{ for all } n \geq N_0'$$

then, $$f(n) = f(n-1) + f(n-2) > c_1 2^{n-1} + c_1 2^{n-2} \geq c_1 2^n$$

but c_1 factors out of this expression, therefore any constant that satisfies the base case will do equally well (poorly).

For the inductive hypothesis to be satisfied, we must have

$$2^{n-1} + 2^{n-2} \geq 2^n$$

$$2^{n-2}(2 + 1) = 3 * 2^{n-2} < 2^n \qquad \text{contradiction; no pair of } c_1, N_0 \text{ exist}$$

SPACE AND TIME COMPLEXITY

SPACE COMPLEXITY.

Space needed by algorithms is sum of following components :

1. A fixed part that is independent of the characteristics

e.g., number, size of the inputs and outputs.

This part typically includes

(*i*) instruction space (i.e., space for the code)

(*ii*) space for simple variables

(*iii*) fixed size component variables (also called *aggregate*)

(*iv*) space for constant,

and so on.

2. A variable part that consist of space needed by component variables whose size is dependent on the particular problem instance being solved, the space needed by referenced variables and the recursion stack space.

The space requirement S(P) of any algorithm P may therefore be written as

$$S(P)\ c + S_P \text{ (instance characteristics)}$$

where c is a constant.

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TIME COMPLEXITY.

The time T(p) taken by a program P is the sum of compile time and the run (or execution) time. The compile time does not depend on the instance characteristics. Run time is denoted by t_P.

$$t_P(n) = c_a\, ADD(n) + c_s\, SUB(n) + c_m\, MUL(n) + c_d\, DIV(n) + ...$$

where, n denotes instance characteristics

c_a, c_s, c_m, c_d and so on respectively, denote the time needed for an addition, subtratction, multiplication, division and so on

ADD, SUB, MUL, DIV and so on, are functions whose values are numbers of additions, subtractions, multiplications, divisions and so on, that are performed when codes for P is used on an instance with character n.

The value of $t_P(n)$ for any given n can be obtained only experimentally. The program is typed, compiled and run on a particular machine. The execution time is physically clocked and $t_P(n)$ obtained.

In a multiuse system, the execution time depends on factors such as system load, the number of other programs running on the computer at the time program P is run.

A program step defined a syntactically or semantically meaningful segment of a program that has an execution time that is independent of the instance characteristics. The number of steps any program statement is assigned depends on type of statement.

e.g., comments count as zero steps; an assignment statement which does not involve any calls to other algorithms is counted as one step ; in an iterative statement such as the **for, while** and **repeat-until** statements, we consider step counts only for the control part of the statement.

The control parts for **for** and **while** statement have following forms.

for i : = <expr> **to** <expr 1> **do**

while (<expr>) **do**

Each execution of the control part of a while statement is given a step count equal to number of step counts assignable to <expr>. The step count for each execution of the control part of a for statement is one, unless to counts attributable to <expr> and <expr 1> are functions of the instance characteristics.

The number of steps needed by a program to solve a particular problem instance can be count by two method. In the first method, we introduce a new variable, **count**, in to the program. This is a global variable with initial value 0. Statements to increment count by the appropriate amount are introduced into the program. This is done so that each time a statement in the original program is executed, count is incremented by the step count of that statement.

Basic Complexity Classes

For a function $f : N \to N$, we define following complexity classes.

1. ***Time(f)* :** It is the set of languages that can be recognized by a Turing machine T with $\tau_T \le f$.
2. ***Space(f)* :** It is the set of languages that can be recognized by a Turing machine T with $s_T \le f$.
3. ***NTime(f)* :** It is the set of languages that can be accepted by an NTM T with nondeterministic time complexity function $\tau_T \le f$.
4. ***NSpace(f)* :** It is the set of languages that can be accepted by an NTM T with nondeterministic space complexity function $s_T \le f$.

In all four cases, the machines are allowed to be multitape TMs.

Here we are concerned with how much time or space is required to solve a problem.

Step-counting functions

A function $f : N \to N$ is a *step-counting* function if there is a TM T, so that for any n and any input string of length n, T halts in exactly $f(n)$ moves.

If f is a step-counting function, T is a TM halting in $f(n)$ moves as in the definition, and T′ is any Turing machine, then a composite TM T can be constructed that executes T′, except that it halts when T halts if that happens first. In other words, T can be used as a *clock* in conjuction with other machines. Similarly way, if we want to constrain the space resources of a TM T during its computation, we can use a step-counting function f in order to mark a space of exactly $f(n)$ squares to be used by T.

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Lemma. If $f : \mathrm{N} \to \mathrm{N}$ is a positive function, and if there is a constant $c > 1$ so that $f(n) > cn$ for all but a finite number of integers n, then f is a step-counting function if and only if f can be computed in time $O(f)$ *i.e.* there is a Turing machine T computing f, and a constant K, so that for every n, T computes $f(n)$ in no more than $\mathrm{K}\,f(n)$ steps.

Theorem 1.

Let $f : \mathrm{N} \to \mathrm{N}$ be any step-counting function. Then for some constant c, $Time(f)$ is a proper subset of $Time\ (cn^2f^2)$.

It is possible to show that, if function f is a step-counting function, then function $cn^2\,(f(n))^2$, or a function that differs from this one at only a finite number of points, is also shown.

This means that by applying the theorem repeatedly, we obtain a sequence of more and more complex languages. This is a simple case of more general "*hierarchy*" results, which specify conditions on functions f and g that are sufficient to obtain languages in $Time(g) - Time(f)$. There are similar *space hierarchy* theorems, which show in particular that there are decision problems whose solutions require Turing machines of arbitrarily great space complexity.

Some relationships among complexity classes in above definition :

Theorem 2.

For any function f, $\quad Time(f) \subseteq NTime(f)$ and $Space(f) \subseteq NSpace(f)$.

Theorem 3.

For any function f, $\quad Time(f) \subseteq Space(f)$ and $NTime(f) \subseteq NSpace(f)$.

Theorem 4.

Let f is a step-counting function. Then

1. $NTime(f) \subseteq Space(f)$.
2. If $L \in NSpace(f)$, then for some constant c, $L \in Time(c^f)$.
3. $NSpace(f) \subseteq Space(cf^2)$ for some constant c (Savitch's Theorem).

By combining Theorems 3 and 4, we see in particular that for a step-counting function f,

$NTime(f) \subseteq Time(c^f)$ for some constant c.

In fact, this result does not require assumption that f be step-counting, and it can easily be obtained directly.

We constructed a Turing machine to try all possible finite sequences of moves of a given NTM.

WORST-CASE AND AVERAGE-CASE ANALYSIS

In the analysis of insertion sort, consider both, the cases in which input array was already sorted, and worst case, in which input array was reverse sorted.

We shall usually concentrate on finding only the **worst-case running time,** *i.e.* longest running time for *any* input of size n.

(*i*) Worst-case running time of an algorithm is an upper bound on the running time for any input. Knowing it gives a guarantee that the algorithm will never take any longer. We need not make some educated guess about the running time and hope that it never gets much worse.

(*ii*) For some algorithms, the worst case occurs fairly often.

e.g., in searching a database for a particular piece of information, the searching algorithm's worst case will often occur when information is not present in the database. In some searching applications, searches for absent information may be frequent.

(*iii*) *Average case* is often roughly as bad as the worst case.

Let us randomly choose n numbers and apply insertion sort.

How long does it take to determine where in subarray $A[1\,.\,.\,i-1]$ to insert element $A[i]$?

On average, half the element in $A[1\,.\,.\,i-1]$ are less than $A[j]$, and half the elements are greater.

Thus on average, we check half of the subarray $A[1\,.\,.\,i-1]$, so $t_i = i/2$.

If we work out the resulting average-case running time, it turns out to be a quadratic function of the input size, just like worst-case running time.

In some particular cases, we are interested in the **average-case** or **expected** running time of an algorithm. One problem with performing an average-case analysis is that it may not be apparent what constitutes an *average* input for a particular problem. Often, we assume that all inputs of a given size are equally likely. In practice, this assumption may be violated, but a randomized algorithm can sometimes force it to hold.

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DESIGN TECHNIQUES

DESIGN TECHNIQUES

Following five basic design techniques aid in creating good algorithms:

1. Divide and Conquer
2. Greedy Method
3. Dynamic Programming
4. Back tracking
5. Branch and Bound

DIVIDE AND CONQUER.

Algorithms that are recursive in nature typically follow divide and conquer.

It involves following three steps at each level of the recursion :

(*i*) **Divide**

Input is divided into a number of sub problems.

(*ii*) **Conquer**

Conquer the sub problems by solving them recursively.

(*iii*) **Combine**

Merge the sub problems into the solution for the original problems.

e.g. Binary search, Mergesort and Quick sort algorithms

(***a***) **Mergesort**.

Consider a list of n items to be sorted in non-decreasing order.

Adopt the following (recursive) strategy:

- Divide the list in half
- Spawn two processes to sort the two halves
- Merge the two sorted halves.

When merging the two halves, worst case, every item in the second half of the list is greater than all items in the first half of the list except for the last item which is the greatest item in the original list. A merge require that the first item of the second half of the list be compared with all $n/2$ items in the first half before it is placed in the merged list. The remaining $n/2 - 1$ items in the second half are then compared with the last of the first half before the merged list is completed.

∴ Total number of compares in the last merge (worst case) = $\frac{n}{2} + (\frac{n}{2} - 1) = n - 1$.

Let T(n) = number of compares to mergesort a list of n items, then

$$T(n) = 2T\left(\frac{n}{2}\right) + n - 1$$

(***b***) **Binary search.**

Strategy: Recursively divide the list in two halves, compare the key with the item

At the front of the second half of the list, if key is greater than or equal to this element, binary search the second half of the list, else binary search the first half of the list.

Let $T(n)$ = number of compares to binary search a list of n items, then

$$T(n) = T\left(\frac{n}{2}\right) + 1 = O(\lg n)$$

(***c***) **Master method.**

These two algorithms have recurrence relations of the form

$$T(n) = a\,T(n/b) + f(n)$$

where, $a \geq 1$ and $b > 1$

and there exists an n_0 such that, $f(n) > 0$ for all $n \geq n_0$

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Theorem (Master Theorem) :

Let $a \geq 1$ and $b > 1$ be constants, let f(n) be an asymptotically non-negative function, and let T(n) be defined on the nonnegative integers by the recurrence

$$T(n) = aT(n/b) + f(n)$$

where we interpret n/b to mean either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$.

Then T(n) can be Bounded asymptotically as follows:

- If $f(n) = O(n^{\log_b a - \varepsilon})$ for some constant $\varepsilon > 0$, then
$$T(n) = \Theta(n^{\log_b a})$$
- If $f(n) = \Theta(n^{\log_b a})$, then
$$T(n) = \Theta(n^{\log_b a} \lg n)$$
- If $f(n) = \Omega(n^{\log_b a + \varepsilon})$ for some constant $\varepsilon > 0$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then
$$T(n) = \Theta(f(n))$$

SEARCHING

Assume that we have a sequential file and we wish to retrieve an element matching with key 'k', then, we have to search the entire file from the beginning till the end to check whether the element matching k is present in the file or not.

Types of Search

There are a number of complex searching algorithms to serve the purpose of searching.

Following are relatively straight forward methods of searching

1. **Sequential search**

   In this method, we start to search from the beginning of the list and examine each element till the end of the list.

   If desired element is found, we stop the search and return the index of that element.

   If item is not found and the list is exhausted, then search returns a zero value.

   In the worst case, the item is not found or the search item is the last (n^{th}) element. For both situations, we must examine all n elements of the array, so the order of magnitude or complexity of the sequential search is n, i.e. O(n). The execution time for this algorithm is proportional to n, i.e. algorithm executes in linear time.

   Algorithm for sequential search.

   Algorithm : sequential search

   Input : A,- vector of n elements

   K,- search element

   Output : i – index of k

   Method : i=1

```
While(i<=n)
{
if(A[i]=k)
{
write("search successful")
write(k is at location i)
exit();
}
else
i++
if end
while end
write (search unsuccessful);
algorithm ends.
```

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2. Binary search

It is necessary to have the vector in an alphabetical or numerically increasing order. A search for a particular item with X resembles the search for a word in the dictionary. The approximate mid entry is located and its key value is examined.

If mid value is greater than X, then list is chopped off at the (mid-1)th location.

Now the list gets reduced to half the original list. The middle entry of the left-reduced list is examined in a similar manner. This procedure is repeated until the item is found or the list has no more elements.

If mid value is lesser than X, then list is chopped off at (mid+1)th location. The middle entry of the right-reduced list is examined and the procedure is continued until desired key is found or the search interval is exhausted.

Algorithm for Binary search.

Algorithm : binary search

Input : A, vector of n elements

K, search element

Output : low –index of k

Method : low=1,high=n

```
While(low<=high-1)
{
mid=(low+high)/2
if(k<a[mid])
high=mid
else
low=mid
if end
}
while end
if(k=A[low])
{
write("search successful")
write(k is at location low)
exit();
}
else
write (search unsuccessful);
if end;
algorithm ends.
```

SORTING

Sorting algorithms arrange items in a set according to a predefined ordering relation. The most common types of data are string information and numerical information. The ordering relation for numeric data simply involves arranging items in sequence from smallest to largest and from largest to smallest, which is called ascending and descending order respectively.

Various methods are available for sorting information, but none is best for all applications.

Performance of the methods depends on following parameters

(*i*) Size of the data set

(*ii*) Degree of relative order already present in the data etc.

Many algorithms have been developed for sorting and to determine efficiency of the algorithm, one should determine its complexity. There are n! orderings of n elements, since n! permutations can be of the correct order. The sorting algorithm based on binary comparisons can be represented by a binary decision tree in which each internal vertex represents a comparison of two elements. Each leaf represents one of the n! permutations of n elements.

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A sorting algorithm based on binary comparisons requires atleast log n! comparisons.

Since log n! is O(n log n) [as log n! > ($\frac{(n \log n}{2}$ for n > 4], no sorting algorithm using binary comparison can have worst case time complexity that is better than O (n log n) . Consequently, a sorting algorithm is as efficient as possible if it has O (n^2 log n) time complexity.

A list of n entries in the correct order needs at least (n – 1) comparisons of keys.

2. GREEDY METHOD

Greedy Algorithm

These are simple and straight forward. These are shortsighted in their approach in the sense that they take decisions on the basis of information at hand without worrying about the effect these decisions may have in the future. These are easy to invent, easy to implement and most of the time quite efficient. Many problems cannot be solved correctly by greedy approach. These are used to solve optimization problems

Greedy Approach

Greedy Algorithm works by making the decision that seems most promising at any moment; it never reconsiders this decision, whatever situation may arise later.

e.g., consider the problem of "*Making Change*".

Coins available are:

- dollars (100 cents)
- quarters (25 cents)
- dimes (10 cents)
- nickels (5 cents)
- pennies (1 cent)

Problem : Make a change of a given amount using the smallest possible number of coins.

Informal Algorithm :

- Start with nothing.
- at every stage without passing the given amount.
    - ➢ add the largest to the coins already chosen.

Formal Algorithm :

Make change for n units using the least possible number of coins.

```
MAKE-CHANGE (n)
    C ← {100, 25, 10, 5, 1}    // constant.
    Sol ← {};                  // set that will hold the solution set.
    Sum ← 0 sum of item in solution set
    WHILE sum not equal n
        x = largest item in set C such that sum + x = n
        IF no such item THEN
            RETURN    No Solution
        S ← S {value of x}
        sum ← sum + x
    RETURN S
```

Functions of Greedy Algorithm

Greedy algorithm consists of four functions :

(*i*) A function that checks whether chosen set of items provide a solution.

(*ii*) A function that checks the feasibility of a set.

(*iii*) Selection function tells which of the candidates is most promising.

(*iv*) An objective function, which does not appear explicitly, gives value of a solution.

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Structure Greedy Algorithm

(1) Initially the set of chosen items is empty, i.e. solution set.

(2) At each step

- item will be added in a solution set by using selection function.
- IF the set would no longer be feasible,
  reject items under consideration (and is never consider again).
- ELSE IF set is still feasible THEN
  add the current item.

General Knapsack problem.

Greedy method is best suited to solve more complex problems such as a knapsack problem. In a knapsack problem there is a knapsack or a container of capacity M n items where, each item i is of weight w_i and is associated with a profit p_i. The problem of knapsack is to fill the available items into the knapsack so that the knapsack gets filled up and yields a maximum profit. If a fraction x_i of object i is placed into the knapsack, then a profit $p_i x_i$ is earned. The constrain is that all chosen objects should sum up to M

Greedy-fractional-knapsack (*w, v, W*)

```
FOR i =1 to n
    do x[i] =0
weight = 0
while weight < W
    do i = best remaining item
       IF weight + w[i] = W
          then x[i] = 1
               weight = weight + w[i]
          else
               x[i] = (w - weight) / w[i]
               weight = W
return x
```

Analysis

- If items are already sorted into decreasing order of v_i / w_i, then the while-loop takes a time in *O(n)* Therefore, total time including the sort is *O(n log n)*.
- If we keep the items in heap with largest v_i/w_i at the root, then
  (*i*) creating the heap takes *O(n)* time
  (*ii*) while-loop now takes *O(log n)* time (since heap property must be restored after the removal of root)

2. DYNAMIC PROGRAMMING

It is an algorithm design method that can be used when the solution to a problem can be viewed as the result of a sequence of decisions.

Perspective

Dynamic programming is an optimization technique.

Greedy method vs. Dynamic programming.

(*i*) Both techniques are optimization techniques, and both build solutions from a collection of choices of individual elements.

(*ii*) Greedy method computes its solution by making its choices in a serial forward fashion, never looking back or revising previous choices (only one decision sequence is ever generated).

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Dynamic programming computes its solution bottom up by synthesizing them from smaller subsolutions, and by trying many possibilities and choices before it arrives at the optimal set of choices.

(*iii*) There is a priori litmus test by which one can tell if Greedy method will lead to an optimal solution, but there is a litmus test for Dynamic programming, called *Principle of Optimality*

Divide and Conquer vs. Dynamic Programming.

(*i*) Both techniques split their input into parts, find subsolutions to the parts, and synthesize larger solutions from smalled ones.

(*ii*) Divide and Conquer splits its input at prespecified deterministic points (e.g., always in the middle)

Dynamic programming splits its input at every possible split points rather than at a pre-specified points. After trying all split points, it determines which split point is optimal.

Principle of Optimality

An optimal sequence of decisions may be found by making decisions one at a time and never making an erroneous decision

A problem is said to satisfy the Principle of optimality if subsolutions of an optimal solution of the problem are themselves optimal solutions for their subproblems.

An optimal sequence of decisions may be found by making decisions one at a time and never making an erroneous decision

Steps of Dynamic Programming

Dynamic programming design involves following four major steps:

(*i*) Develop a mathematical notation that can express any solution and subsolution for the problem at hand.

(*ii*) Prove that the Principle of optimality holds.

(*iii*) Develop a recurrence relation that relates a solution to its subsolutions, using the math notation of step (*i*). Indicate what the initial values are for that recurrence relation, and which term signifies the final solution.

(*iv*) Write an algorithm to compute the recurrence relation.

Note :

- Steps (*i*) and (*ii*) need not be in that order. Do what makes sense in each problem.
- Step (*iii*) is the heart of the design process. In high level algorithmic design situations, one can stop at step (*iii*). In this course, however, we will carry out step (*iv*) as well.
- Without the Principle of optimality, it won't be possible to derive a sensible recurrence relation in step (*iii*).
- When the Principle of optimality holds, the four steps of Dynamic programming are guaranteed to yield an optimal solution. No proof of optimality is needed.

0/1 Knapsack

knapsack problem was to find $\{x_1, x_2, \ldots, x_n\}$ such than $0 \le x_i \le 1$

Adapt the Knapsack problem to consider $\{x_1, x_2, \ldots, x_n\}$ such than $x_i = 0$ OR $x_i = 1$

Find $\{x_1, x_2, \ldots, x_n\}$ subject to feasibility criteria and optimize objective function.

If each x_i can take on only a finite set of values, then we can try all combinations.

```
for all combination of x_i values
        check for constraints
        calculate optimizing function
        keep track of max.
end
```

If b possible values for each x_i, then b^n combinations: $O(b^n)$

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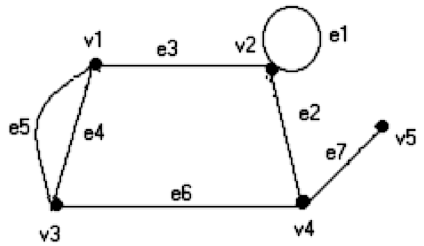
TREE AND GRAPH TRAVERSALS

TREE AND GRAPH TRAVERSALS

GRAPH.

A graph G = (V, E) consists of a set of objects V = {v_1, v_2, .}.called *vertices*, and another set E = {e_1, e_2, .}.whose elements are called *edges*. Each edge e_k in E is identified with an unordered pair (v_i, v_j) of vertices. The vertices v_i, v_j associated with edge e_k are called *end vertices* of e_k. Often this diagram itself is called a *graph*.



Finite and Infinite Graphs

Although in the definition of a graph neither the vertex set V nor the edge set E need be finite, in most of the theory and almost all applications these sets are finite. A graph with a finite number of vertices as well as a finite number of edges is called a *finite graph*; otherwise, it is an *infinite graph*.

Techniques for Graphs

A fundamental problem concerning graphs is the reachability problem. In its simplest form it requires to determine whether there exists a path in the given graph G = (V,E) such that this path starts at vertex v and ends at vertex u. A more general form is to determine for a given starting.

Vertex v belonging to V all vertices u such that there is a path from v to u. This latter problem can be solved by starting at vertex v and systematically searching the graph G for vertices that can be reached from v.

For this, there are two search methods :

1. **Breadth first search**

   In this we start at vertex v and mark it as having been reached. The vertex v at this time is said to be *unexplored*. A vertex is said to have been *explored* by an algorithm when algorithm has visited all vertices adjacent from it. All unvisited vertices adjacent from v are visited next. There are new unexplored vertices. Vertex v has now been explored. The newly visited vertices have not been explored and are put onto the end of the list of unexplored vertices. The first vertex on this list is the next to be explored. Exploration continues until no unexplored vertex is left. The list of unexplored vertices acts as a queue and can be represented using any of the standard queue representations.

2. **Depth first search**

   A depth first search of a graph differs from a breadth first search in that the exploration of a vertex v is suspended as soon as a new vertex is reached. At this time, exploration of the new vertex u begins. When this new vertex has been explored, the exploration of u continues. The search terminates when all reached vertices have been fully explored. This search process is best-described recursively.

```
Algorithm DFS(v)
{
    visited[v]=1
    for each vertex w adjacent from v do
    {
    If (visited[w]=0)then
    DFS(w);
    }
}
```

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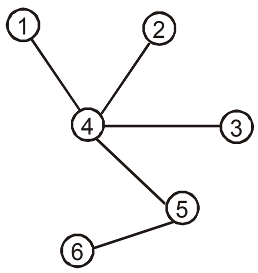
TREES.

The concept of a *tree* is most important in applications of graphs.

Tree is a connected graph without any circuits. The graph shown in the figure below is a tree. It follows immediately from the definition that a tree has to be a simple graph, i.e., having neither a self-loop nor parallel edges (because they both form circuits).



Trees appear in various instances like

(*i*) genealogy of a family is often represented by means of a tree.

(*ii*) river with its tributaries and sub-tributaries can also be represented by a tree.

(*iii*) sorting of mail according to zip code and sorting of punched cards are done according to a tree (called *decision tree* or *sorting tree*).

Properties of Trees

(1) There is one and only one path between every pair of vertices in a tree, T.

(2) A tree with n vertices has n-1 edges.

(3) Any connected graph with n vertices and n-1 edges is a tree.

(4) A graph is a tree if and only if it is minimally connected.

Therefore a graph with n vertices is called a tree if

(*i*) G is connected and is circuit less, or

(*ii*) G is connected and has n-1 edges, or

(*iii*) G is circuit less and has n-1 edges, or

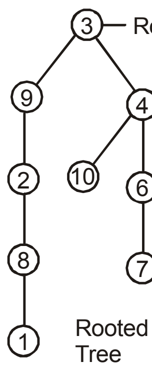
(*iv*) There is exactly one path between every pair of vertices in G, or

(*v*) G is a minimally connected graph.

Rooted and Binary Tree

Rooted tree.

A tree in which one vertex (called *root*) is distinguished from all the others is called *rooted tree*.
e.g., in the figure, vertex named *start*, is distinguished from rest of the vertices. Hence vertex *start* can be considered root of the tree, and so the tree is rooted.
Generally, the term *tree* means trees without any root. However, for emphasis they are sometimes called *free trees* (or *non rooted trees*) to differentiate them from the rooted kind.



Rooted Tree

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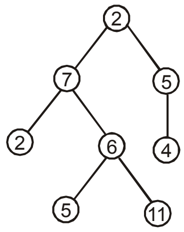
Binary Trees.

It is special class of rooted trees, which is of particular interest, since these are extensively used in the study of computer search methods, binary identification problems, and variable-length binary codes. It is defined as a tree in which there is exactly one vertex of degree two, and each of the remaining vertices of degree one or three. Since vertex of degree two is distinct from all other vertices, this vertex serves as a root. Thus every binary tree is a rooted tree.



Binary Tree

SPANNING TREES

A given graph has numerous subgraphs, from e edges, 2^e distinct combinations are possible. Obviously, some of these subgrphs will be trees. Out of these trees, certain types of trees are called *spanning trees*.

A tree T is said to be a *spanning tree* of a connected graph G if T is a subgraph of G and T contains all vertices of G. Since vertices of G are barely hanging together in a spanning tree, it is a sort of skeleton of the original graph G. Therefore a spanning tree is sometimes called *skeleton* or *scaffolding* of G. Since spanning trees are the largest trees among all trees in G, hence it is also called *maximal tree subgraph* or *maximal tree of* G.

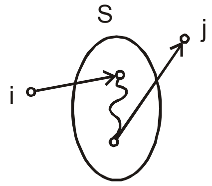
Finding a spanning tree of a connected graph G is simple. If G has no circuit, it is its own spanning tree. If G has a circuit, delete an edge from the circuit. This will still leave the graph connected. If there are more circuits, repeat the operation till an edge from the last circuit is deleted, leaving a connected, circuit-free graph that contains all the vertices of G.

3. All-Pairs Shortest Path Problem

Given a directed graph with edges (E), find shortest path for all (i, j) pairs.

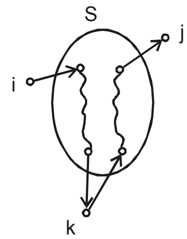
Note that "all (i, j) pairs" would indicate a matrix. That is what we generate - a matrix A such that A(i, j) is the length of the shortest path between i and j.

We allow negative lengths but no negative cycles.



A (i, j) = shortest path starting at i stopping at j passing only through nodes in S. (note i and j)

Each calculation would be *single-source shortest path*.



$$O(n^2) \Rightarrow O(n^3)$$

Starting with S = {} keep adding nodes to S.

Node k being added to S

$A^{k-1}(i,j)$ - before

$A^k(i,j)$ - after

If i to j is the shortest path and k is on this path, then i to k is smallest and k to j is smallest. Principle of optimality works

Input: A weighted graph, represented by its weight matrix W.

Problem: find the distance between every pair of nodes.

Dynamic Programming Design:

- *Notation:* $A^{(k)}(i, j)$ = length of the shortest path from node i to node j, where label of every intermediary node is $\leq$ k.

  $A^{(0)}(i, j) = W[i, j]$. (goes through no vertex - just (i, j) edge)
- *Principle of Optimality:* We already saw that any sub-path of a shortest path is a shortest path between its end nodes.

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- **Recurrence relation:**

  Divide the paths from i to j where every intermediary node is of label < k into two groups:

  (*i*) *Those paths that do not go through node k*

  The shortest path in this group is the shortest path from i to j

  where label of every intermediary node is $\leq$ k–1.

  Therefore, length of the shortest path of group = $A^{(k-1)}(i, j)$

  (*ii*) *Those paths that do go through node k.*

  Each path in this group consists of two portions:

  (*a*) From node i to node k,

  (*b*) from node k to node j.

  The shortest path in this group does not go through k more than once, for otherwise, the cycle around k can be eliminated, leading to a shorter path in this group.

  Therefore, two portions of the shortest path in this group have their intermediary labels $\leq$ k–1.

  Each portion must be shortest of its kind i.e., portion from i to k where intermediary node is $\leq$ k–1 must be the shortest such a path from i to k. If not, we would get a shorter path in this group. Same thing with the second portion (from j to k).

  Therefore, length of the first portion of the shortest path = $A^{(k-1)}(i, k)$

  and length of the second portion of the shortest path = $A^{(k-1)}(k, j)$

  Hence, length of the shortest path in this group = $A^{(k-1)}(i, k) + A^{(k-1)}(k, j)$

  Since shortest path in the two groups is the shorter of the shortest paths of the two groups, we get

  $$A^{(k)}(i, j) = \min(A^{(k-1)}(i, j), A^{(k-1)}(i, k) + A^{(k-1)}(k, j))$$

  Note : $A^{(k)}(i,k) = A^{(k-1)}(i, k)$

  $A^{(k)}(k,j) = A^{(k-1)}(k, j)$

  i.e., k^{th} row and column remain the same.

4. BACKTRACKING

Problems, which deal with searching a set of solutions, or which ask for an optimal solution satisfying some constraints can be solved using backtracking formulation. The backtracking algorithm yields the proper solution in fewer trials.

The basic idea of backtracking is to build up a vector one component at a time and to test whether the vector being formed has any chance of success. The main advantage of this algorithm is that if it is realized that the partial vector generated does not lead to an optimal solution, then that vector may be ignored.

Backtracking algorithm determine the solution by systematically searching the solution space for the given problem. This search is accomplished by using a free organization. Backtracking is a depth first search with some bounding function. All solutions using backtracking are required to satisfy a complex set of constraints. The constraints may be explicit or implicit.

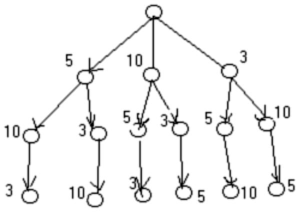
Explicit constraints are rules, which restrict each vector element to be chosen from the given set. Implicit constraints are rules, which determine which of the tuples in the solution space, actually satisfy the criterion function.

BRANCH AND BOUND

The term *branch and bound* refer to all state space search methods in which all possible branches are derived before any other node can become the E-node. In other words, exploration of a new node cannot begin until the current node is completely explored.

Tape filling

The branch and bound tree for the records of length (5,10,3) is as shown below.



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HASHING

It is a practical technique of maintaining a symbol table. Symbol table is a data structure which allows to easily determine whether an arbitrary element is present or not.

Consider a sequential memory shown below. In hashing technique, address X of a variable x is obtained by computing an arithmetic function (hashing function) f(x). Thus f(x) points to the address where x should be placed in the table. This address is called *hash address*.

The memory used to store the variable using hashing technique is assumed to be sequential. The memory is called *hash table*. The hash table is partitioned into several storing spaces called *buckets* and each bucket is divided into *slots*.

The address for an identifier X is obtained by computing f a arithmetic function on X

$$f(X) = \text{address of } X.$$

Memory available to store hash table is sequential. The hash table is partitioned into b buckets with each Bucket capable of holding S records.

$f(\mathrm{X})$ maps the identifier X into 0 to $(b - 1)$

Let n = actual numbers of identifiers, and

T = all possible valid identifiers.

Then $\left(\frac{n}{\mathrm{T}}\right)$ = identifer density

and $\alpha = \left(\frac{n}{\mathrm{S}_b}\right)$ = *loading density* or *loading factor*.

I_1 and I_2 are synonyms, if $f(\mathrm{I}_1) = f(\mathrm{I}_2)$

Overflow occurs when a new identifier X is mapped into a bucket with all slots (in number) are full. *Collision occurs* when two non identical identifiers are mapped into same bucket.

HASHING FUNCTIONS.

1. Uniform hashing function

It results in an unbiased used of hash table of random identifiers X' s.

i.e. $\mathrm{F}(x) = i = 1/b$ for all buckets i.

A random identifier X has an equal chance of hashing in any of the buckets.

Division $f(\mathrm{X}) = \mathrm{X} \text{ Mod M}$.

Bucket address in range 0 to $(n - 1)$ if M is a power of 2 f_{D} (X) depends on last significant bits of X.

e.g. Let $\mathrm{X} = x_1\, x_2$ and $\mathrm{Y} = x_2\, x_1$ be two identifiers with x_1 and x_2 as characters.

Let internal binary representation of x_1 be $\mathrm{C}\,(x_1)$

Let each character be represented by 6 bits.

Numeric value of $\mathrm{X} = 2^6\, [\mathrm{C}\,(x_1) + \mathrm{C}\,(x_2)]$

and numeric value of $\mathrm{Y} = 2^6\, [\mathrm{C}\,(x_2) + \mathrm{C}\,(x_1)]$

Let P be a prime number dividing M, then

$$(f_{\mathrm{D}}\,(\mathrm{X}) - f_{\mathrm{D}}\,(\mathrm{Y})) \bmod \mathrm{P} = (2^6\,((\mathrm{C}(\mathrm{x}_1) + \mathrm{C}\,(\mathrm{x}_2))) \bmod \mathrm{P} - (2^6\,((\mathrm{C}\,(x_2) + \mathrm{C}\,(x_1))) \bmod \mathrm{P}$$

For P = 3

$$(f_{\mathrm{D}}\,(\mathrm{X}) - f_{\mathrm{D}}\,(\mathrm{Y})) \bmod 3 = (64\,\mathrm{C}\,(x_1) \bmod 3 + 64\,\mathrm{C}\,(x_2) \bmod 3 - 64\,\mathrm{C}\,(x_2) \bmod 3 - 64\,\mathrm{C}\,(x_1) \bmod 3)$$

$$= \mathrm{C}\,(x_1) \bmod 3 + \mathrm{C}\,(x_2) \bmod 3 - \mathrm{C}\,(x_2) \bmod 3 - \mathrm{C}\,(x_1) \bmod 3 = 0 \bmod 3.$$

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Permutations of same set of characters are hashed at a distance of 3 apart. This results in a biased use of the table.

Choose M as a prime number then only divisors of M are M and 1.

$$M = (r^k \pm a)$$

where, r = radix of character set

k and a = small numbers

If M does not divide $(r^k \pm a)$ for small values of k and a.

2. Folding

Identifier X is partitioned into many parts all but last being of the same length.

These parts are then added together to give hash address of $\underline{X}$.

Methods of folding.

(*i*) *Shift folding*

P_1	P_2	P_3	P_4	P_5

P_1	123
P_2	203
P_3	241
P_4	112
P_5	20
	699

(*ii*) *Folding at boundaries*

P_1	123
P_2	302
P_3	241
P_4	211
P_5	20
	897

p_1^r = reverse of p_i.

3. Overflow handling

These techniques handle overflows in a hash table.

(*i*) Linear probing or linear open addressing.

The closest unfilled bucket is searched in case of a collision.

(1) Identifiers tend to cluster together

(2) Adjacent clusters tend to collapse

(3) More search time.

Expected average number of comparisons to look up an identifier

$$P \approx \left(\frac{2-\alpha}{2-2\alpha}\right)$$

where, α = loading density = $\frac{n}{S_b}$

(*ii*) Quadratic probing.

In linear search buckets $(f(x) + i) \bmod b \quad 0 \le i \le b - 1$ were searched.

In *chisadre* probing, a quadratic frunction of i is used as an increment.

Buckets examined are

$$f(x)\ (f(x) + i^2) \bmod b \text{ and } (f(x) - i^2) \bmod b \text{ for } 1 \le i \le \frac{(b-1)}{2}.$$

(*iii*) Rehashing .

Involves use a series of hashing functions $(f_1, f_2, \ldots f_n)$.

Buckets $f_i\,(x)$, $1 \le i \le m$ are examined in that order.

(*iv*) Chaining using linked lists.

Expected number of identifier comparisons, $P \approx \left(1+\frac{\alpha}{2}\right)$

where, α = loading density $\left(\alpha = \frac{n}{b}\right)$

n = number of headnodes.

Searching of an identifier involves comparison of identifiers with different values.

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EXERCISE – I

MCQ TYPE QUESTIONS

1. What is the output of the following 'C' program ?

```
main( )
{
    extern int i;
    i = 20;
    print ("%d", size of (i));
}
```

(*a*) 2
(*b*) 4
(*c*) would vary from compiler to compiler
(*d*) Error, *i* underfined

2. What is the output of the following 'C' program ?

```
main( )
    {
    int a[5] = {2, 3};
    printf ("\n%d %d %d", a[2], a[3], a[4]);
    }
```

(*a*) Garbage values (*b*) 2 3 3
(*c*) 3 2 2 (*d*) 0 0 0

3. What is the output of the following 'C' program ?

```
main( )
{
   struct emp
   {
        char name [20];
        int age;
        flat sal;
   };
   struct emp e = {"Tiger"};
   printf ("\n%d%f", e.age, e.sal);
}
```

(*a*) Error (*b*) Garbage values
(*c*) 0.000000 (*d*) 1 0.000000

4. In the following 'C' code, in which order the functions would be called ?

$a = (f1\ (23, 14) * f2\ (12/4)) + f3();$

(*a*) *f*1, *f*2, *f*3
(*b*) *f*3, *f*2, *f*1
(*c*) The order may vary from compiler to compiler
(*d*) None of these

5. What is the correct way to round off *x*, a float, to an int value ?

(*a*) $y = (\text{int})\ (x + 0.5)$
(*b*) $y = \text{int}\ (x + 0.5)$
(*c*) $y = (\text{int})\ x + 0.5$
(*d*) $y = (\text{int})\ (\text{int})\ x + 0.5)$

6. By default, any real number in 'C' is treated as

(*a*) a *float*
(*b*) a *double*
(*c*) a *long double*
(*d*) Depends upon memory model that you are using

7. What error would the following function given on compilation ?

```
f (int a, int b)
        {
            int a;
            a = 20;
            return a;
        }
```

(*a*) Missing parentheses is *return* statement
(*b*) Function should be defined as *int f (int a, int b)*
(*c*) Redeclaration of *a*
(*d*) None of these

8. If a file contains the line "I am a boy\r\n", then on reading this line into the array str using fgets (), what would str contain ?

(*a*) "I am a boy\r\n\0" (*b*) "I am a boy\r\0"
(*c*) "I am a boy\n\0" (*d*) "I am a boy"

9. To print out *a* and *b* given below, which printf() statement would you use ?

```
float a = 3.14;
double b = 3.14;
```

(*a*) printf ("%f%f", a, b);
(*b*) printf ("%Lf%f", a, b);
(*c*) printf ("%Lf%Lf", a, b);
(*d*) printf ("%f%Lf", a, b);

10. To scan a and b given below which scanf() statement would you use ?

```
flat a;
double b;
```

(*a*) scanf ("%f%f", &a, &b);
(*b*) scanf ("%Lf%Lf", &a, &b);
(*c*) scanf ("%f%Lf", &a, &b);
(*d*) scanf ("%f%1f", &a, &b);

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11. If following 'C' program (myprog) is run from the command line as myprog friday tuesday sunday, then what would be the output?

```
main (int argc, char*argv[])
{
    printf("%c", **++argv);
}
```

(*a*) *m* (*b*) *f*
(*c*) myprog (*d*) friday

12. Which of the following statements is incorrect ?
(*a*) C provides no input-output features
(*b*) C provides no file access features
(*c*) Provides no features to manipulate composite objects
(*d*) All of these

13. Integer division in a 'C' program results in
(*a*) truncation (*b*) rounding
(*c*) overflow (*d*) none of these

14. The value of an automatic variable that is declared but not initialized will be
(*a*) 0 (*b*) – 1
(*c*) garbage (*d*) none of these

15. For 'C' programming language,
(*a*) constant expressions are evaluated at compile time
(*b*) string constants can be concatenated at compile time
(*c*) size of array should be known at compile time
(*d*) all of these

16. In a 'C' expression involving | | operator, evaluation
(*a*) will be stopped if one of its components evaluates to false
(*b*) will be stopped if one of its components evaluates to true
(*c*) takes place from right to left
(*d*) takes place from left to right

17. Which of the following statements is (are) correct ?
(*a*) enum variables can be assigned new values
(*b*) enum variables can be compared
(*c*) enumeration feature does not increase the power of C
(*d*) all of these

18. Which of the following comments regarding the reading of a string, using scanf (with option) and gets is true ?
(*a*) Both can be used interchangeably
(*b*) scanf is delimited by end of line, while gets is not
(*c*) scanf is delimited by blank, while gets is not
(*d*) none of these

19. Output of the following program fragment (in C) is

```
for (i = 1; i < 5; i ++);
if (i = = 3) continue;
else printf ("%d", i);
```

(*a*) 1 2 4 5 (*b*) 1 2 4
(*c*) 2 4 5 (*d*) none of these

20. When a variable of data type double is converted into float, then
(*a*) rounding takes place
(*b*) truncation takes place
(*c*) the lower order bits are dropped
(*d*) none of these

21. For loop in a C program, if the condition is missing
(*a*) it is assumed to be present and taken to be false
(*b*) it is assumed to be present and taken to the true
(*c*) it result in a syntax error
(*d*) execution will be terminated abruptly

22. Which of the following statements about for loop are correct ?
(*a*) Index value is retained outside the loop
(*b*) Index value can be changed from within the loop
(*c*) Goto can be used to jump, out of the loop
(*d*) All of these

23. It is necessary to declare the type of a function in the calling program if the function
(*a*) returns an integer
(*b*) returns a non-integer value
(*c*) is not defined in the same file
(*d*) none of these

24. An external variable
(*a*) is globally accessible by all functions
(*b*) has a declaration "extern" associated with it when declared within a function
(*c*) will be initialized to 0 if not initialized
(*d*) all of these

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25. If following variables are set to the values as shown below, then what is the value of the expression following it?

answer = 2;
marks = 10;
! ((answer < 5) // (marks > 2))

(*a*) 1 (*b*) 0
(*c*) –1 (*d*) 2

26. In the following statement

*f*print*f*(fpt, "%*n*", *i*),

the variable fpt is a/an

(*a*) character variable
(*b*) arbitrarily assigned value
(*c*) pointer to a file
(*d*) special kind of variable called "file"

27. How many times will the following loop be executed if the input data item is 01234?

while (*c* = getchar ()! = 0) {
}

(*a*) Infinitely (*b*) never
(*c*) once (*d*) 5 times

28. The declaration "unsigned *u*" indicates *u* is a/an

(*a*) unsigned character (*b*) unsigned integer
(*c*) character (*d*) none of these

29. A declaration "short int" is used for variables

(*a*) which have a short duration in a program
(*b*) which have short names
(*c*) which may require less storage than normal integers
(*d*) all of these

30. The function *f*open ("filename","*r*") returns

(*a*) nothing
(*b*) a value 0 or 1 depending on whether the file could be opened or not.
(*c*) a pointer to FILE filename, if it exits
(*d*) a pointer to a new file after creating it.

31. The functions print*f* () works like print*f* (), but operates on

(*a*) data in a file (*b*) stderr
(*c*) stdin (*d*) string

32. Which of the following 'C' type is not a primitive data structure?

(*a*) int (*b*) float
(*c*) char (*d*) none of these

33. What will be the value returned by the following function, when it is called with 11?

```
recur (int num)
{
    if ((num/2) ! = 0) return (recur (num/2)
    *10+num%2);
    else return 1;
}
```

(*a*) Function does not return any value, because it goes into an infinite loop
(*b*) 11
(*c*) 1011
(*d*) None of these

34. The library function exit () causs an exit from

(*a*) the loop in which it occurs
(*b*) the block in which it occurs
(*c*) the function in which it occurs
(*d*) none of these

35. The getch () library function returns

(*a*) a character when any key is pressed
(*b*) a character when enter is pressed
(*c*) and displays a character on the screen when any key is pressed
(*d*) none of these

36. If integer needs two bytes of storage, then maximum value of a signed integer is

(*a*) $2^{16} - 1$ (*b*) $2^{15} - 1$
(*c*) 2^{16} (*d*) 2^{15}

37. Consider following statements :

putchar (getchar ( )) ;
purchar (getchar ( )) ;

If
a
b
is the input, then output will be

(*a*) an error message
(*b*) this can't be the input
(*c*) *ab*
(*d*) none of these

38. Consider following program fragment :

char *c* = '*a*' ;
while (*c*++ < = '*z*')
putchar (*xxx*) ;

If required output is abcd ..... *xyz*, then *xxx* should be

(*a*) *c* (*b*) $c - 2$
(*c*) $c - 1$ (*d*) $- - c$

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39. Printing a character as an integer
(*a*) results in the printing of a negative integer
(*b*) always prints a positive integer
(*c*) prints a value that is implementation dependent
(*d*) none of these

40. The program fragment
int i = 263 ;
putchar (i) ;
prints
(*a*) 263 (*b*) ASCII equivalent of 263
(*c*) rings the bell (*d*) garbage

41. Feature for accessing a variable through its address is desirable because
(*a*) call by reference is otherwise impossible
(*b*) call by value is otherwise impossible
(*c*) a function can return more than one value using this
(*d*) it can be used to cause side-effects.

42. int i = 5; is a statement in a C program.
(*a*) during execution, value of i may change but not its address
(*b*) during execution both the address and value may change
(*c*) repeated execution may result in different addresses for i
(*d*) i may not have an associated address

43. Prior to using a pointer variable it should be
(*a*) declared
(*b*) intialized
(*c*) both declared and initialized
(*d*) none of these

44. If statement
b = (int *) **c ;
is appended to the above program fragment, then
(*a*) value of b is unaffected
(*b*) value of b will be the address of c
(*c*) value of b becomes 5
(*d*) none of these

45. Consider the declaration :
char x [ ] = "WHATIZIT" ;
char *y = "WHATIZIT" ;
The output of puts (x) and puts (y) will be
(*a*) the same
(*b*) different
(*c*) not related
(*d*) none of these

46. Consider the declarations :
char first (int (*) (char, float)) ;
int second (char, float) ;
Which of the following function invocation is valid ?
(*a*) first (*second) ;
(*b*) first (&second) ;
(*c*) fIrst (second) ;
(*d*) none of these

47. Consider the declaration :
static struct {unsigned a : 5 ;
unsigned b : 5;
unsigned c : 5;
unsigned d : }v = (1, 2, 3, 4) ;
v occupies
(*a*) 4 words (*b*) 2 words
(*c*) 1 word (*d*) none of these

48. If the declaration unsigned c:5; is replaced by unsigned : 6; then
(*a*) it results in a syntax error
(*b*) it is meaningless
(*c*) compiler will give a new name for the field
(*d*) none of these

49. An object oriented language is used because
(*a*) own data types can be defined
(*b*) an object oriented program can be taught to correct its own erros
(*c*) it is easier to conceptualize an object
(*d*) both (*a*) and (*b*)

50. The value of ab if ab & $0 \times 3f$ equals 0×27 is
(*a*) 047 (*b*) $0 \times 0f$
(*c*) $0 \times f3$ (*d*) 0×27

51. In C a pointer variable to an integer can be created by the declaration
(*a*) int p* (*b*) int * p
(*c*) int – p ; (*d*) int $ p ;

52. To instantiate an object with the statement ifstream items ("C:inven.txt");, the file on the disk is identified by the name
(*a*) items (*b*) inven.txt
(*c*) either (*a*) or (*b*) (*d*) none of these

53. A function can make
(*a*) one throw
(*b*) one throw of each scale type
(*c*) one throw of each programmerdefined type
(*d*) as many throws of as many types as necessary.

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54. Function templates
(*a*) must have exactly one parameter
(*b*) may have more than one parameter as long as they are of the same type
(*c*) may have more than one parameters
(*d*) may not have parameters.

55. The function that is actually created from a call to a template function is called
(*a*) generated (*b*) inherited
(*c*) spawned (*d*) declassified.

56. A function that is prototype as int calculate (int num); may
(*a*) receive an integer variable named num from the main () program
(*b*) receive any integer variable from the main () program
(*c*) either (*a*) or (*b*)
(*d*) none of these

57. In the statement template << class T>>,
(*a*) T is a class (*b*) T is a scalr variable
(*c*) either (*a*) or (*b*) (*d*) none of these

58. A function that uses variable types is called
(*a*) overloaded (*b*) template function
(*c*) variable function (*d*) virtual function.

59. A function that is prototyped as double calculate (int num); may
(*a*) receive an integer constant such as 5
(*b*) receive an integer variable
(*c*) either (*a*) or (*b*)
(*d*) none of these

60. In C programming language, which of the following type of operators have the highest precedence
(*a*) relational operators
(*b*) equality operators
(*c*) logical operators
(*d*) arithmetic operators

61. In C programming language, which of the following operators has the highest precedence?
(*a*) unary + (*b*) *
(*c*) ≥ (*d*) = =

62. In C programming language, if the first and the second operands of operator + are of types int and float, respectively, the result will be of type
(*a*) int (*b*) float
(*c*) char (*d*) long int

63. What is the maximum number of dimensions an array in C may have ?
(*a*) Two
(*b*) Eight
(*c*) Sixteen
(*d*) Theoratically no limit. The only practical limits are memory size and compilers.

64. C programming language provides operations which deal directly with objects such as
(*a*) strings and sets
(*b*) lists and arrays
(*c*) characters, integers, and floating point numbers
(*d*) all of these

65. C programming language by itself provides
(*a*) input facility
(*b*) output facility
(*c*) both input and output facilities
(*d*) no input and output facilities

66. The variables which can be accessed by all modules in a program, are called
(*a*) local variables
(*b*) internal variables
(*c*) external variable
(*d*) global variables

67. In what kind of storage structure for strings, one can easily insert, delete, concatenate and rearrange substrings ?
(*a*) Fixed length storage structure
(*b*) Variable length storage with fixed maximum
(*c*) Linked list storage
(*d*) Array type storage

68. The time complexity of linear search algorithm over an array of n elements is
(*a*) $O(\log_2 n)$ (*b*) $O(n)$
(*c*) $O(n \log_2 n)$ (*d*) $O(n^2)$

69. The time required to search an element in a linked list of length n is
(*a*) $O(\log_2 n)$ (*b*) $O(n)$
(*c*) $O(1)$ (*d*) $O(n^2)$

70. The worst case time required to search a given element in a sorted linked list of length n is
(*a*) $O(1)$ (*b*) $O(\log_2 n)$
(*c*) $O(n)$ (*d*) $O(n \log_2 n)$

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71. Consider a linked list of n element which is pointed by an external pointer. What is the time taken to delete the element which is successor of the element pointed to by a given pointer ?

(*a*) O (1) (*b*) O ($\log_2 n$)
(*c*) O (n) (*d*) O ($n \log_2 n$)

72. Consider a linked list of n elements. What is the time taken to insert an element an after element pointed by some pointer ?

(*a*) O (1) (*b*) O ($\log_2 n$)
(*c*) O (n) (*d*) O ($n \log_2 n$)

73. Which of the following is a tabular listing of contents of certain registers and memory locations at different times during the execution of a program ?

(*a*) Loop program
(*b*) Program trace
(*c*) Subroutine program
(*d*) Byte sorting program

74. When key values are reals a similar data representation might be produced by using a hashing function with

(*a*) mod (*b*) div
(*c*) trunc (*d*) log N

75. Which of the following sorting algorithms does not have a worst case running time of O(n^2)?

(*a*) Insertion sort (*b*) Merge sort
(*c*) Quick sort (*d*) Bubble sort

76. For a linear search in an array of n elements the time complexity for best, worst and average case are...., ... and .... respectively

(*a*) O(n), O(1), and O(n/2)
(*b*) O(1), O(n) and O(n/2)
(*c*) O(1), O(n) and O(n)
(*d*) O/(1), O(n) and $\left(\frac{n-1}{2}\right)$

77. The average time required to perform a successful sequential search for an element in an array A(1 : n) is given by

(*a*) $(n + 1)/2$ (*b*) $\log_2{}^n$
(*c*) $n(n + 1)/2$ (*d*) n^2

78. Using the standard algorithm, what is the time required to determine that a number n is prime ?

(*a*) Linear time
(*b*) Logarithmic time
(*c*) Constant time
(*d*) Quadratic time

79. Following is a recursive function for computing the sum of integers from 0 to N.

```
function sum (N : integer) : integer
        begin
        if N = 0 then Sum = 0
        elese
end ;
```

The missing line in the else part is

(*a*) Sum : = N + Sum (N)
(*b*) Sum : = N + Sum (N – 1)
(*c*) Sum : = (N – 1) + Sum (N)
(*d*) Sum : = (N – 1) + Sum (N – 1)

80. The average time required to perform a successful sequential search for an element in an array A (1 : n) is given by

(*a*) $\frac{n+1}{2}$ (*b*) $\frac{n(n+1)}{2}$
(*c*) $\log_2 n$ (*d*) n^2

81. Running out of memory may occur due to

(*a*) non-recursive function call
(*b*) recursive function call
(*c*) use of more global variable
(*d*) none of these

82. A linear collection of data element the linear node is given by mean of pointer is called

(*a*) linked list (*b*) node list
(*c*) primitive list (*d*) none of these

83. Which of the following is most appropriate ?

(*a*) int * matrix; .....; free (void) matrix;
(*b*) int * matrix; .....; free (matrix);
(*c*) int * matrix; .....; if (matrix ! = null) free (void*) matrix
(*d*) None of these

84. Which of the following types of expressions does not require precedence rule for evaluated ?

(*a*) Full parenthesized infix expression
(*b*) Prefix expression
(*c*) Partially parenthesized infix expression
(*d*) More than one of these

85. If space occupied by null terminated string "S_1" and "S_2" in "c" are respectively "m" and "n", the space occupied by the string obtained by concatenating "S_1" and "S_2" is always

(*a*) less than $m + n$
(*b*) equal to $m + n$
(*c*) greater than $m + n$
(*d*) none of these

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86. Given 2 sorted list of size 'm' and 'n' respectively. Number of comparisons needed in the worst case by the merge sort algorithm will be

(*a*) mn (*b*) max (m, n)

(*c*) min (m, n) (*d*) $m + n - 1$

87. The order of an algorithm that finds whether a given boolean function of 'n' variables, produces 1 is

(*a*) constant (*b*) linear

(*c*) logarithmic (*d*) exponential

88. Which of the following is not primitive recursive but partially recursive ?

(*a*) Carnot's function (*b*) Ricmann function

(*c*) Bounded function (*d*) Ackermann's function

89. The following 'C' program

```
main( )
{
    unsigned int num;
    int i;
    scanf ("%u", &num);
    for (i = 0; i < 16; i++)
        printf ("%d", (num << i & 1 << 15)? 1 : 0);
}
```

(*a*) prints all even bits form *num*

(*b*) prints all odd bits from *num*

(*c*) prints binary equivalent of *num*

(*d*) none of these

90. Output of the following 'C' program is

```
main ()
{
    printf ("\n%x", – 1 >> 4);
}
```

(*a*) *ffff* (*b*) 0*fff*

(*c*) 0000 (*d*) *fff*0

91. The following 'C' program

```
{
    unsigned int num;
    int c = 0;
    scanf ("%u", &num);
    for (; num; num >> = 1)
    {
     if (num & 1)
            c++;
    }
    printf ("%d", c);
}
```

(*a*) counts the number of bits which are on in the number *num*

(*b*) sets all bits in the number *num* to 1

(*c*) sets bits in the number *num* to 0

(*d*) none of these

92. The following 'C' program

```
main ( )
{
    unsigned int m[ ] = {0   01, 0   02, 0   04, 0
                    08, 0   10, 0   20, 0   40, 0   80};
    unsigned char n, i;
    scanf ("%d", &n);
    for (i = 0; i < = 7; i++)
    {
    if  (n & m[i])
        printf ("\nyes");
    }
}
```

(*a*) putting off all bits which are on in the number *n*

(*b*) testing whether the individual bits of *n* are on or off

(*c*) would give an error

(*d*) none of these

93. In a 'C' program, constant is defined

(*a*) before main

(*b*) after main

(*c*) anywhere, but starting on a new line

(*d*) none of these

94. Printf ("%d" printf("tim"));

(*a*) result in a syntax error

(*b*) outputs tim3

(*c*) outputs garbage

(*d*) prints tim and terminates abruptly

95. If abc is the input, then following program fragment

```
char x, y, z;
printf ("%d", scanf ("%c%c%c", &x, &y, &z));
```

results in

(*a*) a syntax error

(*b*) a fatal error

(*c*) segmentation violation

(*d*) printing of 3

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96. The rule for implicit type conversion in 'C' is
(*a*) int < unsigned < float < double
(*b*) unsigned < int < float < double
(*c*) int < unsigned < double < float
(*d*) unsigned < int < double < float

97. Output of the following loop is
for (putchar('*c*'); putchar ('*a*'); putchar ('*r*'))
putchar ('*t*');
(*a*) a syntax error (*b*) cartrt
(*c*) catrat (*d*) catratratratrat...

98. Consider the following statement
#define hypotenuse (a, b) sqrt ($a*a + b*b$);
The macro-call hypotenuse ($a + 2$, $b + 3$);
(*a*) finds hypotenuse of triangle with sides $a + 2$ and $b + 3$
(*b*) finds square root of $(a + 2)^2 + (b + 3)^2$
(*c*) is meaningless
(*d*) finds square root of $3*a + 4*b + 5$.

99. Result of the execution of the following 'C' statements is
int *i* = 5;
do { putchar (*i* + 100); print*f* ("%*d*", *i*--); }
while (*i*);
(*a*) *i*5*h*4*g*3*f*2*e*1 (*b*) 14*h*3*g*2*f*1*e*0
(*c*) an error message (*d*) none of these

100. Result of the execution of the following 'C' program fragment is
int *i* = 107, *x* = 5;
printf ((*x* > 7)? "%*d*" : "%*c*:, *i*)
(*a*) an execution error
(*b*) a syntex error
(*c*) printing of *k*
(*d*) none of these

101. Output of the following 'C' fragment is
for (*i* =1, *j* = 10; *i* < 6; ++*i*, – –*j*)
print*f* ("%*d*%*d*", *i*,*j*);
(*a*) 1 10 2 9 3 8 4 7 5 6
(*b*) 1 2 3 4 5 10 9 8 7 6
(*c*) 1 1 1 1 1 9 9 9 9 9
(*d*) none of these

102. The declaration void function 1 (int) indicates the function 1 is a function which
(*a*) has no arguments (*b*) returns nothing
(*c*) both (*a*) and (*b*) (*d*) none of these

103. When a function is recursively called, all automatic variables
(*a*) are initialized during each execution of the function
(*b*) are retained from the last execution
(*c*) are maintained in a stack
(*d*) none of these

104. A static variable
(*a*) cannot be initialized
(*b*) is initialized once at the commencement of execution and cannot be changed at run time.
(*c*) retains its value throughout the file of the program
(*d*) is same as an automatic variable but is placed at the head of a program

105. A "switch" statement is used to
(*a*) switch between functions in a program
(*b*) switch from one variable to another variable
(*c*) to choose from multiple possibilities which may arise due to different values of a single variable
(*d*) to use switching variable

106. The statement
include < math.*h*>
is written at the top of a program to indicate
(*a*) beginning of the program
(*b*) program does heavy mathematical calculations
(*c*) that certain information about mathematical library functions are to be included at the beginning of the program
(*d*) none of these

107. The for statement which can precede a loop to be executed 50 times or till a boolean variable "found" becomes false is given by
(*a*) for (*i* = 0; *i* < = 50 | | found == false; *i*++)
(*b*) for (*i* = 0; *i* < 50 | | found == true; *i*++)
(*c*) for (*i* = 1; *i* < = 50 && found == true; *i*++)
(*d*) none of these

108. The declarations
typedef float hight [100];
height men, women;
(*a*) define men and women as 100 element floating point arrays
(*b*) define men and women as floating point variables
(*c*) define height, men and women as floating point variables
(*d*) are illegal

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109. The function *f*print*f* is used in a program

(*a*) when too many print*f* calls have been already used in the program

(*b*) in place of print*f*, since print*f* uses more memory

(*c*) when output is to be printed on to a file

(*d*) when type of the variables to be printed are not called apriori

110. In case of ordinary int variables

(*a*) leftmost bit is reserved for sign

(*b*) rightmost bit is reserved for sign

(*c*) no bit is reserved for sign

(*d*) none of these

111. The library function sqrt operates on a double precision argument. If, i is an integer variable, then which of the following calls would correctly compute sqrt (i)?

(*a*) sqrt ((double)i) (*b*) (double)sqrt (i)?

(*c*) (double) (sqrt (i)) (*d*) sqrt (i)

112. The function is lower (char)checks whether a character is in lower case or not. Therefore it should return

(*a*) 0 or 1

(*b*) –1, 0 or 1

(*c*) a character

(*d*) nothing

113. The function *f*open ("filename","*r*") returns

(*a*) nothing

(*b*) a value 0 or 1 depending on whether the file could be opened or not.

(*c*) a pointer to FILE filename, if it exits

(*d*) a pointer to a new file after creating it.

114. Expression ((fpt = *f*open ("Samples", "*w*")) = = NULL) would be true if

(*a*) the file sample does not exist while *f*open is being executed

(*b*) the file "sample"could not be created for writing

(*c*) fpt is not deeclared as a FILE pointer

(*d*) the file "sample"is read only

115. If a is a unsigned integer variable whose value is hx6db7, what is the value of ~a?

(*a*) $h11hhhh1$ (*b*) $hxhh1$

(*c*) $hx248$ (*d*) $hx9248$

116. The expression a << 6 shifts all bits of a six places to the left. If a 0x6db7, then what is the value of a << 6?

(*a*) 0xa72b (*b*) 0xa2b

(*c*) 0x6dc0 (*d*) 0x1111

117. The declaration

union *id* {
char color [12];
int size; } shirt, pant;

denotes shirt and pant are variable of type *id* and

(*a*) each can have a value of color and size

(*b*) each can represent either a 12-character color or a integer size at a time

(*c*) shirt and pant are same as struct variables

(*d*) variable shirt and pant cannot be used simultaneously in a statement.

118. Most appropriate sentence to describe unions is

(*a*) Unions are like structures

(*b*) Unions contain members of different data types which share the same storage area in memory

(*c*) Unions are less frequently used in program

(*d*) Unions are used for set operations

119. If space occupied by two strings s_1 and s_2 in 'C' are respectively m and n, then space occupied by string obtained by concatenating s_1 and s_2 is always

(*a*) less than $m + n$

(*b*) equal to $m + n$

(*c*) greater than $m+n$

(*d*) none of these

120. The expression 5 – 2 – 3 * 5 – 2 will evaluate to 18, if – is left associative and

(*a*) * has precedence over *

(*b*) * has precedence over –

(*c*) – has precedence over *

(*d*) – has precedence over –

121. If following program fragment (assume negative numbers are stored in 2's complement form)

unsigned i = 1;
int j = – 4 ;
printf ("%d", 8 * size of (int) ;

output is (lot in the answers are to the base two)

(*a*) an unpredictable value

(*b*) 8 * log (x + 3)

(*c*) log (x + 3)

(*d*) log 8 (x + 3)

122. Consider following program fragment

if (a > b) print f ("a > b") ;
else print f ("else part") ;
print f ("a < = b")

a < = b will be printed if

(*a*) $a > b$ (*b*) $a < b$

(*c*) $a = b$ (*d*) all of these

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123. If max is a function that returns the larger of the two integers, given as arguments, then which of the following statements finds the largest of three given numbers ?

(*a*) max (max (a, b), max (a, c))

(*b*) max (max (a, b), max (b, c))

(*c*) max (b, max (a, c))

(*d*) all of these

124. A pointer variable can be

(*a*) passed to a function as argument

(*b*) changed within a function

(*c*) returned by a function

(*d*) can be assigned an integer value

125. If x is an array of integer, then the value of &x [i] is same as

(*a*) & x [i – 1] + size of (int)

(*b*) x + size of (int) *i

(*c*) $x + i$

(*d*) none of these

126. If initialization is a part of delcaration of a structure, then storage class can be

(*a*) automatic

(*b*) register

(*c*) static

(*d*) anything

127. $a \to b$ is syntactically correct if

(*a*) a and b are structures

(*b*) a is a structure and b is a pointer to a structure

(*c*) a is a pointer to a structure and b is a structure

(*d*) a is a pointer to a structure in which b is a field

128. The statement

*f*seek (*fp*, 0L, 0) i – if syntactically correct, means

(*a*) *fp* is a file pointer

(*b*) position the read – wire–head at the start of the file

(*c*) position the read–write–head at the end of the file

(*d*) erase contents of the file

129. Which of the following is a collection of items into which items can be inserted arbitrarity and from which only the smallest item can be removed?

(*a*) Descending priority queue

(*b*) Ascending priority queue

(*c*) Fifo queue

(*d*) Lifo queue

130. Match List I with List II and select the correct answer from the codes given below the lists :

List I	**List II**
A. m = malloc (5); m = NULL	1. Using doing long pointers
B. Free (n) ; $n \to$ value = 5 ;	2. Using uninitialized pointers
C. char* P; * P = 'a' ;	3. Lost memory

Codes :

	A	**B**	**C**
(*a*)	1	2	3
(*b*)	3	1	2
(*c*)	3	2	1
(*d*)	2	3	1

131. A function can make

(*a*) one throw

(*b*) one throw of each scale type

(*c*) one throw of each programmerdefined type

(*d*) as many throws of as many types as necessary.

132. Which of the following abstract data types can be used to represent a many to many relation ?

(*a*) Tree, only (*b*) Plex, only

(*c*) Graph, only (*d*) Both (*b*) and (*c*)

133. The average search time of hashing, with linear probing will be less if the load factor

(*a*) is for less than one

(*b*) equals one

(*c*) is for greater than one

(*d*) none of these

134. argv is a/an

(*a*) array of character pointers

(*b*) pointer to an array of character pointers

(*c*) array of strings

(*d*) none of these

135. # define max (x, y)x = (x > y)? x : y

is a macro definition. Which can find the maximum of two numbers x and y if

(*a*) x and y are both integers

(*b*) x and y both declared as float

(*c*) x and y are both declared as double

(*d*) all of these

136. In the following declarations

```
typedef struct {
char name [20];
char middlename [5];
char surname [20];
    } NAME
NAME class [20];
```

class is

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(*a*) an array of 20 characters only

(*b*) an array of 20 names where each name consists of a name, middlename and surname

(*c*) a new type

(*d*) none of these

137. What would be the values assigned to a, b, c, if the statement

scanf ("%*d* %*d* %*d*", &*a*, &*b*, &*c*)

is extended with input data item 123456 ?

(*a*) $a = 12$, $b = 34$, $c = 56$

(*b*) $a = 1$, $b = 2$, $c = 3$

(*c*) $a = 123456$ and nothing is assigned to b and c

(*d*) a and b are not assigned anything, $c = 123456$

138. What would be the values assigned to a, b and c if the statement scanf

("%3*d*, %3*d*, %3*d*", &*a*, &*b*, &*c*)

is executed with input data as 1234*b*5678*b*9 (*b* denotes blank)?

(*a*) $a = 123$, $b = 4$, $c = 567$

(*b*) $a = 123$, $b = 567$, $c = 9$

(*c*) $a = 123$, $b = 456$, $c = 789$

(*d*) $a = 1234$, $b = 5678$, $c = 9$

139. What would be the values of i, x and c if

scanf ("%3*d*, %5*f*, %*c*", &*i*, &*x* &*c*)

is executed with input data 10b 256, 875*b*T?

(*a*) $i = 10$, $b = 56.875$, C =T

(*b*) $i = 100$ $b = 256.87$, C = T

(*c*) $i = 010$, $b = 256.87$, C = '5'

(*d*) $i = 10$, $b = 256.8$, C = '7'

140. If S is an array of 80 characters, then the value assigned to S through the statement scan*f* ("%*s*",S) with input 12345 would be

(*a*) "12345"

(*b*) nothing since 12345 is an integer

(*c*) S is an illegal name for a string

(*d*) %*s* cannot be used for reading in values of S.

141. The following declaration

enum color {black = –1, blue, green};

represents

(*a*) black = 0, blue = 1, green = 2

(*b*) color [1] = 'black'or color[2] = 'blue' or color [3] = 'green'

(*c*) color = 'black'or color = 'blue' or color = 'green'

(*d*) none of these

142. The following declaration

enum colors {black, blue, green};

represents

(*a*) black = –1, blue = 2, green = 3

(*b*) black = –1 blue = –2, green = –3

(*c*) black = –1, blue = 0, green = 1

(*d*) an illegal declaration

143. Size of the array need not be specified, when

(*a*) initialization is a part of definition

(*b*) it is a declaration

(*c*) it is a formal parameter

(*d*) all of these

144. The five items : A, B, C, D, and E are pushed in a stack, one after the other starting from A. The stack is popped four times and each element is inserted in a queue. Then two elements are deleted from the queue and pushed back on the stack. Now one item is popped from the stack. The popped item is

(*a*) A (*b*) B

(*c*) C (*d*) D

145. Which of the following statements is false ?

(*a*) Every tree is a bipartite graph

(*b*) A tree contains a cycle

(*c*) A tree with n nodes contains $n–1$ edges

(*d*) A tree is a connected graph

146. A one dimensional array A has indices 1...75. Each element is a string and takes up three memory works. The array is stored starting at location 1120 decimal. The starting address of A[49] is

(*a*) 1267 (*b*) 1164

(*c*) 1264 (*d*) 1169

147. The process of accessing data stored in a tape is similar to manipulating data on a

(*a*) stack (*b*) queue

(*c*) list (*d*) heap

148. Which of the following assertions is most strongly satisfied at the point marked {1} ?

(*a*) list [j] < list [j + 1] for all j such that item $\leq j < n$

(*b*) list [j] < list [j + 1] for all j such that item $< j \leq n$

(*c*) list [j] $\leq$ list [j + 1] for all j such that item $\leq j < n$

(*d*) list [j] < list [j – 1] for all j such that $1 < j \leq$ item

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149. Assume ILAST points to the last node in a singly linked list whose top (i.e. front) is TOPI. Assume TOP2 points to the first node in another singly linked list (which resides in the same physical arrays DATA and LINK). which statement will append list 2 to the end of list I (i.e. make them one linked list with list I's elements before list 2) ?

(*a*) TOP2 : = I LAST;
(*b*) I LAST : = TOP 2;
(*c*) LINK [TOP2] : = I LAST;
(*d*) LINK [I LAST] : = TOP 2;

150. If memory for the run-time stack is only 150 cells (words), how big can N be in Factorial (N) before encountring stack overflow ?

(*a*) 24 (*b*) 15
(*c*) 66 (*d*) 50

151. Which of the following is an illegal array definition ?

(*a*) type COLOGNE : (LIME, PINE, MUSK, MENTHOL);
var a : array [COLOGNE] of REAL;
(*b*) var a : array [REAL] of REAL;
(*c*) var a : array ['A' ..'Z'] of REAL;
(*d*) var a : array [BOOLEAN] of REAL;

152. Using Pop (SI, Item), Push (SI, Item), Read (Item), Print (Item), the variables SI (stack) and Item, and given the input file :

A, B, C, D, E, F < EOF >

Which stacks are possible :

	(*a*)		(*b*)	
	5	A	5	
	4	B	4	
	3	C	3	D
	2	D	2	A
	1	E	1	F

	(*c*)		(*d*)	
	5		5	
	4		4	
	3	F	3	C
	2	D	2	E
	1	B	1	B

153. Using the same functions and files in Problem 255, which of the below stacks are impossible

	(*a*)		(*b*)	
	5	A	5	
	4		4	D
	3	F	3	C
	2	E	2	F
	1	D	1	A

	(*c*)		(*d*)	
	5		5	
	4		4	
	3	F	3	F
	2	E	2	D
	1	C	1	B

154. Using Pop (S1, Item) Push (S1, Item), Getlist (Item), P_{op} (S2, Item), Push (S2, Item), and the variables S1, S2 (stacks with Top 1 and Top 2) and Item and given the input file :

A, B, C, D, E, F <EOF>

Which stacks are possible?

(*a*) All possible stacks with A, B, C, D, E and F
(*b*) No possible stacks with A, B, C, D, E and F
(*c*) Exactly and only those stacks which can be produced with S1 alone
(*d*) Twice as many stacks as can be produced with S1 alone

155. A postfix expression is merely the reverse of the prefix expression.

(*a*) True
(*b*) False

156. Which of the following types of programs would require the program data to be sorted in order for the programs to work correctly ?

(*a*) Programs that display department names in alphabetical order
(*b*) Programs that display sales amounts in numerical order
(*c*) Programs that display subtotals in reports
(*d*) All of these

157. Let f : {a, b}* $\rightarrow$ {a, b}* be given by f (n) = ax for every value of n$\in$ {a, b} then f is

(*a*) one to one not onto
(*b*) one to one and onto
(*c*) not one to one and not onto
(*d*) not one to one and onto

158. Which of the following is false ?

(*a*) A serial search begins with the first array element
(*b*) A serial search continues searching, element by element, either until a match is found or until the end of the array is encountered
(*c*) A serial search is useful when the amount of data that must be search is small
(*d*) For a serial search to work, the data in the array must be arranged in either alphabetical or numerical order

159. A —— search begins the search with the element that is located in the middle of the array.

(*a*) serial (*b*) random
(*c*) parallel (*d*) binary

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160. What can be said about the array representation of a circular queue when it contains only one element ?

(*a*) front = Rear = Null
(*b*) front = Rear + 1
(*c*) front = Rear – 1
(*d*) None of these

161. To sort many large object or structures, it would be most efficient to

(*a*) place reference to them in and array an sort the array
(*b*) place them in a linked list and sort the linked list
(*c*) place pointers to them in an array and sort the array
(*d*) place them in an array and sort the array

162. To sort many large objects or structures, it would be most efficient to place

(*a*) them in an array and sort the array
(*b*) pointers to them in an array and sort the array
(*c*) them in a linked list and sort the linked list
(*d*) references to them in an array and sort the array

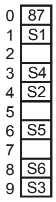
163. A matrix "*a*" is called lower triangular if and only if for all $j > i$, $a_{ij} = 0$. If such a matrix is to be sorted in a one dimensional array, A then a_{ij} could be mapped to which of the following index of A ?

(*a*) $½ * i\,(i + 1)\,j$ (*b*) $i + j$
(*c*) $i\,(i + 1) + j$ (*d*) none of these

164. Minimum number of comparison required to compute the largest and second largest element in an array is

(*a*) $n - [\log_2 n] - 2$ (*b*) $n + [\log_2 n - 2]$
(*c*) $\log_2 n$ (*d*) none of these

165. Following sequence of operations is performed on a stack push (1), push (2), pop, push (1), push (2), pop, pop, pop, push (2), pop. The sequence of popped out values are

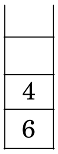


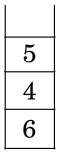
(*a*) 2, 2, 1, 1, 2
(*b*) 2, 2, 1, 2, 2
(*c*) 2, 1, 2, 2, 1
(*d*) 2, 1, 2, 2, 2

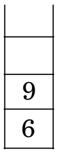
166. Average successful search time for sequential search on '*n*' items is

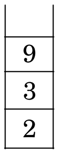
(*a*) $\frac{n}{2}$ (*b*) $\frac{(n-1)}{2}$
(*c*) $\frac{(n+1)}{2}$ (*d*) none of these

167. In evaluating the arithmetic expression 2*3 – (4 + 5), using stacks to evaluate its equivalent postfix form, which of the following stack configuration is not possible ?

(*a*)


(*b*)


(*c*)


(*d*)


168. Stack A has the entries *a, b, c* (with a on top). Stack B is empty. An entry popped out of stack A can be printed immediately or pushed to stack B. An entry popped out of the stack B can only be printed. In this arrangement, which of the following permutations of *a, b, c* are not possible?

(*a*) *b a c* (*b*) *b c a*
(*c*) *c a b* (*d*) *a b c*

169. The expression which accesses the (ij)th entry of a $m \times n$ matrix stored in column major form is

(*a*) $n \times (i - 1) + j$ (*b*) $m \times (j - 1) + i$
(*c*) $m \times (n - j) + j$ (*d*) $n \times (m - i) + j$

170. The average search time of hashing, with linear probing will be less if the load factor

(*a*) is far less than one
(*b*) equals one
(*c*) is far greater than one
(*d*) none of these

171. Stacks can't be used to

(*a*) evaluate an arithmetic expression in postfix form
(*b*) implement recursion
(*c*) convert a given arithmetic expression in infix form to its equivalent postfix form
(*d*) allocate resources (like CPU) by the operating system

172. The information about an array used in a program will be sorted in

(*a*) symbol table (*b*) activation record
(*c*) system table (*d*) dope vector

173. Which of the following remarks about Trie-indexing are true?

(*a*) It is an *m*-ary tree
(*b*) Successful searches should terminate in leaf nodes
(*c*) Unsuccessful searches may terminate in leaf nodes level of the tree structure
(*d*) All of these

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174. Which one the following permutations can be obtained in the output (in the same order), using a stack assuming that the input is the sequence 1, 2, 3, 4, 5 in that order ?

(*a*) 3, 4, 5, 1, 2 (*b*) 3, 4, 5, 2, 1
(*c*) 1, 5, 2, 3, 4 (*d*) 5, 4, 3, 2, 1

175. An advantage of chained hash table over the open addressing scheme is

(*a*) worst case complexity of search operation is less
(*b*) space used is less
(*c*) deletion is easier
(*d*) none of these

176. A priority queue is used to implement a stack S that stores characters PUSH (C) is implemented as INSERT (Q, C, K), where K is an appropriate integer key closen by the implementation. POP is implemented as DELETEMIN (Q). For a sequence of operations, the key chosen are in

(*a*) non-increasing order
(*b*) non-decreasing order
(*c*) strictly increasing order
(*d*) strictly decreasing order

177. An $n \times n$ array V defined as follows :

$v[i, j] = i - j$ for all i, j $1 < i \leq n$, $1 \leq j \leq n$

sum of the elements of the array v is

(*a*) 0 (*b*) $n - 1$
(*c*) $n^2 - 3n + 2$ (*d*) $\frac{n^2(n+1)}{2}$

178. When a new element is inserted in the middle of a linked list, then

(*a*) only elements that appear after the new element need to be moved.
(*b*) only elements that appear before the new element need to be moved.
(*c*) elements that appear before and after the new element need to be moved.
(*d*) none of these

179. If address of the 8th element in a linked list of integers is 1022, then address of 9th element is

(*a*) 1024 (*b*) 1026
(*c*) 1023 (*d*) none of these

180. Which of the following operations is performed more efficiently by doubly linked list than by linear linked list ?

(*a*) Deleting a node whose location is given
(*b*) Searching an unsorted list for a given item
(*c*) Inserting a node after the node with a given location
(*d*) Traversing the list to process each node.

181. Consider a linked list implementation of a queue with two pointers: front and rear. The time needed to insert element in a queue of length n is

(*a*) O (1) (*b*) O ($\log_2 n$)
(*c*) O (n) (*d*) O ($n \log_2 n$)

182. A linear list in which elements can be added or removed at either end but not in the middle is called

(*a*) queue (*b*) deque
(*c*) stack (*d*) tree

183. Queues serve a major role in

(*a*) simulation of recursion
(*b*) simulation of arbitrary linked list
(*c*) simulation of limited resource allocation
(*d*) expression evaluation

184. The linked list implementation of sparse matrices is superior to the generalized dope vector method because it is

(*a*) conceptually easier and completely dynamic
(*b*) efficient if the sparse matrix is a band matrix
(*c*) efficient in accessing an entry
(*d*) all of these

185. In a circularly linked list organization, insertion of a record involves the modification of

(*a*) no pointer (*b*) 1 pointer
(*c*) 2 pointers (*d*) 3 pointers

186. Stack is useful for implementing

(*a*) radix
(*b*) breadth first search
(*c*) recursion
(*d*) none of these

187. Which of the following is useful in traversing a given graph by breadth first search ?

(*a*) Stack (*b*) Set
(*c*) List (*d*) Queue

188. A queue can be defined abstractly according to the following rules where q is a queue, and X, Y, Z are symbols :

(1) e, the empty queue, is a queue
(2) Insert (q, X) is a queue
(3) Remove (Insert (e, X)) = e
Remove (Insert (Insert (q, Y), Z)) = Insert (Remove (Insert (q, Y), Z)
(4) Back (Insert (q, X)) = X
(5) Front (Insert (e, X)) = X
Front (Insert (Insert (q, Y), Z)) = Front (insert (q, Y))

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All of the following are derivable from these rules EXCEPT

(*a*) Front (Insert (Insert (*e*, Z), X), Y) = *i*

(*b*) Remove (Insert (Insert (*e*, Z), Y) = Insert (*e*, Y)

(*c*) Back (Insert (Insert (*e*, Y), Z)) = Z

(*d*) Insert (Insert (*e*, Z), X) is a queue

189. Which of the following lines of code will delete two successive nodes of a singly linked linear list (WITH MORE THAN 2 NODES) ?

Assume this code is in the main program, not a subprocedure?

(*a*) LINK [X] : = LINK [LINK [X]];

(*b*) X : = LINK [LINK [X]];

(*c*) LINK [LINK [X]] : = X;

(*d*) LINK [X]: = LINK [LINK [LINK [X]]];

190. Given two statements

(*i*) Insertion of an element should be done at the last node in a circular list.

(*ii*) Deletion of an element should be done at the last node of the circular list.

Codes :

(*a*) Both are True

(*b*) Both are False

(*c*) First is true and second is false

(*d*) First is false and second is true

191. To insert a node in a circular list at rear position, it should be inserted at —— of the queue

(*a*) Front position (*b*) Front – 1 position

(*c*) Rear position (*d*) Rear – 1 position.

192. To free which of the following list traversing through the entire list is not necessary ?

(*a*) Circular list (*b*) Singly linked list

(*c*) Double linked list (*d*) Both (*b*) and (*c*)

193. Which of the following statement(s) is/are true regarding insertion of node in a linear linked list?

(*a*) Setting the field of the new node means allocating memory to newly created node

(*b*) If node precedes all others in the list, then insert it at the front and return its address

(*c*) Creating a new node depends upon free memory space

(*d*) All of these

194. Which of the following statements are true about a doubly linked list ?

(*a*) It may be either linear or circular

(*b*) It must contain a header node

(*c*) It will occupy same memory space as that of linear linked list, both having same number of nodes

(*d*) All of these

195. Identity the steps to be taken when a first node is to be deleted from linear linked list.

I. Set link of start pointer to the second node in the list

II. Free the space associated with first node

III. Obtain the address of the second node in the list

IV. Count the number of nodes in the list.

Codes :

(*a*) I and II (*b*) I, II and III

(*c*) II and III (*d*) I, II, III and IV

196. Let "*q*" be the queue of integer defined as follows:

```
# define MAX-Q 500
    struct queue {
    int item [MAX-Q];
    int front, rear;
    } q;
```

To insert an element in the queue, we may write operation

(*a*) ++*q*. item [*q*. rear] = X;

(*b*) *q* . item [*q*. rear]++ = X;

(*c*) *q*. item [++ *q*. rear] = X;

(*d*) none of these

197. Which of the following data structure may give overflow error, even though the current number of element in it is less than its size ?

(*a*) Simple queue (*b*) Circular queue

(*c*) Stack (*d*) None of these

198. Number of "ADD" and "REMOVE" operations required to access $n/2$th element of a queue of "n" elements so that the original queue remain the same after the access is (take help of another queue.)

(*a*) $4 * n$ (*b*) $8 * n$

(*c*) $4 * n - 1$ (*d*) $8 * n - 1$

199. In which of the following cases, linked list implementation of sparse matrices consumes the same memory space as the conventional way of storing the entire array ?

(*a*) 5×6 matrix with 9 non-zero entries

(*b*) 5×6 matrix with 8 non-zero entries

(*c*) Efficient in accessing an entry

(*d*) Efficient if the sparse matrix is a band matrix

200. The process of accessing data stored in a tape is similar to manipulating data on a

(*a*) stack (*b*) queue

(*c*) list (*d*) heap

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201. The concatenation of two lists is to be performed in O(1) time. Which of the following implementations of a list should be used ?

(*a*) Singly linked list

(*b*) Doubly linked list

(*c*) Circular doubly linked list

(*d*) Array implementation of list

202. A binary tree in which if all its levels except possibly the last, have the maximum number of nodes and all the nodes at the last level appear as far left as possible, is called

(*a*) full binary tree (*b*) 2-tree

(*c*) threaded tree (*d*) complete binary tree

203. A list of integers is read in, one at a time, and a binary search tree is constructed. Next the tree is traversed and the integers are printed. Which traversed would result in a printout which duplicates the original order of the list of integers?

(*a*) Preorder (*b*) Postorder

(*c*) Inorder (*d*) None of these

204. If each node in a tree has value greater than every value in its left subtree and has value less than every value in its right subtree, the tree is called

(*a*) complete tree (*b*) full binary tree

(*c*) binary search tree (*d*) threaded tree

205. Which of the following sorting procedure is the slowest ?

(*a*) Quick sort (*b*) Heap sort

(*c*) Shell sort (*d*) Bubble sort

206. Which of the following sorting method is stable ?

(*a*) Straight insertion sort

(*b*) Binary insertion sort

(*c*) Shell sort

(*d*) Heap sort

207. A complete binary tree with the property that the value at each node is at least as large as the values at its children is called

(*a*) binary search tree

(*b*) AVL tree

(*c*) Completely balanced tree

(*d*) Heap

208. Which of the following best describes sorting ?

(*a*) Accessing and processing each record exactly ocne

(*b*) Finding the location of the record with a given key

(*c*) Arranging the data (record) in some given order

(*d*) Adding a new record to the data structure

209. A characteristic of the data that binary search uses but the linear search ignores, is

(*a*) order of the list

(*b*) length of the list

(*c*) maximum value in the list

(*d*) mean of data values

210. A sort which compares adjacent elements in a list and switches where necessary is

(*a*) insertion sort (*b*) heap sort

(*c*) quick sort (*d*) bubble sort

211.A sort which iteratively passes through a list to exchange the first element with any element less than it and then repeats with a new first element is called

(*a*) insertion sort

(*b*) selection sort

(*c*) heap sort

(*d*) quick sort

212. A full binary tree with n leaves contains

(*a*) n nodes (*b*) $\log_2 n$ nodes

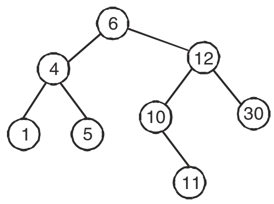
(*c*) $2n - 1$ (*d*) 2^n nodes

213. A full binary tree with n non-leaf nodes contains

(*a*) $\log_2 n$ nodes (*b*) $n + 1$ nodes

(*c*) $2n$ nodes (*d*) $2n + 1$ nodes

214. Consider the tree shown in the figure below.



If this tree is used for sorting, then a new number 8, should be placed as the

(*a*) left child of node labelled 30

(*b*) right child of node labelled 5

(*c*) right child of node labelled 30

(*d*) left child of node labelled 10

215. Consider Fibonacci tree. The depth of a node i in a binary tree is length of the path from the root node to i. Depth of the root node is 0(zero). If d is depth of a Fibonacci tree, and F_d is the d^{th} number in the Fibonacci sequence, then number of nodes in Fibonacci tree is given by

$$|F_d| = |F_{d-1}| + |F_{d-2}| + 1$$

$|F_d|$ is also given by the relationship

(*a*) $|F_d| = F_d$

(*b*) $|F_d| = F_d + d$

(*c*) $|F_d| = F_{d+3} - 1$

(*d*) $|F_d| = F_d * d$

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216. The average search time of hashing with linear probing will be less if the load factor

(*a*) is far less than one

(*b*) equals one

(*c*) is far greater than one

(*d*) none of these

217. Which of the following remarks about Trie Indexing is false ?

(*a*) It is an m-ary tree.

(*b*) It is a search tree of order m.

(*c*) Sucessful searches should terminate in leaf nodes

(*d*) Unsucessful scarches may terminate at any level of the tree structure.

218. A 3-array tree in which every internal node has exactly 3 children. The number of leaf nodes in such a tree with 6 internal nodes will be

(*a*) 10 (*b*) 11

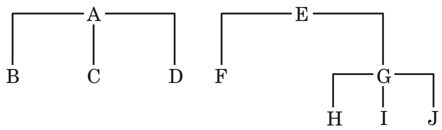
(*c*) 12 (*d*) 13

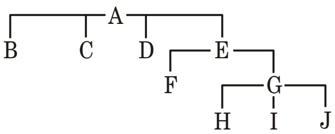
219. A binary tree T has n leaf nodes. The number of nodes of degree 2 in T is

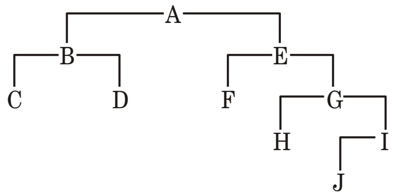
(*a*) $\log_2 n$ (*b*) $2n$

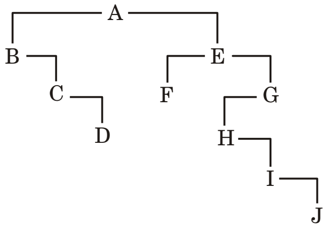
(*c*) n (*d*) 2^n

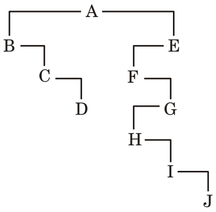
220. The forest (below) as a binary tree can be represented as :



(*a*)


(*b*)


(*c*)


(*d*)


221. If the binary search algorithm determines that the search argument is in the upper half of the array, which of the following statements will set the appropriate variable to the appropriate value?

(*a*) start Sub = middle Sub – 1;

(*b*) start Sub = middle Sub + 1;

(*c*) stop Sub = middle Sub – 1;

(*d*) stop Sub = middle Sub + 1;

222. Which of the following statements is used in the binary search algorithm to halve the array ?

(*a*) middle Sub = (start Sub + stop Sub)/2;

(*b*) middle Sub = start Sub + stop Sub/2;

(*c*) middle Sub = middle Sub/2;

(*d*) middle Sub = (stop Sub - start Sub)/2;

223. The data for which you are searching is called

(*a*) search argument (*b*) sorting argument

(*c*) delection argument (*d*) binary argument.

224. The maximum number of nodes on level i of a binary tree is

(*a*) $2^{i.1}$ (*b*) $3^{i.1}$

(*c*) $i+1$ (*d*) 2^{i+2}

225. The maximum number of nodes in a binary tree of depth k is

(*a*) 2^{k-1} (*b*) 3^{k-2}

(*c*) 2^{k-1} (*d*) $2^k + 2^x + 1$.

226. If the binary search algorithm determines that the search argument is in the lower half of the array, which of the following statements will set the appropriate variable to the appropriate value?

(*a*) start Sub = middle Sub – 1;

(*b*) start Sub = middle Sub + 1;

(*c*) stop Sub = middle Sub – 1;

(*d*) stop Sub = middle Sub + 1.

227. Which of the following is false ?

(*a*) A binary search begins with the middle element in the array

(*b*) A binary search continues having the array either until a match is found or until there are no more elements to search

(*c*) If the search argument is greater than the value located in the middle of the binary, the binary search continues in the lower half of the array

(*d*) For a binary search to work, the data in the array must be arranged in either alphabetical or numerical order.

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228. Identify the correct about a AVL tree

I. In this tree heights of two subtrees of every node never differ by more than 1.

II. Balance factor of each node is – 1, 0, 1.

III. Maximum height of a balanced binary search tree is 1.44 $\log_2{}^4$.

Codes :

(*a*) I and II (*b*) II and III

(*c*) I and III (*d*) All of these

229. Traversing a binary tree first root and then left and right subtrees called —— traversal.

(*a*) postorder (*b*) preorder

(*c*) inorder (*d*) none of these

230. A binary tree having n nodes and depth d will be about complete binary tree if

(*a*) any node *nd* at level less than *d*-1 has two sons

(*b*) it contains $2^{d+1} - 1$ nodes

(*c*) for any node *nd* in the tree with a right descendent at level *d*, *nd* must have a left son

(*d*) all of these

231. Which of the following statements are correct ?

I. If each tree node contains a father field, then it's not necessary to use either stack or threads

II. Traversal using father pointers is more time efficient than traversal of a threaded tree

III. A in-threaded binary tree is defined as binary tree that is both left-in threaded and right-in threaded.

Codes :

(*a*) II, and III (*b*) I and III

(*c*) All of these (*d*) None of these

232. A binary tree of depth "*d*" is an almost complete binary tree if

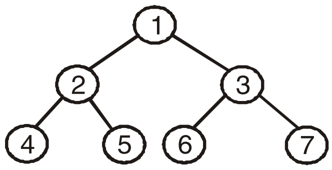
(*a*) each leaf in the tree is either at level "*d*" all or level ($d - 1$)

(*b*) for any node "*n*" in the tree with a right descendent at level "*d*", all the left descendents of "*n*" that are leaves are also at level "*d*".

(*c*) both (*a*) and (*b*)

(*d*) none of these

233. Consider the following tree :



If the post order traversal gives $ab - cd* +$ then the label of the nodes 1, 2, 3, ... will

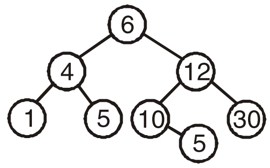
(*a*) +, –, *, *a, b, c, d*

(*b*) *a*, –, *b*, +, *c*, *, *d*

(*c*) *a*, *b*, *c*, *d*, –, *, +

(*d*) –, *a*, *b*, +, *, *c*, *d*

234. Consider the following tree :



If this tree is used for sorting, then a new number 8 should be placed as the

(*a*) left child of node labelled 30

(*b*) right child of node labelled 5

(*c*) right child of node labelled 30

(*d*) left child of node labelled 10

235. A binary tree in which every non-leaf node has non-empty left and right subtrees is called a strictly binary tree. Such a tree with 10 leaves

(*a*) cannot have more than 19 nodes

(*b*) has exactly 19 nodes

(*c*) has exactly 17 nodes

(*d*) cannot have more than 17 nodes

236. The number of possible comutative binary operations that can be defined on a set of *n* elements (for a given *n*) is

(*a*) 2 (*b*) *n*

(*c*) 2^n (*d*) n^2

237. A binary tree is generated by inserting in order the following integers :

50, 15, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24

The number of nodes in the left subtree of the root respectively is

(*a*) (4, 7) (*b*) (7, 4)

(*c*) (8, 3) (*d*) (3, 8)

238. Heap allocation is required for languages

(*a*) that support recursion

(*b*) that support dynamic data structure

(*c*) that use dynamic scope rules

(*d*) none of these

239. The number of binary relations on a set with n elements is

(*a*) n^2 (*b*) 2^n

(*c*) 2^{n^2} (*d*) None of these

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240. The number of binary strings of n zeroes and K ones such that no two ones are adjacent, is

(a) n H C_K (b) n C_K
(c) n C_{KH} (d) none of these

241. Consider the following nested representation of binary trees (xyz) indicates y and z are the left right subtrees, respectively, of node x. Note that y and z may be NULL or further nested. Which of the following represents a valid binary tree ?

(a) (1 2 (4 5 6 7)) (b) (1 ((2 3 4) 5 6) 7)
(c) (1 (234) (567)) (d) (1 (2 3 NULL) (4 5))

242. B+-trees are preferred to binary trees in data bases because

(a) disk capacities are greater than memory capacities.
(b) disk access is much slower than memory Access.
(c) disk data transfer rates are much less than memory data transfer rates.
(d) disks are more reliable than memory.

243. A full binary tree with n non-leaf nodes contains

(a) $\log_2 n$ nodes (b) $n + 1$ nodes
(c) $2n$ nodes (d) $2n + 1$ nodes

244. A full binary tree with n leaves contains

(a) n nodes (b) $\log_2 n$ nodes
(c) $2n - 1$ nodes (d) $2n$ nodes

245. A hash function randomly distributes records one by one in a space that can hold x number of records. The probability that the m^{th} record is the first record to result in collision is

(a) $\dfrac{(x-1)(x-2)\ldots(x-(m-2))(m-1)}{x^{m-1}}$

(b) $\dfrac{(x-1)(x-2)\ldots(x-(m-1))(m-1)}{x^{m-1}}$

(c) $\dfrac{(x-1)(x-2)\ldots(x-(m-2))(m-1)}{x^{m}}$

(d) $\dfrac{(x-1)(x-2)\ldots(x-(m-1))(m-1)}{x^{m}}$

246. Which decision procedure has at least doubly-exponential time complexity ?

(a) Linear programming
(b) Travelling Salesmen Problem
(c) Presburger Arithmetic
(d) Hamiltonian Circuit Problem

247. The running time T (n), where 'n' is the input size of a recursive algorithm is given

$$T(n) = c + T(n-1), \text{ if } n > 1$$
$$= d, \text{ if } n \le 1$$

The order of the algorithm is

(a) n^2 (b) n
(c) n^3 (d) n^n

248. Which of the following shows the correct relationship among some of the more common computing times for algorithms ?

(a) $O(\log n) < O(n) < O(n * \log n) < O(2^n) < O(n^2)$
(b) $O(n) < O(\log n) < O(n * \log n) < O(2^n) < O(n^2)$
(c) $O(n) < O(\log n) < O(n * \log n) < O(n^2) < O(2^n)$
(d) $O(\log n) < O(n) < O(n * \log n) < O(n^2) < O(2^n)$

249. $F(x) = (7x^6 + 3x^4 + 17x + 9) / ((0.01\ x^3 * x^{*-1})))$ is Big - O of WHAT ?

(a) F is $O(n^7)$ but not $O(n^6)$
(b) F is $O(n^6)$ but not $O(n^5)$
(c) F is $O(n^5)$ but not $O(n^4)$
(d) F is $O(n^4)$ but not $O(n^3)$

250. $f(n)$ is of the order of $g(n)$ if there exist positive integers "a" and "b" such that

(a) $f(n) <= a * g(n)$ for all $n >= b$
(b) $f(n) <= a * g(n)$ for all $n <= b$
(c) $g(n) = a * f(n)$ for all $n >= b$
(d) none of these

251. The running time of an algorithm T(n) where (n) is the input size is given by

$$T(n) = 8T(n/2) + qn, \quad \text{if } n > 1$$
$$p, \quad \text{if } n = 1$$

where p, q are constants.

The order of the algorithm is

(a) n^2 (b) n^n
(c) n^3 (d) n

252. The running time T(n), where (n) is the input size of a recursive algorithm is given as follows :

$$T(n) = C + T(n-1); \text{ if } n > 1$$
$$= d, \text{ if } n \le 1$$

The order of algorithm is

(a) n^2 (b) n
(c) n^3 (d) n^n

253. Consider following two functions

$$f(n) = n^3, \quad \text{if } 0 \le n < 10{,}000$$
$$n^2, \quad \text{otherwise}$$
$$g(n) = n, \quad \text{if } 0 \le n < 100$$
$$n^2 + 5n, \text{ otherwise}$$

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Which of the following are true?

(a) $f(n)$ is $O(n^3)$

(b) $g(n)$ is $O(n^3)$

(c) $O(f(n))$ is same as $O(g(n))$

(d) $g(n)$ is $O(n^2)$

254. The complexity of comparison based sorting algorithms is

(a) θ (n log n) (b) θ (n)

(c) θ (n 2) (d) θ (n A n)

255. The recurrence relation that arises in relation with the complexity of binary search is

(a) $T(n) = T(n/2) + K$

(b) $T(n) = 2T(n/2) + K$

(c) $T(n) = T(n/2) + \log n$

(d) $T(n) = T(n/2) + n$

where K is a constant

256. Consider the following two functions

$$g_1(n) = \begin{cases} n^3 & \text{for } 0 \le n < 10{,}000 \\ n^2 & \text{for } n \ge 10{,}000 \end{cases}$$

$$g_2(n) = \begin{cases} n & \text{for } 0 \le n < 100 \\ n^3 & \text{for } n > 100 \end{cases}$$

Which of the following is true ?

(a) $g_1(n)$ is $g_2(n)$) (b) $g_1(n)$ is $0(n^3)$

(c) $g_2(n)$ is $0(g_1(n))$ (d) $g_2(n)$ is $0(n)$

257. The recurrence relation

T (1) = 2

$T(n) = 3T(n/4) + n$

has the solution $T(n)$ equal to

(a) $0(n)$ (b) $0(\log n)$

(c) $0(n^{3/4})$ (d) none of these

258. Let $T(n)$ be the function defined by

$$T(1) = 1,\ T(n) = 2T([n/2]) + \sqrt{n} \ \text{ for } n \ge 2.$$

Which of the following statement is true ?

(a) $T(n) = 0(\sqrt{n})$ (b) $T(n) = 0(n)$

(c) $T(n) = 0(\log n)$ (d) None of these

259. When s be a sorted array of n integers, and $t(n)$ denote the time taken for the most efficient algorithm to determine if there are two elements with sum less than 1000 in s, then which of the following statements is true ?

(a) $t(n)$ is 0 (1)

(b) $n \le t(n) \le n \log_2 n$

(c) $n \log_2 n \le t(n) < \left(\frac{n}{2}\right)$

(d) $t(n) = \left(\frac{n}{2}\right)$

260. Consider the following functions

$f(n) = 3n^{\sqrt{n}}$

$g(n) = 2^{\sqrt{n}\log_2 n}$

$h(n) = \angle n$

Which of the following is true ?

(a) $h(n)$ is $O(F(n))$

(b) $h(n)$ is $O(g(n))$

(c) $g(n)$ is not $O(F(n))$

(d) $F(n)$ is $O(g(n))$

261. Let $f, g, h, k : N \to N$.

If $f = O(h)$ and $g = O(k)$, then

(a) $f + g = O(h + k)$ (b) $fg = O(hk)$

(c) both (a) and (b) (d) none of these

262. Let f is a total function from N to N and that $f = O(p)$ for some polynomial function p. When C and D are constants, then for every n

(a) $f(n) \le Cp(n) + D$ (b) $f(n) \ge Cp(n) + D$

(c) $f(n) \ge Dp(n) + C$ (d) $f(n) \le Dp(n) + C$

263. For any total computable function $f : N \to N$, there is a step-counting function g so that for every n

(a) $g(n) < f(n)$ (b) $g(n) > f(n)$

(c) $g(n) = f(n)$ (d) none of these

264. $\sum_{1 \le k \le n} O(k)$, where $O(n)$ stands for order n is

(a) $O(n)$ (b) $O(n^2)$

(c) $O(n^3)$ (d) $O(3n^2)$

265. Which of the following statements is true ?

I. As the number of entries in a hash table increases, the number of collisions increases

II. Recursive programs are efficient

III. The worst case complexity for Quicksort is $O(n^2)$

IV. Binary search using a linear linked list is efficient

Codes :

(a) I and II (b) II and III

(c) I and IV (d) I and III

266. The running time of an algorithm is given by $T(n) = T(n-1) + T(n-2) - T(n-3)$, if $n > 3$ n, otherwise.

The order is

(a) n (b) $\log n$

(c) n^n (d) n^2

267. What should be the relation between T (1), T (2)

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and T (3), so that above question gives an algorithm ?

(*a*) T (1) = T (2) = T (3)

(*b*) T (1) + T (3) = 2T (2)

(*c*) T (1) – T (3) = T (2)

(*d*) T (1) + T (2) = T (3)

268. Time complexity of an algorithm T (n), where n is the input size is given by

$$T(n) = T(n-1) + 1/n, \text{ if } n > 1$$
$$= 1, \text{ otherwise}$$

The order of this algorithm is

(*a*) log n (*b*) n

(*c*) n^2 (*d*) n^n

269. The concept of order (Big O) is important because

(*a*) it can be used to decide the best algorithm that solves a given problem

(*b*) it determines the maximum size of a problem that can be solved in a given given amount of time

(*c*) it is the lower bound of the growth rate of algorithm

(*d*) both (*a*) and (*b*)

270. The algorithm design technique used in the quick sort algorithm is

(*a*) Dynamic programming

(*b*) Backtracking

(*c*) Divide and conquer

(*d*) Greedy method

271. A hash table can store a maximum of 10 records. Currently there are records in locations 1, 3, 4, 7, 8, 9, 10. The probability of a new record going into location 2, with a hash function resolving collisions by linear probing is

(*a*) 0.6 (*b*) 0.1

(*c*) 0.2 (*d*) 0.5

272. Breadth-first traversal (BFS) is a method to traverse

(*a*) all successors of a visited node before any successors of any of those successors

(*b*) a single path of the graph as far it can go

(*c*) graph using shortest path

(*d*) none of these

273. Identify the correct statements about DFS traversal.

I. It can be used to determine whether a graph is acyclic or not.

II. It identify the connected component of an undirected graph.

III. Traverses a single path of the graph until it visits a node with no successor.

(*a*) I and III (*b*) II and III

(*c*) I and II (*d*) I, II and III

274. A machine took 200 sec to sort 200 names, using bubble sort. In 800 sec, it can approximately sort

(*a*) 400 names (*b*) 800 names

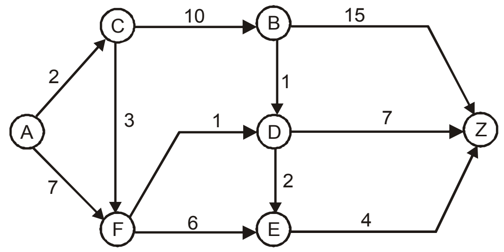
(*c*) 750 names (*d*) 900 names

275. For merging two sorted lists of sizes m and n into a sorted list of size $m + n$, we require comparisons of

(*a*) O (m) (*b*) O (n)

(*c*) O ($m + n$) (*d*) O (log n + log n)

276. Using best first search for a shortest path from A to Z, the order in which nodes are considered best for the path is



(Note that these are node orders, not full paths.)

(*a*) A < C < F < D < E (*b*) A < C < E < B

(*c*) A < C < F < E < B (*d*) A < C < D < F

277. Algorithm which solves the all-pair shortest path problem is

(*a*) Dijkstra's algorithm

(*b*) Floyd's algorithm

(*c*) Prim's algorithm

(*d*) Warshall's algorithm

278. The way a card game player arranges his cards as he picks them up one by one, is an example of

(*a*) bubble sort

(*b*) selection sort

(*c*) insertion sort

(*d*) merge sort

279. If there is an NP-complete language L whose complement is in NP, then complement of any language in NP is in

(*a*) P

(*b*) NP

(*c*) both (*a*) and (*b*)

(*d*) none of these

280. Which of the following is correct ?

(*a*) 3-Sat $\leq p$ CNF-Sat

(*b*) At the language level, *3-Satisfiable* $\leq p$ *CNF-Satisfiable*

(*c*) Both (*a*) and (*b*)

(*d*) None of these

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281. Which of the following is true ?

(*a*) If $k \geq 4$, the k-satisfiability problem is NP-complete.

(*b*) The general satisfiability problem, gives an arbitrary Boolean expression, not necessarily in conjunctive normal form, involving the variables $x_1, x_2, \ldots, x_v$, is it satisfiable ?
Is NP-complete?

(*c*) Consider language L of all strings $e(T)e(x)\,1^n$, where T is a nondeterministic Turing machine, $n \geq 1$, and T accepts x by some sequence of no more than n moves. Then the language L is NP-complete

(*d*) All of these

282. Which of the following decision problems are in P ?

(*a*) *DNF-Satisfiability :* Given a Boolean expression in disjunctive normal form (the disjunction of clauses, each of which is a conjunction of literals), is it satisfiable ?

(*b*) *CNF-Tautology :* Given a Boolean expression in CNF, is it a tautology (*i.e.*, satisfied by every possible truth assignment) ?

(*c*) The 2-colorability problem (Given a graph, is there a 2-coloring of the vertices ?)

(*d*) All of these

283. Let f be a function in PF, the set of functions from Σ^* to Σ^* computable in polynomial time. Let A (a language in Σ^*) be in P. Then f^{-1} (A) is in P, where by definition, $f^{-1}(A) = \{z \in \Sigma^* \mid f(z) \in A\}$.

(*a*) True (*b*) False
(*c*) Uncertain (*d*) None of these

284. Both P and NP are closed under the operation of

(*a*) union (*b*) intersection
(*c*) concatenation (*d*) Kleene

285. Which of the following is correct ?

(*a*) The 2-Satisfiability problem is in P.

(*b*) A *subexponential* function from N to N is one that is $O(2^{nc})$ for every positive real number c. If there is an NP-complete language in *Time*(f) for some subexponential function f, then NP $\subseteq$ *Subexp*, where

$$Subexp = \bigcup \{Time(f) \mid f \text{ is subexponential}\}$$

(*c*) Both (*a*) and (*b*)

(*d*) None of these

286. Which of the following decision problem is NP-complete ?

(*a*) Given a graph G in which every vertex has even degree, and an integer k, does G have a vertex cover with k vertices ? (The degree of a vertex is the number of edges containing it.) Hint: given an arbitrary graph G, find a way to modify it by adding three vertices so that all the vertices of the new graph have even degree.

(*b*) Give a finite set A, a collection C of subsets of A, and an integer k, is there a subset A_1 of A having k or fewer elements so that $A_1 \cap S \neq \theta$ for each S in the collection C ?

(*c*) Both (*a*) and (*b*)

(*d*) None of these

287. Then exact cover problem is the following: Given finite subsets $S_1, \ldots, S_k$ of a set, with $S_i = A$, is there a subset J of $\{1, 2, \ldots k\}$ so that for any two distinct elements i and j of J,

$S_i \cap S_j = \theta$ and $\bigcup_{i \in J} S_i = A$?

(*a*) NP-complete by constructing a reduction from the k-colorability problem.

(*b*) exact cover problem.

(*c*) sum-of-subset problem.

(*d*) None of these.

288. The following: Given a sequence $a_1, a_2, \ldots, a_n$ of integers, and an integer M, is there a subset J of $\{1, 2, \ldots, n\}$ so that $\sum_{i \in J} a_i = M$?

The sum-of-subsets problem is NP-complete by constructing a reduction from the

(*a*) k-colorability problem.

(*b*) exact cover problem.

(*c*) sum-of-subset problem.

(*d*) None of these.

289. Given a sequence $a_1, a_2, \ldots, a_n$ of integers, is there a subset J of $\{1, 2, \ldots, n\}$ so that

$$a_i = \sum i \notin J\, a^i \text{ ?}$$

The partition problem is NP-complete by constructing a reduction from the

(*a*) k-colorability problem.

(*b*) exact cover problem.

(*c*) sum-of-subset problem.

(*d*) None of these.

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290. The following C function takes a singly-linked list as input argument. Let modifies the list by moving the last element to the front of the list and returns the modified list. Some part of the code is left blank.

```
typedef struct node {
int value;
struct node * next;
3 node;
node * move_to_front (Node * head) {
Node * p, * q;
if (head = = NULL) II (head → next == NULL))
return head;
q = NULL; p = head;
white (p → next ! = NULL) {
q = p;
p = p → next;
}
        return head;
}
```

Choose the correct alternative to replace the blank link.

(*a*) q = NULL ; p → next = head; head = p;
(*b*) q → next = NULL; head = p; p → next = head;
(*c*) head = p; p → next = q; q → next = NULL;
(*d*) q → next = NULL; p → next = head; head = p;

291. Which of the given options provides the increasing order of asymptatic complexity of functions F_1, F_2, F_3 and F_4?

$$F_1(n) = 2^n,$$
$$F_2(n) = n^{3/2},$$
$$F_3(n) = n \log_2 n$$
$$F_4(n) = n \log_2 n$$

(*a*) F_3, F_2, F_4, F_1 (*b*) F_2, F_3, F_1, F_4
(*c*) F_3, F_2, F_1, F_4 (*d*) F_2, F_3, F_4, F_1

292. Which is correct matching pair?

Group-1	**Group-2**
A. Level-order traversal	1. Stack
B. BFS	2. Queue
C. Krushkal	3. Linked list
D. DFS	

Codes :

	A	B	C	D
(*a*)	2	2	2	1
(*b*)	3	2	2	1
(*c*)	2	3	2	1
(*d*)	2	2	2	3

293. The most appropriate matching for the following pairs is

X : depth first search	1 : heap
Y : breadth first search	2 : queue
Z : sorting	3 : stack

Codes :

	X	Y	Z
(*a*)	1	2	3
(*b*)	3	1	2
(*c*)	3	2	1
(*d*)	2	3	1

294. In which order the following numbers 7, 5, 1, 8, 3, and should be inserted into an empty binary search tree to get in order and preorder traversal sequence as same?

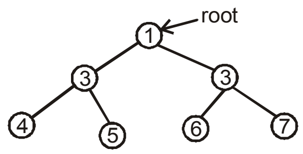
(*a*) 2, 3, 1, 5, 7, 8 (*b*) 3, 2, 1, 8, 7, 5
(*c*) 1, 2, 3, 5, 7, 8 (*d*) 1, 2, 3, 8, 7, 5

295. What would be the running time complexity of the efficient Algorithm that identify A given tree is BST or not?

(*a*) O (log$n$) (*b*) O(n^2)
(*c*) O (n log$n$) (*d*) O(n)

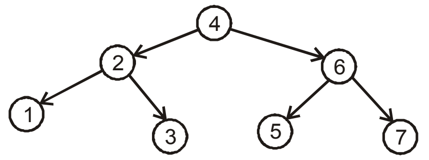
296. Let A (root) is invoked on the root of the following tree. After the function returns the resultant tree is traversed in order sequence.

Which of the following is the correct in-order sequence of the resultant tree?



(*a*) 5, 3, 1, 25, 7, 4, 49 (*b*) 4, 2, 5, 1, 6, 3, 7
(*c*) 5, 3, 25, 1, 7, 4, 49 (*d*) 2, 4, 5, 1, 6, 3, 7

297. Consider the following AVL tree :



Now try to insert the element 16, 15, 14, 13, 12, 11, 10, 8 and 9 in the order. How many single and double rotation operations are required to insert the above sequence into the AVL tree shown above ?

(*a*) 4 single rotation and 4 double rotations
(*b*) 3 single rotation and 4 double rotations
(*c*) 4 single rotation and 3 double rotations
(*d*) 3 single rotation and 2 double rotations.

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298. Consider the following C function which is passed a heap array whose starting location is '1'. Suppose the input heap having post order sequence as follows 4, 5, 2, 6, 7, 3, 1. Then what would printed by this function?

```
Void Foo (int heap { }, int root, int length)
{
    if (root < = length)
    {
    printf ("%d", a [root];
    foo (2* root);
    foo (2* root + 1);
    }
}
```

(*a*) 2, 1, 4, 5, 3, 6, 7 (*b*) 1, 2, 5, 3, 6, 7, 4

(*c*) 1, 2, 4, 5, 3, 6, 7 (*d*) 1, 2, 4, 5, 3, 7, 6

299. Consider a max heap whose inorder is {2, 8, 4, 14, 17, 16, 9, 10, 3}. Let new element '30' is inserted into it. Then which of the following statements is true with respect to the new state of max heap :

(*a*) The preorder of the resultant max heap is 2, 8, 4, 16, 1, 14, 7, 30,10, 3.

(*b*) The inorder of the resultant max heap 2, 8, 4, 16, 1, 14, 7, 30, 93, 10.

(*c*) The postorder of the resultant max heap is 2, 8, 4, 16, 1, 14, 7, 30, 9, 10.

(*d*) None of these

300. Which one of the following is a valid sequence of element in an array representing 3-ary max heap?

(*a*) 1, 3, 5, 6, 8, 9 (*b*) 9, 6, 3, 1, 8, 5

(*c*) 9, 3, 6, 8, 5, 1 (*d*) 9, 5, 6, 8, 3, 1

301. If array A is already sorted in ascending order, then the complexity of the following algorithms are.

Modified Bubble sort, Insertion sort, Quick sort are respectively.

(*a*) $O(n), O(n), O(n^2)$

(*b*) $O(n^2)$, $O(n \log_n)$, $O(n \log_n)$

(*c*) $O(n^2), (n), O(n^2)$

(*d*) $O(n^2), O(n \log_n), O(n^2)$

302. The median of n element can be found in Query time. Which one of the following is correct about the complexity of quick sort, in which median is selected as pivot?

(*a*) $O(n^2 \log_n)$ (*b*) $O(n \log_n)$

(*c*) $O(n)$ (*d*) $O(n^2)$

303. Consider the following algorithm

Algorithm PA(X)

Input : An n-element array X of number

Output : An n-element array A of number such that A[J] is the average of element X[0] ..... X[J]

```
1. For (i = 0; i < = n – 1 ; i + +) {
2. a = 0;
3. For (J = 0; J < = i; J ++) {
4. a = a + X [J];
5. A[J] = a/(i + 1);
6. }
7. }
8. return A
```

In the above statement '4' is executed how many number of times.

(*a*) $(n - 1)$ ^ 2 (*b*) n ^ 2

(*c*) $\frac{n(n-1)}{2}$ (*d*) $\frac{n(n-1)}{2}$

304. What is the no of swaps required to sort n elements using selection sort, in the worst case?

(*a*) $\Theta(n)$ (*b*) $\Theta(n \log_n)$

(*c*) $\Theta(n^2)$ (*d*) $\Theta(n^2 \log_n)$

305. The tightest lower bound on the number of comparisons, in the worst case, for comparisions based sorting is the order of

(*a*) n (*b*) n^2

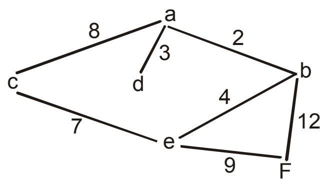
(*c*) $n \log_n$ (*d*) $n \log^2_n$

306. The usual $\Theta(n^2)$ implementation of insertion sort an array uses linear search to identify the position where an element is to be inserted into the already sorted port of the array. If instead we use binary search to identifying the position; the worst ease running time will

(*a*) Remain $\Theta(n^2)$ (*b*) Becomes $\Theta(n \log_n)^2)$

(*c*) Become $\Theta(n \log_n)$ (*d*) Become $\Theta(n)$

307. If an edge with weight 5 is to be inserted in the following simple graph to get 2 minimal spanning trees then where it should be inserted 1.



(*a*) df

(*b*) de

(*c*) ce

(*d*) Not possible to get two minimal trees.

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308. The space complexity of storing a graph in adjacency matrix of n vertices is

(*a*) O (1) (*b*) O (n^2)

(*c*) O (n) (*d*) O ($\log_n$)

309. Which one of the following choices gives a possible order in which the key values could have been inserted in the tablet.

(*a*) 46, 42, 34, 52, 23, 33

(*b*) 34, 42, 23, 52, 33, 46

(*c*) 46, 34, 42, 23, 52, 33

(*d*) 42, 46, 33, 23, 34, 52

NUMERICAL TYPE QUESTIONS

1. The output of the following 'C' program is ______

```
main( )
{
      extern int a;
      printf("\n%d", a);
}
int a = 20;
```

2. The output of the following 'C' program is ______

```
main( )
{
      printf("%f", sqrt(36.0));
}
```

3. If c is a variable initialised to 1, then_____ number of times will the following loop be executed

```
while ((c > 0). && (c < 60)) {
      loop body
       c ++;}
```

4. Following statement

```
printf("%f", 9/5) ;
```

prints_______

5. The value of x and y after execution of the following statement (C language) $n = = 5$; $x = n{+}{+}; y = -x$ is ______

6. The value of F(4) using the following procedure is ______

function F(k : integer) : integer;
begin
  if (k < 3)
then F : = k
else F : = F(k – 1) * F(k – 2) + F (k – 3)
end;

7. A hash table with 10 buckets with one slot per bucket is depicted. The symbols, S1 to S7 are initially emeged using a hashing function with linear probing. Maximum number of comparisons needed in searching an item that is not present is_____

8. The statement,

printf "%d", 10 ?0 ?5:1:12);

prints______

9. For x and y are variables as declared below

double $x = 0.005$, $y = -0.01$;

The value of ceil ($x + y$), where ceil is a function to compute ceiling of a number is _____

10. A short integer occupies 2 bytes, an ordinary integer 4 bytes and a long integer occupies 8 bytes of memory.

If a structure is defined as

```
struct TAB {
short a;
int b;
long c;
}TABLE [10];
```

then total memory requirement for TABLE is_____

11. If i,j,k are integer variable with values 1,2,3 respectively, then the value of the expression is _________

$!((j + k) > (i + 5))$

12. The C declaration

int b [100];

reserves _______ successive memory locations, each large enough to contain single integer.

13. Minimum number of interchange needed to convert the array 89, 19, 14, 40, 17, 12, 10, 2, 5, 7, 11, 6, 9, 70, into a heap with the maximum element at the root is _______

14. A hash function f defined as f (key) = key mod 7, with linear probing, insert the keys 37, 38, 72, 48, 98, 11, 56, into a table indexed from 11 will be stored in the location ______

15. A text is made up of the characters *a, b, c, d, e* each occurring with the probability 0.12, 0.4, 0.15, 0.08 and .25 respectively. The optimal coding technique will have the average length of_____

16. In the previous problem, if the stack A has 4 entries, then the number of possible permutations will be________

17. Suppose DATA array contains 1000000 elements. Using the binary search algorithm, one requires only about n comparisons to find the location of an item in the DATA array, then n is _____

18. The number of nodes in a complete binary tree of level 5 is

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19. The number of binary trees with 3 nodes which when traversed in post-order gives the sequence A, B, C is_______

20. The weight of the minimal spanning tree for the graph in problem 234 is _______

21. A complete binary tree of level 5 has ____ number of nodes

22. The smallest number of key that will force a B-tree of order 3 to have a height 3 is_____

23. Number of possible ordered trees with 3 nodes A, B, C is______

24. Average successful search time taken by binary search on a sorted array of 10 items is_____

25. Number of possible binary trees with 3 nodes is_____

26. A-2-3 tree is a tree such that

1. all internal nodes have either 2 or 3 children.
2. all paths from root to the leaves have the same length.

The number of internal nodes of a 2-3 tree having 9 leaves could be_______

27. The minimum number of inter changes needed to convert the array 89, 19, 40, 17, 12, 10, 2, 5, 7, 11, 6, 9, 70 into a heap with maximum element at the root is _______

28. A polynomial $p(x)$ is such that

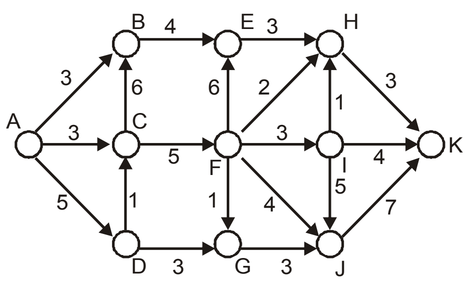
$p(0) = 5, p(1) = 4, \ p(2) = 9$ and $p(3) = 20$

The minimum degree it can have is_____

29. Consider a hashing function that resolves collision by quadratic probing. Assume the address space is indexed from 1 to 8. If a collision occurs at position 4, then the location which will never be probed is________

30. A hash table has space for 100 records. Then the probability of collision before the table is 10% full, is _______

31. Given the diagraph (shown below) whose edges are labelled by flow capacities. The maximum flow for this system from source A to sink K is ______. (All flows are left to right or verticalas shown).



32. A machine needs a minimum of 100 sec to sort 1000 names by quick sort. The minimum time needed to sort 100 names will be approximately_____ sec

33. What is the return value of the function f_{000}. When it is called as f_{00} (345, 10) ?

34. What is the value printed by the following C program ?

```
# include < studio.n >
intf (int * a, int n)
{
    if (n < = 0) return 0;
    else if (*a%2 = = 0) return * a + f (a + 1, n – 1);
    else return * a – f (a + 1, n – 1);
    }
    int main ( )
    {
    int a { } = {12, 7, 13, 4, 11, 6};
    printf (“%d”, F (a 6));
    return 0;
    }
```

35. What does the following program print ?

```
    # include < studio.n)
    void F (int * p, int * q)
    {
    p = q;
    * p = 2;
    }
    int i = 0, J = 1;
    int main ( )
    {
    f(& i, & j);
    printf(“%d%\ d% n”, i, J);
    return 0;
    }
```

36. Consider the polynomial $p(x) = a_0 + a_1x + a_2x^2 + a_3x^2$, where $a_i \neq 0 - \forall i$, the minimum number of multiplications needed to evaluate p on an input x is _______

37. Minimum numbers of queues required to implement a stack data structure is ______

38. The minimum numbers of stacks are required to implement STACK-MAX data structure are _____. This is a similar to stack data structure and thus will support all of stack operation and also supports find-max ( ) operation additionally. The running time of each should not be more.

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39. The following post fix expression with single digit operand is evaluated using a stack.

823 ^ / 23 * + 5 ⊥ * –

Note that ^ is the exponentiation operator. The top elements of the stack after the first * is evaluated are

40. The following keys are inserted into an empty BST. The number of nodes would the right subtree of a have after all insertions are_____

5, 6, 4, 1, 3, 8, 9, 10, 14, 2

41. The total calls that are made to function B ( ) when function A (root) completes its execution are ______

42. In a binary tree with n nodes, every node has an odd number of descendants. Every node is considered to be its own descendant. What is the number of nodes in the tree that have exactly one child?

43. In a binary tree, the number of internal nodes of degree 1 is 5 and the number of internal nodes of degree 2 is 10. The number of leaf nodes in the binary tree is

44. The number of distinct BSTs can be constructed with 3 distinct keys is _______

45. Consider ab$^+$ tree of order m which has 1001 leaf nodes.The number of keys does it have in the internal nodes is______

46. The smallest height of B-Tree of order 5 having 25 keys ______

47. The number of keys does it have in the internal nodes in a B-Tree of order m which has 100 leaf nodes is ______

48. The smallest number of keys that, when inserted in an appropriate order, will force a B-Tree of order 3 to have height _____

49. Consider a B$^+$ – tree in which the maximum number of keys in a node is 5. The minimum number of keys in any non-root node is _____

50. Consider a min Heap which has 4 leaf nodes. Then the number of maximum internal nodes is ______

51. Consider the following C program fragment that has a max heap array as input. Let input heap array is {16, 14, 10, 8, 7, 9, 3, 2, 4, 1} and suppose the root index is 1 then _____ would be the output sequence printed by this function if it runs on the given heap.

```
Void do something (int heap { }, int heap size)
    {
        int current = size;
        For (; current > 1; current / = 2
        {
        if((heap [current] & heap [current[ – 1) = = 0)
        printf (“%d”, heap [current]);
        }
    }
```

52. Minimum number of______ comparison operations required to find the smallest and largest element in an unsorted array of 10 elements.

53. The number of ________ different insertion sequence of the key values using the same hash function and linear probing will result in the hash table shown above.

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. The height of a tree is the length of the longest root-to-leaf path in it. The maximum and minimum number of nodes in a binary tree of height 5 are

(*a*) 63 and 6, respectively

(*b*) 64 and 5, respectively

(*c*) 32 and 6, respectively

(*d*) 31 and 5, respectively

2. Which of the following is/are correct inorder traversal sequence(s) of binary search tree(s)?

I. 3, 5, 7, 8, 15, 19, 25

II. 5, 8, 9, 12, 10, 15, 25

III. 2, 7, 10, 8, 14, 16, 20

IV. 4, 6, 7, 9, 18, 20, 25

(*a*) I and IV only (*b*) II and III only

(*c*) II and IV only (*d*) II only

3. Consider the following pseudo code, where *x* and *y* are positive integers.

```
begin
    q: = 0
    r : = x
    while r ≥ y do
    being
            r : = r – y
            q : = q + 1
    end
end
```

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The post condition that needs to be satisfied after the program terminates is

(*a*) $\{r = qx + y \wedge r < y\}$ (*b*) $\{x = qy + r \wedge r < y\}$

(*c*) $\{y = qx + r \wedge 0 < r < y\}$(*d*) $\{q + 1 < r - y \wedge y > 0\}$

4. What is the output of the following C code? Assume that the address of *x* is 2000 (in decimal) and an integer requires four bytes of memory ?

```
int main( )
{
        unsigened int x[4] [3] = {(1,2,3),
        {4,5,6}, {7, 8, 9}, {10, 11, 12}};
        printf("%u, %u, %u", x + 3, *(x + 3),
            *(x + 2) + 3);
}
```

(*a*) 2036, 2036, 2036 (*b*) 2012, 4, 2204

(*c*) 2036, 10, 10 (*d*) 2012, 4, 6

5. Consider the following C function.

```
int fun1 (int n)
{
int i, j, k, p, q = 0;
for (i = 1; i < n; ++i)
{
    p = 0;
    for(j = n; j > 1; j = j/2)
    ++p;
    for(k =1; k < p; k = k*2)
    ++q;
}
    return q;
}
```

Which one of the following most closely approximates the return value of the function fun1 ?

(*a*) n^3 (*b*) $n (\log n)^2$

(*c*) $n \log n$ (*d*) $n \log (\log n)$

6. Consider the following function written the C programming language.

```
void foo (char * a) {
    if (*a & & * a ! = ' ')
        putchar (*a);
    }
}
```

The output of the above function on input "ABCD EFGH" is

(*a*) ABCD EFGH

(*b*) ABCD

(*c*) HGFE DCBA

(*d*) DCBA

7. In a connected graph, a bridge is an edge whose removal disconnects a graph. Which one of the following statements is true?

(*a*) A tree has no bridges

(*b*) A bridge cannot be part of a simple cycle

(*c*) Every edge of a clique with size ≥ 3 is a bridge (A clique is any compete sub graph of a graph)

(*d*) A graph with bridges cannot have a cycle

8. Suppose you are provided with the following function declaration in the C programming language

int partition (int a [ ], int n);

The function treats the first element of a [ ] as a pivot, and rearranges the array so that all elements less than or equal to the pivot is in the left part of the array , and all elements greater than the pivot is in the right part. In addition, it moves the pivot so that the pivot is the last elements of the left part. The return value is the number of elements in the left part.

The following partially given function in the C programming language is used to find the K^{th} smallest element in an array a [ ] of size n using the partition function We assume $k \leq n$.

```
int kth _ smallest (int a [ ], int n, int k)
{
int left _ end partition (a, n);
if (left _ end + 1 == k){
    return a[left _ end];
}
if (left _ end + 1 > k){
    return kth _ smallest (_______);
} else {
return kth_smallest (_______);
}
}
```

The missing argument lists are respectively

(*a*) (a, left _ end,k) and (a + left _ end +1, n – left _ end – 1, k – left _ end – 1)

(*b*) (a, left _ end,k) and (a, n – left _ end –1, k – left _ end –1)

(*c*) (a + left _ end + 1, n – left end –1, k – left _ end – 1) and (a, left _ end, k)

(*d*) (a, n – left_ end – 1, k – left _ end – 1) and (a, left _ end,k)

9. A graph is self-complementary if it is isomorphic to its complement For all selfcomplementary graphs on *n* vertices, *n* is

(*a*) A multiple of 4

(*b*) Even

(*c*) Odd

(*d*) Congruent to 0 mod 4, or, 1 mod 4

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10. The secant method is used to find the root of an equation $f(x) = 0$. It is started from two distinct estimates, x_a and x_b for the root. It is an iterative procedure involving linear interpolation to a root. The iteration stops if $f(x_b)$ is very small and then x_b is the solution. The procedure is given below. Observe that there is an expression which is missing and is marked by? Which is the suitable expression that is to be put in place of ? so that it follows all steps of the secant method?

```
Secant
Initialize : xa, xb, ε, N     // ε = convergence indicator
                              // N = maximum no. of iterations
fb = –f(xb)
i = 0
While (i < N and |fb| > (ε)do
     i = i + 1                // update counter
     x1 = ?                   // missing expression for
     xa = xb                  // intermediate value
                              // rest xa
     xb = xt                  // rest xb
     fb = f(xb)               // function value at new xb
end while
if |fb| > ε then              // loop is terminated with
                                 i = N
write "Non-convergence"
else
     Write 'Non-convergence"
Else
Write "return xb"
End if
```

(*a*) $x_b - (f_b - f(x_a))\, f_b/(x_b - x_a)$
(*b*) $x_a - (f_b - f(x_a))\, f_a/(x_b - x_a)$
(*c*) $x_b - (x_b - x_a)\, f_b/(f_b - f(x_a))$
(*d*) $x_a - (x_b - x_a)\, f_a/(f_b - f(x_a))$

11. Consider the following C program segment.

```
#include < stdio.h >
int main ( )
{
    char sl [7] = "1234",*p;
    p sl + 2;
    *p = '0';
    print f ("%s",sl)
{
```

What will be printed by the program ?

(*a*) 12 (*b*) 120400
(*c*) 1204 (*d*) 1034

12. The result evaluating the postfix expression 105 + 60 6 / *8 – is

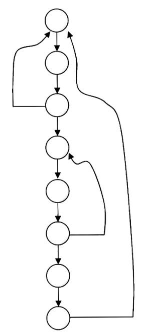
(*a*) 284 (*b*) 213
(*c*) 142 (*d*) 71

13. Consider three software items: Program – X, Control Flow Diagram of Program-Y and Control Flow Diagram of Program – Z as shown below

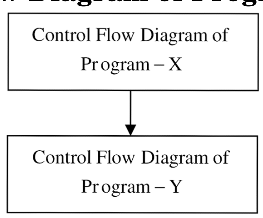
Program – X :

```
Sumcal(int max int, int value)
{
    int result 0, i = 0;
    if (value < 0)
    {
        value = – value;
    }
    while((i < value) AND (result
< = max int))
    {
        i = i + 1;
        result = result + 1;
    }
    if (result < = max int)
    {
        pr int f(result);
    }
    else
    {
        pr int f(" l arg e ");
    }
        pr int f(" end of program") '
}
```

Control Flow Diagram of Program – Y :



Control Flow Diagram of Program – Z :



The values of McCabe's Cyclomatic complexity of Program – X, Program-Y, and Program-Z respectively are

(*a*) 4, 4, 7 (*b*) 3, 4, 7
(*c*) 4, 4, 8 (*d*) 4, 3, 8

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14. Consider the following recursive C function.

```
Void get (int n)
{
    if (n < 1) return;
    get (n – 1)
    get (n – 3);
    print f("% d", n);
}
```

If get (6) function is being called in main () then how many times will the get () function be invoked before returning to the main ( ) ?

(*a*) 15 (*b*) 25

(*c*) 35 (*d*) 45

15. Consider the following two C code segments. Y and X are one and two dimensional arrays of size n and $n \times n$ respectively, where $2 \leq n \leq 10$. Assume that in both code segments, elements of Y are initialized to 0 and each element X[i] [j] of array X is initialized to $i + j$. Further assume that when stored in main memory all elements of X are in same memory page frame.

Code Segment 1:

```
//initialize elements of y to 0
//initialize elements X[i][j] of x to i + j
    for (i = 0; i < n; i++)
    Y[i] + = X[0][i];
```

Code Segment 2:

```
//initialize elements of Y to 0
//initialize elements X[i][j] of X to i + j
    for (i = 0; i < n; i++)
    Y[i] + = X[i][0];
```

Which of the following statements is/are correct?

S1 : Final contents of array Y will be same in both code segments

S2 : Elements of array X accessed inside the for loop shown in code segment 1 are contiguous in main memory

S3 : Elements of array X accessed inside the for loop shown in code segment 2 are contiguous in main memory

(*a*) Only S2 is correct

(*b*) Only S3 is correct

(*c*) Only S1 and S2 are correct

(*d*) Only S1 and S3 are correct

16. Match the following :

P : Prim's algorithm for minimum spanning tree

Q : Floyd – Warshal algorithm for all pairs shortest paths

R : Mergesort

S : Hamiltonian circuit

(I) Backtracking

(II) Greedy method

(III) Dynamic programming

(IV) Divide and conquer

(*a*) P-III, Q-II, R-IV, S-I

(*b*) P-I, Q-II, R-IV, S-III

(*c*) P-II, Q-III, R-IV, S-I

(*d*) P-II, Q-I, R-III, S-IV

17. Which one of the following is the recurrence equation for the worst case time complexity of the Quicksort algorithm for sorting n ($\geq$ 2) numbers ? In the recurrence equations given in the options below, c is a constant.

(*a*) $T(n) = 2T(9n/2) + cn$

(*b*) $T(n) = T(n - 1) + T(1) + cn$

(*c*) $T(n) = 2T(n - 1) + cn$

(*d*) $T(n) = T(n/2) + cn$

18. What are the worst-case complexities of insertion and deletion of a key in a binary search tree ?

(*a*) $\theta(\log n)$ for both insertion and deletion

(*b*) $\theta(n)$ for both insertion and deletion

(*c*) $\theta(n)$ for insertion and $\theta(\log n)$ for deletion

(*d*) $\theta(\log n)$ for insertion and $\theta(n)$ for deletion

19. Let G = (V, E) be a simple undirected graph, and s be a particular vertex in it called the source. For $x \in V$, let $d(x)$ denote the shortest distance in G from s to x. A breadth first search (BFS) is performed starting at s. Let T be resultant BFS tree. If (u, v) is an edge of G that is not in T, then which one of the following CANNOT be the value of $d(u) - d(v)$?

(*a*) – 1 (*b*) 0

(*c*) 1 (*d*) 2

20. Consider a max heap, represented by the array : 40, 30, 20, 10, 15, 16, 17, 8, 4.

Array Index	1	2	3	4	5	6	7	8	9
Value	40	30	20	10	15	16	17	8	4

Now consider that a value 35 is inserted into this heap. After insertion, the new heap is

(*a*) 40, 30, 20, 10, 15, 16, 17, 8, 4, 35

(*b*) 40, 35, 20, 10, 30, 16, 17, 8, 4, 15

(*c*) 40, 30, 20, 10, 35, 16, 17, 8, 4, 15

(*d*) 40, 35, 20, 10, 15, 16, 17, 8, 4, 30

21. An algorithm performs $(\log N)^{1/2}$ find operations, N insert operations, $(\log N)^{1/2}$ delete operations, and $(\log N)^{1/2}$ decreasekey operations on a set of data items with keys drawn form a linearly ordered set. For a delete operation, a pointer is

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provided to the record that must be deleted. For the decrease-key operation, a pointer is provided to the record that has its key decreased. Which one of the following data structures is the most suited for the algorithm to use, if the goal is to achieve the best asymptotic complexity considering all the operations?

(*a*) Unsorted array
(*b*) Min–heap
(*c*) Sorted array
(*d*) Sorted doubly linked list

22. An unordered list contain n distinct elements. The number of comparisons to find an element in this list that is neither maximum nor minimum is

(*a*) $\Theta(n \log n)$ (*b*) $\Theta(n)$
(*c*) $\Theta(\log n)$ (*d*) $\Theta(1)$

23. Consider a complete binary tree where the left and the right subtrees of the root are maxheaps. The lower bound for the number of operations to convert the tree to a heap is

(*a*) $\Omega(\log n)$ (*b*) $\Omega(n)$
(*c*) $\Omega(n \log n)$ (*d*) $\Omega(n^2)$

24. Consider two decision problems Q_1, Q_2 such that Q1 reduces in polynomial time to 3-SAT and 3 -SAT reduces in polynomial time to Q_2. Then which one of following is consistent with the above statement?

(*a*) Q_1 is in NP, Q_2 in NP hard
(*b*) Q_2 is in NP, Q_1 is NP hard
(*c*) Both Q_1 and Q_2 are in NP
(*d*) Both Q_1 and Q_2 are NP hard

25. Given below are some algorithms, and some algorithm design paradigms.

1. Dijkstra's Shortest Path
2. Floyd-Warshall algorithm to compute all pair shortest path
3. Binary search on a sorted array
4. Backtracking search on a graph

i. Divide and Conquer
ii. Dynamic Progamming
iii. Greedy design
iv. Depth-first search
v. Breadth-first search

Match the above algorithms on the left to the corresponding design paradigm they follow.

(*a*) 1 – i, 2 – iii, 3 – i, 4 – v
(*b*) 1 – iii, 2 – iii, 3 – i, 4 – v
(*c*) 1 – iii, 2 – ii, 3 – i, 4 – iv
(*d*) 1 – iii, 2 – ii, 3 – i, 4 – v

26. Which one of the following hash functions on integers will distribute keys most uniformly over 10 buckets numbered 0 to 9 for i ranging from 0 to 2020?

(*a*) $h(i) = i^2 \bmod 10$
(*b*) $h(i) = i^3 \bmod 10$
(*c*) $h(i) = (11 * i) \bmod 10$
(*d*) $h(i) = (12 * i) \bmod 10$

27. Consider the following array of elements.

< 89, 19, 50, 17, 12, 15, 2,5, 7, 11, 6, 9, 100 >

The minimum number of interchanges needed to convert it into a max-heap is

(*a*) 4 (*b*) 5
(*c*) 2 (*d*) 3

28. While inserting the elements 71, 65, 84, 69, 67, 83 in an empty binary search tree (BST) in the sequence shown, the element in the lowest level is

(*a*) 65 (*b*) 67
(*c*) 69 (*d*) 83

29. Consider the equality $\sum_{i=0}^{n} i^3 = X$ and the following choices for X

I. $\theta(n^4)$ II. $\theta(n^5)$
III. $\theta(n^5)$ IV. $\Omega(n^3)$

The equality above remains correct if X is replaced by

(*a*) only I
(*b*) Only II
(*c*) I or III or IV but not II
(*d*) II or III or IV but not I

30. Let $f(n) = n$ and $g(n) = n^{(1 + \sin n)}$, + where n is a positive integer. Which of the following statement is/are correct?

I. $f(n) = O(g(n))$
II. $f(n) = \Omega(g(n))$

(*a*) Only I
(*b*) Only II
(*c*) Both I and II
(*d*) Neither I nor II

31. Assume that a mergesort algorithm in the worst case takes 30 second for an input of size 64.

Which of the following most closely approximates the maximum input size of a problem that can be solved in 6 minutes?

(*a*) 256 (*b*) 512
(*c*) 1024 (*d*) 2048

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32. Language L_1 is polynomial time reducible to language L_2. Language L_3 is polynomial time reducible to L_2, which in turn is polynomial time reducible to language L_4. Which of the following is/are true?

I. if $L_4 \in P$, then $L_2 \in P$

II. if $L_1 \in P$ or $L_3 \in P$, then $L_2 \in P$

III. $L_1 \in P$, if and only if $L_3 \in P$

IV. if $L_4 \in P$, then $L_1 \in P$ and $L_3 \in P$

(*a*) II only (*b*) III only

(*c*) I and IV only (*d*) I only

2014

33. Let G = (V, E) be a directed graph where V is the set of vertices and E the set of edges. Then which one of the following graphs has the same strongly connected components as G?

(*a*) $G_1 = (V, E_1)$ where $E_1 = \{(u, v) \mid (u, v) \notin E\}$

(*b*) $G_2 = (V, E_2)$ where $E_2 = \{(u, v) \mid (v, u) \in$ New income $= 1.20 \times$ E}

(*c*) $G_3 = (V, E_3)$ where $E_3 = \{(u, v) \mid$ there is a path of length ≤ 2 from u to v in E}

(*d*) $G_4 = (V_4, E)$ where V_4 is the set of vertices in G which are not isolated

34. Consider the following program in C language:

```
# include < stdio.h >
main ( )
{
    int i;
    int * pi = & i;
    scanf ("%d", pi);
    print f ("%d \ n", i + 5);
}
```

Which one of the following statements is TRUE?

(*a*) Compilation fails.

(*b*) Execution results in a run-time error.

(*c*) On execution, the value printed is **5** more than the address of variable **i**.

(*d*) On execution, the value printed is **5** more than the integer value entered.

35. Consider the following C function in which size is the number of elements in the array E :

```
int MyX(int *E, unsigned int size)
{
    int Y = 0;
    int Z;
    int i, j, k;
    for(i = 0; i < size; i++)
      Y = Y + E[i];
    for (i = 0; i < size; i++)
    for (j = i; j < size; j++)
    {
      Z = 0;
      f    or (k = i; k <= j; k++)
           Z = Z + E[k];
         if (Z > Y)
           Y = Z;
    }
  return Y;
}
```

The value returned by the function MyX is the

(*a*) maximum possible sum of elements in any sub-array of array E.

(*b*) maximum element in any sub-array of array E.

(*c*) sum of the maximum elements in all possible sub-arrays of array E.

(*d*) the sum of all the elements in the array E.

36. Consider the following pseudo code. What is the total number of multiplications to be performed?

```
D = 2
  for i = 1 to n do
     for j = i to n do
       for k = j + 1 to n do
          D = D * 3
```

(*a*) Half of the product of the 3 consecutive integers

(*b*) One-third of the product of the 3 consecutive integers.

(*c*) One-sixth of the product of the 3 consecutive integers.

(*d*) None of the above.

37. An ordered n-tuple $(d_1, d_2, \ldots, d_n)$ with $d_1 \ge d_2 \ge \ldots \ge d_n$ is called *graphic* if there exists a simple undirected graph with n vertices having degrees $d_1, d_2, \ldots, d_n$ respectively. Which of the following 6-tuples is NOT graphic?

(*a*) (1, 1, 1, 1, 1, 1)

(*b*) (2, 2, 2, 2, 2, 2)

(*c*) (3, 3, 3, 1, 0, 0)

(*d*) (3, 2, 1, 1, 1, 0)

38. Suppose `n` and `p` are `unsigned int` variables in a C program. We wish to set p to ${}^nC_3$. If n is large, which one of the following statements is most likely to set p correctly?

(*a*) p = n * (n – 1) * (n – 2) / 6;

(*b*) p = n * (n – 1) / 2 * (n – 2) / 3;

(*c*) p = n * (n – 1) / 3 * (n – 2) / 2;

(*d*) p = n * (n – 1) * (n – 2) / 6.0;

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39. Consider the procedure below for the *Producer-Consumer* problem which uses semaphores:

```
semaphore n = 0;
semaphore s = 1;
void producer()
{
   while(true)
   {
    produce();
    semWait(s);
    addToBuffer();
    semSignal(s);
    semSignal(n);
   }
}
void consumer()
{
   while(true)
   {
    semWait(s);
    semWait(n);
    removeFromBuffer();
    semSignal(s);
    consume();
   }
}
```

Which one of the following is TRUE?

(*a*) The producer will be able to add an item to the buffer, but the consumer can never consume it.

(*b*) The consumer will remove no more than one item from the buffer.

(*c*) Deadlock occurs if the consumer succeeds in acquiring semaphore `s` when the buffer is empty.

(*d*) The starting value for the semaphore `n` must be 1 and not 0 for deadlock-free operation.

40. For a C program accessing `X[i][j][k]`, the following intermediate code is generated by a compiler. Assume that the size of an `integer` is 32 bits and the size of a `character` is 8 bits.

```
t0 = i * 1024
t1 = j * 32
t2 = k * 4
t3 = t1 + t0
t4 = t3 + t2
t5 = X[t4]
```

Which one of the following statements about the source code for the C program is CORRECT?

(*a*) X is declared as "`int X[32][32][8]`".

(*b*) X is declared as "`int X[4][1024][32]`".

(*c*) X is declared as "`char X[4][32][8]`".

(*d*) X is declared as "`char X[32][16][2]`".

41. Suppose a stack implementation supports an instruction REVERSE, which reverses the order of elements on the stack, in addition to the PUSH and POP instructions. Which one of the following statements is TRUE *with respect to this modified stack*?

(*a*) A queue cannot be implemented using this stack

(*b*) A queue can be implemented where ENQUEUE takes a single instruction and DEQUEUE takes a sequence of two instructions

(*c*) A queue can be implemented where ENQUEUE takes a sequence of three instructions and DEQUEUE takes a single instruction

(*d*) A queue can be implemented where both ENQUEUE and DEQUEUE take a single instruction each

42. Consider the C function given below.

```
int f(int j)
{
 static int i = 50;
 int k;
 if (i == j)
   {
     printf("something");
     k = f(i);
     return 0;
   }
 else return 0;
}
```

Which one of the following is TRUE?

(*a*) The function returns `0` for all values of `j`

(*b*) The function prints the string `something` for all values of `j`

(*c*) The function returns `0` when `j = 50`

(*d*) The function will exhaust the runtime stack or run into an infinite loop when `j = 50`

43. Consider the following combinational function block involving four Boolean variables x, y, a,b where x, a, b are inputs and *y* is the output.

```
f(x, y, a, b)
{
    if (x is 1) y = a;
    else y = b;
}
```

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Which one of the following digital logic blocks is the most suitable for implementing this function?

(a) Full adder

(b) Priority encoder

(c) Multiplexor

(d) Flip-flop

44. Let A be a square matrix size n × n. Consider the following pseudocode. What is the expected output?

```
C = 100;
for i = 1 to n do
for j = 1 to n do
{
    Temp = A[ i ] [ j ] + C ;
      A [ i ] [ j ] = A [ j ] [ i ] ;
    A [ j ] [ i ] = Temp – C ;
}
for i = 1 to n do
for j = 1 to n do
output (A[ i ] [ j ]);
```

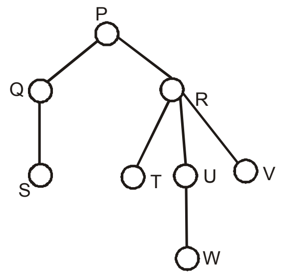
(a) The matrix A itself

(b) Transpose of the matrix A

(c) Adding 100 to the upper diagonal elements and subtracting 100 from lower diagonal elements of A

(d) None of these

45. Consider the following rooted tree with the vertex labeled P as the root :



The order in which the nodes are visited during an in-order traversal of the tree is

(a) SQPTRWUV (b) SQPTUWRV

(c) SQPTWUVR (d) SQPTRUWV

46. Consider the pseudocode given below. The function **Dosomething ( )** takes as argument a pointer to the root of an arbitrary tree represented by the l*eftMostChild-rightSibling* representation. Each node of the tree is of type treeNode.

```
typedef struct treeNode* treeptr;
Struct treeNode
{
    Treeptr leftMostchild, rightSibling;
};
```

```
Int Dosomething (treeptr tree)
 {
    int value =0;
    if (tree ! = NULL) {
       If (tree -> leftMostchild = = NULL)
       value = 1 ;
       else
         value = Dosomething (tree->leftMostchild);
           value = value + Dosometing (tree->rightsibling);
       }
       return (value);
}
```

When the pointer to the root of a tree is passed as the argument to DoSomething, the value returned by the function corresponds to the

(a) number of internal nodes in the tree.

(b) height of the tree.

(c) number of nodes without a right sibling in the tree.

(d) number of leaf nodes in the tree.

47. Consider the C function given below. Assume that the array list A contains n (> 0) elements, sorted in ascending order.

```
int ProcessArray (int * listA, int x, int n)
{
    int i, j, k;
    i = 0;
    j = n – 1;
    do {
           k = (i + j) /2;
               if (x < = listA [k])
                  j = k – 1;
               If (listA [k] < = x)
                  i = k + 1;
       } while (i < = j);
         If (listA [k] = = x)
         return (k) ;
         else
         return -1;
}
```

Which one of the following statements about the function ProcessArray is CORRECT?

(a) It will run into an infinite loop when x is not in listA.

(b) It is an implementation of binary search.

(c) It will always find the maximum element in listA.

(d) It will return – 1 even when x is present in listA.

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48. If G is a forest with n vertices and k connected components, how many edges does G have?

(*a*) $\lfloor n/k \rfloor$ (*b*) $\lceil n/k \rceil$

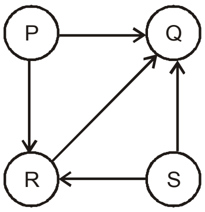
(*c*) $n - k$ (*d*) $n - k + 1$

49. Let G be a graph with n vertices and m edges. What is the tightest upper bound on the running time of Depth First Search on G, when G is represented as an adjacency matrix?

(*a*) $\Theta(n)$ (*b*) $\Theta(n + m)$

(*c*) $\Theta(n^2)$ (*d*) $\Theta(m^2)$

50. Consider the directed graph given below.



Which one of the following is TRUE?

(*a*) The graph does not have any topological ordering

(*b*) Both PQRS and SRQP are topological orderings

(*c*) Both PSRQ and SPRQ are topological orderings.

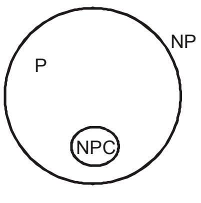
(*d*) PSRQ is the only topological ordering.

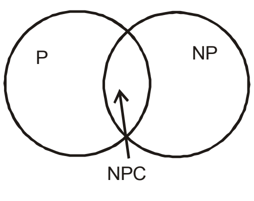
51. Let P be a quick sort program to sort numbers in ascending order using the first element as the pivot. Let t_1 and t_2 be the number of comparisons made by P for the inputs [1 2 3 4 5] and [4 1 5 3 2] respectively. Which one of the following holds?

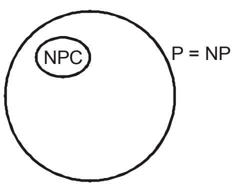
(*a*) $t_1 = 5$ (*b*) $t_1 < t_2$

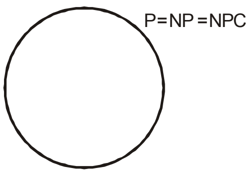
(*c*) $t_1 > t_2$ (*d*) $t_1 = t_2$

52. Suppose a polynomial time algorithm is discovered that correctly computes the largest clique in a given graph. In this scenario, which one of the following represents the correct Venn diagram of the complexity classes P, NP and NP Complete (NPC)?

(*a*)


(*b*)


(*c*)


(*d*)


53. A priority queue is implemented as a Max-Heap. Initially, it has 5 elements. The level-order traversal of the heap is: 10, 8, 5, 3, 2. Two new elements 1 and 7 are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is:

(*a*) 10, 8, 7, 3, 2, 1, 5 (*b*) 10, 8, 7, 2, 3, 1, 5

(*c*) 10, 8, 7, 1, 2, 3, 5 (*d*) 10, 8, 7, 5, 3, 2, 1

54. Which one of the following correctly determines the solution of the recurrence relation with T(1) = 1?

$$T(n) = 2T\left(\frac{n}{2}\right) + \log n$$

(*a*) $\Theta(n)$ (*b*) $\Theta(n \log n)$

(*c*) $\Theta(n^2)$ (*d*) $\Theta(\log n)$

55. Consider the tree arcs of a BFS traversal from a source node W in an unweighted, connected, undirected graph. The tree T formed by the tree arcs is a data structure for computing

(*a*) the shortest path between every pair of vertices

(*b*) the shortest path from W to every vertex in the graph

(*c*) the shortest paths from W to only those nodes that are leaves of T

(*d*) the longest path in the graph

56. You have an array of n elements. Suppose you implement quick sort by always choosing the central element of the array as the pivot. Then the tightest upper bound for the worst case performance is

(*a*) $0(n^2)$ (*b*) $0(n \log n)$

(*c*) $\Theta(n \log n)$ (*d*) $0(n^3)$

57. Consider a hash table with 100 slots. Collisions are resolved using chaining. Assuming simple uniform hashing, what is the probability that the first 3 slots are unfilled after the first 3 insertions?

(*a*) $(97 \times 97 \times 97)/100^3$ (*b*) $(99 \times 98 \times 97)/100^3$

(*c*) $(97 \times 96 \times 95)/100^3$ (*d*) $(97 \times 96 \times 95)/(3! \times 100^3)$

58. Let δ denote the minimum degree of a vertex in a graph. For all planar graphs on n vertices with $\delta \geq 3$, which one of the following is TRUE?

(*a*) In any planar embedding, the number of faces is at least $\frac{n}{2} + 2$

(*b*) In any planar embedding, the number of faces is less than $\frac{n}{2} + 2$

(*c*) There is a planar embedding in which the number of faces is less than $\frac{n}{2} + 2$

(*d*) There is a planar embedding in which the number of faces is at most $\frac{n}{\delta + 1}$

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2013

59. Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is $\frac{1}{2}$. What is the expected number of unordered cycles of length three?

(*a*) $\frac{1}{8}$ (*b*) 1
(*c*) 7 (*d*) 8

60. Which of the following statements is/are **TRUE** for undirected graphs?

P : Number of odd degree vertices is even.
Q : Sum of degrees of all vertices is even.

(*a*) P only (*b*) Q only
(*c*) Both P and Q (*d*) Neither P nor Q

61. Consider the following function:

```
int unknown(int n){
    int i, j, k = 0;
for (i = n/2; i < = n; i++)
for (j = 2; 'j< = n; j = j * 2)
    k = k + n/2;
return (k);
}
```

The return value of the function is

(*a*) $\Theta(n^2)$ (*b*) $\Theta(n^2 \log n)$
(*c*) $\Theta(n^3)$ (*d*) $\Theta(n^3 \log n)$

62. A certain computation generates two arrays *a* and *b* such that $a[i] = f(i)$ for $0 \le i < n$ and $b[i] = g(a[i])$ for $0 \le i < n$. Suppose this computation is decomposed into two concurrent processes: X and Y such that *X* computes the array *a* and Y computes the array *b*. The processes employ two binary semaphores *R* and *S*, both initialized to zero. The array *a* is shared by the two processes. The structures of the processes are shown below.

```
Process X:                  Process Y:
private i;                  private i;
for (i=0; i<n; i++) {       for (i=0; i<n; i++) {
   a[i] = f(i);                EntryY(R, S);
  ExitX(R, S);                 b[i] = g(a[i]);
}                           }
```

Which one of the following represents the CORRECT implementations of Exit X and EntryY?

```
(a) ExitX(R, S) {           (b) ExitX(R, S) {
       P(R);                       V(R);
       V(S);                       V(S);
    }                           }
    EntryY(R, S) {              EntryY(R, S) {
       P(S);                       P(R);
       V(R);                       P(S);
    }                           }
(c) ExitX(R, S) {           (d) ExitX(R, S) {
       P(S);                       V(R);
       V(R);                       P(S);
    }                           }
    EntryY(R, S) {              EntryY(R, S) {
       V(S);                       V(S);
       P(R);                       P(R);
    }                           }
```

63. What is the return value of $f(p, p)$, if the value of *p* is initialized to 5 before the call? Note that the first parameter is passed by reference, whereas the second parameter is passed by value.

```
int f (int &x, int c) {
   C = C – 1;
     if (c==0) return 1;
   x = x + 1;
     return f(x,c) * x;
   }
```

(*a*) 3024 (*b*) 6561
(*c*) 55440 (*d*) 161051

64. Which one of the following is the tightest upper bound that represents the number of swaps required to sort *n* numbers using selection sort?

(*a*) $O(\log n)$ (*b*) $O(n)$
(*c*) $O(n \log n)$ (*d*) $O(n^2)$

65. Which one of the following is the tightest upper bound that represents the time complexity of inserting an object into a binary search tree of *n* nodes?

(*a*) $O(1)$ (*b*) $O(\log n)$
(*c*) $O(n)$ (*d*) $O(n \log n)$

66. A scheduling algorithm assigns priority proportional to the waiting time of a process. Every process starts with priority zero (the lowest priority). The scheduler re-evaluates the process priorities every *T* time units and decides the next process to schedule. Which one of the following is **TRUE** if the processes have no I/O operations and all arrive at time zero?

(*a*) This algorithm is equivalent to the first-come-first-serve algorithm.
(*b*) This algorithm is equivalent to the round-robin algorithm.

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(*c*) This algorithm is equivalent to the shortest-job-first algorithm.

(*d*) This algorithm is equivalent to the shortest-remaining-time-first algorithm.

67. Which of the following statements are TRUE?

1. The problem of determining whether there exists a cycle in an undirected graph is in P.
2. The problem of determining whether there exists a cycle in an undirected graph is in NP.
3. If a problem A is NP-Complete, there exists a non-deterministic polynomial time algorithm to solve A.

(*a*) 1, 2 and 3 (*b*) 1 and 2 only

(*c*) 2 and 3 only (*d*) 1 and 3 only

68. What is the time complexity of Bellman-Ford single-source shortest path algorithm on a complete graph of n vertices?

(*a*) $\Theta(n^2)$ (*b*) $\Theta(n^2 \log n)$

(*c*) $\Theta(n^3)$ (*d*) $\Theta(n^3 \log n)$

69. The line graph $L(G)$ of a simple graph G is defined as follows:

- There is exactly one vertex $v(e)$ in $L(G)$ for each edge e in G.
- For any two edges e and e' in G, $L(G)$ has an edge between $v(e)$ and $v(e')$, if and only if e and e' are incident with the same vertex in G.

Which of the following statements is/are **TRUE**?

(P) The line graph of a cycle is a cycle.

(Q) The line graph of a clique is a clique.

(R) The line graph of a planar graph is planar.

(S) The line graph of a tree is a tree.

(*a*) P only (*b*) P and R only

(*c*) R only (*d*) P, Q and S only

70. The number of elements that can be sorted in $\Theta(\log n)$ time using heap sort is

(*a*) $\Theta(1)$ (*b*) $\Theta\left(\sqrt{\log n}\right)$

(*c*) $\Theta\left(\frac{\log n}{\log \log n}\right)$ (*d*) $\Theta(\log n)$

71. Consider the following operation along with Enqueue and Dequeue operations on queues, where k is a global parameter.

```
MultiDequeue(Q){
        m = k
    while (Q is not empty) and (m > 0) {
            Dequeue(Q)
            m = m - 1
    }
}
```

What is the worst case time complexity of a sequence of n queue operations on an initially empty queue?

(*a*) $\Theta(n)$ (*b*) $\Theta(n + k)$

(*c*) $\Theta(nk)$ (*d*) $\Theta(n^2)$

2012

72. Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph. then the number of bounded faces in any embedding of G on the plane is equal to

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

73. What will be the output of the following C program segment?

```
char inChar = 'A' ;
switch (inChar) {
case 'A' : printf ("Choice A\n");
case 'B' :
case 'C' : print f ("Choice B");
case 'D' :
case 'E' :
default : printf ("No Choice"); }
```

(*a*) No Choice

(*b*) Choice A

(*c*) Choice A, Choice B No Choice

(*d*) Program gives no output as it is erroneous

74. The height of a tree is defined as the number of edges on the longest path in the tree. The function shown in the pseudocode below is invoked as height (root) to compute the height of a binary tree rooted at the tree pointer root.

```
int height (treeptr n)
{ if (n = = NULL) return –1;
    if (n → left = = NULL)
        if (n → right = = NULL) return 0;
        else return [B1] ;          // Box 1
  else { h1 = height (n → left);
        if (n → right = = NULL) return (1 + h1);
        else {h2 = height (n → right);
          return [B2];            // Box 2
          }
        }
}
```

The appropriate expressions for the two boxes B1 and B2 are

(*a*) B1 : (1 + height (n → right)

B2: (1 + max(h1, h2)

(*b*) B1: (height(n → right)), B2: (1 + max(h1, h2))

(*c*) B1: height(n → right), B2: max(h1, h2)

(*d*) B1: (1 + height(n → right)), B2: max(h1, h2)

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75. Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to

(*a*) 15 (*b*) 30

(*c*) 90 (*d*) 360

76. Let G be a weighted graph with edge weights greater than one and G′ be the graph constructed by squaring the weights of edges in G. Let T and T′ be the minimum spanning trees of G and G′, respectively, with total weights t and t′. Which of the following statements is **TRUE**?

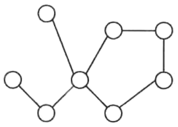
(*a*) T′ = T with total weight $t' = t^2$

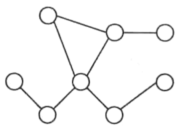
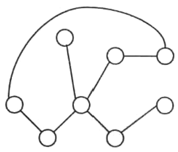
(*b*) T′ = T with total weight $t' < t^2$

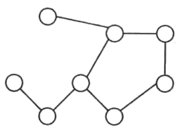
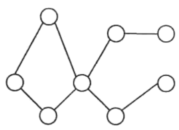
(*c*) T′ ≠ T but total weight $t' = t^2$

(*d*) None of the above

77. Which of the following graphs is isomorphic to



(*a*)  (*b*) 

(*c*)  (*d*) 

78. Let W(*n*) and A(*n*) denote respectively, the worst case and average case running time of an algorithm executed on an input of size *n*. Which of the following is ALWAYS TRUE?

(*a*) $A(n) = \Omega(W(n))$

(*b*) $A(n) = \Theta(W(n))$

(*c*) $A(n) = O(W(n))$

(*d*) $A(n) = o(W(n))$

79. The recurrence relation capturing the optimal execution time of the *Towers of Hanoi* problem with *n* discs is

(*a*) $T(n) = 2T(n-2) + 2$

(*b*) $T(n) = 2T(n-1) + n$

(*c*) $T(n) = 2T(n/2) + 1$

(*d*) $T(n) = 2T(n-1) + 1$

80. The worst case running time to search for an element in a balanced binary search tree with $n2^n$ elements is

(*a*) $\Theta(n \log n)$ (*b*) $\Theta(n2^n)$

(*c*) $\Theta(n)$ (*d*) $\Theta(\log n)$

81. Assuming P ≠ NP, which of the following is TRUE?

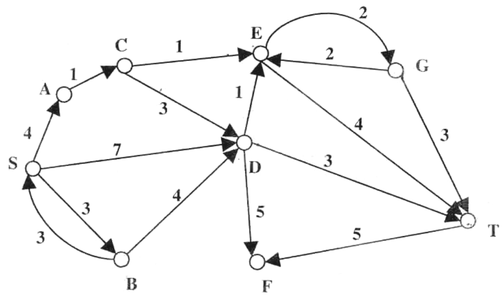
(*a*) NP-complete = NP

(*b*) NP-complete ∩ P = ∅

(*c*) NP-hard = NP

(*d*) P = NP-complete

82. Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijkstra's shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered.



(*a*) SDT (*b*) SBDT

(*c*) SACDT (*d*) SACET

83. A list of *n* strings, each of length *n*, is sorted into lexicographic order using the merge-sort alogrithm. The worst case running time of this computation is

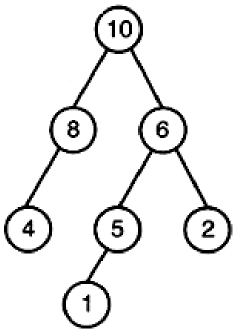
(*a*) $O(n \log n)$ (*b*) $O(n^2 \log n)$

(*c*) $O(n^2 + \log n)$ (*d*) $O(n^2)$

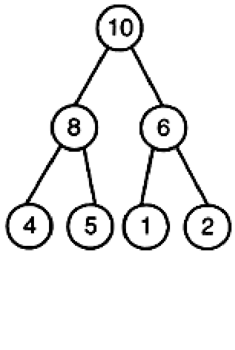
2011

84. A max-heap is heap where the value of each parent is greater than or equal to the value of its children. Which of the following is max-heap?

(*a*)



(*b*)



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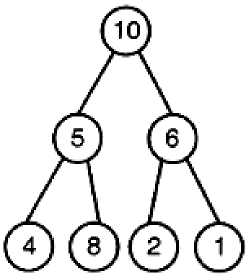
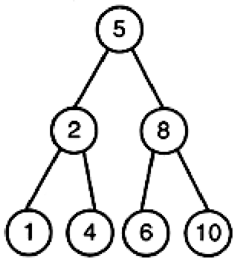
(*c*)



(*d*)



85. What does the following fragment of C program print?

Char c [ ] = "GATE2011";

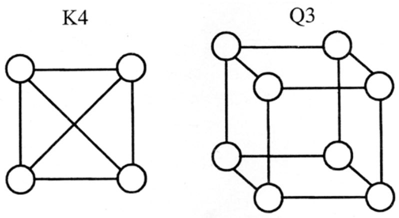
char *p = c ;

printf (:%s", $p + p[3] - p[1]$);

(*a*) GATE2011 (*b*) E2011

(*c*) 2011 (*d*) 011

86. K4 and Q3 are graphs with the following structures.



Which one of the following statements is **TRUE** in relation to these graphs?

(*a*) K4 is planar while Q3 is not

(*b*) Both K4 and Q3 are planar

(*c*) Q3 is planar while K3 is not

(*d*) Neither K4 nor Q3 is planar

87. Consider a relational table with a single record for each registered student with the following attributes.

1. *Registration_Num*: Unique registration number of each registered student
2. *UID* : Unique identity number, unique at the national level for each citizen
3. *BankAccount_Num*: Unique account number at the bank. A student can have multiple accounts or joint accounts. This attribute stores the primary account number.
4. *Name* : Name of the student
5. *Hostel_Room*: Room number of the hostel

Which of the following options is **INCORRECT**?

(*a*) *BankAccount _Num* is a candidate key

(*b*) *Registration_Num* can be primary key

(*c*) *UID* is a candidate key if all students are from the same country

(*d*) If S is a superkey such that S ∩ *UID* is NULL then S ∪ *UID* is also a superkey

88. The following is the comment for a C function.

/ * This function computes the roots of a quadratic equation

a . *x* ^ 2 + *b* . *x* + *c* = 0. The function stores two real roots in "root 1 and *root2 and returns the status of validity of roots. It handles four different kinds of cases.

(*i*) When coefficient *a* is zero irrespective of discriminant

(*ii*) When discriminant is positive

(*iii*) When discriminant is zero

(*iv*) When discriminant is negative.

Only in case (*ii*) and (*iii*), the stored roots are valid. Otherwise 0 is stored in the roots. The function returns 0 when the roots are valid and –1 otherwise.

The function also ensures root 1 > = root 2.

int get_QuadRoots (Float a, float b, float c,
float *root1, float *root2);

* /

A software test engineer is assigned the job of doing black box testing. He comes up with the following test cases, many of which are redundant.

Test	Input set			Expected Output Set		
Case	a	b	c	root1	root2	Return Value
T1	0.0	0.0	7.0	0.0	0.0	–1
T2	0.0	1.0	3.0	0.0	0.0	–1
T3	1.0	2.0	1.0	– 1.0	–1.0	0
T4	4.0	–12.0	9.0	1.5	1.5	0
T5	1.0	–2.0	– 3.0	3.0	–1.0	0
T6	1.0	1.0	4.0	0.0	0.0	–1

Which one of the following options provide the set of non-redundant tests using equivalence class partitioning approach from input perspective for black box testing?

(*a*) T1, T2, T3, T6

(*b*) T1, T3, T4, T5

(*c*) T2, T4, T5, T6

(*d*) T2, T3, T4, T5

89. Consider evaluating the following expression tree on a machine with load-store architecture in which memory can be accessed only through load and store instructions. The variables *a*, *b*, *c*, *d* and *e* are initially stored in memory. The binary operators

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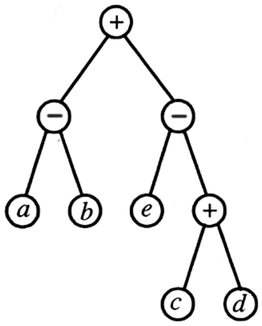
used in this expression tree can be evaluated by the machine only when the operands are in registers. The instructions produce result only in a register. If no intermediate results can be stored in memory, what is the minimum number of registers needed to evaluate this expression?



(*a*) 2 (*b*) 9

(*c*) 5 (*d*) 3

90. Consider the following table of arrival time and burst time for three processes P_0, P_1 and P_2.

Process	Arrival time	Burst Time
P_0	0 ms	9 ms
P_1	1 ms	4 ms
P_2	2 ms	9 ms

The pre-emptive shortest job first scheduling algorithm is used. Scheduling is carried out only at arrival or completion of processes. What is the average waiting time for the three processes?

(*a*) 5.0 ms (*b*) 4.33 ms

(*c*) 6.33 ms (*d*) 7.33 ms

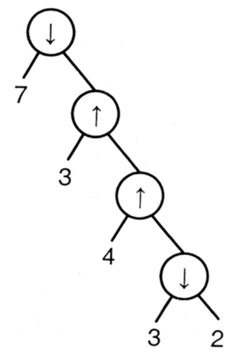
91. We are given a set of n distinct elements and an unlabeled binary tree with n nodes. In how many ways can we populate the tree with the given set so that it becomes a binary search tree?

(*a*) 0 (*b*) 1

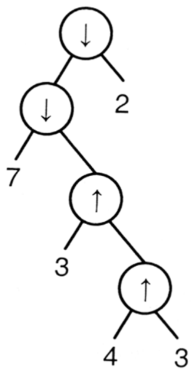
(*c*) $n!$ (*d*) $\frac{1}{n+1}.{}^{2n}C_n$

92. Consider two binary operators '↑' and '↓' with the precedence of operator ↓ being lower than that of the operator ↑. Operator ↑ is right associative while operator ↓ is left associative. Which one of the following represents the parse tree for expression (7 ↓ 3 ↑ 4 ↑ 3 ↓ 2)?

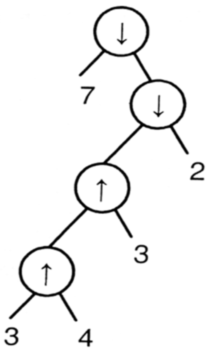
(*a*)



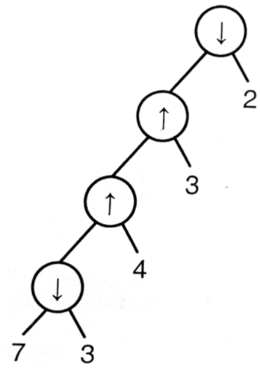
(*b*)



(*c*)



(*d*)



93. An algorithm to find the length of the longest monotonically increasing sequence of numbers in an array A[0 : n – 1] is given below.

Let L_i denote the length of the longest monotonically sequence starting at index i in the array.

Initialize $L_{n-1} = 1$

for all i such that $0 \le i \le n - 2$

$$L_i = \begin{cases} 1 + L_{i+1} & \text{if } A[i] < A[i+1] \\ 1 & \text{Otherwise} \end{cases}$$

Finally the length of the longest monotonically increasing sequence is Max (L_0, L_1, ..., L_{n-1}).

Which of the following statements is TRUE?

(*a*) The algorithm uses dynamic programming paradigm

(*b*) The algorithm has a linear complexity and uses branch and bound paradigm

(*c*) The algorithm has a non-linear polynomial complexity and uses branch and bound paradigm

(*d*) The algorithm uses divide and conquer paradigm.

2010

94. What does the following program print?

```
# include <stdio. h>
void f (int *p, int *q) {
    p = q;
    *p = 2;
}
int i = 0, j = 1;
int main () {
    f (& i, &j);
    printf ("%d %d/n", i, j);
                return 0;
}
```

(*a*) 2 2 (*b*) 2 1

(*c*) 0 1 (*d*) 0 2

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95. What is the appropriate pairing of items in the two columns listing various activities encountered in a software life cycle?

P. Requirements	1. Module Capture Development and Integration
Q. Design	2. Domain Analysis
R. Implementation	3. Structural and Behavioural Modelling
S. Maintenance	4. Performance Tuning

Codes :

	P	**Q**	**R**	**S**
(*a*)	3	2	4	1
(*b*)	2	3	1	4
(*c*)	3	2	1	4
(*d*)	2	3	4	1

96. What is the value printed by the following C program?

```
# include <stdio. h>
int f (int *a, int n)
{
    if (n <=0) return 0;
    else if (*a % 2 == 0) return *a + f (A +
1, n – 1);
    else return *a – f (a + 1, n – 1);
}
int main ( )
{
    int a [ ] = {12, 7, 13, 4, 11, 6};
    print f (“%d”, f (a, 6));
    return 0;
}
```

(*a*) –9 (*b*) 5
(*c*) 15 (*d*) 19

97. The following C function takes a singly-linked list as input argumen.t. It modifies the list by moving the last element to the front of the list and returns the modified list. Some part of the code is left blank.

```
typedef struct node {
                    int value;
        struct node *next;
}   Node;
Node *move_to_front (Node *head) {
    Node *p, *q;
    if ((head = = NULL) | | (head–>next
= = NULL)) return head;
    q = NULL; p = head;
    while(p–>next != NULL) {
                    q = p;
            p = p–>next ;
    }
    ________________________
    return head;
}
```

Choose the correct alternative to replace the blank line.

(*a*) q = NULL; p–>next = head; head = p;
(*b*) q–> next = NULL; head = p; p–>next = head;
(*c*) head = p; p–>next = q; q–>next = NULL;
(*d*) q–>next = NULL; p–>next = head; head = p;

98. The following program is to be tested for statement coverage:

```
begin
if (a == b) {S1; exit;}
else if (c == d) {S2;}
    else {S3; exit;}
S4;
end
```

The test cases T1, T2, T3 and T4 given below are expressed in terms of the properties satisfied by the values of variabls a, b, c and d. The exact values are not given.

T1: a, b, c and d are all equal

T2: a, b, c and d are all distinct

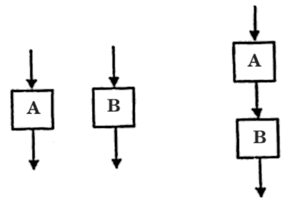
T3: $a = b$ and $c\ != d$

T4: $a\ != b$ and $c = d$

Which of the test suites given below ensures coverage of statements S1, S2, S3 and S4?

(*a*) T1, T2, T3 (*b*) T2, T4
(*c*) T3, T4 (*d*) T1, T2, T4

99. The cyclomatic complexity of each of the modules A and B shown below is 10. What is the cyclomatic complexity of the sequential integration shown on the right hand side?



(*a*) 19
(*b*) 21
(*c*) 20
(*d*) 10

100. Two alternative package A and B are available for processing a database having 10^k records. Package A requires 0.0001 n^2 time units and package B requires 10 $n \log_{10} n$ time units to process n records. What is the smallest value of k for which package B will be preferred over A?

(*a*) 12 (*b*) 10
(*c*) 6 (*d*) 5

101. The weight of a sequence $a_0, a_1, \ldots, a_{n-1}$ of real numbers is defined as

$$a_0 + a_1/2 + \ldots + a_{n-1}/2^{n-1}.$$

A subsequence of a sequence is obtained by deleting some elements from the sequence, keeping the order of the remaining elements the

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same. Let X denote the maximum possible weight of a subsequence of $a_0, a_1, \ldots, a_{n-1}$ and Y the maximum possible weight of a subsequence of $a_1, a_2, \ldots, a_{n-1}$. Then X is equal to

(*a*) max (Y, a_0 + Y) (*b*) max (Y, a_0 + Y)

(*c*) max (Y, a_0 + 2Y) (*d*) a_0 + Y/2

2009

102. Consider the program below:

```
# include <stdio.h>
int fun (int n, int *f_p) {
int t, f;
if (n <=1){
* f_ p = 1;
return 1;
}
t = fun (n-1, f_p) ;
f = t + * f_p ;
* f_p = t;
return f ;
}
int main ( ) {
int x = 15 ;
printf (" % d/n", fun (5, & x));
return  0;
}
```

The value printed is

(*a*) 6 (*b*) 8

(*c*) 14 (*d*) 15

103. What is the maximum height of any AVL-tree with 7 nodes ? Assume that the height of a tree with a single node is 0.

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

104. What is the number of swaps required to sort n elements using selection sort, in the worst case?

(*a*) θ (n) (*b*) θ (n log n)

(*c*) θ (n^2) (*d*) θ (n^2 log n)

105. Which of the following statement (s) is/are correct regarding Bellman-Ford shortest path algorithm?

P. A always finds a negative weighted cycle, if one exists.

Q. Finds whether any negative weighted cycle is reachable from the source.

(*a*) P only

(*b*) Q only

(*c*) both P and Q

(*d*) neither P nor Q

106. If π_A be a problem that belongs to the class NP. then which one of the following is TRUE ?

(*a*) there is no polynomial time algorithm for $\pi_{A.}$

(*b*) If π_A can be solved deterministrically in polynomial time, then P = NP.

(*c*) If π_A is NP - hard , then it is NP - complete.

(*d*) π_A may be undecidable.

107. The running time of an algorithm is represented by the following recurrence relation:

$$T(n) = \begin{cases} n & n \le 3 \\ T\left(\frac{n}{3}\right) + cn & \text{otherwise} \end{cases}$$

Which one of the following represents the time complexity of the algorithm ?

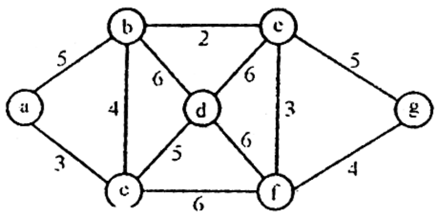
(*a*) θ (n) (*b*) θ (n log n)

(*c*) θ(n^2) (*d*) θ (n^2 log n)

108. The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using open addressing with hash function $h(k) = k$ mode 10 and linear probing. What is the resultant hash table ?

(*a*)		(*b*)		(*c*)		(*d*)	
0		0		0		0	
1		1		1		1	
2	2	2	12	2	12	2	12, 2
3	23	3	13	3	13	3	13, 3,23
4		4		4	2	4	
5	15	5	5	5	3	5	5, 15
6		6		6	23	6	
7		7		7	5	7	
8	18	8	18	8	18	8	18
9		9		9	15	9	

109. Consider the following graph :



Which one of the following is NOT the sequence of edges added to the minimum spannign tree using Kruskal's algorithm ?

(*a*) (b, e) (e, f) (a, c) (b, c) (f, g) (c, d)

(*b*) (b, e) (e, f) (a, c) (f, g) (b, c) (c, d)

(*c*) (b, e) (a, c) (e, f) (b, c) (f, g) (c, d)

(*d*) (b, e) (e, f) (b, c) (a, c) (f, g) (c, d)

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110. In quick sort, for sorting n elements, the $(n/4)^{th}$ smallest element is selected as pivot using an $O(n)$ time algorithm. What is the worst case time complexity of the quick sort ?

(*a*) $\theta(n)$ (*b*) $\theta(n \log n)$

(*c*) $\theta(n^2)$ (*d*) $\theta(n^2 \log n)$

2008

111. Which combination of the integer variables x, y and z makes the variable a get the value 4 in the following expression?

a = (x > y) ? ((x > z) ? x : z) : ((y > z) ? y : z)

(*a*) x = 3, y = 4, z = 2

(*b*) x = 6, y = 5, z = 3

(*c*) x = 6, y = 3, z = 5

(*d*) x = 5, y = 4, z = 5

112. What is printed by the following C program?

```
int f(int x, int *py, int **ppz)
{
    int y, z;
    **ppz + = 1; z = *ppz;
    *py + = 2; y = -py;
    x + = 3;
    return x + y + z;
}
void main ()
{
    int c, *b, **a;
    c = 4; b = &c; a = &b;
    printf ("%d", f(c,b,a));
}
```

(*a*) 18 (*b*) 19

(*c*) 21 (*d*) 22

113. Choose the correct option to fill ?1 and ?2 so that the program below prints an input string in reverse order. Assume that the input string is terminated by a newline character.

```
void reverse (void){
        int c;
    if (?1) reverse ();
        ?2
}
main () {
printf ("Enter Text"); printf ("\n");
reverse (); printf ("\n");
}
```

(*a*) ?1 is (getchar() != ‘|n’), ?2 is getchar (c);

(*b*) ?1 is (c = getchar ()) != ‘\n’), ?2 is getchar (c);

(*c*) ?1 is (c != ‘\n’), ?2 is putchar (c);

(*d*) ?1 is ((c = getchar ()) != ‘\n’), ?2 is putchar (c) ;

114. The following C function takes a singly-linked list of integers as a parameter and rearranges the elements of the list. The function is called with the list containing the integers 1, 2, 3, 4, 5, 6, 7 in the given order. What will be the contents of the list after the function completes execution?

```
struct node {
    int value;
    struct node *next;
};
void rearrange (struct node *list) {
struct node *p, *q;
int temp;
if (! list || !list -> next) return;
p = list; q = list -> next;
while (q) {
temp = p -> value; p -> value = q -> value;
q -> value = temp; p = q -> next;
q = p ? p -> next : 0;
}
}
```

(*a*) 1, 2, 3, 4, 5, 6, 7 (*b*) 2, 1, 4, 3, 6, 5, 7

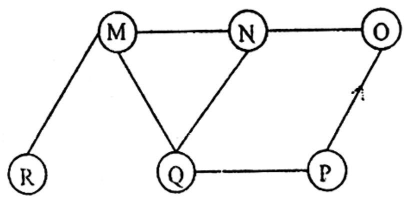
(*c*) 1, 3, 2, 5, 4, 7, 6 (*d*) 2, 3, 4, 5, 6, 7, 1

115. The most efficient algorithm for finding the number of connected components in an undirected graph on n vertices and m edges has time complexity

(*a*) $\Theta(n)$ (*b*) $\Theta(m)$

(*c*) $\Theta(m + n)$ (*d*) $\Theta(mn)$

116. The Breadth First Search algorithm has been implemented using the queue data structure, One possible order of visiting the nodes of the following graph is



(*a*) MNOPQR (*b*) NQMPOR

(*c*) QMNPRO (*d*) QMNPOR

117. Consider the following functions:

$$f(n) = 2^n$$
$$g(n) = n!$$
$$h(n) = n^{\log n}$$

Which of the following statements about the asymptotic behaviour of f(n), g(n), and h(n) is true?

(*a*) $f(n) = O(g(n)); g(n) = O(h(n))$

(*b*) $f(n) = \Omega(g(n)); g(n) = O(h(n))$

(*c*) $g(n) = O(f(n)); h(n) = O(f(n))$

(*d*) $h(n) = O(f(n)); g(n) = \Omega(f(n))$

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118. The minimum number of comparisons required to determine if an interger appears more than $\frac{n}{2}$ times in a sorted array of n integers is

(*a*) Θ(n) (*b*) Θ(log n)

(*c*) Θ(log* n) (*d*) Θ(1)

119. A B-tree of order 4 is built from scratch by 10 successive insertions. What is the maximum number of node sphitting operations that may take place?

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

120. G is a graph on n vertices and 2n-2 edges. The edges of G can be partitioned into two edge-disjoint spannint trees.

Which of the following is NOT true for G?

(*a*) For every subset of k vertices, the induced subgraph has at most 2k-2 edges

(*b*) The minimum cut in G has at least two edges

(*c*) There are two edge-disjoint paths between every pair of vertices

(*d*) There are two vertex-disjoint paths between every pair of vertices

121. Consider the Quicksort algorithm. Suppose there is a procedure for finding a pivot element which splits the list into two sub-lists each of which contatins at least one-fifth of the elements. Let T(n) be the number of comparisons required to sort n elements. Then

(*a*) $T(n) \le 2T\left(\frac{n}{5}\right) + n$

(*b*) $T(n) \le T\left(\frac{n}{5}\right) + T\left(\frac{4n}{5}\right) + n$

(*c*) $T(n) \le 2T\left(\frac{4n}{5}\right) + n$

(*d*) $T(n) \le 2T\left(\frac{n}{2}\right) + n$

122. The subset-sum problem is defined as follows:

Given a set S of n positive integers and a positive integer W, determine whether there is a subset of S whose elements sum to W.

An algorithm Q solves this problem in O(nW) time. Which of the following statements is false?

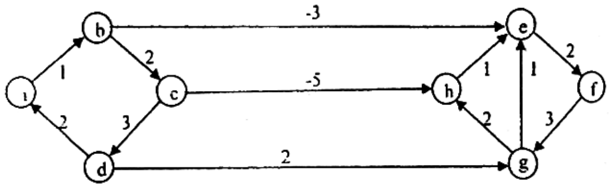
(*a*) Q solves the subset-sum problem in polynomial time when the input is encoded in unary

(*b*) Q solves the subset-sum problem in polynomial time when the input is encoded in binary

(*c*) The subset sum problem belongs to the class NP

(*d*) The subset sum problem is NP-hard

123. Dijkstra's single source shortest path algorithm when run from vertex a in the above graph, computes the correct shortest path distance to



(*a*) only vertex a

(*b*) only vertices a, e, f, g, h

(*c*) only vertices a, b, c, d

(*d*) all the vertices

124. You are given the postorder traversal, P, of a binary search tree on the n elements 1, 2,....., n. You have to determine the unique binary search tree that has P as its postorder traversal. What is the time complexity of the most efficient algorithm for doing this?

(*a*) Θ(log n)

(*b*) Θ(n)

(*c*) Θ(n log n)

(*d*) None of the above, as the tree cannot be uniquely determind.

125. We have a binary heap on n elements and wish to insert n more elements (not necessarily one after another) into this heap. The total time required for this is

(*a*) Θ(log n) (*b*) Θ(n)

(*c*) Θ(n log n) (*d*) $\Theta(n^2)$

2007

126. Consider the following C code segment :

```
int IsPrime(n)
    {
        int i,n;
        for(i=2;i < = sqrt(n);i++)
      if (n % i = = 0)
        {printf('Not Prime\n"};
                        return 0;)
            return 1;
    }
```

Let $T(n)$ denote the number of times the *for* loop is executed by the program on input n. Which of the following is **TRUE** ?

(*a*) $T(n) = O(\sqrt{n})$ and $T(n) = \Omega(\sqrt{n})$

(*b*) $T(n) = O(\sqrt{n})$ and $T(n) = \Omega(1)$

(*c*) $T(n) = O(n)$ and $T(n) = \Omega(\sqrt{n})$

(*d*) None of these

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127. In the following C function, let $n \geq m$.

```
int gcd(n,m)
{
    if(n%m == 0) return m;
    n = n%m;
    return gcd(m,n);
}
```

How many recursive calls are made by this function ?

(*a*) $\Theta(\log_2 n)$ (*b*) $\Omega(n)$

(*c*) $\Theta(\log_2 \log_2 n)$ (*d*) $\Theta(\sqrt{n})$

128. Consider the following segment of C-code :

```
int J, n;
        j = 1;
  while (j <= n)
        j = j* 2;
```

The number of comparisons made in the execution of the loop for any $n > 0$ is

(*a*) $\lceil \log_2 n \rceil + 1$ (*b*) n

(*c*) $\lceil \log_2 n \rceil$ (*d*) $\lfloor \log_2 n \rfloor + 1$

129. The following postfix expression with single digit operands is evaluated using a stack

8 2 3 ^ / 2 3 * + 5 1* –

Note that ^ is the exponentiation operator. The top two elements of the stack after the first * is evaluated are

(*a*) 6, 1 (*b*) 5, 7

(*c*) 3,2 (*d*) 1,5

130. Consider the following C function :

```
int f(int  n)
{static int r = 0;
  if (n < = 0) return 1;
if (n > 3)
   (r = n;
     return f (n–2) + 2;
}
     return f(n–1) + r;
}
```

What is the value of f(5) ?

(*a*) 5 (*b*) 7

(*c*) 9 (*d*) 18

131. Consider the following C program segment where Cell Node represents a node in a binary tree

```
struct CellNode {
struct CellNode * leftchild;
int element;
struct CellNode *rightchild;
};
int GetValue(struct CellNode*ptr){
int value = 0;
if (ptr ! = NULL) {
  if ((ptr->leftChild == NULL) &&
     (ptr->rightChild == NULL))
     value = 1;
else
value = value + GetValue (ptr->leftChild)
                + GetValue
            (ptr->rightChild);
}
return(value);
}
```

The value returned by Get Value when a pointer to the root of a binary tree is passed as its argument is

(*a*) the number of nodes in the tree

(*b*) the number of internal nodes in the tree

(*c*) the number of leaf nodes in the tree

(*d*) the height of the tree

132. The height of a binary tree is the maximum number of edges in any root to leaf path. The maximum number of nodes in a binary tree of height h is

(*a*) 2^h (*b*) $2^{h-1}-1$

(*c*) $2^{h+1}-1$ (*d*) 2^{h+1}

133. The maximum number of binary trees that can be formed with three unlabeled nodes is

(*a*) 1 (*b*) 5

(*c*) 4 (*d*) 3

134. Which of the following sorting algorithms has the lowest worst-case complexity ?

(*a*) Merge sort (*b*) Bubble sort

(*c*) Quick sort (*d*) Selection sort

135. The inorder and preorder traversal of a binary tree are d b e a f c g and a b d e c f g respectively.

The postorder traversal of the binary tree is

(*a*) d e b f g c a (*b*) e d b g f c a

(*c*) e d b f g c a (*d*) d e f g b c a

136. Consider a hash table of size seven, with starting index zero, and a hash function $(3x + 4) \bmod 7$. Assuming the hash table is initially empty, which of the following is the contents of the table when the sequence 1, 3, 8, 10 is inserted into the table using closed hashing ? Note that–denotes an empty location in the table.

(*a*) 8, –, –, –, –, –, 10

(*b*) 1, 8, 10, –, –, –, 3

(*c*) 1, –, –, –, –, –, 3

(*d*) 1, 10, 8, –, –, –, 3

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137. In an unweighted, undirected connected graph, the shortest path from a mode S to every other node is computed most efficiently, in terms of *time complexity,* by

(*a*) Dijkstra's algorithm starting from S.

(*b*) Warshall's algorithm

(*c*) performing a DFS starting from S

(*d*) performing a BFS starting from S

138. A complete n–ary tree is a tree in which each node has n children or no children. Let I be the number of internal nodes and L be the number of leaves in a complete n-ary tree. If L = 41, and I = 10, then what is the value of n?

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

139. What is the *time complexity* of the following recursive function ?

```
int DoSomething ( int n) {
if   (n <= 2)
     return 1;
else
return (DoSomething (floor  (sqrt(n))) + n);
}
```

(*a*) $\Theta(n^2)$ (*b*) $\Theta(n \log_2 n)$

(*c*) $\Theta(\log_2 n)$ (*d*) $\Theta(\log_2 \log_2 n)$

140. Consider the process of inserting an element into a *Max Heap,* where the *Max Heap* is represented by an *array.* Suppose we perform a binary search on the path from the new leaf to the root to find the position for the newly inserted element, the number of *comparisons* performed is

(*a*) $\Theta(\log_2 n)$ (*b*) $\Theta(n \log_2 n)$

(*c*) Θn (*d*) $\Theta(n \log_2 n)$

141. Let w be the minimum weight among all edge weights in an undirected connected graph. Let e be a specific edge of weight w.

Which of the following is **FALSE** ?

(*a*) There is a minimum spanning tree containing e.

(*b*) If e is not in a minimum spanning tree T, then in the cycle formed by adding e to T, all edges have the same weight.

(*c*) Every minimum spanning tree has an edge of weight w

(*d*) e is present in every minimum spanning tree

142. An array of n numbers is given, where n is an even number. The maximum as well as the minimum of these n numbers needs to be determined. Which of the following is **TRUE** about the number of comparisons needed ?

(*a*) At least $2n - c$ comparisons, for some constant c, are needed.

(*b*) At most $1.5n - 2$ comparisons are needed.

(*c*) At least $n \log_2 n$ comparisons are needed.

(*d*) None of these

2006

143. An implementation of a queue Q, using two stacks S1 and S2, is given below :

```
void insert (Q, x) {
        push (S1, x);
}
void delete (Q) {
    if (stack-empty (S2)) then
        if (stack-empty (S1))
then {
            print (Q is empty");
            return;
}
else while (! (stack-empty) (S1))) {
        x = pop (S1);
        push (S2, x);
}
    x = pop (S2);
}
```

Let n insert and $m(\leq n)$ delete operations be performed in an arbitrary order on an empty queue Q. Let x and y be the number of push and pop operations performed respectively in the process. Which one of the following is true for all m and n ?

(*a*) $n + m \leq x < 2n$ and $2m \leq n + m$

(*b*) $n + m \leq x < 2n$ and $2m \leq y \leq 2n$

(*c*) $2m \leq x < 2n$ and $2m \leq y \leq n + m$

(*d*) $2m \leq x < 2n$ and $2m \leq y \leq 2n$

144. Consider the following C-function in which a $[n]$ and b $[m]$ are two sorted integer arrays and c [n + m] be another integer array.

```
void xyz (int a [], int b[], int c []){
   int i, j, k;
   i = j = k = 0;
   while ((i < n) && (j < m))
        if (a[i] < b [j]) c [k ++] = a [i ++ ];
        else c [k ++]  = b [j ++];
   }
```

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Which of the following condition (s) hold(s) after the termination of the while loop ?

(*i*) j < m, k = n + j – 1, and a [n – 1] < b [j] if i = n

(*ii*) i < n, k = m + i – 1, and b [m – 1] ≤ a [i] if j = m

(*a*) only (*i*)

(*b*) only (*ii*)

(*c*) either (*i*) or (*ii*) but not both

(*d*) neither (*i*) nor (*ii*)

145. Consider these two functions and two statements S1 and S2 about them.

```
int work 1(int*a, int i, int j)
{
        int x = a [i + 2];
        a [j] = x + 1;
        return a [i + 2] – 3;}
}
```

```
in twork 2(int*a, int i, int j)
{
        int t1 = i + 2;
        int t2 = a[t1];
        a [j]= t2 + 1;
        return t2 – 3;
}
```

S1 : The transformation from work 1 to work 2 is valid, i.e., for any program state and input arguments, work 2 will compute the same output and have the same effect on program state as work 1

S2 : All the transformations applied to work 1 to get work 2 will always improve the performance (i.e. reduce CPU time) of work 2 compared to work 1

(*a*) S1 is false and S2 is false

(*b*) S1 is false and S2 is true

(*c*) S1 is true and S2 is false

(*d*) S1 is true and S2 is true

146. Consider this C code to swap two integers and these five statements :

The code

```
void swap (int * px, int * py) {
        *px = *px – *py;
       * py = *px + *py;
      * px = *py – * px;
}
```

S1 : will generate a compilation error

S2 : may generate a segmentation fault at runtime depending on the arguments passed

S3 : correctly implements the swap procedure for all input pointers referring to integers stored in memory locations accessible to the process

S4 : implements the swap procedure correctly for some but not all valid input pointers

S5 : may add or subtract integers and pointers

(*a*) S1

(*b*) S2 and S3

(*c*) S2 and S4

(*d*) S2 and S5

147. In a binary max heap containing n numbers, the smallest element can be found in time

(*a*) $\theta(n)$ (*b*) $\theta(\log n)$

(*c*) $\theta(\log \log n)$ (*d*) $\theta(1)$

148. Consider a weighted complete graph G on the vertex set $\{v_1, v_2, \ldots.., v_n\}$ such that the weight of the edge (v_i, v_j) is $2|i - j|$. The weight of a minimum spanning tree of G is

(*a*) $n - 1$ (*b*) $2n - 2$

(*c*) $\binom{n}{2}$ (*d*) n^2

149. To implement Dijkstra's shortest path algorithm on unweighted graphs so that it runs in linear time, then data structure to be used is

(*a*) Queue (*b*) Stack

(*c*) Heap (*d*) B-Tree

150. A scheme for storing binary trees in an array X is as follows. Indexing of X starts at 1 instead of 0. The roots is stored at X [1]. For a node stored at X[i], the left child, if any, is stored in X[$2i$] and the right child, if any, in X [$2i + 1$]. To be able to store any binary tree on n vertices, the minimum size of X should be

(*a*) $\log_2 n$ (*b*) n

(*c*) $2n + 1$ (*d*) $2^n - 1$

151. Which one of the following in place sorting algorithms needs the minimum number of swaps?

(*a*) Quick sort (*b*) Insertion sort

(*c*) Selection sort (*d*) Heap sort

152. Consider the following C-program fragment in which i, j, and n are integer variables.

for $(i = n, j = 0; i > 0; i/ = 2, j + = i)$;

Let val(j) = denote the value stored in the variable j after termination of the *for* loop. Which one of the following is true ?

(*a*) val $(j) = \theta(\log n)$

(*b*) val $(j) = \theta(\sqrt{n})$

(*c*) val $(j) = \theta(n)$

(*d*) val $(j) = \theta(n \log n)$

153. An element in an array X is called a leader if it is greater than all elements to the right of it in X. The best algorithm to find all leaders in an array

(*a*) solves it in linear time using a left to right pass of the array

(*b*) solves in linear time using a right to left pass of the array

(*c*) solves it using divide and conquer in time $\theta(n \log n)$

(*d*) solves it in time $\theta(n^2)$

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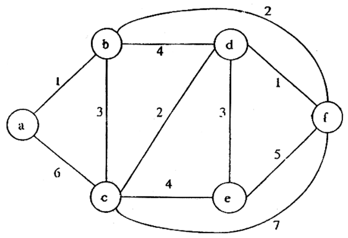
154. Consider the following graph :



Which one of the following cannot be the sequence of edges added, in that order, to a minimum spanning tree using Kruskal's algorithm ?

(*a*) $(a-b), (d-f), (b-f), (d-c), (d-e)$

(*b*) $(a-b), (d-f), (d-c), (b-f), (d-e)$

(*c*) $(d-f), (a-b), (d-c), (b-f), (d-e)$

(*d*) $(d-f), (a-b), (b-f), (d-e), (d-c)$

155. Let T be a depth first search tree in a undirected grpah G. Vertices u and v are leaves of this tree T. The degrees of both u and v in G are at least 2. Which one of the following statements is true ?

(*a*) There must exist a vertex w adjacent to both u and v in G

(*b*) There must exist a vertex w whose removal disconnects u and v in G

(*c*) There must exist a cycle in G containing u and v

(*d*) There must exist a cycle in G containing u and all its neighbours in G

156. A set X can be represented by an array x[n] as follows :

$$x[i] = \begin{cases} 1 & \text{if } i \in X \\ 0 & \text{otherwise} \end{cases}$$

Consider following algorithm in which x, y, and z are boolean arrays of size n :

```
algorithm zzz (x[], y[], z[]){
 int i;
 for (i = 0; i < n; ++ i)
        z[i] = (x [i] ∧ ~ y[i]) ∨ (~ x[i] ∧ y[i])
}
```

The set Z computed by the algorithm is

(*a*) $(X \cup Y)$ (*b*) $(X \cap Y)$

(*c*) $(X - Y) \cap (Y - X)$ (*d*) $(X - Y) \cup (Y - X)$

157. Consider the following recurrence

$$T(n) = 2T\left(\left\lceil \sqrt{n} \right\rceil\right) + 1,$$

$$T(1) = 1$$

Which one of the following is true ?

(*a*) $T(n) = \theta(\log \log n)$ (*b*) $T(n) = \theta(\log n)$

(*c*) $T(n) = \theta(\sqrt{n})$ (*d*) $T(n) = \theta(n)$

158. Given two arrays of numbers $a_1, ..., a_n$ and $b_1, ..., b_n$ where each number is 0 or 1, the fastest algorithm to find the largest span (i, j) such that $a_i + a_{i+1} + ... + a_j = b_i + b_{i+1} + ... + b_j$, or report that there is no such span

(*a*) takes $O(3^n)$ and $\Omega(2^n)$ time if hashing is permitted

(*b*) takes $O(n^3)$ and $\Omega(n^{2.5})$ time in the key comparison model

(*c*) takes $\Theta(n)$ time and space

(*d*) takes $O(\sqrt{n})$ time only if the sum of the $2n$ elements is an even number

159. Consider following code written in a pass-by-reference language like FORTRAN and these statements about the code.

```
        Subroutine swap (ix, iy)
                  it = ix
L1 :                  ix = iy
L2 :                  i y = it
        end
        ia = 3
        ib = 8
          call swap (ia, ib + 5)
        print *, ia, ib
        end
```

S1 : The compiler will generate code to allocate a temporary nameless cell, initialize it to 13, and pass the address of the cell to swap

S2 : On execution the code will generate a runtime error on line L1

S3 : On execution the code will generate a runtime error on line L2

S4 : The program will print 13 and 8

S5 : The program will print 13 and –2

Exactly the following set of statement (s) is correct :

(*a*) S1 and S2 (*b*) S1 and S4

(*c*) S3 (*d*) S1 and S5

NUMERICAL TYPE QUESTIONS

2015

1. The output of the following C program is ______.

```
void f1(int a, int b)
{
    int c;
    c =a; a=b; b=c;
}
void f2(int *a, int *b)
{
    int c;
    c =*a; *a=*b; *b=c;
}
```

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```
int main ( )
{
    int a=4, b=5, c=6;
    f1(a, b);
    f2(&b, &c);
    printf("%d", c–a–b);
}
```

2. Consider the following C program segment.

```
while (first <= last)
{
    if(array[middle] < search)
        first = middle + 1;
    else if (array[middle] = = search)
            found = TRUE;
    else last = middle – 1;
    middle = (first + last)/2;
}
if (first > last) notPresent = TRUE;
```

The cyclomatic complexity of the program segment is ______.

3. A binary tree T has 20 leaves. The number of nodes in T having two children is _________.

4. Consider the following C function.

```
int fun (int n) {
    int x = 1, k;
    if (n == 1) return x;
    for (k = 1; k < n; + +k)
        x = x + fun (k) * fun (n – k);
    return x;
}
```

The return value of fun (5) is _______

5. Consider the C program below.

```
#include <stdio.h>
Int *A, stkTop;
Int stkFunc (int opcode, int val)
{
    Static int size =0, stkTop=0;
Switch (opcode) {
    Case -1 : Size = val; break;
    Case 0 : if (stkTop < size) A (stktop++] = val;
    break;
    Default : if (stktop) return A [--stkTop];
}
return -1;
}
int main ( )
{
int B[20] ; A = B; stkTop = -1;
stkFunc (-1, 10);
stkFunc (0, 5);
stkFunc (0, 10);
print f ("%d\n", stkFunc(1, 0) + stkfunc(1, 0);
}
```

The value printed by the above program is ______.

6. Consider a binary tree T that has 200 leaf nodes. Then, the number of nodes in T that have exactly two children are ____ .

7. Suppose $c = \langle c[0], ..., c[k-1] \rangle$ is an array of length k, where all the entries are from the set {0, 1}. For any positive integers a and n, consider the following pseudo code.

DOSOMETHING (c, a, n)

$z \leftarrow 1$

for $i \leftarrow 0$ to $k - 1$

  do $z \leftarrow z^2 \bmod n$

  if $c[i] = 1$

  then $z \leftarrow (z \times a) \bmod n$

  return z

If $k = 4$, $c = \langle 1.0, 1, 1 \rangle$, $a = 2$ and $n = 8$, then the output of DOSOMETHING (c, a, n) is ________.

8. Consider the following C program

```
# include < stdio.h >
int main ( )
{
    static int a [ ]  = {10, 20, 30, 40, 50};
    static int * p [ ] = {a, a + 3, a + 4, a + 1, a + 2};
    int **ptr = p;
    ptr ++;
    print f (''%d %d'', ptr – p, **ptr):
}
```

The output of the program is _________.

9. Consider the following C program

```
#include < stdio.h >
int main ( )
{
     int i, j, k = 0;
     j = 2*3/4 + 2.0/5 + 8/5;
     k – = – – j;
     for (i = 0; i < 5; i ++)
```

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```
        {
            Switch (i + k)
        {
            case1 :
            case 2 : printf ("\ n % d", i + k)
            case 3 : printf ("\ n % d", i + k);
            default : print f ("\ n%d", i + k) ;
        }
    }
    Return 0 :
}
```

The number of times printf statement is executed is ________.

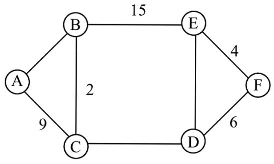
10. Consider the following C program.

```
#include < stdio.h >
int f1 (void);
int f2 (void);
int x = 10;

int main ( )
    {
        int x = 1;
        x + = f1 ( ) + f2 ( ) + f3 ( ) + f2 ( );
        print f ("%d", x);
        return 0;
    }
    int f1 ( ) {int x = 25; x ++; return x;}
    int f2 ( ) {static int x = 50; x + +; return x;}
    int f3 ( ) {x* = 10; return x};
```

The output of program is ________.

11. The graph shown below has 8 edges with distinct integer edge weights. The minimum spanning tree (MST) is of weight 36 and contains the edges: {(A, C), (B, C), (B, E), (E, F), (D, F)}. The edge weights of only those edges which are in the MST are given in the figure shown below. The minimum possible sum of weights of all 8 edges of this graph is _______.



12. Let G be a connected planar graph with 10 vertices. If the number of edges on each face is three, then the number of edges in G is ______.

13. Let G be a connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes __________.

2014

14. Consider an undirected graph G where self-loops are not allowed. The vertex set of G is (i, j): $1 \le i \le 12$, $1 \le j \le 12$}. There is an edge between (a, b) and (c, d) if $|a - c| \le 1$ and $|b - d| \le 1$. The number of edges in the graph is ____________.

15. Consider the function `func` shown below:

```
int func(int num) {
int count = 0;
  while (num) {
          count++;
         num>>= 1;
  }
return (count);
}
```

The value returned by `func(435)` is _________.

16. Three processes A, B and C each execute a loop of 100 iterations. In each iteration of the loop, a process performs a single computation that requires t_c CPU milliseconds and then initiates a single I/O operation that lasts for t_{io} milliseconds. It is assumed that the computer where the processes execute has sufficient number of I/O devices and the OS of the computer assigns different I/O devices to each process. Also, the scheduling overhead of the OS is negligible. The processes have the following characteristics:

Process id	t_c	t_{io}
A	100 ms	500 ms
B	350 ms	500 ms
C	200 ms	500 ms

The processes A, B, and C are started at times 0, 5 and 10 milliseconds respectively, in a pure time sharing system (round robin scheduling) that uses a time slice of 50 milliseconds. The time in milliseconds at which process C would complete its first I/O operation is ___________.

17. Consider the following function

```
double f(double x){
 if( abs(x*x - 3) < 0.01) return x;
 else return f(x/2 + 1.5/x);
}
```

Give a value `q` (to 2 decimals) such that `f(q)` will return `q`:_____.

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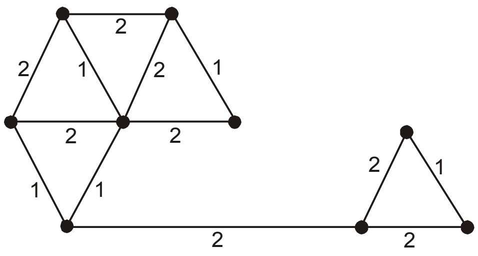
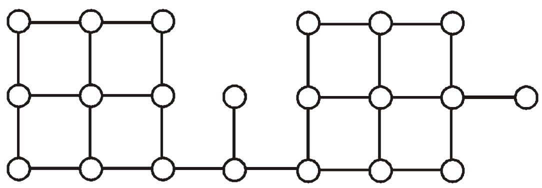
18. The number of distinct minimum spanning trees for the weighted graph below is _____



19. The minimum number of arithmetic operations required to evaluate the polynomial $P(X) = X^5 + 4X^3 + 6X + 5$ for a given value of X, using only one temporary variable is _____.

20. Suppose depth first seach is executed on the graph below starting at some unknown vertex. Assume that a recursive call to visit a vertex is made only after first checking that the vertex has not been visited earlier. Then the maximum possible recursion depth (including the initial call) is _________.



21. Consider a rooted n node binary tree represented using pointers. The best upper bound on the time required to determine the number of subtrees having exactly 4 nodes is $0(n^a \log^b n)$. Then the value of $a + 10b$ is_______

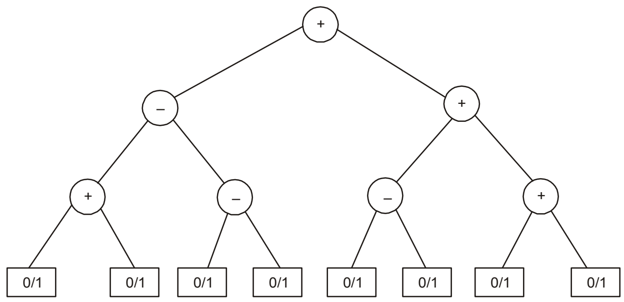
22. The minimum number of comparisons required to find the minimum and the maximum of 100 numbers is ____________.

23. The maximum number of edges in a bipartite graph on 12 vertices is _____.

24. Consider two strings A = "*qpqrr*" and B = "*pqprqrp*". Let x be the length of the longest common subsequence (not necessarily contiguous) between A and B and let y be the number of such longest common subsequences between A and B. Then $x + 10y$ = ________.

25. Suppose P, Q, R, S, T are sorted sequences having lengths 20, 24, 30, 35, 50 respectively. They are to be merged into a single sequence by merging together two sequences at a time. The number of comparisons that will be needed in the worst case by the optimal algorithm for doing this is ______.

26. Consider the expression tree shown. Each leaf represents a numerical value, which can either be 0 or 1. Over all possible choices of the values at the leaves, the maximum possible value of the expression represented by the tree is _________.



27. A cycle on n vertices is isomorphic to its complement. The value of n is _____.

28. Suppose you want to move from 0 to 100 on the number line. In each step, you either move right by a unit distance or you take a *shortcut*. A shortcut is simply a pre-specified pair of integers i, j with $i < j$. Given a shortcut i, j if you are at position i on the number line, you may directly move to j. Suppose $T(k)$ denotes the smallest number of steps needed to move from k to 100. Suppose further that there is at most 1 shortcut involving any number, and in particular from 9 there is a shortcut to 15. Let y and z be such that $T(9) = 1 + \min(T(y), T(z))$. Then the value of the product yz is ___________.

29. Suppose we have a balanced binary search tree T holding n numbers. We are given two numbers L and H and wish to sum up all the numbers in T that lie between L and H. Suppose there are m such numbers in T. If the tightest upper bound on the time to compute the sum is $O(n^a \log^b n + m^c \log^d n)$, the value of $a + 10b + 100c + 1000d$ is _______.

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ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*d*)	**2.** (*d*)	**3.** (*c*)	**4.** (*a*)	**5.** (*a*)	**6.** (*b*)	**7.** (*c*)	**8.** (*c*)	**9.** (*a*)	**10.** (*d*)
11. (*b*)	**12.** (*d*)	**13.** (*a*)	**14.** (*c*)	**15.** (*d*)	**16.** (*b*)	**17.** (*d*)	**18.** (*c*)	**19.** (*b*)	**20.** (*a*)
21. (*b*)	**22.** (*d*)	**23.** (*b*)	**24.** (*d*)	**25.** (*b*)	**26.** (*c*)	**27.** (*a*)	**28.** (*b*)	**29.** (*c*)	**30.** (*c*)
31. (*d*)	**32.** (*d*)	**33.** (*c*)	**34.** (*d*)	**35.** (*b*)	**36.** (*b*)	**37.** (*b*)	**38.** (*c*)	**39.** (*c*)	**40.** (*c*)
41. (*c*)	**42.** (*c*)	**43.** (*c*)	**44.** (*c*)	**45.** (*c*)	**46.** (*c*)	**47.** (*d*)	**48.** (*c*)	**49.** (*d*)	**50.** (*d*)
51. (*b*)	**52.** (*b*)	**53.** (*d*)	**54.** (*b*)	**55.** (*c*)	**56.** (*c*)	**57.** (*a*)	**58.** (*c*)	**59.** (*b*)	**60.** (*d*)
61. (*a*)	**62.** (*b*)	**63.** (*d*)	**64.** (*c*)	**65.** (*d*)	**66.** (*d*)	**67.** (*c*)	**68.** (*b*)	**69.** (*b*)	**70.** (*c*)
71. (*a*)	**72.** (*a*)	**73.** (*b*)	**74.** (*c*)	**75.** (*b*)	**76.** (*c*)	**77.** (*a*)	**78.** (*a*)	**79.** (*b*)	**80.** (*a*)
81. (*b*)	**82.** (*a*)	**83.** (*c*)	**84.** (*d*)	**85.** (*a*)	**86.** (*d*)	**87.** (*d*)	**88.** (*d*)	**89.** (*c*)	**90.** (*a*)
91. (*a*)	**92.** (*b*)	**93.** (*c*)	**94.** (*b*)	**95.** (*d*)	**96.** (*a*)	**97.** (*d*)	**98.** (*d*)	**99.** (*a*)	**100.** (*c*)
101. (*a*)	**102.** (*b*)	**103.** (*a*)	**104.** (*c*)	**105.** (*c*)	**106.** (*c*)	**107.** (*c*)	**108.** (*a*)	**109.** (*c*)	**110.** (*a*)
111. (*a*)	**112.** (*a*)	**113.** (*c*)	**114.** (*b*)	**115.** (*d*)	**116.** (*c*)	**117.** (*b*)	**118.** (*b*)	**119.** (*a*)	**120.** (*c*)
121. (*c*)	**122.** (*d*)	**123.** (*d*)	**124.** (*c*)	**125.** (*c*)	**126.** (*d*)	**127.** (*a*)	**128.** (*b*)	**129.** (*b*)	**130.** (*d*)
131. (*d*)	**132.** (*d*)	**133.** (*a*)	**134.** (*a*)	**135.** (*d*)	**136.** (*b*)	**137.** (*c*)	**138.** (*a*)	**139.** (*d*)	**140.** (*a*)
141. (*a*)	**142.** (*c*)	**143.** (*d*)	**144.** (*d*)	**145.** (*b*)	**146.** (*c*)	**147.** (*b*)	**148.** (*c*)	**149.** (*d*)	**150.** (*c*)
151. (*b*)	**152.** (*c*)	**153.** (*b*)	**154.** (*a*)	**155.** (*a*)	**156.** (*a*)	**157.** (*a*)	**158.** (*d*)	**159.** (*a*)	**160.** (*a*)
161. (*c*)	**162.** (*b*)	**163.** (*a*)	**164.** (*b*)	**165.** (*a*)	**166.** (*c*)	**167.** (*d*)	**168.** (*c*)	**169.** (*b*)	**170.** (*a*)
171. (*d*)	**172.** (*d*)	**173.** (*d*)	**174.** (*b*)	**175.** (*c*)	**176.** (*d*)	**177.** (*a*)	**178.** (*d*)	**179.** (*d*)	**180.** (*a*)
181. (*a*)	**182.** (*b*)	**183.** (*c*)	**184.** (*d*)	**185.** (*c*)	**186.** (*c*)	**187.** (*d*)	**188.** (*a*)	**189.** (*d*)	**190.** (*c*)
191. (*a*)	**192.** (*d*)	**193.** (*d*)	**194.** (*a*)	**195.** (*a*)	**196.** (*c*)	**197.** (*a*)	**198.** (*a*)	**199.** (*c*)	**200.** (*b*)
201. (*c*)	**202.** (*d*)	**203.** (*d*)	**204.** (*c*)	**205.** (*d*)	**206.** (*a*)	**207.** (*d*)	**208.** (*c*)	**209.** (*a*)	**210.** (*d*)
211. (*b*)	**212.** (*c*)	**213.** (*d*)	**214.** (*d*)	**215.** (*c*)	**216.** (*a*)	**217.** (*b*)	**218.** (*d*)	**219.** (*d*)	**220.** (*d*)
221. (*a*)	**222.** (*b*)	**223.** (*a*)	**224.** (*a*)	**225.** (*a*)	**226.** (*c*)	**227.** (*b*)	**228.** (*b*)	**229.** (*b*)	**230.** (*a*)
231. (*b*)	**232.** (*c*)	**233.** (*a*)	**234.** (*d*)	**235.** (*b*)	**236.** (*c*)	**237.** (*b*)	**238.** (*b*)	**239.** (*c*)	**240.** (*a*)
241. (*c*)	**242.** (*b*)	**243.** (*d*)	**244.** (*c*)	**245.** (*a*)	**246.** (*c*)	**247.** (*b*)	**248.** (*d*)	**249.** (*c*)	**250.** (*a*)
251. (*c*)	**252.** (*b*)	**253.** (*c*)	**254.** (*a*)	**255.** (*a*)	**256.** (*c*)	**257.** (*a*)	**258.** (*b*)	**259.** (*a*)	**260.** (*d*)
261. (*c*)	**262.** (*a*)	**263.** (*b*)	**264.** (*b*)	**265.** (*d*)	**266.** (*a*)	**267.** (*a*)	**268.** (*a*)	**269.** (*a*)	**270.** (*c*)
271. (*a*)	**272.** (*a*)	**273.** (*a*)	**274.** (*a*)	**275.** (*c*)	**276.** (*a*)	**277.** (*b*)	**278.** (*c*)	**279.** (*b*)	**280.** (*c*)
281. (*d*)	**282.** (*d*)	**283.** (*a*)	**284.** (*d*)	**285.** (*c*)	**286.** (*c*)	**287.** (*a*)	**288.** (*b*)	**289.** (*c*)	**290.** (*d*)
291. (*a*)	**292.** (*a*)	**293.** (*a*)	**294.** (*c*)	**295.** (*d*)	**296.** (*c*)	**297.** (*c*)	**298.** (*c*)	**299.** (*d*)	**300.** (*d*)
301. (*a*)	**302.** (*b*)	**303.** (*c*)	**304.** (*a*)	**305.** (*c*)	**306.** (*a*)	**307.** (*d*)	**308.** (*b*)	**309.** (*c*)	

Numerical Type Questions

1. 20	**2.** 6.000000	**3.** 59	**4.** 2.0	**5.** 5, 4	6. 5	**7.** 5
8. 11	**9.** 1	**10.** 140	**11.**1	**12.** 100	**13.** 2	**14.** 6
15. 2.15	**16.** 14	**17.** 20	**18. 63**	**19.** 5	**20.** 21	**21.** 63
22. 7	**23.** 12	**24.** 2.9	**25.**15	**26.** 4	**27.** 2	**28.** 2
29. 2	**30.** 0.45	**31.** 10	**32.** 6.7 sec	**33.** 12	**34.** 15	**35.** 02
36. 3	**37.** 1	**38.** 2	**39.** 6, 1	**40.** 2	**41.** 9	**42.** 0
43. 11	**44.** 5	**45.** 1000	**46.** 1	**47.** 99	**48.** 21	**49.** 3
50. 4	**51.** 1	**52.** 15	**53.** 30			

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EXERCISE – II

MCQ Type Questions

1. (*a*)	**2.** (*a*)	**3.** (*b*)	**4.** (*a*)	**5.** (*d*)	**6.** (*d*)	**7.** (*b*)	**8.** (*a*)	**9.** (*d*)	**10.** (*c*)
11. (*c*)	**12.** (*c*)	**13.** (*a*)	**14.** (*b*)	**15.** (*c*)	**16.** (*c*)	**17.** (*b*)	**18.** (*b*)	**19.** (*d*)	**20.** (*b*)
21. (*a*)	**22.** (*d*)	**23.** (*a*)	**24.** (*a*)	**25.** (*c*)	**26.** (*b*)	**27.** (*d*)	**28.** (*b*)	**29.** (*c*)	**30.** (*d*)
31. (*b*)	**32.** (*c*)	**33.** (*b*)	**34.** (*d*)	**35.** (*a*)	**36.** (*c*)	**37.** (*c*)	**38.** (*b*)	**39.** (*c*)	**40.** (*a*)
41. (*c*)	**42.** (*d*)	**43.** (*c*)	**44.** (*a*)	**45.** (*a*)	**46.** (*d*)	**47.** (*b*)	**48.** (*c*)	**49.**(*c*)	**50.** (*c*)
51. (*c*)	**52.** (*d*)	**53.** (*a*)	**54.** (*a*)	**55.** (*b*)	**56.** (*a*)	**57.** (*a*)	**58.** (*a*)	**59.** (*c*)	**60.** (*c*)
61. (*b*)	**62.** (*b*)	**63.** (*b*)	**64.** (*b*)	**65.** (*c*)	**66.** (*b*)	**67.** (*a*)	**68.** (*c*)	**69.** (*b*)	**70.** (*a*)
71. (*a*)	**72.** (*d*)	**73.** (*)	**74.** (*a*)	**75.** (*)	**76.** (*)	**77.** (*b*)	**78.** (*c*)	**79.** (*d*)	**80.** (*c*)
81. (*b*)	**82.** (*d*)	**83.** (*b*)	**84.** (*b*)	**85.** (*c*)	**86.** (*b*)	**87.** (*a*)	**88.** (*c*)	**89.** (*d*)	**90.** (*a*)
91. (*d*)	**92.** (*b*)	**93.** (*a*)	**94.** (*d*)	**95.** (*b*)	**96.** (*c*)	**97.** (*d*)	**98.** (*d*)	**99.** (*a*)	**100.** (*c*)
101. (*)	**102.** (*b*)	**103** (*b*)	**104.** (*a*)	**105.** (*b*)	**106.** (*c*)	**107.** (*a*)	**108.** (*c*)	**109.** (*d*)	**110.** (*b*)
111. (*a*)	**112.** (*d*)	**113.** (*d*)	**114.** (*b*)	**115.** (*c*)	**116.** (*c*)	**117.** (*d*)	**118.** (*a*)	**119.** (*a*)	**120.** (*d*)
121. (*b*)	**122.** (*b*)	**123.** (*d*)	**124.** (*d*)	**125.** (*a*)	**126.** (*b*)	**127.** (*a*)	**128.** (*d*)	**129.** (*a*)	**130.** (*d*)
131. (*c*)	**132.** (*c*)	**133.** (*b*)	**134.** (*a*)	**135.** (*a*)	**136.** (*b*)	**137.** (*d*)	**138.** (*c*)	**139.** (*d*)	**140.** (*a*)
141. (*d*)	**142.** (*d*)	**143.** (*a*)	**144.** (*c*)	**145.** (*c*)	**146.** (*b*)	**147.** (*a*)	**148.** (*b*)	**149.** (*c*)	**150.** (*d*)
151. (*c*)	**152.** (*c*)	**153.** (*c*)	**154.** (*d*)	**155.** (*d*)	**156.** (*d*)	**157.** (*a*)	**158.** (*c*)	**159.** (*a*)	

Numerical Type Questions

1. (–5) **2.** (5) **3.** (19) **4.** (51) **5.** (15) **6.** (199) **7.** (0) **8.** (140) **9.** (10) **10.** (230)

11. (69) **12.** (24) **13.** (995) **14.** (506) **15.** (9 to 9) **16.**(1000 to 1000) **17.** (1.72 to 1.74)

18. (6 to 6) **19.**(7) **20.** (19) **21.** (1) **22.** (148) **23.** (36 to 36) **24.** (34 to 34)

25. (358 to 358) **26.** (6 to 6) **27.** (5 to 5) **28.** (150) **29.** (110)

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EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. Error, i is undefined in this program.

2. Integer value in array is only two so the value are print 000.

3. In this program given value in character form and print the value integer and flat so ans is 0.000000

4. The program output show the value of f_1, f_2, f_3.

5. The round of the x, is the float is in value of x.

So here y = (int) (x + 0.5)

6. The real number in C is treated as a double.

7. Because this programe are not completed so the compile time redeclaration of a program.

8. The string contain are mainly of the print output is "I am a boy \n\0".

9 . The print*f* ("%*f* %*f* ", *a*,*b*); because two floating value printed.

10 . When the print statement scan*f* () are true mainly of the data are Scan*f* ("% *f* %1*f*", &*a*, &b).

11. If the program for print the number and main of two the only of the data printf ("%c", **+ + arg*v*);

12. They all are wrong because these are not perform in c program.

13. The integer division in a C program then results in truncation.

14 . This type of variable declaration but not initalized will be garbage collection error.

15 . 'C' programing language constant expressions are evaluated at compile time, and string constants and size of array should be known, then all of these.

16 . 'C' expression and operator, evalution will be stopped if one of its components evaluates to true.

17 . They all are the correct and because the following are available in 'C' program enum variables can be assigned new values, enum variable can be compared.

18 . In scan*f* reading of string scan*f* is delimited by blank, while gets is not.

19 . The output of 124.

20 . Double is converted into float, then rounding takes place.

21 . When the condition is missing then it is assumed to be present and taken to the true.

22 . Loop are index value that is retained outside the loop and the index value can be changed from within the loop or goto can be used to jump out of the loop.

23 . The necessary to declare the type of function in the calling program if the function returns a non-interger value.

24 . The external variable is globally accessible by all function, has a declaration "extern" associated with it when declared within a function or will be initialized to 0 if not initialized. Then all of these.

25 . The answer and marks are mainly 0 is the value.

26 . The statement showing the pointer to a file.

27 . if while (*c* = getchar ( )! = 0) {

}

then the loop executed infinitely time.

28 . "unsigned *u*" indicates u is a/an unsigned integer number.

29 . "Short int" is used for variable which may require less storage than normal integers.

30 . *f*open retuns a pointer to FILE Filename, if it exists.

31 . if sprint*f* ( ) works like print*f*( ) but operator on used string.

32 . The primitive data structures are int, float and char.

33 . The program called the value of the data are intnum there is no value in this program then print none of these.

34 . The library function exit ( ) causes an exit from the block in which it occurs.

35 . The getch ( ) library function returns a character when enter is pressed.

36. In signed magnitude form, one bit is dedicated to store the sign. (*e.g.*, 1 for negative and otherwise). Only the remaining 15 bits are available to store the magnitude. Hence the answers.

37. The input is actually *a*\n*b*. Since we are reading only two characters, only *a* and \n will be read and printed.

38 . The program output is c-1.

39 . Printing a character as an integer prints a value that is implementation dependent.

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40. 263 in binary form is 100000111. If one tries to print an integer as a character, only the last 8 bits will be considered – the rest chopped off. So, in this case the ASCII value of 00000111 (*i.e.*, decimal 7) will be printed. Look in the ASCII table. It is ringing a bell !

41. The accessing a variable through its address is desirable because a function can return more than one value using this.

42. if int i = 5; then repeated execution may result in different addresses for i.

43. Using a pointer variable, without initializing it, will be disastrous, as it will have a garbage value.

44. b = (int *) **c; then the value of b becomes 5.

45. The first declaration means, first is a function (returning a character), whose only argument is, a pointer to a function that takes a character and float as arguments and returns an integer. The name of a function can be used as the starting address of the function (*i.e.* a pointer to it). So, option c is correct.

46 . According to declaration

char first (int (*) (char, float));

int second (char, float); is valid, then first (second);

48. The declaration unsigned c : 5 is replaced by unsigned : 6; then compiler will give a new name for the field.

49. The object oriented language is used to

1. own data types can be defined

2. an object oriented program can be taught to correct its own errors.

50. Let ab be 0 × MN. N&f should yield 7 *i.e*. N&1111 should produce 0111. So, N should be 0111, *i.e.*, 7. Similarly, M can be found to be 2. So, ab is 0 × 27.

51. If the pointer variable is the integer value the int * p showing.

52. The field on the disk is identified by the name inven.txt.

53. A function can make as many throws of as many types as necessary.

54. Function templates may have more than one parameter as long as they are of the same type.

55. The function that is actually created from a call to a templete spawned.

56. Function prototype as int calculate (int num);

may receive any integer variable from the main ( ) program.

57. If this type of statement template is << class T >>, this type of class is T.

58. A function that uses variable types is called variable function.

59. The prototyped as double calculate (int num); receive an integer variable.

60. In programming language arithmetic operators are used.

61. In 'C' programming language is used to only highest predence "unary +" operator.

62. This type of condition create then the operator (int num); are finally result find output float value.

63. The maximum number of dimensions an array in C there is theoritically no limit. The only practical limits are memory size and compilers.

64. In C programming operations deal directly with objects characters, integers and floating point number.

65. In the C programming language provides no input and output facilities.

66. The variable can be accessed by all modules in a program are called global variables.

67. Linked list storage structure for string can easily insert, delete, concatenate and rearrange substrings.

68. Linear search algorithm over an array of n elements is O(n).

69. Linked list of length n is O(n).

70. Worst case time complexity of linked list of length n is O(n).

71. The deleted element which is successor of the element pointed to by a given pointer O(1).

72. If the element n is inserted the linked list according to time taken, then some pointer is O(1).

73. The different times during the execution of program trace are used.

74. The key value is produced by trunc.

75. The worst case sorting algorithm is O(n^2) is the merge sort.

76. The linear search in an array of n elements the time complexity for best of the O/1, O(n) and O(n).

77. The average time required to sequential search for an element in an array A (1 : n) is given the (n + 1)/2 is always.

78. The time required to determine to the linear time.

79. The function sum (N : integer) : integer

begin

if N = 0 then Sum = 0

else

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end;

Sum = N + Sum (N – 1)

80. Average time required to perform successful sequential search for a given element of an array A (1 : n) is $\left(\frac{n+1}{2}\right)$ is the value.

81. Running out of memory may occur due to recursive function call.

82. The linear node is given by mean of pointer is called linked list.

83. The most appropiate is int * matrix; -------, if (matrix ! = null) free (void *) matrix.

84. This type of expression does not require precedence rule for evaluated more than one of these.

85. The finally result of S_1 and S_2 in "*c*" respectively of the data less than $m + n$.

86. Each comparison puts 1 element in the final sorted array. So, in the worst case $m + n - 1$ comparisons are necessary.

87. In the worst case it has to check all the 2^n possible input combinations, which is exponential.

88. The primitive recursive but partically recursive Ackermann's function.

89. In this program is output prints is prints binary equivalent of *num*.

90. main ( )

{

print*f* ("ln%*x*", – 1 >>4);

}

output : = ffff

91. In this program is the data are output is the counts the number of bits which are on in the number *num*.

92. The following program output is putting off all bits which are on in the number *n*.

93. In C program the constant is defined after main.

94. Print*f* ("%d" print*f* ("tim"));

output : result in a syntax error.

95. This program result in "segmentation violation".

96. The rule for implicit type conversion is 'C' is int < unsigned < double < float.

97. The following loop is output catrat.

98. According to the data one finds square root of 3 * a + 4 * b + 5.

99. print f ("%d", 10 ? 0 ? 5:1:12);

Result of the following statement 'C' is output *i*5*h*4g3*f*2 *e*1

100. The following program printing of k.

101. Print ouptut : 1 10 29 38 47 56 110 29

102. The function which returns nothing.

103. The function recursively called are initialized during each execution of the function.

104. A static variable retains its value throughout the file of the program.

105. A "switch" statement is used to choose from multiple possibilities which may arise due to different values of a single variable.

106. The statement that certain information about mathematical library functions are to be included at the beginning of the program.

107. for (i = 1; i < = 50 && found = = true; i ++) are boolean variable 'found".

108. The declaration define men and women as 100 element floating point arrays.

109. The *f*print*f* is used when ouput is to be printed on to a file.

110. The ordinary int variable is leftmost bit is reserved for sign.

111. The i is sqrt ((double) i)

112. Therefore it should return 0 or 1.

113. The function *f*open ("filename", "r") returns a pointer to FILE filename, if it exits.

114. The file "sample" could not be created for writing.

115. The value of ~ a is h*x*9248.

116. The expression a << 6 shifts all bits of a six places to the left then 0*x*6dc0.

118. The unions contain members of different data types which share the same storage area in memory.

119. The s_1 and s_2 is always less then $m + n$.

120. 5 – 2 – 3 * 5 – 2 will yield 18, if it is treated as (5 – (2 – 3)) * (5 – 2). *i.e.*, if–has precedence over* and if it associates from the right.

121. Let size of (int) = 1. So, – 4 will be stored as 11111100. Since we are adding unsigned and signed integers, the signed gets coverted to unsigned. So, $i + j$ will become 11111101. We are trying to print this as an unsigned integer.

So, what is printed will be 2^8 - 1 - 2. So, log (x + 3) = 8 (*i.e.*, *size of (int)).

122. The else clause has no brackets *i.e.,* {and}. This means the else clause is made up only one statement. So, printf ("a < = b"); will be executed anyway, *i.e*. if a>b a< = b. Hence the answer.

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123. If max is a function that returns the larger of the two integer then the max (max (a, b), max (a, c)), max (max (a, b), max (b, c)), max (b, max (a, c)) then all of these.

124. A pointer variable can be returned by a function.

125. The value of & x[i] is same as $x + i$.

126. Part of declaration of a structure, then storage class can be anything.

127. $a \to b$ is correct if a and b are structures.

128. The statement position the read-wire-head at the start of the file.

129. The item can removed ascending priority queue.

130. The matching is 2, 3, 1.

131. A function can make as many throws of as many types as necessary.

132. The represent the data many to many relation only, Plex and Graph only.

133. The average search time of hashing with linear probing is for less than one.

134. argv is a/an array of character pointers.

135. # define max (x, y) $x = (x > y)$? x : y

then the value x and y are both integers and x and y both declaration as float, x and y are both declarations as double. The all of these.

136. In the following declarations an array of 20 names where each name consists of a name, middlename and surname.

137. The value assigned to a, b, c if then a = 1 2 3 4 5 6 and nothing is assigned to b and c.

138. The value of outut a = 123, b = 4, c = 567 then are the value of the data.

139. The value of the output (i = 10, b = 256.8, C = "7")

140. scan$f$ ("%s", S)which input 12345 would be output "12345".

141. According to declaration black = 0, blue = 1, green = 2

142. The following declaration is black = – 1, blue = 0, green = 1

143. Size of the array need not be specified, when initialization is a part of definition, it is a declaration, it is a formal parameter.

144. The popped items are D.

145. The false statement is a tree contains a cylce.

146. The starting address of A[49] is then 1264 size consume.

147. The manipulating data on a queue.

148. The point marked [1] list [j] ≤ list [j + 1] for all j such that item ≤ j < n.

149. The list element before list 2 the

LINK [I LAST] : = TOP 2;

150. The stack overflow is 66.

151. The illegal array defination is

var a : array [REAL] of REAL,

152. According to the program A, B, C, D, E, F <EOF> stack are possible is

5	
4	
3	F
2	D
1	B

153. The stack are impossible are not possible.

5	
4	D
3	C
2	F
1	A

154. The all possible stacks with A, B, C, D, E, F < EOF > are possible stackes.

155. True

156. Programs that display department names in alphabetical order.

157. The every value of $n \in \{a, b\}$ then f is one to one not onto.

158. A serial search begins with the first array element is true but for a serial search to work the data in the array that must be search is small.

159. Serial

160. The contain only one element front = Rear = Null.

161. The sort many large object or structure it would be most efficient to place pointers to them in an array and sort the array.

162. The pointers to them in an array and sort the array.

163. A matrix "a" is called lower triangular if and only if for all $j > i$, $a_{ij} = 0$ then the value is

$1/2 * i(i + 1)j$.

164. The second largest element in an array is

$$n + [\log_2 n - 2]$$

165. Pop, Push, Pop, then the answer is the 2, 2, 1, 1, 2 are mainly.

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166. If search key matches the very first item, with one comparison we can terminate. If it is second, two comparisons, etc.

$$\therefore \quad \text{Average} = \frac{(1+2+....+n)}{n} = \frac{(n+1)}{2}$$

167. The postfix equivalent is 2 3 * 4 5 + – .

For evaluating this using stack, starting from the left, we have to scan one by one, if it is an operand push.

If it is an operator, pop it twice, apply operator on the poppedout entries and push the result onto the stack.

If we follow this, we can find configuration in option (d) is not possible.

169. The expression which accesses the (i, j)th entry of a m×n is $m \times (j - 1) + i$.

170. Load factor is the ratio of number of records that are currently present and the total number of records that can be present. If the load factor is less, free space will be more. This means probability of collision is less. So, search time will be less.

171. The stacks can't be used to allocate resource (like CPU) by the operating system.

172. The Array used in a program will be sorted in dope vector.

173. The trie-indexing are true is an m-ary tree and successful searches should terminate in leaf nodes or on successful searches may terminate in leaf nodes level of the tree structure then all of these.

174. The sequence is 3, 4, 5, 2, 1

175. An addressing scheme is deletion is easier.

176. The sequence of operations, the key chosen are in strictly decreasing order.

177. The sum of elements are 0.

178. This type of operation is not possible so they all are not possible, none of these option.

179. None of these

180. The doubly linked list than by linear linked list deleting a node whose location is given.

181. The time needed to insert element in a queue of length n is O(1).

182. The middle is called deque.

183. Queues serve a major role in simulation of limited resource allocation.

184. Link list implementation of sparse matrices is superior to the generalized dope vector method because it is conceptually easier and completely dynamic, efficient if the sparse matrix is a band matrix and efficient in accessing an entry are all of these.

185. Circularly linked list organization insertion of a record inovolves the modification of 2 pointers.

186. Stack is useful for implementing recursion.

187. Immediately after visiting a node, append it to the queue. After visiting all its children, the node currently in the head of the queue is deleted. This process is recursively carried out on the current head of the queue, till the queue becomes empty.

188. The queue defined abstractly are the mainly of the data are Front (Insert (Insert (e, Z), X)Y) = i;

189. The delete two node is successive main program is the LINK [X] : = [LINK [LINK [LINK [X]]];

190. The statements is 1. Insertion of an element should be done at the last node in a circular list then first is true.

191. It should be inserted at front position of the queue.

192. List traversing through the entire list is not necessary singly linked list and double linked list.

194. The doubly link list it may be either linear or circular.

197. Data structure may simple queue.

198. Data 4 * n is the data of 'n' elements.

199. Conventional way needs storage of $m \times n$

In the case of linked list implementation of sparse matrices, storage needed will be (the number of non-zero entries).

Only in case (c), both the methods need the same storage of 30.

200. Similar to manipulating data on a queue.

201. Circular doubley linked list.

202. This binary tree is complete binary tree.

204. The binary search tree is the left subtree and has value less than every value in its right subtree.

205. The sorting procedure is the Bubble Sort.

206. The following sorting method is stable in straight insertion sort.

207. Heap

208. The best describes sorting is arranging the data (record) in some given order.

209. Binary search uses order of the list.

210. A bubble sort that compares adjacement elements in a list and switches where necessary.

211. Selection sort is the first element with any element less than it and then repeats with a new first element.

212. A full binary tree with n leaves contains $2n - 1$ nodes.

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213. A binary tree with n non-leaf nodes contains $2n + 1$ nodes.

214. The 8 should be placed as the left child of node labelled 10.

215. The fibonacci tree is the relationship is $|F_d| = F_{d+3} - 1$.

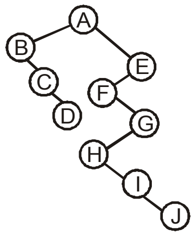
216. Average search time of hashing with linear probing is far less than one.

217. Trie-indexing it is a search tree of order m is false.

219. It can be proved by induction that a strictly binary tree with 'n' leaf nodes will have a total of $2n - 1$ nodes.

$\therefore$ Number of non-leaf nodes $= (2n - 1) - n = n - 1$.

220 . The binary tree is represented.



221. The binary search algorithm determines that the search argument is start Sub = middle Sub – 1 ;

222. Middle Sub = start Sub + stop Sub/2;

223. The data are searching is called search argument.

224. Maximum number of nodes on level i of a binary tree is $2^i–1$.

225. Maximum node is binary tree is $2^k–1$.

226. Stop sub = middle sub – 1;

227. The binary search continues having the array either until a match is found or until there are no more elements to search this is false statement.

228. AVL tree is I. In this tree heights of two subtrees of every node never differ by more than 1. II. balance factor of each node is – 1, 0, 1, then I and II is correct.

229. Subtree is called pre-order traversal.

230. Binary tree having n nodes and depth d will be about complete binary tree if any node nd at level less than d – 1 has two sons.

231. I and III statement is correct.

232. Binary tree depth 'd' is an almost complete binary tree if

(*a*) each leaf in the tree is either at level "d" all or level (d – 1)

(*b*) for any node "n" in the tree with a right descendent at level 'd', all the left descendents of "n" that are leaves are also at level "d".

233. The post order traversal ab-cd * + then the lable, +, –, *, a, b, c, d.

234. If it is to be used for sorting, label of left child should be less than the label of the current node. So, coming down the tree, we get left child of node labelled 10 as the correct slot for 8.

235. A strictly binary tree with 'n' leaves must have $(2n – 1)$ nodes. Verify for some small 'n'. This can be proved by the principle of mathematical induction.

236. Defined on a set of n elements is 2^n.

237. Left subtree of the root (7, 4).

238. Heap allocation language required that support dynamic data structure.

239. The binary relation n element is 2^{n^2}.

241. Valid binary tree (1(234) (567).

242. B^+ trees are disk access is much slower than memory access.

243. The full binary tree with n non-leaf notes contains $2n + 1$ nodes.

244. A full binary tree with n leaves contains 2n – 1 nodes.

245. Probability for the first record not colliding $= \frac{x}{x}$.

Probability for the second record not colliding

$= x - 1/x$.

(This is because one place is already occupied. So, favourable number of cases is $x - 1$).

Probability for the third record not colliding

$= x - 2/x$.

Probability for the $(m - 1)$ th record not colliding

$$= x - \frac{(m-2)}{x}.$$

Now next (m^{th}) record is resulting in a collision. So, out of x places, it should hash to one of the $(m - 1)$ places, already filled. So,

$$\text{probability} = \frac{(m-1)}{x}.$$

$$\therefore \quad \text{Required probability} = \left(\frac{x}{x}\right)\left(\frac{x-1}{x}\right)\left(\frac{x-2}{x}\right) \cdots \left(\frac{x-(m-2)}{x}\right)\left(\frac{m-1}{x}\right).$$

247. Recursively applying the relation, we finally get

$$\begin{aligned} T(n+1) &= c(n-1) + T(1) \\ &= c(n-1) + d \end{aligned}$$

hence order is n.

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271. If new record hashes onto one of the six locations 7, 8, 9, 10, 1 or 2, the location 2 will receive a new record. The probability is $\frac{6}{10}$ (as 10 is total possible number of locations).

274. For sorting 200 names, bubble sort makes $200 \times \frac{199}{2} = 19900$ comparisons.

So, time needed for 1 comparison is 200 sec (approximately).

In 800 sec it can make 80,000 comparisons.

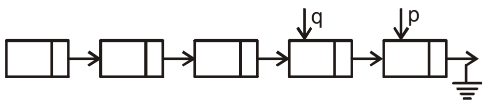
We have to find n, such that

$$n\frac{(n-1)}{2} = 80{,}000.$$

Solving, we get $n \approx 400$.

275. Each comparison will append one item to the existing merge list. So, in worst case, one needs $m + n - 1$ comparisons which is of order $m + n$.

290. When 'while' loop is over the states of p and q variables are as given in the diagram. Now node will become last node. Thus, q → next = NULL will be required.



Since, P is inserting in the front of the list p → next = head is required and p become first node of the modification list hence = p is required for the successful operation.

291. $$n\log \le n^{3/2} \le n^{\log n} \le 2^n$$

This is clear if we take logarithm of expressions

$$\log n + \log\log n \le (3/2)\log n = (\log n)^2 \le n$$

292. By the definition of the given data structures we can answer the question.

294. The get order of the mainly of the data and final the data are 1, 2, 3, 5, 7, 8

295. Boolean is BST (Tree * root)

{

int min = find min (root); .... > O(n)

int max = find max (root); .... > O(n)

return is BST 2 (root, min-1, max+1); ....> O(n)

}

boolean is BST 2 (Tree * root, int min, int max)

{

if (root)

}

return (min < root → val < max) $$ is BST 2 (root → left, min, root → val) ZZ is BST 2 (root → val, max);

{

return true;

}

is BST 2 is implemented using preorder traversal and that takes up not more n unit in worst case. Thus is BST 2 running time in worst case is O(n). Which implies is BST running time is of O(n) in worst case. If tree were having a balanced structure then it would be O(log n) in worst case–

∴ Consider the following code to answer the question.

297. Inserting 16 requires no rotations.

Inserting 15 requires 1 double rotations.

Inserting 14 requires 1 double rotations.

Inserting 13 requires 1 single rotations.

Inserting 12 requires 1 single rotations.

Inserting 11 requires 1 single rotations.

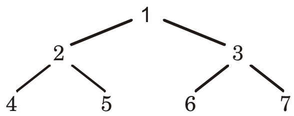
Inserting 10 requires 1 single rotations.

Inserting 8 requires no rotations.

Inserting 9 requires 1 single rotations.

So, we need 4 single and 3 double rotation operation.

298. Since for given number of elements the structure of binary heap is always fixed hence unique binary heap can always be identified using post order (or any traversal for that matter). The following is the binary heap for which post order is given.



Now run the algo. as given in the question on this tree structure.

299. Firstly let construct binary tree structure of the given max heap using its inorder. Then apply insert operation for 30. Notice that this is not level order sequence of that max heap.

300. Use the heap property to identify the correct heap.

302. If median is always used as pivot, then recussion remains $T(n) = T\left(\frac{n}{2}\right) + cn$ for all the cases where cn is combined time for median finding and partition. So worst case time complexity of the qick sort becomes 0 (n $\log_n$).

306. Binary search is just locating the correct position however, still have to shift elements towards right for vacating the position for the each current element. Thus by, it will remain n^2.

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307. Please observe that the weight of all the edges is unique. Hence, such graph cannot have more than one minimal spanning tree.

308. By the definition of space complexity.

309. n (46) = 6 location 6 is free hence placed.

n (34) = 4 location 2 is not free as 42 is already there.

n (42) = 2 linear probing will return index 5.

n (23) = 3

n (52) = 2

Hence 52 will be placed at 52.

n (33) = 3 linear probe will return index 7.

NUMERICAL TYPE QUESTIONS

1. In this program int value is 20 then the output is 20.

2. Output of sqrt of 36.0 is floating and will be 6.000000.

3. Because the loop body < 60, then the answer is 59.

5. if $n == 5;$

then $x = n++$

y $= -x$

then $x = 5$

and increment the value

and $y = -5$

then result will be

$x = 5, y = 4$

8 . The statement

print$f$ "% d ", 10 ? 0 ? 5 : 1 : 12);

print$f$?

output : ⇒ 11

9 . x and y are variable then declared number is 1.

10 . Following the program structure defined is output 140

11 . The value of the expression is 1.

12 . The reserves 100 successive memory.

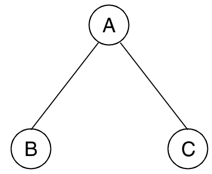
13 . The maximum element of the root is 2.

17 . The data array is n = 20 then find location.

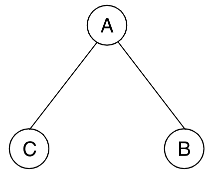
18 . The binary tree is the complete binary tree $n \times n$ then 5 is 63.

20 . Minimal spanning tree for the graph in problem 234 is the 21.

23. The tree may be of depth 2 or 1. If 2, we have 6 possible trees. Because one of the three nodes A, B, C may be the root. The next level may be one of the remaining two. So, 6. If the depth is 2, the root may be one of the 3 nodes A, B, C corresponding to a root say A, two trees are possible as this.

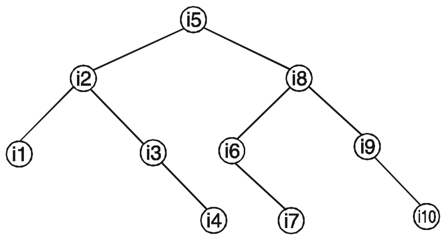


and



So, 6 possibilities. So, altogether 12 possible trees

24. The 10 items $i1$, $i2$ .... $i10$ may be arranged in a binary search tree as shown in the figure below. So, to match $i5$, the number of comparison needed is 1; for $i2$, it is 2, for $i8$ it is 2, for $i1$ it is 3 and so on.



$$\therefore \text{ Average} = \frac{(1+(2+2)+(3+3+3+3)+(4+4+4))}{10} = 2.9$$

26. 4

27. The maximum element of the root is 2.

29. We can verify that the 1st, 3rd, 5th, 7th ... probes check at location 5.

2nd, 6th, 10th ... probes check at location 8.

4th, 8th, 12th ... probes check at location 4.

Rest of the address space will never be probed.

30. If there is only one record, then

probability of a collision = $\frac{1}{100}$

If there are two records,

then probability of collision = $\frac{2}{100}$

and if there are 9 records,

then probability of collision =. $\frac{9}{100}$

∴ Required probability

$$= \frac{1+2+3+......+9}{100} = 0.45$$

33.

$$= f_{00}\,(345, 10)$$
$$= 5 + f_{00}\,(34, 10)$$
$$= 5 + 4 + f_{00}\,(3, 10)$$
$$= 5 + 4 + 3 + f_{00}\,(0, 10) = 12$$

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34.

a ↓ 15

$F = (\{12, 7, 13, 4, 11, 6\}, 6)$

$12 + \frac{3}{2}$ a

$F = (\{7, 13, 4, 11, 6\}, 5)$

$7 - \frac{4}{4}$ a

$F = (\{13, 4, 11, 6\}, 4)$

$13 - \frac{9}{9}$ a

$F = (\{4, 11, 6\}, 3)$

$4 + \frac{5}{5}$ a

$F = (\{11, 6\}, 2)$

$11 - \frac{6}{6}$

$F = (\{6\}, 1)$

$6 + \frac{0}{0}$

$F = (\{\ \}, 0)$

35. i, j, variables are global to main function p and q will also be pointing to same memory location where i and j respectively, when $f(\&i, \&j)$ is called Up.

Now p = q statement will move p to the memory location being pointed by q

Thus * p = 2 will replace 1 by 2

There is no change in the value of variable i

(which is still 0);

36. Use the hornor algorithm

1st iter	$a_3 * x$	....	1 multiplication
2nd iter	$(a_2 + a_3 * x) * x$	....	1 multiplication
3rd iter	$(a_1 + (a_2 + a_3 * x) * x)$	....	1 multiplication
4th iter	$a_0 + (a_1 + (a_2 + a_3 * x) * x) * x$	....	1 multiplication

The expression in bold represents that is already computed.

37. Use one queue and a counter to keep track number of elements present in the queue.

38. Use two stacks one stack to keep the elements as usual and another one for keeping maximum element. Thus push code is modified to see if the new element is greater than the top element of S_{max} if so the same elements will be push in both stacks. Therefore top of S_{max} is always the maximum element. And pop will see if deleting element is on S_{max} then that also be deleted.

39. Given postfix expression is

823 □ / 23 * + 51 * –

Expression symbol	OP1	OP2	Value	Top S(RL)
8				8
2				8,2
3	2	3	8	8,2,3
^	8	8	1	8,8
/				1
2				1,2
3				1,2,3
*	2	3	6	1,6

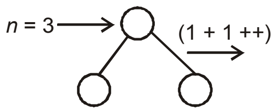
So the top two elements of the stack are 6, 1 after the first * is evaluated.

40. The inserted empty BST, then the nodes right subtree after instruction of 2.

42. We will use induction on such trees.

$n = 1 \rightarrow 0$ numbers of descendant is 1

$n = 2 \rightarrow$ tree is not possible as per given defination.



Number of required nodes are zero which is not possible in such tree structure. Hence, zero is right.

46. You should assume input such that being distributed among leaf nodes so that splits differed as much as possible for getting smallest height for a given set of keys. Remember more splits at leaf level cause more keys to be promoted to higher levels hence may result in more splits to be promoted to higher level, therefore there is a high probability for getting more height.

47. Look at any B-Tree for any number of leaf nodes, if L is leaf node (L > = 2) then keys in the internal nodes is always L – 1. This is because each split at leaf level gives birth to new leaf node and promote one key value into internal nodes.

48. You have to assume an input order such that there will be higher chances of node split possibly at each key insertion. Now follow the insert algorithm with this assumption. Certainly 21st key is the first key that will increase the height from 1 to two.

Using N_{min} formula you will get 17 but that's the different of degree 3 of height 2 certainly will have at least 17 keys however to bring the height to 2 from one at least 21 keys are required.

49. As per B^+ tree definition each node to be full at least 50% of its total capacity.

50. Only two tree structure are possible that has 4 leaf nodes, only right tree produces the maximum internal nodes.

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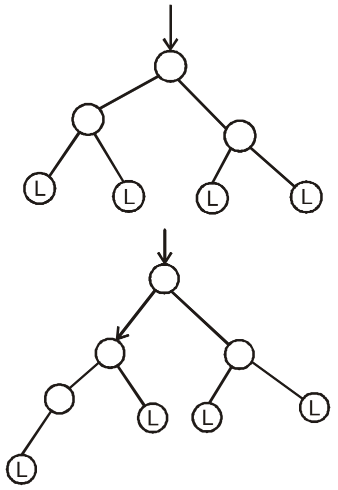
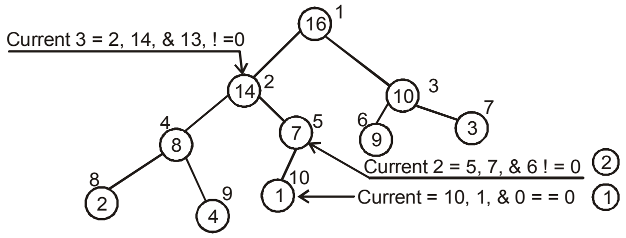


51. The function visits each node in the path starting from the last node to root node except the root node.

It prints the value that is power of 2 (XZX – 1 = = 0; means X is power of 2 Z).

So the nodes in the path are [1, 7 and 14]. And only '1' (2^0) is power of 2.



52. Compare all the elements pair wise, total 5 comparisons. We got two seats, first seat has all greater elements and second set has all smaller elements. Find the largest in the first in 5 comparision and smallest in the second set with 5 comparisons. Total 15 comparisons.

53. Please notice 52 cannot be appeared in any of first three position of any input sequence otherwise we will not get the needed hash table. Also, 33 can only be appeared in the last position.

EXERCISE II

MCQ TYPE QUESTIONS

1. Maximum number of nodes with height

$h \Rightarrow 2^{5+1} - 1 = 63$ nodes is $2^{h+1} - 1$

and minimum no. of nodes with height h is $h + 1$.

Hence 5 + 1 = 6 nodes.

2. 1. 3, 5, 7, 8, 15, 19, 25

4. 4, 6, 7, 9, 18, 20, 25

All nodes are in ascending order inorder traversal of a binary search tree.

4. 2036, 2036, 2036,

It will return initial address for all three

i.e 2036

5. n log(log n)

First for loop is running n times nested loops are running log (log n) times.

Hence total running time is n log(log n)

6. DCBA is the output

The condition will fail on encountering (' '-space), and printed character will be DCBA

7. A bridge can not be part of simple cycle

11. 1204

S_1 is

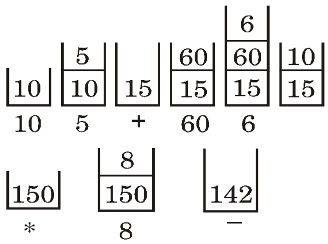
1	2	3	4
1	2	3	4

$P = S_1 + 2$

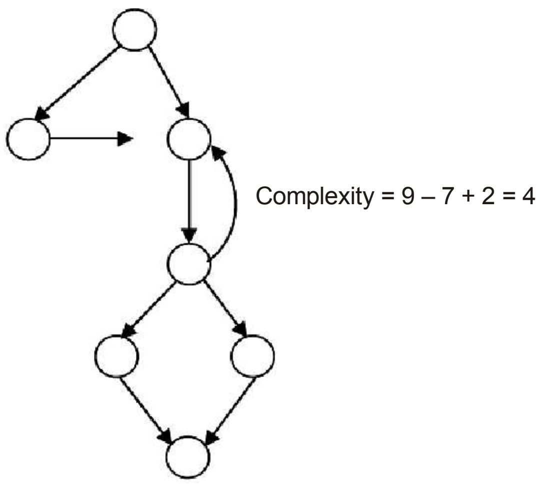
P = 0

1	2	0	4

12.



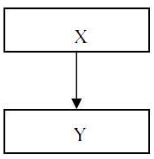
13. **Program-X :**



Program-Y :

Cyclomatic complexity = 10 – 8 + 2 = 4

Program Z :



Total number of edges = X + Y + 1 ⇒ 20

Total number of vertices = 15

Hence cyclomatic complexity = 20 – 15 + 2 = 7

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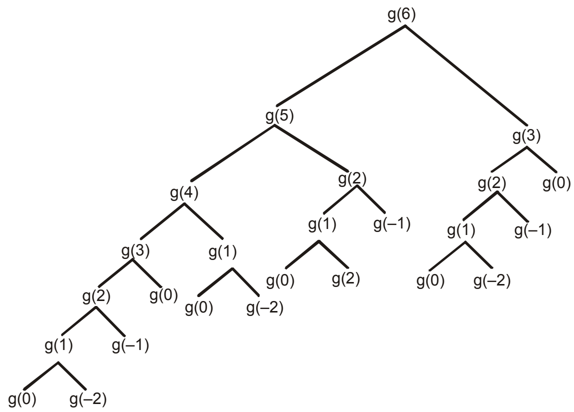
14.



Total calls = 25

16. Prims algorithm is greedy method because it choose minimum distance node.

Floyd warshal's is dynamic because it changes the distance on each iteration.

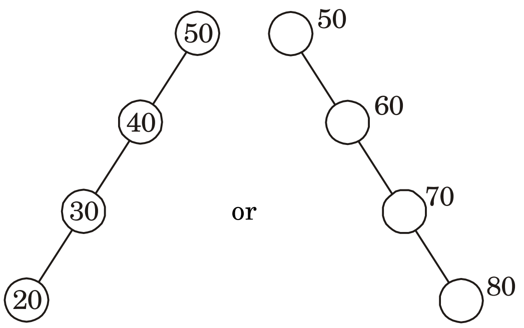
Merge sort is divide and conquer because it divide the hole list is sublist and combine after sorting each sublist Hamiltonian cycle is based on backtracking.

17. The recurrence equation is

$$T(n) = T(n-1) + T(1) + cn$$

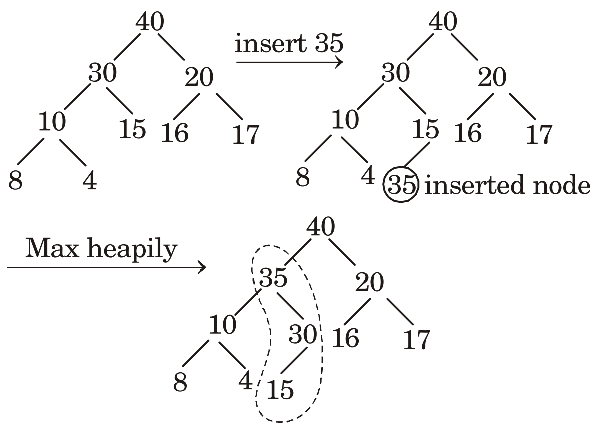
In quick sort, worst case divide the list into two list with 1 and $(n-1)$ elements each time.

18. Both insertion and deletion of a key is a binary search tree is Q(n) in worst case as both are worst case of BST will take Q(n) for insertion and deletion.



19. $d(u) - d(v) \neq 2$

20. The max heap is



Then list is

40, 35, 20, 10, 30, 16, 17, 8, 4, 15

21. Unsorted array

The algorithm perform

find operation $(\log N)^{½}$

Insert operation N

delete operation $(\log N)^{½}$

decrease key operations $(\log N)^{½}$

Hence unsorted array is best data structure for all above operations.

22. Suppose a list contains n elements, consider first three element and find middle element which will be neither maximum nor minimum.

Hence it is $\Theta(1)$.

23. The subtrees are already Max-heap, so to make it half, we have to heapify the root, which takes $\Omega(\log n)$ time.

24. Q_1 is in NP and Q_2 is in NP-hard

25. Dijkstra use gready approach, warshals use dynamic programming approach, Binary search is based on divide and conquer and Back tracking is depth first search.

26. $h(i) = l^3 \bmod 10$

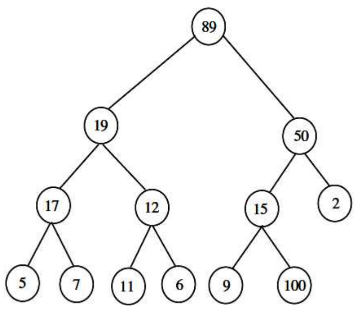
i = 1	=	1^3	mod 10	=	1
2	=	2^3	mod 10	=	8
3	=	3^3	mod 10	=	7
4	=	4^3	mod 10	=	4
5	=	5^3	mod 10	=	5
6	=	6^3	mod 10	=	6
7	=	7^3	mod 10	=	3
8	=	8^3	mod 10	=	2
9	=	9^3	mod 10	=	9
10	=	10^3	mod 10	=	0

Hence the hash function will be $h(i) = l^3 \bmod 10$

27. 1st swap is : 100 and 15

2nd swap is : 100 and 50

3rd swap is : 100 and 89



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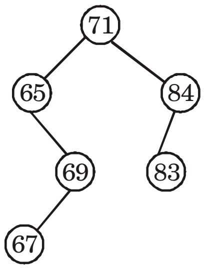
28.



Lowest level element is 67.

29. I or III or IV but not II

as $\sum_{i=0}^{n} i^3 \Rightarrow \frac{n^2(n+1)^2}{4}$,

this can be represent by $Q(n^4)$, $O(n^5)$ 4 $\Omega(n^3)$ but not $Q(n^5)$.

30. As $-1 \le \sin x \le 1$, neither of them is true

31.

$$\begin{aligned} O(n \log n) &= 30s \\ n &= 64 \\ O(64 \log 64) &= 30 \end{aligned}$$

Hence will get factor of 12.8 for 6 min

$$\begin{aligned} &= 6 \times 60 = 360 \text{ s} \\ O(256 \log 256) &= 360 \\ O(512 \log 512) &= 360 \\ O(1024 \log 1024) &= 360 \\ O(2048 \log 2048) &= 360 \end{aligned}$$

So for 512 will get 12.8 as a factor

32.

$$L_2 \le_p L_4$$
$$L_1 \le_p L_2$$

If $L_4 \in P$

then $L_2 \in P$

$L_1 \in P$,

Hence option (*c*).

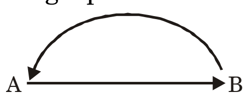
33. Take an example for Graph G

$\overset{\cdot}{A}$ $\overset{\cdot}{B}$

Then option A and D will be eliminated.

Let G is below graph

Then G_3 is a graph with below structure



In G the numbers of strongly connected components are 2 where as in G_3 it is only one.

34. pi contains the address of *i*. So scanf("%d",pi) places the value entered in console into variable i.e So printf("%d\n", *i* + 5),prints 5 more than the value entered in console.

35.

```
int Myx (int*E, un sin ged int sixe)
{
    int Y= 0 ;
    int Z ;
    int i, j, k ;
    for (i = 0; i < size ; i++)

            Y = Y + E[i]

    for (i = 0; i < size ; i++)
    for (j = i; j < size ; j++)
        {
        Z = 0
        for (k = i; k <= j ; k++)
            Z = Z + E[k];

        if (Z >Y) → Checks whether sum of
                    elements of each subarry
        Y = Z ;     is greater than the sum of
                    elements of array if so, that
                    sum is assigned to Y, if not
                    'Y' will be the sum of
                    elements of complete array
        }
    return Y ;
}
```

Ultimately returns the maximum possible sum of elements in any sub array of given array E.

36. $i = 1, j = 1, k = 2$ to $n \Rightarrow n - 1$ times

$i = 1, j = 2, k = 3$ to $n \Rightarrow n - 2$ times

$i = 1, j = 3, k = 4$ to $n \Rightarrow n - 3$ times

:

$i = 1, j = n-2, k = n-1$ to $n \Rightarrow$ 2 times

$i = 1, j = n-1, k = n$ to $n \Rightarrow$ 1 time

$i = 2, j = 2, k = 3$ to $n \Rightarrow n - 2$ times

$i = 2, j = 3, k = 4$ to $n \Rightarrow n - 3$ times

: :

: : $\Sigma (n-1)$ times

$i = 2, j = n-1, k = n$ to $n \Rightarrow$ 1 time

:

:

$i = n-1, j = n-1, k = n$ to $n \Rightarrow$ 1 time $\}\Sigma$ 1 times

$\therefore$ Total number of multiplications

$\Rightarrow \Sigma 1 + \Sigma 2 + \Sigma 3 + \ldots \Sigma(n-1)$

$$= \underset{S_1}{\underset{\downarrow}{1}} + \underset{S_2}{\underset{\downarrow}{(1+2)}} + \underset{S_3}{\underset{\downarrow}{(1+2+3)}} + \ldots + \underset{S_{n-1}}{\underset{\downarrow}{(1+2+3+\ldots n-1)}}$$

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$$= \sum_{i=1}^{n-1} S_i = \sum \frac{n(n-1)}{2}$$

$$= \frac{1}{2}\sum n^2 - \frac{1}{2}\sum n$$

$$= \frac{1}{2}\frac{n(n+1)(2n+1)}{6} - \frac{1}{2}\frac{n(n+1)}{2}$$

$$= \frac{(n-1)(n)(n+1)}{6}$$

37. According to havel-hakimi theorem

(1,1,1,1,1,1) is graphic iff<1,1,1,1,0> is graphic

(0,1,1,1,1) is graphic iff (0,1,1,0) is graphic

(0,0,1,1) is graphic iff (0,0,0) is graphic

Since (0,0,0) is graphic (1,1,1,1,1,1) is also graphic.

(The process is always finding maximum degree and removing it from degree sequence, subtract 1 from each degree for d times from right to left where d is maximum degree)

(2,2,2,2,2,2) is graphic iff (2, 2, 22 -1, 2 -1)

= (2, 2, 2,1,1) is graphic.

(1,1,2,2,2) is graphic iff (1,1,1,1) is graphic.

(1,1,1,1) is graphic iff (0,1,1)

(0,1,1) is graphic iff (0,0) is graphic.

Since (0,0) is graphic (2,2,2,2,2,2) is also graphic.

Consider option C now.

(3,3,3,1,0,0)→(0,0,1,3,3,3) is graphic iff (0,0,0,2,2) is graphic.

Note that before applying the havel-hakimi step degree sequence should be in non-increasing order.

(0,0,0,2,2) is graphic iff (0,0,-1,1) is graphic.

Since (0,0,–1,1) is not graphic (3,3,3,1,0,0) is also not graphic.

38. $^nC_3$ will stand for

$$= n * \frac{n-1}{2} \quad \text{... }(i)$$

$$\frac{n-1}{3} \quad \text{... }(ii)$$

Multiplying (i) by (ii), we get

$$\rho = n * \frac{(n-1)}{2} * \frac{(n-2)}{3}$$

39. Since the consumer will always execute wait(s) following by wait (n) due to which it goes to sleep by continuously decreasing value of n. After the consumer sleeps, the turn comes to producer which goes to sleep by executing wait(s). This process continues and causes dead lock.

40. X is declared as

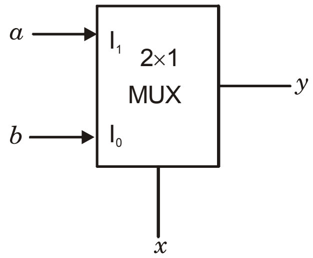
"int X [32] [32] [8]"

41. Since enqueue means to add an item from the back called push, So it will take all the three instructions ie PUSH, POP & REVERSE, but DEQUEUE doesn't have to take any thing from PUSH, REVERSE because it removes item from front called POP. Hence, a queue will be implemented ENQUEUE takes 3 instructions, DEQUEUE takes 1 instruction.

42. When f = 50, the f^n will exhaust run time stack, because function will be called infinite times which results into an infinite loop at j = 50.

43. $y = \bar{x}b + xa$

'x' is working as selection line, where the two input lines are 'a' and 'b', so the function F(x, y, a,b) can be implemented using (2 × 1) multiplexer as follows:



44. In the computation of given pseudo code for each row and column of Matrix A, each upper triangular element will be interchanged by its mirror image in the lower triangular and after that the same lower triangular element will be again re-interchanged by its mirror image in the upper triangular, resulting the final computed Matrix A same as input Matrix A.

45. In order Traversal of Ternary Tree is done as follows:

Left → Root → Middle → Right

So the nodes are visited in SQPTRWUV order.

46. The key to solving such questions is to understand or detect where/by what condition the value (or the counter) is getting incremented each time. Here, that condition is if

(tree → leftMostchild == Null)

⇒ Which means if there is no left most child of the tree (or the sub-tree or the current nodeas called in recursion)

⇒ Which means there is no child to that particular node (since if there is no left most child, there is no child at all).

⇒ Which means the node under consideration is a leaf node.

⇒ The function recursively counts, and adds to value, whenever a leaf node is encountered.

⇒ The function returns the number of leaf nodes in the tree.

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47. By the logic of the algorithm it is clear that it is an attempted implementation of Binary Search. So option (*c*) is clearly eliminated. Let us now check for options (*a*) and (*d*).

A good way to do this is to create small dummy examples (arrays) and implement the algorithm as it is. One may make any array of choice. Running iterations of the algorithm would indicate that the loop exits when the x is not present. So option (*a*) is wrong. Also, when x is present, the correct index is indeed returned. (*d*) is also wrong.

Correct answer is (*b*). It is a correct implementation of Binary Search.

48. Let $n_1, n_2, \ldots n_k$ be the number of vertices respectively in K connected components of a forest G, then $n_1 - 1, n_2 - 1, \ldots, n_k - 1$ be the number of edges respectively in K connected components and $n_1 + n_2 + \ldots + n_k = n$ (number of vertices in G)

Hence, number of edges in G = number of edges in K connected components

$$= (n_1 - 1) + (n_2 - 1) + \ldots + (n_k - 1) = n - k$$

49. DFS visits each vertex once and as it visits each vertex, we need to find all of its neighbours to figure out where to search next. Finding all its neighbours in an adjacency matrix requires O(V) time, so overall the running time will be $O(V^2)$.

50. Topological ordering of a directed graph is a linear ordering of its vertices such that for every directed edge uv from vertex u to vertex v, u comes before v in the ordering. Topological ordering is possible iff graph has no directed cycles.

(*a*) As the given graph doesn't contain any directed cycles, it has at least one topological ordering. So option (A) is false

(*b*) PQRS cannot be topological ordering because S should come before R in the ordering as there is a directed edge from S to R. SRQP cannot be topological ordering, because P should come before Q in the ordering as there is a directed edge from P to Q

(*c*) PSRQ and SPRQ are topological orderings as both of then satisfy the above mentioned topological ordering conditions.

(*d*) PSRQ is not the only one topological ordering as SPRQ is other possibility.

51. Partition algorithm for quick sort

Partition (A, P, q)// A [P, ...q]

$x \leftarrow$ A [P] // pivot = A [P]

$i \leftarrow$ P

for j = P + 1 to q

  do if A [j] $\leq x$

    then $i \leftarrow i + 1$

      exchange A [i] $\leftrightarrow$ A [j]

exchange A [P] $\leftrightarrow$ A [i]

return i [returning where pivot element is there after partitioning]

Recursively call the above algorithm for the two sub arrays [elements before and after pivot element] to complete the sorting.

x = pivot

```
1  2  3  4  5        2 ≤ 1 ? NO
i  j

1  2  3  4  5
i     j              3 ≤ 1 ? NO

1  2  3  4  5
i        j           4 ≤ 1 ? NO

1  2  3  4  5
i           j        5 ≤ 1 ? NO
```

↗ Pivot =x = A [B]

```
1  2  3  4  5        3 ≤ 2 ? NO
   i  j

1  2  3  4  5
   i     j           4 ≤ 2 ? NO

1  2  3  4  5
   i        j        5 ≤ 2 ? NO
```

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exchange A [P] & A [i]

1 2 3 4 5

Pivot ≥ Pivot Call recursively for this subarray

x = Pivot = A [P]

1 2 3 4 5

i j $\quad$ 4 ≤ 3 ? NO

1 2 3 4 5

i j $\quad$ 5 ≤ 3 ? NO

exchange A [P] & A [i]

1 2 3 4 5

Pivot ≥ Pivot

x = Pivot = A [P]

4 1 5 3 2

i j $\quad$ 1 ≤ 4 ? Yes

i ← i +1exchange A[i] & A[j] & increment j

4 1 5 3 2

i j $\quad$ 5 ≤ 4 ? NO

4 1 5 3 2

i j $\quad$ 3 ≤ 4 ? Yes

i ← i +1 exchange A[i] & A[j] & increment j

4 1 3 5 2

i j $\quad$ 2 ≤ 4 ? Yes

i ← i + 1

4 1 3 2 5

i j

exchange A[P] & A[i]

2 1 3 4 5

≤Pivot Pivot ≥Pivot

exchange A [P] & A [J]

1 2 3 4 5

Pivot ≥ Pivot

Call recursively for this

x = Pivot

1 2 3 4 5

i j $\quad$ 5 ≤ 4 ? NO

exchange A [P] & A[i]

1 2 3 4 5

Pivot ≥ Pivot

∴ total 10 conparisons

x = pivot = A (P)

2 1 3 |4| 5

i j $\quad$ 1 ≤ 2 ? Yes

2 1 3 |4| 5

i j $\quad$ 3 ≤ 2 ? NO

exchange A [P] & A[i]

1 2 3 4 5

Pivot

∴ 6 conparisons

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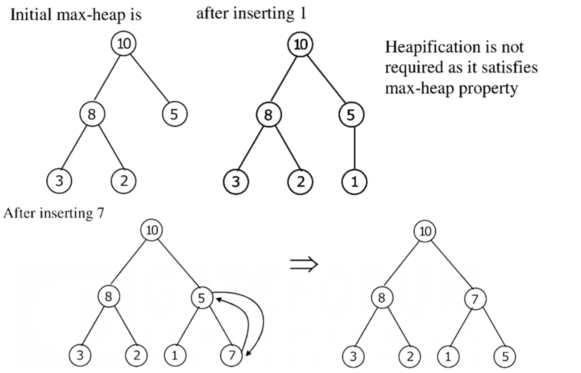
52. The most important open question in complexity theory is whether the **P** = **NP** , which asks whether polynomial time algorithms actually exist for **NP**-complete and all **NP** problems (since a problem "C" is in NP-complete, iff C is in NP and every problem in NP is reducible to C in polynomial time). In the given question it is given that some polynomial time algorithm exists which computes the largest clique problem in the given graph which is known NP-complete problem. Hence P = NP = NP-Complete.

53.



Hence level order traversal is 10, 8, 7, 3, 2, 1, 5

54. Since $f(n) = \log n$

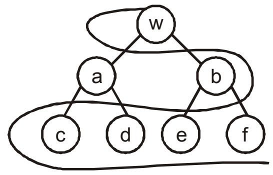
$a = 2, b = 2$

finding $\log_b a$ using master's method = $\log_2 2 = 1$

Hence, $f(n) = n'$

So, $T(n) = O(n)$

55.



Shortest path from w to every vertex in graph.

56. The Worst case time complexity of quick sort is O (n^2). This will happen when the elements of the input array are already in order (ascending or descending), irrespective of position of pivot element in array.

57. A

P(First insertion in such a way that first 3 slots are unfilled) $= \dfrac{97C_1}{100C_1} = \dfrac{97}{100}$

B

P(second insertion in such a way that first 3 slots are unfilled) $= \dfrac{97C_1}{100C_1} = \dfrac{97}{100}$ (∵ chaining is used to resolve collision, so second insertion can be done at same index as first index]

C

P (Third insertion in such a way that first 3 slots are unfilled) $= \dfrac{97C_1}{100C_1} = \dfrac{97}{100}$ (∵ Third insertion can be done at same index as first or second index]

So total prob. P (A) × P (B) ×P(C)

$$= \frac{97}{100} \times \frac{97}{100} \times \frac{97}{100} = \frac{(97 \times 97 \times 97)}{100^3}$$

58. We know that $v + r = e + 2 \Rightarrow e = n + r - 2$ ...(1)

Where V = n(number of vertices);

r = number of faces and

e = number of edges

Given, $\delta \geq 3$ then $3n \leq 2e$

$\Rightarrow e \geq \dfrac{3n}{2}$ $\Rightarrow$ $n + r - 2 \geq \dfrac{3n}{2}$ (using (1))

$\Rightarrow r \geq \dfrac{3n}{2} - n + 2 \Rightarrow r \geq \dfrac{n}{2} + 2$ ∴ Number of faces is atleast $\dfrac{n}{2} + 2$

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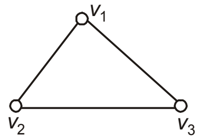
59. To find the expected number of unordered cycles of length three.

To compute:

Look at all the possible triplets of 3 vertices out of 8 and see if they form a cycle.

Possible triplets: We are looking at unordered cycles, so it suffices to chose any 3 vertices out of the 8 in any order. This can be done in ${}^8C_3$ ways.

If they have a cycle among them? To form a cycle between 3 chosen vertices, we need to form edges between all pairs



So, probability of this happening

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

So, by definition of Expectation, which is if X is a random variable oveer sample space Ω

$$\underline{|F}[x] = \sum_{x \in \Omega} x\,P(x)$$

Here $P(x) = \frac{1}{8} \forall x$

& x spans over all ${}^8C_3$ configurations

Hence, expected value $= {}^8C_3 \times \frac{1}{8} = 7$

Hence, the solution is (c)

60. P : Number of odd degree vertices is even

Q : Sum of degrees of all vertices is even

Q is true:

Reason: Calculating the sum of degrees of all vertices.

Take any edge, it is joining two vertices (not necessarily distinct), hence contribution 2 in the sum of degrees. Hence, for 'e' edges, the sum of degrees of all vertices is $2e$ (is even).

Note: It is volid even in case of self loops

P is also true:

Reason: We have established that sum of degrees of all vertices is even.

Let us assume, number of odd degree vertices is odd, so, the contribution of odd degree vertices in total sum is odd.

Now, the contribution of even degree vertices is also even (whether the number is even or odd).

So, total sum becomes odd, which is not possible. Hence P is true.

Note: Include vertex of degree zero in even degree vertices.

61. So, this is in $\theta(\log n)$

Hence Answer is (c)

The outer for-loop goes for $\frac{n}{2}+1$ iterations. The inner for-loop runs independent of the otuer loop.

And for each inner iteration. $\frac{n}{2}$ gets added to k.

$\therefore$ Our answer $= \frac{n}{2} \times$ # outer loops × #Inner loops per outer loop

Inner loops $= \theta(\log n)$ $[\because 2^{\theta(\log n)} = \theta(n)]$

$$\therefore \quad \text{Our answer} = \frac{n}{2} + \left[\frac{n}{2}+1\right] \cdot \theta(\log n) = \theta(n^2 \log n)$$

$L_1 = \{0^p\, 1^q\, 0^r \mid p, q, r \geq 0\}$

$L_2 = \{0^p\, 1^q\, 0^r \mid p, q, r \geq 0, p \neq r\}$

62. Sol. Option (a) : Suppose process X executes Exit X then it will wait for R, and then process Y executes Entry Y then it will wait for S. Since initially both binary semaphore are 0, no one will increment it and both process will stuck in dead lock.

Hence option (a) is wrong.

Option (b) : Here if process X executes for n times repeadetly it will set both semaphorse to 1 (since only two values are possible) and after that process Y executes. First time it passes the Entry Y and make both semaphores to 0. And on second time it finds both semaphores to 0 and cannot pass the Entry Y barrier. Hence it will stuck.

So option (b) is wrong.

Option (d) : Suppose first process X executes it sets R = 1 and then waits for S. Now after that process Y executes. It first sets S = 1 and then decrement R = 0. It comes again and then again sets S = 1 (i.e. it overlaps the value of S) and then wait for R.

Clearly here we lost one iteration of process X due to overlapping of value of S. and after n–1 iteration process X will stuck.

So option (d) is wrong.

Option (c) : Here take any sequence of operation of process X and process Y, first process X will wait for S which is increment by process Y and then process Y waits for R which is incremented by process X.

There is no sequence of operation in which the value of R or S overlaps.

Hence both process executes one after another.

So option (c) is correct.

To find : Tighest upper bound on number of swaps required to sort n numbers using selection sort.

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63. Since, reference of p is passed as 'x'

Thus, any change in value of x in f would be reflected globally.

The recursion can be broken down as:

$f\begin{pmatrix} 5 & 5 \\ x, & c \end{pmatrix}$

$\hookrightarrow x * f\begin{pmatrix} 6 & 4 \\ x, & c \end{pmatrix}$

$\hookrightarrow x * f\begin{pmatrix} 7 & 3 \\ x, & c \end{pmatrix}$

$\hookrightarrow x * f\begin{pmatrix} 8 & 2 \\ x, & c \end{pmatrix}$

$\hookrightarrow x * f\begin{pmatrix} 9 & 1 \\ x, & c \end{pmatrix}$

$\hookrightarrow 1 \qquad [\because c = 0$ in this code]

$\therefore$ Our answer $= x * x * x * x$

The final value of $x = 9$

$\therefore \quad 9^4 = \boxed{6561}$

64. Given: n numbers

To find: Tighest upper bound on number of swaps required to sort n numbers using selection sort.

Nalysis: In selection sort, in the unsorted part of the array, we find the minimum element and swap it with the value placed at the index where the unsorted array starts.

Hence, for each element to put it in its sorted position, we will do some swaps. In each itenation, when we find the minimum and place it in its sorted position, we do only one swap.

There are n such iterations, since maximum number of positions to sort is n.

Hence, there are n. 0(1) swaps

$\Rightarrow$ 0(n) swaps.

$\therefore$ The solution is (b)

65. Given: A binary tree with n nodes

To find: Tighest upper bound on time complexity of inserting an object.

Nalysis: When inserting a new element, we need to find its position in the tree. The time for this operation is upper bounded by the height of the binary tree. Since, the tree is unbalanced, we don't need to perform balancing operations upon insertion, so only time consums is in finding its right position. The time consumed in finding the position is equal to height of the tree.

Max height of tree = 0(n)

[All elements are on one side of root & no root has two non empty subtrees]

$\Rightarrow$ Answer is (c)

66. Given: priority of a process α Waiting Time i.e. process scheduling is independent of the time it takes for a process to execute. Thus, options C & D can be discarded.

Now, initially when all the processes arrived simultaneously, waiting time for each process = 0.

Any process can be scheduled first, say P_1.

After T units of time,

Waiting time for P_1 = 0 & waiting time for all others = T. Thus, in priority all other processes supercede P_1 and hence some other process gets scheduled now.

Now, at every time = kT (K > 0), if there is any process which has not been given CPU atleast once, gets scheduled. Once every process got scheduled one time each (for T time units) all processes have now the same waiting time = (n – 1) T where n = # processes.

Till now, the scheduling was Round-Robin with pre-emptions at periods of T.

The same analycis can be extended for further schedulings as all the processes have the same priority (= (n – 1)T) which was O, earlier.

Option (A) is incorrect because the scheduling is independent of the arrival time of the processes. Actually, all arrived at the same time & among all processes with same priority any one can get scheduled at an instant.

67. 1. We can use "Depth First Search" Algorithm, to check it there is a cycle is an undirected graph. It we encounter any "back edge" in "Depth first search" then given undirected graph has a cycle. Also, If there is a cycle in the Undirected graph, we must encounter a "back edge" in DFS. And, DFS can be done in O(|E| + |V|) time for graph G = (V, E). So, it can is in F.

2. P $\leq$ NP, So, it is also in NP.

3. NP - complete problem A $\in$ NP. By, Definition Every problem in NP can be solved in polynomial time using non-deterministic turing machine.

So, Answer is (a) i.e. 1, 2, and 3 are true.

68. Given: A complete graph on n vertices?

To find: Time complexity of Bellman-Ford Single source shortest path.

Analysis:

Time complexity of Bellman-ford algorithm on a graph with n vertices and m edges is O(nm)

For a complete graph, $m = {}^nC_2 = 0(n^2)$ (Since, there is an edge between all pair of vertices)

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$\therefore$ Time Complexity $= 0(n^2 \cdot n)$

$= 0(n^3)$

$\therefore$ Solution is (c)

Note : Include vertex of degree zero in even degree vertices.

69. (P) Let 'C' be a cycle. In the live graph L(C) :

(i) There will be an edge between two vertices corresponding to adjacent edges in the cycle.

(ii) Degree of each vertex in L(C) will be 2

(iii) L(C) will be connected.

Hence, L(C) will also be a cycle. So, (P) is TRUE

(Q) Let 'K_n' be a clique on 'n' vertices. Consider line graph $L(K_n)$:

– $L(K_n)$ has n_{c2} vertices at least say e_1, e_2

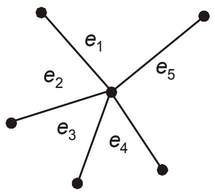
– For $n > 3$, there will be two edges in "K_n" which do not share a vertex in "K_n".

– So, $V(e_1)$, $V(e_2)$ will not be adjacent in $L(K_n)$.

– Hence, $L(K_n)$ is not always a clique.

(Q) is FALSE.

(R) Let 'G' be a star graph consider line graphical



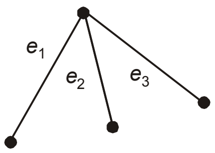
$\Rightarrow$ In L(G): $V(e_1)$, $V(e_2)$, $V(e_3)$, $V(e_4)$, $V(e_5)$ will "K5" form a clique because they all share a vertex.

A planar graph cannot have a "K5" minor. Hence L(G) is not a pllanar graph.

Even though G is planar

So, R is FALSE.

(S) Let 'T' be the following tree, and L(T) is its line graph



– In L(T): e_1, e_2, e_3 will form a $K_3(\Delta L_e)$ i.e., a 3-cycle.

– Tree is an acyclic graph i.e. doesn't have a cycle.

– So, L(T) is not a tree.

So, S is False

$\therefore$ Answer is "P" only.

70. Let the number of element is "K", they can be sorted in θ(k log k) time.

We try the options in decreasing order of complexity since, we need a tight band i.e. θ

i.e. $\theta(\log n) \cdot \theta\left(\frac{(\log n)}{\log\log n}\right), \theta\left(\sqrt{\log n}\right), \theta(1)$

So if $K \in \theta(\log n)$ time required for loop sort is. θ(k log k) i.e.

$\theta(\log n \times \log\log n)$, But this is not in $\theta(\log n)$

if $k \in \theta\left(\frac{\log n}{\log\log n}\right)$ time required for loop sort:

$$\theta\left(\frac{\log n}{\log\log n} \times \log\left(\frac{\log n}{\log\log n}\right)\right),$$

i.e. $$\theta\left(\log n \times \underbrace{\left[\frac{\log\left(\frac{\log n}{\log 1 > n}\right)}{\log\log n}\right]}_{\leq 1}\right)$$

71. Complexity of a sequence of 'n' operations

= Total Complexity of Enqueue operations ($\propto$)

+ Total complexity of Dequeue operations (β)

total complexity of Dequeue operations $(\beta) \leq$ Total complexity of Enqueue operations ($\propto$).

$\beta \leq \propto \ldots(i)$

Total complexity of Enquence operations

$(\propto) \leq n \ldots(2)$

$\therefore$ Total complexity of n operations

$= \propto + \beta$

$\leq \propto + \propto$ (from (1))

$\leq n + n$ (from (2)) $\leq 2n$.

72. $r = e - v + 2$ for bound + non bound

$r = e - v + 2 - 1$ for bounded

$r = 15 - 10 + 2 - 1$

$r = 6$.

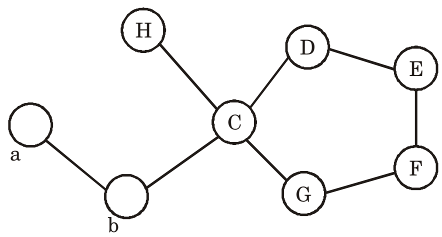
74. If right child is present

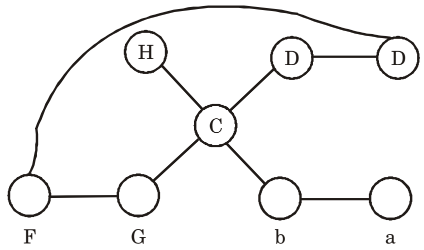
B1 = 1 + height ($n \rightarrow$ right)

Other if both are present

B2 = 1 + max (h_1, h_2)

77.





78. If B(n), XXA(n) and W(n) denote best case , average case and worst case time complexities of an algorithm P respectively then B(n) = O(A(n)), A(n) = O(W(n))

79. Let the three pegs be A,B and C, the goal is to move n pegs from A to C usingpeg B

The following sequence of steps are executed recursively

1. move $n-1$ discs from A to B. This leaves disc n alone on peg A ..... T($n-1$)
2. move disc n from A to C 1
3. move $n-1$ discs from B to C so they sit on disc n T($n-1$)

So, $T(n) = 2T(n-1) + 1$

80. The worst case running time of search in BST is log n where n is total number of elements in the tree. The total number of elements are $n.\ 2^n$ thus search R. T. will be long $(n.\ 2^n)$.

$$\log n\ .\ 2n = \log n + \log 2^n$$

$$\therefore \quad = \log n + n \log 2$$

$$= \theta(n)$$

81. If P ! = NP, then it implies that no NP-Complete problem can be solved in polynomial time which implies that the set P and the set NPC are disjoint.

82.

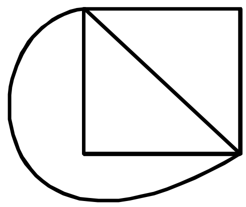
S	A	B	C	D	E	F	G	T
∞	∞	∞	∞	∞	∞	∞	∞	∞
4	$\boxed{3}$		7					
$\boxed{4}$	3		7					
4	3	$\boxed{5}$	7					
4	3	5	7	$\boxed{6}$				
4	3	5	7	6		8	10	
4	3	5	$\boxed{7}$	6	12	8	10	
4	3	5	7	6	12	$\boxed{8}$	10	
4	3	5	7	6	12	8	10	

SACET

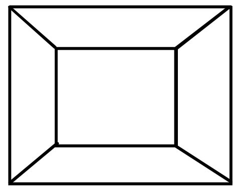
84. Heap is a complete binary tree.

86.

K4



Q3



Both K4 and Q3 are planar

87. In case two students hold joint account then Bank Account_Num will not uniquely determine other attributes

88. T1 and T2 : checking same condition $a = 0$

hence, any one of T1 and T2 is redundant.

T3, T4 : in both case discriminant

(D) = $b^2 - 4ac = 0$.

Hence any one of it is redundant.

T5 : D > 0

T6 : D < 0

89. Load R_1, a; $R_1 \leftarrow M[a]$

Load R_2, b; $R_2 \leftarrow M[b]$

Sub R_1, R_2; $R_1 \leftarrow R_1 - R_2$

Load R_2, c; $R_2 \leftarrow M[c]$

Load R_3, d; $R_3 \leftarrow M[d]$

Add R_2, R_3; $R_2 \leftarrow R_2 + R_3$

Load R_3, e; $R_3 \leftarrow M[e]$

Sub R_3 R_2 : $R_3 \leftarrow R_3 - R_2$

Add R_1, R_3; $R_1 \leftarrow R_1 + R_3$

Total 3 Registers are required minimum

90.

P_0	P_1	P_0	P_2

0 1 5 13 22

Average waiting time = $\dfrac{4+11}{3}$ = 5 ms

92. $7 \downarrow 3 \uparrow 4 \uparrow 3 \downarrow 2$

$\Rightarrow$ $7 \downarrow 3 \uparrow (4 \uparrow 3) \downarrow 2$ as $\uparrow$ is right associative

$\Rightarrow$ $7 \downarrow (3 \uparrow (4 \uparrow 3)) \downarrow 2$

$\Rightarrow$ $(7 \downarrow (3 \uparrow (4 \uparrow 3))) \downarrow 2$ as $\downarrow$ is left associative

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94. Given :

# include < stdio.h>
void f (int * p, int * q){
{ p = q;
* p = 2;
}
int i = 0, j = 1;
int main() {
{ f (& i, & j); → Passing the address of i, j
print f ("% d % n", i, j);
retur, 0;
}

Let address of i and j are 200, 400

means $\boxed{i = 0}$ $\boxed{j = 0}$
200 400

in function f pRq are pointers that have address of i, j than by $p = q$ copy address of j in p

means initially $\boxed{200}$ $\boxed{400}$
p q

after $p = q$ $\boxed{400}$ $\boxed{400}$
p q

by * p it represent value of j now
then * p = 2 changes value of j
So after print f statement $i = 0, j = 2$.

95. P-2 means Requirment Capture in Domain Analysis.
Q-3 means Design considered as Structural and Behavioral modeling.
R-1 Implementation Modules are developed and integration are done between modules.
S-4 Maintenance is Performance Tuning after deployment of system.

96. a [ ] =

12	7	13	4	11	6

So $f(a, 6)$ → first this call

Thus is the number stored in the array and we can access these by using *a-

$f(a, 6)$
↓ 12
So 12 + f (a + 1), n – 1) {even}
↓
7 – f (a + 1, n – 1) {odd}
↓
13 – f (a + 1, n – 1) {odd}
↓
4 + f (a + 1, n – 1) {even}
↓
11 – f (a + 1, n – 1) {odd}
↓
6 + f (a + 1, 0) {even}
↓
0{n = 0 then 0}

= 12 + [7 – [13 – [4 + [11 – (6 + 0)]]]]
= 12 + [7 – [13 – [4 + [11 – 6]]]
= 12 + [7 – [13 – [4 + 5]]]
= 12 + (7 – (13 – 9))
= 12 + (7 – 4) = 12 + 3 = 15

97. We assigned initial q = NULL, p = head

Now we checked 0 is there p → next ≠ NULL if it is then break means if we are at the end of linked-list.

Now we are at p → next = NULL and q → next = p and we are out of loop

Now we want p as a head and q is last node

Then we have to p → next = head means we assign first node as the next node of p and declare p = head means at head is p means **head = p**

Now it is ring because q → next = p till now

So declare q → next = NULL

then we have to write at blank line

q → next = NULL, p → next = head, head = p

Alternately

```
1.  typedef struct node
2.  {    int value;
3.  struct node * next;
4.  } Node;
5.  Node * move-to-front (Node * head)
6.  }
7.  Node * p, * q;
8.  if((head = = NULL) || (head -> next = = NULL))
9.  return head
10. q = NULL;
11. p = head;
12. while (p -> next ! = NULL)
13. {
14. q = p;
15. p = p -> next;
16. }
17. q -> next = NULL;
18. p -> next = head
19. head = p ;
20. return head
21. }
```

Line 1-4 declare the structure

Line 5 function definition

Line 7 declare two pointer variable of type NODE

Line 8 check whether list is empty or contain only 1 node

Line 10-11 if condition at line 8 fails, then it set p and q

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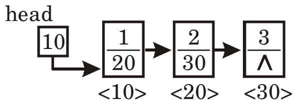
Line 12-16 iteration till node with NULL is fetched and each time p and q gets update

e.g.



initially

q – NULL, 10, 20

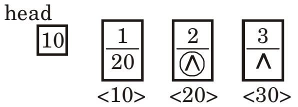
p – 10, 20, 30 – loop terminate

Now, we have to move 3rd node at 1st place.

So,

Line 17 $\quad q \rightarrow$ next = NULL

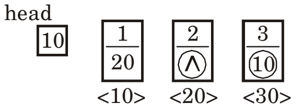
i.e. $\quad 20 \rightarrow$ next = NULL



Line 18 $\quad p \rightarrow$ next = head

[$\because$ head = 10]

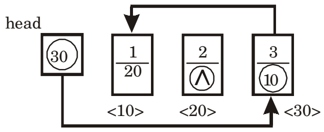
$30 \rightarrow$ next = 10



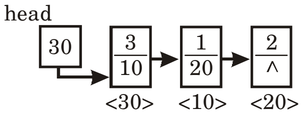
Line 19

head = p

head = 30



OR



98. begin

if (a = = b) {S1; exit;}

else if (c = = d) {S2 ;}

else {S3; exit;}

S4;

end

99. Cyclomatic complexity always

(number of comparisions +1)

So for A (number of comparisons + 1)

9 + 1 = 10

for B $\quad$ 9 + 1 = 10

When we summarise both, then

total number comparisions = 9 + 9 = 18

$\therefore$ Cyclomatic complexity = 18 + 1 = 19

100. B must be preferred on A as

$0.0001\, n^2 < 10\, n \log_{10} n$

$\Rightarrow \quad 10^{-5}\, n < \log_{10} n$

$\Rightarrow \quad 10^{-5} < \log_{10} n^{1/n}$

$\Rightarrow \quad n\, 10^{-5} < \log_{10} n$

We know $\quad \log_{10} n = k$

$\Rightarrow \quad 10^k = t$

$\therefore \quad k > 10^{-5}\, 10k$

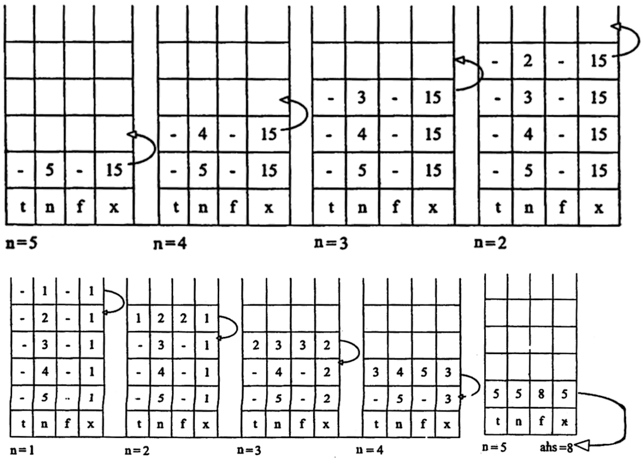
$\Rightarrow \quad k > 10^{k-5}$

$\Rightarrow \quad k - 5 > 0$

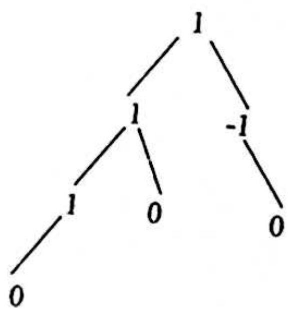
$\Rightarrow \quad k > 5$

Then $\quad$ min $k = 6$

102. *Stack Positions :*



103.



Maximum height = 3

104. If there are n elements, then in worst case, total swaps will be (n-1) in selection sort. So the number of swaps is θ(n).

Alternately

The selection sort is similar to bubble sort except it does not swap elements with every move. The sorting algorithm first finds the smallest element in the list and then puts it in to place in single swap. So n swap required for n elements.

105. In Bellman Ford shortest path algorithm, we choose a node first and then we find any negative weighted cycle is rechable from source.

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106. If π_A is NP-hard, then it is NP-complete

Alternately

π_A be a problem (given) in class NP. we say that π_A is NP complete if the following statements are true about L.

(1) π_A is in NP

(2) For every π_A' in NP there is polynomial time reduction of π_A' to π_A.

107. Applying Master's Theorem

$cn > n^{\ln_3 1}$

$cn > n^0$

Now checking $af(n/b) \le kf(n)$ for some $k < 1$

$1*cm/3 \le kcn$

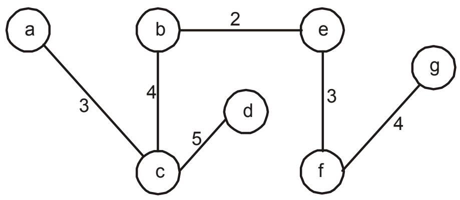
This is true for $k > 1/3$

Hence solution is $\theta(n)$.

108. In linear probing, if there is already an element then the new element is fed into the next blank value field in the table.

109. In Kruskal's Algorithm we choose on edge of G which has smallest weight among the edges of G.

So (b, e) = 2, (e, f) = 3, (b, c) = 4, (a, c)= 8, (f,g) = 4 (c,d) = 5



Alternately

Order the edges in non-decreasing order and pick edge one by one until all the nodes are completed with no edge making cycle on addition.

110. Average complexity of quick sort is θ (n log n).

111.

$a = (\overset{3}{x} > \overset{4}{y})\ ?\ ((x > z)\ ?\ x : z) : ((\overset{4}{y} > \overset{2}{z}\ ?\ \underset{4}{y} : z))$

x = 3, y = 4, z = 2 satisfy the values, so we get a value 4.

112. Suppose 100 $\boxed{4}$ b $\boxed{100}$ 4 $\boxed{1000}$

C 1000

$**ppz += 1$

$c = 5$

$z = 5$

$**py += 2$

$c = 7$

$y = 7$

$x += 3$

$x + y + z = 22$

113. Whenever character input is not return key (in) it accept character input ? 1 (c = getchar()) = ! (in) ? 2 for bringing all character after reverse functions so ?2 is putchar (c).

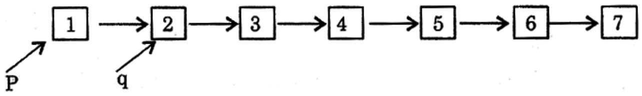
Alternately

To print reverse of the input, this programme basically collects the input in buffer with the help of some f^n (i.e. f^n is getcher ()) and by using stack, then where ↵ is pressed stack is popped off and characters are printed in reverse order as that of the process call, i.e. last process call to reverse is processed first because of stack.

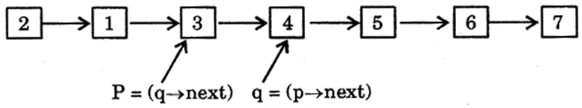
114. In function rearrange the string of integer 1, 2, 3, 4, 5, 6, 7 converted in to 2, 1, 4, 3, 6, 5, 7 we can check this by execute this program.

Alternately

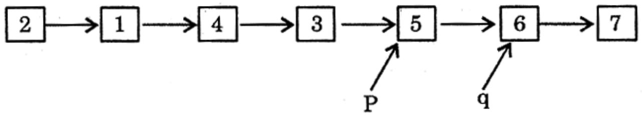
Initially



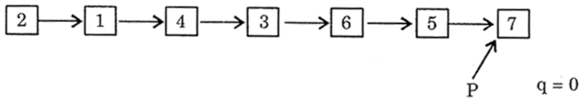
1^{st} iteration of while



2^{nd} iteration



3^{rd} iteration



so loop terminates

115. The most efficient algorithm for finding the number of connected components (articulation point) in an undirected graph on n vertices and n edges using depth-first search takes $O(m + n)$ time. Assume $n \le m$.

116. *Applying BFS Algorithms*

Starting from Q

Queue Q

Making Q's child's status 1.

Queue

Q | M | N | P

Making M's child 1 (which are not 1)

Status

Q	M	N	O	P	R
2	1	1		1	
2	2	1		1	1
2	2	2	1	1	1

Q | M | N | P | R

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Making N's Child 1

Q|M|N|P|R|O

(other are already 1)

117.

$$h(n) = n^{\log n}$$
$$f(n) = 2^n$$
$$n \log n \leq C2^n$$

for all $n \geq n_o$

$$C = 1 \text{ and } n_o = 2$$
$$2 \log 2 \leq 2^2$$

So $h(n) = O(f(n))$

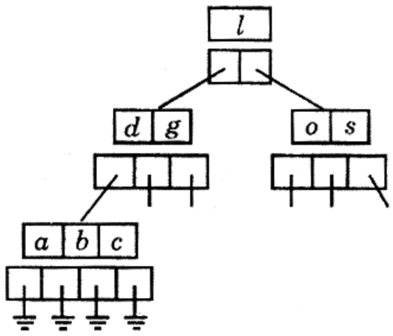
$$g(n) = n!$$
$$f(n) = 2^n$$
$$n! \geq C2^n$$

for all $n \geq n_o$

$$C = 1 \text{ and } n = 4$$
$$4! = 24 \geq 2^4$$
$$\geq 16$$
$$g(n) = \Omega(f(n))$$

118. To find an integer appears more than $n/2$ times, minimum comparisons are of order $O(n)$.

119. A B-tree of order 4. All internal nodes except root have at most 4 children and at least 2 children. For 10 instruction, it will stake 3 node splitting operation.

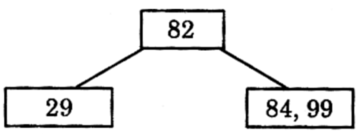


Alternately

Order 4-B tree can have maximum 3 keys in one node and each node have max 4 children so lets take values 84, 82, 29, 99, 65, 12, 50, 28, 58, 71

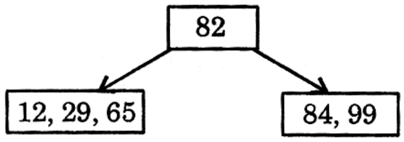
Inserting 84, 82, 29 $\boxed{29,\ 82,\ 84}$

Inserting 99

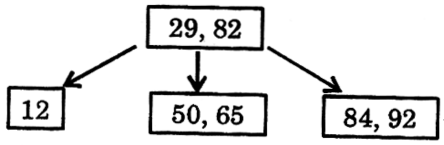


1st Splitting

Inserting 65, 12

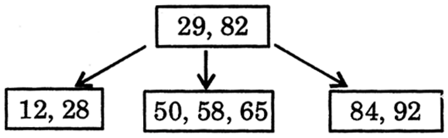


Inserting 50

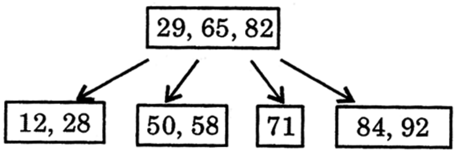


2nd Splitting

Inserting 28, 58



Inserting 71



3rd Splitting

For any sequence of insertion, maximum spilliting is 3.

120. Minimum number of edges for a connected graph = n

and for a connected tree = $n - 1$

Here given that $2\,n - 2$ edges and the edges of graph is partitioned into two edge disjoint trees. So there are two vertex disjoint paths between every pair of vertices.

121. In quicksort a set of number is reduced to sorting two smaller set. We take first element as key value and combine all other with this. A pivot element which splits list into two sublists each of which at least one fifth of element only (B), i.e.

$T(n) \leq T(n/5) + T(4n/5) + n$ the problem.

Alternately

If one sublist contains 1/5 elements other contains 4/5 elements.

If T(n) number of comparisons for sorting n elements.

So, for 1/5 elements = $T(1/5n)$

and for 4/5 elements = $T(4n/5)$

So, $T(n) \leq T(n/5) + T(4n/5) + n.$

Here, n = time to spilt

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122. Given a set S of n positive integer and a positive integer W. And also a algorithm Q solves in $0(x\text{W})$ time. So only (b) is false.

123. In given graph, the graph is directed and from vertex a to all the vertices there exist a weighted path so Dijkstra's single source shortest path algorithm when run from vertex a in the above graph it includes all the vertices. Dijkstra's don't work properly if the edge has negative weight. $(b, e), (c, h)$ are negative edges so algorithm results correct result only for *b, c, d.*

124. To find binary tree two of the orders, i.e. preorder, postorder and inorder should be given. By using only post order unique b –, tree cannot be found.

125. It takes only Θ (log (n)) time because it is not addition of n more elements in list.

126. Upper bound may be $\sqrt{n}$ times it gets executed.

Lower bound may be that if gets executed only once or twice.

127. Let T (m, n) be the total number of steps.

So $\text{T}(m, 0) = 0$ $\text{T}(m, n) = 1$ $(n, m \text{ nod } n)$ on average

$$\text{T}_n = \frac{1}{n}\sum_{0\le k\le n} \text{T}(k,n)$$

$$\text{T}_n \approx 1 + \frac{1}{n}(\text{T}_0\ \text{T}_1 + \ldots + \text{T}_{n-1})$$

$$\text{T}_n \approx \text{S}_n$$

$$\text{S}_n = 1 + \frac{1}{n}(\text{S}_0\ \text{S}_1 + \ldots + \text{S}_{n-1})$$

$$\text{S}_n = 1 + \frac{1}{n+1}(\text{S}_0\ \text{S}_1 + \ldots + \text{S}_n)$$

$$= 1 + \frac{1}{n+1}(n(\text{S}_{n-1}) + \text{S}_n)$$

$$= 1 + \frac{1}{n+1} = \text{S}_n + \frac{1}{n+1}$$

So $\quad \text{T}_n \approx \theta(\log_2 n + 0(1)$

$\quad\quad \text{T} \approx \theta\,(\log_2 n)$

128. Since, j increases in power of 2′s.

if statement $j = j * 2$ executes k times, then

$$2^{\text{k}} \le \text{n}$$

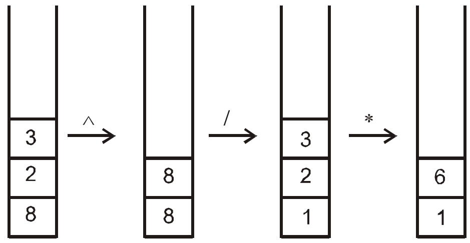
$$\Rightarrow \quad \text{k} \le \log_2 n$$

Since k will be integer,

total number of comparison

$$= \lfloor \log_2 n \rfloor + 1 \text{ (when loop exits)}$$

129. Evaluation will proceed as



So after evaluation of first *top elements will be 6, 1.

130. *Calbing sequence will be*

Sequence returns

$f(5)$ $\quad f(3) + 2 = 18$

$\downarrow\ \uparrow$

$f(3)$ $\quad f(2) + 5 = 16$

$\downarrow\ \uparrow$

$f(2)$ $\quad f(1) + 5 = 11$

$\downarrow\ \uparrow$

$f(1)$ $\quad f(0) + 5 = 6$

$\downarrow\ \uparrow$

$f(0)$ $\quad 1 = 1$

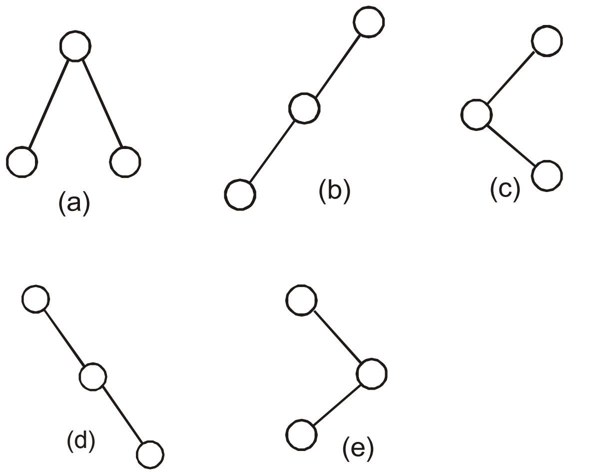
131. Since value = 1 only if nodes has no left as well as right node otherwise value = 0. Therefore it returns number of leg nodes in tree.

132. Maximum nodes in binary tree of height 0 $= 1 = (2^{0+1} - 1)$

Maximum nodes in binary tree of height 1 $= 3 = (2^{1+1} - 1)$

$\therefore$ Maximum nodes in binary tree of height h $= (2^{h+1} - 1)$

133. 5 binary trees can be formed with three unlabeled nodes as shown below.



134. Merge sort has lowest worst – case complexity, i.e. $O(n \log n)$, whereas all remaining three has $O(n^2)$.

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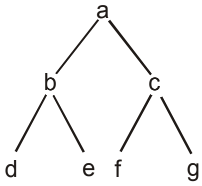
135. From given inorder and preorder traversal, the binary tree can be formed.



Since a is first node traversed in preorder, so it will be root of tree elements (nodes) left to a and right to a in inorder traversal will be left subtree and right subtree respectively of the tree.

In left subtree elements d, b, e since bcomes first in preorder, so b will be root of left subtree d is left to b in inorder, so it will be left child of d and e will be right child of d. Tree will be like

136. 1 will occupy location 0.

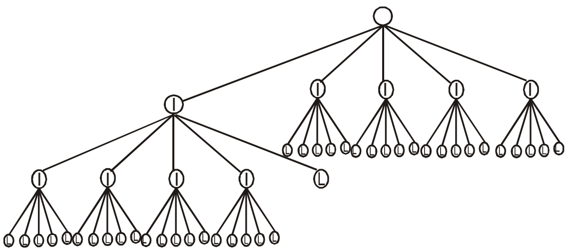
3 will occupy location 6.

8 hashed to location 0 which is already occupied, so it will be hashed to one location next to it (0) i.e. to location 1.

Since 10 also clashes, so it will be hashed to location 2.

137. In case of unweighted, undirected graphs, BFS gives the most time efficient computation for shortest path. It is guaranteed to find first shortest path.

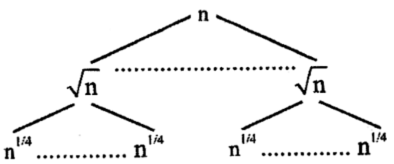
138.



139. The given function is recursive so the equivalent recursion equation is

$$T(n) = 1 \quad n \le 2$$

$$T(n) = \lfloor\sqrt{n}\rfloor + n \quad n > 2$$



All the level sums are equal to n. The problem size at level k of the recursion tree is n^{2-k} and we stop recursing when this value is a constent. Setting $n^{2-k} = 2$ and solving for k gives us

$$2^{-k} \log_2 n = 1$$

$$\Rightarrow \quad 2^k = \log_2 n$$

$$\Rightarrow \quad k = \log_2 \log_2 n$$

So $\quad T(n) = \theta(\log_2 \log_2 n)$

140. Since tree for Max Heap is binary tree and binary search is being performed.

141. e is need not to be present in every spanning tree since there may be edges (in cycle formed by adding e), which has same weight as e has.

142. Since,

$2n - c$ = average number of comparison needed.

$1.5\, n - 2$ = number of comparison in case.

$n \log_2 n$ = also doesn't conform with number of comparison needed.

143. Number of push operations will be minimum when m alternate insert and delete *op* are performed with $n–m$, incerts in the end.

For each alternate insert and delete *op* 1 push is needed

$\therefore \quad 2m$

Rest of the $n - m$ inserts need $n - m$ push

$\therefore$ Minimum push $= 2m + n^{-n}$

$= m + n$

Number of *pop* operations will be maximum when n insertions are done consecutively and then m deletions are performed.

For the first deletion, first n element have to be popped from S_1, then all the m deletions can be performed each by a single pop.

$\therefore$ Maximum *pop* $= n + m$

144. (*i*) will occur, if $i = n$ occurs before $j = n$ so that the while loop terminates till that time j–1 elements of b and n elements of a would have been inserted in c. $i = n$ would have occured only if i would have been incremented i.e,

$a[i] < b[j]$ for the last i .

$\Rightarrow \quad a[n-1] < b[j]$

$\therefore \quad k = j - 1 + n$

145. Only an extra variable t 1 has been added in work 2 instead of directly computing the subscript as in work 1. The output will be the same. S1 is true.

The addition of variables $t1$ and $t2$ will not improve the performance in any way. i.e. S2 is false.

146. S3 is true S4 is false S1 is false S5 is false

147. MAX Heap used to identity max element in O(1) time for to identity min element require O(n).

148. Minimum spanning tree will be formed when $\forall$ $i. \in. \{1, 2, ... n\}$

V_i is connected to V_{i+1}, as in that case each edge will have the minimum weight of 2.

For example, for $n = 4$, the minimum spanning tree is as follows :

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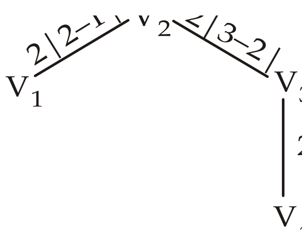


In such a tree, weight of tree consisting of n–1 edges each of weight 2 will be

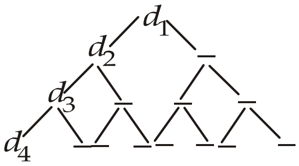
$$2\,(n-1) = 2\,n-2$$

149. In order to find minimum size of X capable of storing any kind of binary tree, the worst cose size of the tree has to be taken.

In the worst case size of the tree has to be taken.

In the worst case only a single node will contain the data at each level of the tree.

For example, for 4 data items, the worst case binary tree will be as follows



Generalizing the concept, for n vertices, the levels of tree that are required will be n.

Memory required at each level (starting from root) will be

$$2^0, 2^1, 2^2, \ldots\ldots\ 2^{n-1}$$

$$\therefore \text{ Minimum size} = 2^0 + 2^1 \ldots\ldots\ldots\ldots\ldots\ldots 2^{n-1}$$

$$= \frac{2^0\left(2^n-1\right)}{2-1} = 2^{n-1}$$

152. If n is a power of 2, then

$$j = n + \frac{n}{2} + \frac{n}{2} + \ldots\ldots\ldots\ldots\ldots + \log_2 n \text{ term}$$

If n is not a power of 2, then there will be minor differences of 1 at $\frac{n}{2^i}$ wherever $\frac{n}{2^{i-1}}$ is odd.

Hence val (j) computed on the basis of $n = 2^n$ will give a fair answer

$$j = n + \frac{n}{2} + \ldots\ldots\ldots\ldots 1 \text{ G.P.}$$

$$= \frac{n\left(\left(\frac{1}{2}\right)^{\log n} - 1\right)}{\frac{1}{2}-1}$$

$$= 2n\left(1 - \frac{1}{2^{\log n}}\right)$$

$$= 2n\left(1-\frac{1}{n}\right) = 2\,(n-1) = \theta\,(n)$$

153. Let x contains n elements divie it by $n/2$ repeatedly so the recurrence become

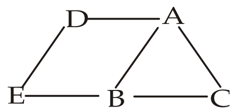
$$T(1) = 1$$
$$T(n) = 2T(n/2) + 1$$
$$T(n) = O(n \log n)$$

154. Ordering the weights, the sequence obtained is 1, 1, 2, 2, 3, 3, 4, 4, 5, 6, 7

Now, in order to obtain minimum spanning tree using Kruskal's algorithm, we have to add edges with weights in increasing order such that weight of the spanning tree is minimum.

In option (d), weights of the edges are 1, 1, 2, 3, 2 which contradicts Kruskal's algorithm.

155.



For the above graph, conducting depth First scarch:

Stack : A

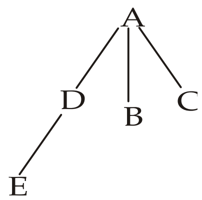
Stack : DBC Tree : A

Stack : EBC Tree : A / D

Stack : BC Tree : A / D / E

Stack : BC

Stack : ϕ Tree :



Considering leaf nodes E and C, the given condition is satisfied as their degrees are more than one. But none of (a), (b) or (c) hold.

But a cycle consisting of E and all its neighbour hold.

156. The statement inside the for loop is similar to X–OR operation such that the set obtained is $(X \cap Y') \cup (X' \cap Y)$.

It is equivalent to $(X - Y) \cup (Y - X)$.

157.

$$T(1) = 1$$
$$T(n) = 2T\left(\left\lceil \sqrt{n} \right\rceil\right) + 1$$

We know that $\log_2^2 = 1$

So, all the level sums are equal to $\log_2^2$. The problem size at level k of the recursion tree is $n^{2^{-k}}$ and we stop recursing this value is a constant. Setting $n^{2^{-k}} = 2$ and solving for k gives us $2^{-k}\log_2 n = 1 \Rightarrow 2^k = \log_2 n \Rightarrow k = \log\log_2 n)$

So $T(n) = \theta(\log\log n)$

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158. a_1, a_2 ....a_n

b_1, b_2 ....b_n

In order to find out the largest span, check the sums of

a_i +...........a_j and b_i +.............b_j at each step.

If $a_1 + a_2 = b_1 + b_2$ go on, check $a_1 + a_2 + a_3$ and $b_1 + b_2 + b_3$.

If not, then check $a_2 + a_3$ and $b_2 + b_3$

Similarly a check is done at each of the n places during traversal. A separate variable has to be kept that contains the maximum span observed hitherto

Hence fastest algorithm computes with (~) (n) time and space.

159. S1 is true S_4 and S5 are false as the program will print 3 and 8. The option left is (a).

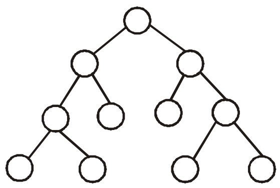
NUMERICAL TYPE QUESTIONS

1. The output is – 5

2. The cyclomatic complexity of given code segment is 5.

6. The number of nodes in a binary tree have exectly two childrens are $n - 1$ if n is number of leaf.

Hence (200 – 1) = 199 nodes



Number of leaf nodes is 6

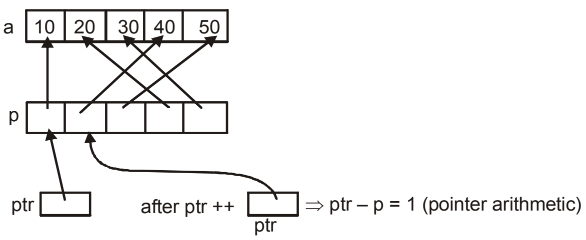
Number of nodes have exectly two children is 5

7. C $\boxed{i \mid 0 \mid 1 \;\; 1}$

20 something

{ $z = 1$				
for $i = 0 + 0.3$	$k = 0$			
do	$z = 1$			
$z \leftarrow z^2$ mod	$z = 1$	$z = 4$	$z = 0$	$z = 0$
if $c[i] = 1$	$c[0] = 1$	$c[1] = 0$	$c[1] = 1$	$c[1] = 1$
$z \leftarrow 2 \times z \bmod 8$	$z = 2$		$z = 0$	$z = 0$
end				
return 2				
}				

8.



**ptr = 40

∴ printf ('' %d%d'', p + r – p, p + r) will print 140

9. j and k will be evaluated to 2 and -1 respectively.

In for loop :

When i = 0; 1 time printed (– 1)

When i = 1; 1 time printed (0)

When i = 2; 3 times printed (1, 1, 1)

When i = 3; 3 times printed (2, 2, 2)

When i = 4; 2 times printed (3, 3)

∴ on the whole printf is executed 10 times

10. In function main x will be updated as follows

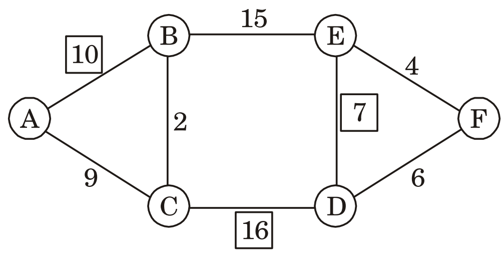
$x = x + f_1\,(\,) + f_2\,(\,) + f_3\,(\,) + f_2\,(\,)\ x$

$\downarrow \quad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$

1 26 51 100 52

Note : static variable in f_2 ( ) will be initialized only once & it retains value in between function calls.

11. MST, The complete graph should be



Total sum = 10 + 9 + 2 + 15 + 16 + 7 + 4 + 6 = 69.

12. as |V| + |R| = |E| + 2, where |V| is vertices, |E| is edges and |R| is region given for a connected planner graph |V| = 10 and no of edges are 3.

Then 3|R| = 2|E|

and then $|R| = \frac{2}{3}|E|$

Hence

$10 + \frac{2}{3}|E| = |E| + 2$

$\frac{|E|}{3} = 8$ and $|E| = 24$

13. G has 100 verticles ⇒ spanning tree contain 99 edges given, weight of a minimum spanning tree of G is 500 since, each edge of G is increased by five

∴ Weight of a minimum spanning tree becomes 500 + 5 × 99 = 995

14. The graph formed by the description contains 4 vertices of degree 3 and 40 vertices of degree 5 and 100 vertices of degree 8.

According to sum of the degrees theorem 4 *3 + 40* 5 + 100 * 8 = 2|E|

|E| = 1012/2 = 506

15. Since, the binary value of 435 = 110110011. We know that for the num > = 1, the operator implies that num will be shifted right for every time when loop

is executed. Since, in the given code the loop is incremented 9 times and at last 10^{th} time it is zero. That's why, count will be incremented 9 times.

Hence total count is **9**.

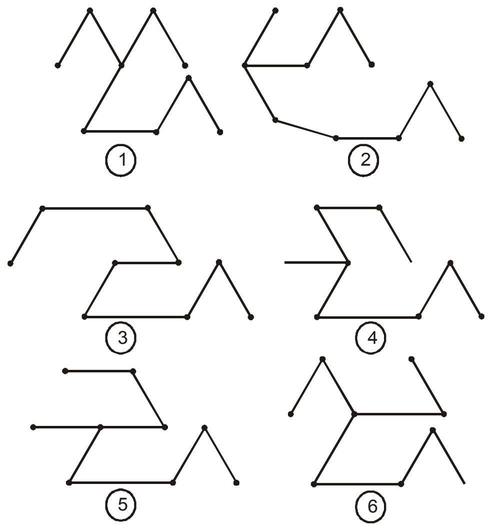
16. C would complete its first I/O operation in 1000 ms as it is using round robin scheduling (as per IIT).

But it is seems to be wrong.

17. When we take the value from 1·72 to 1·74, we will see that else condition will be true *i.e.* return $f(x/2 + 1.5/x)$;

That's why value to $f(q)$ such that it returns q is 1·72 to 1·74.

18. The MST are



19. $P(x) = x^5 + 4x^3 + 6x + 5$ can be rewritten as follows

$P(x) = x^3 (x^2 + 4) + 6x + 5$

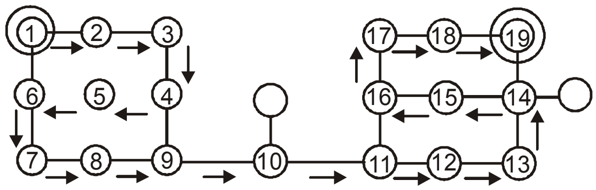
Now using only one temporary variable t and any number of data transfer as well as memory related operation the polynomial can be evaluated as follows

1.	$t = x * x$	[Evaluate x^2 and store in memory]
2.	$t = t + 4$	[Evaluate $(x^2 + 4)$ and store in memory]
3.	$t = x^2$	[Retinue x^2 from memory]
4.	$t = t^*.X$	[Evaluate x^3 and store in memory]
5.	$t = t^*(x^2 + 4)$	[Evaluate $x^3 (x^2 + 4)$ amd store in memory]
6.	$t = 6^* x$	[Evaluate $6x$ and store in memory]
7.	$t = t + 5$	[Evaluate $(6x + 5)$ and store in memory]
8.	$t = t + x^3 (x^2 + 4)$	[Retrieve $x^2 (x^2 + 4)$ from memory and evaluate $\{x^3 (x^2 + 4) + 6x + 5\}$

In the above 8 steps of evaluation, the total number of arithmetic operations required and 7 [4 Multiplications, 3 Additions]

So answer is 7 arithmetic operations.

20.



Suppose, we start DFS at vertex numbered as 1 and continue calling recursive function for DFS on subsequent nodes numbered in ascending order. The recursive calling sequence is shown as marked line in the above diagram which shows maximum possible recursion depth including the initial call is 19.

21.
```
int print_subtrees_size_4(node *n)
{
int size=0;
if(node==null)
return 0;
size=print_subtrees_size_4 (node->left) +
          print_subtrees_size_4(node ->right) + 1;
if(size ==4)
printf("this is a subtree of size 4");
return size;
}
```

The above function on taking input the root of a binary tree prints all the subtrees of size 4 in O(n) time

so a =1 , b = 0 and then a + 10 b =1

22. From the list of given n numbers [say n is even],

Pick up first two elements, compare them

assign Current – min = min of two numbers

Current – max = max of two numbers

From the remaining n – 2 numbers, take pairs wise and follow this process given below.

1. Compare two elements

   Assign min = min of two numbers

   max = max of two numbers

2. Compare min and current – min

   Assign current – min

   = min{current–min,min}

3. Compare max and current – max

   Assign current – max

   = max{current – max, max}

Repeat above procedure for all the remaining pairs of numbers. We can observe that each of pair requires 3 comparisons

1. for finding min and max
2. For updating current – min
3. for updating current – max

But for initial pair we need only one comparison not 3.

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$\therefore$ Total number of comparisons $= \dfrac{3(n-2)}{2} + 1$

$$= \frac{3n}{2} - 3 + 1 = \frac{3n}{2} - 2$$

Here n =100, so number of comparisons = 148.

23. Since each vertices in A meet with B. That's why total number of edges = 6 × 6 = 36.

24.

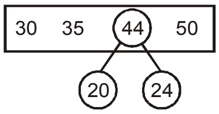
	•	p	q	p	r	q	r	p
	0	0	0	0	0	0	0	0
q	0	0↑	↖1	1←	1←	↖1	1←	1←
p	0	↖1	↑1	↖2	2←	2←	2←	↖2
q	0	↑1	↖2	↑2	↑2	↖3	3←	3←
r	0	↑1	↑2	↑2	↖3	↑3	↖4	4←
r	0	↑1	↑2	↑2	↖3	↑3	↖4	4←

Longest common subsequence : *qpqr*

$x + 10y = 4 + 10 \times 3 = 34$

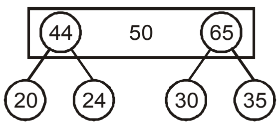
25. 20, 24, 30, 35, 50

Step 1 : We take



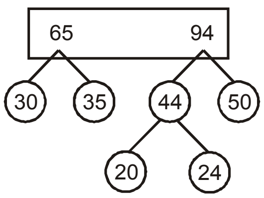
Comparisons = (20 + 24 – 1) Max = 43

Step 2 :



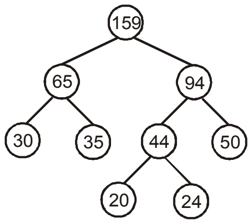
Total comparison = 43 + (65 – 44) = 64

Step 3 :



Total comparison = 64 + 19 = 83

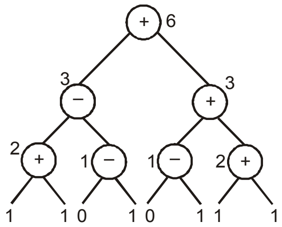
Step 4 :



Total comparison = 93 + 65 = 158

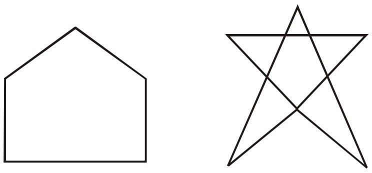
= 158 + 93 + 64 + 43 = 358

26.



For ⊖, we can take maximum value 0 or 1, but ⊕ we can take maximum value 1, Max. value of tree will be 6.

27. When we take n = S, ie



G G′

We can see that in this case G & G′ will be isomorphic to each other.

28. By definition, T(9) = Dist. From 9 to 100

As given, T(9) = 1+min (T(y), T()(z) = 1 + min (Dist. from y to 100, Dist. From z to 100)

⇒ 1 = Dist. from 9 to y/Dist. From 9 to z

⇒ There are only two such values-one is the simple one step on number line i.e. 10, and the other is the shortcut associated with 9 i.e. 15.

⇒ Therefore, y and z are 10 and 15 (in any order)

⇒ Product yz = 150.

29. It takes (log n) time to determine numbers n_1 and n_2 in balanced binary search tree T such that

1. n_1 is the smallest number greater than or equal to L and there is no predecessor n'_1 of n_1 such that n'_1 is equal to n_1.
2. n_2 is the largest number less than or equal to H and there is no successor of n'_2 of n_2 such that is equal to n_2.

Since there are m elements between n_1 and n_2, it takes 'm' time to add all elements between n_1 and n_2.

So time complexity is O (log n + m)

So the given expression becomes

$$O(n^{\circ} \log' n + m' \log^{\circ} n)$$

And $a + 10b + 100c + 1000d$ = 0 + 10*1 + 100*1 + 1000*1 = 10 + 100 = 110

Because a = 0, b = 1, c = 1 and d = 0

■■

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4

CHAPTER

Theory of Computation

REGULAR LANGUAGES & FINITE AUTOMATA

SET

Set is a group of elements, having a property that characterizes those elements.

One way to specify a set is to enumerate the elements completely. All the elements belonging to the set are explicitly given.

e.g., $A = \{1,2,3,4,5\}$

Another way to specify a set is to give the properties that characterize elements of the set.

e.g., $B = \{x | x$ is a positive integer less than or equal to $5\}$

SET TERMINOLOGY

Belongs To ($\in$)

$x \in B$ means that x is an element of set B.

Using this notation we can specify the set {0,1,2,3,4} call it Z by writing

$$Z = \{x \mid x \in N \mid x \leq 5\}$$

where N represents set of natural numbers.

It is read as *"the set of natural numbers that are less than or equal to 5"*.

Subset

Let A and B be two sets.

A is a subset of B, if every element of A is an element of B.

A** is a **subset** of **B is represented as $\mathbf{A \subseteq B}$.

Note : If A is a subset of B and B is a subset of A, then **A = B**.

Also, if A is a subset of B, but not equal to B represented as $\mathbf{A \subset B}$.

Universal Set

The set **U** of all the elements we might ever consider in the discourse is called the **universal set**.

Complement

If A is a set, then complement of A is the set consisting of all elements of the universal set that are not in A. It is denoted by A′ or $\overline{A}$. Thus

$$A' = \{x | x \in U \wedge x \notin A\},$$

where $\notin$ means "*is not an element of*" or "*does not belongs to*"

e.g., If U is set of natural numbers and A = {1, 2, 3}, then

$$A' = \{x \mid x \in U \wedge x > 3\}.$$

Set Operations

Following operations can be performed on sets are:

1. **Union :** If A and B are two sets, then union of A and B is the set that contains all the elements that are in A and B including ones in both A and B. It is denoted by $A \cup B$.
   e.g., If A = {1, 2, 3} and B = {3, 4, 5} then $A \cup B$ = {1, 2, 3, 4, 5}
2. **Difference :** If A and B are two sets, then the difference of A from B is the set that consists of the elements of A that are not in B. It is denoted by A – B.
   e.g., If A = {1, 2, 3} B = {3, 4, 5}, then A – B = {1, 2}
   Note : In general, $A - B \neq B - A$.
   For A and B of the above example B – A = {4, 5}.

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3. **Intersection :** If A and B are two sets, then intersection of A and B is the set that consists of the elements in both A and B. It is denoted by $A \cap B$.

   e.g., If A = {1, 2, 3, 8} B = {3, 4, 5, 8}, then $A \cap B = \{3, 8\}$.

Disjoint sets

A and B are said to be *disjoint* if they contain no elements in common, i.e. $A \cap B = \phi$,

where ϕ is the Empty set.

e.g., A = {1, 2, 3, 4, 5} and B = {6, 8, 9} are disjoint, become $A \cap B = \phi$

Some standard Set Identities

A, **B**, **C** represent arbitrary sets and ϕ is the empty set and **U** is the Universal Set.

Commutative laws: $A \cup B = B \cup A$

$A \cap B = B \cap A$

Associative laws: $A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

Distributive laws: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Idempotent laws: $A \cup A = A$

$A \cap A = A$

Absorptive laws: $A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

De Morgan laws: $(A \cup B)' = A' \cap B'$

$(A \cup B)' = A' \cup B'$

Laws *involving Complements* :

$(A')' = A$

$A \cap A' = \phi$

$A \cup A' = U$

Laws *involving empty set* :

$A \cup \phi = A$

$A \cap \phi = \phi$

Laws *involving Universal Set* :

$A \cup U = U$

$A \cap U = A$

RELATIONS

Definition Relation

Let A and B be sets. A binary relation from A into B is any subset of the Cartesian product $A \times B$.

Example 1 :

Let's assume that a person owns three shirts and two pairs of slacks.

More precisely, let A = {blue shirt, black shirt, mint green shirt} and B = {gray slacks, tan slacks}.

Then certainly $A \times B$ is the set of all possible combinations (six) of shirts and slacks that the individual can wear. However, the individual may wish to restrict himself to combinations which are color coordinated, or **"related"**. This may not be all possible pairs in $A \times B$ but will certainly be a subset of $A \times B$. For example, one such subset may be {(blue shirt, gray slack), (black shirt, tan slacks), (mint green shirt, tan slacks)}.

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Example 2 :

Let A = {2, 3, 5, 6} and define a relation R from A into A by (a, b) $\in$ R if and only if a *divides evenly into b*. So, R = {(2, 2), (3, 3), (5, 5), (6, 6), (2,6), (3, 6)}.

A typical element in R is an ordered pair (x, y). In some cases R can be described by actually listing the pairs which are in R, as in the previous example. This may not be convenient if R is relatively large. Other notations are used depending on the past practice.

Consider the following relation on real numbers.

R = $\{(x, y) \mid y$ is the square of $x\}$ and S = $\{(x, y) \mid x <= y\}$.

R could be more naturally expressed as $R(x) = x^2$. or $R(x) = y$ where $y = x^2$.

Relation on a Set

A relation from a set A into itself is called a *relation on A*.

R of Example 2 above is relation on A = {2, 3, 5, 6}.

Let A be a set of people and let P = $\{(a, b) \mid a \in A \wedge b \in A \wedge a$ is a child of $b\}$. Then P is a relation on A which we might call a parent-child relation.

Composition

Let R be a relation from a set A into set B, and S be a relation from set B into set C. The composition of R and S, written as RS, is the set of pairs of the form $(a, c) \in A \times C$, where $(a, c) \in RS$ *if and only if* there exists $b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$.

For example PP, where P is the parent-child relation given above, is the composition of P with itself and it is a relation which we know as grandparent-grandchild relation.

Properties of Relations

Assume R is a relation on set A; in other words, $R \subseteq A \times A$. Let us write a R b to denote $(a, b) \in R$.

1. **Reflexive :** R is reflexive if for every $a \in A$, a R a.
2. **Symmetric :** R is symmetric if for every a and b in A, if aRb, then bRa.
3. **Transitive :** R is transitive if for every a, b and c in A, if aRb and bRc, then aRc.
4. **Equivalence :** R is an equivalence relation on A if R is *reflexive, symmetric* and *transitive.*

FUNCTIONS

Definition

A **function**, denoted by f, from a set A to a set B is a relation from A to B that satisfies

1. for each element a in A, there is an element b in B such that $<a, b>$ is in the relation, and
2. if $<a, b>$ and $<a, c>$ are in the relation, then $b = c$.

The set A in the above definition is called the domain of the function and B its codomain.

Thus, f is a function if it **covers** the domain and it is **single valued**.

The relation given by f between a and b represented by the ordered pair $<a, b>$ is denoted as $f(a) = b$, and b is called the image of a under f. The set of images of the elements of a set S under a function f is called the image of the set S under f, and is denoted by $f(S)$, that is, $f(S) = \{f(a) \mid a \in S\}$, where S is a subset of the domain A of f. The image of the domain under f is called the range of f.

Example : Let f be the function from the set of natural numbers N to N that maps each natural number x to x^2 . Then the domain and codomain of this f are N, the image of, say 3, under this function is 9, and its range is the set of squares, *i.e.,* (0, 1, 4, 9, 16, ...}.

Definition (sum and product) : Let $\boldsymbol{f}$ and $\boldsymbol{g}$ be functions from a set $\boldsymbol{A}$ to the set of real numbers $\boldsymbol{R}$. Then the **sum** and the **product** of $\boldsymbol{f}$ and $\boldsymbol{g}$ are defined as follows:

For all x, $(f + g)(x) = f(x) + g(x)$, and

for all x, $(f*g)(x) = f(x)*g(x)$,

where $f(x)*g(x)$ is the product of two real numbers $f(x)$ and $g(x)$.

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LANGUAGE

BASIC CONCEPTS

Alphabet

It is a finite set of symbols. e.g., {0, 1} is an alphabet with two symbols, {*a, b*} is another alphabet with two symbols and English alphabet is also an alphabet.

String

It is also called word. It is a finite sequence of symbols of an alphabet.

e.g., b, a and *aabab* are examples of string over alphabet {*a, b*} and 0, 10 and 001 are examples of string over alphabet {0, 1}.

Language

It is a set of strings over an alphabet.

Thus {*a, ab, baa*} is a language (over alphabert {*a,b*}) and {0, 111} is a language (over alphabet {0,1}).

The number of symbols in a string is called **length of the string**. For a string w its length is represented by $|w|$. It can be *defined more formally by recursive definition.*

Empty string (also called *null string*).

It is the string with length 0, *i.e.*, it has no symbols.

The empty string is denoted by Λ(capital lambda). Thus $|\Lambda| = 0$.

Let u and v be strings. Then uv denotes the string obtained by **concatenating** u with v, *i.e.*, uv is the string obtained by appending the sequence of symbols of v to that of u.

e.g., if $u = aab$ and $v = bbab$, then $uv = aabbbab$.

Note : $vu = bbabaab \neq uv$.

We are going to use first few symbols of English alphabet such as a and b to denote symbols of an alphabet and those toward the end such as u and v for strings.

A string x is called a **substring** of another string y if there are strings u and v such that $y = uxv$. Note that u and v may be an empty string. So a string is a substring of itself. A string x is a **prefix** of another string y if there is a string v such that $y = xv$. v is called a **suffix** of y.

SOME SPECIAL LANGUAGES

The empty set ϕ is a language which has no strings. The set {Λ} is a language which has one string, namely Λ. Though Λ has no symbols, this set has an object in it. So it is not empty, but it is empty string.

For any alphabet Σ, the set of all strings over Σ (including empty string) is denoted by Σ^*. Thus a language over alphabet Σ is a subset of Σ^*. It is known as closure of alphabet Σ.

OPERATIONS ON LANGUAGES

Since languages are sets, all the set operations can be applied to languages. Thus the union, intersetion and difference of two languages over an alphabet Σ are languages over Σ. The complement of a language L over an alphabet Σ is $\Sigma^* - L$ and it is also a language.

Another operation on languages is concatenation. Let L_1 and L_2 be languages. Then the **concatenation** of L_1 with L_2 is denoted as L_1L_2 and it is defined as $L_1L_2 = \{uv \mid u \in L_1 \text{ and } v \in L_2\}$.

e.g., L_1L_2 is the set of strings obtained by concatenating strings of L_1 with those of L_2.

e.g., $\{ab, b\}\ \{aaa, abb, aaba\} = \{abaaa, ababb, abaaba, baaa, babb, baaba\}$.

POWERS.

For a symbol $a \in \Sigma$ and a natural number k, a^k represents concatenation of k. For a string $u \in \Sigma^*$ and a natural number k, u^k denotes concatenation of k u's. Similarly for a language L, L^k means concatenation of k L's. Hence L^k is the set of strings that can be obtained by concatenating k strings of L. These powers can be formally defined recursively.

Recursive definition of L^k:

Basis Clause: $L^0 = \{\Lambda\}$

Inductive Clause: $L^{(k+1)} = L^k\, L$.

Since L^k is defined for natural numbers k, the extremal clause is not necessary.

a^k and u^k can be defined similarly.

Here, $a^0 = \Lambda$ and $u^0 = \Lambda$.

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Generalization of Σ^*.

Following two types of languages are generalizations of Σ^*

(*i*) **Recursive definition of L^*:**

Basis Clause : $\Lambda \in L^*$

Inductive Clause : For any $x \in L^*$ and any $w \in L$, $xw \in L^*$.

Extremal Clause : Nothing is in L^* unless it is obtained from the above two clauses.

L^* is the set of strings obtained by **concatenating zero or more** strings of L. This * is called **Kleene star** or star closure.

e.g., if $L = (aba, bb\}$, then $L^* = (\Lambda, aba, bb, ababb, abaaba, bbbb, bbaba, ...\}$

* in Σ^* is also the same Kleene star defined above.

(*ii*) **Recursive definition of L^+:**

Basis Clause : $L \subseteq L^+$

Inductive Clause : For any $x \in L^+$ and any $w \in L$, $xw \in L^+$.

Extremal Clause : Nothing is in L^+ unless it is obtained from the above two clauses.

Thus L^+ is the set of strings obtained by **concatenating one or more** strings of L.

e.g., if $L = \{aba, bb\}$, then $L^+ = \{aba, bb, ababb, abaaba, bbbb, bbaba, ...\}$

Also define $\mathrm{U}_{i=0}^{\infty} L^i$ (i.e. $L^0 \cup L \cup L^2 ...$) as $\mathrm{U}_{i=0}^{\infty} L^i = (x \mid x \in L^k$ for some natural number $k\}$.

Then following relationships hold on L^* and L^+.

Theorem 1 : $L^n \subseteq L^*$

Theorem 2 : $L^* = \mathrm{U}_{i=0}^{\infty} L^i$

Theorem 3 : $L^+ = \mathrm{U}_{i=0}^{\infty} L^i$

Theorem 4 : $L^+ = L\, L^* = L^* L$

Note: According to Theorems 2 and 3, any nonempty string in L^* or L^+ can be expresssed as the concatenation of strings of L, i.e. $w_1 w_2 \ldots w_k$ for some k, where w_i's are strings of L.

Theorem 5 : $L^* = (L^*)^*$.

Proof: Applying $L^* = \mathrm{U}_{i=0}^{\infty} L^i$, to the language L^*, we see

$$L^* \subseteq (L^*)^*.$$

Let x be an arbitrary nonempty string of $(L^*)^*$. Then there are nonempty strings $w_1, w_2, ..., w_k$ in L^* such that

$$x = w_1 w_2 \ldots w_k .$$

Since $w_1, w_2, ..., w_k$ are strings of L^*, for each w_i there are strings $w_{i1}, w_{i2}, ..., w_{imin}$ in L such that

$$w_i = w_{i1} w_{i2} \ldots w_{imi}$$

Hence $$x = w_{11} \ldots w_{1m1} w_{21} \ldots .w_{2m2} \ldots w_{m1} \ldots w_{mmk} .$$

Hence x is in L^* .

If x is an empty string, then it is obviously in L^* .

Hence $$(L^*)^* \subseteq L^* .$$

Since $$L^* \subseteq (L^*)^* \text{ and } (L^*)^* \subseteq L^*, L^* = (L^*)^* .$$

REGULAR LANGUAGE AND REGULAR EXPRESSION

REGULAR LANGUAGE.

The set of regular languages over an alphabet Σ is defined recursively as below. Any language belonging to this set is a *regular language* over Σ.

Definition of Set of Regular Languages.

Basis Clause : ϕ, $\{\Lambda\}$ and $\{a\}$ for any symbol $a \in \Sigma$ are regular languages.

Inductive Clause : If L_r and L_s are regular languages, then $L_r \cup L_s$, $L_r L_s$ and L_r^* are regular languages.

$$\{a + b\} = (\{a\} \cup \{b\}) \text{ and } \{ab\} = (\{a\}\{b\})$$

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Extremal Clause : Nothing is a regular language unless it is obtained from the above two clauses.

e.g., Let $\Sigma = \{a, b\}$. Then since $\{a\}$ and $\{b\}$ are regular languages,

$$\{a + b\} = (\{a\} \cup \{b\}) \text{ and } \{ab\} = (\{a\}\{b\}) \text{ are regular languages.}$$

Also since $\{a\}$ is regular, $\{a\}^*$ is a regular language which is the set of strings consisting of *a's* such as Λ, *a, aa, aaa, aaaa* etc.

Note : Σ^*, which is the set of strings consisting of *a's* and *b's*, is a regular language because $\{a, b\}$ is regular.

REGULAR EXPRESSION.

These are used to denote regular languages. They can represent regular languages and operations on them succinctly.

The set of regular expressions over an alphabet Σ is defined recursively as below. Any element of that set is a *regular expression.*

Basis Clause : ϕ, Λ and a are regular expressions corresponding to languages ϕ, $\{\Lambda\}$ and $\{a\}$, respectively, where a is an element of Σ.

Inductive Clause : If R and S are regular expressions corresponding to languages L_r and L_s, then (R + S), (RS) and (R^*) are regular expressions corresponding to languages $L_r \cup L_s$, $L_r L_s$ and $L_r^{\ *}$, respectively.

Extremal Clause : Nothing is a regular expression unless it is obtained from the above two clauses.

Conventions on Regular expressions

(1) When there is no danger of confusion, bold face may not be used for regular expressions.

(2) The operation * has precedence over concatenation, which has precedence over union (+). Thus the regular expression $(a + (b(c^*)))$ is written as $a + bc^*$,

(3) The concatenation of k *r's*, where r is a regular expression, is written as r^k.

e.g., $rr = r^2$.

The language corresponding to r^k is L_r^k, where L_r is language corresponding to the regular expression r.

For a recursive definition of L_r^{k}

(4) (r^+) is used as a regular expression to represent L_r^+ .

Examples of Regular expression and corresponding Regular languages

(*i*) $(a + b)^2$ corresponds to the language $\{aa, ab, ba, bb\}$, that is the set of strings of length 2 over the alphabet $\{a, b\}$.

In general $(a+b)^k$ corresponds to the set of strings of length k over the alphabet $\{a, b\}$. $(a+b)^*$ corresponds to the set of all strings over the alphabet $\{a, b\}$ including $\wedge$.

(*ii*) a^*b^* corresponds to the set of strings consisting of zero or more *a's* followed by zero or more *b's*.

(*iii*) $a^*b^+a^*$ corresponds to the set of strings consisting of zero or more *a's* followed by one or more *b's* followed by zero or more *a's*.

(*iv*) $(ab)^+$ corresponds to the language $\{ab, abab, ababab, ...\}$, that is, the set of strings of repeated *ab's*.

AUTOMATA.

It is a tool for modelling automata and experimenting with grammars.

The environment deals with :

- Finite state automata-FSA (both deterministic and non-deterministic)
- Push-down automata-PDA (both deterministic and non-deterministic)
- Finite state transducers (Moore and Mealy machines)
- Grammars
- Regular expressions
- Pattern matching (a module to generate automata given a pattern matching problem).

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It provides the visualisation of some important algorithms, such as the transformation of non-deterministic automata into deterministic ones or the minisisation. The automata are defined by entering their state diagram in a graphic editor or by giving a grammar. An example of a finite state automaton, accepting strings of even number of a's and b's, and its equivalent regular grammar.

Finite state automaton is a mathematical model of systems which have a finite number of internal states and respond to external world just by changing their internal state. A finite state machine can be crouch.

Automata theory is a mathematical subject that have serveral practical and useful applications but it is usually introduced very formally to students.

DETERMINISTIC FINITE AUTOMATON (DFA)

Let Q be a finite set of states and let Σ be a finite set of symbols. Also let δ be a transition function from $Q \times \Sigma$ to Q, let q_0 be a state in Q and let A be a subset of Q. Then elements of Q are called a state, δ the transition function, q_0 the initial state and A the set of accepting states or final states.

Then deterministic finite automaton is a 5-tuple : $< Q, \Sigma, q_0, \delta, A >$

Notes :

(1) Set Q in the above definition is simply a set with a finite number of elements. Its elements can be interpreted as a state that the system (automaton) is in.

(2) The transition function is also called **next state function** meaning that the automaton moves into the state $\delta(q, a)$ if it receives the input symbol a while in state q.

(3) δ is a function. Thus for **each state** q of Q and for **each symbol** Σ, $\delta(q, a)$ must be specified.

(4) The accepting states are used to distinguish sequences of inputs given to the finite automaton. If finite automaton is in an accepting state when the input ceases to come, the sequence of input symbols given to the finite automaton is "accepted". Otherwise it is not accepted.

(5) A deterministic finite automaton is also called a "*finite automaton*". Abbreviations such as **FA** and **DFA** are used to denote deterministic finite automaton.

TYPES OF GRAMMAR.

(*i*) 'Type 0' grammar or Grammar of Phrase-Structure Grammar (PSG)

A grammar denoted by G consists of 4 components.

i.e. $G = (V_N, V_T, P, S)$

where, V_N = a set of non-terminals *e.g.* S, A, B

V_T = a set of terminals *e.g.* a, b

P = a set of production rules *e.g.* $\alpha \to \beta$

S = start symbol, where $S \in V$

Also, $V_N \cup V_T = V$ and $V_N \cap V_T = \phi$

All grammars are type 0 grammar.

(*ii*) 'Type 1' grammar or Context Sensitive Grammar (CSG)

This is a grammar in which each production rule is of the form $\alpha \to \beta$, where

length of $\alpha \leq$ length of β.

Let $\phi A \psi \to \phi \alpha \psi$

Here ϕ and ψ are left and right context respectively. If there is any left or right context in a production then the grammar is context sensitive grammar

e.g. consider the grammar

$$
\begin{aligned}
A &\to aABc \\
cB &\to Bc \\
A &\to abc \\
bB &\to bb \\
B &\to cba
\end{aligned}
$$

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(*iii*) 'Type 2' grammar or Context-Free Grammar (CFG)

This is a grammar in which every production rule is of the form $\alpha \to \beta$, where $\alpha \in V_N$ and $\beta \in V$ or $\beta \in (V_N U V_T)^*$.

e.g. consider the grammar, G = (V_N, V_T, P, S)

where $V_N = \{S\}$

$V_T = \{a, b\}$

P:

$S \to aSaSb$

$S \to a$

S is the starting symbol.

(*iv*) 'Type 3' grammar or Regular Grammar or Linear Grammar

This is a grammar in which every production rule is either of the form, $A \to aB$ or $A \to Ba$ and $A \to a$

where A, B $\in V_N$ and $a \in V_T$

e.g. Right Linear Grammar

$A \to aB$

$A \to a$

Left linear grammar

$A \to Ba$

$A \to a$

- Let G be a grammar. A string x is called a *sentential form* if $S \xrightarrow{*} x$
- Let G be a grammar. The language of the grammar G, denoted by L(G) and is

$$L(G) = \{x | S \xrightarrow{*} x \text{ and } x \in V_T^*\} \ \{x | S \xrightarrow{*} x \text{ and } x \in V_T^*\}$$

Ambiguous sentence.

If a particular sentence has more than one leftmost or rightmost derivation, then the sentence is called *ambiguous sentence*.

If a grammar generates an ambiguous sentence, then it is called *ambiguous* grammar.

e.g. consider the grammar. $E \to E + E \mid E * E \mid (E) \mid id$.

Consider the sentence "id + id *id". *It has two leftmost derivation as shown below :*

(*i*) $E \to E + E$

$E \to id + E$

$E \to id + E * E$

$E \to id + id * E$

$E \to id + id * id$

(*ii*) $E \to E * E$

$E \to E + E * E$

$E \to id + E * E$

$E \to id + id * E$

$E \to id + id * id$

Unambiguous sentence.

A sentence in a language is said to be unambiguously derivable, if it has a unique derivation tree or unique-leftmost or rightmost derivation.

Ambiguous language.

A language is said to be ambiguous if it has ambiguous grammar.

Unambiguous language.

A language is said to be unambiguous if there is at least one unambiguous grammar generating it.

- A string of terminals w is in L (G) if and only if $S \xrightarrow{*} w$. This string w is called a *sentence of G*.
- If $S \xrightarrow{*} \alpha$, where may contain non-terminals, then α is a *sentential form* of G.
  If this sentential form follows a LMD scheme, then it is called *left sentential form*, otherwise right *sentential form*.

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Transition Diagram

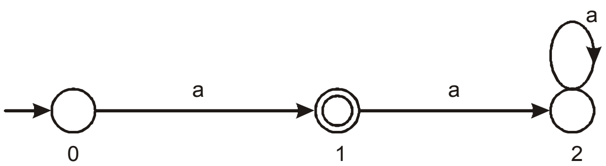
DFAs are often represented by digraphs called **(state) transition diagram**. The vertices (denoted by single circles) of a transition diagram represent the states of the DFA and the arcs labeled with an input symbol correspond to the transitions. An arc (p, q) from vertex p to vertex q with label σ represents the transition $\delta(p, \sigma) = q$. The accepting states are indicated by double circles.

Transition functions can also be represented by tables as seen below. They are called **transition table**.

Example 1. Q = {0, 1, 2}, $\Sigma = \{a\}$, A = {1}, the initial state is 0 and δ is as shown in the following table :

State (q)	Input (a)	Next State ($\delta(q, a)$)
0	a	1
1	a	2
2	a	2

A state transition diagram for this DFA is given below.



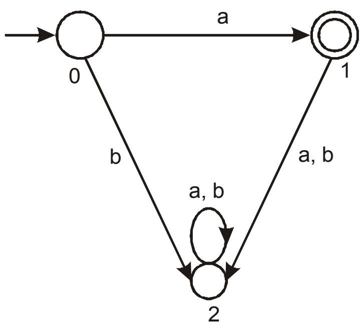
If alphabet Σ of the example is changed to $\{a, b\}$ in stead of $\{a\}$, then a DFA is needed such as shown in the following example to accept the same string a. It is a little more complex DFA.

Example 2. Q = {0, 1, 2}, $\Sigma = \{a, b\}$, A = {1}, the initial state is 0 and δ is as shown in the following table.

State (q)	Input (a)	Next State ($\delta(q, a)$)
0	a	1
0	b	2
1	a	2
1	b	2
2	a	2
2	b	2

For each state there are two rows in the table for δ corresponding to the symbols a and b, while in the Example 1 there is only one row for each state.

A state transition diagram for this DFA is given below.



LANGUAGE ACCEPTED BY DFA

- *A string w is accepted by a DFA* $(Q, \Sigma, q_0, \delta, A)$, if and only if $\delta^*(q_0, w) \in A$.

  i.e., a string is accepted by a DFA if and only if DFA starting at the initial state ends in an accepting state after reading the string.

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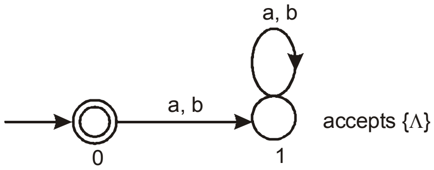
- *A language L is accepted by a DFA* < Q, Σ, q_0, δ, A >, if and only if L = {w | $\delta^*(q_0, w) \in A$}.
  i.e., the language accepted by a DFA is the set of strings accepted by the DFA.

  Example 1 :



This DFA accepts {Λ} because it can go from initial state to the accepting state (also initial state) without reading any symbol of the alphabet, *i.e.* by reading an empty string Λ. It accepts nothing else because any non-empty symbol would take it to state 1, which is not an accepting state, and it stays there.

Definition of δ^*.

Basis Clause: For any state q of Q, $\delta^*(q, \Lambda) = q$,

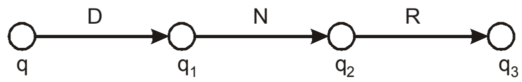
where Λ denotes the empty string.

Inducitve Clause: For any state q of Q, any string $y \in \Sigma^*$ and any symbol $a \in \Sigma$,

$$\delta^*(q, ya) = \delta(\delta^*(q, y), a) .$$

In the definition, Basis Clause says that a DFA stays in state q when it reads an empty string at state q and Inductive Clause says that the state DFA reaches after reading string ya starting at state q is the state it reaches by reading symbol a after reading string y from state q.

Example. Let a DFA contains the transitions shown below.



Then $\delta^*(q, DNR)$ can be calculated as follows:

$\delta^*(q, \text{DNR})$ = $\delta(\delta^*(q, \text{DN}), \text{R})$ by the Inductive Clause.
= $\delta(\delta(\delta^*(q, \text{D}), \text{N}), \text{R})$ by applying the Inductive Clause to $\delta^*(q, \text{DN})$.
= $\delta(\delta(\delta^*(q, \Lambda\text{D}), \text{N}), \text{R})$ since D = ΛD .
= $\delta(\delta(\delta(\delta^*(q, \Lambda), \text{D}), \text{N}), \text{R})$ by applying the Inductive Clause to $\delta^*(q, \Lambda\text{D})$.
= $\delta(\delta(\delta(q, \text{D}), \text{N}), \text{R})$, since $\delta(q, \Lambda) = q$.
= $\delta(\delta(q_1, \text{N}), \text{R})$, since $\delta(q, \text{D}) = q_1$ as seen from the diagram.
= $\delta(q_2, \text{R})$, since $\delta(q_1, \text{N}) = q_2$ as seen from the diagram.
= q_3 since $\delta(q_2, \text{R}) = q_3$ as seen from the diagram.

Properties of δ^*

Following are two properties of δ^*.

Theorem 1: For any state q of Q and any symbol a of Σ for a DFA < Q, Σ, q_0, δ, A >,

$$\delta^*(q, a) = \delta(q, \text{a})$$

Proof : Since a = Λa, $\delta^*(q, a) = \delta^*(q, \Lambda\text{a})$.

By definition of δ^*, $\delta^*(q, \Lambda\text{a}) = \delta(\delta^*(q, \Lambda), a)$

But by the definition of δ^*, $\delta^*(q, \Lambda) = q$

Hence $\delta(\delta^*(q, \Lambda), a) = \delta(q, a)$.

Theorem 2: This theorem states that state reached from any state, say q, by reading a string, say w, is same as the state reached by first reading a prefix of w, call it x, and then by reading rest of the w, call it y.

i.e., For any state q of Q and any strings x and y over Σ for a DFA < Q, Σ, q_0, δ, A >,

$$\delta^*(q, xy) = \delta^*(\delta^*(q, x), y).$$

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Proof : *This is going to be proven by induction on string y, i.e., the statement to be proven is the following:*

For an arbitrary fixed string x, $\delta^*(q, xy) = \delta^*(\delta^*(q, x), y)$ holds for any arbitrary string y.

Recursive definition of Σ^*:

Basis Clause: $\Lambda \in \Sigma^*$.

Inductive Clause: If $x \in \Sigma^*$ and $a \in \Sigma$, then $xa \in \Sigma^*$.

Extremal Clause: Nothing is in Σ^* unless it is obtained from the above two clauses.

Proof of the theorem.

Basis step: If y = Λ, then $\delta^*(q, xy) = \delta^*(q, x\Lambda) = \delta^*(q, x)$.

Also, $\delta^*(\delta^*(q, x), y) = \delta^*(\delta^*(q, x), \Lambda) = \delta^*(q, x)$ by definition of δ^*.

Hence theorem holds for $y = \Lambda$.

Inductive step: Assume that $\delta^*(q, xy) = \delta^*(\delta^*(q, x), y)$ holds for an arbitrary string y. This is the induction hypothesis.

We are going to prove that $\delta^*(q, xya) = \delta^*(\delta^*(q, x), ya)$ for any arbitrary symbol a of Σ.

$\delta^*(q, xya) = \delta(\delta^*(q, xy), a)$ by the definition of δ^*

$= \delta(\delta^*(\delta^*(q, x), y), a)$ by the induction hypothesis.

$= \delta^*(\delta^*(q, x), ya)$ by the definition of δ^*.

NON DETERMINISTIC FINITE AUTOMATON (NFA)

Let Q be a finite set of states and let Σ be a finite set of symbols. Also let δ be a function from $Q \times \Sigma$ to 2^Q, let q_0 be a state in Q and let A be a subset of Q. Then elements of Q is called a **state**, δ the **transition function**, q_0 the **initial state** and A the set of **accepting states**.

Then a **nondeterministic finite automaton** is a 5-tuple

$$< Q, \Sigma, q_0, \delta, A >$$

Notes :

(1) As in the case of DFA, the set Q in the above definition is simply a set with a finite number of elements. Its elements can be interpreted as a state that the system (automaton) is in.

(2) The transition function is also called a **next state function**. Unlike DFAs an NFA moves into one of the states given by $\delta(q, a)$ if it receives the input symbol a while in state q, which one of the states in $\delta(q, a)$ to select is determined non deterministically.

(3) δ is a function. Thus for **each state** q of Q and for **each symbol** a of Σ $\delta(q, a)$ must be specified. But it can be the empty set, in which case NFA aborts its operation.

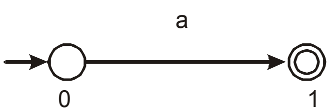
(4) As in the case of DFA, accepting states are used to distinguish sequences of inputs given to the finite automaton. If finite automaton is in an accepting state when input ends, *i.e.* ceases to come, the sequence of input symbols given to the finite automaton is "*accepted*", otherwise it is not accepted.

(5) For every NFA, there is an equivalent DFA.

Example 1. Q = {0, 1}, Σ = {a}, A = (1}, the initial state is 0 and δ is as shown in the following table :

State (q)	*Input (a)*	*Next State (δ (q, a))*
0	a	{ 1 }
1	a	ϕ

A state transition diagram for this finite automaton is given below :



If alphabet Σ is changed to {a, b} in stead of {a}, this is still an NFA that accepts {a} .

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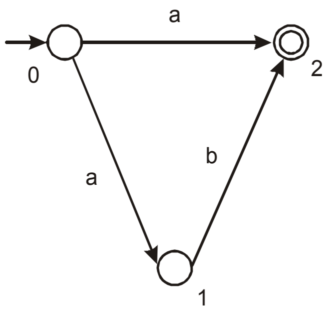
Example 2. Q = {0, 1, 2}, Σ = {a, b}, A = {2}, the initial state is 0 and δ is as shown in the following table.

State (q)	*Input (a)*	*Next State ($\delta(q, a)$)*
0	a	{1, 2}
0	b	ϕ
1	a	ϕ
1	b	{2}
2	a	ϕ
2	b	ϕ

For each state, there are two rows in the table for δ corresponding to the symbols a and b, while in Example 1, there is only one row for each state.

A state transition diagram for this finite automaton is given below :



Operation of NFA :

Consider the automaton of Example 2 above.

Initially it is in state 0. When it reads the symbol a, it moves to either state 1 or state 2.

Since state 2 is accepting state, if it moves to state 2 and no more inputs are given, then it stays in the accepting state. This automaton accepts the string a but if it moves to state 1 after reading a, if next input is b and if no more inputs are given, then it goes to state 2 and remains there. Thus string ab is also accepted by this NFA. If any other strings are given to this NFA, it does not accept any of them.

DIFFERENCE BETWEEN DFA AND NFA.

(1) When processing a string in a DFA, there is always a unique state to go next when each character is read and in an NFA, several choice (or no choice) may exist for the next state

(2) For each state in DFA, there is exactly one state that corresponds to each character being read and in an NFA *can move to more than 1 states, or nowhere can move to a state without reading anything*

(3) Null moves are possible and may be specified in NFA but not in DFA.

LANGUAGE ACCEPTED by NFA.

Definition of δ^*

For a state q and string w, $\delta^*(q, w)$ is the set of states that the NFA can reach when it reads the string w starting at the state q. In general, an NFA nondeterministically goes through a number of states from the state q as it reads the symbols in the string w.

Thus for an NFA < Q, Σ, q_0, δ, A >, the function

$$\delta^* : Q \times \Sigma^* \to 2^Q$$

is defined recursively as follows:

Basis Clause: For any state q of Q, $\delta^*(q, \Lambda) = \{q\}$, where Λ denotes the empty string.

Inductive Clause: For any state q of Q, any string $y \in \Sigma^*$ and any symbol $a \in \Sigma$,

$$\delta^*(q, ya) = \cup_{p \in \delta^*(q, y)} \delta(p, a)$$

In the definition, Basis Clause says that an NFA stays in state q when it reads an empty string at state q and the Inductive Clause says that the set of states NFA *can* reach after reading string ya starting at state q is the set of states it can reach by reading symbol a after reading string y starting at state q.

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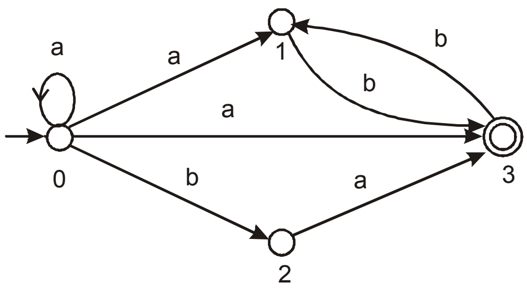
Example. *Consider NFA with the following transition table:*

State (q)	Input (a)	Next State ($\delta(q, a)$)
0	a	{0, 1, 3}
0	b	{2}
1	a	ϕ
1	b	{3}
2	a	{3}
2	b	ϕ
3	a	ϕ
3	b	{1}

The transition diagram for this NFA is as given below.



Let state 3 is an accepting state of this NFA.

Then $\delta^*(0, ab)$ can be calculated as follows:

$\delta^*(0, ab)$ is the union of $\delta(p, b)$ for all $p \in \delta^*(0, a)$ by the Inductive Clause of the definition of δ^*.

Now, $\delta^*(0, a)$ is the union of $\delta(p, a)$ for all $p \in \delta^*(0, \Lambda)$ again by the Inductive Clause of the definition of δ^*.

By Basis Clause of the definition of δ^*, $\quad \delta^*(0, \Lambda) = \{0\}$.

Hence $\quad \delta^*(0, a) = \delta(0, a) = \{0, 1, 3\}$.

and $\quad \delta^*(0, ab) = \delta(0, b) \cup \delta(1, b) \cup \delta(3, b) = \{2\}\ \delta\{3\}\ \delta\{1\} = \{1, 2, 3\}$.

So, a string $x \in \Sigma^*$ is accepted by an NFA $(Q, \Sigma, q_0, \delta, A)$ if and only if $\delta^*(q_0, x) \cap A$ is not empty, *i.e.*, if and only if it can reach an accepting state by reading x starting at the initial state. The language accepted by an NFA $(Q, \Sigma, q_0, \delta, A)$ is set of strings that are accepted by the NFA.

Some of the strings accepted by the NFA given above are *a*, *ab, aaa, abbbb* etc. and language it accepts is $a^*(ab + a + ba)(bb)^*$.

δ^* for NFA has properties similar to that for DFA.

Theorem 1: For any state q of Q and any symbol a of Σ for an NFA $(Q, \Sigma, q_0, \delta, A)$,

$$\delta^*(q, a) = \delta(q, a)$$

Theorem 2: For any state q of Q and any strings x and y over Σ for an NFA $(Q, \Sigma, q_0, \delta, A)$,

$$\delta^*(q, xy) = \cup_{p \in \delta^*(q, x)} \delta^*(p, y)$$

Non deterministic Finite Automata with Λ-Transitions

Let Q be a finite set and let Σ be a finite set of symbols. Also let δ be a function from $Q \times \{\Sigma \cup \Lambda\}$ to 2^Q, let q_0 be a state in Q and let A be a subset of Q. We call the elements of Q a **state**, δ the **transition function**, q_0 the **initial state** and A the set of **accepting states**.

Then **nondeterministic finite automaton with Λ-Transitions** is a 5-tuple $(Q, \Sigma, q_0, \delta, A)$

Notes :

(1) A transition on reading Λ means NFA-Λ makes the transition without reading any symbol in the input. Thus tape head does not move when Λ is read.

(2) Any NFA is also a NFA-Λ.

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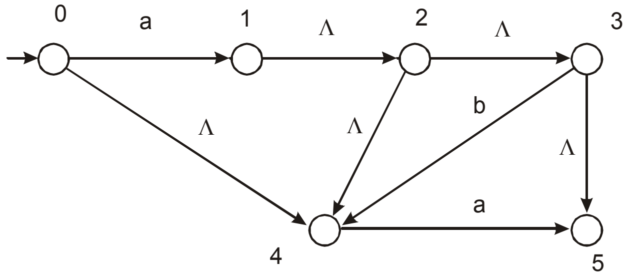
Example. Q = {0, 1, 2, 3, 4, 5}, Σ = {*a, b*}, A = φ, the initial state is 0 and φ is as shown in the following table.

State (q)	*Input (a)*	*Next State (δ(q, a))*
0	*a*	{1}
0	Λ	{4}
1	Λ	{2}
2	Λ	{3, 4}
3	Λ	{5}
3	*b*	{4}
4	*a*	{5}

Here transitions to φ are omitted from the table. A state transition diagram for this finite automaton is given below.



For example when ‘a’ symbol a is read at the initial state 0, it can move to any of the states other than 0. For example, for once you are in state 1, you can go to state 2, 3, 4 and 5 without reading any symbol on the tape. If you read string ab, then you come to state 4. For though you go to states 1, 2, 3, 4 and 5 by reading a, there are no transitions on reading b except from state 3. Thus 4 is the only state you can go to from the initial state by reading *ab*.

Language Accepted by NFA-Λ

Definition of Λ-closure

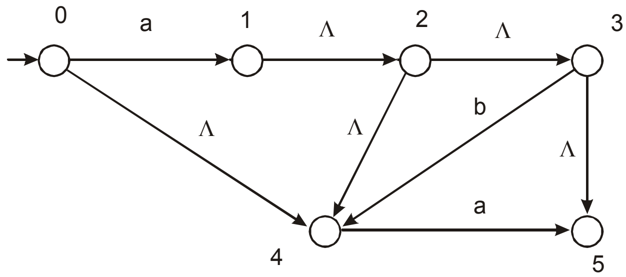
Let (Q, Σ, q_0, φ, A) be an NFA-Λ. Denote Λ-closure of a set S of states of Q by Λ(S).

Then Λ(S) is defined recursively as follows :

Basis Clause: S ⊆ (S)

Inductive Clause: For any state *q* of Q, if *q* ∈ Λ(S), then δ(q, Λ) ⊆ Λ(S) .

Extremal Clause: Nothing is in Λ(S) unless it is obtained by the above two clauses.



For NFA-Λ of the above figure, Λ({2}) is obtained as follows:

First {2} ⊆({2}), *i.e.*, 2 ∈({2}) .

Then since 2 ∈ Λ((2}), by the Inductive Clause, δ(2, Λ) ⊆ Λ({2}).

Since δ(2, Λ) = {3, 4}, we have (2, 3, 4} ⊆ Λ({2}) .

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Since 3 and 4 have been added to $\Lambda(\{2\})$, $\delta(3, \Lambda) = \{5\}$ and $\delta(4, \Lambda) = \phi$ must be included in $\Lambda(\{2\})$.

Thus $\{2, 3, 4, 5\} \subseteq \Lambda(\{2\})$.

Though 5 has become a memeber of the closure, since $\delta(5, \Lambda)$ is empty, no new members are added to $\Lambda(\{2\})$. Since $\delta(q, \Lambda)$ has been examined for all the states currently in $\Lambda(\{2\})$ and no more elements are added to it, this process of generating the Λ closure terminates and $\Lambda(\{2\}) = \{2, 3, 4, 5\}$ is obtained.

As we see, $\Lambda(S)$ is the set of states that can be reached from the states of S by traversing any number of Λ arcs, *i.e.*, it is set of states that can be reached from the states of S without reading any symbols in Σ.

Now with this Λ-closure, δ^* can be defined recursively as follows:

As in the cases of DFA and NFA, δ^* gives the result of applying the transition function δ repeatedly as dictated by the given string.

Definition of δ^*

δ^* is going to be defined recursively.

Let $(Q, \Sigma, q_0, \delta, A)$ be an NFA-Λ.

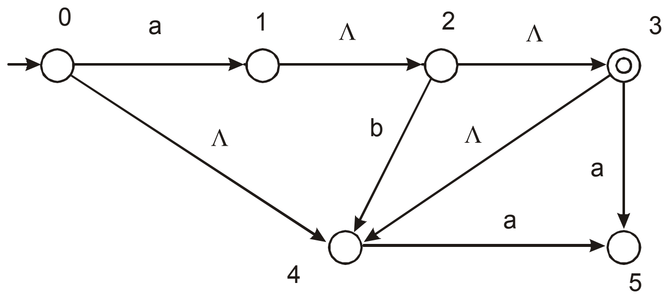
Basis Clause: For any state q of Q, $\delta^*(q, \Lambda) = \Lambda(\{q\})$.

Inductive Clause: For any state q, a string y in Σ^* and a symbol a in Σ,

$$\delta^*(q, ya) = \Lambda(\cup_{\in \delta^*(q,\, y)} \delta(p, a)).$$

The Inductive Clause means that $\delta^*(q, ya)$ is obtained by first finding the states that can be reached from q by reading y ($\delta^*(q, y)$), then from each of those states p by reading a (i.e. by finding $\delta(p, a)$), and then by reading Λ's (i.e. by taking Λ closure of the $\delta(p, a)$'s) .

Example. For NFA-Λ of the following figure, $\delta^*(0, ab)$ can be obtained as below:



First compute $\Lambda^*(0, a)$, which required $\Lambda(\{0\})$.

Since it is the set of states reached by traversing the Λ arcs from state 0, $\Lambda(\{0\}) = \{0, 3, 4\}$.

Next from each of the states in $\Lambda(\{0\})$, read symbol a and move to another state (*i.e.* apply δ).

They are $\delta(0, a) = \{1\}$, $\delta(3, a) = \delta(4, a) = \{5\}$.

Hence $\cup_{\in \delta^*(q,\, y)} \delta(p, a) = \{1, 5\}$ for $q = 0$.

Then traverse the Λ arcs from {1, 5} to get to the states in $\delta^*(0, a)$.

Since $\Lambda(\{1\}) = (1, 2, 3\}$ and $\Lambda(\{5\}) = \{5\}$, $\delta^*(0, a) = \{1, 2, 3, 5\}$.

Then to find $\delta^*(0, ab)$, read b from each of the states in $\delta^*(0, a)$ and then take Λ arcs from there.

Now $\delta(1, b)$, $\delta(3, b)$ and $\delta(5, b)$ are empty sets, and $\delta(2, b) = \{4\}$.

Thus since $\Lambda(\{4\}) = \{3, 4\}$, $\delta^*(0, ab) = \{3, 4\}$.

Note :

A string x is accepted by an NFA-Λ $(Q, \Sigma, q_0, \delta, A)$ if and only if $\delta^*(q_0, x)$ contains at least one accepting state.

The language accepted by an NFA-Λ $(Q, \Sigma, q_0, \delta, A)$ is the set of strings accepted by the NFA-Λ.

e.g., NFA-Λ of the figure given above accepts the language (Λ, a, ab}.

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Conversion of NFA-Λ to NFA

Let $M_1 = (Q_1, \Sigma, q_{1,0}, \delta_1, A_1)$ be an NFA-Λ that recognizes a language L.

Then NFA $M_2 = (Q_2, \Sigma, q_{2,0}, \delta_2, A_2)$ that satisfies following conditions recognizes L:

$$\begin{aligned} Q_2 &= Q_1, \\ q_{2,0} &= q_{1,0}, \\ \delta_2(q, a) &= \delta_1^*(q, a) = \Lambda(\cup_{p \in \Lambda(q)} \delta_1(p, a)) \\ A_2 &= A_1 \cup \{q_{1,0}\} \text{ if } \Lambda(\{q_{1,0}\}) \cap A_1 \neq \phi \\ &= A_1 \text{ otherwise.} \end{aligned}$$

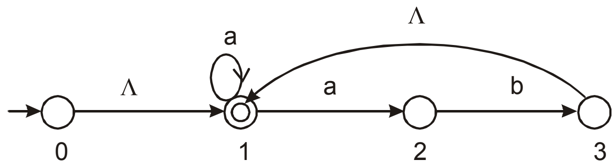
Thus to obtain an NFA, $M_2 = (Q_2, \Sigma, q_{2,0}, \delta_2, A_2)$ which accepts same language as the given NFA-Λ $M_1 = (Q_1, \Sigma, q_{1,0}, \delta_1, A_1)$ does, first copy the states of Q_1 into Q_2.

Then for each state q of Q_2 and each symbol a of Σ find $\delta_2(q, a)$ as follows:

Find Λ({q}), *i.e.,* all the states that can be reached from q by traversing Λ arcs. Then collect all the states that can be reached from each state of Λ({q}) by traversing one arc labeled with the symbol a. The Λ closure of the set of those states is $\delta_2(q, a)$.

The set of accepting states A_2 is the same as A_1 if no accepting states can be reached from the initial state $q_{1,0}$ through Λ arcs in M_1. Otherwise, if an accepting state can be reached from the initial state $q_{1,0}$ through Λ arcs in M_1, then all the accepting states of M_1 plus state $q_{1,0}$ are accepting states of M_2 .

Example. Convert following NFA-Λ to NFA.



The set of states Q_2 of NFA is {0, 1, 2, 3), initial state is 0 and accepting states are 1 and 0, since 1 is in Λ({0}).

The transition function δ_2 is obtained as follows:

$$\delta_2(0, a):$$

First Λ({0}) = {0, 1} . Then from the transition function of the NFA-Λ

$$\delta_1(0, a) = \phi, \text{ and } \delta_1(1, a) = \{1, 2\}.$$

Hence $\delta_2(0, a) = \Lambda(\{1, 2\}) = \{1, 2\}$.

For $\delta_2(0, b)$, since Λ({0})= {0, 1} and $\delta_1(0, b) = \delta_1(1, b) = \phi$, $\delta_2(0, b) = \phi$.

Similarly δ_2 can be obtained for other states and symbols. They are given in the following table together with Λ({*q*}) and $\cup_{p\in\Lambda(q)} \delta_1(p, \sigma)$.

State q	*Input σ*	*Λ({q})*	$\cup_{p\in\Lambda(q)} \delta_1(p, \sigma)$	$\delta_2(q, \sigma)$ $(=\Lambda(\cup_{p\in\Lambda(q)} \delta_1(p, \sigma))$
0	*a*	{0, 1}	{1, 2}	{1, 2}
0	*b*	{0, 1}	φ	φ
1	*a*	{1}	{1, 2}	{1, 2}
1	*b*	{1}	φ	φ
2	*a*	{2}	φ	φ
2	*b*	{2}	{3}	{1, 3}
3	*a*	{1, 3}	{1, 2}	{1, 2}
3	*b*	{1, 3}	φ	φ

Conversion of NFA to DFA

Let $M_2 = (Q_2, \Sigma, q_{2,0}, \delta_2, A_2)$ be an NFA that recognizes a language L.

Then DFA $M = (Q, \Sigma, q_0, \delta, A)$ satisfies following conditions recognizes L :

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$\mathbf{Q} = 2^{Q_2}$, *i.e.,* set of all subsets of Q_2,

$q_0 = \{q_{2,0}\}$,

$\delta(q, a) = \cup_{p \in q} \delta(p, a)$ for each state q in $\mathbf{Q}$ and each symbol a in Σ and

$A = \{q \in Q \mid q \cap A_2 \neq \phi\}$

To obtain a DFA, $M = (Q, \Sigma, q_0, \delta, A)$ which accepts same language as the given NFA $M_2 = (Q_2, \Sigma, q_{2,0}, \delta_2, A_2)$ does, proceed as follows:

Initially $\mathbf{Q} = \phi$.

First put $\{q_{2,0}\}$ into $\mathbf{Q}$. $\{q_{2,0}\}$ is the initial state of the DFA M.

Then for each state q in $\mathbf{Q}$ *do the following:*

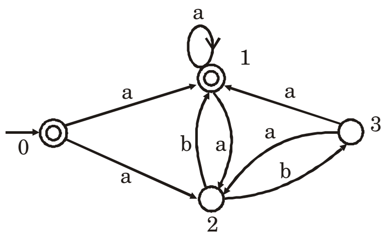
Add the set $\cup_{p \in q} \delta(p, a)$, where δ here is that of NFA M_2, as a state to $\mathbf{Q}$ if it is not already in $\mathbf{Q}$ for each symbol a in Σ.

For this new state, add $\delta(q, a) = \cup_{p \in q} \delta(p, a)$ to δ, where δ on the right hand side is that of NFA M_2.

When no more new states can be added to $\mathbf{Q}$, the process terminates. All the states of $\mathbf{Q}$ that contain accepting states of M_2 are accepting states of M.

Note: The states that are not reached from the initial state are not included in $\mathbf{Q}$ obtained by this procedure. Thus set of states $\mathbf{Q}$ thus obtained is not necessarily equal to $2^{Q}{}_2$.

Example. Convert the following NFA to DFA.



Initially Q is empty. Then since initial state of the NDFA is {0}, {0} is added to Q.

Since $\delta_2(0, a) = \{1, 2\}$, (1, 2} is added to $\mathbf{Q}$ and $\delta(\{0\}, a) = \{1, 2\}$.

Since $\delta_2(0, b) = \phi$, ϕ is added to $\mathbf{Q}$ and $\delta(\{0\}, b) = \phi$.

At this point, $Q = (\{0\}, \{1, 2\}, \phi)$.

Then since {1, 2} is now in $\mathbf{Q}$, the transitions from {1, 2} on symbols a and b are computed.

Since $\delta_2(1, a) = (1, 2\}$, and $\delta_2(2, a) = \phi$, $\delta(\{1, 2\}, a) = \delta(\{1\}, a) \cup \delta(\{2\}, a) = \{1, 2\} \cup \{\phi\} = \{1, 2\}$

Similarly $\delta(\{1, 2\}, b) = \{1, 3\}$.

Thus (1, 3} is added to $\mathbf{Q}$.

Similarly $\delta(\{1, 3\}, a) = \{1, 2\}$ and $\delta(\{1, 3\}, b) = \phi$.

Thus no new states are added to $\mathbf{Q}$.

Since transitions from all states of $\mathbf{Q}$ have been computed and no more states are added to $\mathbf{Q}$, the conversion process stops here.

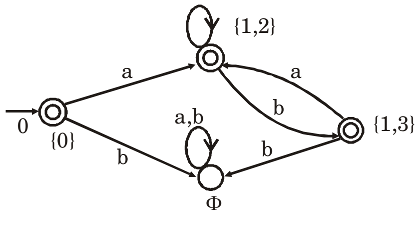
Note : There are no states of Q_2 in ϕ.

Hence there are no states that M_2 can go to from ϕ.

Hence $\delta(\phi, a) = \delta(\phi, b) = \phi$.

For accepting states of M, since states 0 and 1 are accepting states of the NFA, all the states of $\mathbf{Q}$ that contain 0 and/or 1 are accepting states. Hence {0}, {1, 2} and {1, 3} are accepting states of M.

Thus DFA thus obtained is shown below.



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CONTEXT-FREE LANGUAGES

The class of context-free languages generalizes the class of regular languages *i.e.,* every regular language is a context-free language.

The reverse of this is not true *i.e.,* every context-free language is not necessarily regular.

e.g., $\{0^k 1^k | k>=0\}$ is context-free but not regular.

Many issues and questions for regular languages will be the same for context-free languages:

Machine model – PDA (Push-Down Automata)

Descriptor – CFG (Context-Free Grammar)

Pumping lemma for context-free languages

Closure of context-free languages with respect to various operations

Algorithms and conditions for finiteness or emptiness

Some analogies don't hold, *e.g.* non-determinism in a PDA makes a difference and, in particular, deterministic PDAs define a subset of the context-free languages.

Informally a Context-Free Language (CFL) is a language generated by a Context-Free Grammar (CFG).

Context-Free Grammar (CFG)

Informally, CFG is a set of rules for deriving (or generating) strings (or sentences) in a language.

Informally CFG consists of :

(*i*) a set of replacement rules, each having a Left-Hand Side (LHS) and a Right-Hand Side (RHS).

(*ii*) two types of symbols; variables and terminals.

(*iii*) LHS of each rule is a single variable (no terminals).

(*iv*) RHS of each rule is a string of zero or more variables and terminals.

(*v*) a string consists of only terminals.

Note : G "generates" the language $\{0^k1^k \mid k \geq 0\}$

Formally, a CFG is a 4-tuple:

$$G = (V, T, P, S)$$

where, V - A finite set of variables or *non-terminals*

T - A finite set of *terminals* (V and T do not intersect)

P - A finite set of *productions*, each of the form $A \to \alpha$, where A is in V and α is in $(V \cup T)^*$

Note that α may be $\wedge$

S - A starting non-terminal (S is in V)

Example CFG:

$G = (\{S\}, \{0, 1\}, P, S)$

P : Consist of S

(1) $S \to 0S1$ or just simply $S \to 0S1 \mid \Lambda$

(2) $S \to \wedge$

Example Derivations:

$S \Rightarrow 0S1$

$S \Rightarrow 0S1$

$\Rightarrow 01$

$S \Rightarrow 0S1$

$\Rightarrow 00S11$

$\Rightarrow 000S111$

$\Rightarrow 000111$

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Example CFG:

$G = (\{A, B, C, S\}, \{a, b, c\}, P, S)$

P: (1) $S \rightarrow ABC$

(2) $A \rightarrow aA$

(3) $A \rightarrow \wedge$

(4) $B \rightarrow bB$

(5) $B \rightarrow \wedge$

(6) $C \rightarrow cC$

(7) $C \rightarrow \wedge$

Example Derivations:

$$
\begin{aligned}
S &\Rightarrow ABC \\
&\Rightarrow aABC && \{A \Rightarrow aA\} \\
&\Rightarrow aaABC \\
&\Rightarrow aaBC && \{A \Rightarrow \wedge\} \\
&\Rightarrow aabBC && \{B \Rightarrow bB\} \\
&\Rightarrow aabC && \{B \Rightarrow \wedge\} \\
&\Rightarrow aabC \\
&\Rightarrow aabcC \\
&\Rightarrow aabBC \\
&\Rightarrow aabC \\
&\Rightarrow aabcC && \{C \Rightarrow cC\} \\
&\Rightarrow aabc && \{C \Rightarrow \wedge\}
\end{aligned}
$$

Note : G generates the language $a^*b^*c^*$.

Definition.

A CFG is a regular grammar if each rule is of the following form:

$$A \rightarrow a$$
$$A \rightarrow aB$$
$$A \rightarrow \wedge$$

where A and B are in V, and a is in T

Theorem : A language L is a regular language iff there exists a regular grammar G such that $L = L(G)$.

Observation : The grammar $S \rightarrow 0S1 \mid \wedge$ is not a regular grammar.

Observation : A language may have several CFGs, some regular, some not (The fact that the preceding grammar is not regular does not in and of itself prove that $L = \{0^n1^n | n \geq 0\}$ is not a regular language).

Definition.

Let $G = (V, T, P, S)$ be a CFG.

A tree is a derivation (or parse) tree if:

- Every vertex has a label from $V \cup T \cup \{\wedge\}$
- Label of the root is S
- If a vertex with label A has children with labels $X_1, X_2, \ldots, X_n$, from left to right, then
  $$A \rightarrow X_1, X_2, \ldots, X_n$$
  must be a production in P
- If a vertex has label $\wedge$, then that vertex is a leaf and the only child of its' parent

More Generally, a derivation tree can be defined with any non-terminal as the root.

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Example.

S → AB
A → aAA
A → aA
A → a
B → bB
B → b
yield = aaab
yield = aaab

Note:

(1) Root can be any non-terminal
(2) Leaf nodes can be terminals or non-terminals
(3) A derivation tree with root S shows productions used to obtain a sentential form

Observation : Every derivation corresponds to one derivation tree.

$$\begin{aligned} S &\Rightarrow AB \\ &\Rightarrow aAAB \\ &\Rightarrow aaAB \\ &\Rightarrow aaaB \\ &\Rightarrow aaab \end{aligned}$$

Observation: Every derivation tree corresponds to one or more derivations.

$$\begin{aligned} S &\Rightarrow AB \\ &\Rightarrow aAAB \\ &\Rightarrow aaAB \\ &\Rightarrow aaaB \\ &\Rightarrow aaab \end{aligned}$$

Definition.

A derivation is *leftmost (rightmost)* if at each step in the derivation a production is applied to the leftmost (rightmost) non-terminal in the sentential form.

- The first derivation above is leftmost, second is rightmost, the third is neither.

Definition.

Let G be a CFG. Then G is said to be ambiguous if there exists an x in L(G) with >1 leftmost derivations. Equivalently, G is said to be ambiguous if there exists an x in L(G) with >1 parse trees, or >1 rightmost derivations.

Note : Given a CFL L, there may be more than one CFG G with L = L(G). Some ambiguous and some not.

Definition.

Let L be a CFL. If every CFG G with L = L(G) is ambiguous, then L is inherently ambiguous.

Information for the Pumping Lemma for Context-Free Languages

Definition.

Let G = (V, T, P, S) be a CFL.

If every production in P is of the form

$$A \to BC$$
$$\text{or}$$
$$A \to a$$

where A, B and C are all in V and a is in T,

then G is in Chomsky Normal Form (CNF).

Example: (not quite!)

S → AB | BA | aSb
A → a
B → b

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Theorem: Let L be a CFL. Then

$L - \{\wedge\}$ is a CFL.

Theorem: Let L be a CFL not containing $\{\wedge\}$. Then there exists a CNF grammar G such that L = L(G).

Definition: Let T be a tree. Then height of T, denoted h(T), is defined as follows:

- If T consists of a single vertex then h(T) = 0
- If T consists of a root r and subtrees T1, T2, ..Tk, then

h(T) = maxi{h(Ti)} + 1

Lemma: Let G be a CFG in CNF. In addition, let w be a string of terminals, where $A \overset{*}{\Rightarrow} w$ and w has a derivation tree T.

If T has height h(T)≥1, then

$|w| \leq 2h(T)\text{-}1$.

Pumping Lemma for Context-Free Languages

Lemma: Let G = (V, T, P, S) be a CFG in CFL, and let n > = |V|.

If z is a string in L(G) and $|z| \geq n$, then there exist strings u, v, w, x and y in T* such that

$$z = uvwxy$$

and

- $|vx| \geq 1$ (*i.e.,* $|v| + |x| \geq 1$)
- $|vwx| \leq n$
- uv^iwx^iy is in L(G), for all $i \geq 0$

PUSHDOWN AUTOMATA (PDA)

Consider a new notion of automata *Pushdown Automata*. PDAs are finite automata with a stack, *i.e.* a data structure which can be used to store an arbitrary number of symbols (hence PDAs have an infinite set of states) but which can be only accessed in a *Last-In-First-Out* (LIFO) fashion. The languages which can be recognized by PDA are precisely the context free languages.

A Pushdown Automaton P — (Q, Σ, Γ, δ. q_0, Z_0, F) is given by the following data :

(*i*) A finite set *Q* of states,

(*ii*) A finite set Σ of symbols (the alphabet).

(*iii*) A finite set τ of stack symbols.

(*iv*) A transition function δ is defined as

$$Q \times (\Sigma \cup \{\wedge\}) \times \Gamma \rightarrow (Q \times \Gamma^*)$$

- An initial state $q_0 \wedge Q$.
- An initial stack symbol $Z_0 \wedge \Gamma$.
- A set of final states $F \subseteq Q$.

Consider a PDA P which recognizes language of even length palindromes over Σ = {0, 1}

$$L = \{ww^R \mid w \wedge \{0, 1\}^*\}.$$

Intuitively, this PDA pushes the input symbols on the stack until it *guesses* it is in the middle and then it compares that input with what is on the stack, popping of symbols from the stack as it goes. If it reaches the end of the input precisely at the time when the stack is empty, it accepts.

$$P_0 = (\{q_0, q_1, q_2\}, \{0, 1\}, \{0, 1\, Z_0\}, \delta, q_0, Z_0, \{q_2\})$$

where δ is given by the following equations:

$$\delta(q_0, 0, Z_0) = \{(q_0, 0\,Z_0)\}$$
$$\delta(q_0, 1, Z_0) = \{(q_0, 1\,Z_0)\}$$
$$\delta(q_0, 0, 0) = \{(q_0, 0\,0)\}$$
$$\delta(q_0, 1, 0) = \{(q_0, 10)\}$$

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$$\delta(q_0, 0, 1) = \{(q_0, 01)\}$$
$$\delta(q_0, 1, 1) = \{(q_0, 11)\}$$
$$\delta(q_0, \wedge, Z_0) = \{(q_1, Z_0)\}$$
$$\delta(q_0, \wedge, 0) = \{(q_1, 0)\}$$
$$\delta(q_0, \wedge, 1) = \{(q_1, 1)\}$$
$$\delta(q_1, 0, 0) = \{(q_1, \wedge)\}$$
$$\delta(q_1, 1, 1) = \{(q_1, \wedge)\}$$
$$\delta(q_1, \wedge, Z_0) = \{(q_2, \wedge)\}$$

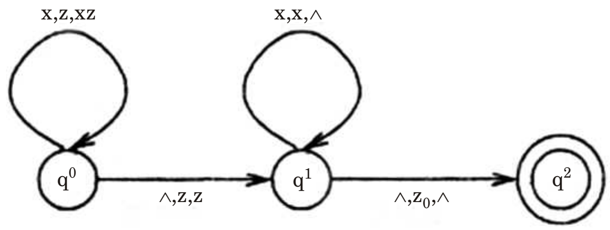
q_2 is final state

To save space, we may abbreviate this by writing :

$$\delta(q_0, x, z) = \{(q_0, xz)\}$$
$$\delta(q_0, \wedge, z) = \{(q_1, z)\}$$
$$\delta(q_1, x, x) = \{(q_1, \wedge)\}$$
$$\delta(q_1, \wedge, Z_0) = \{(q_2, \wedge)\}$$

We obtain previous table by expanding all the possibilities for *q, x, z.*

We draw transition diagram of P by labelling each transition with a triple $x, Z, \gamma \wedge$ with $x \wedge \Sigma, Z \wedge \Gamma, \gamma \wedge \Gamma^*$:



Working of PDA

At any time of the computation of a PDA P = (Q, Σ, Γ, δ, q_0, Z_0, F) is given *by :*

- the state $q \wedge Q$, the PDA is in,
- the input string $w \wedge \Sigma^*$ which still has to be processed,
- the contents of the stack $\gamma \wedge \Gamma^*$.

Such a triple $(q, w, \gamma) \wedge Q \times \Sigma^* \times \Gamma^*$ is called *Instantanous Description (ID).*

We define a relation $\vdash p \subseteq ID \times ID$ between IDs which describes how the PDA can change from one ID to the next one. Since PDAs in general arc nondeterministic this is a relation (not a function], *i.e.* there may be more than one possibility.

There are two possibilities for $\vdash p$ *:*

(1) if δ(q, w, z) = (q′, wz)

Then (q, w, zγ) ⊢ (q′, wz)

In this case, PDA reads an input symbol and consults the transition function δ to calculate a possible new state *q′* and a sequence of stack symbols α which replace current symbol on the top *z*.

(2) if δ(q, w, z) = (q′, z)

Then (q, w, zγ) ⊢ (q′, zγ)

In this case, PDA ignores the input and silently moves into a new state and modifies the slack as above. The input is unchanged.

Consider the word 0110 — what are possible sequences of IDs fur P_0 starting with (q_0, 0110. Z_0) ?

Example.

$(q_0, 0110, Z_0)$	$\vdash p_0, (q_0, 110, 0Z_0)$	1. with $(q_0, 0Z_0) = \wedge\ \delta(q_0, 0, Z_0)$
	$\vdash p_0, (q_0, 110, 0Z_0)$	1. with $(q_0, 10) = \wedge\ \delta(q_0, 1, 0)$
	$\vdash p_0, (q_0, 10.\ 10Z_0)$	2. with $(q_1, 1) = \wedge\ \delta(q_0, \wedge, 1)$
	$\vdash p_0, (q_1, 0, 0Z_0)$	1. with $(q_1, \wedge) = \wedge\ \delta(q_1, 1.\ 1)$
	$\vdash p_0, (q_1, \wedge, Z_0)$	1. with $(q_1, \wedge) = \wedge\ \delta(q_1, 0, 0)$
	$\vdash p_0, (q_2, \wedge, \wedge)$	2. with $(q_2, \wedge) = \wedge\ \delta(q_1, \wedge, Z_0)$

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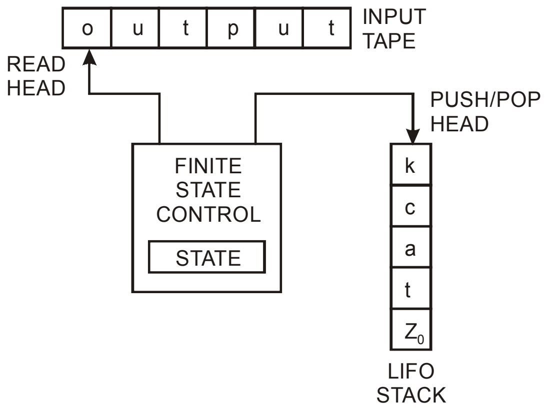
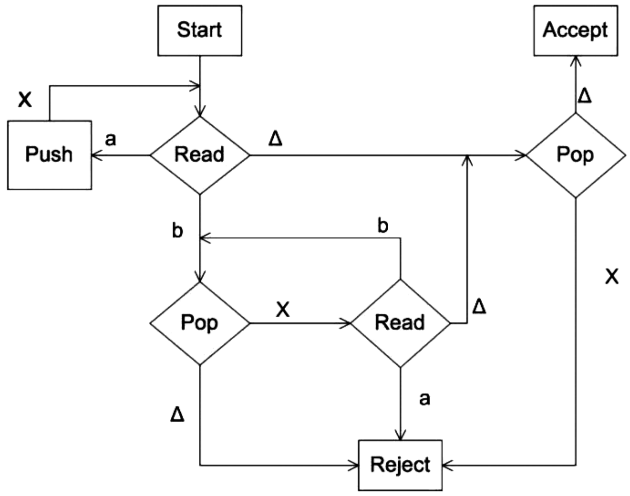
Model

A typical mental model for a PDA is shown below :





We write $(q, w, \gamma) \vdash^{*} p\ (q', w', \gamma)$ if PDA can move from (q, w, γ) to (q', w', γ') in a (possibly empty) sequence of moves.

Above we have shown that

$$(q_0, 0110, Z_0) \vdash^{*} p_0, (q2, \wedge, \wedge).$$

However, this is not the only possible sequence of IDs for this input.

e.g. PDA may just guess the middle wrong :

$(q_0, 0110, Z_0) \quad \vdash p_0, (q_0, 110, 0Z_0)$ 1. with $(q_0, 0Z_0) = \delta(q_0, 0, Z_0)$

$\qquad\qquad\qquad\quad \vdash p_0, (q_1, 110, 0Z_0)$ 2. with $(q_{\wedge}, 0) = \delta(q_0, \wedge, 0)$

We have shown $(q_0, 0110, Z_0) \vdash^{*} p_0, (q_1, 110, 0Z_0)$.

Here PDA gets stuck there is no state after $(q_1, 0110, 0Z_0)$.

If we start with a word which is not in the language L (like 0011), then automaton will always get stuck before reaching a final state.

Language of a PDA

There arc two ways to define the language of a PDA

$$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)\ (L(P) \subseteq \Sigma^*)$$

because there are two notions of acceptance :

Acceptance by Final state

$$L(P) = \{w \mid (q_0, w, Z_0) \vdash^{*} (q, \wedge, \gamma) \wedge q \wedge F\}$$

i.e., PDA accepts the word w if there is any sequence of IDs starting from (q_0, w, Z_0) and leading to $(q, \wedge, \gamma)$, where $q \wedge F$ is one of the final states. Here it doesn't play a role what the contents of the stack are at the end.

Acceptance by Empty stack

$$L(P) = \{w \mid (q_0, w, Z_0) \vdash^{*} (q, \wedge, Z_0)\}$$

i.e., PDA accepts the word w if there is any sequence of IDs starting from (q_0, w, Z_0) and leading to $(q, \wedge, Z_0)$. In this case the final state plays no role.

If we specify a PDA for acceptance by empty stack, we leave out the set of final states F and just use

$$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0).$$

We can always turn a PDA which use one acceptance method into one which uses the other. Hence, both acceptance criteria specify the same class of languages.

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Deterministic PDAs

Define Deterministic Pushdown Automata (DPDA) those which never have a choice.

To be precise we say that a PDA $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ is deterministic (is a DPDA) iff

$$|\delta(q, x, z)| + |\delta(q, \wedge, z)| \leq 1$$

Here, $|X|$ stands for the number of elements in a finite set X.

i.e., a DPDA may gel stuck but it lias never any choice.

Unlike the situation for finite automata, there is in general no way to translate a nondeterministic PDA into a deterministic. Indeed, there is no DPDA which recognizes the language L ! Nondeterministic PDAs are more powerful than deterministic PDAs.

However, we can define a similar language L′ over $\Sigma = \{0, 1, \$)$ which can be recognized by a deterministic PDA :

$$L' = \{w\$w^R \mid w \wedge \{0, 1\}^*\}$$

i.e., L′ contains palindroms with a marker \$ in the middle,

e.g., 01\$10 $\wedge$ L′.

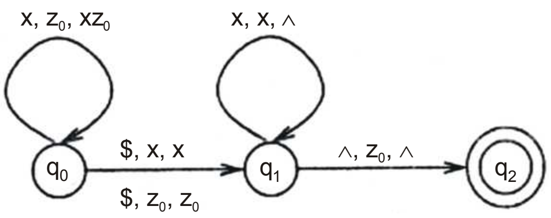
We define a DPDA P_1 for *L′:*

$$P_1 = (\{q_0, q_1, q_2\}, (0, 1, \$\}, \{0, 1, z_0\}, \delta', q_0, z_0, \{q_2\})$$

where δ is given by:

$$\delta(q_0, x, z_0) = \{(q_0, xz_0)\ x \wedge \{0, 1\}\}$$
$$\delta(q_0, \$, z_0) = \{(q_1, z_0)\}$$
$$\delta(q_1, x, x) = \{(q_1, \wedge)$$
$$\delta(q_1, \wedge, z_0) = \{(q_2, \wedge)\}$$

Its transition graph is shown below :



It can be checked that this automaton is deterministic In particular, 3rd and 4th line cannot overlap because z_0 is not an input symbol.

Different to PDAs in general the two acceplance methods are not equivalent for DPDAs — acceptance by final state makes it possible to define a bigger class of langauges. Hence, we shall always use acceptance by final stale for DPDAs.

Context free grammars and push-down-Automata

Theorem : For *a language* $L \subseteq \Sigma^*$ *the following is equivalent:*

1. L is given by a CFG G, L = L(G).
2. L is the language of a PDA P, L = L(P).

To summarize : Context Free Languages (CFLs) can be described by a Context Free Grammar (CFG) and can be processed by a pushdown automaton.

We will only show how to construct a PDA from a grammar.

Given a CFG $G = (V, \Sigma, S, P)$, we define a PDA

$$P(G) = (\{q_0\}, \Sigma, V \cup \Sigma, \delta, q_0, S)$$

where δ is defined as follows:

$$\delta(q_0, \wedge, A) = \{(q_0, \alpha) \mid A \rightarrow \alpha \text{ is a production in P}\}$$

for all $A \wedge V$.

$$\delta(q_0, a, a) = \{(q_0, \wedge)\}$$

for all $a \wedge \Sigma$.

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We haven't given a set of final states because we use acceptance by empty stack.

Yes, we use only one state!

Take as an example $G = (\{E, T, F\}, \{(,), a, +, *\}, E, P)$ where

$$
\begin{aligned}
P = \; & E \rightarrow T \mid E + T \\
& T \rightarrow F \mid T*F \\
& F \rightarrow a \mid (E)
\end{aligned}
$$

we define with

$$
\begin{aligned}
P(G) &= (\{q_0\}, \{(,), a, +,*\}, \{E, T, F, (,). a, +, *\}, \delta, q_0, E) \\
\delta(q_0, \wedge, E) &= \{(q_0, T), (q_0, E + T)\} \\
\delta(q_0, \wedge, T) &= \{(q_0, F), (q_0, T * F)\} \\
\delta(q_0, \wedge, F) &= \{(q_0, a), (q_0, (E))\} \\
\delta(q_0, (, () &= \{(q_0, \wedge)\} \\
\delta(q_0,),)) &= \{(q_0, \wedge)\} \\
\delta(q_0, a, a) &= \{(q_0, \wedge)\} \\
\delta(q_0, +, +) &= \{(q_0, \wedge)\} \\
\delta(q_0, *, *) &= \{(q_0, \wedge)\}
\end{aligned}
$$

How does P(G) accept $a + (a*a)$?

$$
\begin{aligned}
(q_0, a + (a*a), E) &\vdash (q_0, a + (a*a), E + T) \\
&\vdash (q_0, a + (a*a), T + T) \\
&\vdash (q_0, a + (a*a), F + T) \\
&\vdash (q_0, a + (a*a), a + T) \\
&\vdash (q_0, + (a*a), + T) \\
&\vdash (q_0, (a*a), T) \\
&\vdash (q_0, (a*a), F) \\
&\vdash (q_0, (a*a), (E)) \\
&\vdash (q_0, a*a), E)) \\
&\vdash (q_0, a*a), T)) \\
&\vdash (q_0, a*a), T * F)) \\
&\vdash (q_0, a*a), F * F) \\
&\vdash (q_0, a*a), a * F)) \\
&\vdash (q_0, *a), *F)) \\
&\vdash (q_0, a), F)) \\
&\vdash (q_0, a), a)) \\
&\vdash (q_0,),)) \\
&\vdash (q_0, \wedge, \wedge)
\end{aligned}
$$

Hence $a + (a*a) \wedge L(P(G))$.

This example hopefully allready illustrates the general idea:

$$
\begin{aligned}
& w \wedge L(G) \\
& S \Leftrightarrow \ldots \Leftrightarrow w \\
& (q_0, w, S) \vdash \ldots \vdash (q_0, \wedge, \wedge) \\
& \Leftrightarrow w \wedge L(P(G))
\end{aligned}
$$

The automaton we have constructed is very non-deterministic : Whenever we have a choice between different rules the automaton may silently choose one of the alternative.

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RECURSIVE AND RECURSIVELY ENUMERABLE LANGUAGES

There are *three* possible outcomes of executing a Turing machine over a given input :

Turing machine may

(*i*) halt and accept the input;

(*ii*) halt and reject the input; or

(*iii*) never halt.

A language is *recursive* if there exists a Turing machine that accepts every string of the language and rejects every string (over the same alphabet) that is not in the language.

Note : If a language L is recursive, then its complement L^C must also be recursive.

A language is *recursively enumerable* if there exists a Turing machine that accepts every string of the language, and does not accept strings that are not in the language. (Strings that are not in the language may be rejected or may cause the Turing machine to go into an infinite loop.)

Clearly, every recursive language is also recursively enumerable. It is not obvious whether every recursively enumerable language is also recursive.

Non-Recursively Enumerable Languages

A language is a subset of Σ^* A language is *any* subset of Σ^*.

Turing machines are enumerable. Since recursively enumerable languages are those whose strings are accepted by a Turing machine, the set of recursively enumerable languages is also enumerable.

The powerset of an infinite set is *not* enumerable. Each of these subsets represents a language. Therefore, there must be languages that are not computable by a Turing machine.

According to Turing's thesis, a Turing machine can compute any effective procedure. Therefore, there are languages that cannot be defined by any effective procedure.

We can find a non-recursively enumerable language X by diagonalization. Using enumerations, let string i belong to language X if and only if it does *not* belong to language i.

When Recursively Enumerable Implies Recursive

Let a language L is recursively enumerable, *i.e.,* there exists a Turing machine T_1 that gives any string of the language, halts and accepts that string.

Also, let complement of L, $L^C = \Sigma^* - L$, is recursively enumerable, *i.e.,* there is some other Turing machine T_2 that gives any string of L^C, halts and accepts that string.

Clearly, any string (over the appropriate alphabet Σ.) belongs to either L or L^C. Hence, any string will cause either T_1 or T_2 (or both) to halt.

We construct a new Turing machine that emulates both T_1 and T_2, alternating moves between them. When either one stops, we can tell (by whether it accepted or rejected the string) to which language the string belongs. Thus, we have constructed a Turing machine that, for each input, halts with an answer *-whether or not* the string belongs to L. Therefore L and L^C are recursive languages.

Recursively Enumerable But Not Recursive

The recursive languages are a subset of the recursively enumerable languages, Now, we will show that they are *aproper* subset.

We have shown how to enumerate strings for a given alphabet, W_1, W_2, W_3,.... and how to enumerate Turing machines, T_1, T_2, T_3,....

Consider the language $L = \{w_i : w_i \in L(T_i)\}$

$$L^C = \{w_i : w_i \notin L(T_i)\}$$

Here, L is itself recursively enumerable, but now consider its complement:

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If L^C is recursively enumerable, then there must exist a Turing machine that recognizes it. This Turing machine must be in the enumeration somewhere - call it T_k.

- If W_k belongs to L, then (by the way we have defined L) T_k accepts this string. But T_k accepts only strings that do not belong to L, so we have a contradiction.
- If W_k does not belong to L, then it belongs to L^C and is accepted by T_k. But since T_k accepts W_k, W_k must belong to L. Again, a contradiction.

We have now defined a recursively enumerable language L and shown by contradiction that L^C is not recursively enumerable.

We know, if a language is recursive, its complement must also be recursive. If language L above were recursive, then L^C would also be recursive, hence recursively enumerable. But L^C is not recursively enumerable; therefore L must not be recursive.

We have therefore shown that L is recursively enumerable but not recursive, therefore set of recursive languages is a proper subset of the set of recursively enumerable languages.

TURING MACHINES AND UNDECIDABILITY

An algorithm is a computational procedure for systematic execution.

Languages describing Algorithms

(*i*) Programming languages (e.g. Algal)

(*ii*) Flow charts

(*iii*) Decision tables

(*iv*) Markov algorithms

(*v*) Post canonical systems

(*vi*) Recursive functions

(*vii*) Lambda calculus

(*viii*) Turing machines

Definition of a Turing machine uses an infinite two-way tape, quintuples, and three halt states.

Other definitions use a one-way infinite tape and/or quadruples, and one halt state. However, all the definitions are equivalent.

Basic definitions.

A Turing machine M involves three disjoint nonempty sets:

(1) A finite state set, $S = \{a_1, a_2, ...a_m\} \cup \{B\}$
Here $B = a_0$ is the *blank* symbol.

(2) A finite state set, $S = \{s_1, s_2, ...s_m\} \cup \{s_0\} \cup \{s_H, s_Y, s_N\}$
Here, S_0 is initial state
S_H(HALT) is halting state
S_Y(YES) is accepting state
S_N(NO) is nonaccepting state.

(3) A direction set, $d = \{L, R, N\}$
Here, L denotes *left*
R denotes right
N denotes *no movement* or *stay*

Definition 1.

An expression is a finite (possibly empty) sequence of elements from $A \cup S \cup d$.

In other words, an expression is a word whose letters (symbols) come from the sets A, S and d.

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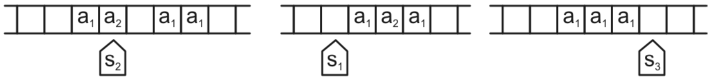
Definition 2.

A tape expression is an expression using only elements from the tape set A.

Turing machine M may be viewed as a read/write tape head which moves back and forth along an infinite tape. The tape is divided lengthwise into squares (cells), and each square may be blank or hold one tape symbol. At each step in time, the Turing machine M is in a certain internal state s_i scanning one of the tape symbols a_j on the tape. Assume that only a finite number of non-blank symbols appear on the tape.



$\alpha = a_1\ s_2\ a_2\ Ba_1\ a_1$ $\beta = s_1\ Ba_1\ a_2\ a_1$ $\gamma = a_1\ a_1\ a_1\ s_3\ B$

(*a*) (*b*) (*c*)

Fig. (a) shown in a picture of a Turing machine M in state s_2 scanning the second symbol where $a_1 a_2\ Ba_1 a_1$ is printed on tape (B is blank symbol). This picture may be represented by the expression $a = a_1 s_2 a_2\ Ba_1 a_1$ where we write the state s_2 of M before the tape symbol a_3 that M is scanning. α is an expression using only the tape alphabet A except for the state symbol s_2 which is not at the end of the expression since it appears before the tape symbol a_3 that M is scanning.

Definition 3.

A picture α is an expression of the form $\alpha = Ps_i a_k\ Q$

where P and Q are tape expressions (possibly empty).

Definition 4.

Let $\alpha = Ps_i a_k\ Q$ be a picture.Turing machine M is in state s_i scanning the letter a_k and that the expression on the tape is the expression $Pa_k\ Q$, i.e. α without its state symbol s_i.

At each step in time, the turing machine M is in a certain state s_i and is scanning a tape symbol a_k.

Turing machine M can do following three things simultaneously:

(*i*) M erases the scanned symbol a_k and writes in its place a tape symbol a_i (where we permit $a_i = a_k$).

(*ii*) M changes its internal states s_i to a state s_j (where we permit $s_j = s_i$).

(*iii*) M moves one square to the left, moves one square to the right, or does not move at all.

The above action by M may be described by a five-letter expression called a *quintuple*.

Definition 5.

A quintuple q is a five-letter expression of the following form:

$$q = \left\{ s_i,\ a_k,\ a,\ s_j,\ \begin{pmatrix} L \\ R \\ N \end{pmatrix} \right\}$$

i.e. first letter of q is a state symbol, second letter is a tape symbol, third letter is a tape symbol, fourth letter is a state symbol, and last letter is a direction symbol L, R or N.

Definition 6 (Formal definition of a Turing machine).

A turing machine M is a finite set of quintuples such that

(*i*) no two quintuples begin with the same first two letters

(*ii*) no quintuple begins with S_H, S_Y or S_N.

Condition (*i*) in the definition guarantees that the machine M cannot do more than one thing at any given step, and condition (*ii*) guarantees that M halts in state S_H, S_Y or S_N.

Definition 7 (Alternative equivalent definition).

A Turing machine M is a partial function from

$$S/\{S_H, S_Y, S_N\} \times A \text{ into } A \times S \times d$$

The term partial function simply means that domain of M is a subset of $S/\{S_H, S_Y, S_N\} \times A$

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Definition 8 (Action of theTuring machine).

Let α and β be pictures.

We write $\alpha \to \beta$ if one of the following holds, where a, b, c are tape letters and P and Q are tape expressions (possibly empty):

(*i*) $\alpha = Ps_i aQ$, $\beta = Ps_j bQ$ and M contains the quintuple $q = s_i\, abs_j N$.

(*ii*) $\alpha = Ps_i acQ$, $\beta = Pbs_j cQ$ and M contains the quintuple $q = s_i abs_j R$.

(*iii*) $\alpha = Pcs_i a\, Q$, $\beta = Ps_j cbQ$ and M contains the quintuple $q = s_i abs_j L$.

(*iv*) $\alpha = Ps_i a$, $\beta = Pbs_i B$ and M contains the quintuple $q = s_i abs_j\, R$.

(*v*) $\alpha = s_i aQ$, $\beta = s_j BbQ$ and M contains the quintuple $q = s_i abs_j L$.

Here in all five cases, m replaces a on the tape by b (where we permit $b = a$), and M changes its state from s_i to s_j (where we permit $s_j = s_i$).

Again here

(*i*) M does not move.

(*ii*) M moves to the right.

(*iii*) M moves to the left.

(*iv*) M moves to the right; however, since M is scanning the rightmost letter, it must add the blank symbol B on the left.

(*v*) M moves to the left; however, since mis-scanning the leftmost letter, it must add the blank symbol B on the left.

Definition 9.

A picture α is called *terminal*, if there is no picture β such that $\alpha \to \beta$

In particular, any picture α in one of the three halt states must be terminal since no quintuple begins with S_H, S_Y or S_N.

COMPUTING WITH A TURING MACHINE

Definition 10.

A computation of a Turing machine M is a sequence of pictures $a_0, a_1, \ldots, a_m$ such that

$$a_{i-1} \to \alpha_i \text{ for } i = 1, \ldots, m, \text{ and } \alpha_m \text{ is a terminal picture.}$$

In other words, a computation is a sequence

$$a_0 \to \alpha_1 \to \alpha_2 \to \ldots \to \alpha_m$$

which cannot be extended since α_m is terminal.

Let term (α) denote the final picture of a computation beginning with α.

Thus in the above computation, term $(\alpha_0) = \alpha_m$.

TURING MACHINES WITH INPUT

Definition 11.

An input for a Turing machine M is a tape expression W. The initial picture for an input W is $\alpha(W)$ where

$$\alpha(W) = s_0\, W.$$

The initial picture $\alpha(W)$ of the input W is obtained by placing the initial state s_0 in front of the input tape expression W. In other words, Turing machine M begins in its initial state s_0 and it is scanning the first letter of W.

Definition 12.

Let M be a Turing machine M and let W be an input. We say M halts on W if there is a computation beginning with the initial picture $\alpha(W)$.

i.e. given an input W, we can form the initial picture $\alpha(W) = s_0$ W and apply M to obtain the sequence

$$\alpha(W) \to \alpha_1 \to \alpha_2 \to \ldots$$

Two things can happen :

(1) M halts on W, *i.e.* sequence ends with some terminal picture α_r.

(2) M does not halt on W, *i.e.* sequence never ends.

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GRAMMARS AND TURING MACHINES

Turing machines may be used to recognize languages specifically.

Let m be a Turing machine with tape set A.

Let L be the set of words W in A such that M halts in the accepting state s_Y when W is the input.

Then write L = L(M), and say that M recognizes the language L. Thus an input W does not belong to L(M) if M does not halt on W or if M halts on W but not in the accepting state S_Y.

Following theorem is the main result of this subsection :

Theorem 1.

A language L is recognizable by a Turing machine M if and only if L is a type 0 language.

Remark. The reason for three halt states is that s_Y and s_N are used for recognizing languages, whereas s_H is used for computations

e.g. Let a Turing machine M with tape set A = $\{a, b, c\}$ contains the following four quintuples :

$$q_1 = s_0\, aas_0\mathrm{R},$$
$$q_2 = s_0 bbs_0\mathrm{R},$$
$$q_3 = s_0\mathrm{BB}s_N\mathrm{R},$$
$$q_4 = s_0 ccs_Y\mathrm{N}.$$

- Let W = W(a, b, c) is an input without any c's.

  By the quintuples q_1 and q_2, M stays in state and moves to the right until it encounters a blank symbol B. Then M changes its state to the nonaccepting state s_N and halts.

- Let W = W(a, b, c) is an input with at least one c symbol.

  By the quintuple q_4, when M initially meets the first on W it changes its state to the accepting state by s_Y and halts.

  Thus M recognizes the language L of all words W in a, b, c with atleast one letter c.

  i.e. L = L(M)

Turing machine also has a tape head, which allows it to read from and write to individual squares of the tape.

If $q \in$ Q, $r \in$ Q $\cup \{h\}$ X, Y$\in \Gamma \cup \wedge$, and D$\in$ {R, L, S}, then

$$\delta(q, \mathrm{X}) = (r, \mathrm{Y}, \mathrm{D})$$

i.e. when TM is in state q and symbol on the current tape square is X, then machine replaces X by Y on that square, change to state r, and either moves the tape head one square left, moves it one square right, or leaves it stationary, depending on whether D is L, R or S, respectively. If in this situation D = L, but the tape head is scanning square 0, the left most square, the tape head is not allowed to move. This is one of the situations in which the TM is said to crash. If this does not happen, and r = h, the move causes the TM to halt. Once it has halted, the machine can not move farther because δ is not defined at any pair (h, X).

Definition 13.

If T = (Q, Σ, Γ, q_0, δ) is a TM, and $x \in \Sigma^*$, x is accepted by T, if starting in the initial configuration corresponding to input x, T eventually reaches a halting configuration.

In other words, x is accepted if there exist $y, z \in (\Gamma \cup \{\wedge\})^*$ and $\in \Gamma \cup \{\wedge\}$, so that

$$(q_0, \wedge x) \vdash^* (h, y\ \underline{a}\, z)$$

In this situation we say T halts on input x. The language accepted by T is the set L(T) of input strings on which T halts.

Ways in which input string might fail to be accepted by a TM.

(*i*) It can lead to some non-halting configuration from which the TM cannot move (*i.e.* to some combination of non-halting state and current tape symbol for which the transition function is undefined).

(*ii*) At some point in the processing of the string, the tape head is scanning square 0 and the next move specifies moving the head left, off the end of the tape.

In either of these cases, we say that the TM crashes, which is not the same as saying that it halts.

(*iii*) An input string might cause the TM to enter an infinite loop, a never-ending sequence of moves.

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Above three cases are described informally as follows :

A string can fail to be accepted by being explicitly rejected, as a result of the machine's crashing or it can fail to be accepted because the machine is unable to make up its mind whether to accept. The difference between these outcomes is that in the second case there is no outcome. Someone waiting for the answer is left in suspense. The machine continues to make moves, but the observer is never sure that it is not about to halt or crash. Because of the possibility of infinite loops, potential indecisiveness of a TM has significantly implication in the theory of computation.

COMBINING TURING MACHINES

A TM is often described most simply by specifying several smaller TMs that combine to form it. In the simplest case, we can construct a composite machine by executing first one TM and then another. If T_1 and T_2 are TMs, with disjoint sets of non-halting states and transition functions δ_1 and δ_2, respectively, we write T_1T_2 to deonte this composite TM. The set of states is the union of the two sets. The TM T_1T_2 begins in the initial state of T_1 and executes the moves of (T_1 using the function δ_1) up to the point at which T_1 halts or crashes; for any move that causes T_1 to halt, T_1T_2 executes the same move, except that it moves instead to the initial state T_2. At this point, the tape head is positioned at the square on which T_1 halted. From this point on, the moves of T_1T_2 are the moves of T_2 (using the function δ_2). If either T_1 or T_2 crashes during this process, then T_1T_2 crashes; T_1T_2 halts precisely if and when T_2 halts.

To use this composite machine in a larger context, in a manner similar to a transition diagram but without the states explicitly, we might also write

$$T_1 \rightarrow T_2$$

The composition can be also made conditional, depending on the current tape symbol when T_1 halts. Thus, for an 'a' that is either blank or a symbol in the tape alphabet of T_1, we can write

$$T_1 \xrightarrow{a} T_2$$

to stand for the composite machine $T_1\, T'T_2$.

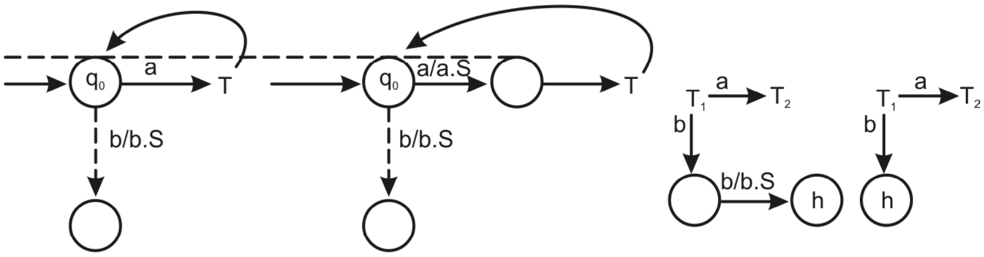
This composite machine can be described informally as follows:

it executes T_1; if and when T_1 halts,

it executes T_2 if the current tape symbol is 'a' and crashes otherwise.

In order to be able to describe composite TMs without having to describe every primitive operation as a separate TM, it is sometimes helpful to use mixed notation in which some but not all of the states of a TM are shown.

If the current tape symbol is 'a' execute the TM T; if it is 'b', halt; and if it is anything else, crash. In the first case, assuming T halts, repeat the execution of T until T halts, scanning some symbol other than a; if at that point the tape symbol is b, halt, otherwise crash. (machine might also crash during one of the iterations of T, and it might loop forever, either because one of the iterations of T does or because T halts with current tape symbol 'a' every time).



(*a*) (*b*) (*c*) (*d*)

Although giving a completely precise definition of an arbitary combination of TMs would be complicated, what is involved is usually clear in specific examples. There is one possible source of confusion in the notation we are adopting.

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Consider a TM of the form $T = T_1 \xrightarrow{a} T_2$.

If T_1 halts scanning some symbol not specified explicitly (i.e. other than a), T crashes.

If T_2 halts, T halts even though no tape symbols are specified explicitly. This could be avoid apparent inconsistency by saying that if T_1 halts scanning a symbol other than a, T halts; however, then T would not be equivalent to the composition $T_1 T'T_2$ described above, and this seems undesirable. In our notation, if at the end of one sub-TM's operation atleast one way is specified for the composite TM to continue, then any option that allows halting at that point must be shown explicitly, as in Fig. (*c*) and Fig. (*d*).

Some TMs that would only crash when viewed as self-contained machines can be used successfully in combination with others. If a TM halts when run independently, then it will not crash when it is used as a component of a larger machine, provided that the tape has been prepared properly before its use.

A TMT expecting to find an input string x needs to begin in a configuration of the form $(q, y\,\underline{\Delta}x)$. The subsequent action of T does not depend on y, because if T does not crash when started in the configuration $(q, \underline{\Delta}x)$, then in particular, T will never attempt to move its tape head to the left of the blank. Therefore, starting in the configuration $(q, y\,\underline{\Delta}x)$, T will never actually see any of symbols of y.

The use of a TM as a component of a larger machine, is to change in some specific way the tape contents, head position, or both, to create a beginning configuration appropriate for the component that follows.

There are a number of standard tape operation that will obviously be useful in many settings and therefore constitute basic TM building blocks. *Some of these are :*

moving the head a specified number of positions in one direction or the other,

writing a specific symbol in the current square,

searching in one direction or the other for a specified symbol.

COMPUTING A PARTIAL FUNCTION WITH A TURING MACHINE

Any computer program whose purpose is to produce a specified output string for every legal input string can be thought of as computing a function from one set of strings to another. Similarly, a TM with input alphabet Σ can compute a function whose domain is a subset of Σ^*.

Now consider partial functions on Σ^*, rather than functions on subsets of Σ^*. This is largely a matter of convenience, and there is no real difference except in some of the terminology. A partial function f on Σ^* may be undefined at certain points, if it happens that f is defined everywhere on Σ^*, we often emphasize the fact by referring to f as a total function. In order for a TM to compute f, it is appropriate for the values of f to be strings over the tape alphabet of the machine.

A TM can also handle a function of several variables. If the input is to represent the k-tuple

$$(x_1, x_2, \ldots\ldots x_k) \in (\Sigma^*)^k \text{ (k-fold Cartesian product of Σ^*)},$$

the only change required is to relax slightly the rule for the input to a TM and to allow the initial tape to contain all k strings, separated by blanks.

Definition 14.

Let $T = (Q, \Sigma, \Gamma, q_0, \delta)$ be a Turing machine, and let f be a partial function on Σ^* with values in Γ^* with values in Γ^*. We say that T computes f if for every $x \in \Sigma^*$ on which f is defined

$$(q_0. \wedge x) \vdash^*_T (h, \wedge f(x))$$

and for every other $x \in \Sigma^*$. T fails to halt on input x.

Definition 15.

Let $T = (Q, \{1\}, \Gamma, q_0, \delta)$ be a Turing machine. If f is a partial function from N, the set of natural numbers, to itself, T computes f if for every n at which f is defined

$$(q_0. \underline{\Delta}\, 1^n) \vdash^*_T (h, \underline{\Delta}\, I^{f(n)})$$

and for every other natural number n, T fails to halt on input 1^n.

Similarly, if f is a partial function from N^k to N, T computes f if for every k-tuple $(n_1, n_2 \ldots\ldots\ldots n_k)$ at which f is defined

$$(q_0. \underline{\Delta} 1_{n1} \underline{\Delta} 1^{n}{}_{2} \Delta \ldots\ldots \underline{\Delta} 1^{n}{}_{k}) \vdash^*_T (h, \underline{\Delta}\, 1^{f(n_1, n_2 \ldots\ldots\ldots n_k)})$$

and T fails to halt if the input is any k-tuple at which f is not defined.

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VARIATIONS OF TURING MACHINES MULTITAPE TMs

There are analogies between the way a TM works and the actions of a human computer.

Some variations on the TM model, and investigate the multitape TM.

(*i*) In many cases, they might seem to have the potential for increasing the computing power of the basic model; however, they can be shown not to do so. Thus they constitute a sort of evidence for the generality of the TM, evidence to support the Church-Turing thesis. By considering more sophisticated models and by seeing in each case that an ordinary TM can do everything these models can, one can better appreciate the flexibility and power of the TM.

(*ii*) Some of the variants are often more convenient to work with than the basic model. An algorithm that would require considerable book keeping machinery if carried out on an ordinary TM might be very easy to describe on a three-tape machine.

When two classes of abstract machines are compared with regard to *computing power*, then be precise about the criteria used. Don't consider speed, efficiency, or convenience; but concern only with whether the two types of machines can solve the same problems and get the same answers. A machine in the extended family of 4 TMs gives an "answer," first by halting or failing to halt, and second by producing a particular output when it halts. This means if we want to show that a machine of type B is atleast as powerful as one of type A, we need to show that for any machine T_A of type A, there is a machine T_B of type B that halts on exactly the same input strings as T_A and, when it halts on a particular input, produces exactly the same output as T_A.

Some minor variations on the basic model, each slightly more restrictive.

(*i*) First possibility is to require that in each move, the tape head move either to the right or to the left.

In this version, values of the transition function δ are elements of $Q \times (\Gamma \cup \{\wedge\}) \times \{L, R\}$ instead of $Q \times (\Gamma \cup \{\wedge\}) \times \{L, R, S\}$.

(*ii*) Second possibility is that a move can include writing a tape symbol or moving the tape head, but not both.

In this case, δ would take values in $Q \times (\Gamma \cup \{\wedge\}) \cup \{L, R\}$, where L and R are assumed not to be elements of Γ.

In both cases, restrictions do not reduce the power of machine.

One identifiable difference between a TM and a typical human computer is that a TM has a one-dimensional tape with a left end, rather than sheets of paper that might be laid out in both directions. Thus one way to try to increase the power of machine, might be to remove one or both of these restrictions: to make the "Tape" two-dimensional, or to remove the left end make the tape potentially infinite in both directions. In order to investigate these generalizations precisely, we must first specify the rules under which the machine would operate and the conventions that would be followed with regard to input and output. Thus it is concluded that *power of the machine is not significantly changed by either of these additions.*

Rather than modifying the tape, we might instead add extra tapes. We could decide in that case whether to have a single tape head, which would at any time scan the same position on all the tapes, or a head on each tape that could move independently of the others. We choose the second option for closer study.

An n-type TM is specifiable by a 5-tuple

$$T_1 = (Q, \Sigma, \Gamma, q_0, \delta).$$

The TM makes a move on the basis of its current state and the n-tuple of tape symbols currently being examined; because the tape heads move independently, we describe the transition function as a partial function.

$$\delta : Q \times (\Gamma \cup \{\Delta\})^n \to (Q \cup \{h\}) \times (\Gamma \cup \{\Delta\})^n \times \{R, L, S\}^n$$

Here, superscript denotes the n-fold cartesion product.

Notion of configuration generalizes in a straightforward way:

A configuration of an n-type TM is specified by an $(n + 1)$-tuple of the form

$$(q, x_1\, \underline{a_1}\, y_1, x_2\, \underline{a_2}\, y_2, \ldots\ldots x_n\, \underline{a_n}\, y_n)$$

with the same restrictions as before on the strings x_i and y_i.

We take initial configuration corresponding to input string x to be

$$(q_0, \underline{\Delta}x, \underline{\Delta}, \ldots\ldots, \underline{\Delta})$$

i.e. first tape is the one used for the input.

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We will also say that output of an n-tape TM is the final contents of tape 1. We could define output by taking into account the final contents of all the tapes; however, tapes 2 through n as auxiliary working space and to disregard their contents when the TM halts. In particular, such a TM computes a function f if, whenever it begins with an input string x in or representing an element in the domain of f, it halts in some configuration $(h, \underline{\Delta}f(x), \ldots\ldots.)$ where contents of tapes 2 through n are arbitrary, and otherwise it fails to halt.

It is obvious that for any $n \geq 2$, n-tape TMs are at least as powerful as ordinary 1-tape TMs. To simulate an ordinary TM exactly, a TM with n tapes simply acts as if tape 1 were its only one and leaves the others blank.

Theorem.

Let $n > 2$, and $T_1 = \{Q_1, \Sigma, \Gamma_1, q_1, \delta_1\}$ be an n-tape Turing machine. Then there is a one-tape TM $T_2 = \{Q_2, \Sigma, \Gamma_2, q_2, \delta_2\}$, with $\Gamma_1 \subseteq \Gamma_2$, satisfying the following two conditions :

1. $L(T_2) = L(T_1)$; i.e. for any $x \in \Sigma^*$, T_2 halts on input x if and only if T_1 halts on input x.

2. For any $x \in \Sigma^*$, if $(q_1, \underline{\Delta}x, \underline{\Delta}\ldots\ldots,\underline{\Delta}) \vdash^*_{T_1} (h, y\underline{a}\ z\, y_2\underline{a_2}\, z_2, \ldots\ldots y_n\ \underline{a_n}\, z_n)$ for some $a, a_i \in \Gamma_1 \cup \{\Delta\}$

and $y, z, y_i, z_i, \in (\Gamma_1 \cup \{\Delta\})^*)$,

then $(q_2, \underline{\Delta}x,) \vdash^*_{T_2} (h, y\,\underline{a}\,z)$

In other words, if T_1 halts on input x, then T_2 halts on input x and produces the same output as T_1.

NON-DETERMINISTIC TURING MACHINES

In the case of FAs, simplest model of computation, non-determinism was introduced primarily for convenience. It did not change the class of language that could be recognized. For PDAs it did; the language pal is an example of a CFL that cannot be accepted by any deterministic PDA. A TM, even in its most basic form, has enough computing power that once again, non-determinism fails to add any more. Any language that can be accepted by a non-deterministic, FM can be accepted by an ordinary one. The argument showing this requires a simulation more complex, even though we will take advantage of the result established thereby allowing our ordinary TM to have several tapes. Nevertheless, the proof depends ultimately on the same sorts of routine manipulations, and it illustrates the fact that by combining enough of these simple manipulations, we can produce TMs able to implement very complex algorithms.

A non-deterministic Turing machine (NTM) $T = (Q, \Sigma, \Gamma, q_0, \delta)$ is defined exactly, the same way as an ordinary TM, except that the transition function δ takes values that are subsets, rather single elements of the set $(Q \cup \{h\}) \times (\Gamma \cup \{\Delta\}) \times \{R, L, S\}$. We do not need to say that δ is a partial function because now $\delta(q, a)$ is allowed to take the value 0.

We can refer to a configuration of an NTM in the same way. To say that

$$(p, x\,\underline{a}\,y) \vdash (q, w\,\underline{b}\,z)$$

means that beginning in the first configuration, there is atleast one move that will produce the second.

Similarly, $$(p, x\,\underline{b}\,y) \vdash^*_T (q, w\,\underline{b}\,z)$$

means that there is atleast one sequence of zero or more moves that takes T from the first configuration to the second. With this definition, we may still say that a string $x \in \Sigma^*$ is accepted by T if for some $a \in \Gamma \cup \{\Delta\}$ and some $y, z \in (\Gamma \cup \{\Delta\})^*$,

$$(q_0, \underline{\Delta}x) \vdash^*_T (h, y\,\underline{a}\,z)$$

here, L(T) denotes the language of strings accepted by T.

It is less obvious what an appropriate definition of output would be in the non-deterministic case. Conceivably, depending on the sequence of moves chosen, the final tape contents could be any one of an infinite number of possibilities. TMs of this type are useful but primarily as components of larger machines. When we compare NTMs to ordinary ones, we are concerned with them only as language acceptors.

Because every TM can be interpreted as an NTM, it is obvious that any language that can be accepted by a TM can be accepted by an NTM. The converse is the less obvious statement, and this is what we must prove.

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Theorem 2.
Let $T_1 = (Q_1, \Sigma, \Gamma_1, q_1, \delta_1)$ be a non-deterministic Turing machine. Then there is an ordinary (deterministic) TM $T_2 = (Q_2, \Sigma, \Gamma_2, q_2, \delta_2)$ with $L(T_2) = L(T_i)$.

UNIVERSAL TURING MACHINES

Alan Turing described a *universal computing machine.*

It is a TM T_u' whose input consists of two parts:

(*i*) string specifying some other (special-purpose) TM T_1

(*ii*) second string z that is interpreted as input to T_1.

TM T_u then simulates the processing of z by T_1.

First step in constructing a universal TM is to formulate a notational system in which we can encode both an arbitary TM T_1 and an input string z over an arbitrary alphabet as strings $e(T_1)$ and $e(z)$ over some fixed alphabet. We can do this arbitrarily, as long as the encoding does not destroy any information; given the strings $e(T_1)$ and $e(z)$, we must be able to reconstruct the TM T_1 and the string z. We use the alphabet {0, 1} even though the TM we are encoding may have a much larger alphabet. We start by assigning positive integers to each state, each tape symbol and each of the three *directions* S, L, and R in the TM T_1 we want to encode.

At this point, ε slight technical problem arises. We want the string of 0's and 1's that results to be the encoding of at most one TM. In other words, we want the encoding function e to be one-to-one. Consider two TM's T_1 and T_2 that are identical except that the tape symbols of T_1 are a and b, and those of T_2 are a and c. If we really want to call these two TMs different, then in order to guarantee that their encodings are different, we must make sure that the integers assigned to b and c are different. To accommodate any TM and still ensure that the encoding is one-to-one, we must somehow fix it so that no symbol in any TM's alphabet receives the same number as any other symbol in any other TM's alphabet. The easiest way to handle this problem is to fix, once and for all, the set of symbols that can be used by TMs and to number these symbols at the outset. While we are at it, we do the same with states. This is the reason for the following convention.

Definition 16. (Encoding function *e*)

First we associate to each tape symbol (including Δ), to each state (including h), and to each of the three directions, a string of 0'*s*.

Let $s(\Delta) = 0$

$s(a_i) = 0^{i+1}$ (for each $a_i \in$ S)

$s(h) = 0$

$s(q_i) = 0^{i+1}$ (for each $q_i \in$ Q)

$s(S) = 0$

$s(L) = 00$

$s(R) = 000$

Each move m of a TM, described by the formula

$$\delta(p, a) = (q, b, D)$$

is encoded by the string

$$e(m) = s(p)\, 1s(a)\, 1s(q)\, 1s(b)\, 1s(D)1$$

and for any TM T with initial state q, T is encoded by the string

$$e(T) = s(q)\, 1e(m_1)\, 1e(m_2)1......e(m_k)1$$

where $m_1, m_2, ..., m_k$ are the distinct moves of T, arranged in some arbitrary order.

Finally, any string $z = z_1z_2.......z_k$, where each $z_i \in$ S, is encoded by

$$e(z) = 11s(z_1)1s(z_2)1......\, s(z_k)1$$

The 11 at the beginning of the string $e(z)$ is there, so that in a composite string of the form $e(T)$ stops.

Now one consequence is that the encoding $s(a)$ of a single symbol a e S is different from the encoding $e(a)$ of the one-character string a.

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FINITE STATE MACHINES WITH OUTPUT

Finite state machines normally include a finite set of states, with a designated starting state, an input alphabet and a transition function that assigns a next state and input pair. Again there are finite state machines with output.

e.g. consider a vending machine which accepts ₹ 1, ₹ 2, and ₹ 5 coins. When a total of ₹ 6 or more has been deposited, the machine immediately returns the amount in excess of ₹ 6. When ₹ 6 has been deposited and any excess refunded, the customer can push a green button and receive Coca Cola or a red button and receive Fanta.

The functioning can be described by specifying these states, how it changes states when input is received and the output that is produced for every combination of input and current states.

The possible states are :

S_0 – initial state – no coin has been deposited
S_1 – ₹ 1 has been deposited
S_2 – ₹ 2 have been deposited
S_3 – ₹ 3 have been deposited
S_4 – ₹ 4 have been deposited
S_5 – ₹ 5 have been deposited
S_6 – ₹ 6 have been deposited

State table for a vending machine

Initial State	State reached									
	Input					Output				
	₹ 1	₹ 2	₹ 5	G	R	₹ 1	₹ 2	₹ 5	G	R
S_0	S_1	S_2	S_5	S_0	S_0	*n*	*n*	*n*	*n*	*n*
S_1	S_2	S_3	S_6	S_1	S_1	*n*	*n*	*n*	*n*	*n*
S_2	S_3	S_4	S_6	S_2	S_2	*n*	*n*	1	*n*	*n*
S_3	S_4	S_5	S_6	S_3	S_2	*n*	*n*	2	*n*	*n*
S_4	S_5	S_6	S_6	S_4	S_4	*n*	*n*	3	*n*	*n*
S_5	S_6	S_6	S_6	S_5	S_5	*n*	1	4	*n*	*n*
S_6	S_6	S_6	S_6	S_6	S_6	1	2	4	CC	FA

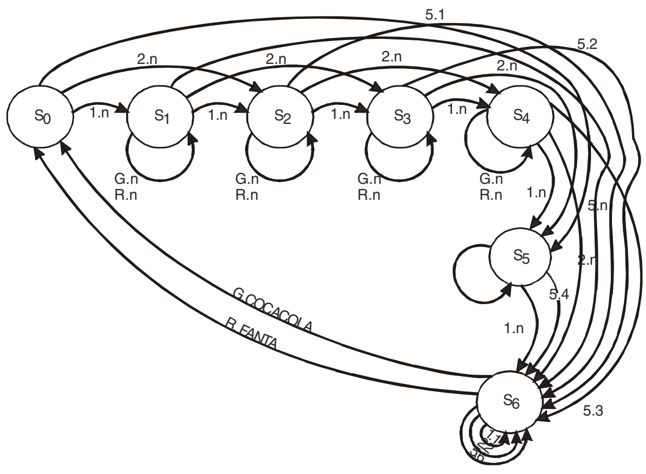

Fig. Vending machine state diagram

G – Green Button
R – Red Button
CC – Coca Cola
FA – Fanta

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EXERCISE – I

MCQ TYPE QUESTIONS

1. The major difference between a moore and mealy machine is that
(*a*) output of the former depends on the present state and present input
(*b*) output of the former depends only on the present state
(*c*) output of former depends only on the present input
(*d*) all of these

2. Any given transition graph has an equivalent
(*a*) regular expression (*b*) DFSM
(*c*) NDFSM (*d*) all of these

3. For which of the following application, regular expressions can not be used ?
(*a*) Designing computers
(*b*) Designing compilers
(*c*) Both (*a*) and (*b*)
(*d*) Developing computers

4. If S be an infinite set and S_1 ...., S_n be sets such that $S_1 \cup S_2 \cup ..... \cup S_n = S$, then
(*a*) atleast one of the set S_i is a finite set
(*b*) not more than one of the sets S_i can be finite
(*c*) atleast one of the sets S_i is an infinite set
(*d*) not more than one of the sets S_i can be infinite

5. Vienna Definition Language is an example of language definition facility based on
(*a*) Mathematical semantics
(*b*) Interpretative semantics
(*c*) Translational semantics
(*d*) Axiomatic semantics

6. Which of the following regular expressions denotes a language comprising all possible strings over the alphabet $\{a, b\}$?
(*a*) $a^* b^*$ (*b*) $(a \mid b)^*$
(*c*) $(ab)^+$ (*d*) $(a \mid b^*)$

7. An FSM (Finite State Machine) can be considered to be a TM (Turing Machine) of finite tape length
(*a*) without rewinding capability and unidirectional tape movement.
(*b*) rewinding capacity, and unidirectional tape movement.
(*c*) without rewinding capability and bidirectional tape movement.
(*d*) rewinding capability and bidirectional tape movement.

8. Palindromes can't be recognized by any FSM because
(*a*) FSM can't remember arbitrarily large amount of information
(*b*) FSM can't deterministically fix the mid-point
(*c*) even if mid-point is known, FSM can't be found whether second half of the string matches the first half
(*d*) all of these

9. A language L is accepted by a finite automaton if and only if it is
(*a*) context-free (*b*) context-sensitive
(*c*) recursive (*d*) right-linear

10. Can a DFA simulate NFA ?
(*a*) No (*b*) Yes
(*c*) Sometimes (*d*) Depends on NFA

11. Which of the following statements is wrong ?
(*a*) The language accepted by finite automata are the languages denoted by regular expressions.
(*b*) For every DFA there is a regular expression denoting its language.
(*c*) For a regular expression *r*, there does not exist any NFA with transit that accepts L(*r*)
(*d*) None of these

12. Regular expression a/b denotes the set
(*a*) $\{a\}$ (*b*) $\{\in, a, b\}$
(*c*) $\{a, b\}$ (*d*) $\{ab\}$

13. Regular expression $(a \mid b)(a \mid b)$ denotes the set
(*a*) $\{a, b, ab, aa\}$ (*b*) $\{a, b, ba, bb\}$
(*c*) $\{a, b\}$ (*d*) $\{aa, ab, ba, bb\}$

14. Which of the following regular expressions denotes zero or more instances of an *a* or *b* ?
(*a*) $a \mid b$ (*b*) $(ab)^*$
(*c*) $(a \mid b)^*$ (*d*) $a^* \mid b$

15. Which of the following regular expressions denotes a language comprising all possible strings of even length over the alphabet (0, 1) ?
(*a*) $(0 \mid 1)^*$ (*b*) $(0 \mid 1)(0 \mid 1)^*$
(*c*) $(00 \mid 01 \mid 1 \mid 10)^*$ (*d*) $(0 \mid 1)(0 \mid 1)(0 \mid 1)^*$

16. The regular expression $(a \mid b)^*$ denotes the set of all strings
(*a*) with zero or more instances of *a* or *b*
(*b*) with one or more instances of *a* or *b*
(*c*) equal to regular expression $(a^* b^*)^*$
(*d*) both (*a*) and (*c*)

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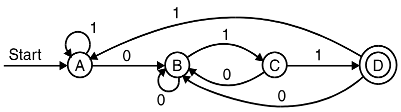
17. The string $(a) \mid ((b) * (c))$ is equivalent to
(*a*) set of strings with either a or zero or more b's and one c
(*b*) set of strings with either a or one or more b's and one c
(*c*) $b^* \, c \mid a$
(*d*) both (*a*) and (*c*)

18. An automation is a ..... device and a grammar is a ..... device.
(*a*) generative, cognitive
(*b*) generative, acceptor
(*c*) acceptor, cognitive
(*d*) cognitive, generative

19. In the figure given below, a deterministic finite automation M has start state A and accepting state D. Which of the following regular expression denoted the set of all words accepted by M ?
(*a*) 001
(*b*) 10 * 1 * 0
(*c*) (0 | 1) * 011
(*d*) 1 * 0 * 001



20. The regular sets are closed under
(*a*) union (*b*) concatenation
(*c*) Kleenes closure (*d*) all of these

21. Dynamic errors can be detected at
(*a*) compile time (*b*) run time
(*c*) both (*a*) and (*b*) (*d*) none of these

22. If a and b be the regular expressions, then $(a^* \cup b^*)^*$ is equivalent to
(*a*) $(a \cup b)^*$ (*b*) $(b^* \cup a^*)^*$
(*c*) $(b \cup a)^*$ (*d*) $a \cup b$

23. Finite state machine .... recognize palindromes.
(*a*) can (*b*) can't
(*c*) may (*d*) may not

24. If S and T be language over $\Sigma = \{a, b\}$ represented by regular expression $(a + b^*)^*$ and $(a + b)^*$, respectively, then
(*a*) $S \subset T$ (*b*) $T \subset S$
(*c*) $S = T$ (*d*) $S \cap T = \phi$

25. Consider regular expression (0 + 1) (0 + 1) .... n times. Minimum state finite automaton that recognizes the language represented by this regular expression contains
(*a*) n states (*b*) $n + 1$ states
(*c*) $n + 2$ states (*d*) none of these

26. If regular set A is represented by $A = (01 + 1)^*$ and the regular set 'B' is represented by $B = ((01)^*1^*)^*$, then
(*a*) $A \subset B$
(*b*) $B \subset A$
(*c*) A and B are uncomparable
(*d*) $A = B$

27. Which of the following can be recognized by a Deterministic Finite-state Automaton ?
(*a*) Numbers, 1, 2, 4, ... z^n ..... written in binary.
(*b*) Numbers 1, 2, 4, ..., z^n, ... written in unbinary.
(*c*) Set of binary string in which number of zeros is same as the number of ones.
(*d*) Set {1, 101, 11011, 1110111, ...}

28. Regarding power of recognition of languages, which of the following statements is false ?
(*a*) Non deterministic finite-state automata are equivalent to deterministic finite-state automata.
(*b*) Non-deterministic push-down automata are equivalent to deterministic push-down automata.
(*c*) Non-deterministic Turing Machines are equivalent to deterministic push-down automata.
(*d*) Multi-tape Turing Machines are equivalent to Single-tape Turing Machines.

29. Which of the following regular expressions over {0, 1} denotes the set of all strings not containing 100 as a substring ?
(*a*) $0^* (1^+ 0)^*$ (*b*) $0^* 1010^*$
(*c*) $0^* 1^* 01$ (*d*) $0^* (10 + 1)^*$

30. Two of the following four regular expressions are equivalent. Which two is the empty string?
I. $(00)^* (\in + 0)$
II. $(00)^*$
III. 0^*
IV. $0(00)^*$
Codes :
(*a*) I and II (*b*) II and III
(*c*) I and III (*d*) III and IV

31. If $L \subseteq \Sigma^*$, where $\Sigma = \{a, b\}$; then which of the following is regular ?
(*a*) $L = \{x \mid x$ has an equal number of a's and b's$\}$
(*b*) $L = \{a^n b^n \mid n \geq 1\}$
(*c*) $L = \{x \mid x$ has more a's than b's$\}$
(*d*) $L = \{a^m b^n \mid m \geq 1, n \geq 1\}$

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32. Which of the following is false ?
Read $\wedge$ as AND, $\vee$ as OR, ~ as NOT, $\rightarrow$ as one way implication and $\leftrightarrow$ as two way implication.
(*a*) $((x \rightarrow y) \wedge x) \rightarrow y$
(*b*) $((\sim x \rightarrow y) \wedge (\sim x \wedge \sim y)\ 1 \rightarrow x$
(*c*) $(x \rightarrow (x \vee y))$
(*d*) $((x \vee y) \leftrightarrow (\sim x \rightarrow \sim y))$

33. Which of the following is false ?
(*a*) Set of all bijective functions on a finite set forms a group under function composition.
(*b*) Set {1, 2, ... p – 1} forms a group under multiplication mod p where p is a prime number.
(*c*) Set of all strings over a finite alphabet forms a group under concatenation.
(*d*) Subset $s \neq \phi$ of G is a subgroup of the group < G*, > if and only if for any pair of elements $a, b \in S$, $a * b^{-1} \in S$

34. Match List I with List II and select the correct answer from the codes given below the lists.

	List I		**List II**
A.	Groups	1.	Associativity
B.	Semigroups	2.	Identity
C.	Monoids	3.	Commutativity
D.	Abelian groups	4.	Left inverse

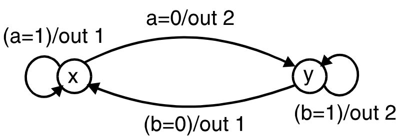
Codes:

	A	**B**	**C**	**D**
(*a*)	2	1	2	3
(*b*)	4	1	2	3
(*c*)	2	4	1	3
(*d*)	3	4	1	2

35. If R_1 and R_2 be regular sets defined over the alphabet Σ, then
(*a*) $R_1 \cap R_2$ is not regular
(*b*) $R_1 \cup R_2$ is regular
(*c*) $\Sigma^* - R_1$ is regular
(*d*) both (*a*) and (*b*)

36. Which of the following well-formed formula are valid ?
(*a*) $(P \Rightarrow Q) \wedge (Q \Rightarrow R) \Rightarrow (P \Rightarrow R)$
(*b*) $(P \Rightarrow Q) \Rightarrow (1\ P \Rightarrow 7\ Q)$
(*c*) $(P \wedge (1\ P \vee 7\ Q)) \Rightarrow Q$
(*d*) Both (*a*) and (*b*)

37. If state machine shown in the figure should have a stable state, then restriction on the inputs is given by
(*a*) $a \cdot b = 1$
(*b*) $a + b = 1$
(*c*) $\overline{a} + \overline{b} = 0$
(*d*) $\overline{a \cdot b} = 1$



38. Which of the following pairs of regular expressions are required ?
(*a*) 1 (01)* and (10)* 1
(*b*) × (××)* and (××)*×
(*c*) $(ab)^*$ and $a^*\ b^*$
(*d*) All of these

39. Which of the following pairs of regular expression are not equivalent ?
(*a*) $(ab)^*a$ and $a\,(ba)^*$
(*b*) $(a + b)^*$ and $(a^* + b)^*$
(*c*) $b^*\ ab^*\ a\ (a + b)^*$
(*d*) All of these

40. Power of
(*a*) DFSM and NDFSM are same
(*b*) DFSM and NDFSM are different
(*c*) DPDM and NDPDM are different
(*d*) both (*a*) and (*c*)

41. Number of states of the FSM required to simulate behaviour of a computer with a memory capable of storing "m" words, each of length 'n' bits is
(*a*) $m \times 2^n$
(*b*) 2^{mn}
(*c*) 2^{m+n}
(*d*) all of these

42. In an incompletely specified automata
(*a*) no edge should be labelled epsilon
(*b*) from any given state, there can't be any token leading to two different states
(*c*) some states have no transition on some tokens
(*d*) start state may not be there

43. The word 'formal' in formal languages means
(*a*) the symbols used have well-defined meaning
(*b*) they are unnecessary, in reality
(*c*) only form of the string of symbols is significant
(*d*) none of these

44. Finite state machine can recognize
(*a*) any grammar
(*b*) only context-free grammar
(*c*) any unambiguous grammar
(*d*) only regular grammar

45. Which of the following are not regular ?
(*a*) String of 0's whose length is a perfect square
(*b*) Set of all palindromes made up of 0's and 1's
(*c*) Strings of 0's, whose length is a prime number
(*d*) All of these

46. Which of the following pairs of regular expressions are equivalent ?
(*a*) 1 (01) * and (10) * 1
(*b*) $x\ (xx)^*$ and $(xx)^*\ x$
(*c*) x^+ and x^*x^+
(*d*) All of these

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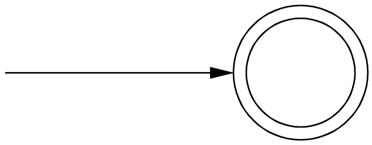
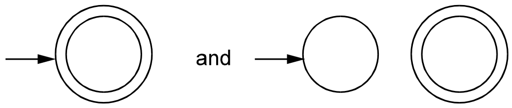
47. FSM shown in the figure



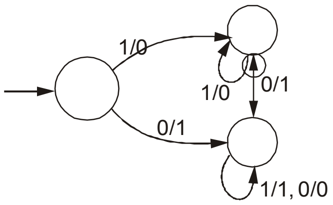
(*a*) all strings (*b*) no string
(*c*) ε - alone (*d*) none of these

48. Consider following two FSM shown in the figure.



(*a*) Both are equivalent
(*b*) Second FSM accepts ε - only
(*c*) First FSM accepts nothing
(*d*) None of these

49. For the machine shown in the figure



(*a*) complements a given bit pattern
(*b*) finds 2's complement of a given bit pattern
(*c*) increments a given bit pattern by 1
(*d*) changes the sign bit

50. For which of the following applications regular expressions can't be used ?

(*a*) Designing compilers
(*b*) Developing text editors
(*c*) Simulating sequential circuits
(*d*) All of these

51. An FSM with

(*a*) 1 stack is more powerful than an FSM with no stack
(*b*) 2 stacks is more powerful than a FSM with 1 stack
(*c*) both (*a*) and (*b*)
(*d*) none of these

52. A finite state machine with the following state table has a single input x and a single output z.

Present state	Next state, z	
	$x = 1$	$x = 0$
A	D, 0	B, 0
B	B, 1	C, 1
C	B, 0	D, 1
D	B, 1	C, 0

If initial state is unknown, then shortest input sequence to reach the final state C is

(*a*) 01 (*b*) 10
(*c*) 10 (*d*) 110

53. Which of the following statements is/are false ?

(*a*) The task of lexical analyzer is to translate the input source language text into tokens and determine the groups of tokens are inter-related.
(*b*) Two basic approaches to translation are generation and interpretation.
(*c*) A load-and-go compiler is capable of translating the source language text on a host machine A that can be later run on any target machine B.
(*d*) None of these

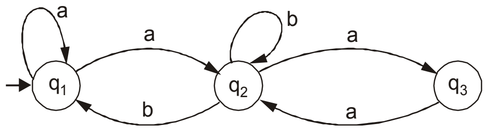
54. If $w \in (a, b)^*$ satisfy $abw = wab$, then (w) is

(*a*) even (*b*) odd
(*c*) null (*d*) none of these

55. If $f : \{a, b\}^* \rightarrow \{a, b\}^*$ be given by $f(n) = ax$ for every value of $n \in \{a, b\}$, then f is

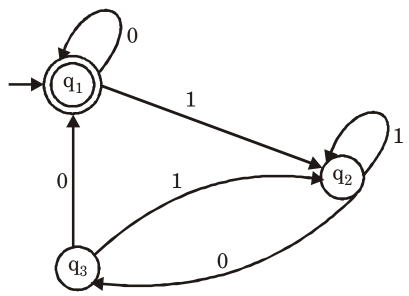
(*a*) one to one not onto
(*b*) one to one and onto
(*c*) not one to one and not onto
(*d*) not one to one and onto

56. Which string recognize it ?



(*a*) $(a + a(b + aa)^* b)^* (b + bb)^* a$
(*b*) $a + a(b + aa)^* b a (b + aa)^* a$
(*c*) $a + b(a + bb)^* (a + (b + aa)^*)^*$
(*d*) None of these

57. Regular expression corresponding to the state diagram given in the figure is



(*a*) $(0 + 1(1 + 01)^* 00)^*$ (*b*) $(1 + 0(0 + 10)00)^*$
(*c*) $(0 + 1(1 + 10)00)^*$ (*d*) $(1 + 0(1 + 00)11)^*$

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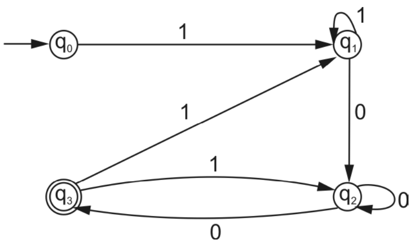
58. L = {a^p | p; is prime, is
(*a*) regular (*b*) not regular
(*c*) accepted by DFA (*d*) accepted by PDA

59. Regular expression corresponding to the automata given in the figure below are



(*a*) 1 (1 + 0 (0 + 10)* 11)* 0 (0 + 10)* 1
(*b*) 0 (1 + 0 (1 + 01)* 11)* 1 (1 + 10)* 1
(*c*) 1 (1 + 0 (1 + 01)* 00)* 1 (1 + 01)*
(*d*) 0 (1 + 0 (1 + 01)* 00)* 1 (1 + 01)* 1

60. The main difference between a DFSA and an NDFSA is
(*a*) in DFSA, ε transition may be present
(*b*) in NDFSA, ε transitions may be present
(*c*) in DFSA, from any given state, there can't be any alphabet leading to two different states
(*d*) in NDFSA, from any given state, there can't be any alphabet leading to two different states

61. Two finite state machines are said to be equivalent if they
(*a*) have same number of states
(*b*) have same number of edges
(*c*) have same number of states and edges
(*d*) recognize same set of tokens

62. Running time of NFA to DFA conversion including the case where NFA has e-transition is
(*a*) O (n^3) (*b*) O ($n^3 3^2$)
(*c*) O ($n^3 2^n$) (*d*) O ($n^2 2^n$)

63. If two finite state machines are equivalent, they should have the same number of
(*a*) states (*b*) edges
(*c*) states and edges (*d*) none of these

64. If two finite states machine M and N are isomorphic, then
(*a*) M can be transformed to N, merely re-labelling its states
(*b*) M can be transformed to N, merely re-labelling its edges
(*c*) M can be transformed to N, merely re-labelling its edges
(*d*) none of these

65. Define for a context free language
L ≤ {0 ; 1} init (L) = {u/uv ∈ L for some v in {0,1}}
(in other words, init (L) is the set of prefixes of L)
Let L {w/w is noempty and has an equal number of 0's and 1's}
Then init (L) is
(*a*) set of all binary strings with unequal number of 0's and 1's
(*b*) set of all binary strings including the null string
(*c*) set of all binary strings with exactly one more 0's than the number of 1's or th emore 1 than the number of 0's
(*d*) none of these

66. If L_1 and L_2 are context free language and R a regular set, then which one of the languages below is not necessarily a context free language?
(*a*) $L_1 L_2$ (*b*) $L_1 \cap L_2$
(*c*) $L_1 \cap R$ (*d*) $L_1 \cup L_2$

67. Consider a grammar with the following productions
S → a a b | b a c| aB
S → α S | b
S → α bb | ab
S α → bdb | b
The above grammar is
(*a*) Context free (*b*) Regular
(*c*) Context sensitive (*d*) LR (k)

68. What can be said about a regular language L over {a} whose minimal finite state automaton has two states?
(*a*) L must be {a^n | n is odd}
(*b*) L must be {a^n | n is even}
(*c*) L must be {a^n | ≥ 0}
(*d*) Either L must be {a^n | n is odd}, or L must be {a^n | n is even}

69. In a context-sensitive grammar, number of grammar symbols on the left hand side of a production can't be greater than the number of
(*a*) grammar symbols on the right hand side
(*b*) terminals on the right hand side
(*c*) non-terminals on the right hand side
(*d*) all of these

70. In a context-free grammar
(*a*) ε can't be the right hand side of any production
(*b*) terminal symbols can't be present in the left hand side of any production
(*c*) number of grammar symbols in the left hand side is not greater than the number of grammar symbols in the right hand side
(*d*) all of these

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71. CFG can be recognized by a

(*a*) push-down automata

(*b*) 2-way linear bounded automata

(*c*) both (*a*) and (*b*)

(*d*) none of these

72. Which of the following statements are true?

I. The set of all odd integers is a monoid under multiplication.

II. The set of all complex number is a group under multiplication

III The set of all integers under the operation * given by a * b = a+b–ab is a monoid

IV Z^S under symmetric difference $\overline{Z}$ defined by $A \overline{Z} B = (A–B) \cup (B–A)$ is an abelian group

(*a*) I and II (*b*) I, III and IV

(*c*) I, II and III (*d*) I, II and IV

73. A given grammar is called *ambiguous* if

(*a*) two or more productions have the same non-terminal on the left hand side

(*b*) a derivation tree has more than one associated sentence

(*c*) there is a sentence with more than one derivation tree corresponding to it

(*d*) brackets are not present in the grammar

74. Let L be a language recognizable by a finite automaton. The language

REVERSE (L) = {*w* such that *w* is the reverse of *v* where $v \in$ L } is a

(*a*) regular language

(*b*) context-free language

(*c*) context-sensitive language

(*d*) recursively enumerable language

75. The grammars G = ({*s*}, {0, 1}, *p*, *s*)

where $p = \{s \to 0S1, S \to 0S, S \to S1, S \to 0\}$ is a

(*a*) recursively enumerable language

(*b*) regular language

(*c*) context-sensitive language

(*d*) context-free language

76. The logic of pumping lemma is a good example of

(*a*) pigeon-hole principle

(*b*) divide-and-conquer technique

(*c*) recursion

(*d*) iteration

77. The intersection of CFL and regular language

(*a*) is always regular

(*b*) is always context free

(*c*) both (*a*) and (*b*)

(*d*) need not be regular

78. For two regular languages

$$L_1 = (a + b)^* a \text{ and } L_2 = b (a + b)^*,$$

the intersection of L_1 and L_2 is given by

(*a*) $(a + b)^* ab$ (*b*) $ab (a + b)^*$

(*c*) $a (a + b)^* b$ (*d*) $b (a + b)^* a$

79. Context free grammar is not closed under

(*a*) product (*b*) union

(*c*) complementation (*d*) kleen star

80. If L be a language recognizable by a finite automaton, then language front {L} = {*w* such that *w* is prefix of *v* where $v \in$ L }, is a

(*a*) regular language

(*b*) context-free language

(*c*) context-sensitive language

(*d*) recursive enumeration language

81. For which of the following application, regular expressions can not be used ?

(*a*) Designing computers

(*b*) Designing compilers

(*c*) Both (*a*) and (*b*)

(*d*) Developing computers

82. Consider the following grammar :

$$S \to Ax/By$$
$$A \to By/Cw$$
$$B \to x/Bw$$
$$C \to y$$

Which of the regular expressions describe the same set of strings as the grammar ?

(*a*) $xw * y + xw * yx + ywx$

(*b*) $xwy + xw * xy + ywx$

(*c*) $xw * y + xw \times yx + ywx$

(*d*) $xw\, xy + xww * y + ywx$

83. Which of the following statement(s) is (are) correct ?

(*a*) Recursive languages are closed under complementation.

(*b*) If a language and its complement are both regular, the language is recursive.

(*c*) Set of recursively enumerable language is closed under union.

(*d*) All of these

84. Which of the following statement is wrong ?

(*a*) Any regular language has an equivalent context-free grammar.

(*b*) Some non-regular languages can't be generated by any context-free grammar.

(*c*) Intersection of context free language and a regular language is always context-free.

(*d*) All languages can be generated by context-free grammar.

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85. Consider a grammar :

$$G = (\{x, y\}, \{s, x, y\}, p, s)$$

where elements of parse :

$$S \rightarrow xy$$
$$S \rightarrow yx$$
$$x \rightarrow xz$$
$$x \rightarrow x$$
$$y \rightarrow y$$
$$z \rightarrow z$$

The language L generated by G most accurately is called

(*a*) Chomsky type 0
(*b*) Chomsky type 1
(*c*) Chomsky type 2
(*d*) Chomsky type 3

86. Consider a grammar :

$$G = \{\{S\}, \{0, 1\}, p, s\}$$

where elements of p are:

$$S \rightarrow SS$$
$$S \rightarrow 0S1$$
$$S \rightarrow 1S0$$
$$S \rightarrow \text{empty}$$

The grammar will generate

(*a*) regular language
(*b*) context-free language
(*c*) context-sensitive language
(*d*) recursive enumerable language

87. A grammar that produces more than one parse tree for some sentence is called

(*a*) ambiguous (*b*) unambiguous
(*c*) regular (*d*) none of these

88. Given a grammar G a production of G with a dot at some position of the right side is called

(*a*) LR (0) item of G (*b*) LR (1) item of G
(*c*) both (*a*) and (*b*) (*d*) none of these

89. If a language is denoted by a regular expression

$L = (x)^* (x \mid yx)$,

then which of the following is not a legal string within L ?

(*a*) yx (*b*) xyx
(*c*) x (*d*) $xyxyx$

90. If every string of a language can be determined, whether it is legal or illegal in finite time, the language is called

(*a*) decidable (*b*) undecidable
(*c*) interpretive (*d*) non-deterministic

91. The defining language for developing a formalism in which language definitions can be stated, is called

(*a*) syntactic meta language
(*b*) decidable language
(*c*) intermediate language
(*d*) high level language

92. If L be set of strings from alphabet, then kleen closure of L is given as

(*a*) $L^+ = \bigcup_{i=0} L^i$ (*b*) $L_* = \bigcup_{i=0} L^i$

(*c*) $L^* = \bigcup_{i=0}^{\infty} L^i$ (*d*) $L^+ = \bigcup_{i=1} L^i$

93. If e_1 and e_2 are the regular expressions denoting the languages L_1 and L_2 respectively, then which of the following is wrong ?

(*a*) $(e_1) \mid (e_2)$ is a regular expression denoting $L_1 \cup L_2$
(*b*) $(e_1)\ (e_2)$ is a regular expression denoting $L_1 . L_2$
(*c*) ϕ is not a regular expression
(*d*) $\{e_1\}$ is a regular expression denoting L_1^*

94. Context-free grammar can be recognized by

(*a*) finite state automaton
(*b*) 2-way linear bounded automata
(*c*) push down automata
(*d*) both (*b*) and (*c*)

95. The language $L = (0^n\ 1^n\ 2^n$ where $n > 0)$ is a

(*a*) context free language
(*b*) context-sensitive language
(*c*) regular language
(*d*) recursive enumerable language

96. Context free language are closed under

(*a*) union, intersection
(*b*) union, kleene closure
(*c*) intersection, complement
(*d*) complement, kleene closure

97. In the following grammar :

$x ::= x \oplus y \mid 4$
$y ::= z * y \mid 2$
$z ::= id$

which of the following is true ?

(*a*) '$\oplus$' is left associative while '*' is right associative.
(*b*) Both '$\oplus$' and '*' are left associative.
(*c*) '$\oplus$' is right associative while '*' is left associative.
(*d*) None of these

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98. Define for the context free language

$L \leq \{0 ; 1\}$ init (L) = $\{u \mid uv \in L$ for some v in $\{0, 1\}\}$.

If L $\{w \mid w$ is nonempty and has an equal number if 0's and 1's$\}$, then init (L) is set of all binary strings

(*a*) with unequal numbers of 0'*s* and 1'*s*.

(*b*) including the null string.

(*c*) with exactly one more 0'*s* than the number of 1'*s* or one more 1 than the number of 0'*s*.

(*d*) none of these

99. The productions

$$E \rightarrow E + E \mid E \rightarrow E - E \mid E \rightarrow E * E,$$
$$E \rightarrow E \mid E, E \rightarrow id$$

(*a*) generate an inherently ambiguous language.

(*b*) generate an ambiguous language but not inherently so.

(*c*) are unambiguous.

(*d*) can generate all possible fixed length valid computation for carrying out addition, subtraction, multiplication and division, which can be expressed in one expression.

100. Correct hierarchical relationship among context-free, right-linear, and context-sensitive language is

(*a*) context-free $\subset$ right-linear $\subset$ context-sensitive

(*b*) context-free $\subset$ context-sensitive $\subset$ right-linear

(*c*) context-sensitive $\subset$ right-linear $\subset$ context-free

(*d*) right-linear $\subset$ context-free $\subset$ context-sensitive

101. If $\Sigma = \{0, 1\}$, $L = \Sigma^*$ and $R = \{0^n 1^n$ such that $n > 0\}$ then languages $L \cup R$ and R respectively are

(*a*) Regular, Regular

(*b*) Regular, Not regular

(*c*) Not regular, Not regular

(*d*) both (*b*) and (*c*)

102. Consider the grammar :

$$\begin{aligned} S &\rightarrow ABCc \mid Abc \\ BA &\rightarrow AB \\ Bb &\rightarrow bb \\ Ab &\rightarrow ab \\ Aa &\rightarrow aa \end{aligned}$$

Which of the following sentences can be derived by this grammar ?

(*a*) abc (*b*) aab

(*c*) abcc (*d*) abbc

103. Which of the following definitions below generates the same language as L, where

$L = \{x^n y^n$ such that $n >= 1\}$?

I. $E \rightarrow xEy \mid xy$

II. $xy \mid (x^+ xyy^+)$

III. $x^+ y^+$

(*a*) I only (*b*) I and II

(*c*) II and III (*d*) II only

104. FSM can recognize

(*a*) any grammar (*b*) only CG

(*c*) any unambiguous (*d*) only regular grammar

105. Pumping lemma is generally used for proving that

(*a*) given grammar is regular

(*b*) given grammar is not regular

(*c*) whether two given regular expressions are equivalent or not

(*d*) none of these

106. Which of the following statements is correct?

(*a*) $A = \{a^n b^n \mid n = 0, 1, 2, 3 ..\}$ is regular language

(*b*) Set B of all strings of equal number of a's and b's defines a regular language

(*c*) $L(A^* B^*) \cap B$ gives the set A

(*d*) None of these

107. Basic limitation of FSM is that it

(*a*) cannot remember arbitrary large amount of information

(*b*) sometimes fails to recognize grammars that are regular

(*c*) sometimes recognizes grammars are not regular

(*d*) all of these

108. The language of all words with at least 2 a's can be described by the regular expression

(*a*) $(ab)^*a$ and $a(ba)^*$

(*b*) $(a + b)^* ab^* a (a + b)^*$

(*c*) $b^* ab^* a (a + b)^*$

(*d*) all of these

109. Set of regular languages over a given alphabet set is not closed under

(*a*) union (*b*) complementation

(*c*) intersection (*d*) all of these

110. The CFG $s \rightarrow as \mid bs \mid a \mid b$ is equivalent to regular expression

(*a*) $(a + b)$ (*b*) $(a + b)(a + b)^*$

(*c*) $(a + b)(a + b)$ (*d*) all of these

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111. Any string of terminals that can be generated by the following CFG is

$S \rightarrow XY$

$X \rightarrow aX \mid bX \mid a$

$Y \rightarrow Ya \mid Yb \mid a$

(*a*) has atleast one '*b*'
(*b*) should end in a '*a*'
(*c*) has no consecutive *a*'s or *b*'s
(*d*) has atleast two *a*'s

112. Which of the following statement is correct?
(*a*) All languages can be generated by CFG
(*b*) Any regular language has an equivalent CFG
(*c*) Some non regular languages can't be generated by CFG
(*d*) both (*b*) and (*c*)

113. Which of the following CFG's can't be simulated by an FSM ?
(*a*) $s \rightarrow sa \mid a$
(*b*) $s \rightarrow abX$ $X \rightarrow cY, Y \rightarrow a \mid ax$
(*c*) $s \rightarrow a\,sb \mid ab$
(*d*) none of these

114. $L = \{a^n b^n a^n \mid n = 1, 2, 3 ....\}$ is an example of a language that is
(*a*) context free
(*b*) not context free
(*c*) not context free but whose complement is CF
(*d*) both (*b*) and (*c*)

115. P, Q, R are three languages, if P and R are regular and if PQ = R, then
(*a*) Q has to be regular
(*b*) Q cannot be regular
(*c*) Q need not be regular
(*d*) Q cannot be a CFL

116. Given $A = \{0, 1\}$ and $L = A^*$. If $R = \{0^n 1^n, n > 0\}$, then language $L \cup R$ and R are respectively
(*a*) regular, regular
(*b*) not regular, regular
(*c*) regular, not regular
(*d*) not regular, not regular

117. Which of the following is not possible algorithmically ?
(*a*) Regular grammar to context free grammar
(*b*) Non-deterministic FSA to deterministic FSA
(*c*) Non-deterministic PDA to deterministic PDA
(*d*) Non-deterministic turing machine to deterministic turing machine

118. A DG is said to be in Chomsky Form (CNF), if all the productions are of the form $A \rightarrow BC$ or $A \rightarrow a$. Let G be a CFG in CNF. To derive a string of terminals of length *x*, the number of productions to be used is
(*a*) $2x - 1$ (*b*) $2x$
(*c*) $2x + 1$ (*d*) $2x$

119. A class of language that is closed under
(*a*) union and complementation has to be closed under intersection
(*b*) intersection and complement has to be closed under union
(*c*) union and intersection has to be closed under complementation
(*d*) both (*a*) and (*b*)

120. Following context free grammar

$S \rightarrow aB \mid bA$

$A \rightarrow b \mid aS \mid bAA$

$B \rightarrow b \mid bS \mid aBB$

generates strings of terminals that have
(*a*) equal number of *a*'s and *b*'s
(*b*) odd number of *a*'s and odd number *b*'s
(*c*) even number of *a*'s and even number of *b*'s
(*d*) odd number of *a*'s and even number of *a*'s

121. The set $\{a^n b^n \mid n = 1, 2, 3 ...\}$ can be generated by the CFG
(*a*) $S \rightarrow ab \mid aSb$
(*b*) $S \rightarrow aaSbb + ab$
(*c*) $S \rightarrow ab \mid aSb \mid E$
(*d*) $S \rightarrow aaSbb \mid ab \mid aabb$

122. Which of the following CFG's can't be simulated by an FSM ?
(*a*) $S \rightarrow Sa \mid a$
(*b*) $S \rightarrow abX, X \rightarrow cY, Y \rightarrow -d \mid aX$
(*c*) $S \rightarrow aSb \mid ab$
(*d*) None of these

123. If $L_1 = \{x \mid x$ is a palindrome in $(0 + 1)^*\}$

$L_2 = \{\text{letter (letter + digit)}^*\}$;

$L_3 = (0^n 1^n 2^n \mid n \geq 1\}$

$L_4 = \{a^m b^n a^{m+n} \mid m, n \geq 1\}$

then which of the following statement is incorrect ?
(*a*) L_1 is context free language and L_3 is context sensitive language
(*b*) L_2 is a regular set and L_4 is not a context free language
(*c*) Both L_1 and L_2 are regular sets
(*d*) Both L_3 and L_4 are context-sensitive languages

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124. Consider the grammar

$$S \to PQ \mid SQ \mid PS$$
$$P \to x$$
$$Q \to y$$

To get a string of n terminals, the number of productions to be used is

(*a*) n^2 (*b*) $n + 1$
(*c*) $2n$ (*d*) $2n - 1$

125. If G = ({S}, {a}, {S → SS}, S), then language generated by G is

(*a*) L (G) = ϕ
(*b*) L (G) = a^n
(*c*) L (G) = a^*
(*d*) L (G) = a^nba^n

126. What is the highest type number which can be applied to the following grammar ?

$$S \to Aa, A \to Ba, B \to abc$$

(*a*) Type 0 (*b*) Type 1
(*c*) Type 2 (*d*) Type 3

127. Grammar

$S \to a, S \to A_3A_4, A_3 \to A_1 A_3, A_2, A_3 \to A_1 A_2, A_1$
$A_2 \to aA_2A_1, A_1a \to aA_1, A_2a \to aA_2, A_1A_4 \to A_4a$
$A_2A_4 \to A_5a, A_2A_5 \to A_5a, A_5 \to a$

generates

(*a*) a^{n^2} (*b*) n^{2a}
(*c*) $2a^n$ (*d*) none of these

128. A grammar to generate

$$\{(ab)^n \mid n \geq 1\} \cup \{(ba)^n \mid n \geq 1\}$$

is constructed as

(*a*) $S \to S_1, S_1 \to abS_1, S_1 \to ab, S \to S_2,$
$S_2 \to baS_2, S_2 \to ba$
(*b*) $S \to S_1, S_1 \to aS_1, S_1 \to ab, S \to S_2, S_2 \to bS_2,$
$S_2 \to bc$
(*c*) $S \to S_1, S_1 \to S_2, S_2 \to S_1a, S_1 \to ab, S_2 \to ba$
(*d*) None of these

129. Following syntax-directed translation scheme is used with a shift reduction (bottom up) parser that perform the action in braces immediately after a reduction by the corresponding production

$A \to aB$ {print "(1)" $A \to c$ {print "1"},
$B \to Ab$ {print "2"}.

When parser is *aaacbbb,* then string printed

(*a*) 0202021
(*b*) 1202020
(*c*) 1020202
(*d*) none of these

130. Which of the following is not accepted by Deterministic Push Down Machine (DPDM) but accepted by Non Deterministic Push Down Machine (DPDM) ?

(*a*) Strings end with a particular alphabet
(*b*) All strings in which a given symbol is present atleast twice
(*c*) Even palindromes
(*d*) None of these

131. Which of the following instances of the post correspondence problem have a viable sequence ?

(*a*) $\{(b, bb), (bb, bab), (bab, abb) (abb, babb)\}$
(*b*) $\{(ab, aba), (baa, aa) (aba, baa)\}$
(*c*) $\{(ab, abb), (ba, aaa) (aa, a)\}$
(*d*) None of these

132. If L_D is set of all language accepted by a PDA by final state and L_E set of all languages accepted by empty stack, then

(*a*) $L_D = L_E$ (*b*) $L_D \supset L_E$
(*c*) $L_D \subset L_E$ (*d*) none of these

133. A PDM behaves like a TM when the number of auxiliary memory it has, is

(*a*) 0 (*b*) 1 or more
(*c*) 2 or more (*d*) none of these

134. Which of the following is complement of a?

(*a*) Recursive language is recursive.
(*b*) Recursively enumerable language is recursively enumerable.
(*c*) Recursive language is either recursive or recursively enumerable.
(*d*) Context-free language is context-free.s

135. If nL can be recognized by a multitape TM with time complexity *f*, then L can be recognized by a one-tape machine with time complexity

(*a*) $O(f^2)$ (*b*) $o(f^2)$
(*c*) $o(h)$ (*d*) $O(h^2)$

136. If T is a TM recognizing L, and T reads every symbol in the input string, $\tau_T(n) \geq 2n + 2$, then any language that can be accepted by a TM T with $\tau_T(n) = 2n + 2$ is

(*a*) regular (*b*) not regular
(*c*) uncertain (*d*) none of these

137. Consider an alternate Turing machine model, in which there is an input tape on which the tape head can move in both directions but cannot write, and one or more work tapes, one of which serves as an output tape. For a function *f*, denoted by DSpace(*f*), the set of languages that can be recognized by a Turing machine of this type which uses no more than *f*(*n*) squares on any work tape

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for any input string of length n. The only restriction we need to make on f is that $f(n) > 0$ for every n. The language of balanced strings of parentheses are in

(a) $DSpace\ (1+\lceil \log_2 (n+1) \rceil)$. ($\lceil x \rceil$) means the smallest integer greater than or equal to x.)

(b) $DSpace\ (1+\lceil \log_2 n \rceil)$

(c) $DSpace\ (1+\lceil \log_2 n^2 \rceil)$

(d) none of these

138. Which of the following problems is solvable ?

(a) Writing a universal Turing machine.

(b) Determining of an arbitrary turing machine is an universal turing machine.

(c) Determining of a universal turing machine can be written for fewer than k instructions for some k.

(d) Determining of a universal turing machine and some input will halt.

139. Which of the following is not primitive recursive but partially recursive ?

(a) Carnot's function (b) Ricmann function

(c) Bounded function (d) Ackermann's function

140. Turing machine (TM) is more powerful than FMS (Finite State Machine) because

(a) tape movement is confined to one direction

(b) it has no finite state

(c) it has the capability to remember arbitrarily long sequences of input symbols

(d) none of these

141. If $f : N \to N$. If L can be recognized by a TM T, so that $\tau_T(n) \leq f(n)$ for all but finitely many n, then (Recall our convention that Time(f) means Time (max(f, $2n + 2$)).)

(a) $L \in Time(f)$ (b) $L \in Time(cf)$

(c) $L \in Time(h)$ (d) none of these

142. Let s is a step-counting function satisfying $s(n) \geq n$, and L be a language accepted by a (multitape) TM T. If tape heads of T do not move past square $s(n)$ on any of the tapes for an input string of length n, then T $\in$

(a) Space(s) (b) $f(n)$

(c) Time(f) (d) Time(h)

143. Which of the following statements is false ?

(a) Halting problem of Turing machines is undecidable

(b) Determining whether a context-free grammar is ambiguous is undecidable

(c) Given two arbitrary context-free grammars G_1 G_2 and it is undecidable whether $L(G_1) = L(G_2)$.

(d) Given two regular grammars G_1 G_2 and it is undecidable whether $L(G_1) = L(G_2)$.

144. Bounded minimalization is a technique for

(a) proving whether a promotive recursive function is turning computable or not

(b) proving whether a primitive recursive function is a total function or not

(c) generating primitive recursive functions

(d) generating partial recursive functions.

145. If there exists a language L, for which there exists a TM, T, that accepts every word in L and either rejects or loops for every word that is not in L, is called

(a) recursive

(b) recursively enumerable

(c) NP-HARD

(d) none of these

146. Which of the following statement(s) is/are correct?

(a) $L = \{a^n\ b^n\ a^n \mid n = 1, 2, 3...\}$ is recursively enumerable

(b) Recursively languages are closed under union

(c) Every recursive are closed under union.

(d) All of these

147. Universal TM influenced the concept of

(a) stored program computers

(b) interpretative implementation of programming language

(c) computability

(d) all of these

148. The statement, "A TM can't solve halting problem" is

(a) true

(b) false

(c) still at open question

(d) all of these

149. If there exists a TM which when applied to any problem in the class, terminates, if correct answer is yes and may or may not terminate otherwise is called

(a) stable (b) unsolvable

(c) partially solvable (d) unstable

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150. Given a Turing machine T and a step-counting function f, is the language accepted by T in Time(f) ? This decision problem is

(a) solvable (b) unsolvable

(c) uncertain (d) none of these

151. A total recursive function is also a

(a) partial recursive function

(b) primitive recursive function

(c) both (a) and (b)

(d) none of these

152. The running time T (n), where 'n' is input size of a recursive algorithm, is given as

$$T(n) = c + T(n-1), \text{ if } n > 1$$
$$= d, \text{ if } n \le 1$$

The order of the algorithm is

(a) n^2 (b) n

(c) n^3 (d) n^n

153. Next move function δ of a Turing machine M = (Q, Σ, Γ, δ, q_0 , B, F) is a mapping

(a) $\delta : Q \times \Sigma \to Q \times \Gamma$

(b) $\delta : Q \times \Gamma \to Q \times \Sigma \times \{L, R\}$

(c) $\delta : Q \times \Sigma \to Q \times \Gamma \times \{L, R\}$

(d) $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

154. Turing machine with transition table is shown below.

Present state	0	1	b
q_0	0 Rq_0	1 Rq_1	bRq_f
q_1	0 Rq_1	1 Rq_0	
q_f			
accept			

In this

(a) set of all even palindromes over {0, 1}

(b) string over {0, 1} containing even number of 1

(c) set of all string with even number of 1 and even number of 0

(d) none of these

155. If L can be recognized by a TM T with a doubly infinite tape, and $\tau_t = f$, then L can be recognized by an ordinary TM with time complexity

(a) O(f) (b) $o(f)$

(c) O(h) (d) $o(h)$

NUMERICAL TYPE QUESTIONS

1. If Σ = {a, b, c, d, e, f}, then number of strings in Σ of length 4 such that no symbol is used more than once in a string is _______

2. An FSM can be used to add _______ number of given integers

3. A PDM behaves like an FSM when the number of auxiliary memory it has, is ______

4. The number of auxiliary memory required for a push down machine (PDM) to behave like a finite state machine (FSM) is _______

5. A Push Down Machine (PDM) behaves like a Turing Machine (TM) when number of auxiliary memory it has, is_________

6. Number of external states of a UTM should be atleast ______

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. For any two languages L_1 and L_2 such that L_1 is context-free and L_2 is recursively enumerable but not recursive, which of the following is/are necessarily true ?

I. $\bar{L}_1$ (complement of L_1) is recursive

II. $\bar{L}_2$ (complement of L_2) is recursive

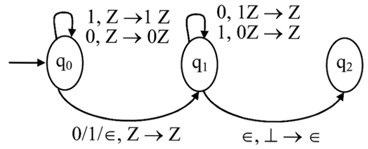
III. $\bar{L}_1$ is context – free

IV. $\bar{L}_1 \cup L_2$ is recursively enumerable

(a) I only (b) III only

(c) III and IV only (d) I and IV only

2. Consider the NPDA ⟨ Q = {q_0, q_1, q_2}, Σ = {0, 1}, Γ = {0, 1, Γ} ⊥δ, q_0, ⊥, F = {q_2}⟩, where (as per usual convention) Q is the set of states, Σ is the input alphabet, δ is the stack alphabet, δ is the state transition function, q_0 is the initial state, ⊥ is the initial stack symbol, and F is the set of accepting states. The state transition is as follows :



Which one of the following sequences must follow the string 101100 so that the overall string is accepted by the automation ?

(a) 10110 (b) 10010

(c) 01010 (d) 01001

3. Consider the following statements

I. The complement of every Turing decidable language is Turing decidable

II. There exists some language which is in NP but is not turing decidable

III. If L is a language in NP, L is turing decidable

Which of the above statements is/are true?

(*a*) Only I

(*b*) Only III

(*c*) Only I and II

(*d*) Only I and III

4. Consider the alphabet $\Sigma = \{0.1\}$, the null/empty string λ and the sets of strings X_0, X_1, and X_2 generated by the corresponding non-terminals of regular grammar. X_0, X_1, and X_2 are related as follows:

$X_0 = 1X_1$

$X_1 = 0X_1 + 1X_2$

$X_2 = 0X_1 + \{\lambda\}$

Which one of the following choices precisely represents the strings in X_0?

(*a*) $10(0^* + (10)^*)1$

(*b*) $10(0^* + (10)^*)^*1$

(*c*) $1(0 + 10)^*1$

(*d*) $10(0 + 10^*)^*1 + 110(0 + 10)^*1$

5. Which of the following languages is/are regular?

L_1 : $\{wxw^R \mid w, x \in \{a, b\}^*$ and $|w|, |x| > 0\}$, w^R is the reverse of string w

L_2 : $\{a^n b^m \mid m \neq n$ and $m, n \geq 0\}$

L_3 : $\{a^p b^q c^r \mid p, q, r \geq 0\}$

(*a*) L_1 and L_3 only (*b*) L_1 only

(*c*) L_2 and L_3 only (*d*) L_3 only

6. Let L be the language represented by the regular expression $\sum^* 0011 \sum^*$ where $\Sigma = \{0,1\}$.

What is the minimum number of states in a DFA that recognizes $\overline{L}$ (complement of L) ?

(*a*) 4 (*b*) 5

(*c*) 6 (*d*) 8

7. Consider the following grammar G

$S \to F \mid H$

$S \to p \mid c$

$S \to d \mid c$

where S, F, and H are non-terminal symbols, p, d, and c are terminal symbols. Which of the following statements(*s*) is/are correct?

S1 : LL(1) can parse all strings that are generated using grammar G

S2 : LR(1) can parse all strings that are generate using grammar G

(*a*) Only S1 (*b*) Only S2

(*c*) Both S1 and S2 (*d*) Neither S1 nor S2

8. Which of the following languages are context-free ?

$$L_1 = \left\{a^m b^n a^n b^m \middle| m, n \geq 1\right\}$$

$$L_2 = \left\{a^m b^n a^m b^n \middle| m, n \geq 1\right\}$$

$$L_3 = \left\{a^m b^n \middle| m = 2n + 1\right\}$$

(*a*) L_1 and L_2 only (*b*) L_1 and L_3 only

(*c*) L_2 and L_3 only (*d*) L_3 only

2014

9. Which one of the following is TRUE?

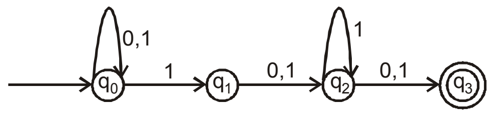
(*a*) The language $L = \{a^n\ b^n \mid n \geq 0\}$ is regular.

(*b*) The language $L = \{a^n \mid n$ is prime$\}$ is regular.

(*c*) The language $L = \{w \mid w$ has $3k + 1$b's for some $k \in \mathbb{N}$ with $\Sigma = \{a, b\}\}$ is regular.

(*d*) The language $L = \{ww \mid w \in \Sigma^*$ with $\Sigma = \{0,1\}\}$ is regular.

10. Consider the finite automaton in the following figure.



What is the set of reachable states for the input string 0011?

(*a*) $\{q_0, q_1, q_2\}$ (*b*) $\{q_0, q_1\}$

(*c*) $\{q_0, q_1, q_2, q_3\}$ (*d*) $\{q_3\}$

11. Let L be a language and $\overline{L}$ be its complement. Which of the following is NOT a viable possibility?

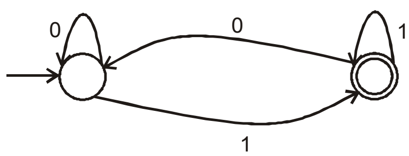
(*a*) Neither L nor $\overline{L}$ is recursively enumerable (r.e.).

(*b*) One of L and $\overline{L}$ is r.e. but not recursive; the other is not r.e.

(*c*) Both L and $\overline{L}$ are r.e. but not recursive.

(*d*) Both L and $\overline{L}$ are recursive.

12. Which of the regular expressions given below represent the following DFA?



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I. 0*1(1 + 00*1)*

II. 0*1*1 + 11*0*1

III. (0 + 1)*1

(*a*) I and II only

(*b*) I and III only

(*c*) II and III only

(*d*) I, II, and III

13. If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, consider

I. $L_1 \cdot L_2$ is a regular language

II. $L_1 \cdot L_2 = \{a^n b^n \mid n \geq 0\}$

Which one of the following is CORRECT?

(*a*) Only (I)

(*b*) Only (II)

(*c*) Both (I) and (II)

(*d*) Neither (I) nor (II)

14. Let A $\leq_m$ B denotes that language A is mapping reducible (also known as many-to-one reducible) to language B. Which one of the following is FALSE?

(*a*) If A $\leq_m$ B and B is recursive then A is recursive

(*b*) If A $\leq_m$ B and A is undecidable then B is undecidable

(*c*) If A $\leq_m$ B and B is recursively enumerable then A is recursively enumerable

(*d*) If A $\leq_m$ B and B is not recursively enumerable then A is not recursively enumerable

15. Consider the grammar defined by the following production rules, with two operators $*$ and +

$S \rightarrow T * P$

$T \rightarrow U \mid T * U$

$P \rightarrow Q + P \mid Q$

$Q \rightarrow Id$

$U \rightarrow Id$

Which one of the following is TRUE?

(*a*) + is left associative, while $*$ is right associative

(*b*) + is right associative, while $*$ is left associative

(*c*) Both + and $*$ are right associative

(*d*) Both + and $*$ are left associative

16. Let $L_1 = (w \in \{0, 1\}^* \mid$ w has least as many occurrences of (110)'s as (011)'s}. Let $L_2 = \{w \in \{0, 1\}^* \mid$ w has at least as many occurrences of (000)'s as (111)'s}. Which one of the following is TRUE?

(*a*) L_1 is regular but not L_2

(*b*) L_2 is regular but not L_1

(*c*) Both L_1 and L_2 are regular

(*d*) Neither L_1 nor L_2 are regular

17. Let Σ be a finite non-empty alphabet and let 2^{Σ^*} be the power set of Σ^*. Which one of the following is **TRUE**?

(*a*) Both 2^{Σ^*} and Σ^* are countable

(*b*) 2^{Σ^*} is countable Σ^* is uncountable

(*c*) 2^{Σ^*} is uncountable and Σ^* is countable

(*d*) Both 2^{Σ^*} and Σ^* are uncountable

18. Which one of the following problems is undecidable?

(*a*) Deciding if a given context-free grammar is ambiguous.

(*b*) Deciding if a given string is generated by a given context-free grammar.

(*c*) Deciding if the language generated by a given context-free grammar is empty.

(*d*) Deciding if the language generated by a given context-free grammar is finite.

19. Consider the following languages over the alphabet $\Sigma = \{0, 1, c\}$:

$L_1 = \{0^n 1^n \mid n \geq 0\}$

$L_2 = \{wcw^r \mid w \in \{0, 1\}^*\}$

$L_3 = \{ww^r \mid w \in \{0, 1\}^*\}$

Here w^r is the reverse of the string w. Which of these languages are deterministic Context-free languages?

(*a*) None of the languages

(*b*) Only L_1

(*c*) Only L_1 and L_2

(*d*) All the three languages

20. Consider the decision problem 2CNFSAT defined as follows :

$\{\phi \mid \phi$ is a satisfiable propositional formula in CNF with at most two literal per clause}

For example, $\phi = (x_1 \vee x_2) \wedge (x_1 \vee \overline{x_3}) \wedge (x_2 \vee x_4)$ is a Boolean formula and it is in 2CNFSAT.

The decision problem 2CNFSAT is

(*a*) NP-Complete.

(*b*) solvable in polynomial time by reduction to directed graph reachability.

(*c*) solvable in constant time since any input instance is satisfiable.

(*d*) NP-hard, but not NP-complete.

2013

21. Which of the following statements is/are **FALSE**?

1. For every non-deterministic Turing machine, there exists an equivalent deterministic Turing machine.
2. Turing recognizable languages are closed under union and complemen-tation.

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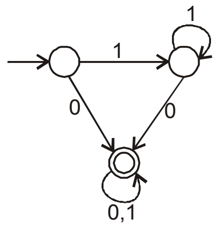
3. Turing decidable languages are closed under intersection and complemen-tation.
4. Turing recognizable languages are closed under union and intersection.

(*a*) 1 and 4 only (*b*) 1 and 3 only
(*c*) 2 only (*d*) 3 only

22. Consider the DFA A given below.



Which of the following are ***FALSE***?

1. Complement of L(A) is context-free.
2. L(A) = L((11 *0+0)(0 + 1)*0*1*)
3. For the language accepted by A, A is the minimal DFA.
4. A accepts all strings over {0, 1} of length at least 2.

(*a*) 1 and 3 only (*b*) 2 and 4 only
(*c*) 2 and 3 only (*d*) 3 and 4 only

23. Which of the following is/are undecidable?

1. G is a CFG. Is L(G) = Φ?
2. G is a CFG. Is L(G) = ∑*?
3. M is a Turing machine. Is L(M) regular?
4. A is a DFA and N is an NFA. Is L(A) = L(N)?

(*a*) 3 only (*b*) 3 and 4 only
(*c*) 1, 2 and 3 only (*d*) 2 and 3 only

2012

24. Given the language $L = \{ab, aa, baa\}$, which of the following strings are in L^*?

1. abaabaaabaa
2. aaaabaaaa
3. baaaaabaaaab
4. baaaaaba

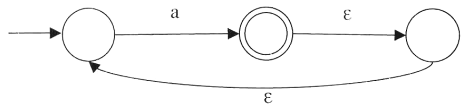
(*a*) 1, 2 and 3 (*b*) 2, 3 and 4
(*c*) 1, 2 and 4 (*d*) 1, 3 and 4

25. Consider a random variable X that takes values +1 and – 1 with probability 0.5 each. The values of the cumulative distribution function F(x) at x = – 1 and + 1 are

(*a*) 0 and 0.5
(*b*) 0 and 1
(*c*) 0.5 and 1
(*d*) 0.25 and 0.75

26. What is the complement of the language accepted by the NFA shown below?

Assume Σ = {a} and ε is the empty string.



(*a*) ∅ (*b*) {ε}
(*c*) a (*d*) {a, ε}

27. Consider the following logical inferences.

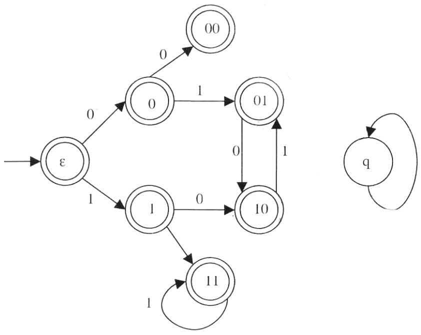
I_1 : If it rains then the cricket match will not be played.
The cricket match was played.
Inference : There was no rain.

I_2 : If it rains then the cricket match will not be played.
It did not rain.
Inference : The cricket match was played.

Which of the following is TRUE?

(*a*) Both I_1 and I_2 are correct inferences
(*b*) I_1 is correct but I_2 is not a correct inference
(*c*) I_1 is not correct but I_2 is a correct inference
(*d*) Both I_1 and I_2 are not correct inferences

28. Consider the set of strings on {0, 1} in which, *every substring of 3 symbols* has at most *two* zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially completed DFA that accept this language is shown below.



The missing arcs in the DFA are

(*a*)

	00	01	10	11	q
00	1	0			
01				1	
10	0				
11			0		

(*b*)

	00	01	10	11	q
00		0			1
01		1			
10				0	
11		0			

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(c)

	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

(d)

	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

2011

29. Let P be a regular language and Q be a context-free language such that Q ⊆ P. (For example, let P be the language represented by the regular expression p^*q^* and Q be $[p^n q^n \mid n \in N]$. Then which of the following is **ALWAYS** regular?

(a) P ∩ Q (b) P – Q

(c) Σ* – P (d) Σ* – Q

30. The lexical analysis for a modern computer language such as Java needs the power of which one of the following machine models in a necessary and sufficient sense?

(a) Finite state automata

(b) Deterministic pushdown automata

(c) Non-deterministic pushdown automata

(d) Turing machine

31. Which of the following pairs have DIFFERENT expressive power?

(a) Deterministic finite automata (DFA) and Non-deterministic finite automata (NFA)

(b) Deterministic push down automata (DPDA) and Non-deterministic push down automata (NPDA)

(c) Deterministic single-tape Turing machine and Non-deterministic single-tape Turing machine

(d) Single-tape Turing machine and multi-tape Turing machine

32. A company needs to develop a strategy for software product development for which it has a choice of two programming languages L1 and L2. The number of lines of code (LOC) developed using L2 is estimated to be twice the LOC developed with L1. The product will have to be maintained for five years. Various parameters for the company are given in the table below.

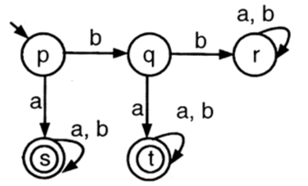
Parameter	Language L1	Language L2
Man years needed for development	LOC/10000	LOC/10000
Development Cost per man year	₹10,00,000	₹7,50,000
Maintenance time	5 years	5 years
Cost of maintenance per year	₹10,00,000	₹50,000

Total cost of the project includes cost of development and maintenance. What is the LOC for L1 for which the cost of the project using L1 is equal to the project using L2?

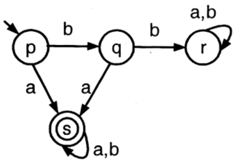
(a) 4000 (b) 5000

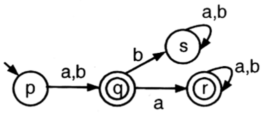
(c) 4333 (d) 4667

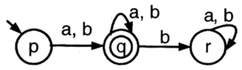
33. A deterministic finite automaton (DFA) D with alphabet Σ = {a, b} is given below.

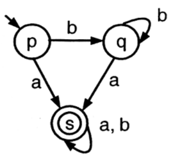


Which of the following finite state machines is a valid minimal DFA which accepts the same language as D?

(a)


(b)


(c)


(d)


34. Definition of a language L with alphabet {a} is given as following.

L = $\{a^{nk} \mid k < 0$, and n is a positive integer constant$\}$

What is the minimum number of states needed in a DFA to recognize L?

(a) $k + 1$ (b) $n + 1$

(c) 2^{n+1} (d) 2^{k+1}

35. Consider the languages L1, L2 and L3 as given below.

L1 = $\{0^p 1^q \mid p, q \in N\}$

L2 = $\{0^p 1^q \mid p, q \in N$ and $p = q\}$ and

L3 = $\{0^p 1^q 0^r \mid p, q, r \in N$ and $p = q = r\}$. Which of the following statements is NOT TRUE?

(a) Push Down Automate (PDA) can be used to recognize L1 and L2

(b) L1 is a regular language

(c) All the three languages are context free

(d) Turing machines can be used to recognize all the languages

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2010

36. Let L1 be a recursive language. Let L2 and L3 be languages that are recursively enumerable but not recursive. Which of the following statements is not necessarily true?

(*a*) L2 – L1 is recursively enumerable

(*b*) L1 – L3 is recursively enumerable

(*c*) L2 $\bigcap$ L3 is recursively enumerable

(*d*) L2 $\bigcup$ L3 is recursively enumerable

37. Let L = $\{w \in (0 + 1)^* \mid w$ has even number of 1s$\}$, i.e., L is the set of all bit strings with even number of 1s. Which one of the regular expressions below represents L?

(*a*) (0*10*1)*

(*b*) 0* (10*10*)*

(*c*) 0* (10*1)*0*

(*d*) 0* 1(10*1)*10*

38. Consider the languages

$$L1 = \{0^i1^j \mid i \neq j\},$$
$$L2 = \{0^i1^j \mid i = j\},$$
$$L3 = \{0^i1^j \mid i = 2\ j + 1\},$$
$$L4 = \{0^i1^j \mid i \neq 2j\}.$$

Which one of the following statements is true?

(*a*) Only L2 is context free

(*b*) Only L2 and L3 are context free

(*c*) Only L1 and L2 are context free

(*d*) All are context free

39. Let w be any string of length n in $\{0, 1\}^*$.

Let L be the set of all substrings of w.

What is the minimum number of states in a non-deterministic finite automaton that accepts L?

(*a*) $n - 1$ (*b*) n

(*c*) $n + 1$ (*d*) 2^{n-1}

2009

40. $S \rightarrow aSa \mid bSb \mid a \mid b$

The language generated by the above grammar over the alphabet {a, b} is the set of

(*a*) all palindromes.

(*b*) all odd length palindromes.

(*c*) strings that begin and end with the same symbol.

(*d*) all even length palindromes.

41. Which one of the following languages over the alphabet {0, 1} is described by the regular expression (0 + 1)*0(0 + 1) * 0(0 + 1)*?

(*a*) The set of all strings containing the substring 00

(*b*) The set of all strings containing at most two 0's.

(*c*) The set of all strings containing at least two 0's.

(*d*) The set of all strings that begin and end with either 0 or 1.

42. Which one of the following is FALSE ?

(*a*) There is a unique minimal DFA for every regular language.

(*b*) Every NFA can be converted to an equivalent PDA.

(*c*) Complement of every context-free language is recursive.

(*d*) Every nondeterministic PDA can be converted to an equivalent deterministic PDA.

43. Match all items in Group 1 with correct options from those given in Group 2.

Group 1	Group 2
P. Regular expression	**1.** Syntax analysis
Q. Pushdown automata	**2.** Code generation
R. Dataflow analysis	**3.** Lexical analysis
S. Register allocation	**4.** Code Optimization

Codes :

(*a*) P -4, Q -1, R -2, S -3

(*b*) P -3, Q -1, R -4, S -2

(*c*) P -3, Q -4, R -1, S -2

(*d*) P -2, Q -1, R -4, S -3

44. Given the following state table of an FSM with two states A and B, one input and one output:

Present State A	Present State B	Input State C	Next State A	Next State B	Output
0	0	0	0	0	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	1	0	0
0	0	1	0	1	0
0	1	1	0	0	1
1	0	1	0	1	1
1	1	1	0	0	1

If the initial state is A = 0, B = 0, what is the minimum length of an input string which will take the machine to the state A= 0, B = 1 with Output = 1?

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

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45. Let $= L_1 \cap L_2$, where L_1 and L_2 are languages as defined below:

$L_1 = \{a^m b^m c\, a^n b^n \mid m, n^3 \geq 0\}$

$L_2 = \{a^i b^i c^k \mid i, j, k \geq 0\}$

The L is

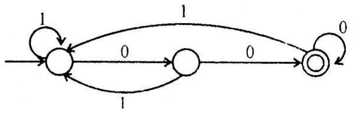
(*a*) not recursive

(*b*) regular

(*c*) context-free but not regular

(*d*) recursively enumerable but not context-free.

46. In the following figure, DFA accepts the set of all strings over $\{0,1\}$ that



(*a*) begin either with 0 or 1.

(*b*) end with 0.

(*c*) end with 00.

(*d*) contain the substring 00.

2008

47. Which of the following is true for the language $\{a^p \mid p \text{ is a prime}\}$?

(*a*) It is not accepted by a Turing Machine

(*b*) It is regular but not context-free

(*c*) It is context-free but not regular

(*d*) It is neither regular nor context-free, but accepted by a Turing machine

48. Which of the following are decidable?

I. Whether the intersection of two regular languages is infinite

II. Whether a given context-free language is regular

III. Whether two push-down automata accept the same language

IV. Whether a given grammar is context-free

(*a*) I and II

(*b*) I and IV

(*c*) II and III

(*d*) II and IV

49. If L and $\overline{L}$ are recursively enumerable, then L is

(*a*) regular

(*b*) context-free

(*c*) context-sensitive

(*d*) recursive

50. Which of the following statements is false?

(*a*) Every NFA can be convrted to an equivalent DFA

(*b*) Every non-deterministic Turing machine can be converted to an equivalent deterministic Turing machine

(*c*) Every regular language is also a context-free language

(*d*) Every subset of a recursively enumerable set is recursive

51. Given below are two finite state automata (→ indicates the start state and F indicates a final state)

Y:

	a	*b*
→ 1	1	2
2(F)	2	1

Z:

	a	*b*
→ 1	2	2
2(*f*)	1	1

Which of the following represents the product automaton $Z \times Y$?

(*a*)

	a	*b*
→ P	S	R
Q	R	S
R(F)	Q	P
S	Q	P

(*b*)

	a	*b*
→ P	S	Q
Q	R	S
R(F)	Q	P
S	P	Q

(*c*)

	a	*b*
→ P	Q	S
Q	R	S
R(F)	Q	P
S	Q	P

(*d*)

	a	*b*
→ P	S	Q
Q	S	R
R(F)	Q	P
A	Q	P

52. Which of the following statements are true?

I. Every left-recursive grammar can be converted to a right-recursive grammar and vice-versa

II. All ε-productions can be removed from any context-free grammar by suitable transformations

III. The language generated by a context-free grammar all of whose productions are of the form $X \to w$ or $X \to wY$ (where, w is a string of terminals and Y is a non-terminal), is always regular

IV. The derivation trees of strings generated by a context-free grammar in Chomsky Normal From are always binary trees

(*a*) I, II, III and IV

(*b*) II, III and IV only

(*c*) I, III and IV only

(*d*) I, II and IV only

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53. Match the following

List - I

E. Checking that identifiers are declared before their use

F. Number of formal para-meters in the declaration of a function agrees with the number of actual parameters in a use of that function

G. Arithmetic expressions with matched pairs of

H. Palindromes

List - II

P. $L = \{a^n b^m c^n d^m \mid n \geq 1, m \geq 1\}$

Q. $X \to X\,b\,X \mid X\,c\,X \mid d\,X\,f \mid g$

R. $L = \{wcw \mid w \in (a \mid b)^*\}$

S. $X \to b\,X\,b \mid c\,X\,c \mid \varepsilon$

Codes :

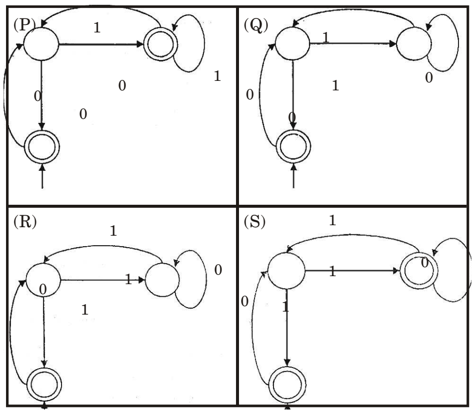
(*a*) E – P, F – R, G – Q, H – S

(*b*) E – R, F – P, G – S, H – Q

(*c*) E – R, F – P, G – Q, H – S

(*d*) E – P, F – R, G – S, H – Q

54. Match the following NFAs with the regular expressions they correspond to.



1. $\varepsilon + 0(01^*1 + 00)^*01^*$
2. $\varepsilon + 0(10^*1 + 00)^*0$
3. $\varepsilon + 0(10^*1 + 10)^*1$
4. $\varepsilon + 0(10^*1 + 10)^*10^*$

Codes :

(*a*) P – 2, Q – 1, R – 3, S – 4

(*b*) P – 1, Q – 3, R – 2, S – 4

(*c*) P – 1, Q – 2, R – 3, S – 4

(*d*) P – 3, Q – 2, R – 1, S – 4

55. Which of the following are regular sets?

I. $\{a^n b^{2m} \mid n \geq 0, m \geq 0\}$

II. $\{a^n b^m \mid n = 2m\}$

III. $\{a^n b^m \mid n \neq m\}$

IV. $\{xcy \mid x, y \in \{a, b\}^*\}$

(*a*) I and IV only

(*b*) I and III only

(*c*) I only

(*d*) IV only

2007

56. Which of the following problems is undecidable ?

(*a*) Membership problem for CFGs.

(*b*) Ambiguity problem for CFGs.

(*c*) Finiteness problem for FSAs

(*d*) Equivalence problem for FSAs

57. Which of the following is **TRUE**?

(*a*) Every subset of a regular set is regular

(*b*) Every finite subset of a non-regular set is regular

(*c*) The union of two non-regular sets is not regular

(*d*) Infinite union of finite sets is regular

58. A minimum state deterministic finite automaton accepting the language

$L = \{w \mid w \in \{0,1\}^*$, number of 0s and 1s in w are divisible by 3 and 5, respectively$\}$ has

(*a*) 15 states (*b*) 11 states

(*c*) 10 states (*d*) 9 states

59. The language $L = \{0^i 21^i \mid i \geq 0\}$ over the alphabet $\{0, 1, 2\}$ is

(*a*) not recursive

(*b*) is recursive and is a deterministic CFL

(*c*) is a regular language

(*d*) is not a deterministic CFL but a CFL

60. Which of the following languages is regular ?

(*a*) $\{ww^R \mid w \in \{0,1\}^+\}$

(*b*) $\{ww^R x \mid x, w \in \{0,1\}^+\}$

(*c*) $\{wxw^R \mid x, w \in \{0,1\}^+\}$

(*d*) $\{xww^R \mid x, w \in \{0,1\}^+\}$

2006

61. Let S be an NP-complete problem Q and R be two other problems not known to be in NP. Q is polynomial-time reducible to S and S is polynomial-time reducible to R.

Which of the following statements is true ?

(*a*) R is NP-complete

(*b*) R is NP-hard

(*c*) Q is NP-complete

(*d*) Q is NP-hard

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62. Let $L_1 = \{0^{n+m} 1^n 0^m \mid n, m \geq 0\}$,

$L_2 = \{0^{n+m} 1^{n+m} 0^m \mid n, m \geq 0\}$, and

$L_3 = \{0^{n+m} 1^{n+m} 0^{n+m} \mid n, m \geq 0\}$.

Which of these languages are NOT context free?

(*a*) L_1 only

(*b*) L_3 only

(*c*) L_1 and L_2

(*d*) L_2 and L_3

63. If s is a string over $(0 + 1)^*$, then let $n_0(s)$ denote the number of 0's in s and $n_1(s)$ the number of 1's in s. Which one of the following languages is not regular ?

(*a*) $L = \{s \in (0+1)^* \mid n_0(s) \text{ is a 3-digit prime}\}$

(*b*) $L = \{s \in (0+1)^* \mid \text{for every prefix } s' \text{ of } s, \; |n_0(s') - n_1(s')| \leq 2\}$

(*c*) $L = \{s \in (0+1)^* \mid |n_0(s) - n_1(s)| \leq 4\}$

(*d*) $L = \{s \in (0+1)^* \mid n_0(s) \bmod 7 = n_1(s) \bmod 5 = 0\}$

64. For $s \in (0+1)^*$ let $d(s)$ denote the decimal value of s (e.g. $d(101) = 5$).

Let $L = \{s \in (0+1)^* \mid d(s) \bmod 5 = 2 \text{ and } d(s) \bmod 7 \neq 4\}$

Which one of the following statements is true ?

(*a*) L is recursively enumerable, but not recursive

(*b*) L is recursive, but not context-free

(*c*) L is context-free, but not regular

(*d*) L is regular

65. Consider the following statements about the context-free grammar :

$G = \{S \to SS, S \to ab, S \to ba, S \to \in\}$

I. G is ambiguous

II. G produces all strings with equal number of *a's* and *b's*

III. G can be accepted by a deterministic PDA.

Which combination below expresses all the true statements about G ?

(*a*) I only

(*b*) I and III only

(*c*) II and III only

(*d*) I, II, and III

66. Let L_1 be regular language, L_2 be a deterministic context-free language and L_3 a recursively enumerable, but not recursive, language. Which one of the following statements is false ?

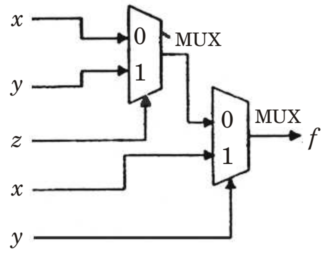
(*a*) $L_1 \cap L_2$ is a deterministic CFL

(*b*) $L_3 \cap L_1$ is recursive

(*c*) $L_1 \cup L_2$ is context free

(*d*) $L_1 \cap L_2 \cap L_3$ is recursively enumerable

67. Consider the circuit shown below.



Which one of the following options correctly represents $f(x, y, z)$?

(*a*) $x\bar{z} + xy + \bar{y}z$ (*b*) $x\bar{z} + xy + \overline{yz}$

(*c*) $xz + xy + \overline{yz}$ (*d*) $xz + x\bar{y} + \bar{y}z$

68. Consider the regular language $L = (111 + 11111)^*$. The minimum number of states in any DFA accepting this languages is

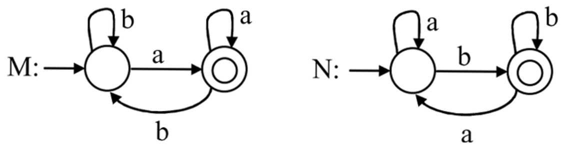
(*a*) 3 (*b*) 5

(*c*) 8 (*d*) 9

NUMERICAL TYPE QUESTIONS

2015

1.



Consider the DFAs M and N given above. The number of states in a minimal DFA that accepts the languages $L(M) \cap L(N)$ is ________.

2. The number of states in the minimal deterministic finite automaton corresponding to the regular expression $(0 + 1)^* (10)$ is __________

2014

3. The length of the shortest string NOT in the language (over $\Sigma = \{a,b\}$) of the following regular expression is _________.

$a^*b^*(ba)^*a^*$

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ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*b*)	**2.** (*d*)	**3.** (*c*)	**4.** (*c*)	**5.** (*a*)	**6.** (*b*)	**7.** (*a*)	**8.** (*d*)	**9.** (*d*)	**10.** (*b*)
11. (*c*)	**12.** (*c*)	**13.** (*d*)	**14.** (*b*)	**15.** (*c*)	**16.** (*d*)	**17.** (*c*)	**18.** (*d*)	**19.** (*c*)	**20.** (*d*)
21. (*b*)	**22.** (*a*)	**23.** (*b*)	**24.** (*c*)	**25.** (*b*)	**26.** (*d*)	**27.** (*a*)	**28.** (*b*)	**29.** (*c, d*)	**30.** (*c*)
31. (*d*)	**32.** (*b*)	**33.** (*b*)	**34.** (*b*)	**35.** (*d*)	**36.** (*d*)	**37.** (*b*)	**38.** (*d*)	**39.** (*c*)	**40.** (*d*)
41. (*c*)	**42.** (*d*)	**43.** (*c*)	**44.** (*d*)	**45.** (*d*)	**46.** (*c*)	**47.** (*c*)	**48.** (*d*)	**49.** (*c*)	**50.** (*c*)
51. (*c*)	**52.** (*b*)	**53.** (*d*)	**54.** (*a*)	**55.** (*a*)	**56.** (*c*)	**57.** (*a*)	**58.** (*b*)	**59.** (*a*)	**60.** (*c*)
61. (*c*)	**62.** (*c*)	**63.** (*d*)	**64.** (*a*)	**65.** (*b*)	**66.** (*b*)	**67.** (*c*)	**68.** (*b*)	**69.** (*c*)	**70.** (*d*)
71. (*d*)	**72.** (*b*)	**73.** (*c*)	**74.** (*b*)	**75.** (*b*)	**76.** (*a*)	**77.** (*c*)	**78.** (*d*)	**79.** (*c*)	**80.** (*a*)
81. (*c*)	**82.** (*a*)	**83.** (*d*)	**84.** (*d*)	**85.** (*d*)	**86.** (*a*)	**87.** (*a*)	**88.** (*a*)	**89.** (*b*)	**90.** (*a*)
91. (*a*)	**92.** (*b*)	**93.** (*c*)	**94.** (*d*)	**95.** (*b*)	**96.** (*c*)	**97.** (*b*)	**98.** (*b*)	**99.** (*b*)	**100.** (*d*)
101. (*d*)	**102.** (*a*)	**103.** (*a*)	**104.** (*d*)	**105.** (*b*)	**106.** (*c*)	**107.** (*a*)	**108.** (*d*)	**109.** (*d*)	**110.** (*b*)
111. (*d*)	**112.** (*d*)	**113.** (*c*)	**114.** (*d*)	**115.** (*c*)	**116.** (*b*)	**117.** (*c*)	**118.** (*a*)	**119.** (*d*)	**120.** (*a*)
121. (*a,d*)	**122.** (*c*)	**123.** (*a*)	**124.** (*d*)	**125.** (*a*)	**126.** (*c*)	**127.** (*a*)	**128.** (*c*)	**129.** (*a*)	**130.** (*c*)
131. (*c*)	**132.** (*a*)	**133.** (*c*)	**134.** (*c*)	**135.** (*a*)	**136.** (*c*)	**137.** (*a*)	**138.** (*a*)	**139.** (*d*)	**140.** (*c*)
141. (*a*)	**142.** (*a*)	**143.** (*d*)	**144.**(*c*)	**145.** (*b*)	**146.** (*d*)	**147.** (*d*)	**148.** (*a*)	**149.** (*c*)	**150.** (*b*)
151. (*d*)	**152.** (*b*)	**153.** (*d*)	**154.** (*b*)	**155.** (*a*)					

Numerical Type Questions

1. 360	**2.** 3	**3.** 0	**4.** 0	**5.** 2	**6.** 2

EXERCISE – II

MCQ Type Questions

1. (*d*)	**2.** (*b*)	**3.** (*d*)	**4.** (*c*)	**5.** (*a*)	**6.** (*b*)	**7.** (*d*)	**8.** (*b*)	**9.** (*c*)	**10.** (*a*)
11. (*c*)	**12.** (*b*)	**13.** (*a*)	**14.** (*d*)	**15.** (*b*)	**16.** (*a*)	**17.** (*c*)	**18.** (*a*)	**19.** (*c*)	**20.** (*b*)
21. (*a*)	**22.** (*d*)	**23.** (*d*)	**24.** (*c*)	**25.** (*c*)	**26.** (*b*)	**27.** (*b*)	**28.** (*d*)	**29.** (*c*)	**30.** (*a*)
31. (*b*)	**32.** (*b*)	**33.** (*a*)	**34.** (*b*)	**35.** (*c*)	**36.** (*b*)	**37.** (*b*)	**38.** (*d*)	**39.** (*c*)	**40.** (*b*)
41. (*c*)	**42.** (*d*)	**43.** (*b*)	**44.** (*a*)	**45.** (*c*)	**46.** (*c*)	**47.** (*b*)	**48.** (*c*)	**49.** (*c*)	**50.** (*d*)
51. (*b*)	**52.** (*c*)	**53.** (*a*)	**54.** (*b*)	**55.** (*c*)	**56.** (*b*)	**57.** (*b*)	**58.** (*a*)	**59.** (*b*)	**60.** (*c*)
61. (*c*)	**62.** (*d*)	**63.** (*b*)	**64.** (*b*)	**65.** (*d*)	**66.** (*d*)	**67.** (*a*)	**68.** (*d*)		

Numerical Type Questions

1. (1)	**2.** (3)	**3.** (3)

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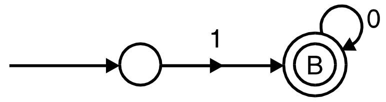
EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

27. 

Deterministic Automata of the series

45. Strings of odd number of zeroes can be generated by the regular expression (00) *0.

Pumping lemma can be used to prove the non-regularity of the other options.

46. Two regular expressions R1 and R2 are equivalent if any string that can be generated by can be generated by R2 and *vice-versa*.

In option (*c*), (*ab*) * will generate *abab*, which is not the form a *nbn* (because *a*'s and *b*'s should come together). All other options are correct.

47. Here final state and the start state are one and the same. No transition is there. But by definition, there is an (implicit) ε-transition from any state to itself. So, the only string that could be accepted is ε.

48. In the second diagram, the final state is unreachable from the start state. So, not even ε could be accepted.

49. Let 011011 be the input to the FSM and let it be fed from the right (*i.e.* least significant digit first). If we add 1 to 011011, we should get 011100.

Whenever we add 1 to a 1, we make it 0 and carry 1 to the next stage (State) and repeat the process. If we add 1 to a 0, then first make it 1 and all the more significant digits will remain the same, *i.e.*, a 0 will be 0 and a 1 will be 1.

The given machine does this.

Hence the answer is (*c*).

103. II generates strings like *xxyyy*, which are not supposed to be.

III generates strings like *xyy*, which are not supposed to be.

I can be verified to generate all the strings in L and only those.

120. $S \rightarrow aB \rightarrow aaBB \rightarrow aabB \rightarrow aabb$

So (*b*) is wrong.

We have

$$S \rightarrow aB \rightarrow ab$$

So (*c*) is wrong.

A careful observation of the productions will reveal a similarity. Change A to B, B to A, *a* to *b* and *b* to *a*. The new set of productions will be the same as the original set. So (*d*) is false and (*a*) is the correct answer.

121. Option (*b*) is wrong because it can't generate *aabb* (in fact any even power).

Option (*c*) is wrong since it generates ε also.

Both options (*a*) and (*d*) are correct.

152. Recursively applying the relation, we get

$$\begin{aligned} T(n+1) &= C(n-1) + T(1) \\ &= C(n-1) + d \end{aligned}$$

Hence order is *n*.

EXERCISE – II

MCQ TYPE QUESTIONS

1. By the property.

If L_1 is context free, $\bar{L}_1$ is recursive.

If L_2 is recursively enumerable and $\bar{L}_1$ is recursive then union of recursive and recursively enumerable language is recursively enumerable.

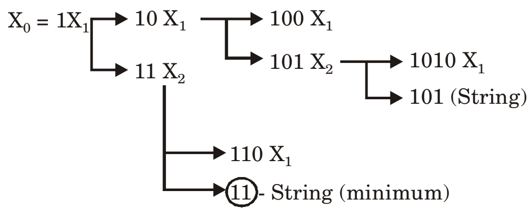
II and III are false because complement of a context free language is not context free and complement of a recursively enumerable language is not recursive.

2. The overall string accepted by automata is 10010

3. By property,

Complement of every Turing decidable language is also turing decidable

If L is in NP, L is Turing decidable.

4. 

All string are starting with 1 with min string 11. which can be generated by 1(0* + 10)*1.

5. $L_1 = \left\{W \times W^R \,\middle|\, W, x \in (a,b)^*, |W|, |X| > 0\right\}$

W^R is reverse of string of W

It can not be regular, because we can not compare W is regular expression.

$L_2 = \left\{a^n b^m \,\middle|\, m \neq n, \text{and } m, n \geq 0\right\}$ is also not regular due to comparision in *a* and *b*

$L_3 = \left\{a^p b^q c^r \,\middle|\, p, q, r \geq 0\right\}$

Here *a*, *b*, *c* are independent and regular exp is $a^*b^*c^*$. Hence L_3 is regular.

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8. $a^m b^n a^n b^m \Rightarrow$ This one is CFL

$a^m b^n a^m b^n \Rightarrow$ by pumping lemma this one is not CFL.

$\{a^m b^n \mid m = 2n + 1\}$ This is CFL.

9. (*a*) L = $\{a^n b^n \mid n \geq 0\}$ is a CFL but not regular, it requires memory for the representation.

(*b*) L = $\{a^n \mid n \text{ is prime}\}$ is neither regular nor CFL

(*c*) L = $\{w \mid w \text{ has } 3K + 1 \, b\text{'s for some } k \in \text{N with}\}$

$\Sigma = \{a, b\}$

is a regular language, since the total count of b's are multiple of 3 plus one. The regular expression is $a * ba *(a *ba *ba * ba *)*$

$+ (a * ba * ba * ba *)*a * ba *$

(*d*) L = $\{ww \mid w \in \Sigma^* \text{with } \Sigma = \{0,1\}\}$ is neither regular nor CFL

10.

$$\begin{aligned} \delta(q_0, 0011) &= \delta(q_0, 011) \\ &= \delta(q_0, 11) \\ &= \delta(\{q_0, q_1\}, 1) \\ &= \delta(q_0, 1) \cup \delta(q_1, 1) \\ &= \{q_0, q_1\} \cup \{q_2\} \\ &= \{q_0, q_1, q_2\} \end{aligned}$$

11. Recursive languages are closed under complement. If a language L is recursive enumerable but not recursive then its complement is not a recursive enumerable, so both L and $\overline{\text{L}}$ are recursive enumerable but not recursive is not a viable possibility.

12. Given DFA will accept all the strings over ε ={0,1} which are ending with 1.

0*1(1+ 00*1)* and(0 +1)*1, are the regular expressions for ending with 1.

13. $L_1 . L_2$ will be regular language, concatenation of two regular language is language.

It will be like $L_1 . L_2 \neq \{a^n b^n \mid n \geq 0\}$

It will be like $L_1 . L_2 = \{a^n b^m \mid n, m \geq 0\}$

That's why only (I) will satisfy.

14. Since A $\leq_m$ B and since B is not enumerable recursively creates a situation for A to not to be recursively enumerable.

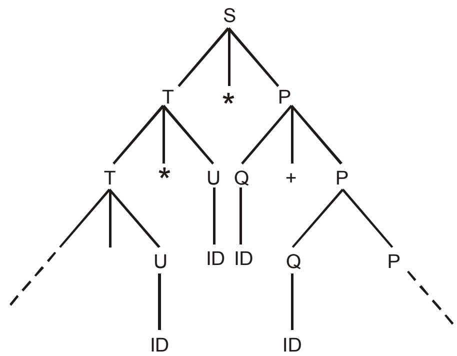
15. S → T * P

T → U/T * U

P → Q + P/Q

Q → ID

U → ID



16. L_1 is regular because we can design FA for L_1 but we can not design FA for L_2. Hence L_1 is regular but not L_2.

17. 2^{ε^*} is the power set of ε*

ε* is countabily infinite

The power set of countabily infinite set is uncountable.

So 2^{ε^*} is uncountable, and ε* is countable.

18. There were algorithms to find the membership of CFG (using CYK algorithm) and finiteness of CFG (using CNF graph) and emptiness. But there is no algorithm for ambiguity of CFG, so it is undecidable.

19. For the languages L_1 and L_2 we can have deterministic push down automata, so they are DCFL's, but for L_3 only non-deterministic PDA possible. So the language L_3 is not a deterministic CFL.

20. 2 SAT is in P. This we can prove by reducing 2 SAT to directed graph reachability problem which is known to be in P.

<u>Procedure for reducing 2 SAT to reachability problem:</u>

1. Let φ be CNF with clauses of length 2 and let P be the set of propositional variables(literals) in φ
2. Build a graph G = (V,E) with V= P∪{¬p | p∈P} and $(x, y) \in$ E iff there is a clause in φ that is equivalent to $x \rightarrow y$ (all the clauses are converted to equivalent implications and the graph built is called as implication graph)
3. Observe that φ is unsatisfiable iff there is a p∈P such that there is both a path from p to ¬p and from ¬p to p in G.

   This condition can be tested by running the reachability algorithm several times.

21. 1. TRUE. Non deterministic turing machine can be simulated by a Deterministic turing machine with exponential time.

2. Turing Recognizable languages are "not" closed under complementation. For any Toring recognizable language, the toring machine 'T' recognizing 'L', may not terminate on inputs $x \notin$ L. Hence this is FALSE

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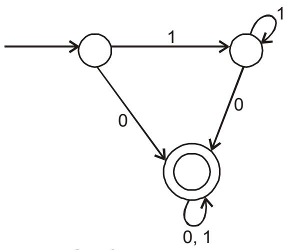
3. Toring Decidable languages are CLOSED under unioin and complementation. It is easy to construct a turing machine to decide $L_1 \cup L_2$, L_1' if L_1, L_2 are turing decidable.

   So, this is TRUE.

   (*c*) is the only option which contains (2).

22. **Given:** DFA :



Q: Which are False?

1. Complement of L(A) is context free – <u>True</u>

   – As there exist a DFA, L(A) is regular. Regular languages are closed under complemer,

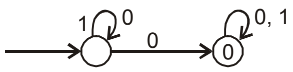
   ∴ Complement of L(A) is regular and therefore context free.

2. $L(A) = L((11^* 0 + 0)(0 + 1)^* 0^* 1^*)$ – True

   – As $L(A) = L((11^* 0 + 0)(0 + 1)^*)$ is equivalent to $L((11^*0 + 0)(0 + 1)^* 0^* 1^*)$

3. For the language accepted by A, A is minimal DFA – <u>FALSE</u>

   – Minimal DFA is



4. A accepts all strings over {0, 1} of length at least 2 – <u>FALSE</u>

   – The given DFA A accepts the string "0" which is of length less than

Ans. (*d*) 3/4 only FALSE.

23. Which are/is undecidable

1. G is CFG. Is $L(G) = \phi$?

   – Decidable

2. G is a CFG. Is $L(G) = \Sigma^*$?

   – Undecidable

3. M is a turing machine. Is L(M) regular?

   – Undecidable (Present in all options)

4. A is a DFA and N is a NFA.

   Is L(A) = L(N)?

   – Decidable

Ans. (*d*) 2 and 3 only.

PS. Refer closere properties (wiki)

24. $L = \{ab, aa, baa\}$

Let $S1 = ab$, $S2 = aa$

and $S3 = baa$

abaabaaabaa can be written as S1 S2 S3 S1 S2

aaaabaaaa can be written as S1 S1 S3 S1

baaaabaa can be written as S3 S2 S1 S2

25. The cumulative distribution function

$$F(x) = P(X \leq x)$$
$$F(-1) = P(X \leq -1)$$
$$= P(X = -1) = 0.5$$
$$F(+1) = P(X \leq +1)$$
$$= P(X = -1) + P(X = +1)$$
$$= 0.5 + 0.5 = 1$$

26. Language accepted by NFA is a^+, so complement of this language is $\{\varepsilon\}$

27. $I_1 : R \to \sim C \quad = \sim RV \sim C$

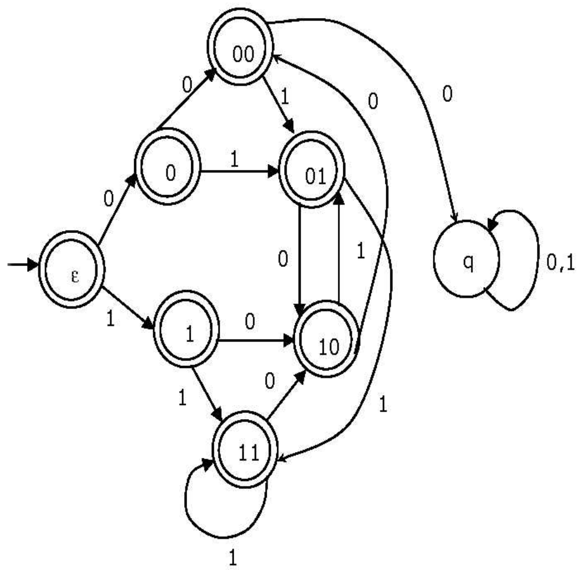
$$\frac{\begin{array}{c} C \\ \hline -R \end{array}}{}$$ (there was no rain)

$I_2 : R \to C \quad = \sim Rv \sim C$

$\sim R \qquad \dfrac{\sim R}{-RvC}$

(I_1 is correct and I_2 is not correct inference)

28. The complete DFA is



29. $\Sigma^* - P$ is the complement of P so it is always regular,

Since regular languages are closed under complementation

30. Lexical Analysis is implemented by finite automata

31. NPDA is more powerful than DPDA.

Hence answer is (*b*)

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32. LOC L1 = x

L2 = $2x$

Total cost of project

$$= \frac{x}{10000} \times 1000000 + 5 \times 100000$$

$$= \frac{2x}{10000} \times 750000 + 50000 \times 5$$

$100x + 500000 = 150x + 250000$

$\Rightarrow \quad 50\,x = 500000 - 250000$

$\therefore \quad x = \frac{250000}{50} = 5000$

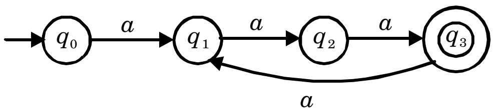
33. Options (b) and (c) will accept the string b

Option (d) will accept the string "bba"

Both are invalid strings.

So minimized DFA is option a

34. Let n = 3 and k = 1



35. L1 : regular language

L2 : context free language

L3 : context sensitive language

36. L1 is recursive and L2, L3 are recursively enumerable. So L1$\cap$L3 and L2$\cup$L3 can recursively enumerable and also L2 – L1 can recursively enumerable. But we can't say that **L1-L3** is also **recursively enumerable.**

37. Every string should contain even number of 1's min string are

$\in$, 0, 11, 101, 1010, 0101, 0110, ...

We can't get it by

0* (10 * 1) 0* and 0 * 1 (10 * 1)* 10*

because these don't give $\in$ and 0 string.

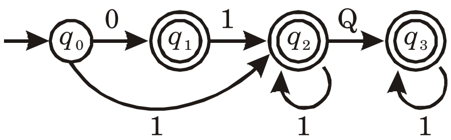
38. Since L1, L2, L3, L4 can be recognised by push down automata, so these all are context-free grammer.

39. w be any string of length n in {0, 1} * and L is substring of L

means if **101** is the w

Then L = 01, 10, 01, 11

Then

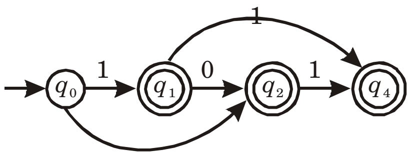


Then it should be n + 1, where n is length of the string.

Alternately

Let w = 101

$\therefore$ sub strings = 1, 0, 10, 11, 01, 101



It requires 4 states.

If a string is of 'n' length, it requires n + 1 states

40. S – > aSa | bSb | a | b

will generate all odd length palindromes

e.g. S = > aSa = > abSba = > abbSbba = > abbabba

41. R.E. = (0+1)*0(0+1)*0(0+1)*

Accepting Language will be

L = {00, 000, 100, 001, 010, 0000, 0001, 1000, 1001, 0100, 1100, 0010, 0011, 0110, 0101, 1010, ...}

atleast two zeros.

42. (a) There is a unique minimal DFA for every regular language. TRUE

(b) Every NFA can be converted to an equivalent PDA. TRUE

(c) Complement of every context-free language is recursive. TRUE

Because context-free language is not closed under Complementation.

(d) Every nondeterministic PDA can be converted to an equivalent deterministic PDA. FALSE

Because ww^R can not be accepted by deterministic PDA but nondeterministic PDA can accept it. So there does not exist any equivalent deterministic PDA.

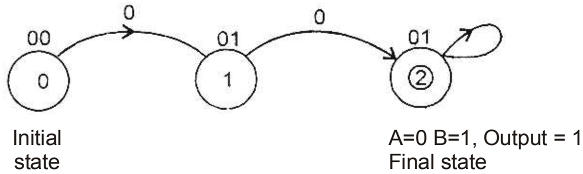
43. Regular expression denote structure of data specially text string-lexical analyser break input text into logical unit such as identifiers.

Push down automater- Syntax analysis. Study with the help of syntax.

Data flow analysis – Code optimization

Register allocation – Lexical analysis

44.



Hence 3 stakes required

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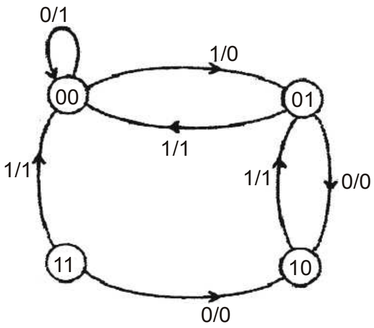
Alternately



Fom state 00 to 01 with output 1 required minimum 3 length of string i.e. 101.

45. Intersection of two regular language is regular

Given : $L_1 = \{a^m b^m c a^n b^n \mid m, n \geq 0\}$

If $\begin{matrix} n=1 \\ m=1 \end{matrix}$ { a, b, c, a, b} not regular

If n/1 $a^m, b^m c a^n b^n$ is regular

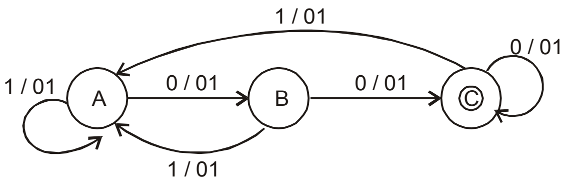
46. Let A be the starting state

A to A input 1 0 output 0 0

B to A input 1 0 output 0 0

C to A input 1,0 output 0 0

so end with 0 0



Alternately

R.E. = (0 + 1)* 00

47. L = $a^P \mid$ p is prime.

L is not be regular, can be proved by pumping Lemma, so it cannot be context free language.

48. Intersection of two regular languages can be determined by a given algorithm, it can be determined whether a given grammer is context free.

Context free grammar should have all productions of the type

$$A \rightarrow x$$

where, $A \in V_N$ {set of varibale}

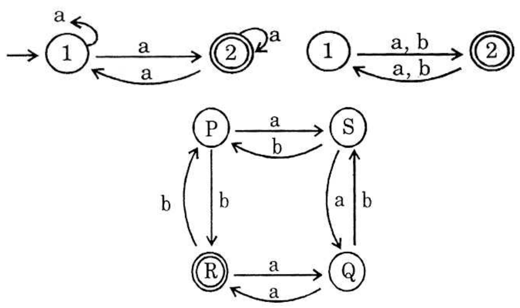
$\alpha \in (V_N Uz)^*$ {z = set of terminal}

49. If a language is recursively enumerable, and its complement is also, then the language is recursive.

50. I, II and III are true

Instead true statement is, "every recursive set is recursively enumerable".

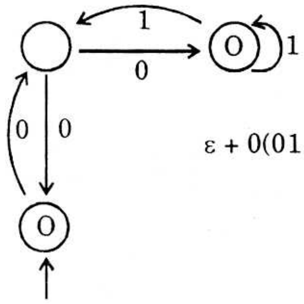
51.



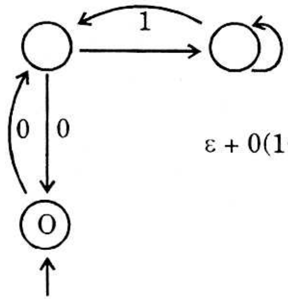
52. Statement III is false. In a context free grammar, right side of the productions should either have single terminal or one non-terminal followed by terminal or λ .

53. Here X $\rightarrow$ b x b$\mid$ c x c$\mid$ ε surely repesent palindrome and x $\rightarrow$ x b x $\mid$ $\mid$ x c x $\mid$ d x f $\mid$ g Arithmetic expressions with match pair are simple & L = {wcw/wε (a/b)*}checking that identifier are declared before their use.

54.



$\varepsilon + 0(01 * 1 + 00) * 01 *$



$\varepsilon + 0(10 * 1 + 00) * 0$

Alternately

Converting NFA into regular expression.

P says ε moves or 0, followed by zero and 1*1 or 00 any number of times followed by zero, again followed by 1 any times.

$$P \rightarrow \varepsilon + 0\,(01 * 1 + 0\,0)^* \,01^*$$

Similarly for Q, R & S.

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55. Only $\{a^n b^{2m} \mid n \geq 0, m \geq 0\}$

& $\{x_{cy}$, x, y ε (a, b)*} are regular sets.

Alternately

In a regular language, counting of numbers is not possible but counting that a input is repeated even times or odd times is possible; to get the number (even / odd say 4/7) is not possible.

I can be written as a*(bb)*

IV can be (a+b)* c (a+b)*

while for other regular expressions is not possible as they need the counting as number of times.

56. As we know that Ambiguity problem for CFGs is undecidable.

57. Every finite subset of a non – regular set is regular.

58. The minimum state deterministic finite automata accepting specified language will have 15 states.

59. Language $L = \{ 0^i 21^i \mid i \geq 0\}$

over alphabet { 0, 1, 2} is recursive and is a deterministic CFL (every CFL is recursive)

60. language $L = \{wxw^R \mid x_1 w \in \{0, 1\}^+\}$ is a regular language.

61. **Note :** If $P_1 \leq_p P_2$ then P_1 is Np-complete $\Rightarrow P_2$ is Np-complete.
Given, S is Np-complete.
$Q \leq_p S$ and $S \leq_p R$
Since $S \leq_p R$
S is Np-complete ⇒ R is Np-complete
∴ (*a*) is true.

62. In L_1, L_2 and L_3, only L_1 can be written as a context free language, the production for L_1 are

$$S \to 0S0 \mid 0A1 \mid \in$$
$$A \to 0A1 \mid \in$$

to generate $0^{n+m} 1^n 0^m$

i. e. $0^m 0^n 1^n 0^m$

first we apply, $S \to 0S0$, m times

So $S \xrightarrow{*} 0^m S 0^m$

Now we apply, $S \longrightarrow 0A1$

$$S \xrightarrow{*} 0^m 0 A 1 0^m$$

Now apply, $A \to 0A1$, $(n-1)$ times

$$S \xrightarrow{*} 0^m 0 0^{n-1} A 1^{n-1} 10^m$$

i.e. $S \xrightarrow{*} 0^m 0^n A 1^n 0^m$

Now $A \longrightarrow \in$

$$S \xrightarrow{*} 0^m 0^n 1^n 0^m$$

If $m = 0$ and $n \neq 0$
start from $S \to 0A1$
If $m \neq 0$ and $n = 0$
start from $S \to 0S0$ and never use
$S \to 0A1$
If $m = n = 0$
use $S \to \in$

Hence G_1 can be written as a context free grammar and

$$L_1(G_1) = L_1 \{0^{n+m} 1^n 0^m \mid n, m \geq 0\}$$

For L_2 and L_3 we can not generate a grammer that is context free like L_1,

63. Choice (*a*) is regular since it is finite.

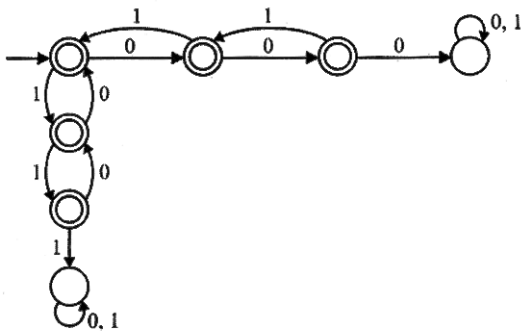
Choice (*b*) is regular since although comparison is made between 0's and 1's, it is for all prefixes and this can be done by dfa.

Note : $|n_0(s') - n_1(s')| \leq 2$ is same as $n_0(s') - n_1(s') \geq 2$ or $n_1(s') - n_0(s') \geq 2$.

Choice (*c*) involves comparison of number of 0's and 1's, but for the string as a whole, and this cannot be done by a dfa, since it has finite memory and has no stack for counting upto infinity. Therefore, choice (*c*) is not regular.

Choice (*d*) is regular since $n_0(s) \bmod 7 = n_1(s) \bmod 5 = 0$ means number of 0's is divisible by 7 and number of 1's is divisible by 5 and this can be accepted by a dfa with 7 × 5 = 35 states.

A dfa that will accept the language of choice (*b*) is shown below :



64. By observation, all the numbers of the form $5x + 2$, $x \in N$ except of the form $7(5n + 4) + 4n \in N$, belong to the language, Since we have an algorithm to check whether a given number belongs to L or not. It is recursive.

65. Let's take a string *abba* generated by G. The two leftmost derivations are

(i) $S \to SS \to \in S \to \in SS \to SS \to abS \to abba$

(ii) $S \to SS \to abS \to abba$

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Hence G is ambiguous

As any replacement of S with terminals will add equal number of *a*'s and *b*'s to the string

Hence All strings generated by G will have equal number of *a's* and *b's*

G is also accepted by a deterministic PDA.

66. Since L_1 may or may not be content – free, hence options (*a*) and (*c*) are ruled out.

Since L_3 is not recursive, hence option *(b)* is also ruled out.

67. Output of the top MUX

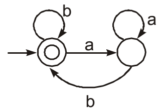
$$M_1 = x\bar{z} + \bar{y}z$$

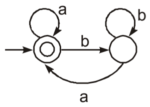
Output of the second MUX

$$\begin{aligned} M_2 &= M_1\bar{y} + xy \\ &= (x\bar{z} + \bar{y}z)\,\bar{y} + xy \\ &= x\bar{y}\bar{z} + \bar{y}z + xy \\ &= \bar{y}z + x\bar{y}\bar{z} + xy\,(z + \bar{z}) \\ &= \bar{y}z + x\bar{y}\bar{z} + xy\bar{z} + xy\bar{z} + xyz \\ &= \bar{y}z + x\bar{z}\,(\bar{y} + y) + xy\,(z + \bar{z}) \\ &= \bar{y}z + x\bar{z} + xy \end{aligned}$$

NUMERICAL TYPE QUESTIONS

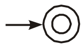
1. $L(M) \cap L(N) = \Sigma^* - (\Sigma^* - L(M) \cup (\Sigma^* - L(N))$

$\Sigma^* - L(M) \Rightarrow$ 

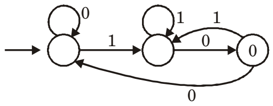
$\Sigma^* - L(N) \Rightarrow$ 

M accept all string end with a and N accept all string end with *b*. The intersection of both will accept empty language.

Hence



Then no. of state is 1.

2. 

Ans = 3

3. R .E = a*b* (ba) *a*

Length 0 is present as it accepts $\in$ all length 1 strings are present (a,b) also aa, ab, ba, bb are present, But 'bab ' is not present. So it is 3.

■■

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5
CHAPTER

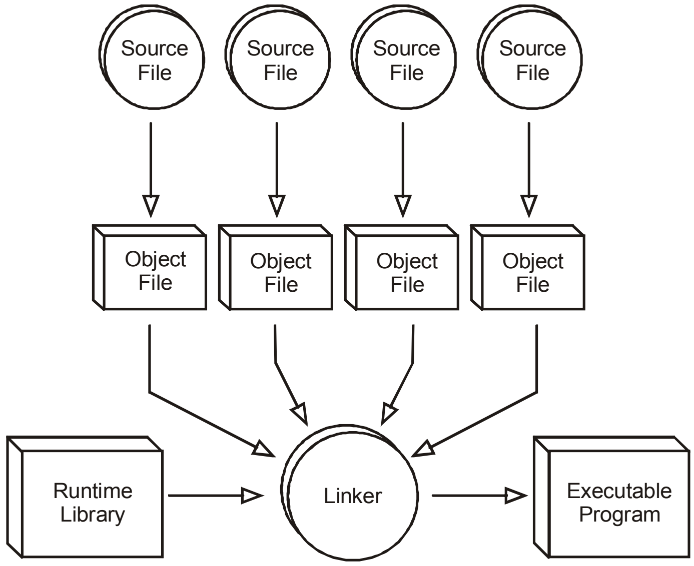
Compiler Design

LIXICAL ANALYSIS, PARSING

COMPILER

It is a program that translates high level language (*i.e.,* source code) into low level language object code. It works, looking at the entire piece of source code and collecting and reorganizing the instructions. Thus, a compiler differs from an interpreter, which analyzes and executes each line of source code in succession, without looking at the entire program.

Advantage of interpreters is that they can execute a program immediately. Compilers require some time before an executable program emerges. However, programs produced by compilers run much faster than the same programs executed by an interpreter.



Parts of compilation

There are two parts of compilation :

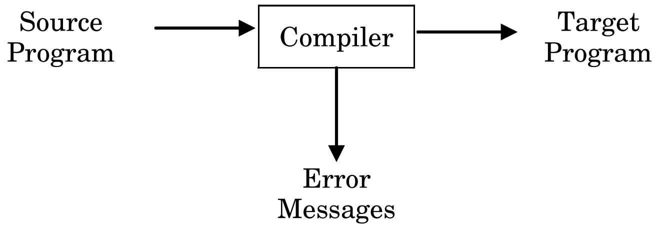
(1) Analysis part.

It breaks up the source program into constant piece and creates an intermediate representation of the source program.

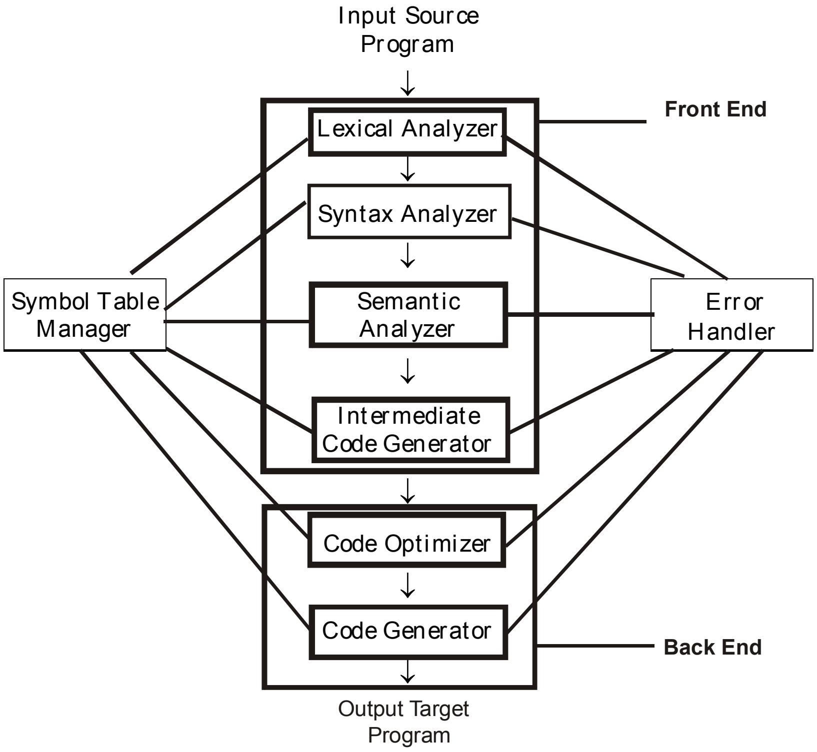
(2) Synthesis part.

It constructs the desired target program from the intermediate representation.

The translation process should also report the presence of errors in the source program (if any).



Phases of Compiler

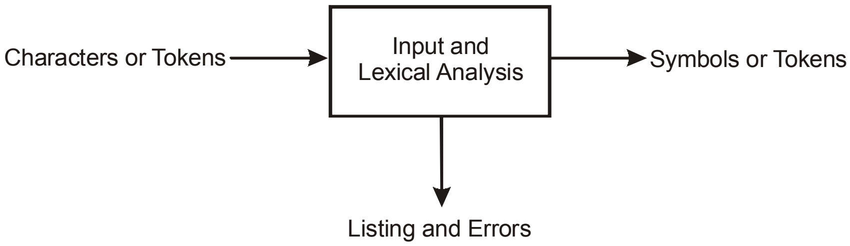


LEXICAL ANALYSIS

In a compiler, linear analysis is called *lexical analysis* or *scanning*.

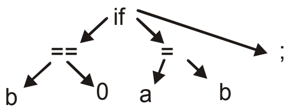
It is about breaking a sequence of characters into tokens with attributes such as kind, type, value, etc. The process of searching for matched tokens is typically described using Finite Automata (FA) and Regular Expression (RE).

This group of routines is responsible for reading the input, producing a listing, reporting errors, and splitting the input stream into distinct 'symbols' to be passed on to the next stage of the compiler.



It is based on the finite state automata and hence finds the lexicons from the input on the basis of corresponding regular expressions. If there is some input which it can't recognize then it generates error. The lexical analyzer recognizes identifiers, numbers, brackets etc.

if	(	b	= =	0	)	a	=	b	:



- Error reporting and recovery
- Model using context free grammars
- Recognize using Push down automata/Table Driven Parsers

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SYNTAX DIRECTED TRANSLATION

SYNTAX ANALYSIS

It is modeled on the basis of context free grammars. Programming languages can be written using context free grammars. Based on the rules of the grammar, a syntax tree can be made from a correct code of the language. A code written in a CFG is recognized using Push Down Automata. If there is any error in the syntax of the code, then an error is generated by the compiler. Some compilers also tell that what exactly is the error, if possible.

Scanner Generator.

The specification of input based on regular expression.

The organization is based on finite automation.

Syntax-Directed Translation.

It walks the parse tee and as a result generate intermediate code.

Automatic Code Generators.

Translates intermediate rampage into machine language.

Data-Flow Engines.

It does code optimization using data-flow analysis.

SYNTAX DEFINITION

Contex Free Grammar (CFG) is a common notation for specifying syntax of a languages

e.g. an "IF-ELSE" statement in c-language has the form

IF (Expr) stmt ELSE stmt

It is the concatenation of :

(*i*) the keyword IF

(*ii*) an opening parenthesis (

(*iii*) an expression Expr

(*iv*) a closing parenthesis)

(*v*) a statement stmt

(*vi*) a keyword ELSE

(*vii*) another statement stmt.

The syntax of an 'IF-ELSE' statement can be specified by following 'production rule' in the CFG :

$$\text{stmt} \rightarrow \text{IF (Expr) stmt ELSE stmt}$$

The arrow ($\rightarrow$) is read as "can have the form".

Components of CFG

A context-free grammar (CFG) has four components :

(1) A set of tokens called terminals.

(2) A set of variable called nonterminals.

(3) A set of production rules.

(4) A designation of one of the nonterminals as the start symbol.

Multiple production with the same nonterminal on the left like :

$$\text{list} \rightarrow + \text{digit}$$
$$\text{list} \rightarrow - \text{digit}$$
$$\text{list} \rightarrow \text{digit}$$

may be grouped together separated by vertical bars, like:

$$\text{list} \rightarrow \text{list + digit} \mid \text{list - digit} \mid \text{digit}$$

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AMBIGUITY.

A grammar is ambiguous if two or more different parse trees can be desire the same token string. Equivalently, an ambiguous grammar allows two different derivations for a token string.

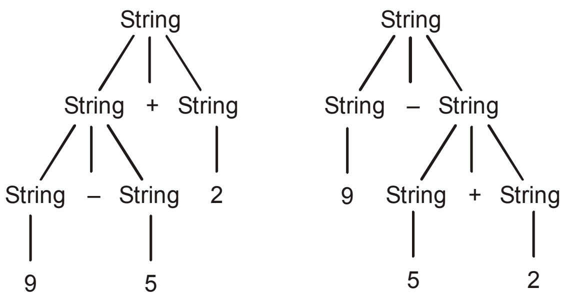
Grammar for complier should be unambiguous since different parse trees will give a token string different meaning.

Consider the following grammar :

string $\rightarrow$ string + string

| string - string ; 0 | 1 | 2 ...... | 9

To show that a grammar is ambiguous all find a "single" string that has more than one perse tree.



Above figure show two different parse trees for the token string 9 – 5 + 2 that corresponds to two different way of parenthesizing the expression :

$$(9 - 5) + 2 \text{ and } 9 - (5 + 2).$$

The first parenthesization evaluates to 6 and second parenthesis evaluate to 2.

Removal of Ambiguity

1. Associativity of Operators.

If operand has operators on both side then by connection, operand should be associated with the operator on the left.

In most programming languages arithmetic operators like addition, subtraction, multiplication, and division are left associative.

- *Token string:* 9 - 5 + 2
- *Production rules :*

  list $\rightarrow$ list - digit | digit

  digit $\rightarrow$ 0 | 1 | 2 | . . . | 9

  In the C programming language, the assignment operator, =, is right associative, *i.e.*, token string a = b = c should be treated as a = (b = c).

- Token string : a = b =c.
- Production rules :

  right $\rightarrow$ letter = right | letter

  letter $\rightarrow$ a | b | . . . | z

- Parse tree for right-associative operator is :

2. Precedence of Operators.

*An expression 9 + 5 * 2 has two possible interpretation :*

(*i*) (9 + 5) * 2, and

(*ii*) 9 + (5 * 2)

Associativity of '+' and '*' do not resolve this ambiguity, hence, we should know relative precedence of operators.

The convention is to give multiplication and division higher precedence than addition and subtraction.

Only when we have the operations of equal precedence, we apply the rules of associative.

So, in the example expression : 9 + 5 * 2

We perform operation of higher precedence i.e., * before operations of lower precedence i.e., +.

Therefore, correct interpretation is 9 + (5 * 2).

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3. Separate Rule.

Consider following grammar and language again :

S → IF b THEN S ELSE S
| IF b THEN S
| a

An ambiguity can be removed if we arbitrary decide that an ELSE should be attached to the last preceding THEN, like :

We can revise the grammar to have two nonterminals S1 and S2. We insist that S2 generates IF-THEN-ELSE, while S1 is free to generate either kind of statements.

Rules of the new grammar are:

S_1 → IF b THEN S1
| IF b THEN S2 THEN S1
| a

S_2 → IF b THEN S2 ELSE S2
| a

Syntax of Expressions

A grammar of arithmetic expressions looks like:

Expr → expr + term | expr - term | term
term → term * factor | term/factor | factor
factor → id | num | (expr)

Here expr is a string of terms separated by '+' and '-'.

A term is a string of factors separated by '*' and '/' and a factor is a single operand or an expression wrapped inside of parenthesis.

SEMANTIC ANALYSIS

- Check semantics
- Error reporting
- Disambiguate overloaded operators
- Type coercion
- Static checking
- Type checking
- Control flow checking
- Unique ness checking
- Name checks

Semantic analysis should ensure that the code is unambiguous. Also it should do the type checking wherever needed.

e.g. int y = "Hi"; should generate an error.

Type coercion : int y = 5.6 + 1; The actual value of y used will be 6 since it is an integer. The compiler knows that since y is an instance of an integer it cannot have the value of 6.6 so it down-casts its value to the greatest integer less than 6.6. This is called *type coercion.*

COMMONLY USED TERMS.

Token.

Sentences consist of string of tokens (a syntactic category)

A token is a syntactic category. Sentences consist of a string of tokens.

e.g. number, identifier, keyword, string etc.

Lexeme.

Sequence of characters in a token is a lexeme.

e.g. 100.01, counter, const, "How are you?" etc.

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Pattern.

Rule of description is a pattern

e.g. letter (letter | digit)* is a pattern to symbolize a set of strings which consist of a letter followed by a letter or digit. In general, there is a set of strings in the input for which the same token is produced as output. This set of strings is described by a rule called *pattern* associated with the token. This pattern is said to match each string in the set. Lexeme is a sequence of characters in the source program that is matched by the pattern for a token. Patterns are specified using regular expressions.

e.g., in the Pascal statement

Const pi = 3.1416;

The substring pi is a lexeme for the token "*identifier*".

Discard whatever does not contribute to parsing like white spaces (blanks, tabs, new lines) and comments.

When more than one pattern matches a lexeme, the lexical analyzer must provide additional information about the particular lexeme that matched to the subsequent phases of the compiler, e.g., the pattern num matches both 1 and 0 but it is essential for the code generator to know what string was actually matched.

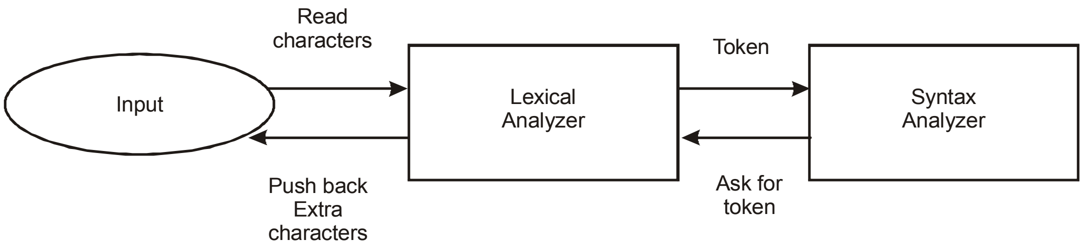
The lexical analyzer collects information about tokens into their associated attributes.

e.g., integer 31 becomes <num, 31>.

So, the constants are constructed by converting numbers to token 'num' and passing the number as its attribute

Similarly, we recognize keywords and identifiers.

e.g., count = count + in becomes id = id + id.



- Push back is required due to lookahead

  e.g., > = and >
- It is implemented through a buffer
- Keep input in a buffer
- Move pointers over the input

Construct constants : convert numbers to token num and pass number as its attribute, for example, integer 31 becomes <num, 31>

Recognize keyword and identifiers for example counter = counter + increment becomes id = id + id / *check if id is a keyword*/

We often use the terms "token", "pattern" and "lexeme" while studying lexical analysis. Lets see what each term stands for.

Example. Consider the following grammar :

program	→	program id is block
block	→	stmt$^+$
stmt	→	assn | if
assn	→	id := num Expr :
if	→	if bool Expr then stmt else stmt
num Expr	→	num Expr + term | numExpr - term | term
term	→	term * factor | term/factor | factor
factor	→	number |'(' numExpr')'
bool Expr	→	numExpr relop num Expr
relop	→	<|<=|==|!=|>=|>
id	→	letter let Digund*
letDigUnd	→	letter|digit|_
number	→	zero | nonZero digit*
digit	→	zero | nonZero
zero	→	0
non-zero	→	[1, ..9]
letter	→	[a...zA..Z]

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PARSING

Parsing is a process of finding a parse tree for a string of tokens. Equivalently, it is a process of determining whether a string of tokens can be generated by a grammar.

Worst-case time of parsing algorithms are : $O(n^3)$

But typical is : O(n) time.

e.g., Production rules of grammar G is

$$\text{list} \rightarrow \text{list} + \text{digit} \mid \text{list} - \text{digit} \mid \text{digit}$$
$$\text{digit} \rightarrow 0 \mid 1 \mid \ldots \mid 9$$

Given token string is 9-5+2.

PARSE TREES.

It is a graphical representation of a derivation of a string.

In general, a parse tree is a tree where each node is labeled with $v \in V$ such that

(*i*) root of a parse tree is initial symbol

(*ii*) leaves of a parse three are terminals

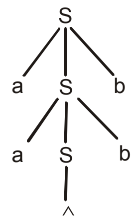
(*iii*) internal node of a parse tree are non-terminals.

(*iv*) children of each internal node N are the symbols on the right-hand side of a rule that has N as the left-hand side.

Parse tree is :

If grammar is s $\rightarrow$ asb/$\wedge$

Then Parse free for aabb is



- Each node in the parse tree is labeled by a grammar symbol.
- Interior node corresponds to the left side of the production.
- Children of the interior node corresponds to the right side of production.
- Language defined by a grammar is the set of all token strings can be derived from its start symbol.

  Language defined by the grammar:

  $$\text{list} \rightarrow \text{list} + \text{digit} \mid \text{list} - \text{digit} \mid \text{digit}$$
  $$\text{digit} \rightarrow 0 \mid 1 \mid 2 \mid \ldots \mid 9$$

  contains all lists of digits separated by plus and minus signs.

  Null symbol $\wedge$, on the right side of the production denotes the empty string.

  Parsing is the process of determining a string of tokens can be generated by a grammar. A parser must be capable of constructing the tree, or else the translation cannot be guaranteed correct. For any language that can be described by CFG, the parsing requires $O(n^3)$ time to parse string of n token. However, most programming languages are so simple that a parser requires just O(n) time with a single left-to-right scan over the input string of n tokens.

TYPES OF PARSING.

There are two types of Parsing :

1. Top-down Parsing (start from start symbol and derive string).

Top-down parser builds a parse tree by starting at the root and working down towards the leaves.

It is easy to generate by hand.

e.g. Recursive-descent, Predictive.

Consider the CFG with productions:

expr $\rightarrow$ term rest

rest $\rightarrow$ + term rest | - term rest | term | $\wedge$

term $\rightarrow$ 0 | 1 | . . . | 9

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Step 0 : Initialization: Root must be starting symbol
Step 1 : expr → term rest
Step 2 : term → 9
Step 3 : rest → term rest
Step 4 : term → 5
Step 5 : rest → term rest
Step 6 : term → 2
Step 7 : rest → ∧
Here the grammar made it easy for the top-down parser to pick the correct production in each step.

Predictive Parsing

Recursive-descent parsing is a top-down method of syntax analysis that executes a set of recursive procedure to process the input. A procedure is associated with each nonterminal of a grammar.

Predictive parsing is a special form of recursive-descent parsing, in which current input token unambiguously determines the production to be applied at each step.

Let the grammar be:

expr → term rest
rest → + term rest | - term rest | ∧
term → 0 | 1 | . . . | 9

In a recursive-descent parsing, we write code for each nonterminal of a grammar. In case of above grammar, we should have three procedure, correspond to nonterminals expr, rest, and term.

Since there is only one production for nonterminal expr, the procedure expr is

```
expr ( )
{
term ( );
rest ( );
return;
}
```

Since there are three productions for rest, procedure rest uses a global variable, 'lookahead', to select the correct production or simply selects "no action" i.e.,

E - production, indicating that lookahead variable is neither + nor -

```
rest ( )
{
IF (lookahead = = '+') {
match ( ' + ' );
term ( );
rest ( );
return;
}
ELSE IF ( lookahead = = '-') {
match (' - ');
term ( );
rest ( );
return;
{
ELSE {
return;
}
}
```

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The procedure term checks whether global variable lookahead is a digit.

```
term ( ) {
IF ( isdigit (lookahead)) {
match (lookahead);
return;
}
else{
ReportError ( );
}
```

After loading first input token into variable '*lookahead*' predictive parser is stared by calling starting symbol, 'expr'.

If input is error free, then parser conducts a depth-first traversal of the parse tree and return to caller routine through expr.

Problem with Predictive Parsing.

Checking for given grammer is LL(1) or not

1. If G does not contain $\in$ i.e. $A \to \alpha_1/\alpha_2/\alpha_3$ then G is said to be LL(1) iff

(*a*) $f_i(\alpha_1) \wedge f_i(\alpha_2) = \phi$

(*b*) $f_i(\alpha_2) \wedge f_i(\alpha_3) = \phi$

(*c*) $f_i(\alpha_3) \wedge f_i(\alpha_1) = \phi$

2. If G contain $\in$ i.e. $A \to \alpha_1/\in/\alpha_2$ then G is said to be LL(1) iff

(*a*) $f_i(\alpha_1) \wedge f_0(A) = \phi$

(*b*) $f_i(\alpha_1) \wedge f_i(\alpha_2) = \phi$

(*c*) $f_i(\alpha_2) \wedge f_0(A) = \phi$

Left Recursion

Production is left-recursive if leftmost symbol on the right side is same as the non terminal on the left side.

e.g., expr $\to$ expr + term.

If one were to code, this production in a recursive-descent parser, the parser would go in an infinite loop.

diagram

We can eliminate left-recursion by introducing new nonterminals and new productions rules.

For example, left-recursive grammar is :

$$E \to E + T \mid T$$
$$T \to T * F \mid F$$
$$F \to (E) \mid id.$$

We can redefine E and T without left-recursion as:

$$E \to TE'$$
$$E' \to + TE' \mid \in$$
$$T \to FT'$$
$$T \to * FT' \mid \in$$
$$F \to (E) \mid id$$

Getting rid of such immediate left recursion is not enough but we must get rid of indirect left recursion too, where two or more nonterminals are mutually left-recursive.

Left factoring

$$A \to \beta\alpha_1 | \beta\alpha_2 | \beta\alpha_3$$

Left factoring

$$A \to \beta A'$$
$$A' \to \alpha_1 \mid \alpha_2 \mid \alpha_3$$

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2. BOTTOM-UP PARSING.

Bottom-up Parsing (start from string and reduce to start symbol).

Bottom-up parser builds a parser tree by starting at the leaves and working up towards the root.

It is not easy to handle by hands, usually compiler-generating software generate bottom up parser

but handles larger class of grammar

e.g. LR parser.

In this, we start with the sentence and try to apply the production rules in reverse, in order to finish up with the start symbol of the grammar. This corresponds to starting at the leaves of the parse tree, and working back to the root.It can be thought of a process of reducing the string in question to the start symbol of the grammar. Bottom-up parsing is also called *shift-reduce parsing*.

Consider a grammar :

$$S \rightarrow aABe$$
$$A \rightarrow Abc \mid b$$
$$B \rightarrow d$$

And the input string

abbcde

Then an instance of bottom-up parsing can be given as

$$aAde \rightarrow aABe \rightarrow S$$

Here in the first step we used the derivation

$$B \rightarrow d$$

and then the derivation $S \rightarrow aABe$

Thus this process is like tracing out the right most derivations in reverse.

Right Derivation

Expanding the rightmost non-terminal

In context of the above example, right derivations can be shown as

$$S \rightarrow aABe \rightarrow aAde \rightarrow aAbcde \rightarrow abbcde$$

Here in each step we have expanded the rightmost terminal.

Operator Precedence Parser

For this grammer should be operator grammer.

Operator garmmer :- It does not contain

(*i*) $\in$ ruler

(*ii*) Two adjacent variables on R.H.S. of production

LR PARSING.

It is a type of bottom-up parsing which includes SLR, LALR, and LR parsers.

LR means that the input is scanned *left-to-right*, and that a rightmost derivation, in reverse, is constructed.

SLR means "simple" LR.LALR means "*look-ahead*" LR.

Every SLR(1) grammar is also LALR(1), and every LALR(1) grammar is also LR(1), so SLR is the most limited of the three, and LR is the most general. In practice, it is pretty easy to write an LALR(1) grammar for most programming languages (i.e., "power" of an LR parser isn't usually needed).

Advantages of LR Parsers

(*i*) Almost all programming languages have LR grammars.

(*ii*) It take time and space linear in the size of input (with a constant factor determined by the grammar).

(*iii*) It is strictly more powerful than LL

e.g., every LL(1) grammar is also both LALR(1) and LR(1), but not vice versa.

(*iv*) LR grammars are more "natural" than LL grammars

e.g., grammars for expression languages get mangled when we remove left recursion to make them LL(1), but that isn't necessary for an LALR(1) or an LR(1) grammar.

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Disadvantages of LR Parsers.

(*i*) Although an LR grammar is usually easier to understand than the corresponding LL grammar, the parser itself is harder to understand and to write (thus, LR parsers are built using parser generators, rather than being written by hand).

(*ii*) When we use a parser generator, if the grammar that we provide is not LALR(1), it can be difficult to figure out how to fix that.

(*iii*) Error repair may be more difficult using LR parsing than using LL.

(*iv*) Table sizes may be larger (about a factor of 2) than those used for LL parsing.

Bottom-up parsers.

These use a stack, but in this case, the stack represents a summary of the input already seen, rather than a prediction about input yet to be seen.

Now, we pretend that the stack symbols are :

(*a*) terminals

(*b*) nonterminals (as they are for predictive parsers).

This isn't quite true, but it makes our introduction to bottom-up parsing easier to understand.

Bottom-up parser is also called *shift-reduce parser* because it performs two types of operations :

***(i)* Shift operation:** It simply shifts the next input token from the input to the top of the stack.

***(ii)* Reduce operation :** It is only possible when the top N symbols on the stack match the right-hand side of a production in the grammar. A reduce operation pops those symbols off the stack and pushes the non-terminal on the left-hand side of the production.

Derivation (using a given grammar)

It is performed as follows :

(1) Start with the start symbol (i.e., the current string is the start symbol)

(2) Repeat :

- Choose a nonterminal X in the current string
- Choose a grammar rule.
- Replace X in the current string with alpha

until there are no more nonterminals in the current string

Constructing a Reverse Rightmost derivation

Rightmost derivation is one in which rightmost nonterminal is always the one chosen.

e.g. Grammar

$$\begin{array}{lllllll} E & \rightarrow & E & + & T & | & T \\ T & \rightarrow & T & * & F & | & F \\ F & \rightarrow & id & & & | & (E) \end{array}$$

Rightmost derivation

$$E \Rightarrow^1 E + \mathbf{T} \Rightarrow^2 E + T * \mathbf{F} \Rightarrow^3 E + \mathbf{T} * id \Rightarrow^4 E + \mathbf{F} * id \Rightarrow^5$$
$$\mathbf{E} + id * id \Rightarrow^6 \mathbf{T} + id * id \Rightarrow^7 \mathbf{F} + id * id \Rightarrow^8 id + id * id$$

In this example, the nonterminal that is chosen at each step is shown in bold, and each derivation step is numbered. The corresponding bottom-up parse is shown below by showing the parse tree with its edges numbered to show the order in which the tree was built

e.g., first step was to add the nonterminal F as the parent of the leftmost parse-tree leaf "id", and last step was to combine the three subtrees representing "id", "+", and "id * id" as the children of a new root node E.

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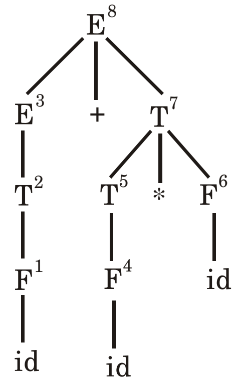

Both rightmost derivation and bottom-up parse have 8 steps.

Step 1 of the derivation corresponds to step 8 of the parse; step 2 of the derivation corresponds to step 7 of the parse; etc. Each step of building the parse tree (adding a new nonterminal as the parent of some existing subtrees) is called a reduction (that's where the "reduce" part of "shift-reduce" parsing comes from).

BASIC LR PARSING ALGORITHM.

All LR parsers use the same basic algorithm based on :

(*i*) top-of-stack symbol

(*ii*) current input symbol (token), and

(*iii*) entry in one of the parse tables (indexed by the top-of-stack and current-input symbols)

Functions of Parser.

Parser performs one of the following actions :

1. **Shift :** Push the current input symbol onto the stack, and go on to the next input symbol (i.e., call the scanner again)
2. **Reduce :** A grammar rule's right-hand side is on the top of the stack, pop it off and push the grammar rule's left-hand-side nonterminal
3. **Accept :** Accept the input (parsing has finished successfully)
4. **Reject :** Input is not syntactically correct

SLR PARSING.

SLR means simple LR; it is the weakest member of the LR family (i.e., every SLR grammar is also LALR and LR, but not vice versa). To understand SLR parsing we'll use a new example grammar (a very simple grammar for parameter lists) :

```
plist   → ( idlist )
idlist  → ID
        → idlist ID
```

Difference between SLR, LALR and LR Parsers

It is in the tables that they use. Those tables use different techniques to determine when to do a reduce step, and, if there is more than one grammar rule with the same right-hand side, which left-hand-side nonterminal to push.

Building the Action and Goto Tables for an SLR Parser

Definition of an SLR item.

An item (for a given grammar) is a production with a dot somewhere on the right-hand side.

e.g.,

```
plist → ( idlist )   <-- the production

  plist → . ( idlist )   --+
  plist → ( . idlist )     | the possible items
  plist → ( idlist . )     |
  plist → ( idlist ) .   --+
```

Note : For the production X → epsilon, there is just one item: X → . epsilon

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The item

"plist $\rightarrow$. (idlist)" can be thought of as meaning "we may be parsing a plist, but so far we haven't seen anything".

"plist $\rightarrow$ (. idlist) " means "we may be parsing a plist, and so far we've seen a left paren".

plist $\rightarrow$ (idlist .) means "we may be parsing a plist, and so far we've seen a left paren and parsed an idlist".

Operations required on sets of items.

Two operations on sets of items are :

(1) Closure

(2) Goto

To compute Closure (I), where I is a set of items:

***(i)* Closure.**

1. put I itself into Closure(I)
2. while

   there exists an item in Closure(I) of the form

   X $\rightarrow$ alpha . B beta

   such that there is a production B $\rightarrow$ gamma, and B $\rightarrow$. gamma is not in Closure(I)

   do

   add B $\rightarrow$. gamma to Closure(I)

   The idea is that the item "X $\rightarrow$ alpha . B beta" means "we may be trying to parse an X, and so far we've parsed all of alpha, so the next thing we'll parse may be a B". And item "B $\rightarrow$. gamma" also means that the next thing we'll parse may be a B (in particular, a B that derives gamma), but we haven't seen any part of it yet.

   Building FSM :

   1. Augment the grammar

      (*a*) add new start nonterminal S^1

      (*b*) add new production $S^1 \rightarrow S$ (where S is the old start nonterminal)

   2. I_0 = closure({$S^1 \rightarrow . S$})
   3. For each grammar symbol X such that there is an item in I_0 containing ".X" do add a transition on X from state I_0 to state Goto(I_0,X)
   4. Repeat step (3) for each new state until there are no more new states

   Note : In step (3), when we say "state Goto(...)" we mean the state that contains exactly that set of items; if there is not already such a state, then create a new one.

   Action Table :

   1. if state i includes item

      A $\rightarrow$ alpha . a beta

      where a is a terminal, and the transition out of state i on a is to state j, then

      set Action[i,a] = shift j

   2. if state i includes item

      A $\rightarrow$ alpha.

      where A is not the new start symbol S^1, then

      for every a in FOLLOW(A), set Action[i, a] = reduce by A $\rightarrow$ alpha

   3. if state i includes item

      $S^1 \rightarrow S$

      then set Action[i,$] = accept

   4. set all other entries of Action table to error

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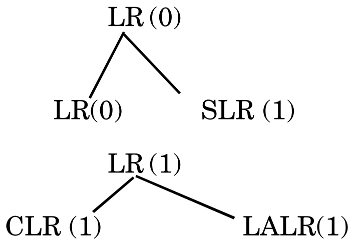
(*ii*) Goto Table :

For every nonterminal X
if there is a transition from state i to state j on X
then set Goto[i, X] = j



LR(1) = LR(0) + look ahead symbol

* Minimization of CLR(1) is done by LALR(1)

(Q) If number of stales in LR(0) = n_1,

SLR (1) = n_2, LALR(1) = n_3, CLR (1) = n_4

then $n_1 = n_2 = n_4 \leq n_3$

CONFLICTS.

Not every grammar is SLR(1). If a grammar is not SLR(1), there will be a conflict in the SLR Action table. There is a conflict in the table if there is a table entry with more than 1 rule in it.

Types of Conflicts

There are two possible types of conflicts:

1. **Shift/reduce conflicts** (*a shift and a reduce action in the same table entry*) :
   It means that it is not possible to determine, based only on the top-of-stack state symbol and the current token, whether to shift or to reduce.
   This type of conflict arises when one state contains two items of the following forms:
   X → alpha . a beta
   Y → alpha
   and a is in FOLLOW(Y).
2. **Reduce/reduce conflicts** (*two reduce actions in the same table entry*) :
   It means that it is not possible to determine, based only on the top-of-stack state symbol and the current token, whether to reduce by one grammar rule or by another grammar rule.
   This type of conflict arises when one state contains two items of the following forms:
   X → alpha.
   Y → beta.
   and there is some symbol a that is in both FOLLOW(X) and FOLLOW(Y).
   Note :- In term of power SLR(1) < LALR(1) < CLR(1)

PARSE TABLE

A parse is a blueprint for the creation of a recursive descent parser. The rows of the table are labeled with non-terminals and its columns are labeled with terminals. Each table entry is placed with a corresponding grammar rule.

A parse table is created as follows :

(*i*) Rows of the parse table are labeled with the non-terminals of the grammar.

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(*ii*) Columns of the parse table are labeled with the terminals of the grammar.

(*iii*) Each entry of the parse table is either empty, or contains a grammar rule :

(*a*) Place each rule of the form $S \to \gamma$ in row S, in each column in First (γ).

(*b*) Place each rule of the form $S \to \gamma$, where First(γ) contains $\in$, in row S in each column in, first (γ)

GRAMMAR RESTRICTIONS.

Restriction 1. Consider sentences of the form N = E | F.

To decide whether the next symbol is the beginning of an E or F sentence, the parser must assume that the two types of sentences always begin with different symbols. So the EBNF grammar must satisfy the following condition :

$$\text{First (E)} \cap \text{First (F)} = \text{Symbols []}.$$

Restriction 2.

If some of the sentences described by a BNF rule N may be empty,

like $N \to E \mid n \mid f$

then $f_i\,(E) \cap fo\,(N) = \phi$

$f_i(f) \cap f_o\,(N) = \phi$

$f_i\,(E) \cap f_i\,(f) = \phi$

the grammar must be constrained as Follows :

$$\text{First (N)} \cap \text{Follow (N)} = \text{Symbols []}$$

Any rule that contains [E] must be considered.

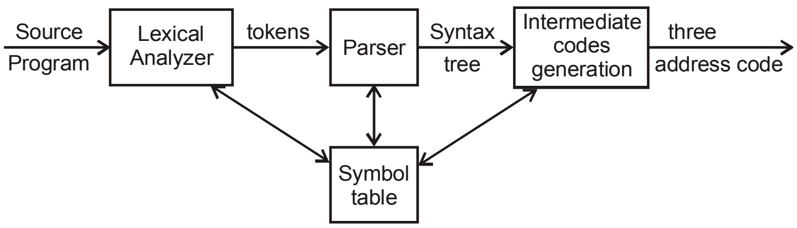
SYNTAX DIRECTED TRANSLATION

There are two notations for associating semantic rules with productions :

(1) Syntax directed definitions

(2) Translation schemes.

Conceptually, with both, we pass the input token stream, build the parse tree, and then traverse the tree as required to evaluate the semantic rules at the parse tree nodes. Evaluation of the semantic rules may generate code, save information in a symbol table, issue error messages, or perform any other activities. The translation of the token stream is the result obtained by evaluating the semantic rules.



SYNTAX DIRECTED DEFINITIONS.

It is a generalization of a context free grammar in which each grammar symbol has an associated set of attributes, partitioned into two subsets called *synthesized* and inherited attributes of that grammar symbol.

An attribute can represent anything we choose :

a string, a number, a type, a memory location, or whatever. The value of an attribute at a parse tree node is defined by a semantic rule associated with a production used at that node. The value of a synthesized attribute at a node is computed from

(*i*) the values of attributes at the children of that node in the parse tree

(*ii*) the value of an inherited attribute is computed from the values of attributes at the siblings and parent of that node.

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Semantic rules set up dependencies between attributes that will be represented by a graph. From the dependency graph, we derive an evaluation order for the semantic rules. Evaluation of the semantic rules defines the values of the attributes at the nodes in parse tree for the input string.

Parse tree showing the values of attributes at each node is called *annotated parse* tree. The process of computing the attributes at the nodes is called *annotating* or *decorating the parse tree*.

Note : A syntax directed definition is said to be circular if the dependency graph for some parse tree generated by its grammar has a cycle.

Form of a Syntax Directed Definition

In a syntax directed definition, each grammar production A has associated with it a set of semantic rules of the form

$$b := f(c_1, c_2, \ldots \ldots .., c_k),$$

where f is a function,

and either

(*i*) b is a synthesized attribute of A and $c_1, c_2, \ldots, c_k$ are attributes belonging to the grammar symbols of the production, or,

(*ii*) b is an inherited attribute of one of the grammar symbols on the right side of the production, and $c_1, c_2, \ldots, c_k$ are attributes belonging to the grammar symbols of the production.

In either case, the attribute b depends on attributes $c_1, c_2, \ldots, c_k$. An attribute grammar is a syntax directed definition in which functions in semantic rules cannot have side effects.

Synthesized Attributes

A syntax directed definition that uses synthesized attributes exclusively is said to be an S-attributed definition. A parse tree for an S-attributed definition can always be annotated by evaluating the semantic rules for the attributes at each node bottom up, from the leaves to the root.

Inherited Attributes

It is one whose value at a node in a parse tree is defined in terms of attributes at the parent and/or siblings of that node. These are convenient for expressing the dependence of a programming language construct on the context in which it appears.

e.g., we can use an inherited attribute to keep track of whether an identifier appears on the left or the right side of an assignment in order to decide whether the address or the value of the identifier is needed. Although it is always possible to rewrite a syntax directed definitions with inherited attributes.

Syntax Directed definition with Inherited attribute.

Production	Semantic Rules
$\rightarrow$ D *TL*	L.*in* :=T.*type*
$\rightarrow$ T int	T.*type* := *integer*
$\rightarrow$ T real	T.*type* := *real*
$\rightarrow$ L L_1,id	L_1.*in* := L.*in* addtype(id.entry,L.in)
$\rightarrow$ L id	*addtype*(id.*entry*,L.*in*)

The figure shows an annotated parse tree for the sentence real id1, id2, id3. The value of L.in at the three L nodes gives the type of identifiers id1, id2, and id3. These values are determined by computing the value of the attribute T.type at the left child of the root and then evaluate L.in top-down at the three L nodes in the right subtree of the root. At each L node we also call the procedure addtype to insert into the symbol table the fact that the identifier at the right child of this node has type real.

Parse tree with inherited attributed in at each node labeled L

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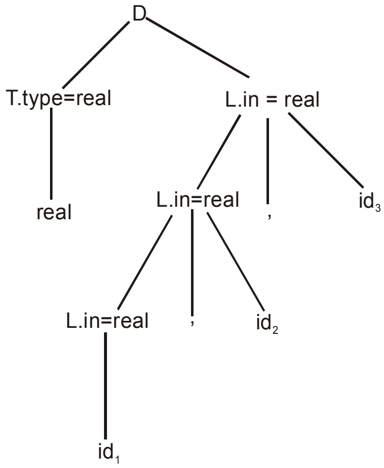

Dependency Graphs

If an attribute b at a node in a parse tree depends on an attribute c, then the semantic rule for b at that node must be evaluated after the semantic rule that defines c. The interdependencies among the inherited and synthesized attributes at the nodes in a parse tree can be depicted by a directed graph called *dependency graph*.

Before constructing a dependency graph for a parse tree, we put each semantic rule in the form

$$b := f(c_1, c_2, \ldots, c_k),$$

by introducing a dummy synthesized attribute b for each semantic rule that consists of a procedure call. The graph has a node for each attribute and an edge to the node for b from the node for c if attribute b depends on attribute c.

Dependency graph for a given parse tree is constructed as follows :

```
for each node n in the parse tree do
   for each attribute a of the grammar symbol at n do
       construct a node in the dependency graph for a;
for each node n in the parse tree do
   for each semantic rule b := f(c1, c2,..., ck) associated with
       the production used at n do
           for i:=1 to k do
     construct an edge from the node for ci to the node for b;
```

Methods for evaluating Semantic rules.

1. **Parse tree methods.**
   At compile time, these methods obtain an evaluation order from a topological sort of the dependency graph constructed from the parse tree for each input. These methods will fail to find an evaluation order only if the dependency graph for the particular parse tree under construction has a cycle.
2. **Rule based methods.**
   At compiler construction time, the semantic rules associated with productions are analyzed, either by hand, or by specialized tool. For each production, the order in which the attributes associated with that production are evaluated is predetermined at compiler construction time.
3. **Oblivious methods.**
   An evaluation order is chosen without considering the semantic rules. For example, if translation takes place during parsing, then the order of evaluation is forced by the parsing method, independent of the semantic rules. An oblivious evaluation order restricts the class of syntax directed definitions that can be implemented.

Note : Rule-based and oblivious methods need not explicitly construct the dependency graph at compile time, so they can be more efficient in their use of compile time and space

SYNTAX TREES.

It is a condensed form of parse tree useful for representing language constructs.They are created with the help of syntax-directed definitions. The use of syntax trees as an intermediate form helps to dissociate translation from parsing.

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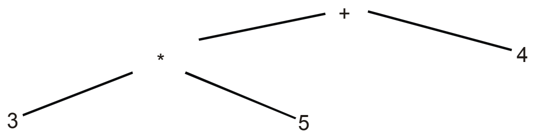
Translation routines that are invoked during parsing must operate under two types of restrictions.

(1) A grammar that is suited for parsing may not reflect the natural hierarchical structure of the constructs in the language.Second,the

(2) Parsing method constrains the order in which nodes in a parse tree are considered.This order may not match the order in which information about a construct becomes available.

In a syntax tree,operators and keywords do not appear as leaves,but rather are associated with the interior node that would be the parent of those leaves in the parse tree.Another simplification found in syntax trees is that chains of single productions may be collapsed (see figure below). Syntax-directed translation can be based on syntax trees as well as on parse trees.The approach is the same in each case;we attach attributes to the nodes as in a parse tree.



Construction Syntax Trees for Expressions

It is similar to the translation of the expression into postfix form.We construct subtrees for the subexpressions by creating a node for each operator and operand.The children of an operator node are roots of the nodes representing the subexpresions constituting the operands of that operator.

Each node in a syntax tree can be implemented as a record with several fields.

In the node for an operator,one field identifies the operator and the remaining fields contain pointers to the nodes for the operands. The operator is often called *label of the node*.

When used for translation, the nodes in a syntax tree may have additional fields to hold the values of attributes attached to the node. Usually there are a number of functions defined to create the nodes of syntax trees. Each function returns a pointer to a newly created node.

Consider the expression a – 4 + c.

Following functions are used to create nodes of syntax trees for expressions with binary operators :

(*i*) mknode (op, left , right)

creates an operator node with label op and two fields containing pointers to left and right.

(*ii*) mkleaf (id, entry)

creates a identifier node with label id and a field containing entry,a pointer to the symbol-table entry for the identifier.

(*iii*) mkleaf (num, val)

creates a number node with label num and a field containing val, the value of the number.

Following sequence of function calls creates syntax tree for the expression $a - 4 + c$:

(*i*) p_1 := mkleaf(id , entry a);

(*ii*) p_2 := mkleaf(num , 4);

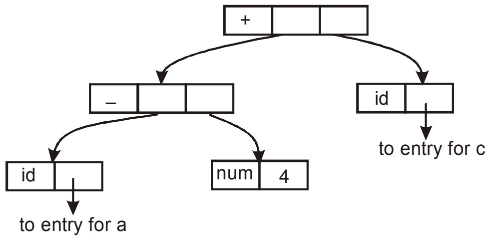
(*iii*) p_3 := mknode(' - ', p_1 , p_2);

(*iv*) p_4 := mkleaf(id , entry c);

(*v*) p_5 := mknode('+', p_3 , p_4);

p_1, p_2, p_3, p_4, p_5 are pointers to nodes, and entrya and entyrc are pointers to the symbol-table entries for identifiers a and c, respectively.

Tree is constructed bottom up. The function calls mkleaf (id, entrya) and mkleaf (num , 4) construct the leaves for a and 4; the pointers to these nodes are saved using p1 and p2. The call mknode (' - ', p_1, p_2) then constructs the interior node with the leaves for a and 4 as children. After two mor steps, p5 is left pointing to the root.



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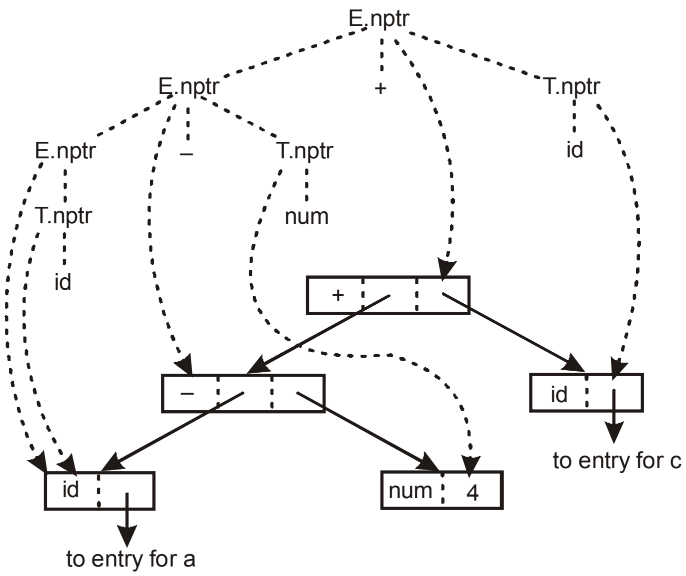
Syntax-Directed Definition for Constructing Syntax Trees

Figure contains an S-attributed definition for constructing a syntax tree for an expression containing the operators + and -.It uses the underlying productions of the grammar to schedule the calls of the functions mknode and mkleaf to construct the tree.The synthesized attribute nptr for E and T keeps track of the pointers returned by the function calls.

Production	Semantic Rules
E → E1 + T	E.nptr : = mknode (‘+’, E1 .nptr, T.nptr)
E → E1 – T	E.nptr : = mknode (‘–‘, E1 .nptr, T.nptr)
E → T	E. nptr : = T.nptr
T → (E)	T. nptr : = E.nptr
T → id	T.nptr : = mkleaf (id, id, entry)
T → num	T.nptr : = mkleaf (num, num, val)

An annotated parse tree depicting the construction of a syntax tree for the expression a – 4 + c is shown in below. The parse tree is shown dotted.The parse-tree nodes labeled by the nonterminals E and T use the synthesized attribute nptr to hold a pointer to the syntax-tree node for the expression represented by the nonterminal.



The semantic rules associated with the productions T → id and T → num define attribute T.nptr to bea pointer to a new leaf for an identifier and a number,respectively. Attributes id.entry and num.val are the lexical values assumed to be returned by the analyzer with the tokens id and num.

In above figure, when an expression E is a single term,corresponding to a use of the production E → T, the attribute E.nptr gets the value of T.nptr. When the semantic rule E.nptr := mknode(‘–’, E1.nptr, T.nptr) associated with the production E → E1 – T is invoked,previous rules have set E1.nptr and T.nptr to be pointers to the leaves for a and 4, respectively.

SYNTHESIZED ATTRIBUTES ON THE PARSER STACK

A bottom up parser uses a stack to hold information about he subtrees that have been parsed, how ?

1. Add additional fields to the stack to hold the attribute values.
2. Each child node pushes its synthesized attributes on the stack.
3. When reducing, parents pops them, uses them to evaluate its own attributes and pushes them on the stack.

Let the stack be implemented by a pair of arrays state and val.

Each state entry is a pointer to LR(1)parsing table.

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If i^{th} state symbol is A, hen val[I] will hold the value of the attribute associated with parse tree n ode corresponding to this A.

Table : Parser stack with attributes.

	State	Val
	...	...
	X	X.x
	Y	Y.x
top →	Z	Z.x
	...	...

L-ATTRIBUTED DEFINITIONS

1. Definition

These define rules for determining inherited attributes in which each inherited attribute depends only on

(*i*) attributes of the symbols to the left of it in the production

(*ii*) inherited attributes of the non-terminal on the left-side of the production

Every L-attributed definition is L-attributed, as the rules stated above apply only to inherited attributes. L-attributed definitions are said to be L-attributed as information flows from left to right in the syntax tree.

2. Order of Evaluation

The L-attributed definitions can be evaluated by a depth first traversal of the tree. The procedure can be outlined as below.

```
dfvisit(node n)
{
for each child m of n, from left to right
evaluate inherited attributes of m
dfvisit(m)
}
evaluate synthesized attributes of n
```

3. Translation Scheme

It is a context free grammar in which there are attributes associated with the grammar symbols and semantic actions enclosed within braces are inserted with the right hand sides of productions. These are useful for specifying translations during parsing. These must be designed carefully if we have both inherited and synthesized attributes. The rules that must be followed by a translation scheme involving both synthesized and inherited attributes.

Following rules have to be followed :

(*i*) An inherited attribute for a symbol on the right hand side of a production must be computed in an action before that symbol.

(*ii*) An action must not refer to a synthesized attribute of a symbol to the right of the action.

(*iii*) A synthesized attribute for the non-terminal on the left can only be computed after all attributes it references have been computed. The action for computing such attributes can usually be placed at the end of the right side of the production.

It is always possible to start with an L-attributed definition and construct a translation scheme that satisfies all the above requirements. Given an L-attributed grammar in which no inherited attributes depend on synthesized attributes, a recursive-descent parser can evaluate all attributes by turning inherited attributes into parameters and synthesized attributes into return values.

4. Top-Down translation

A major issue in this case is left recursion. For top-down predictive parsing, we can assume that an action be executed at the time that a symbol in the same position would be expanded. All first, left recursion is to be eliminated from the translation scheme.

Translation Scheme with left recursion

$A \rightarrow A_1Y$ {$A.a = g(A_1.a, Y.y)$}

$A \rightarrow X$ { $A.a = f(X.x)$}

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Translation scheme with left recursion eliminated

$A \rightarrow X$ {R.i = f(X.x)} R {A.a = R.s}

$R \rightarrow Y$ { R_1.i = g(R.i,Y.y)} R_1 {R.s = R_1.s}

$R \rightarrow e$ {R.s = R.i}

A translation scheme should be eliminated to left-recursion as per the above algorithm (just as left-recursion is eliminated from context free grammars when they are to be subjected to predictive top-down translator creation) and the translation scheme can be implemented in a very straightforward manner. A top-down translation nicely adapts to top-down and left-right flow of information.

5. Bottom-up translation

It lends itself naturally to passage of information up the tree rather than down the tree. So some modifications have to be made to the translation schemes based on various issues.

(*i*) Moving attribute rules from the middle to the end of productions.

This requires addition of new non-terminals called marker non-terminals and additional productions. The additional productions introduced are of the form $M \rightarrow e$ for each marker non-terminal M.

$A \rightarrow B$ {print("A");} C	$A \rightarrow BMC$
	$M \rightarrow$ {print("A");}

The rule in the left column can be replaced by the set of rules in the right column. This evidently does not make any change at all.

(*ii*) Inheriting attributes on the parser stack

If translation scheme inherits attributes by copy rules, then value can just be inherited from the stack. A bottom-up parser reduces the right side of a $A \rightarrow XY$, by removing X and Y from the top of the stack and replacing them by A.

Let X has a synthesized attribute X.s, which is kept along with X in the stack, this can be inherited directly by Y if copy rule is of the form Y.i=X.s etc.

But this works if and only if we can tell where a synthesized attribute value is is on the stack i.e., grammar allows the attribute value to be predicted.

Example. Case where the position on parser stack cannot be predicted.

$S \rightarrow aAC$ {C.i=A.s;}

$S \rightarrow bABC$ {C.i=A.s;}

$C \rightarrow c$ {C.s=g(C.i);}

In this case, when reduction by $C \rightarrow c$ occurs, C.i is either top-1 or top-2 of the stack. This cannot be predicted as there may or may not be an intervening B between C and A.

Systematic introduction of markers, can make it possible to evaluate L-attributed definitions during LR parsing. Since there is only one production for each marker, a grammar, a grammar remains LL(1) when markers are added.

(*iii*) Replacement of inherited by synthesized attributes

It is sometimes possible to avoid the use of inherited attributes by changing the grammar.

e.g., a declaration in Pascal can consist of a list of identifiers followed by a type e.g., a,b:integer.

A grammar for such declarations may include productions of the form :

Grammar for which synthesized attributes alone will not work :

$D \rightarrow L : T$

$T \rightarrow$ integer | char

$L \rightarrow$ L,id | id

Modifcation to remove the need for inherited attributes:

$D \rightarrow$ id L

$L \rightarrow$, id L | :T

$T \rightarrow$ integer | char

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RUNTIME ENVIRONMENT, INTERMEDIATE AND TARGET CODE GENERATION

RUNTIME ENVIRONMENT

When one starts running the program, then some data is only available at run time, so we must relate the static source text of a program to the actions that must occur at run time to implement the program. We need to understand the relationship between names and data objects (address and value).

Allocation and de-allocation is managed by run time support package which consists of routines loaded with the generated target code.

Each execution of a procedure is referred to as an activation of the procedure. If procedure is recursive, several of its activations may be alive at the same time.

We will be dealing with activations of two procedures whose lifetimes are either non overlapping or nested, i.e., if a and b are procedure activations and b is entered before a is left, then control must leave b before leaves a. We will not be dealing with partially overlapping activations (threads).

A procedure is recursive if a new activation can begin before an earlier activation of the same procedure has ended.

Procedure

Procedure definition is a declaration that associates an identifier with a statement. The identifier is the procedure name and the statement is the procedure body. Procedures that return value are also referred as procedures so a complete program is also a procedure.

When a procedure name appears within an executable statement, the procedure is said to be called at that point. Basically, this procedure call executes the procedure body. The identifiers appearing in the procedure definition are called

formal parameters (or just formals) of the procedure. Arguments, known as actual arguments may be passed to a called procedure, they are substituted for the formal parameters in the procedure body.

Activation tree

- Control flows sequentially
- Execution of a procedure starts at the beginning of body
- It returns control to place where procedure was called from
- A tree can be used, called an activation tree, to depict the way control enters and leaves activations
- The root represents the activation of main program
- Each node represents an activation of procedure
- The node a is parent of b if control flows from a to b
- The node a is to the left of node b if lifetime of a occurs before b

Assumptions about the flow of control among procedures during execution of the program :

(1) Control flows sequentially, i.e., execution of a program consists of a sequence of steps and the control does not change arbitrarily but only on explicit calls.

(2) Each execution of a procedure starts at the beginning of the procedure body and eventually returns control to the point immediately following the place where procedure was called. A tree like data structure can be used to depict the way control enters and leaves activation this tree is called *activation tree*.

(3) Root represents activation of the main program.

(4) Each node represents activation of a procedure.

(5) Node for a is the parent of the node for b if and only if control flows from activation a to b .

(6) Node for a is to the left of the node for b if and only if the lifetime of a occurs before the lifetime of b.

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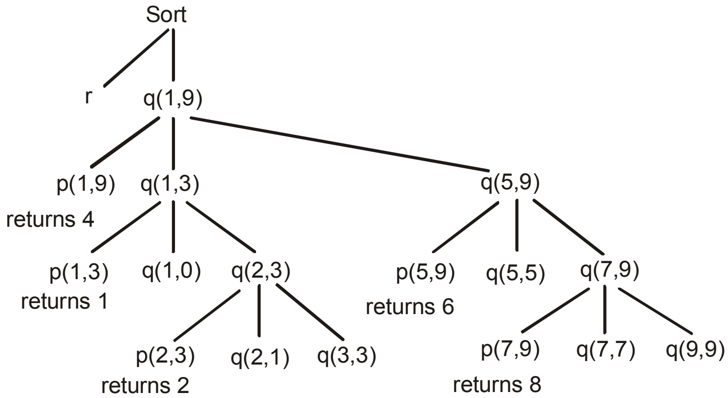
Example. *Flow of control of the call quicksort(1,9) would be like*

Execution begins ..
enter readarray
exit readarray
enter quicksort (1,9)
enter partition (1,9)
exit partition (1,9)
enter quicksort (1,3)
exit quicksort (1,3)
enter quicksort (5,9)
exit quicksort (5,9)
exit quicksort (1,9)
Execution terminates

When information is represented as an activation tree, we get the tree as shown below :



Control stack

Flow of control in a program corresponds to a depth first traversal of the activation tree that starts at the root, visits a node before its children, and recursively visits children at each node in a left to right fashion.

A stack called *control stack* is used to keep track of live procedure activations. The idea is to push the node for an activation onto the control stack as the activation begins and to pop the node when activation ends. Then contents of the control stack are related to paths to the root of the activation tree.

When node n is at the top of the control stack, the stack contains the nodes along the path from n to the root.

Scope of declaration

Declaration in a language is a syntactic construct associating information with a name.

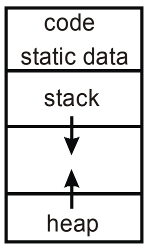
There can be two types of declarations :

***(i)* Explicit declaration :** Pascal (Algol class of languages) e.g.- var i : integer.

***(ii)* Implicit declaration:** e.g., In Fortran, the variable i is assumed to be integer unless declared.

There may be independent declarations of the same name in different parts of a program. The portion of the program to which a declaration applies is called *scope of the declaration*. An occurrence of a name in a procedure is said to be local to the procedure if it is in the scope of a declaration within the procedure, or else it is called *nonlocal*. Scope rules determine which declaration applies to a name

Storage organization



Runtime storage might be subdivided into :

(*i*) Target code

(*ii*) Data objects

(*iii*) Stack to keep track of procedure activation

(*iv*) Heap to keep all other information

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This type of organization of run-time storage is used for languages such as Fortran, Pascal and C. The size of the generated target code, as well as that of some of the data objects, is *compile time*. Thus, these can be stored in statically determined areas in the memory. Pascal and C use the stack for procedure activations. Whenever a procedure is called, execution of an activation gets interrupted, and information about the machine state (like register values) is stored on the stack. When the called procedure returns, the interrupted activation can be restarted after restoring the saved machine state. The heap may be used to store dynamically allocated data objects, and also other stuff such as activation information (in the case of languages where an activation tree cannot be used to represent lifetimes). Both the stack and the heap change in size during program execution, so they cannot be allocated a fixed amount of space. Generally they start from opposite ends of the memory and can grow as required, towards each other, until the space available has filled up.

Activation Record

(*i*) **Temporaries:** Used in expression evaluation

(*ii*) **Local data:** Field for local data

(*iii*) **Saved machine status:** Holds info about machine status before procedure call

(*iv*) **Access links :** To access non local data

(*v*) **Control links :** Points to activation record of caller

(*vi*) **Actual parameters :** Field to hold actual parameters

(*vii*) **Returned value :** Field for holding value to be returned

Temporaries
Local data
Machine status
Access links
Control links
Parameters
Return value

Activation record is used to store the information required by a single procedure call. Not all the fields shown in the figure may be needed for all languages. The record structure can be modified as per the language/compiler requirements. For Pascal and C, the activation record is generally stored on the run-time stack during the period when the procedure is executing.

Of the fields shown in the figure, access link and control link are optional

e.g. Fortran doesn't need access links.

Also, actual parameters and return values are often stored in registers instead of the activation record, for greater efficiency. The activation record for a procedure call is generated by the compiler. Generally, all field sizes can be determined at compile time. However, this is not possible in the case of a procedure which has a local array whose size depends on a parameter. The strategies used for storage allocation in such cases will be discussed in the coming slides.

Layout of Local data

- Assume byte is the smallest unit
- Multi-byte objects are stored in consecutive bytes and given address of first byte
- Amount of storage needed is determined by its type
- Memory allocation is done as the declarations are processed
- Data may have to be aligned (in a word) padding is done to have alignment.
- Complier may pack the data so no padding is left
- Additional instructions may be required to execute packed data

Storage Allocation Strategies

(1) Static allocation:

Lays out storage at compile time for all data objects

(2) Stack allocation:

Manages the runtime storage as a stack

(3) Heap allocation :

Allocates and de-allocates storage as needed at runtime from heap

These represent different storage-allocation strategies used in the distinct parts of the run-time memory organization. Now look at the possibility of using these strategies to allocate memory for activation records. Different languages use different strategies for this purpose.

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e.g., old FORTRAN used static allocation, Algol type languages use stack allocation, and LISP type languages use heap allocation.

Static allocation

Fundamental characteristics of Static allocation :

(*i*) Names are bound to storage as the program is compiled

(*ii*) No runtime support is required

(*iii*) Bindings do not change at run time

(*iv*) On every invocation of procedure names are bound to the same storage

(*v*) Values of local names are retained across activations of a procedure

Since name binding occurs during compilation, there is no need for a run-time support package. The retention of local name values across procedure activations means that when control returns to a procedure, the values of the locals are the same as they were when control last left.

e.g., Let we had following code, written in a language using static allocation:

```
function F( )
        {
        int a;
        print(a);
        a = 10;
        }
```

After calling F( ) once, if it was called a second time, the value of a would initially be 10, and this is what would get printed.

Type of a name determines the amount of storage to be set aside

- Address of a storage consists of an offset from the end of an activation record
- Compiler decides location of each activation
- All the addresses can be filled at compile time
- Constraints
- Size of all data objects must be known at compile time
- Recursive procedures are not allowed
- Data structures cannot be created dynamically
- The address for this storage is an offset from the procedure's activation record, and the compiler positions the records relative to the target code and to one another. After this position has been decided, the addresses of the activation records, and hence of the storage for each name in the records, are fixed. Thus, at compile time, the addresses at which the target code can find the data it operates upon can be filled in. The addresses at which information is to be saved when a procedure call takes place are also known at compile time. Static allocation does have some limitations:
- Size of data objects, as well as any constraints on their positions in memory, must be available at compile time.
- No recursion, because all activations of a given procedure use the same bindings for local names.
- No dynamic data structures, since no mechanism is provided for run time storage allocation.

Run Time Storage Management

Run time allocation and de-allocation of activations occurs as part of procedure call and return sequences

Assume four types of statements

(*i*) call,

(*ii*) return,

(*iii*) halt

(*iv*) action

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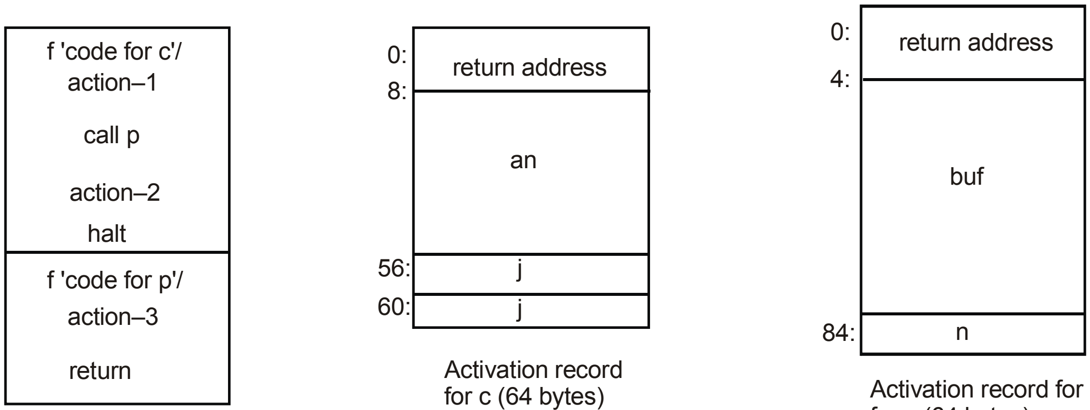


Three address code

Activation record for c (64 bytes)

Activation record for for c (64 bytes)

These by themselves give sufficient insight into the behavior shown by functions in calling each other and returning. The run-time allocation and de-allocation of activations occur on the call of functions and when they return.

There are mainly two types of run-time allocation systems:

(**1**) **Static allocation :** is used by the FORTRAN class of languages,

(**2**) **Stack allocation :** It used by the ADA class of languages.

INTERMEDIATE CODE GENERATION

Intermediate codes are machine independent codes, but they are close to machine instructions.

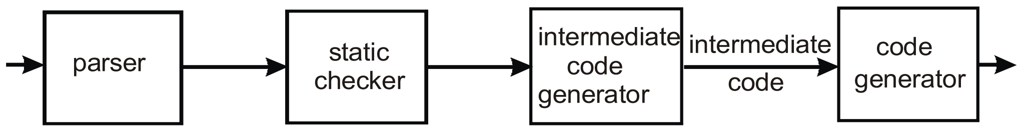
The given program in a source language is converted to an equivalent program in an intermediate language by the intermediate code generator.

Front end translates a program into intermediate representation.Its good to translate in IR because different back ends can be used to convert the same code in different destination language ,so it give an ease to the construction of different compiler. Therefore every compiler programmer wants its middle phase to be intermediate representation.

Advantages of IR

(1) Retargeting

(2) A machine independent optimizer to optimize produced intermediate code.



TYPES OF IR

(1) Syntax tree (DAG : a compact form of Syntax tree)

It depicts the hierarchical structure of source program

(2) Postfix

It is a lineralised representation of syntax tree

(3) Three address code

A three address code is a sequence of statements of general form:

$$Z = x \text{ op } y$$

Intermediate language

It can be many different languages, and designer of the compiler decides this intermediate language.

- Syntax trees can be used as an intermediate language.
- Postfix notation can be used as an intermediate language.
- Three-address code (Quadraples) can be used as an intermediate language

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Some programming languages have well defined intermediate languages :

(*i*) **JAVA** - Java virtual machine

(*ii*) **Prolog** - Warren abstract machine

In fact, there are byte-code emulators to execute instructions in these intermediate languages

Quadraples

Quadraples are close to machine instructions, but they are not actual machine instructions.

A quadraple is x := y op z

where x, y and z are names, constants or compiler-generated temporaries;op is any operator.

But we may also have following notation for quadraples (much better notation because it looks like a machine code instruction)

p y, z, x apply operator op to y and z, and store the result in x.

The term "*three-address code*" is used because each statement usually contains three addresses (two for operands, one for the result).

Three-Address Statements.

Binary Operator :

op y, z, resultor result := y op z

where op is a binary arithmetic or logical operator.

This binary operator is applied to

y and z, and the result of the operation is stored in result.

Unary Operator :

op y,,resultor result := op y

where op is a unary arithmetic or logical operator.

This unary operator is applied to y, and result of the operation is stored in result.

Syntax-Directed Translation into Three-Address.

Code :

S→id := E

S.code = E.code || gen('mov' E.place ',,' id.place)

E→E_1 + E_2

E.place = newtemp();

E.code = E_1.code || E_2.code || gen('add' E_1.place ',' E_2.place ',' E.place)

E→E_1 * E_2

E.place = newtemp();

E.code = E_1.code || E_2.code || gen('mult' E_1.place ',' E_2.place ',' E.place)

E→- E_1

E.place = newtemp();

E.code = E_1.code || gen('uminus' E_1.place ',,' E.place)

E→(E_1)

E.place = E_1.place;

E.code = E_1.code

E→ i d E.place = i d.place;

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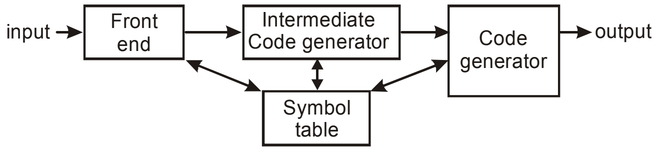
Translation scheme to produce Three-Address Code.

S→ id := E
{ p= lookup(id.name);
if (p is not nil) then emit('mov' E.place ',,' p)
else error("undefined-variable") }
E→ E_1 + E_2
{ E.place = newtemp();
emit('add' E_1.place ',' E_2.place ',' E.place) }
E→ E_1 * E_2
{ E.place = newtemp();
emit('mult' E_1.place ',' E_2.place ',' E.place) }
E→ - E_1
{ E.place = newtemp();
emit('uminus' E1.place ',,' E.place) }
E→ (E_1)
{ E.place = E_1.place; }
E→ id { p= lookup(id.name);
if (p is not nil) then E.place =id.place
else error("undefined-variable") }

Code generation and Instruction Selection

The final phase in any compiler is *code generator*. It takes input as an intermediate representation of the source program and produces as output an equivalent target program, as shown in the figure.



For compiler's correct working optimization phase is optional

Conditions for a Good compiler.

(1) Output code must be correct :

Meaning of the source and the target program must remain the same i.e., given an input, we should get same output both from the target and from the source program. We have no definite way to ensure this condition. What all we can do is to maintain a test suite and check.

(2) Output code must be of high quality :

Target code should make effective use of the resources of the target machine.

(3) Code generator must run efficiently :

It is also of no use if code generator itself takes hours or minutes to convert a small piece of code.

Design of Code generator

It considers following issues :

(1) Input to the code generator :

Input to the code generator consists of intermediate representation of the source program produced by the front end, together with the information in the symbol table that is used to determine runtime addresses of the data objects denoted by the names in the intermediate representation. We assume that prior to code generation, the input has been validated by the front end i.e., type checking, syntax, semantics etc. have been taken care of. Thus code generation phase can proceed on the assumption that the input is free of errors.

(2) Target programs. Output of the code generator is the target program. This output may take following forms :

(*i*) Absolute machine language :

Producing an absolute machine language as output has the advantage that it can be placed in a fixed location in memory and immediately executed. A small program can be thus compiled and executed quickly.

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(*ii*) Relocatable machine language :

Producing a relocatable machine code as output allows subprograms to be compiled separately. Although we must pay the added expense of linking and loading if we produce relocatable object modules, we gain a great deal of flexibility in being able to compile subroutines separately and to call other previously compiled programs from an object module.

(*iii*) Assembly language :

Producing an assembly code as output makes the process of code generation easier as we can generate symbolic instructions. The price paid is the assembling, linking and loading steps after code generation.

INSTRUCTION SELECTION

Nature of the instruction set of the target machine determines difficulty of instruction selection. Uniformity and completeness of the instruction set are important factors.

Instruction selection depends upon following factors :

(**1**) **Instructions used**, i.e. which instructions should be used in case there are multiple instructions that do the same job.

(**2**) **Uniformity**, i.e. support for different object/data types, what op-codes are applicable on what data types etc.

(**3**) **Completeness :** Not all source programs can be converted/translated in to machine code for all architectures/machines. E.g., 8086 doesn't support multiplication.

(**4**) **Instruction Speed :** This is needed for better performance.

(**5**) **Register Allocation :**

- Instructions involving registers are usually faster than those involving operands memory.
- Store long life time values that are often used in registers.

(**6**) **Evaluation Order :** The order in which the instructions will be executed. This increases performance of the code.

TARGET MACHINE

- **Addressing modes**

MODE	FORM	ADDRESS
Absolute	M	M
register	R	R
index	c(R)	c+cont(R)
indirect register	*R	cont(R)
indirect index	*c(R)	cont(c+cont(R))
literal	#c	c

Familiarity with the target machine and its instruction set is a prerequisite for designing a good code generator. Our target computer is a byte addressable machine with four bytes to a word and n general purpose registers, R_0, R_1,... R_{n-1} .

It has two address instructions of the form :

(*i*) source

(*ii*) destination

In which op is an op-code, and source and destination are data fields.

It has the following op-codes among others :

MOV (move source to destination)

ADD (add source to destination)

SUB (subtract source from destination)

The source and destination fields are not long enough to hold memory addresses, so certain bit patterns in these fields specify that words following an instruction

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Code Generation Algorithm

It takes as input a sequence of three-address statements constituting a basic block.

For each three-address statement of the form x := y op z, we perform following actions:

(1) Invoke a function getreg to determine location L, where result of the computation y op z should be stored. L will usually be a register, but it could also be a memory location. We shall describe getreg shortly.

(2) Consult the address descriptor for u to determine y', the current location(s) of y. Prefer the register for y' if value of y is currently both in memory and a register. If value of u is not already in L, generate the instruction MOV y', L to place a copy of y in L.

(3) Generate the instruction OP z', L where z' is a current location of z. Again, prefer a register to a memory location if z is in both. Update the address descriptor to indicate that x is in location L. If L is a register, update its descriptor to indicate that it contains the value of x, and remove x from all other register descriptors.

(4) If current values of y and/or y have no next uses, are not live on exit from the block, and are in registers, alter the register descriptor to indicate that, after execution of x := y op z, those registers no longer will contain y and/or z, respectively.

Function getreg

(1) If Y is in register (that holds no other values) and Y is not live and has no next use after

$$X = Y \text{ op } Z$$

then return register of Y for L.

(2) Failing (1) return an empty register

(3) Failing (2) if X has a next use in the block or op requires register then get a register R, store its content into M (by Mov R, M) and use it.

(4) Else select memory location X as L

Conditional Statements

Branch if value of R meets one of six conditions :

negative, zero, positive, non-negative, non-zero, non-positive

if X < Y goto Z	Mov X, R_0
	Sub Y, R_0
	Jmp negative Z

Condition codes:

Indicate whether last quantity computed or loaded into a location is negative, zero, or positive

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EXERCISE – I

MCQ TYPE QUESTIONS

1. Which of the following system program forgoes the production of object code to generate absolute machine code and load it into the physical main storage location from which it will be executed immediately upon completion of the assembly ?
(*a*) Two pass assembler
(*b*) Load-and-go assembler
(*c*) Macroprocessor
(*d*) Compiler

2. System program such as compiler are designed so that they are
(*a*) re-enterable
(*b*) non-reusable
(*c*) serially usable
(*d*) recursive

3. A system program that set-up an executable program in main memory ready for execution is
(*a*) assembler (*b*) linker
(*c*) loader (*d*) text editor

4. Assembler is a program that
(*a*) places programs into memory and prepares them for execution
(*b*) automates the translation of assembly language into machine language
(*c*) accepts a program written in a high level language and produces an object program.
(*d*) appears to execute a resource as if it were machine language.

5. An interpreter is a program that
(*a*) places programs into memory and prepares them for execution.
(*b*) automates the translation of assembly language into machine language.
(*c*) accesses a program written in a high level language and produces an object program.
(*d*) appears to execute a source program as if it were machine language.

6. A compiler is a program that
(*a*) places programs into memory and prepares them for execution.
(*b*) automates the translation of assembly language into machine language.
(*c*) accepts a program written in a high level language and produces an object program.
(*d*) appears to execute a source program as if it were machine language.

7. A loader is a program that
(*a*) places programs into memory and prepares them for execution.
(*b*) automates the translation of assembly language into machine language.
(*c*) accepts a program written in a high level language and produces as object program.
(*d*) appears to execute a source program as if it were machine language.

8. A compiler program written in a high level language is called
(*a*) source program
(*b*) object program
(*c*) machine language program
(*d*) none of these

9. Compiler can diagnose
(*a*) grammatical errors only
(*b*) logical errors only
(*c*) grammatical as well as logical errors
(*d*) neither grammatical nor logical errors

10. A programmer, by mistake, writes an instruction to divide, instead of a multiply, such error can be detected by *a*/*an*
(*a*) compiler
(*b*) interpreter
(*c*) compiler or interpreter test
(*d*) none of these

11. A series of statements explaining how the data is to be processed is called
(*a*) assembly (*b*) machine
(*c*) pascal (*d*) program

12. The computer language generally translated to pseudocode is
(*a*) assembly (*b*) machine
(*c*) pascal (*d*) PL I I

13. Language which have many types, but the type of every name and expression must be calculated at compile time are
(*a*) strongly-type languages
(*b*) weakly typed languages
(*c*) loosely typed languages
(*d*) none of these

14. A system program that setup an executable program in main memory ready for execution is
(*a*) assembler (*b*) linker
(*c*) loader (*d*) load and go

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15. A system program that combines separately compiled modules of a program into a form suitable for execution is
(*a*) assembler (*b*) linking loader
(*c*) cross compiler (*d*) load and go

16. Advantage of using assembly language rather than machine language is that
(*a*) it is mneomonic and easy to read
(*b*) addresses any symbolic not absolute
(*c*) introduction of data to program is easior
(*d*) all of these

17. Advantage of incorporating the macro-processor into pass I is that
(*a*) many functions donot have to be implemented twice.
(*b*) functions are combined and it is not necessary to create intermediate files as output from the macro-processor and input to the assembler.
(*c*) more flexibility is available to the programmer in that he may use all the features of the assembler in conjunction with macros.
(*d*) all of these

18. In which way a macro processor for assembly language can be implemented?
(*a*) Independent two-pass processor
(*b*) Independent one-pass processor
(*c*) Processor incorporated into pass 1 of a standard two pass assembler
(*d*) All of these

19. In an absolute loading scheme, which loader function is accomplished by programmer ?
(*a*) Allocation
(*b*) Linking
(*c*) Reallocation
(*d*) Both (*a*) and (*b*)

20. Uniform symbol table
(*a*) contains all constants in the program.
(*b*) is a permanent table of decision rules in the form of patterns for matching with the uniform symbol table to discover syntactic structure.
(*c*) consists of full or partial list of the token's as they appear in the program. Created by Lexical analysis and used for syntax analysis and interpretation.
(*d*) a permanent table which lists all key words and special symbols of the language in symbolic form.

21. Terminal table
(*a*) contains all constants in the program.
(*b*) is a permanent table of decision rules in the form of patterns for matching with the uniform symbol table to discover syntactic structure.
(*c*) consists of a full or partial list of the token is as they appear in the program created by lexical analysis and used for syntax analysis and interpretation.
(*d*) is a permanent table which lists all key words and special symbols of the language in symbolic form.

22. Which of the following is a phase of a compilation process ?
(*a*) Lexical analysis (*b*) Code generation
(*c*) Both (*a*) and (*b*) (*d*) None of these

23. Resolution is externally defined symbols is performed by
(*a*) Linker (*b*) Loader
(*c*) Compiler (*d*) Assembler

24. A simple two-pass assembler does which of the following in the first pass ?
(*a*) It allocates space for the literals.
(*b*) It computes the total length of the program.
(*c*) It builds the symbol table for the symbols and their values.
(*d*) All of these

25. Which of the following grammar rules violate the requirements of an operator grammar ? P, Q, R are nonterminals, and *r*, *s*, *t* are terminals.
(*i*) $P \rightarrow Q\ R$
(*ii*) $P \rightarrow Q\ s\ R$
(*iii*) $P \rightarrow \wedge$
(*iv*) $P \rightarrow Q\ t\ R\ r$
(*a*) (*i*) only (*b*) (*i*) and (*iii*) only
(*c*) (*ii*) and (*iii*) only (*d*) (*iii*) and (*iv*) only

26. The symbol table implementation is based on the property of locality of reference is
(*a*) linear list (*b*) search tree
(*c*) hash table (*d*) self-organization list

27. Access time of the symbol table will be logarithmic, if it is implemented by
(*a*) linear list (*b*) search tree
(*c*) hash table (*d*) self-organization list

28. Recursive descent parsing is an example of
(*a*) top-down parsers
(*b*) bottom-up parsers
(*c*) predictive parsing
(*d*) none of these

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29. In operator precedence parsing, precedence relations are defined
(*a*) for all pair of non-terminals
(*b*) for all pair of terminals
(*c*) to delimit the handle
(*d*) none of these

30. LR parsers are attractive because
(*a*) it can be constructed to recognize CFG corresponding to almost all programming constructs
(*b*) it does not backtrack
(*c*) both (*a*) and (*b*)
(*d*) none of these

31. The most powerful parser is
(*a*) SLR
(*b*) LALR
(*c*) Canonical LR
(*d*) Operator-precedence

32. YACC builds up
(*a*) SLR parsing table
(*b*) canonical LR parsing table
(*c*) LALR parsing table
(*d*) none of these

33. Object program is a
(*a*) program written in machine language.
(*b*) program to be translated into machine language.
(*c*) translation of high-level language into machine language.
(*d*) none of these

34. Let (Z,*) be an algebraic structure, where Z is set of integers and the operation* is defined by

$$n*m = \text{maximum}(n, m).$$

Which of the following statements is true for (Z, *)?
(*a*) (Z, *) is a monoid
(*b*) (Z, *) is an Abelian group
(*c*) (Z, *) is a group
(*d*) None of these

35. Which of the following translation program converts assembly language programs to object program ?
(*a*) Assembler (*b*) Compiler
(*c*) Macroprocessor (*d*) Linker

36. If E be a shifting operation applied to a function f, such that $E(f) = f(x + \beta)$, then
(*a*) $E(\alpha f + \beta g) = \alpha E(f) + \beta E(g)$
(*b*) $E(\alpha f + \beta g = (\alpha + \beta) E(f + g)$
(*c*) $E(\alpha f + \beta g) = \alpha E(f + g\beta)$
(*d*) $E(\alpha f + \beta g) = \alpha\beta E(f + g)$

37. In a single pass assembler, most of the forward references can be avoided by putting the restriction
(*a*) on the number of strings/lifereacts.
(*b*) that the data segment must be defined after the code segment.
(*c*) on unconditional rump.
(*d*) that the data segment be defined before the code segment.

38. The segment base is specified using the register named is
(*a*) ORG instructions
(*b*) TITLE instruction
(*c*) ASSUME instruction
(*d*) SEGMENT instruction

39. Which is not true about syntax and semantic parts of a computer language?
(*a*) Syntax is generally checked by the programmer.
(*b*) Semantics is the responsibility of the programmer.
(*c*) Semantics is checked mechanically by a computer.
(*d*) Both (*b*) and (*c*)

40. Left factoring is the process of factoring out common
(*a*) prefixed of alternates
(*b*) suffixes of alternates
(*c*) predictive parsing
(*d*) none of these

41. The method which merges the bodies of two loops is
(*a*) loop unrolling
(*b*) loop ramming
(*c*) constant folding
(*d*) none of these

42. Loop is a collection of nodes that is
(*a*) strongly connected
(*b*) loosely connected and has a unique entry
(*c*) strongly connected and has a unique entry
(*d*) none of these

43. Pass I
(*a*) assign address to all statements in the program
(*b*) save the values assigned to all labels for use in pass 2
(*c*) perform some processing of assembler directives
(*d*) all of these

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44. Pass II
(*a*) assemble instruction and generate data
(*b*) perform processing of assembler directives not done during pass 1
(*c*) write the object program and assembly listing
(*d*) all of these

45. The process manager has to keep track of
(*a*) status of each program
(*b*) priority of each program
(*c*) information management support to a programmer using the system.
(*d*) both (*a*) and (*b*)

46. Which of the following functions is performed by loader ?
(*a*) Allocate space in memory for the programs and resolve symbolic references between objects decks.
(*b*) Adjust all address dependent locations, such as address constants, to correspond to the allocated space.
(*c*) Physically place the machine instructions and data into memory.
(*d*) All of these.

47. In an absolute loading scheme, which loader function is accomplished by assembler ?
(*a*) Reallocation (*b*) Allocation
(*c*) Linking (*d*) Loading

48. In an absolute loading scheme, which loader function is accomplished by loader ?
(*a*) Reallocation (*b*) Allocation
(*c*) Linking (*d*) Loading

49. Assembly code data base is associated with
(*a*) assembly language version of the program which is created by the code.
(*b*) a permanent table of decision rules in the form of patterns for matching with the uniform symbol table to discover syntactic structure.
(*c*) consists of full or partial list or the token is as they appear in the program. Created by lexical analysis and used for syntax analysis and interpretation.
(*d*) a permanent table which lists all key words and special symbols of the language in symbolic form.

50. Which table is permanent databases that has an entry for each terminal symbol ?
(*a*) Terminal table
(*b*) Literal table
(*c*) Identifier table
(*d*) Reduction

51. The table created by lexical analysis to describe all literals used in the source program is
(*a*) terminal table
(*b*) literal table
(*c*) identifier table
(*d*) reductions

52. Function of the syntax phase is to
(*a*) recognize the major constructs of the language and to call the appropriate action routines that will generate the intermediate form or matrix for these constructs.
(*b*) build a literal table and a identifier table.
(*c*) build a uniform symbol table.
(*d*) parse the source program into the basic elements or tokens of the language.

53. Software that measures, monitors, analyzes and controls real world events is called
(*a*) System software (*b*) Real time software
(*c*) Scientific software (*d*) Business software

54. Dividing a project into segments and smaller units in order to simplify analysis, design and programming efforts is called
(*a*) modular approach
(*b*) top down approach
(*c*) bottom up approach
(*d*) left right approach

55. The part of the machine level instruction, which tells the central processor what has to be done is
(*a*) operation code (*b*) address
(*c*) locator (*d*) flip-flop

56. A linker is given object module for a set of programs that were compiled separately. What information need not be included in an object module ?
(*a*) Object code
(*b*) Relocation bits
(*c*) Names and locations of all external symbols defined in the object module ?
(*d*) Absolute addresses of internal symbols.

57. The root directory of a disk should be placed
(*a*) at a fixed address in main memory
(*b*) at a fixed location on the disk
(*c*) anywhere on the disk
(*d*) on a fixed location on system disk

58. Which of the following is true for machine language?
(*a*) Repeated execution of program segments
(*b*) Depicting flow of data in a system
(*c*) A sequence of instructions which, when followed properly, solves a problem
(*d*) The language which communicates with the computer using only the binary digits 1 and 0.

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59. In what module multiple instances of execution will yield the same result even if one instance has not terminated before the next one has begun ?

(*a*) Non reusable module
(*b*) Serially usable
(*c*) Re-entrable module
(*d*) Recursive module

60. The conditional expansion facility of macro procesors is provided to

(*a*) test a condition during the execution of the expanded program
(*b*) to expand certain model statements depending upon the value of a condition during the execution of the expanded program
(*c*) to implement recursion
(*d*) to expand certain model statements depending upon the value of a condition during the process of macro expansion.

61. Which of the following statement is ture?

(*a*) SLR parser is more powerful than LALR.
(*b*) LALR parser is more powerful than Canonical LR parser.
(*c*) Canonical LR parser is more powerful than LALR parser.
(*d*) The parsers SLR, Canonical LR, and LALR have the same power.

62. Which of the following features cannot be captured by CFG ?

(*a*) Syntax of if-then-else statements
(*b*) Syntax of recursive procedures
(*c*) Whether a variable is declared before its use
(*d*) Matching nested parenthesis

63. Which of the following strings can definitely be said to be tokens without looking at the next input character while compiling a Pascal program?

(*a*) Begin (*b*) Program
(*c*) < > (*d*) All of these

64. Relative mode of addressing is most relevant to writing

(*a*) coroutines
(*b*) position-independent code
(*c*) shareable code
(*d*) interrupt handlers

65. Type checking is normally done during

(*a*) lexical analysis
(*b*) syntax analysis
(*c*) syntax directed translation
(*d*) code optimization

66. An interpreter is preferred to a compiler because

(*a*) it is much helpful in the initial stages of program development
(*b*) debugging can be faster and easier
(*c*) both (*a*) and (*b*)
(*d*) none of these

67. A compiler is preferable to an interpreter because

(*a*) it can generate stand-alone programs that often take less time for execution
(*b*) it is much helpful in the initial stages of program development
(*c*) debugging can be faster and easier
(*d*) all of these

68. An ideal compiler should

(*a*) be smaller in size and take less time for compiling
(*b*) be written in a high level language
(*c*) produce object code that is smaller in size and executes faster
(*d*) all of these

69. In a syntax directed translation scheme, if value of an attribute of a node is a function of the values of the attributes of its children, then it is called

(*a*) synthesized attribute
(*b*) inherited attribute
(*c*) canonical attribute
(*d*) none of these

70. Inherited attribute is a natural choice in

(*a*) keeping track of variable declaration
(*b*) checking for the correct use of L-values and R-values
(*c*) both (*a*) and (*b*)
(*d*) none of these

71. Syntax directed translation scheme is desirable becuase

(*a*) it is based on the syntax
(*b*) its description is independent of any implementation
(*c*) it is easy to modify
(*d*) all of these

72. Any transcription error can be repaired by

(*a*) insertion alone
(*b*) deletion alone
(*c*) insertion and delection alone
(*d*) replacement alone

73. In which of the following no information hiding is done?

(*a*) compile prog 1, prog 2
(*b*) run test, prog
(*c*) load R_1, A
(*d*) 001001000010101

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74. The identification of common sub-expression and replacement of run-time computations by compile-time computations is
(*a*) local optimization
(*b*) loop optimization
(*c*) constant folding
(*d*) data flow analysis

75. The graph that shows basic blocks and their successor relationship is called
(*a*) DAG (*b*) Flow graph
(*c*) Control graph (*d*) Hamiltonion graph

76. A base register table
(*a*) allows the programmer to write base registers and displacements explicitly in the source program.
(*b*) is used to remember which of the following general purpose registers are currently available as base registers, and what base addresses they contain.
(*c*) allows multiple programs to reside in separate areas of care at the same time.
(*d*) is a term that refers to control programs of an O.S.

77. The specific tasks storage manager performs are
(*a*) allocation/deallocation of storage to programs
(*b*) protection of storage area allocated to a program from illegal access by other programs in the system.
(*c*) the status of each program
(*d*) both (*a*) and (*b*)

78. When a computer is first turned on or restarted, a special type of absolute loader is executed called
(*a*) "Compile and GO" loader
(*b*) Boot loader
(*c*) Boot strap loader
(*d*) Relating loader

79. Disadvantage of "*Compile and go*" loading scheme is that
(*a*) a portion of memory is wasted because the case occupied by the assembler is unavailable to the object program.
(*b*) it is necessary to retranslate the users program deck every time it is run.
(*c*) it is very difficult to handle multiple segments, especially if the source programs are in different languages and to produce orderly modular programs.
(*d*) all of these

80. Function of the storage assignment is
(*a*) assign storage to all variables referenced in the source program.
(*b*) assign storage to all temporary locations that are necessary for intermediate results.
(*c*) assign storage to literals, and to ensure that the storage is allocated and appropriate locations are initialized.
(*d*) all of these

81. A non relocatable program is one which
(*a*) cannot be made to execute in any area of storage other than the one designated for it at the time of its coding or translation.
(*b*) consists of a program and relevant information for its relocation.
(*c*) can itself perform the relocation of its address sensitive portions.
(*d*) all of these

82. A relocatable program form is one which
(*a*) cannot be made to execute in any area of storage other than the one designated for it at the time of its coding or translation.
(*b*) consists of a program and relevant information for its relocation.
(*c*) can itself perform the relocation of its address sensitive portions.
(*d*) all of these

83. A self relocating program is one which
(*a*) cannot be made to execute in any area of storage other than the one designated for it at the time of its coding or translation.
(*b*) consists of a program and relevant information for its relocation.
(*c*) can itself perform the relocation of its address-sensitive portions.
(*d*) all of these

84. Scissoring enables
(*a*) a part of data to be displayed
(*b*) entire data to be displayed
(*c*) full data display on full area of screen
(*d*) no data to be displayed

85. Which of the following can be accesses by transfer vector approach of linking ?
(*a*) External data segments
(*b*) External subroutines
(*c*) Data located in other procedure
(*d*) All of these

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86. Relocation bits used by relocating loader are specified by
(*a*) Relocating loader itself
(*b*) Linker
(*c*) Assembler
(*d*) Macro processor

87. Generation of intermediate code based on a abstract machine model is useful in compilers because
(*a*) it makes implementation of lexical analysis and syntax analysis easier.
(*b*) syntax-directed translations can be written for intermediate code generation.
(*c*) it enhances the portability of the front end of the compiler.
(*d*) it is not possible to generate code for real machines directly from high level language programs.

88. Which of the following module does not incorporate initialization of values changed by the module ?
(*a*) non reusable module
(*b*) serially reusable module
(*c*) re-entrable module
(*d*) all of these

89. In some programming languages, an identifier is permitted to be a letter followed by any number of letters or digits. If L and D denotes the sets of letters and digits respectively, which of the following expressions define an identifier ?
(*a*) $(L \cup D)^{+}$ (*b*) $L(L \cup D)^{*}$
(*c*) $(L . D)^{*}$ (*d*) $L.(L . D)^{*}$

90. A language L allows declaration of arrays whose sizes are not known during compilation. It is required to make efficient use of memory. Which one of the following is true?
(*a*) A compiler using static memory allocation can be written for L
(*b*) A compiler cannot be written for L; an interpreter must be used.
(*c*) A compiler using dynamic memory allocation can be written for L
(*d*) None of these

91. Match List I with List II and select correct answer from the codes given below the lists:

List I	List II
A. Activation record	1. Linking loader
B. Location counter	2. Garbage collection
C. Reference counts	3. Subroutine call
D. Address relocation	4. Assembler

Codes :

	A	B	C	D
(*a*)	3	4	1	2
(*b*)	4	3	1	2
(*c*)	4	3	2	1
(*d*)	3	4	2	2

92. What are x and y in the following macro definition?
```
macro          Add x, y
               Load y
               Mul x
               Store y
end macro
```
(*a*) variables
(*b*) identifiers
(*c*) actual parameters
(*d*) formal parameters

93. A language with string manipulation facilities uses the following operations
head (s) : first character of a string
tail (s) : all but the first character of a string
concat (s1, s2) : s1 s2
For the string acbc, what will be the output of
Concat (head (s), head (tail (tail(s))))?
(*a*) ac (*b*) bc
(*c*) ab (*d*) cc

94. Which of the following macros can put a macro assembler into an infinite loop?
(*a*)
```
MACRO          M1, X
IF EQ, X :     if X = 0 then .....
M1 X + 1
ENDC
IF NE, X :     if X ≠ O then .......
WORD X :       address (X) is stored here
ENDC
ENDM
```
(*b*)
```
MACRO M2, X
IF EQ, X
M2 X
ENDC
IF NE, X
WORD X + 1
ENDC
ENDM
```
(*c*) both (*a*) and (*b*)
(*d*) none of these

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95. Given the following Pascal-like program segment

```
Procedure A;
  x, y : integer;
  Procedure B;
        x, z : real;
              S1
  end B;
  Procedure C;
        i : integer;
              S2
  end C
  end A;
```

The variables accessible in S1 and S2 are

(*a*) x of A, y, x of B and z in S1 and x of B, y and i in S2

(*b*) x of B, y and z in S1 and x of B, i and z in S2

(*c*) x of B, z and y in S1 and x of A, i and y in S2

(*d*) None of these

96. An intermediate code form is

(*a*) postfix notation

(*b*) syntax trees

(*c*) three address codes

(*d*) all of these

97. Three address code involves

(*a*) exactly 3 address

(*b*) at the most 3 address

(*c*) no unary operators

(*d*) none of these

98. The best way to compare the different implementations of symbol table is to compare the time required to

(*a*) add a new name

(*b*) make an inquiry

(*c*) add a new name and make an inquiry

(*d*) all of these

99. Advantage of panic mode of error recovery is that

(*a*) it is simple to implement

(*b*) it never gets into an infinite loop

(*c*) both (*a*) and (*b*)

(*d*) none of these

100. To recover from an error, the operator precedence parser may

(*a*) insert symbols onto the stack and onto the input

(*b*) delete symbols from the stack

(*c*) delete symbols from the input

(*d*) all of these

101. Reduction in strength means

(*a*) replacing run-time computation by compile time computation

(*b*) removing loop invariant computation

(*c*) removing common sub-expressions

(*d*) replacing a costly operation by a relatively cheaper one

102. Running time of a program depends on

(*a*) the way the registers and addressing modes are used

(*b*) the order in which computations are performed

(*c*) the usage of machine idioms

(*d*) all of these

103. If control signals $\{a, b, c, d, e, f, g\}$ for some micro instructions and their corresponding MCCS (Maximum Compatibility Classes) are

$$\{a\,d\,f \cdot\; g, abd, bcg, b\,deg\}$$

then to determine minimal MCC cover, we need to remove

(*a*) *adfg* (*b*) *afd*

(*c*) *bcg* (*d*) *bdeg*

104. Assume that X and Y are nonzero positive integers. What does the following Pascal program segment do?

```
while     X < > Y do
if        X > Y then
          X : = X – Y
else
          Y : = Y – X;
write (X);
```

(*a*) Computes LCM of two numbers

(*b*) Divides larger number by the smaller number

(*c*) Computes GCD of two numbers

(*d*) None of these

105. A variant record in Pascal is defined by

```
type varirec = record
                number : integer;
                case (var1, var2) of
                var1 : (x, y : integer);
                var2 : (p.q. : real)
                end
end
```

Let an array of 100 records was declared on a machine which uses 4 bytes for an integer and 8 bytes for a real. How much space would the compiler have to reserve for the array?

(*a*) 2800 (*b*) 2400

(*c*) 2000 (*d*) 1200

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106. What is printed by the print statements in the program P1 assuming call by reference parameter passing?

```
Program P1()
{
        x = 10;
        y = 3;
        func 1(y, x, x);
        print x;
        print y;
}
func1 (x, y, z)
{
        y = y +4;
        z = x + y + z;
}
```

(*a*) 10, 3 (*b*) 31, 3
(*c*) 27, 7 (*d*) None of these

107. Consider the following three C functions:

```
[P1]            int * g (void)
              {
                int x = 10;
                 return (& x);
              }
[P2]            int * g (void)
              {
                int * px;
                *px = 10;
                return px;
              }
[P3]            int * g (void)
              {
                int * px;
                px = (int *) malloc (size of
(int));
                *px = 10;
                return px;
              }
```

Which of the above three functions are likely to cause problems with pointers?

(*a*) Only P3 (*b*) Only P1 and P3
(*c*) Only P1 and P2 (*d*) P1, P2, and P3

108. Consider the following program ;

```
        Program P2
            var n: int:
            procedure W (var x : int)
            begin
                    x = x + 1;
                    print x;
            end
            Procedure D
            begin
                    var n : int;
                    n = 3;
                    W (n);
            end
        begin           \\ begin P2
            no = 10;
                 D;
        end
```

If the language has dynamic scoping and parameters are passed by reference, what will be printed by the program?

(*a*) 10 (*b*) 11
(*c*) 3 (*d*) None of these

109. Which of the following does not interrupt a running process?

(*a*) A device (*b*) Timer
(*c*) Scheduler process (*d*) Power failure

110. A grammar will be meaningless

(*a*) if terminal set and non-terminal set are not disjoint
(*b*) if left hand side of a production is a single terminal
(*c*) if left hand side of a production has no non-terminal
(*d*) all of these

111. What does the following code do?

```
var a, b : integer;
begin
        a : = a + b;
        b : = a – b;
         a : = a – b
end;
```

(*a*) exchanges a and b
(*b*) doubles a and stores in b
(*c*) doubles b and stores in a
(*d*) leaves a and b unchanged

112. For the program segment given below, which of the following are true?

```
program main (output);
type link = ^ data;
data  = record
      d : real;
      n : link
end
var ptr : link;
begin
new (ptr);
```

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ptr : = nil;
.ptr^.d : = 5.2;
write ln (ptr)
end.
The program
(*a*) leads to compile time error
(*b*) leads to run time error
(*c*) outputs 5.2
(*d*) produces error relating to nil pointer dereferencing

113. In analyzing the compilation of PL/I program, the term "*Machine independent optimization*" is associated with
(*a*) recognition of basic syntactic construction through reductions.
(*b*) recognition of basic elements and creation of uniform symbols.
(*c*) creation of more optimal matrix.
(*d*) use of macro processor to produce more optimal assembly code.

114. In analyzing the compilation of PL/I program the description "*resolving symbolic address (lables) and generating machine language*" is associated with
(*a*) assembly and output (*b*) code generation
(*c*) storage assignment (*d*) syntax analysis

115. In analyzing the compilation of PL/I program the description "*creation of more optimal matrix*" is associated with
(*a*) assembly and output
(*b*) code generation
(*c*) syntax analysis
(*d*) machine independent optimization

116. Peephole optimization is a form of
(*a*) loop optimization
(*b*) local optimization
(*c*) constant folding
(*d*) data flow analysis

117. Substitution of values for names whose values are constant, is done in
(*a*) local optimization
(*b*) loop optimization
(*c*) constant folding
(*d*) none of these

118. Local and loop optimization inturn provide motivation for
(*a*) data flow analysis
(*b*) constant folding
(*c*) peephole optimization
(*d*) DFA and constant folding

119. A compiler for a high level language that runs on one machine and produces code for a different machine is called
(*a*) optimizing compiler
(*b*) one pass compiler
(*c*) cross compiler
(*d*) multipass compiler

120. A linker reads four modules whose lengths are 200, 800, 600 and 500 words, respectively. If they are loaded in that order, what are the relocation constants?
(*a*) 0, 200, 500, 600
(*b*) 0, 200, 1000, 1600
(*c*) 200, 500, 600, 800
(*d*) 200, 700, 1300, 2100

121. An optimizing compiler
(*a*) is optimized to occupy less space
(*b*) is optimized to take less time for execution
(*c*) optimizes the code
(*d*) none of these

122. Whether a given pattern constitutes a token or not, depends on the
(*a*) source language
(*b*) target language
(*c*) compiler
(*d*) all of these

123. The optimization technique which is typically applied on loops is
(*a*) removal of invariant computation
(*b*) peephole optimization
(*c*) constant folding
(*d*) all of these

124. Ud-chaining is useful for
(*a*) determining whether a particular definition is used anywhere or not
(*b*) constant folding
(*c*) checking whether a variable is used, without prior assignment
(*d*) all of these

125. Concept which can be used to identify loops is
(*a*) dominators
(*b*) reducible graphs
(*c*) depth first ordering
(*d*) all of these

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126. The expression (a *b) * c op ....

where ' op' is one of '+', '*' and '↑' (exponentiation) can be evaluated on a CPU with a single register without storing the value of (a * b) if

(*a*) 'op' is '+' or '*'

(*b*) 'op' is '↑' or '*'

(*c*) 'op' is '↑' or '+'

(*d*) not possible to evaluate without storing

127. Given the programming constructs

(*i*) assignment

(*ii*) for loops where the loop parameter cannot be changed within the loop

(*iii*) If-then-else

(*iv*) forward goto

(*v*) arbitrary goto

(*vi*) non-recursive procedure call

(*vii*) recursive procedure/function call

(*viii*) repeat loop

Which constructs will you not include in a programming language such that it should be possible to program the terminates (i.e. halting) function in the same programming language?

(*a*) (*ii*), (*iii*), (*iv*) (*b*) (*v*), (*vii*), (*viii*)

(*c*) (*vi*), (*vii*), (*viii*) (*d*) (*iii*), (*vii*), (*viii*)

128. Consider SLR(1) and LR(0), which of the following is true?

(*a*) Goto of both tables may be diff.

(*b*) Shift entries are identical in both tables.

(*c*) Reduces entries may be diff. in both tables.

(*d*) None

129. Consider the following grammer

$S \rightarrow C'C'$

$C' \rightarrow CC/d$

Which one is true?

(*a*) It is LL(1)

(*b*) SLR (1) but not LL(1)

(*c*) LL(1) but not SLR(1)

(*d*) Both SLR(1) and LL(1)

130. Which of the following is true ?

(*a*) A left recursive grammer can never be LL(1) but can be LR(0).

(*b*) A left recursive grammer can be LL(1) but not LR(0).

(*c*) A left recursive grammer can be LL(1) and LR(0).

(*d*) A left recursive grammer can never be LL(1) nor LR(0)

131. $A \rightarrow AA/a/E$

For converting it to LR(0), which is better to eliminate?

(*a*) Anbiguity (*b*) Left recursion

(*c*) Both a & b (*d*) None

132. Which of the following strings can definitery be said to be tokens without looking or new i/p character.

(*a*) main (*b*) ib

(*c*) < > (*d*) All

133. For n variables and m terminals, what is the size of LL(1). Parsing table?

(*a*) n.m (*b*) n(m + 1)

(*c*) (n + 1)m (*d*) (n + 1) (m + 1)

NUMERICAL TYPE QUESTIONS

1. A shift reduce parser carries out the actions specified within braces immediately after reducing with the corresponding rule of grammar

$S \rightarrow$ xxW {print "1"}

$S \rightarrow$ y { print "2"}

$W \rightarrow$ Sz {print "3"}

The translation of xxxxyzz using the syntax directed translation scheme described by the above rules is ________

2. The value of X printed by the following program is _____

```
program COMPUTE ( input, output);
var
        X : integer;
procedure FIND (X: real);
        begin
        X : = sqrt (X);
        end;
begin
        X : = 2
        FIND (X)
        writeln (X)
end
```

3. The number of token is ___________.

4. Check whether the following grammer is ambiguous or not

$R \rightarrow R + R$ / RR/R * /a/b

W = a * b + a

5. Find whether the following grammer in LL (1) or not

6. Check whether the following grammer is LR(O) or not

$E \rightarrow T + E/T$

$T \rightarrow i$

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EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. Which one of the following is TRUE at any valid state in shift – reduce parsing ?

(*a*) Viable prefixes appear only at the bottom of the stack and not inside

(*b*) Viable prefixes appear only at the top of the stack and not inside

(*c*) The stack contains only a set of viable prefixes

(*d*) The stack never contains viable prefixes

2. For computers based on three-address instruction formats, each address field can be used to specify which of the following.

S1 : A memory operand

S2 : A processor register

S3 : An implied accumulator register

(*a*) Either S1 or S2

(*b*) Either S2 or S3

(*c*) only S2 and S3

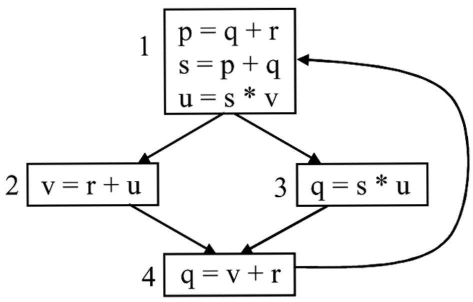
(*d*) All of S1, S2 and S3

3. A variable x is said to be live at a statement S_i in a program if the following three conditions hold simultaneously :

I. There exists a statement S_j that uses x

II. There is a path from S_i to S_j in the flow graph corresponding to the program

III. The path has no intervening assignment to x including at S_i and S_j



The variables which are live at the statement in basic block 2 and at the statement in basic block 3 of the above control flow graph are

(*a*) p, s, u

(*b*) r, s, u

(*c*) r, u

(*d*) q, v

4. In the context of abstract-syntax-tree (AST) and control-flow-graph (CFG), which one of the following is TRUE?

(*a*) In both AST and CFG, let node, N_2 be the successor of node N_1. In the input program, the code corresponding to N_2 is present after the code corresponding in N_1.

(*b*) For any input program, neither AST nor CFG will contain a cycle

(*c*) The maximum number of successors of a node in an AST and a CFG depends on the input program

(*d*) Each node is AST and CFG corresponds to at most one statement in the input program

5. Match the following :

(P) Lexical analysis	1. Graph coloring
(Q) Parsing	2. DFA minimization
(R) Register allocation	3. Post-order traversal
(S) Expression evaluation	4. Production tree

(*a*) P – 2, Q – 3, R – 1, S – 4

(*b*) P – 2, Q – 1, R – 4, S – 3

(*c*) P – 2, Q – 4, R – 1, S – 3

(*d*) P – 2, Q – 3, R – 4, S – 1

6. Consider the intermediate code given below.

1. i = 1	2. j = 1
3. t1 = 5 * i	4. t2 = t1 + j
5. t3 = 4 * t2	6. t4 = t3
7. a[t4] = –1	8. j = j + 1
9. if j <= 5 goto (3)	10. i = i + 1
11. if i < 5 goto 2	

The number of nodes and edges in the control-flow-graph constructed for the above code, respectively, are

(*a*) 5 and 7 (*b*) 6 and 7

(*c*) 5 and 5 (*d*) 7 and 8

7. Among simple LR (SLR), canonical LR, and look ahead LR (LALR), which of the following pairs identify the method that is very easy to implement and the method that is the most powerful, in that order

(*a*) SLR. LALR

(*b*) Canonical LR, LALR

(*c*) SLR, canonical LR

(*d*) LALR, canonical LR

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2014

8. A canonical set of items is given below

$S \to L. > R$

$Q \to R.$

On input symbol < the set has

(*a*) a shift-reduce conflict and a reduce-reduce conflict.

(*b*) a shift-reduce conflict but not a reduce-reduce conflict.

(*c*) a reduce-reduce conflict but not a shift-reduce conflict.

(*d*) neither a shift-reduce nor a reduce-reduce conflict.

9. Which one of the following is NOT performed during compilation?

(*a*) Dynamic memory allocation

(*b*) Type checking

(*c*) Symbol table management

(*d*) Inline expansion

10. One of the purposes of using intermediate code in compilers is to

(*a*) make parsing and semantic analysis simpler.

(*b*) improve error recovery and error reporting

(*c*) increase the chances of reusing the machine-independent code optimizer in other compliers.

(*d*) improve the register allocation.

11. Consider the basic block given below.

$a = b + c$

$c = a + d$

$d = b + c$

$e = d - b$

$a = e + b$

The minimum number of nodes and edges present in the DAG representation of the above basic block respectively are

(*a*) 6 and 6 (*b*) 8 and 10

(*c*) 9 and 12 (*d*) 4 and 4

2013

12. Consider the languages $L_1 = \Phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1 L_2^* \cup L_1^*$?

(*a*) $\{\varepsilon\}$ (*b*) Φ

(*c*) a^* (*d*) $\{\varepsilon, a\}$

13. What is the maximum number of reduce moves that can be taken by a bottom-up parser for a grammar with no epsilon- and unit-production (i.e., of type $A \to \varepsilon$ and $A \to a$) to parse a string with n tokens?

(*a*) $n/2$ (*b*) $n - 1$

(*c*) $2n - 1$ (*d*) 2^n

14. Consider the following languages.

$L_1 = \{0^p 1^q 0^r \mid p, q, r \geq 0\}$

$L_2 = \{0^p 1^q 0^r \mid p, q, r \geq 0, p \neq r\}$

Which one of the following statements is FALSE?

(*a*) L_2 is context-free.

(*b*) $L_1 \cap L_2$ is context-free.

(*c*) Complement of L_2 is recursive.

(*d*) Complement of L_1 is context-free but not regular.

15. Consider the following two sets of LR(1) items of an LR(1) grammar.

$X \to c.X, c/d$	$X \to c.X, \$$
$X \to .cX, c/d$	$X \to .cX, \$$
$X \to .d, c/d$	$X \to .d, \$$

Which of the following statements related to merging of the two sets in the corresponding LALR parser is/are **FALSE**?

1. Cannot be merged since look aheads are different.
2. Can be merged but will result in S-R conflict.
3. Can be merged but will result in R-R conflict.
4. Cannot be merged since goto on c will lead to two different sets.

(*a*) 1 only

(*b*) 2 only

(*c*) 1 and 4 only

(*d*) 1, 2, 3 and 4

2012

16. Which of the following problems are decidable?

1. Does a giv.en program ever produce an output?
2. If L is a context-free language, then, is $\bar{L}$ also context-free?
3. If L is a regular language, then, is $\bar{L}$ also regular?
4. If L is a recursive language, then, is $\bar{L}$ also recursive?

(*a*) 1, 2, 3, 4 (*b*) 1, 2

(*c*) 2, 3, 4 (*d*) 3, 4

17. Consider the program given below, in a block-structured pseudo-language with lexical scoping and nesting of procedures permitted.

```
Program main;
    Var ---
    Procedure A1;
        Var ...
        Call A2;
    End A1
```

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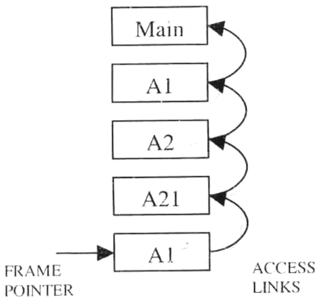
```
    Procedure A2;
        Var ...
        Procedure A21;
            Var ...
            Call A1;
        End A21
        Call A21;
    End A2
    Call A1;
End main.
```

Consider the calling chain:

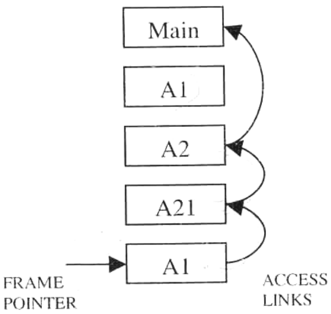
Main $\rightarrow$ A1 $\rightarrow$ A2 $\rightarrow$ A21 $\rightarrow$ A1

The correct set of activation records along with their access links is given by

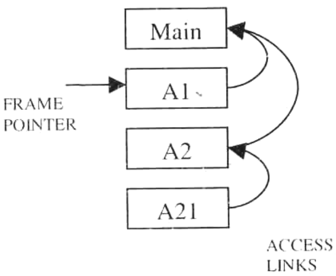
(*a*)



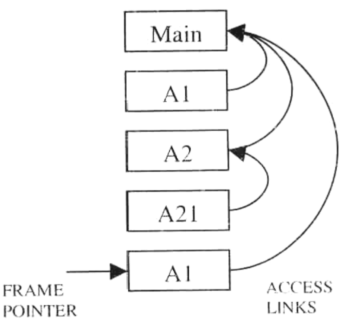
(*b*)



(*c*)



(*d*)



2011

18. In a compiler, keywords of a language are recognized during

(*a*) parsing of the program

(*b*) the code generation

(*c*) the lexical analysis of the program

(*d*) dataflow analysis

2010

19. Which data structure in a compiler is used for managing information about variables and their attributes?

(*a*) Abstract syntax tree

(*b*) Symbol table

(*c*) Semantic stack

(*d*) Parse table

20. Which languages necessarily need heap allocation in the runtime environment?

(*a*) Those that support recursion

(*b*) Those that use dynamic scoping

(*c*) Those that allow dynamic data structures

(*d*) Those that use global variables

21. The grammar $S \rightarrow aSa \mid bS \mid c$ is

(*a*) LL(1) but not LR(1)

(*b*) LR(1) but not LL(1)

(*c*) Both LL(1) and LR(1)

(*d*) Neither LL(1) nor LR(1)

2009

22. What of the following statements are TRUE ?

I. There exist parsing algorithms for some pro gramming languages whose complexities are less than $\theta(n^3)$.

II. A programming language which allows recur sion can be implemented with static storage allocation.

III. No L-attributed definition can be evaluated in the framework of bottom-up parsing.

IV. Code improving transformations can be per formed at both source language and itermediate code level.

(*a*) I and II (*b*) I and IV

(*c*) III and IV (*d*) I, III and IV

2008

23. Which of the following describes a handle (as applicable to LR-parsing) appropriately?

(*a*) It is the position in a sentential form where the next shift or reduce operation will occur

(*b*) It is a non-terminal whose production will be used for reduction in the next step

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(*c*) It is a production that may be used for reduction in a future step along with a position in the sentential form where the next shift or reduce operation will occur

(*d*) It is the production p that will be used for reduction in the next step along with a position in the sentential form where the right hand side of the production may be found

24. Some code optimizations are carried out on the intermediate code because

(*a*) they enhance the portability of the compiler to other target processors

(*b*) program analysis is more accurate on intermediate code than on machine code

(*c*) the information from dataflow analysis cannot otherwise be used for optimization

(*d*) the information from the front end cannot otherwise be used for optimization

25. Which of the following are true?

I. A programming language which does not permit global variables of any kind and has no nesting of procedures/functions, but permits recursion can be implemented with static storage allocation

II. Multi-level access link (or display) arrangment is needed to arrange activation records only if the programming language being implemented has nesting of procedures/functions

III. Recursion in programming languages cannot be implemented with dynamic storage allocation

IV. Nesting of procedures/functions and recursion require a dynamic heap allocation scheme and cannot be implemented with a stack-based allocation scheme for activation records

V. Programming languages which permit a function to return a function as its result cannot be implemented with a stack-based storage allocation scheme for activation records

(*a*) II and V only

(*b*) I, III and IV only

(*c*) I, II and V only

(*d*) I, III and V only

26. An LALR(1) parser for a grammar G can have shift-reduce (S-R) conflicts if and only if

(*a*) the SLR(1) parser G has S-R conflicts

(*b*) The LR(1) paser for G has S-R conflicts

(*c*) The LR(0) parser for G has S-R conflicts

(*d*) The LALR(1) parser for G has reduce-reduce conflicts

2007

27. Which one of the following is a top-down parser ?

(*a*) Recursive descent parser

(*b*) Operator precedence parser

(*c*) An LR(k) parser

(*d*) An LALR(k) parser

28. Consider the grammer with non-terminals N = {S, C, S_1}, terminals T = {*a, b, i, t, e*}, with S as the start symbol, and the following set of rules

$$S \to iCtSS_1 \mid a$$
$$S_1 \to eS \mid \varepsilon$$
$$C \to b$$

The grammar is **NOT** LL(1) because :

(*a*) It is left recursive (*b*) it is right recursive

(*c*) it is ambiguous (*d*) it is not context-free

29. Consider the following two statements :

P : Every regular grammar is LL (1)

Q : Every regular set has a LR (1) grammar

Which of the following is **TRUE** ?

(*a*) Both P and Q are true

(*b*) P is true and Q is false.

(*c*) P is false and Q is true.

(*d*) Both P and Q are false.

30. In a simplified computer the instructions are :

OP R_j, R_i – Performs R_j OP R_i and stores the result in register R_i.

OP *m*, R_i – Performs *val* OP R_i and stores the result in R_i. *val* denotes the content of memory location *m*.

MOV *m*, R_i – Moves the content of memory location *m* to register R_i.

MOV R_i,*m* – Moves the content of register R_i to memory location *m*.

The computer has only two registers, and OP is either ADD or SUB. Consider following basic block :

$$t_1 = a + b$$
$$t_2 = c + d$$
$$t_3 = e - t_2$$
$$t_4 = t_1 - t_3$$

Assume that all operands are initially in memory. The final value of the computation should be in memory. What is the minimum number of MOV instructions in the code generated for this basic block ?

(*a*) 2 (*b*) 3

(*c*) 5 (*d*) 6

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2006

31. Consider the following grammar :

$$S \to S^*E$$
$$S \to E$$
$$E \to F + E$$
$$E \to F$$
$$F \to id$$

Consider the following LR(0) items corresponding to the grammar above :

(*i*) $S \to S * .E$ (*ii*) $E \to F. + E$

(*iii*) $E \to F + .E$

Given the items above, which two of them will appear in the same set in the canonical sets-of-items for the grammar ?

(*a*) (*i*) and (*ii*) (*b*) (*ii*) and (*iii*)

(*c*) (*i*) and (*iii*) (*d*) None of these

32. Consider the following translation scheme :

$S \to ER$

$R \to {}^*E$ {print ('*'); R | ε

$E \to F + E$ {print ('+'); | F

$F \to$ (S) | *id* {print (*id*. value);}

Here *id* is a taken that represents an integer and *id. value* represents the corresponding integer value. For an input '2 * 3 + 4', this translation scheme prints

(*a*) 2 * 3 + 4 (*b*) 2 * + 3 4

(*c*) 2 3 * 4 + (*d*) 2 3 4 + *

33. Consider the following C code segment :

```
for (i = 0; i < N ; i ++ )
        for (j = 0; j < N ; j ++)
            if (i % 2)
                    {x + = (4 * j + 5 * i);
                     y + = (7 + 4 * j);
                    }
            }
}
```

Which one of the following is false ?

(*a*) The code contains loop-in variant computation

(*b*) There is scope of common sub-expression elimination in this code

(*c*) There is scope of strength reduction in this code

(*d*) There is scope of dead code elimination in this code

NUMERICAL TYPE QUESTIONS

2015

1. The least number of temporary variables required to create a three-address code in static single assignment form for the expression $q + r/3 + s - t * 5 + u * v/w$ is ______.

ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*b*)	**2.** (*a*)	**3.** (*c*)	**4.** (*b*)	**5.** (*d*)	**6.** (*c*)	**7.** (*a*)	**8.** (*a*)	**9.** (*a*)	**10.** (*a*)
11. (*d*)	**12.** (*a*)	**13.** (*a*)	**14.** (*c*)	**15.** (*b*)	**16.** (*d*)	**17.** (*d*)	**18.** (*d*)	**19.** (*d*)	**20.** (*c*)
21. (*d*)	**22.** (*c*)	**23.** (*a*)	**24.** (*d*)	**25.** (*a*)	**26.** (*c*)	**27.** (*b*)	**28.** (*b*)	**29.** (*a*)	**30.** (*d*)
31. (*c*)	**32.** (*c*)	**33.** (*c*)	**34.** (*d*)	**35.** (*a*)	**36.** (*a*)	**37.** (*b*)	**38.** (*a*)	**39.** (*d*)	**40.** (*a*)
41. (*b*)	**42.** (*c*)	**43.** (*d*)	**44.** (*d*)	**45.** (*d*)	**46.** (*d*)	**47.** (*a*)	**48.** (*d*)	**49.** (*a*)	**50.** (*a*)
51. (*b*)	**52.** (*a*)	**53.** (*b*)	**54.** (*a*)	**55.** (*a*)	**56.** (*d*)	**57.** (*b*)	**58.** (*d*)	**59.** (*c*)	**60.** (*d*)
61. (*c*)	**62.** (*d*)	**63.** (*c*)	**64.** (*b*)	**65.** (*c*)	**66.** (*a*)	**67.** (*c*)	**68.** (*a*)	**69.** (*a*)	**70.** (*a*)
71. (*c*)	**72.** (*c*)	**73.** (*d*)	**74.** (*c*)	**75.** (*b*)	**76.** (*b*)	**77.** (*d*)	**78.** (*c*)	**79.** (*d*)	**80.** (*d*)
81. (*a*)	**82.** (*c*)	**83.** (*c*)	**84.** (*a*)	**85.** (*b*)	**86.** (*b*)	**87.** (*a*)	**88.** (*a*)	**89.** (*b*)	**90.** (*c*)
91. (*d*)	**92.** (*d*)	**93.** (*c*)	**94.** (*b*)	**95.** (*b*)	**96.** (*d*)	**97.** (*d*)	**98.** (*d*)	**99.** (*d*)	**100.** (*c*)
101. (*a*)	**102.** (*d*)	**103.** (*b*)	**104.** (*c*)	**105.** (*c*)	**106.** (*a*)	**107.** (*b*)	**108.** (*c*)	**109.** (*d*)	**110.** (*a*)
111. (*a*)	**112.** (*b,d*)	**113.** (*c*)	**114.** (*a*)	**115.** (*d*)	**116.** (*b*)	**117.** (*c*)	**118.** (*a*)	**119.** (*c*)	**120.** (*c*)
121. (*d*)	**122.** (*c*)	**123.** (*d*)	**124.** (*a*)	**125.** (*d*)	**126.** (*c*)	**127.** (*a*)	**128.** (*b,c*)	**129.** (*d*)	**130.** (*a*)
131. (*a*)	**132.** (*d*)	**133.** (*b*)							

Numerical Type Questions

1. 23131	**2.** 2	**3.** 7	**4.** Yes/ No	**5.** Yes/ No	**6.** Yes/ No

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EXERCISE – II

MCQ Type Questions

1. (*c*)	**2.** (*a*)	**3.** (*c*)	**4.** (*c*)	**5.** (*c*)	**6.** (*b*)	**7.** (*c*)	**8.** (*d*)	**9.** (*a*)	**10.** (*c*)
11. (*a*)	**12.** (*d*)	**13.** (*a*)	**14.** (*d*)	**15.** (*d*)	**16.** (*d*)	**17.** (*d*)	**18.** (*c*)	**19.** (*b*)	**20.** (*c*)
21. (*c*)	**22.** (*c*)	**23.** (*d*)	**24.** (*a*)	**25.** (*d*)	**26.** (*b*)	**27.** (*a*)	**28.** (*c*)	**29.** (*a*)	**30.** (*b*)
31. (*c*)	**32.** (*d*)	**33.** (*b*)							

Numerical Type Questions

1. (8)

EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. Load and go assembly for goes the production of object code to generate absolute machine code.

2. Compiler are designed so that they are re-enterable.

3. Loader is a system program that set up an executable program.

4. Assembler is a program that automatis the translation of assembly language into m/c language.

5. An interforces is a program that attempts to execute a source program as if it were machine language.

6. A compiler is a program that accepts a program in high level language and produces an object program.

7. A loader is a program that places program into memory and prepares them for execution.

8. A compiler program written in high level language in called source program.

9. Compiler can diagnose gramatical errors only.

10. A programmer by mistake writes an instruction to divide instead of multiply, such an error can be detected by an compiler.

11. A series of statements explaining how the data is to be processed is called program.

12. The computer language generally translated to pseudo code is assembly.

13. Such languages are called strongly type language.

14. The system program is loader.

15. Such a system program is called linking loader.

16. The advantages are all of the above.

17. The advantages are all of the above.

18. Microprocessor for assembly language can be implemented in all the three given ways.

19. Allocation and linking is accomplished by programmer.

20. Uniform symbol tables consists of few or partial lists of the token's as they appear in the program created by lexical analysis and used for syntax analysis and interpretation.

21. Terminal table in a permanent table which lists all key words and special symbols of the language in symbolic form.

22. Phaser of a complitation process are levical analysis and code generation.

23. Resolution is performed by linker.

24. A simple two pass assembler does all of the given task.

25.
$$P \to QR$$
$$P \to Q\ SR$$
$$P \to \wedge$$
$$P \to Q\ t\ R\ r$$

$P \to QR$ violate the condition of operator because no terminal symbol.

26. Symbol table implementation based on locality of reference in hash table.

27. If memory space is not the constraint, then by increasing the number of bins to K, the access time can be reduced by a factor of K. So, average number of items in a bin will decrease as the number of bins increases. In the case of list, access time will be proportional to n, the number of items, but we will be using as much memory space as is absolutely necessary. In the case of search tree implementation, the access time will be logarithmic.

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28. Any shift-reduce parser typically works by shifting entries onto the stack. If a handle is found in the top of the stack, it is popped and replaced by the corresponding left hand side of the production. If ultimately we have only the starting non-terminal on the stack, when there are no more tokens to be scanned, the parsing will be successful. So, it is bottom-up.

29. To operater precedence parsing, precedence relations are defined for all pair of non-terminals.

31. The most powerful parses in canonical LR.

32. YACC being up LALR parsing table.

33. Object program is a translation of high level language into machine language.

35. Assembler is a program which converts assembly language program to object program.

36. $$E(f) = f(x + \beta)$$

then for shifting operation,

$$E(\alpha f + \beta g) = \alpha E(f) + \beta E(g)$$

37. Forward reference are avoided by putting restriction that the data segment must be defined after the code segment.

38. The segment base is specified using the register named an ORG instruction.

39. (*b*) & (c) are not true about system and semantic parts.

40. Left factor is the process of factoring out common prefired of alternates.

41. The method which marges the bodies of two loops is called loop ramming.

42. Loop is a collection of nodes that is strongly connected and has a unique entry.

43. Pass I perform all of the given functions.

44. Pass II perform all of the given functions.

45. The process manager has to keep track of both (*a*) & (*b*).

46. Loader perform all of the given function.

47. Reallocation is accomplished by assembles.

48. Loading is accomplished by loader.

49. Assembly code data base in associated with assembly language version of the program which is created by the code.

50. Terminal table is permanent database that has an entry for each terminal symbol.

51. Literal table is created by lexical analysis.

52. Syntax phase recognize the major constructs of the language and to call the appropriate action routiner that will generate the intermediate form or matrix for these constructs.

53. Such softwere is called real time software.

54. Dividing a project is called modular approach.

55. Operation code tells the central processor what has to be done.

56. Absolute addresses of internal symbols need not be included in an object module.

57. The root directory of a disk should be placed at a fixed location on the disk.

58. Machine language is a language which communicate with the computer using only the binary digits 1 & 0.

59. Such module is called re-enterable module.

60. The facility is provided to expand certain model statements depending upon the value of a condition during the process of macro expansion.

61. LR is powerful than LALR parsers and LALR are powerful than SLR parsers.

62. It is because, it is equivalent to recognizing wcw, where the first w is the declaration and the second is its use. wcw is not a CFG.

63. < > is definitely a token.

64. Relative mode of addressing in most relevant to writing position independent code.

65. Type checking is normally done during syntax directed translation.

66. An interpretes is prefered because it is much helpful in initial stages of program developement.

67. A compiler is preferred because debugging can be faster and easier.

68. A ideal compiler should be small in size and take ten time for compiling.

69. It is called synthesized attribute.

70. Inherited attribute is a natural choice in keeping track a variable declaration.

71. Syntax directed translation scheme is desirable because it is easy to modify.

72. Any transcription error can be repaired by insertion and deletion alone.

73. No information hiding is done in 001001000010101

74. The identification is called constant folding.

75. The graph that shows basic blocks and their successor relationship is called flow graph.

76. A base register table is used to remember which of the following general purpose registers are curently available as base registers and what base addresses they contain.

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77. The specific task storage manager perform both (*a*) & (*b*).

78. Loader executed is called boot strap loader.

79. Disadvantages of compile and go loading scheme is all of the above.

80. Function of the storage assignment is all of the given.

81. A non-relocatable program is one which cannot be made to execute in any area of storage other than the one designated for it at the time of its coding or translation.

82. A relocatable program form is one which can itself of perform the relocation of its addresses sensitive portions.

83. A self relocating program is one which can itself perform the relocation of its addresses sensitive portions.

84. Scissoring enables a part of data to be displayed.

85. External subroutines can be accesses by transfer vector approach of linking.

86. Relocation bits used by relocating loader are specified by linker.

87. It makes implementation of lexical analysis and syntax analysis easier.

88. Non-reusable module doesnot incorporate initialization of values changed by the module.

89. The identifier is L(LUD)*

90. A compiter using dynamic memory allocation can be written for L.

91.

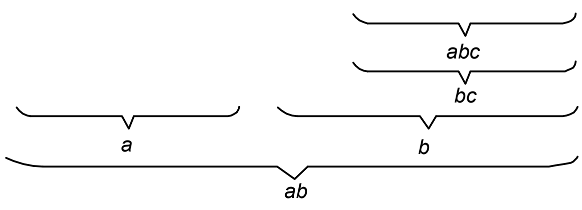
Activation record	Subroutine call
Location counter	Assembles
Reference counts	Garbage collection
Address relocation	Linking loader.

92. X and Y are formal parameters.

93. Consider

s = acbc

and acbc acbc



Concat (head(s), head (tail (tail(s))))

96. An intermediate code form is all of the given.

97. Three address code involves none of the above.

98. The best way to compare is all of the given.

99. Advantage of panic mode of error recovery is none of the given.

100. To recover from error, the operator precedence parser may delete symbols from the input.

101. Reduction is strength means replacing run time computation by compile time computation.

102. Running time of program depends upon all of the given.

103. We need to remove afd.

104. To computes GCD of two number.

105. The array has to reserve 2000 space.

106. X = 10

Y = 3

107. Only P_1 and P_3 will cause problems.

108. 3 will be printed by the program.

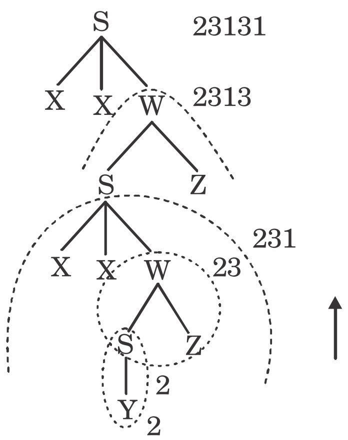
109. Power failure doesnot interrupts a running process.

110. A grammer will be meaning glasss if terminal set and nontechnical set are hot disjoints.

127. The two sets are the same.

NUMERICAL TYPE QUESTIONS

1.



Evaluation will be from bottom to top.

So first S → Y will give 2

then W – SY will give 23

then S → XXW will give 231

then W → SZ will give 2313

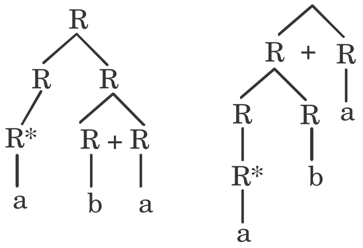
then S → XXW give 23131

2. X = 2

3. $\underbrace{\text{Print f}}_{1}$ ("Hai x = %d", i);
2 $\underbrace{\longmapsto}_{3}$ 4 5 6 7

Total = 7

4.



Two parse trees are there

$\therefore$ It is ambiguous.

5. $S \to E/a$

$E \to a$

	a	$
S	$S \to E$ $S \to a$	
E		

Two entries is one column

So not LL (1)

6. Not LR(0)

EXERCISE – II

MCQ TYPE QUESTIONS

1. In shift reduce parser, the stack contains only a set of variable prefix.

2. In three address instruction format, each address field can be used to specify memory operand and processor register

e.g. $R = R_1 + R_2$

5. The match are as follows

Lexical analysis – DFA minimization

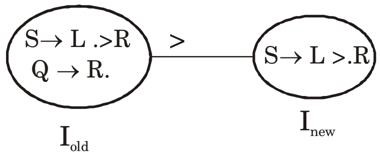
Parsing – Production Tree

Register allocation – Graph coloring

expression evaluation – Post order traversal

7. SLR and CLR are easy to implement. CLR is more powerful to any of the parsac.

8.



Form above diagram, we can see that there is no shift-reduce or reduce-reduce conflict.

9. At compilation time dynamic memory allocation is not feasible.

10. Intermediate code is machine independent code which makes it easy to retarget the compiler to generate code for newer and different processors.

11. The given basic block can be rewritten as

$a = b + c$ $\quad$ $a = b + c$

$c = a + d$ $\quad$ $c = b + c + d$

$d = b + c \Rightarrow$ $\quad$ $d = b + b + c + d = 2b + c + d$

$e = d - b$ $\quad$ $e = b + b + c + d - b = b + c + d$

$a = e + b$ $\quad$ $a = b + b + c + d = 2b + c + d$

From above simplification it is visible that e is same as c and final value of a is same as d. So the final basic block can be written as follows:

$a = b + c$

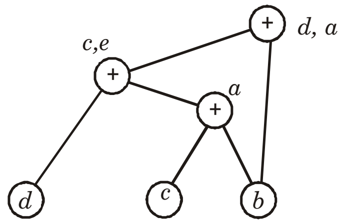
$c = a + d$

$d = 2b + c + d$

$e = c$

$a = d$

The DAG generated for the above basic block in as



Maximum number of modes and edges in above DAG is (6,6)

12. $L_2^* \cup L_1^* = (L_1 \cdot L_2) \cup L_1^*$

We know that $\phi \cdot L = \phi$ for all languages L.

This is because,

Suppose $\exists a$ string $s \in \phi \cdot L$

$\Rightarrow \exists s'$ such that $s = s' \cdot s''$

and $s' \in \phi$ & $s'' \in L$

But $s' \notin \phi$

$\because$ ϕ is an empty language

$\therefore$ $\phi \cdot L = \phi$.

also $\{\in\} \subseteq L^*$ for all languages L. as it means that take letters from the language and concatenate them 'O' time creating a length string which can be done for all language.

$\therefore$ E is in ϕ^*

$L_1 \quad L_2 = f$

& $L_1^* = \{\in\}$

$\therefore$ $\phi \cup \{E\} = \{\in\}$

14. Which is FALSE?

(*a*) L_2 is context free – <u>TRUE</u>

– We can accept or reject L_2 with single stack.

– Insert P O'S into stack skip q 1's

For each 0 corresponding to r, remove 0 from stack.

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(*b*) $L_1 \cap L_2$ is context free – <u>TRUE</u>

– Here $L_1 \cap L_2 = L_2$ which is context free.

(*c*) Complement of L_2 is recursive – <u>TRUE</u>

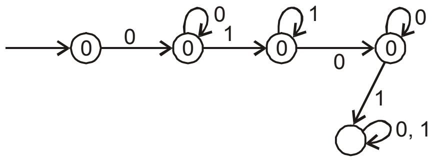
– L_2 is CFL ($\because$ Given)

– Complement of CFL may or may not be CFL.

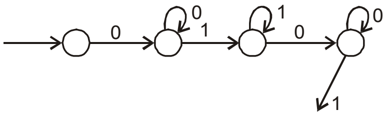
– Complement of CFL is definately recursive.

(*d*) Complenent of L1 is context free but not Regular – FALSE.

$L_1 = \{0^p\ 1^q\ 0^r \mid p, q, r \geq 0\}$

– 

– which is regular?

– $\overline{L_1}$ =  → Regular

Regular are closed

16. CFL's are not closed under complementation. Regular and recursive languages are closed under complementation.

18. Any identifier is also a token so it is recognized in lexical Analysis

19. Symbol table is a Data structure containing record for each identifier, with fields for the attribute of the identifier.

The symbols entered into symbol table are nothing but identifier and operators. It contain information about variables and their attribute.

20. When dynamic data structure is used, heap should be used for runtime environment.

21. It is LL(1) and LR(1)

because in $S \rightarrow aSa$

$S \rightarrow bS$

$S \rightarrow c$

S	a	b	c
	$s \rightarrow sSa$	$s \rightarrow bS$	$s \rightarrow c$

no multiple entries for 1 iteration, so it is LL(1) grammer

mean by LR(1) that it is Left shift Reduced grammer and so it is also LR(1).

Hence it is both LL(1) and LR(1)

22. It is not necessary there exists parsing algo for some programming language whose complexities are less than θ (n^3)

A programming language which allow recursion can be implemented with static storage allocation.

23. According to definition, handle is a production rule $A \rightarrow \beta$ and a position in the sentential form where string β may be found and replaced to produce the previous right sentiential form in a reightmost derivation of sentiential form.

24. Code optimizations are carried out on the intermediate code so that same source code can be converted to machine language of the target machine depending upon the back end tools. It also increases the probability of the compiler, i.e. compiler can generate machine code of different machine codes.

25. Consider each statement separately

I. If a programming language doesn't permit global variables and no nesting procedures then we can't use the recursion with static storage because there is no stack support for example Fortran doesn's support the recursion. So statement is false.

II. Statement is true.

III. Recursion in programming languages will be implemented with dynamic storage for example C language implement recursion with the help of heap if size is also increases because heap stores the all activation record of a recursion. So statement if false.

IV. Statement is false because we can implement recursion either with the help of heap or a stack.

V. Statement is true.

26. An LALR(1) parser for a grammer G can have shift reduce S – R confilict if and only if the LR(1) parser for G has S – R conflicts.

27. Recursive descent parser is a kind of top – down parser.

28. It is ambiguous

29. Regular grammar is well recognised by LL(1) parsers and LR (1) parser is stronger and more than powerful than LL (1). So regular grammar is also accepted by LR (1) parser. Every regular set has LR (1) grammar.

Hence both statement are correct.

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30. Instructions will be following :

MOV	b,	R_1
ADD	a,	R_1
MOV	d,	R_2
ADD	c,	R_2
SUB	e,	R_2
SUB	R_1,	R_2
MOV	R_2,	t_4

It takes 3 MOV instructions.

31.

$S \to S * \bullet E$

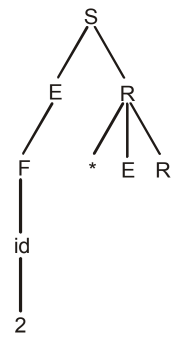
$S \to \bullet E$

$E \to F + \bullet E$

Hence $S \to S * \bullet E$ and $E \to F + \bullet E$

belong to the same set of canonical LR (0) items because in both productions the dot is left to E.

32.



So an inoput 2 * 3 + 4 it prints

234+*

33. The code can be converted to the following in order to reduce the number of cycles needed for execution

for ($i = 0$, $i < N$, i ++)

if (i % 2)

for ($j = 0$; $j < N$, j ++)

{ x + = (4 * j + 5 * c);

{ y + = (7 + 4 * j); }

∴ The only false statement is B since no sub expression can be eliminated

■■

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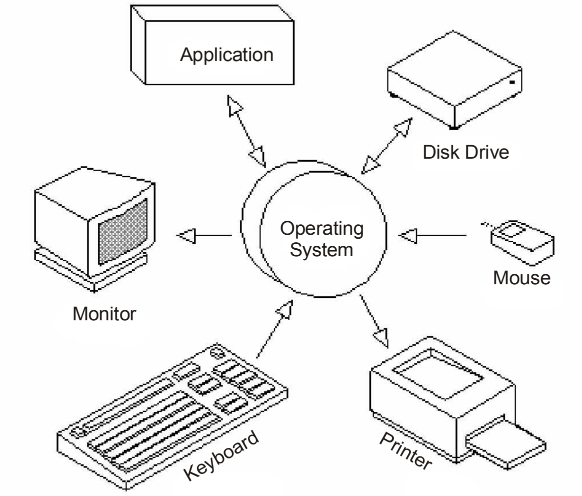
6 CHAPTER Operating System

PROCESSES, THREADS, INTER-PROCESS COMMUNICATION

OPERATING SYSTEM

It is the set of programs that control a computer.

e.g., UNIX, Mach, MS-DOS, MS-Windows, Windows/NT, Chicago, OS/2, MacOS, VMS, MVS, and VM.



Controlling the computer involves software at several levels.

Parts of operating system

(*i*) **Kernel services**

(*ii*) **Library services**

(*iii*) **Application-level services.**

Processes run applications are linked together with libraries that perform standard services.

Kernel supports the processes by providing a path to the peripheral devices. The kernel responds to service calls from the processes and interrupts from the devices. Core of the operating system is the kernel, a control program that functions in **privileged state** (an execution context that allows all hardware instructions to be executed), reacting to interrupts from external devices and to service requests and traps from processes. Generally, kernel is a permanent resident of the computer. It creates and terminates processes and responds to their request for service.

Operating Systems are resource managers. The main resource is computer hardware in the form of processors, storage, input/output devices, communication devices, and data.

Operating system functions

(*i*) implementing the user interface

(*ii*) sharing hardware among users

(*iii*) allowing users to share data among themselves

(*iv*) preventing users from interfering with one another

(*v*) scheduling resources among users

(*vi*) facilitating input/output

(*vii*) recovering from errors

(*viii*) accounting for resource usage

(*ix*) facilitating parallel operations

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(*x*) organizing data for secure and rapid access
(*xi*) handling network communications.

TYPES OF OPERATING SYSTEM

1. Single user

It allows one single user to login at a time. There is no user account database which makes the level of security loc and so users cannot protect their files from being viewed, copied or deleted.

e.g., **DOS** and **Windows 98**.

2. Multi-user

It has a user database account which states the right that users have on certain resources. They are more secure than the single user since access is limited.

e.g., **UNIX**

3. Networked/Work alone

Stand alone are usually not connected to a network and thus cannot access networked resources. They are usually more secured than remate users and remote users cannot log into the computer.

Network operating system uses a standard communication protocol for **UNIX** networks and over the internet (**TCP/IP**). For Novell Netware **internetwork Packet Exchange** /Sequenced Packet Exchange (IPX/SPX). They are less secure than Standalone and should be protected most of the times by creating user accounts.

e.g., **Windows NT 5.0, Windows 98**,

4. Multi-tasking/Single Tasking

Multi-tasking allows one or more programs to run, at time. Each process is given a prioritised amount of time on the processor.

Single-user allows are programs to run at a time. It is usually faster than multitasking systems as some time is taken to in switching in multi-user. But multi-user is more efficient as they allow other tasks to run when a task is not performing any operation.

Single-users are more robust, as multiprograms are require to communicate with each other which can cause synchronization problems (**deadlock)**

5. Multi processor/Single processor

Some Operating System allows for more than one **processor** to use on the system. This allows more than one task to be run, at a time, on different processors. Windows NT/2000 supports multi processors (up to 4).

Single processor, multitasking involves running each of the processor for a given **time slice** or a single processor where with multi processor, they all can run at the same.

FUNCTION OF OPERATING SYSTEM

Usually operating system is used to manage all pieces of a complex system.

If three programs running on a computer, all try to print their output simultaneously on the same printer, then first few lines might be from program one and the next few lines might be from program two and so on with the result resulting in chaos. The operating system can bring order to potential chaos by buffering all the output destined for the printer on the disk.

The primary task of operating system is to keep track of who is using which resource, to grant resource request, to account for usage, and to meditate conflicting request from different programs and users.

When a computer has multiple users, then need for managing and protecting the memory, input/output devices and other resources are even more apparent. This need arises because it is usually necessary to share expensive resources such as tapes drives and phototypesetters.

Following basic tasks :

(*i*) recognizing **input** from the **keyboard**
(*ii*) sending **output** to the **display screen**
(*iii*) keeping track of files and **directories** on the **disk**
(*iv*) controlling **peripheral devices** such as **disk drives** and printers.

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For large systems, operating system has even greater responsibilities and power. It is like a traffic cop — it makes sure that different program and **users** running at the same time do not interfere with each other. The operating system is also responsible for **security**, ensuring that unauthorized users do not **access** the system. *e.g.* Unix, Windows.

Device Interfacing.

Operating system should try and hide the complexity of interfacing to devices from user program and the user. Typically an operating system should also try and configure device to start up rather than getting the user to set them up.

File System.

An operating System can create and maintain a file System, where users can create, delete and move files around a structured file system.

Many systems organize the files in directories (or folders). In multi-users system; these folders can have associated user ownership, and associated access rights.

Multi-user.

This allows one or more user to log into a system. Thus operating system must contain a user account database, which contains user name, default home directory, user passwords and user rights.

Multiprocessing.

This allows two or more processes to be used at a time. Here the operating system must decide if it can run the different processes on individual processors. It must also manage the common memory between processors.

Memory Management.

This involves allocating, and often to create a virtual memory for program. Paging which means organizing data so that the program data is loaded into pages of memory. Another method of managing memory is swapping. This involves swapping the content of memory to disk storage.

Multi-threading.

Processes are often split into smaller tasks, named threads. These threads allow smoother operations.

OBJECTIVES OF OPERATING SYSTEMS

Modern operating systems generally accomplish following three major goals by running processes in low privilege and providing service calls that invoke the operating systems kernel in high-privilege state.

1. **To hide details of hardware by creating Abstraction**

   Abstraction is a software that hides lower level details and provides a set of higher-level functions. An operating system transforms physical world of devices, instructions, memory, and time into virtual world that is the result of abstractions built by the operating system.

   Reasons for Abstraction.

   (*i*) The code needed to control peripheral devices is not standardized. Operating systems provide subroutines called *device drivers* that perform operations on behalf of programs.
   e.g., input/output operations.

   (*ii*) Operating system introduces new functions as it abstracts the hardware. Operating system introduces the file abstraction so that programs do not have to deal with disks.

   (*iii*) Operating system transforms the computer hardware into multiple virtual computers, each belonging to a different program. Each program that is running is called a *process*. Each process views the hardware through the lens of abstraction.

   (*iv*) Operating system can enforce security through abstraction.

2. **To allocate resources to processes (Manage resources)**

   An operating system controls how **processes** (the active agents) may access **resources** (passive entities).

3. **Provide a pleasant and effective user interface**

   The user interacts with the operating systems through the user interface and usually interested in the "*look and feel*" of the operating system.

   Important components of the user interface :

   (*i*) command interpreter

   (*ii*) the file system

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(*iii*) on-line help

(*iv*) application integration.

The recent trend has been toward increasingly integrated graphical user interfaces that encompass the activities of multiple processes on networks of computers.

Resource manager and Extended Form Resource manager point of view

Operating Systems manage different parts of the system efficiently

Extended machines point of view

Operating Systems provide a virtual machine to users that is more convenient to use.

Structurally Operating Systems can be design as a

(*i*) monolithic system

(*ii*) hierarchy of layers

(*iii*) virtual machine system

(*iv*) exokernel

(*v*) using the client-server model

Basic concepts of Operating Systems

(*i*) processes

(*ii*) memory management

(*iii*) I/O management

(*iv*) file systems

(*v*) security.

SYSTEM COMPONENTS

1. **Process Management**

The operating system manages many types of activities ranging from user programs to system programs like printer spooler, name servers, file server etc. Each of these activities is encapsulated in a process. A process includes the complete execution context (code, data, PC, registers, OS resources in use etc.).

A process is not a program but a process is only ONE instant of a program in execution. There are many processes can be running the same program.

Five major activities of an operating system in regard to process management are :

(*i*) creation and deletion of user and system processes.

(*ii*) suspension and resumption of processes.

(*iii*) mechanism for process synchronization.

(*iv*) mechanism for process communication.

(*v*) mechanism for deadlock handling.

2. **Main-Memory Management**

Primary-memory or *Main-Memory* is a large array of words or bytes. Each word or byte has its own address. Main-memory provides storage that can be access directly by the CPU, *i.e.,* to say for a program to be executed, it must in the main memory.

Major activities of an operating in regard to memory-management are:

(*i*) keep track of which part of memory are currently being used and by whom.

(*ii*) decide which process are loaded into memory when memory space becomes available.

(*iii*) allocate and deallocate memory space as needed.

3. **File Management**

File is a collected of related information defined by its creator. Computer can store files on the disk (secondary storage), which provide long term storage.

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Some examples of storage media are

magnetic tape, magnetic disk and optical disk. Each of these media has its own properties like speed, capacity, data transfer rate and access methods.

A file systems normally organized into directories to ease their use. These directories may contain files and other directions.

Five main major activities of an operating system in regard to file management are :

(*i*) The creation and deletion of files.

(*ii*) The creation and deletion of directions.

(*iii*) The support of primitives for manipulating files and directions.

(*iv*) The mapping of files onto secondary storage.

(*v*) The back up of files on stable storage media.

4. I/O System Management

I/O subsystem hides the peculiarities of specific hardware devices from the user. Only device driver knows the peculiarities of the specific device to whom it is assigned.

5. Secondary-Storage Management

Systems have several levels of storage, including primary storage, secondary storage and cache storage.

(*i*) *Primary storage:* Instructions and data must be placed in primary storage or cache to be referenced by a running program. Because main memory is too small to accommodate all data and programs, and its data are lost when power is lost, the computer system must provide secondary storage to back up main memory.

(*ii*) *Secondary storage :* It consists of tapes, disks, and other media designed to hold information that will eventually be accessed in primary storage (primary, secondary, cache) is ordinarily divided into bytes or words consisting of a fixed number of bytes. Each location in storage has an address; the set of all addresses available to a program is called an address space.

Three major activities of an operating system in regard to secondary storage management are:

(*i*) managing the free space available on the secondary-storage device.

(*ii*) allocation of storage space when new files have to be written.

(*iii*) scheduling the requests for memory access.

6. Networking

Distributed systems is a collection of processors that do not share memory, peripheral devices, or a clock. The processors communicate with one another through communication lines called *network*. The communication-network design must consider routing and connection strategies, and the problems of contention and security.

7. Protection System

If a computer systems has multiple users and allows the concurrent execution of multiple processes, then the various processes must be protected from one another's activities. Protection refers to mechanism for controlling the access of programs, processes, or users to the resources defined by a computer systems.

8. Command Interpreter System

Command interpreter is an interface of the operating system with the user. The user gives commands with are executed by operating system (usually by turning them into system calls). The main function of a command interpreter is to get and execute the next user specified command. Command-Interpreter is usually not part of the kernel, since multiple command interpreters (shell, in UNIX terminology) may be support by an operating system, and they do not really need to run in kernel mode.

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Advantages to separating command interpreter from the Kernel.

(*i*) If we want to change the way to command interpreter looks, i.e., I want to change the interface of command interpreter, I am able to do that if the command interpreter is separate from the kernel. I cannot change the code of the kernel so I cannot modify the interface.

(*ii*) If command interpreter is a part of the kernel it is possible for a malicious process to gain access to certain part of the kernel that it showed not have to avoid this ugly scenario it is advantageous to have the command interpreter separate from kernel.

OPERATING SYSTEMS SERVICES

Following five services are provided by an operating systems to the convenience of users.

1. **Program Execution**

The purpose of a computer system is to allow the user to execute programs. So operating system provides an environment where the user can conveniently run programs. The user does not have to worry about the memory allocation or multitasking or anything. These things are taken care of by the operating system.

Running a program involves allocating and deallocating memory, CPU scheduling in case of multiprocess. These functions cannot be given to the user-level programs. So user-level programs cannot help the user to run programs independently without the help from operating systems.

2. **I/O Operations**

Each program requires an input and produces output. This involves use of I/O. The operating systems hides the user the details of underlying hardware for the I/O. All the user sees is that the I/O has been performed without any details. So operating systems by providing I/O makes it convenient for the users to run programs.

For efficiently and protection users cannot control I/O, so this service cannot be provided by user-level programs.

3. **File System Manipulation**

Output of a program may need to be written into new files or input taken from some files. Operating system provides this service. The user does not have to worry about secondary storage management. User gives a command for reading or writing to a file and sees his task accomplished. Thus operating systems makes it easier for user programs to accomplished their task.

This service involves secondary storage management. The speed of I/O that depends on secondary storage management is critical to the speed of many programs and hence I think it is best relegated to the operating systems to manage it than giving individual users the control of it. It is not difficult for the user-level programs to provide these services but for above mentioned reasons it is best if this service s left with operating system.

4. **Communications**

There are instances where processes need to communicate with each other to exchange information. It may be between processes running on the same computer or running on the different computers. By providing this service, operating system relieves the user of the worry of passing messages between processes. In case where messages need to be passed to processes on the other computers through a network, it can be done by the user programs. The user program may be customized to the specifics of the hardware through which message transmits and provides service interface to the operating system.

5. **Error Detection**

Error is one part of the system, which may cause malfunctioning of the complete system. To avoid such a situation, operating system constantly monitors the system for detecting the errors. This relieves the user of the worry of errors propagating to various part of the system and causing malfunctioning.

This service cannot allowed to be handled by user programs because it involves monitoring and in cases altering area of memory or deallocation of memory for a faulty process. Or may be relinquishing the CPU of a process that goes into an infinite loop. These tasks are too critical to be handed over to the user programs. A user program if given these privileges can interfere with the correct (normal) operation of the operating systems.

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SYSTEM CALLS AND SYSTEM PROGRAMS

System calls

It provide an interface between process and the operating system. System calls allow user-level processes to request some services from the operating system which process itself is not allowed to do. In handling the trap, operating system will enter in the kernel mode, where it has access to privileged instructions, and can perform the desired service on behalf of user-level process, because of the critical nature of operations that the operating system itself does them every time they are needed.

e.g., for I/O, a process involves a system call telling the operating system to read or write particular area and this request is satisfied by the operating system.

System programs

It provide basic functioning to users so that they do not need to write their own environment for program development (editors, compilers) and program execution (shells). In some sense, they are bundles of useful system calls.

PROCESS

Term process, used somewhat interchangeably with '*task*' or '*job*'.

Definition of Process

(1) *Program in Execution :* It seem to be most frequently used.

(2) An asynchronous activity.

(3) '*Animated sprit*' of a procedure in execution.

(4) The entity to which processors are assigned.

(5) The '*dispatchable*' unit.

Relation between process and program.

It is same beast with different name or when this beast is sleeping (not executing) it is called *program* and when it is executing becomes *process*. Process is not the same as program.

Difference between Process and Program.

Process is more than a program code. Process is an *active entity* as oppose to program which consider to be a *passive entity*.

Program is an algorithm expressed in some suitable notation,
e.g., programming language

Being a passive, a program is only a part of process.

Process includes:

(*i*) Current value of Program Counter (PC)

(*ii*) Contents of the processors registers

(*iii*) Value of the variables

(*iv*) Process stack (SP) which typically contains temporary data such as subroutine parameter, return address, and temporary variables.

(*v*) A data section that contains global variables.

Note : Process is the unit of work in a system.

In *process model,* all software on the computer is organized into a number of sequential processes.

Process includes

(*i*) PC

(*ii*) Registers

(*iii*) Variables.

Conceptually, each process has its own virtual CPU. In reality, CPU switches back and forth among processes. (Rapid switching back and forth is called *multiprogramming*).

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PROCESS STATE.

It consists of everything necessary to resume the process execution if it is somehow put aside temporarily.

Process state consists of atleast following:

(1) Code for the program.

(2) Program's static data.

(3) Program's dynamic data.

(4) Program's procedure call stack.

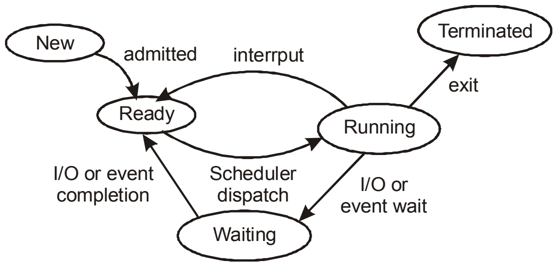
(5) Contents of general purpose registers.

(6) Contents of program counter (PC)

(7) Contents of program status word (PSW).

(8) Operating Systems resource in use.

A process goes through a series of discrete process states as follows :



(1) **New state:** The process being created.

(2) **Running state:** A process is said to be *running* if it has the CPU, *i.e.,* process actually using CPU at that particular instant.

(3) **Blocked (or waiting) state:** A process is said to be *blocked* if it is waiting for some event to happen such that as an I/O completion before it can proceed.

A process is unable to run until some external event happens.

(4) **Ready state:** A process is said to be *ready* if it uses a CPU if one was available. A ready state process is runable but temporarily stopped running to let another process run.

(5) **Terminated state:** The process has finished execution

Three Process States

1. Running state

2. Ready state

3. Blocked state

Logically, '*Running*' and '*Ready*' states are similar. In both cases, process is willing to run only in the case of '*Ready*' state, there is temporarily no CPU available for it. The 'Blocked' state is different from the '*Running*' and '*Ready*' states in that the process cannot run, even if the CPU is available.

The three states are shown below :

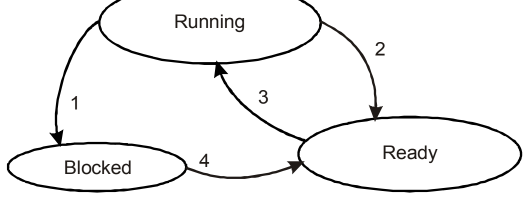


Fig. Transitions between processes states in Running, Ready and Blocked

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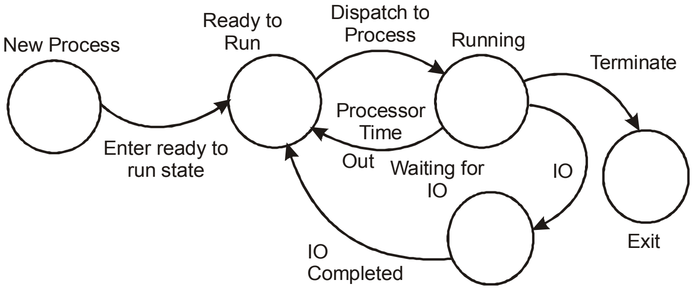
Process State Transitions

Following are six(6) possible transitions among above mentioned five (5) states



Transition 1 : It occurs when process discovers that it cannot continue. If running process initiates an I/O operation before its allotted time expires, the running process voluntarily relinquishes the CPU.

This state transition is: Block (process-name): Running ? Block.

Transition 2 : It occurs when the scheduler decides that the running process has run long enough and it is time to let another process have CPU time.

This state transition is: Time-Run-Out (process-name): Running ? Ready.

Transition 3 : It occurs when all other processes have had their share and it is time for the first process to run again

This state transition is: Dispatch (process-name): Ready ? Running.

Transition 4 : It occurs when the external event for which a process was waiting (such as arrival of input) happens.

This state transition is: Wakeup (process-name): Blocked ? Ready.

Transition 5 : It occurs when the process is created.

This state transition is Admitted (process-name): New ? Ready.

Transition 6 : It occurs when the process has finished execution.

This state transition is: Exit (process-name): Running ? Terminated.

PROCESS OPERATIONS.

1. **Process Creation**

   Following four principal events led to processes creation :

   (*i*) System initialization.

   (*ii*) Execution of a process : Creation of System calls by a running process.

   (*iii*) A user request to create a new process.

   (*iv*) Initialization of a batch job.

   Foreground processes interact with users.

   Background processes that stay in background sleeping but suddenly springing to life to handle activity such as email, webpage, printing, and so on. Background processes are called *daemons*. This call creates an exact clone of the calling process.

   A process may create a new process by some create process such as '*fork*'. It chooses to does so, creating process is called *parent process* and the created one is called *child processes*. Only one parent is needed to create a child process.

   Note. Unlike plants and animals that use sexual representation, a process has only one parent. This creation of process (processes) yields a hierarchical structure of processes like one in the figure. Each child has only one parent but each parent may have many children. After the fork, the two processes, *parent* and the *child*, have same memory image, same environment strings and the same open files. After a process is created, both parent and child have their own distinct address space. If either process changes a word in its address space, the change is not visible to the other process.

   Reasons for process creation.

   (*i*) User logs on.

   (*ii*) User starts a program.

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(*iii*) Operating system creates process to provide service, e.g., to manage printer.

(*iv*) Some program starts another process, e.g., Netscape calls *xv* to display a picture.

2. Process Termination

A process terminates when it finishes executing its last statement. Its resources are returned to the system, it is purged from any system lists or tables, and its process control block (PCB) is erased *i.e.,* PCB's memory space is returned to a free memory pool.

New process terminates existing process, due to following reasons:

(i) **Normal exist :** Most processes terminates because they have done their job. This call is exist in UNIX.

(ii) **Error exist :** When process discovers a fatal error.
e.g., a user tries to compile a program that does not exist.

(iii) **Fatal Error :** An error caused by process due to a bug in program
e.g., executing an illegal instruction, referring non-existing memory or dividing by zero.

(iv) **Killed by another Process :** A process executes a system call telling the Operating Systems to terminate some other process. In UNIX, this call is kill. In some systems when a process kills all processes it created are killed as well (UNIX does not work this way).

PROCESS CONTROL BLOCK (PCB)

Process in an operating system is represented by a data structure called *process control block (PCB)* or *process descriptor*.

PCB contains following information about specific process :

(1) Current state of the process i.e., whether it is ready, running, waiting, or whatever.

(2) Unique identification of the process in order to track "*which is which*" information.

(3) A pointer to parent process.

(4) Pointer to child process (if it exists).

(5) Priority of process (a part of CPU scheduling information).

(6) Pointers to locate memory of processes.

(7) Register save area.

(8) Processor it is running on.

PCB is a certain store that allows the operating systems to locate key information about a process. Thus, PCB is the data structure that defines a process to the operating systems.

THREADS

The process and its associated threads are different concept. Processes are used to group resources together and threads are the entities scheduled for execution on the CPU.

Thread is a single sequence stream within in a process. As threads have some of the properties of processes, they are sometimes called *lightweight processes*. In a process, threads allow multiple executions of streams. In many respect, threads are popular way to improve application through parallelism. CPU switches rapidly back and forth among the threads giving illusion that the threads are running in parallel. Like a traditional process i.e., process with one thread, a thread can be in any of several states (Running, Blocked, Ready or Terminated). Each thread has its own stack. Since thread will generally call different procedures and thus a different execution history. Thus, thread needs its own stack. An operating system that has thread facility, the basic unit of CPU utilization is a thread.

Thread consists of following :

(*i*) A program counter (PC)

(*ii*) A register set

(*iii*) A stack space.

Threads are not independent of one other like processes as a result threads shares with other threads their code section, data section, operating system resources also called task, such as open files and signals.

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Similarities between Processes and Threads.

(*i*) Both CPU and only one thread active (running) at a time.

(*ii*) Within a processes, both execute sequentially.

(*iii*) Both can create children.

(*iv*) If one thread or process is blocked, another thread or process can run.

Differences between Processes and Threads

(*i*) Unlike processes, threads are not independent of one another.

(*ii*) Unlike processes, all threads can access every address in the task .

(*iii*) Unlike processes, thread are design to assist one other.

Note : Processes might or might not assist one another because processes may originate from different users.

Use of Threads

Threads are used in designing operating system due to following reasons:

(*i*) A process with multiple threads make a great server for example printer server.

(*ii*) Because threads can share common data, they do not need to use interprocess communication.

(*iii*) Because of the very nature, threads can take advantage of multiprocessors.

Advantages of Threads.

Threads are cheap because

(*i*) they only need a stack and storage for registers therefore, threads are cheap to create.

(*ii*) they use very little resources of an operating system in which they are working. That is, threads do not need new address space, global data, program code or operating system resources.

(*iii*) context switching are fast when working with threads. because only and/or restore PC, SP and registers have to be saved.

Drawback of Threads.

There is no protection between threads.

User-level Threads

These implement in user-level libraries, rather than via system calls, so thread switching does not need to call operating system and to cause interrupt to the kernel. In fact, kernel knows nothing about user-level threads and manages them as if they were single-threaded processes.

Advantages.

(1) User-level threads package can be implemented on an Operating system that does not support threads.

(2) User-level threads does not require modification to operating system.

(3) Simple representation : Each thread is represented simply by a PC, registers, stack and a small control block, all stored in the user process address space.

(4) Simple management : Creating a thread, switching between threads and synchronization between threads can all be done without intervention of the kernel.

(5) Fast and Efficient : Thread switching is not much more expensive than a procedure call.

Disadvantages.

(1) There is a lack of coordination between threads and operating system kernel. Therefore, process as whole gets one time slice irrespect of whether process has one thread or 1000 threads within. It is up to each thread to relinquish control to other threads.

(2) User-level threads requires non-blocking system call i.e., a multithreaded kernel. Otherwise, entire process will blocked in the kernel, even if there are runable threads left in the processes.
e.g., if one thread causes a page fault, the process blocks.

KERNEL-LEVEL THREADS

In this method, kernel knows about and manages the threads. No runtime system is needed in this case. Instead of thread table in each process, the kernel has a thread table that keeps track of all threads in the system. In addition, kernel also maintains the traditional process table to keep track of processes. Operating Systems kernel provides system call to create and manage threads.

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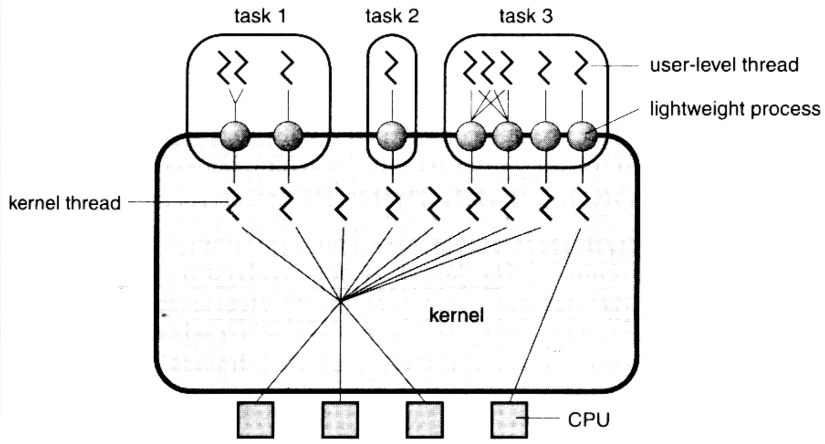
Implementation of general structure of kernel-level thread :



Advantages.

(1) Because kernel has full knowledge of all threads, scheduler may decide to give more time to a process having large number of threads than process having small number of threads.

(2) These are especially good for applications that frequently block.

Disadvantages.

(1) These are slow and inefficient. Threads operations are hundreds of times slower than that of user-level threads.

(2) Since kernel must manage and schedule threads as well as processes, it require a full thread control block (TCB) for each thread to maintain information about threads. As a result there is significant overhead and increased in kernel complexity.

Advantages of Threads over Multiple Processes

(1) Context switching.

Threads are very inexpensive to create and destroy, and they are inexpensive to represent.

e.g., they require space to store, PC, SP, and general-purpose registers, but they do not require space to share memory information, Information about open files of I/O devices in use, etc. With so little context, it is much faster to switch between threads. In other words, it is relatively easier for a context switch using threads.

(2) Sharing.

Threads allow sharing of a lot of resources that cannot be shared in process,

e.g., sharing code section, data section, Operating system resources like open file etc.

Disadvantages of Threads over Multiprocesses

(1) Blocking.

If kernel is single threaded, a system call of one thread will block the whole process and CPU may be idle during the blocking period.

(2) Security.

Since there is an extensive sharing among threads there is a potential problem of security. It is quite possible that one thread over writes the stack of another thread (or damaged shared data) although it is very unlikely since threads are meant to cooperate on a single task.

Application that Benefits from Threads

A proxy server satisfying the requests for a number of computers on a LAN would be benefited by a multi-threaded process.

In general, any program that has to do more than one tasks at a time could benefit from multitasking.
e.g., a program that reads input, process it, and outputs could have three threads, one for each task.

Application that cannot Benefit from Threads

Any sequential process that cannot be divided into parallel tasks will not benefit from thread, as they would block until the previous one completes.
e.g., a program that displays the time of the day would not benefit from multiple threads.

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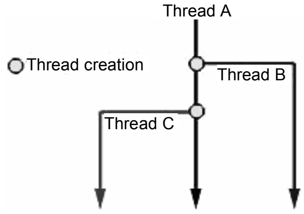
Resources used in Thread Creation and Process Creation

When a new thread is created it shares its code section, data section and operating system resources like open files with other threads. But it is allocated its own stack, register set and a program counter.

The creation of a new process differs from that of a thread mainly in the fact that all the shared resources of a thread are needed explicitly for each process. So though two processes may be running the same piece of code they need to have their own copy of the code in the main memory to be able to run. Two processes also do not share other resources with each other. This makes the creation of a new process very costly compared to that of a new thread.



CONTEXT SWITCH

To give each process on a multiprogrammed machine a fair share of the CPU, a hardware clock generates interrupts periodically. This allows the operating system to schedule all processes in main memory (using scheduling algorithm) to run on the CPU at equal intervals. Each time a clock interrupt occurs, the interrupt handler checks how much time the current running process has used. If it has used up its entire time slice, then CPU scheduling algorithm (in kernel) picks a different process to run. Each switch of the CPU from one process to another is called *context switch*.

Steps of Context Switching

(1) Values of the CPU registers are saved in the process table of the process that was running just before the clock interrupt occurred.

(2) Registers are loaded from the process picked by the CPU scheduler to run next.

In a multiprogrammed uniprocessor computing system, context switches occur frequently enough that all processes appear to be running concurrently. If a process has more than one thread, then Operating system can use the context switching technique to schedule the threads so they appear to execute in parallel. This is the case if threads are implemented at the kernel level. Threads can also be implemented entirely at the user level in run-time libraries. Since in this case no thread scheduling is provided by the Operating system, it is the responsibility of the programmer to yield the CPU frequently enough in each thread so all threads in the process can make progress.

Action of Kernel to Context Switch

(1) Among Threads.

The threads share a lot of resources with other peer threads belonging to the same process. So a context switch among threads for the same process is easy. It involves switch of register set, the program counter and the stack. It is relatively easy for the kernel to accomplished this task.

(2) Among Processes

Context switches among processes are expensive. Before a process can be switched its process control block (PCB) must be saved by the operating system.

PCB consists of following information:

(*i*) Process state.

(*ii*) Program counter, PC.

(*iii*) Values of the different registers.

(*iv*) CPU scheduling information for the process.

(*v*) Memory management information regarding the process.

(*vi*) Possible accounting information for this process.

(*vii*) I/O status information of the process.

When PCB of the currently executing process is saved, operating system loads PCB of the next process that has to be run on CPU. This is a heavy task and it takes a lot of time.

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INTER-PROCESS COMMUNICATION (IPC)

It is a set of interfaces, which is usually programmed in other for a programmer to communicate between a series of processes. This allows running of programs concurrently in an operating system.

Methods used in IPC

1. **Pipes.**
   This allows flow of data in one direction only. Data from the output is usually buffered until input process receives it which must have a common origin.
2. **Named pipes.**
   This is a pipe with a specific name. It can be used in processes that do not have a shared common process origin.
   e.g., is **FIFO** where the data is written to a pipe is first named.
3. **Message queuing.**
   This allows messages to be passed between messages using either a single queue or several message queues. This is managed by the system **kernel.** These messages are co-ordinated using an application program interface **(API)**
4. **Semaphores.**
   This is used in solving problems associated with synchronization and avoiding race conditions. They are integer values which are greater than or equal to zero
5. **Shared memory.**
   This allows interchange of data through a defined area of memory. Semaphore value has to be obtained before data can get access to shared memory.
6. **Sockets.**
   This method is mostly used to communicate over a network, between a **client** and a **server.** It allows for a standard connection which is computer and operating system independent.

Primitive interprocesses that block instead of wasting processor time.

(1) Sleep and Wakeup

SLEEP is a system call that causes the caller to block, *i.e.,* be suspended until another process wakes it up.

WAKEUP call has one parameter, the process to be awakened.

(2) Producer-Consumer Problem

In this case, two processes share a common, fixed-size buffer. One of the processes puts information into the buffer, and the other one, the consumer, takes it out. This could be easy with 3 or more processes in which one wakeup waiting bit is insufficient, another patch could be made, and a second wakeup waiting bit is added of 8 or 32 but the problem of race condition will still be there.

(3) Events Counter

This involves programming a program without requiring mutual exclusion. Event counters value can only increase and never decrease.

There are three operations defined on an event counter (for example, E) :

(*i*) **Read (E):** Return value of E

(*ii*) **Advance (E):** Atomically increment E by 1.

(*iii*) **Await (E, v):** Wait until E has a value of v or more.

Two events counters used are :

(*i*) **in :** To count the cumulative number of items that the producer discussed above has put into the buffer since the program started running.

(*ii*) **out :** To count the cumulative number of items that the consumer has removed from the buffer so far.

Therefore, **in** must be greater than or equal to **out,** but not more than size of the buffer.

(4) Monitors

This is the best way of achieving mutual exclusion. Monitor is a collection of procedures, variables, and data structures that are grouped together in a special kind of module or package. The monitor uses the *wait and signal***.**

WAIT is to indicate to the other process that the buffer is full and so causes the calling process to block and allows the process that was earlier prohibited to enter the process at this point.

SIGNAL allow other process to be awakened by the process that entered during the "WAIT".

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CONCURRENCY & SYNCHRONIZATION

An operating system can have a simple design, if computer it controls has just a *single user* running a *single process* the whole of which is small enough to *fit into memory* running on a *single processor* because many design problems are avoided. The system, however, is far too simplistic to be useful, is extremely wasteful of resources and is operating far below potential.

An operating system can be following :

1. Multitasking or Multiprogramming.
Appear to be running several processes at once.

2. Multiuser.
Have several users using it at the same time.

3. Multiprocessor capable.
Able to handle multiple CPU systems, and together with hardware can also support.

4. Virtual memory addressing.
The address space (apparently accessible memory) of a process does not need to be identical to the physical address space. Virtual address space can be much larger than the physical space. Each process can have the same view of memory, for example, its program code is at (the same) very low virtual addresses even though physically this would be possible for at most one process.

These features can be achieved by giving each process a slice of time on a processor and a slice of memory for running and allocating resources as necessary.

Scheduler.

It schedule the process and allocates time. It decides when a process has used up enough time and should be forced to relinquish a processor. Often processes are forced off a processor before their alloted time is up because they are doing I/O and have to wait for I/O to complete - devices are typically very slow compared to the CPU. Processes can also be forced off as a result of signals or because higher priority processes want time.

Pager.

It moves pages (contents of blocks of memory) in and out from memory to disk so that a process can appear to have a large address space, independent of all other processes but also shared where desired, and much larger than actual memory.

Swapper.

It moves processes to and from memory, by moving process pages and process data that the kernel has for the process.

CONCURRENCY

Concurrency cannot be avoided due to following reasons :

(*i*) **Users are concurrent :** A person can handle several tasks at once and expects the same from a computer.

(*ii*) Multiprocessors are becoming more prevalent. Internet is perhaps a huge multiprocessor.

(*iii*) Distributed system (client/server system) is naturally concurrent.

(*iv*) Windowing system is naturally concurrent.

(*v*) I/O is often slow because it involves slow devices such as disks, printers; many network operations are essentially (slow) I/O operations. When doing I/O it is helpful to handle I/O concurrently with other work.

Concurrency involve following issues :

(*i*) Writing an operating system.

(*ii*) When interacting with the kernel, for example, when performing I/O.

(*iii*) When generating multiple processes, *e.g.,* with forks and pipelines.

(*iv*) When using multiple threads.

Concurrency issues (expressed using processes).

(1) Atomic.

An operation is atomic if steps are done as a unit. Operations that are not atomic, but interruptible and done by multiple processes can cause problems.

e.g., an lseek followed by a write is not atomic. A process is likely to lose its time quantum between lseek (a slow operation if distance seeked is large!) and the write. If another process has the file open and does a write, then result is not what is intended.

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(2) Race conditions.

It occurs if outcome depends on which of several processes gets to a point first.

e.g., fork( ) can generate a race condition if result depends on whether the parent or the child process runs first. Other race conditions can occur if two processes are updating a global variable.

(3) Blocking and Starvation.

While neither of these problems is unique to concurrent processes, their effects must be carefully considered.

Processes can *block* waiting for resources. A process could be blocked for a long period of time waiting for input from a terminal.

If process is required to periodically update some data, this would be very undesirable.

Starvation occurs when a process does not obtain sufficient CPU time to make meaniful progress.

(4) Deadlock.

It occurs when two processes are blocked in such a way that neither can proceed. The typical occurrence is where two processes need two non-shareable resources to proceed but one process has acquired one resource and the other has acquired the other resource. Acquiring resources in a specific order can resolve some deadlocks.

PROCESS SYNCHRONIZATION

In the multiprogramming system, some processes perform read and some processes perform write on the file simultaneously. This lead to data inconsistency as data is being read as well as modified by many processes at the same time. To prevent such data inconsistency process synchronization is required. All process that want to perform read operation can do the reading simultaneously but process that needs to perform write operation must do it one at a time. The process that exist at the same time are *called concurrent processes.*

Types of processes.

There are two process.

(*i*) *Producer* : Places an item in the buffer

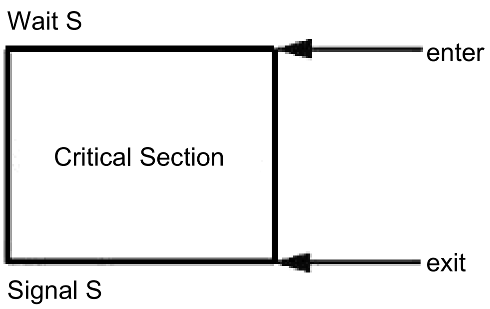
(*ii*) *Consumer* : Consumer consumes an item from the buffer.

Race Conditions

In operating systems, processes that are working together share some common storage (main memory, file etc.) that each process can read and write. When two or more processes are reading or writing some shared data and the final result depends on who runs precisely when, are called *race conditions*. Concurrently executing threads that share data need to synchronize their operations and processing in order to avoid race condition on shared data. Only one 'customer' thread at a time should be allowed to examine and update the shared variable.

Race conditions are also possible in Operating Systems. If ready queue is implemented as a linked list and if ready queue is being manipulated during the handling of an interrupt, then interrupts must be disabled to prevent another interrupt before the first one completes. If interrupts are not disabled than the linked list could become corrupt.

CRITICAL SECTION PROBLEM.



The key to preventing trouble involving shared storage is find some way to prohibit more than one process from reading and writing the shared data simultaneously. That part of the program where shared memory is accessed is called *Critical section*. To avoid race conditions and flawed results, one must identify codes in *Critical sections* in each thread.

Characteristic of the code that form a Critical section

(1) Codes that reference one or more variables in a "read-update-write" fashion while any of those variables is possibly being altered by another thread.

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(2) Codes that alter one or more variables that are possibly being referenced in "read-updata-write" fashion by another thread.

(3) Codes use a data structure while any part of it is possibly being altered by another thread.

(4) Codes alter any part of a data structure while it is possibly in use by another thread.

Solution to Critical section problem

It must satisfy following three requirements.

***(i)* Mutual exclusion.**

If process P_i is executing in critical section, then no other processes can be executing in their critical sections.

***(ii)* Progress.**

If no process is executing in its critical section and some process exist whcih wish to execute their critical section, then those process ie not executing in remainder section can participate is decision of which will enter critical section next.

***(iii)* Bounded waiting.**

There exist a bound on the number of times that other processes are allowed to enter their critical section after a process has made a request is granted to enter its critical enter their critical section before the request is granted.

When one process is executing shared modifiable data in its critical section, no other process is to be allowed to execute in its critical section. Thus, execution of critical sections by the processes is mutually exclusive in time.

MUTUAL EXCLUSION

A way of making sure that if one process is using a shared modifiable data, the other processes will be excluded from doing the same thing.

Formally, while one process executes the shared variable, all other processes desiring to do so at the same time moment should be kept waiting; when that process has finished executing the shared variable, one of the processes waiting; while that process has finished executing the shared variable, one of the processes waiting to do so should be allowed to proceed. In this fashion, each process executing the shared data (variables) excludes all others from doing so simultaneously. This is called *Mutual exclusion*.

Mutual exclusion needs to be enforced only when processes access shared modifiable data. When processes are performing operations that do not conflict with one another they should be allowed to proceed concurrently.

Mutual Exclusion conditions

If matter can be arranged such that no two processes were ever in their critical sections simultaneously, can be avoided.

Four conditions are required to hold to have a good solution for critical section problem (mutual exclusion).

(*i*) No two processes may at the same moment inside their critical sections.

(*ii*) No assumptions are made about relative speeds of processes or number of CPUs.

(*iii*) No process should outside its critical section should block other processes.

(*iv*) No process should wait arbitrary long to enter its critical section.

Proposals for Achieving Mutual Exclusion

The mutual exclusion problem is to devise a pre-protocol (or entry protocol) and a post-protocol (or exist protocol) to keep two or more threads from being in their critical sections at the same time. Tanenbaum examine proposals for critical-section problem or mutual exclusion problem.

Problem : When one process is updating shared modifiable data in its critical section, no other process should allowed to enter in its critical section.

Proposal 1 -Disabling Interrupts (Hardware Solution).

Each process disables all interrupts just after entering in its critical section and re-enable all interrupts just before leaving critical section. With interrupts turned off the CPU could not be switched to other process. Hence, no other process will enter its critical and mutual exclusion achieved.

Conclusion :

Disabling interrupts is sometimes a useful interrupts is sometimes a useful technique within the kernel of an operating system, but it is not appropriate as a general mutual exclusion mechanism for users process. The reason is that it is unwise to give user process the power to turn off interrupts.

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Proposal 2 - Lock Variable (Software Solution).

In this solution, consider a single, shared, (lock) variable, initially 0. When a process wants to enter in its critical section, it first test the lock. If lock is 0, the process first sets it to 1 and then enters the critical section. If the lock is already 1, the process just waits until (lock) variable becomes 0. Thus, a 0 means that no process in its critical section, and 1 means hold your horses - some process is in its critical section.

Conclusion :

The flaw in this proposal can be best explained by example. Suppose process A sees that the lock is 0. Before it can set the lock to 1 another process B is scheduled, runs, and sets the lock to 1. When process A runs again, it will also set the lock to 1, and two processes will be in their critical section simultaneously.

Proposal 3 - Strict Alteration.

In this proposed solution, the integer variable 'turn' keeps track of whose turn is to enter the critical section. Initially, process A inspect turn, finds it to be 0, and enters in its critical section. Process B also finds it to be 0 and sits in a loop continually testing 'turn' to see when it becomes 1.Continuously testing a variable waiting for some value to appear is called *Busy-Waiting*.

Conclusion :

Taking turns is not a good idea when one of the processes is much slower than the other. Let process 0 finishes its critical section quickly, so both processes are now in their noncritical section. This situation violates above mentioned condition 3.

Using Systems calls 'sleep' and 'wakeup'

Basically, in the above mentioned solution; when a processes wants to enter in its critical section, it checks to see if then entry is allowed. If it is not, the process goes into tight loop and waits (i.e., start busy waiting) until it is allowed to enter. This approach waste CPU-time.

Now consider some interprocess communication primitives is the pair of sleep-wakeup.

- **Sleep.** It is a system call that causes the caller to block, *i.e.,* be suspended until some other process wakes it up.
- **Wakeup.** It is a system call that wakes up the process.

Both 'sleep' and 'wakeup' system calls have one parameter that represents a memory address used to match up 'sleeps' and 'wakeups' .

Bounded Buffer Producers and Consumers

It assumes that there is a fixed buffer size i.e., a finite numbers of slots are available.

Statement

To suspend the producers when buffer is full, to suspend the consumers when buffer is empty, and to make sure that only one process at a time manipulates a buffer so there are no race conditions or lost updates.

As an example how sleep-wakeup system calls are used, consider the producer-consumer problem also called *bounded buffer problem*.

Two processes share a common, fixed-size (bounded) buffer. The producer puts information into the buffer and the consumer takes information out.

Problem :

(*i*) *When producer wants to put a new data in the buffer, but buffer is already full.*

Solution. Producer goes to sleep and to be awakened when the consumer has removed data.

(*ii*) *When consumer wants to remove data the buffer but buffer is already empty.*

Solution. Consumer goes to sleep until the producer puts some data in buffer and wakes consumer up.

Conclusion :

This approaches also leads to same race conditions. Race condition can occur due to the fact that access to '*count*' is unconstrained. The essence of the problem is that a wakeup call, sent to a process that is not sleeping, is lost.

SEMAPHORES.

E.W. Dijkstra (1965) abstracted the key notion of mutual exclusion in his concepts of semaphores.

Definition.

Semaphore is a process synchronization tool.

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It can be accessed using two atomic operations :

(*i*) Wait (P)

(*ii*) Signal(V)

A semaphore is a protected variable whose value can be accessed and altered only by the operations P and V and initialization operation called '*Semaphorinitislize*'.

Binary Semaphores can assume only the value 0 or the value 1 counting semaphores also called *general semaphores* can assume only nonnegative values.

P (or wait or sleep or down) operation on semaphores S, written as P(S) or wait (S), operates as follows:

P(S) : IF S > 0
  THEN S := S – 1
  ELSE (wait on S)

V (or signal or wakeup or up) operation on semaphore S, written as V(S) or signal (S), operates as follows:

V(S): **IF** (one or more process are waiting on S)
  THEN (let one of these processes proceed)
  ELSE S := S +1

Operations P and V are done as single, indivisible, atomic action. It is guaranteed that once a semaphore operations has stared, no other process can access the semaphore until operation has completed. Mutual exclusion on the semaphore, S, is enforced within **P(S)** and **V(S)**.

If several processes attempt a P(S) simultaneously, only process will be allowed to proceed. The other processes will be kept waiting, but the implementation of P and V guarantees that processes will not suffer indefinite postponement.

Semaphores solve the *lost-wakeup problem*.

Producer-Consumer Problem using Semaphores

Solution to producer-consumer problem uses three semaphores : namely, full, empty and mutex.

(i) Semaphore 'full' : It is used for counting the number of slots in the buffer that are full.

(*ii*) Semaphore empty : It is used for 'empty' for counting the number of slots that are empty

(*iii*) Semaphore 'mutex' : It is used to make sure that the producer and consumer do not access modifiable shared section of the buffer simultaneously.

Initialization

- Set full buffer slots to 0.
  i.e., semaphore Full = 0.
- Set empty buffer slots to N.
  i.e., semaphore empty = N.
- For control access to critical section set mutex to 1.
  i.e., semaphore mutex = 1.

```
Producer ( )
WHILE (true)
        produce-Item ( );
        P (empty);
        P (mutex);
        enter-Item ( )
        V (mutex)
        V (full);
Consumer ( )
WHILE (true)
        P (full)
        P (mutex);
        remove-Item ( );
        V (mutex);
        V (empty);
        consume-Item (Item)
```

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CPU SCHEDULING & DEADLOCK

SCHEDULING

It involves determining which thread should be run on the processor at a given time. This is called *time slice*, and its actual value depends on the system configuration.

Each thread currently has a base priority which is set by the programmer who created the program. It defines how the thread is executed in relation to other system threads, and the thread with the highest priority gets use of the processor

Scheduler is concerned with deciding on policy, not providing a mechanism.

According to A.S. Tanenbaum and A.S. wood hull, a good scheduling algorithm should include following:

1. **Fairness.**
   Makes sure each process gets its fair share of the CPU.
2. **Efficiency.**
   Keep the CPU busy 100 percent of the time.
3. **Response time.**
   Minimise response time for interactive users.
4. **Turnaround.**
   Minimise the time batch users must wait for output.
5. **Throughput.**
   Maximise the number of jobs processed per hour.

Scheduling Objectives

These should:

1. predictable
2. enforce priorities
3. balance resource use
4. avoid indefinite postponement
5. degrade gracefully under heavy load

SCHEDULER

Parts of a Scheduler

Scheduler is made up of two main parts:

1. **Primary scheduler.**
   This determines priority numbers of the threads which are currently running. It then compares their priority and assigns resources to them, depending on their priority. Threads with the highest priority will be executed for the current time slice. With two or more threads with the same priority, they are put on a stack to allow each run on a given time slice.
2. **Secondary scheduler**
   While primary scheduler runs threads with the highest priority, the secondary scheduler is responsible for increasing the priority of non-executing threads. It is important that those low-priority threads are given a chance to run on the operating system and this is the function of this type of scheduler. This will prevent the blocking of I/O operations.

Process Scheduling

Scheduler operates on a queue of processes, each of which can either be of following types:

1. **Running.**
   This is where actually currently running on the processor.
2. **Waiting.**
   This is where the process is waiting on another process to run and provide it with some data, or if a process is waiting to access a resource.

   A waiting process can sometimes turn into a zombie process, where a process terminates for some reason, but whose parent process has not yet waited for it to terminate. A zombie process is not a big problem, as it has no resources allocated to it.

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3. **Ready.**

   This is where the process is ready to run on the processor, and is not waiting for any other process or has terminated.

4. **Terminated.**

   This is where a process has finished its run, and all resources that have been allocated to it must be taken away from it.

Scheduler makes a decision whenever a change occurs, such as:

(*i*) Running to waiting running to ready.

(*ii*) Waiting to ready running to terminate.

A preemptive scheduler uses a timer to allow each process some time on the processor coordinates access to shared data. Along with this, it requires a kernel designed to protect the integrity of its own data structures.

SCHEDULING QUEUES

There are three main system queues:

(1) Job Queue-incoming jobs

(2) Ready Queues

(3) Device Queues (blocked processes).

Selection Process.

Selection of type of scheduler depends on the type of system that is required, such as:

(*i*) **Long-term (job) scheduler.** This type of scheduler is used in batch systems.

(*ii*) **Short-term scheduler.** This type of scheduler typically uses a FIFO (First In, First Out) queue, or a priority queue.

(*iii*) **Medium-term scheduler.** This type of scheduler swaps processes out to improve job mix.

Interactive processes

These are processes that require some user input, such as from the keyboard or mouse. It is important that user must feel that these processes are running with a high priority, otherwise they may try to delete them, and try to rerun the process.

Interactive editing processes

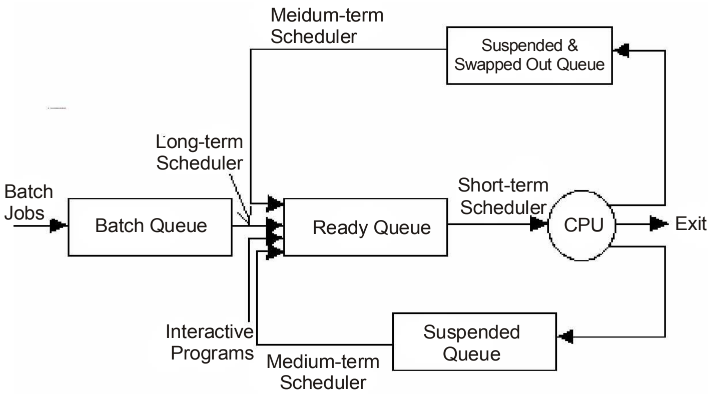
These processes tend to run without user input for long periods, but occasionally require some guidance on how they run.

Batch processes (Lowest priority)

These tend to be less important processes that do not require any user input.

Multilevel Feedback Queue Scheduling

This scheme is the most complex, and allows processes to move between a number of different priority queues. Each queue has an associated scheduling algorithm. To support this there must be a way to promote and relegate processes for their current queues.



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SCHEDULING MECHANISMS

A multiprogramming operating system allows more than one process to be loaded into the executable memory at a time and for the loaded process to share the CPU using time-multiplexing. Multiprogramming is

(*i*) operating system itself is implemented as one or more processes, so there must be a way for the operating system and application processes to share the CPU.

(*ii*) need for processes to perform I/O operations in the normal course of computation.

Since I/O operations ordinarily require orders of magnitude more time to complete than do CPU instructions, multiprograming systems allocate the CPU to another process whenever a process invokes an I/O operation

GOALS FOR SCHEDULING

Scheduling strategy must be good enough with the following criteria:

(1) Utilization/Efficiency: Keep the CPU busy 100% of the time with useful work.

(2) Throughput: Maximize the number of jobs processed per hour.

(3) Turnaround time: from the time of submission to the time of completion, minimize the time batch users must wait for output

(4) Waiting time: Sum of times spent in ready queue - Minimize this

(5) Response Time: Time from submission till the first response is produced, minimize response time for interactive users

(6) Fairness: Make sure each process gets a fair share of the CPU

CONTEXT SWITCHING

Typically there are several tasks to perform in a computer system.

So if one task requires some I/O operation, we want to initiate I/O operation and go on to the next task. We will come back to it later.

This act of switching from one process to another is called "*Context Switch*"

When we return back to a process, we should resume where we left off. For all practical purposes, this process should never know there was a switch, and it should look like this was the only process in the system.

To implement this, on a context switch, we have to

(*i*) save the context of the current process

(*ii*) select the next process to run

(*iii*) restore the context of this new process.

Context of a process

(1) Program Counter

(2) Stack Pointer

(3) Registers

(4) Code + Data + Stack (also called Address Space)

(5) Other state information maintained by the operating system for the process (open files, scheduling info, I/O devices being used etc.)

All this information is usually stored in a structure called Process Control Block (PCB).

All the above has to be saved and restored.

A context_switch() routine look like given below :

```
context_switch()
{
     Push registers onto stack
     Save ptrs to code and data.
     Save stack pointer
     Pick next process to execute
     Restore stack ptr of that process /* You have now switched the stack */
     Restore ptrs to code and data.
 Pop registers
 Return
}
```

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NON-PREEMPTIVE VS PREEMPTIVE SCHEDULING

Preemptive and Non-preemptive Scheduler

A complication that schedulers have to deal with is that every process is unique and unpredictable. Some spend a lot of time waiting for file I/O, while others would use the CPU for hours at a time if given the chance. When the scheduler starts running some process, it never knows for sure how long it will be until that process blocks, either for I/O, or on a semaphore, or for some other reason. To make sure that no process runs too long, nearly all computers have an electronic timer or clock built in, which causes an interrupt periodically.

The strategy of allowing processes that are logically runnable to be temporarily suspended is called *preemptive scheduling*, while the run to completion method of the early batch systems is called *non-preemptive scheduling*.

1. ***Non-Preemptive algorithms.***

   These are designed so that once a process enters the running state (is allowed a process), it is not removed from the processor until it has completed its service time (or it explicitly yields the processor).

   context_switch() is called only when the process terminates or blocks.

2. ***Preemptive algorithms.***

   These are driven by the notion of prioritized computation. The process with the highest priority should always be the one currently using the processor. If a process is currently using the processor and a new process with a higher priority enters, the ready list, the process on the processor should be removed and returned to the ready list until it is once again the highest-priority process in the system.

   context_switch() is called even when the process is running usually done via a timer interrupt.

SCHEDULING ALGORITHMS

1. **First In First Out (FIFO)**

   This is a *Non-Premptive* scheduling algorithm. FIFO strategy assigns priority to processes in the order in which they request the processor.The process that requests the CPU first is allocated the CPU first.When a process comes in, add its PCB to the tail of ready queue. When running process terminates, dequeue the process (PCB) at head of ready queue and run it.

```
Consider the example with P1=24, P2=3, P3=3

      Gantt Chart for FCFS : 0 - 24 P1 , 25 - 27 P2 , 28 - 30 P3

      Turnaround time for P1 = 24
      Turnaround time for P2 = 24 + 3
      Turnaround time for P3 = 24 + 3 + 3
      Average Turnaround time = (24*3 + 3*2 + 3*1) / 3

      In general we have (n*a + (n-1)*b + ....) / n

      If we want to minimize this, a should be the smallest, followed by b and so on.
```

   Comments: While FIFO algorithm is easy to implement, it ignores the service time request and all other criteria that may influence the performance with respect to turnaround or waiting time.

   Problem with F1F0 is that one process can monopolize CPU

   For this limit the amount of time a process can run without a context switch. This time is called a time slice.

2. **Round Robin**

   It calls for the distribution of the processing time equitably among all processes requesting the processor.Run process for one time slice, then move to back of queue. Each process gets equal share of the CPU. Most systems use some variant of this.

   Choosing Time Slice

   For example, consider two processes, one doing 1 ms computation followed by 10 ms I/O, and other doing all computation. Let we use 20 ms time slice and round-robin scheduling:

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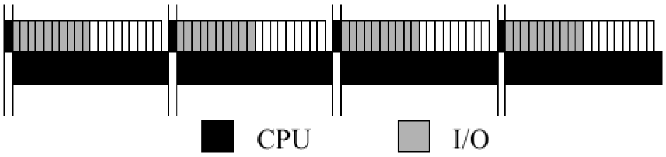
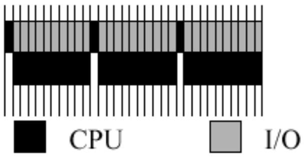
I/O process runs at 11/21 speed, I/O devices are only utilized 10/21 of time.



Let we use 1 ms time slice. Then compute-bound process gets interrupted 9 times unnecessarily before I/O-bound process is runnable



3. **Priority Based Scheduling**

Run highest-priority processes first, use round-robin among processes of equal priority. Re-insert process in run queue behind all processes of greater or equal priority.

- Allows CPU to be given preferentially to important processes.
- Scheduler adjusts dispatcher priorities to achieve the desired overall priorities for the processes, e.g. one process gets 90% of the CPU.

Comments: In priority scheduling, processes are allocated to the CPU on the basis of an externally assigned priority. The key to the performance of priority scheduling is in choosing priorities for the processes.

Problem: Priority scheduling is that it may cause low-priority processes to starve

Solution: (AGING) This starvation can be compensated for if the priorities are internally computed. Suppose one parameter in the priority assignment function is the amount of time the process has been waiting. The longer a process waits, the higher its priority becomes. This strategy tends to eliminate the starvation problem.

4. **Shortest Job First (SJF)**

Maintain Ready queue in order of increasing job lengths. When a job comes in, insert it in the ready queue based on its length. When current process is done, pick the one at the head of the queue and run it.

This is provably the most optimal in terms of turnaround/response time.

But, how do we find the length of a job?

Make an estimate based on the past behavior.

```
Say the estimated time (burst) for a process is E0, suppose the actual time is measured to be
T0.
Update the estimate by taking a weighted sum of these two ie. E1 = aT0 + (1-a)E0

in general, E(n+1) = aTn + (1-a)En (Exponential average)

if a=0, recent history no weightage
if a=1, past history no weightage.
typically a=1/2.
       E(n+1) = aTn + (1-a)aTn-1 + (1-a)^jatn-j + ...
Older information has less weightage
```

Comments: SJF is proven optimal only when all jobs are available simultaneously.

Problem: SJF minimizes the average wait time because it services small processes before it services large ones. While it minimizes average wiat time, it may penalize processes with high service time requests. If the ready list is saturated, then processes with large service times tend to be left in the ready list while small processes receive service. In extreme case, where the system has little idle time, processes with large service times will never be served. This total starvation of large processes may be a serious liability of this algorithm.

For this Multi-Level Feedback Queques

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5. Multilevel queue scheduling

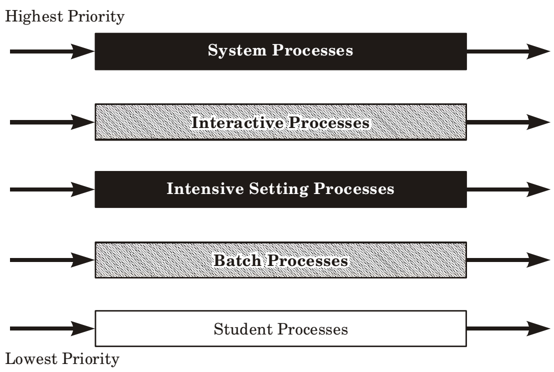
Under this type of scheduling, processes are easily classified into different groups.

Common division is made between

(*i*) *Foreground* (interactive) processes

(*ii*) *Background* (batch) processes.

These two types of processes have different response-time requirements, and so might have different scheduling needs. Foreground processes may have priority (externally defined) over back ground processes.



A *multilevel queue-scheduling algorithm* partitions the ready queue into several separate queues. The processes are permanently assigned to the queue

The drawback of this algorithm is starvation of processes at low priority level.

6. Multilevel feedback queue scheduling

It allows a process to move between queues. The idea is to separate processes with different CPU-burst characteristics. If a process uses too much CPU time, it will be moved to a lower-priority queue. This scheme leaves I/O-bound and interactive processes in the higher-priority queues. Similarly, a process that waits too long in a lower-priority queue may be moved to a higher-priority queue. This form of aging prevents starvation.

In general, a multilevel feedback queue scheduler is defined by the following parameters :

(*i*) Number of queues

(*ii*) Scheduling algorithm for each queue

(*iii*) Method used to determine when to upgrade a process to a higher-priority queue

(*iv*) Method used to determine when to denote a process to a lower-priority queue

(*v*) Method used to determine which queue a process will enter when that process needs service

The definition of a multilevel feedback queue scheduler makes it the most general CPU scheduling algorithm.

DEADLOCK

A set of process is in a deadlock state if each process in the set is waiting for an event that can be caused by only another process in the set, or each member of the set of deadlock processes is waiting for a resource that can be released only by a deadlock process. None of the processes can run, none of them can release any resources, and none of them can be awakened. The number of processes and the number and type of resources possessed and requested are unimportant.

Resources may be

(*i*) *either physical resource*

e.g., printers, Tape Drivers, Memory Space, and *CPU* Cycles.

(*ii*) *logical resources*

e.g., Files, Semaphores, and Monitors.

The simplest example of deadlock is where process 1 has been allocated non-shareable resources *A*, say, a tap drive, and process 2 has be allocated non-sharable resource *B*, say, a printer. Now, if it turns out that process 1 needs resource *B* (printer) to proceed and process 2 needs resource *A* (the tape drive) to proceed and these are the only two processes in the system, each is blocked the other and all useful work in the system stops. This situation is called *deadlock*. The system is in deadlock state because each process holds a resource being requested by the other process neither process is willing to release the resource it holds.

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Preemptable and Non-preemptable Resources

Resources come in two flavors:

(*i*) **Preemptable resource :** It is one that can be taken away from the process with no ill effects, *e.g.,* Memory

(*ii*) **Non-preemptable resource :** It cannot be taken away from process (without causing ill effect) *e.g., CD* resources are not preemptable at an arbitrary moment.

Reallocating resources can resolve deadlocks that involve preemptable resources. Deadlocks that involve nonpreemptable resources are difficult to deal with.

NECESSARY AND SUFFICIENT DEADLOCK CONDITIONS.

Coffman (1971) identified following four conditions that must hold simultaneously for there to be a deadlock.

1. **Mutual Exclusion condition**

   The resources involved are non-shareable.

   Explanation: At least one resource (thread) must be held in a non-shareable mode, *i.e.,* only one process at a time claims exclusive control of the resource. If another process requests that resource, the requesting process must be delayed until the resource has been released.

2. **Hold and Wait condition**

   Requesting process hold already, resources while waiting for requested resources.

   Explanation: There must exist a process that is holding a resource already allocated to it while waiting for additional resource that are currently being held by other processes.

3. **No-Preemptive condition**

   Resources already allocated to a process cannot be preempted.

   Explanation: Resources cannot be removed from the processes are used to completion or released voluntarily by the process holding it.

4. **Circular Wait condition**

   The processes in the system form a circular list or chain where each process in the list is waiting for a resource held by the next process in the list.

Example : Consider traffic deadlock in the following figure

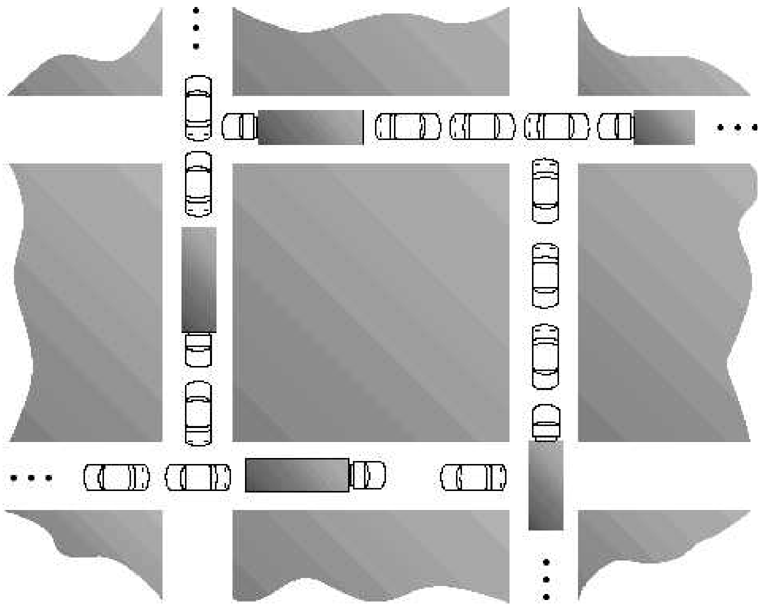
Consider each section of the street as a resource.

(1) *Mutual exclusion condition applies,* since only one vehicle can be on a section of the street at a time.

(2) *Hold-and-wait condition applies,* since each vehicle is occupying a section of the street, and waiting to move on to the next section of the street.

(3) *No-preemptive condition applies,* since a section of the street that is a section of the street that is occupied by a vehicle cannot be taken away from it.

(4) *Circular wait condition applies,* since each vehicle is waiting on the next vehicle to move, *i.e.,* each vehicle in the traffic is waiting for a section of street held by the next vehicle in the traffic.



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The simple rule to avoid traffic deadlock is that a vehicle should only enter an intersection if it is assured that it will not have to stop inside the intersection.

It is not possible to have a deadlock involving only one single process. The deadlock involves a circular "*hold-and-wait*" condition between two or more processes, so "one" process cannot hold a resource, yet be waiting for another resource that it is holding. In addition, deadlock is not possible between two threads in a process, because it is the process that holds resources, not the thread, *i.e.,* each thread has access to the resources held by the process.

DEALING WITH DEADLOCK PROBLEM

In general, there are four strategies of dealing with deadlock problem:

(1) Ostrich approach : Just ignore the deadlock problem altogether.

(2) Deadlock detection and recovery : Detect deadlock and, when it occurs, take steps to recover.

(3) Deadlock avoidance : Avoid deadlock by careful resource scheduling.

(4) Deadlock prevention : Prevent deadlock by resource scheduling so as to negate at least one of the four conditions.

Now consider each strategy in order of decreasing severity.

Deadlock Prevention : Havender in his pioneering work showed that since all four of the conditions are necessary for deadlock to occur, it follows that deadlock might be prevented by denying any one of the following conditions :

(1) Elimination of "Mutual Exclusion" condition.

The mutual exclusion condition must hold for non-sharable resources, *i.e.,* several processes cannot simultaneously share a single resource. This condition is difficult to eliminate because some resources, such as the tap drive and printer, are inherently non-shareable. Shareable resources like read-only-file do not require mutually exclusive access and thus cannot be involved in deadlock.

(2) Elimination of "Hold and Wait" condition.

There are two possibilities for elimination of the second condition :

(*i*) A process request be granted all of the resources it needs at once, prior to execution.

(*ii*) To disallow a process from requesting resources whenever it has previously allocated resources. This strategy requires that all of the resources a process will need must be requested at once. The system must grant resources on "*all or none*" basis. If complete set of resources needed by a process is not currently available, then the process must wait until the complete set is available, while the process waits, however, it may not hold any resources. Thus "*wait for*" condition is denied and deadlocks simply cannot occur. This strategy can lead to serious waste of resources.

(3) Elimination of "No-preemption" condition.

The nonpreemption condition can be alleviated by forcing a process waiting for a resource that cannot immediately be allocated to relinquish all of its currently held resources, so that other processes may use them to finish. Let a system does allow processes to hold resources while requesting additional resources. Consider what happens when a request cannot be satisfied. A process holds resources a second process may need in order to proceed while second process may hold the resources needed by the first process. This is a deadlock. This strategy require that when a process that is holding some resources is denied a request for additional resources. The process must release its held resources and, if necessary, request them again together with additional resources. Implementation of this strategy denies the "*no-preemptive*" condition effectively.

High cost. When a process release resources, the process may lose all its work to that point. One serious consequence of this strategy is the possibility of indefinite postponement (starvation). A process might be held off indefinitely as it repeatedly requests and releases the same resources.

(4) Elimination of "Circular Wait" condition

This condition can be denied by imposing a total ordering on all of the resource types and than forcing, all processes to request the resources in order (increasing or decreasing). This strategy impose a total ordering of all resources types, and to require that each process requests resources in a numerical order (increasing or decreasing) of enumeration. With this rule, the resource allocation graph can never have a cycle.

Rule: Processes can request resources whenever they want to, but all requests must be made in numerical order. A process may request first printer and then a tape drive (order: 2, 4), but it may not request first a plotter and then a printer (order: 3, 2). The problem with this strategy is that it may be impossible to find an ordering that satisfies everyone.

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DEADLOCK AVOIDANCE.

This approach to the deadlock problem anticipates deadlock before it actually occurs. This approach employs an algorithm to access the possibility that deadlock could occur and acting accordingly. This method differs from deadlock prevention, which guarantees that deadlock cannot occur by denying one of the necessary conditions of deadlock.

If necessary conditions for a deadlock are in place, it is still possible to avoid deadlock by being careful when resources are allocated. The most famous deadlock avoidance algorithm, due to Dijkstra [1965], is the Banker's algorithm. In this process is analogous to that used by a banker in deciding if a loan can be safely made.

Banker's Algorithm

In this analogy

Customers	≡	processes
Units	≡	resources, say, tape drive
Banker	≡	Operating System

Customers	Used	Max	
A	0	6	
B	0	5	Available Units = 10
C	0	4	
D	0	7	

In the above, we see four customers each of whom has been granted a number of credit units. The banker reserved only 10 units rather than 22 units to service them. At certain moment, the situation becomes

Customers	Used	Max	
A	1	6	
B	1	5	Available Units = 2
C	2	4	
D	4	7	

Safe state. The key to a state being safe is that there is at least one way for all users to finish. In other analogy, the state of above is safe because with 2 units left, the banker can delay any request except *C's*, thus letting *C* finish and release all four resources. With four units in hand, the banker can let either *D* or *B* have the necessary units and so on.

Unsafe state. Consider what would happen if a request from B for one more unit were granted in above.

We would have following situation

Customers	Used	Max	
A	1	6	
B	2	5	Available Units = 1
C	2	4	
D	4	7	

This is an unsafe state.

If all the customers namely *A, B, C,* and *D* asked for their maximum units, then banker could not satisfy any of them and we would have a deadlock.

Note: An unsafe state does not imply the existence or even the eventual existence a deadlock. What an unsafe state does imply is simply that some unfortunate sequence of events might lead to a deadlock.

Thus Banker's algorithm is to consider each request as it occurs, and see if granting it leads to a safe state. If it does, the request is granted, otherwise, it postponed until later. Haberman has shown that executing of the algorithm has complexity proportional to N^2, where N is number of processes and since the algorithm is executed each time a resource request occurs, the overhead is significant.

DEADLOCK DETECTION.

It is the process of actually determining that a deadlock exists and identifying the processes and resources involved in the deadlock.

The basic idea is to check allocation against resource availability for all possible allocation sequences to determine if system is in deadlocked state a. Deadlock detection algorithm is only half of this strategy. *Once a deadlock is detected, there needs to be a way to recover several alternatives exists:*

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Methods.

(1) Temporarily prevent resources from deadlocked processes.

(2) Back off a process to some check point allowing preemption of a needed resource and restarting the process at the checkpoint later.

(3) Successively kill processes until the system is deadlock free.

These methods are expensive in the sense that each iteration calls the detection algorithm until the system proves to be deadlock free. The complexity of algorithm is $O(N^2)$, where N is number of proceeds. Another potential problem is starvation; same process killed repeatedly.

MEMORY MANAGEMENT AND VIRTUAL MEMORY

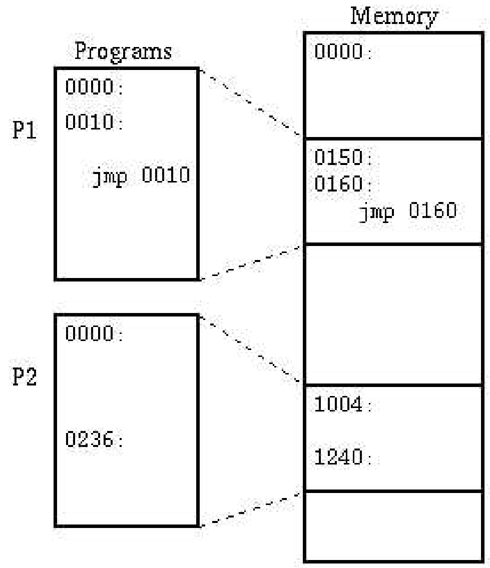
SHARING MAIN MEMORY

Issues

1. Want to let several processes coexist in main memory.
2. No process should need to be aware of the fact that memory is shared. Each must run regardless of the number and/or locations of processes.
3. Processes must not be able to corrupt each other.
4. Efficiency (both of CPU and memory) should not be degraded badly by sharing. After all, the purpose of sharing is to increase overall efficiency.

RELOCATION.

Draw a simple figure of memory with some processes in it.



Because several processes share memory, it cannot be predicted in advance where a process will be loaded in memory. This is similar to a compiler's inability to predict where a subroutine will be after linking.

Relocation adjusts a program to run in a different area of memory. Linker is an example of static relocation used to combine modules into programs.

Relocation Techniques

These allow several programs to share one main memory.

1. Static software relocation, no protection.

(*i*) Lowest memory holds OS.

(*ii*) Processes are allocated memory above the operating system.

(*iii*) When a process is loaded, relocate it so that it can run in its allocated memory area (just like linker: linker combines several modules into one program, OS loader combines several processes to fit into one memory; only difference is that there are no cross-references between processes).

Problem: Any process can destroy any other process and/or the operating system.

e.g., early batch monitors where only one job ran at a time and all it could do was wreck the operating system, which would be rebooted by an operator. Many of today's personal computers also operate in a similar fashion.

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Protection Keys :

Protection key is a small integer stored with each chunk of memory. The chunks are likely to be 1k-4k bytes.

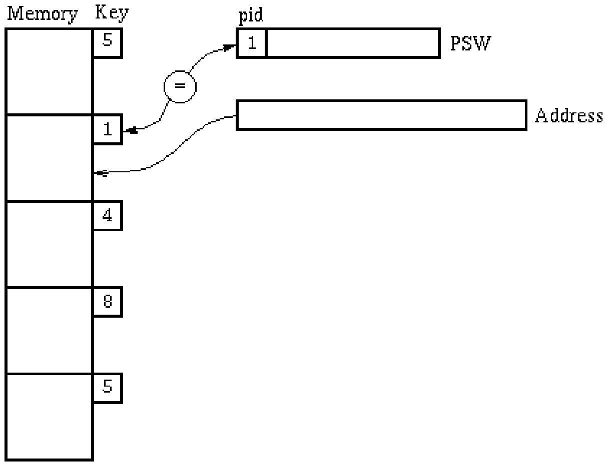

Fig. Static memory relocation with protection keys (IBM S/360 approach)

Process id, or PID.0 : Keep an extra hardware register to identify the current process. This is called *process id*, or PID. 0 is reserved for the operating system's process id.

On every memory reference, check PID of the current process against the key of the memory chunk being accessed. PID 0 is allowed to touch anything, but any other mismatch results in an error trap.

Additional control:

Who is allowed to set the PID?

How does OS regain control once it has given it up?

Scheme used for IBM S/360 family is safe but inconvenient due to following reasons:

(*i*) Programs have to be relocated before loading. In some systems (e.g. MPS) this requires complete relinking.

(*ii*) Expensive.

(*iii*) Cannot share information between two processes very easily

(*iv*) Cannot swap a process out to secondary storage and bring it back to a different location

2. **Dynamic memory relocation.**

Instead of changing addresses of a program before it is loaded, change the address dynamically *during every reference*.

Under dynamic relocation, each program-generated address (called a *logical* or *virtual* address) is translated in hardware to a *physical*, or *real* address. This happens as part of each memory reference.

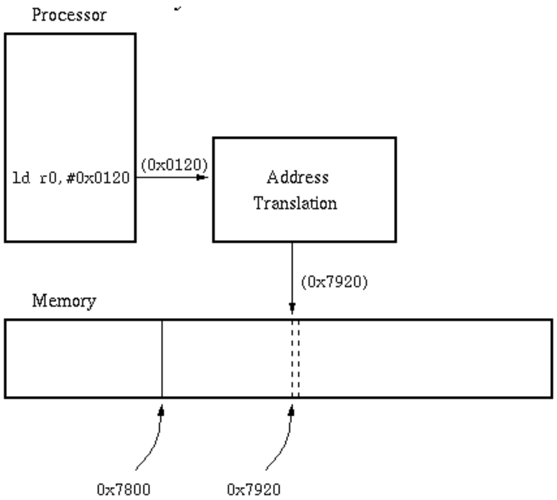

Fig. Dynamic translation

Show how dynamic relocation leads to two views of memory, called *address spaces*. With static relocation we force the views to coincide. That there can be several levels of mapping.

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SWAPPING

It is a process of removing the currently running program from memory (called *swap out*) and bringing in another program in to the memory (called *swap in*).

A program can be swapped out undere following conditions :

1. If its time quantum has expired
2. If higher priority process has arrived
3. If using preemptive SJF scheduling algorithm
4. If a program has completed its execution.

Note : A process with pending I/O should never be swapped out.

Swapping requires backing store which could be fast disk. It should be large enough to accomodate copies of all memory images for all users.

The context-switch time in such a swapping system is fairly high.

Consider an example (using RR scheduling algorithm) :

Assume that the user process is of size 200 K and backing store is a hard disk with transfer rate 10 megabyte per second (assume no head seeks and an average latency of 8 millisecond).

Actual transfer of the 200 K process to or from memory takes

$$\frac{200}{10,000} = \frac{1}{50} \text{ second} = 2 \text{ millisecond}$$

Swap time takes = 8 + 2 = 10 millisecond

Since a process is swapped out, total swap time taken = 10 + 10 = 20 milliseconds.

Thus, in a RR scheduling algorithm, time quantum should be more than 20 milliseconds for efficient CPU utilization.

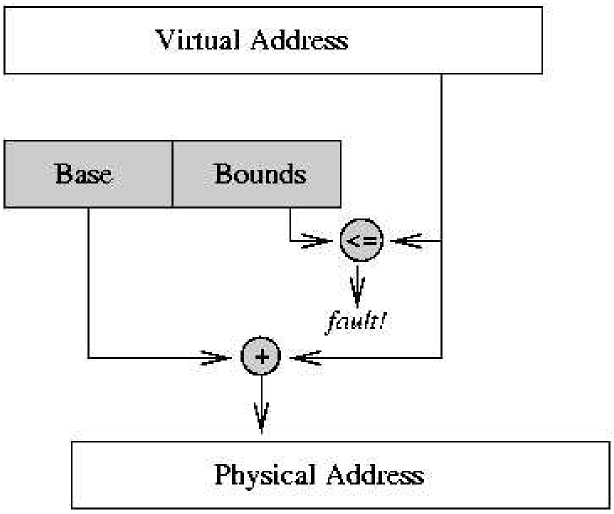
BASE AND BOUNDS, SEGMENTATION

Base and Bounds Relocation

Two Hardware registers.

Base address for process, bounds register that indicates last valid address the process may generate.

Each process must be allocated contiguously in real memory.



On each memory reference, virtual address is compared to the bounds register, then added to the base register. A bounds violation results in an error trap.

Each process appears to have a completely private memory of size equal to the bounds register plus 1. Processes are protected from each other. No address relocation is necessary when a process is loaded.

Typically, operating system runs with relocation turned off, and there are special instructions to branch to and from the operating system while at the same time turning relocation on and off. Modification of the base and bounds registers must also be controlled.

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Advantages of Base and Bounds :

(*i*) is cheap

(*ii*) Only 2 registers

(*iii*) Fast

(*iv*) *Add* and *compare* can be done in parallel.

Explain how swapping can work.

e.g., CRAY-1.

Problem with Base & Bound relocation:

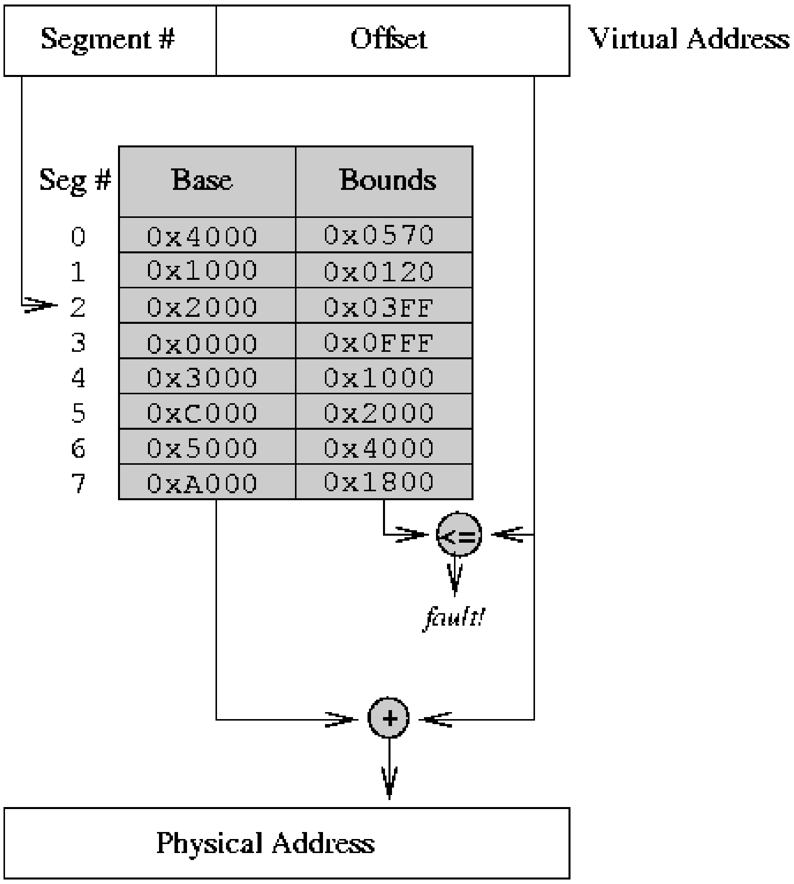
Only one segment. How can two processes share code while keeping private data areas?

e.g. shared editor

Draw a picture to show that it cannot be done safely with a single-segment scheme.

MULTIPLE SEGMENTS.

Permit process to be split between several areas of memory. Each area is called a *segment* and contains a collection of logically-related information, e.g. code or data for a module.



It use a separate base and bound for each segment, and also add a protection bit (read/write).

Each memory reference indicates a segment and offset in one or more of following three ways:

(*i*) Top bits of address select segment, low bits the offset. This is the most common, and the best.

(*ii*) Segment is selected implicitly by the operation being performed
e.g. code vs. data, stack vs. data.

(*iii*) Segment is selected by fields in the instruction
e.g., as in Intel x86 prefixes.

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Note : Last two alternatives are kludges used for machines with such small addresses that there is not room for both a segment number and an offset.

Segment table holds the bases and bounds for all the segments of a process.

Show memory mapping procedure, involving table lookup + add + compare.

e.g. PDP-10 with high and low segments selected by high-order address bit.

Example: 8-bit segment number, 16-bit offset.

- Segment table (use above picture — all numbers in hexadecimal):
- Code in segment 0 (addresses are virtual):
- `0×00242:    mov 0×60100,%r1`
- `0×00246:    st %r1,0×30107`
- `0×0024A:    b 0x20360`
- `Code in segment 2:`
- `0×20360:    ld [%r1+2],%r2`
- `0×20364:    ld [%r2],%r3`
- `...`
- `0×203C0:    ret`

Advantage :
Segments can be swapped and assigned to storage independently.

Problems.

1. *External fragmentation:* segments of many different sizes.
2. Segments may be large, have to be allocated contiguously.
3. These problems also apply to base and bound schemes.

Example: In PDP-10's when a segment gets larger, it may have to be shuffled to make room. If things get really bad it may be necessary to compact memory.

PAGING

It is a technique to solve the problem of external fragmentation. The logical address of a process can occupy non-contiguous space which map on to the physical address when space in the main memory is available.

Goal of Paging

1. To make allocation and swapping easier

   Keep a free list of available pages and grab the first one. Easy to swap since everything is the same size, which is usually the same size as disk blocks to and from which pages are swapped.
2. To reduce memory fragmentation.

Pages

Make all chunks of memory the same size, call them *pages*. Typical sizes range from 512-8k bytes.

Page table.

For each process, it defines base address of each of that process' pages along with read/only and existence bits.

Problems in Paging

1. Internal fragmentation.

Page size does not match up with information size. The larger the page, the worse this is.

2. Table space.

If pages are small, the table space could be substantial. In fact, this is a problem even for normal page sizes.

Consider a 32-bit address space with 1k pages. What if the whole table has to be present at once?

Partial solution: Keep base and bounds for page table, so only large processes have to have large tables.

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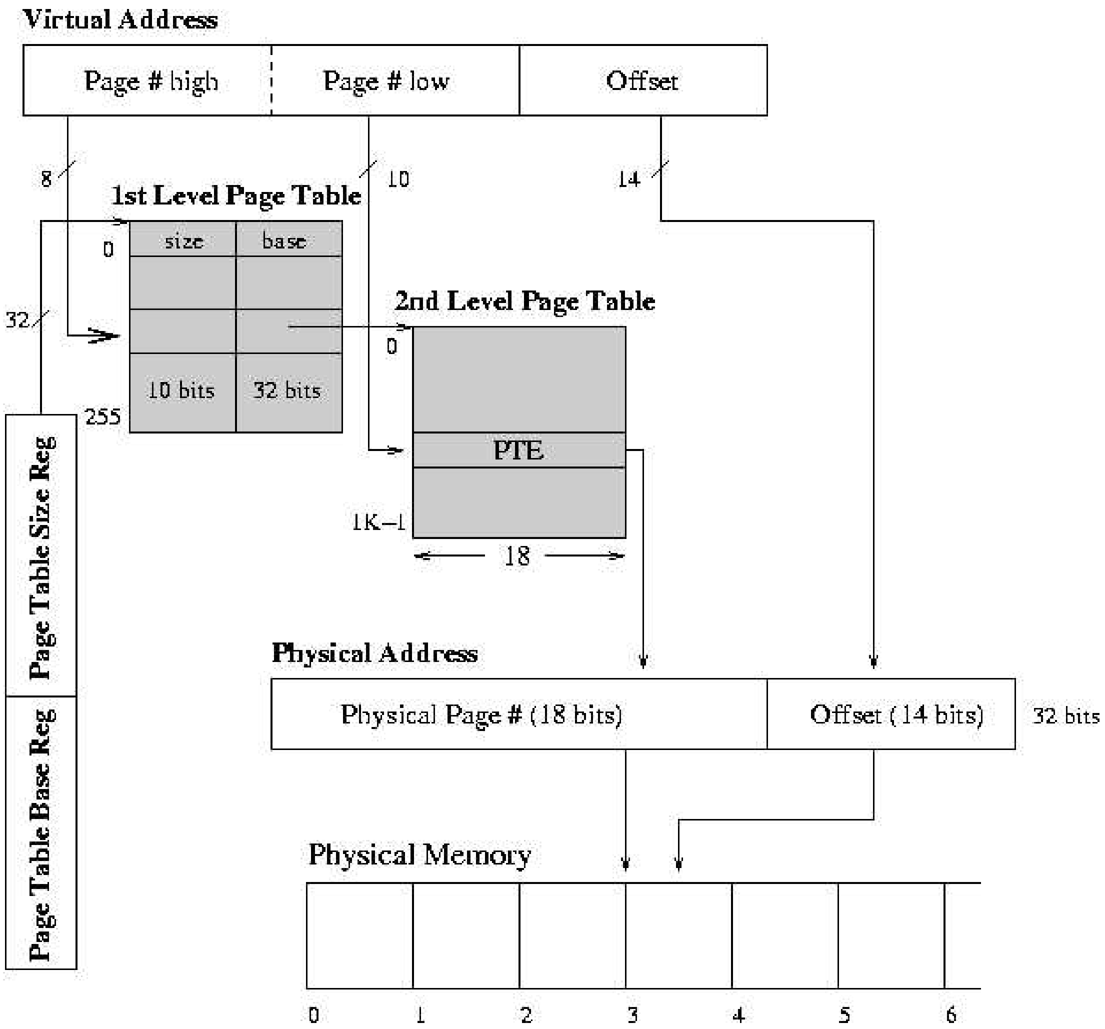
3. Efficiency of access.

It may take one overhead reference for every real memory reference (page table is so big it has to be kept in memory).

Two-Level (Multi-Level) Paging



Segmentation and Paging

- Use two levels of mapping, with logical sizes for objects, to make tables manageable.
- Each segment contains one or more pages.
- *Segment correspond to logical units:*

  (*i*) Code

  (*ii*) Data

  (*iii*)Stack.

  Segments vary in size and are often large. Pages are used for the operating system; they are of fixed size to make it easy to manage memory.
- Going from paging to Paging + Segmentation is like going from Single segment to Multiple segments, except at a higher level. Instead of having a single page table, have many page tables with a base and bound for each. Call the stuff associated with each page table a segment.

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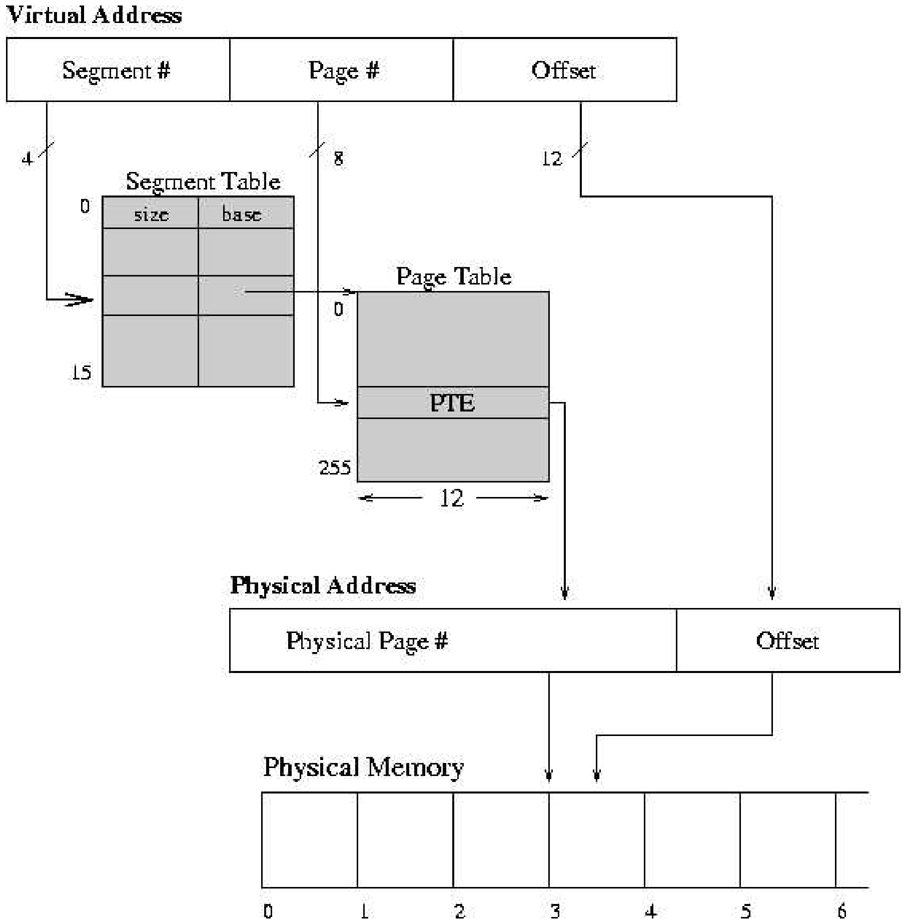

System 370

Example: 24-bit virtual address space, 4 bits of segment number, 8 bits of page number, and 12 bits of offset. Segment table contains real address of page table along with the length of the page table (a sort of bounds register for the segment). Page table entries are only 12 bits, real addresses are 24 bits.

- If a segment is not used, then there is no need to even have a page table for it.
- Can share at two levels: single page, or single segment (whole page table).

Pages eliminate external fragmentation, and make it possible for segments to grow without any reshuffling.

If page size is small compared to most segments, then internal fragmentation is not too bad.

The user is not given access to the paging tables.

If translation tables are kept in main memory, overheads could be very high: 1 or 2 overhead references for every real reference.

Example: VAX.

- Address is 32 bits, top two select segment. Three base-bound pairs define page tables (system, P0, P1).
- Pages are 512 bytes long.
- Read-write protection information is contained in the page table entries, not in the segment table.
- One segment contains operating system stuff, two contain stuff of current user process.

Potential problem:

Page tables can get big. Do not want to have to allocate them contiguously, especially for large user processes.

Solution:

System base-bounds pairs are physical addresses, system tables must be contiguous.

User base-bounds pairs are virtual addresses in the system space. This allows the user page tables to be scattered in non-contiguous pages of physical memory. The result is a two-level scheme.

In current systems, we will see three and even four-level schemes to handle 64-bit address spaces.

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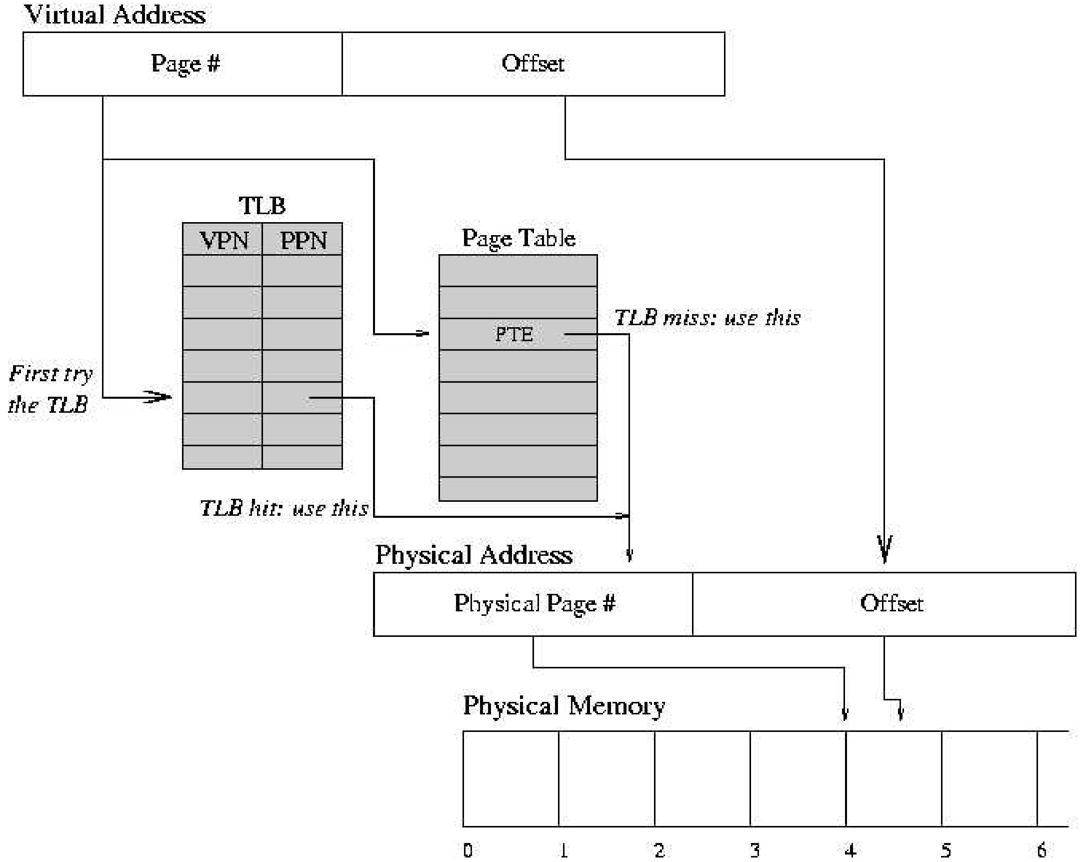
Problem with Segmentation and Paging

Extra memory references to access translation tables can slow down programs by a factor of two or three. Too many entries in translation tables to keep them all loaded in fast processor memory.

Solution is r*e-introduce fundamental concept of locality:*

At any given time, a process is only using a few pages or segments.

TRANSLATION LOOKASIDE BUFFER (TLB).



It is used to store a few translation table entries. It is very fast, but only remembers a small number of entries.

On each memory reference:

- First ask TLB if it knows about the page. If so, the reference proceeds fast.
- If TLB has no info for page, must go through page and segment tables to get info. Reference takes a long time, but give the info for this page to TLB, so it will know it for next reference (TLB must forget one of its current entries in order to record new one).

TLB Organization.

It show picture of black box. Virtual page number goes in, physical page location comes out. It is similar to a cache, usually direct mapped.

TLB is just a memory with some comparators.

Typical size of memory: 128 entries.

Each entry holds a virtual page number and corresponding physical page number.

Organising memory to find an entry quickly?

There are two possibilities :

(*i*) Search whole table from start on every reference.

(*ii*) Restrict the info for any given virtual page to fall in exactly one location in the memory. Then only need to check that one location.

e.g. use low-order bits of the virtual page number as the index into the memory. This is the way real TLB's work.

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Disadvantage of TLB scheme.

If two pages use same entry of the memory, only one of them can be remembered at once. If process is referencing both pages at same time, TLB does not work very well.

Example. TLB with 64 (100 octal) slots.

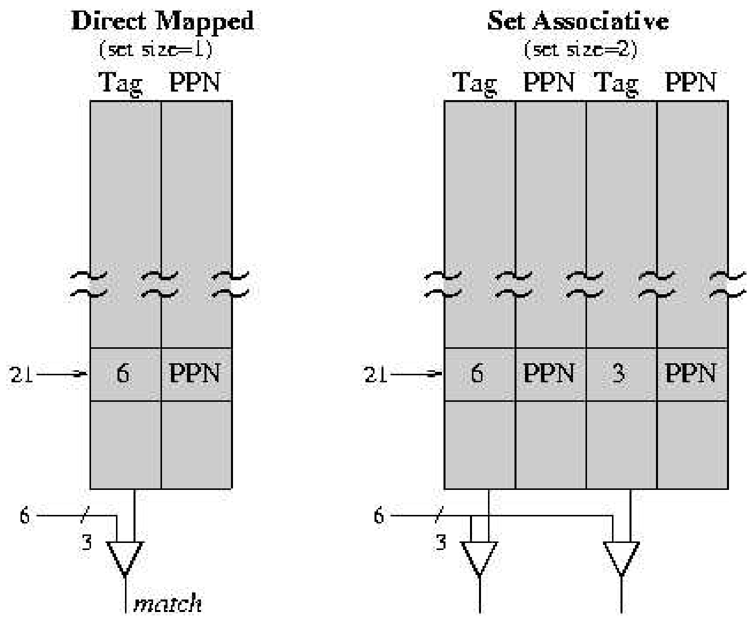
Suppose following virtual pages are referenced (octal):

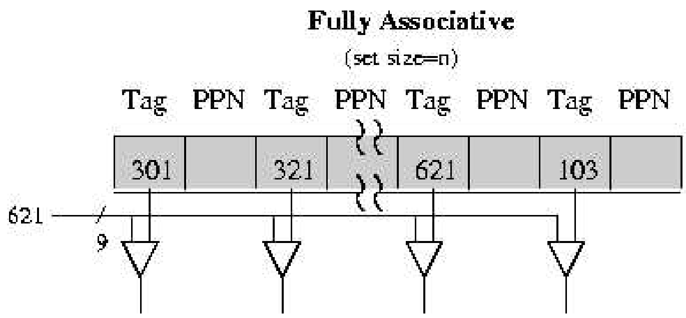
621, 2145, 621, 2145, ... 321, 2145, 321, 621.

TLBs are a lot like hash tables except simpler (must be to be implemented in hardware). Some hash functions are better than others.

(*i*) Is it better to use low page number bits than high ones?

(*ii*) Is there any way to improve on the TLB hashing function?





Another approach:

Let any given virtual page use either of *two* slots in the TLB. Make memory wider, use two comparators to check both slots at once.

This is as fast as the simple scheme, but a bit more expensive (two comparators instead of one, also have to decide which old entry to replace when bringing in a new entry).

Advantage.

Less likely that there will be conflicts that degrade performance. *Takes following three pages falling in the same place, instead of two :*

(*i*) Direct mapped

(*ii*) Set associative

(*iii*) Fully associative

Note : Must be careful to flush TLB during each context swap because in practice, TLB's have been extremely successful with 95% or great hit rates for relatively small sizes.

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INVERTED PAGE TABLES.

As address spaces have grown to 64 bits, the side of traditional page tables becomes a problem. Even with two-level (or even three or four!) page tables, the tables themselves can become too large.

A solution to this problem has two parts:

(1) *Physical page table instead of a logical one.*

The physical page table is often called an *inverted* page table. This table contains one entry per page frame. An inverted page table is very good at mapping from physical page to logical page number (as is done by the operating system during a page fault), but not very good at mapping from virtual page number to physical page number (as is done on every memory reference by the processor).

(2) *TLB fixes above problem.*

Since there is no other hardware or registers dedicated to memory mapping, TLB can be quite larger so that missing-entry faults are rare.

With an inverted page table, most address translations are handled by the TLB. When there is a miss in the TLB, operating is notified (via an interrupt) and TLB miss-handler is invoked.

SHADOW TABLES

Operating system can sometimes be thought of as an extension of the abstractions provided by the hardware. However, when table format is defined by the hardware (such as for a page table entry), that format cannot be changed. So, if it is required to store additional information, such as last reference time or sharing pointer, in each entry, general technique is adopted called a *shadow table*.

Idea of a shadow (simple and familiar to Fortran programmers):

(*i*) Consider hardware defined data structure as an array.

(*ii*) For the new information that is required to add, define a new (shadow) array.

(*iii*) There is one entry in the shadow array for each entry in the hardware array.

(*iv*) For each new item required to add to the data structure, add a new data member to the shadow array.

Example : Consider the hardware defined page table to be an array of structures:

```
struct Page_Entry {
     unsigned PageFrame_hi  : 10; // 42-bit page frame number
     unsigned PageFrame_mid : 16;
     unsigned PageFrame_low : 16;
     unsigned UserRead      : 1;
     unsigned UserWrite     : 1;
     unsigned KernelRead    : 1;
     unsigned KernelWrite   : 1;
     unsigned Reference     : 1;
     unsigned Dirty         : 1;
     unsigned Valid         : 1;
 }
struct Page_Entry pageTable[TABLESIZE];
```

If wanted to added a couple of data members, **cannot** simply change it to the following:

```
 struct Page_Entry {
     unsigned PageFrame_hi  : 10;
     unsigned PageFrame_mid : 16;
     unsigned PageFrame_low : 16;
     unsigned UserRead      : 1;
     unsigned UserWrite     : 1;
     unsigned KernelRead    : 1;
     unsigned KernelWrite   : 1;
     unsigned Reference     : 1;
     unsigned Dirty         : 1;
     unsigned Valid         : 1;
     Time_t lastRefTime;
     PageList *shared;
 }
```

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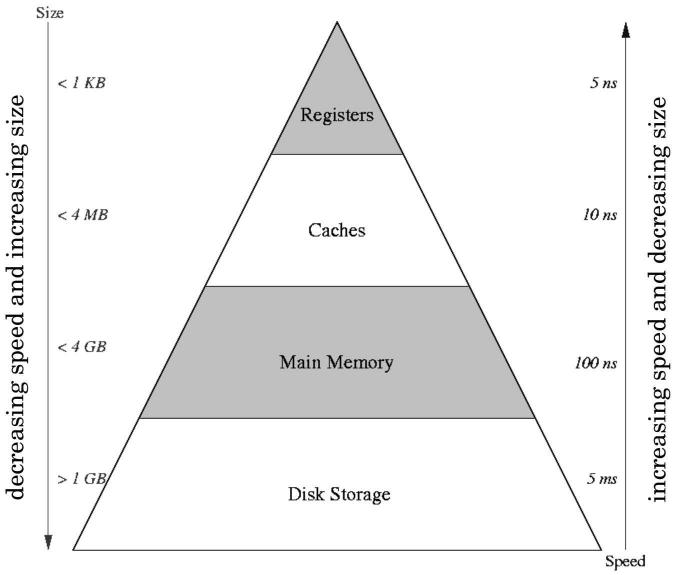
Instead, would define a second array based on this type:

```
struct Page_Entry {                    struct PE_Shadow {
     unsigned PageFrame_hi  : 10;           Time_t lastRefTime;
     unsigned PageFrame_mid : 16;           PageList *shared;
     unsigned PageFrame_low : 16;      }
     unsigned UserRead      : 1;
     unsigned UserWrite     : 1;
     unsigned KernelRead    : 1;
     unsigned KernelWrite   : 1;
     unsigned Reference     : 1;
     unsigned Dirty         : 1;
     unsigned Valid         : 1;
 }
 struct Page_Entry pageTable[TABLESIZE];
 struct PE_Shadow pageShadow[TABLESIZE];
```

MEMORY HIERARCHY.

It's idea is to produce illusion of a memory with the size of the disk and speed of main memory.

Data can be in registers (very fast), caches (fast), main memory (not so fast, or disk (slow), keep the things that are used frequently as close (and as fast to access) as possible.



Because this works is that most programs spend most of their time in only a small piece of the code. Give Knuth's estimate of 90% of the time in 10% of the code. Introduce again the principle of *locality*.

PAGE FAULTS

If not all of process is loaded when it is running, and when it references a byte that is only in the backing store, hardware and software cooperate to make things work anyway.

First, extend the page tables with an extra bit "*present*". If present is not set, then a reference to the page results in a trap. This trap is called *page fault*.

Any page not in main memory right now has the *present bit* cleared in its page table entry.

Effects of page fault

(*i*) Operating system brings page into memory.

(*ii*) Page table is updated, "*present*" bit is set.

(*iii*) Process is continued.

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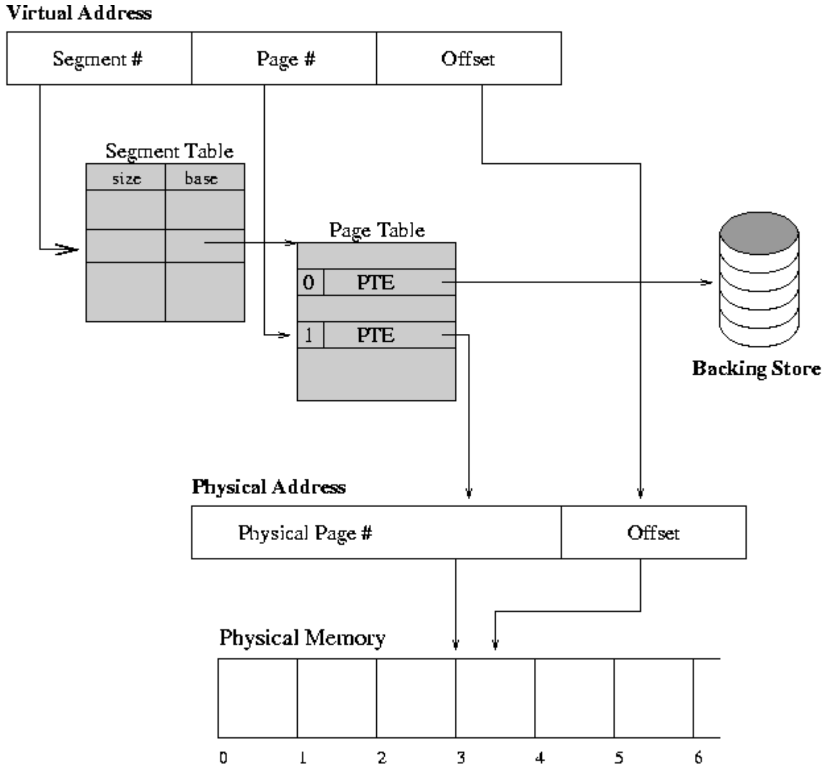

Continuing process is very tricky, since it may have been aborted in the middle of an instruction. Do not want user process to be aware that the page fault even happened.

Let the instruction is restarted from the *beginning*.

Even if beginning is found, what about instructions with side effects, like:

ld [%r2], %r2

Without additional information from the hardware, it may be impossible to restart a process after a page fault. Machines that permit restarting must have hardware support to keep track of all the side effects so that they can be undone before restarting.

When designing instruction set, it is not too hard to make a machine virtualizable. It is much harder to do after the fact. VAX is example of doing it right.

EFFECTIVE ACCESS TIME (EAT).

Estimated cost of page faults can be calculated by performing an *effective access time* calcuation. Sometimes location accessed quickly (there is no page fault) and sometimes more slowly (to wait for a page to come into memory). Cost of each type of access and the pecentage of time that it occurs to compute the average time to access a word of memory is used.

Let h = fraction of time that a reference does **not** require a page fault.

t_{mem} = time it takes to read a word from memory.

t_{disk} = time it takes to read a page from disk.

then $$EAT = h * t_{mem} + (1 - h) * t_{disk}.$$

If there a multiple classes of memory accesses, such as no disk access, one disk access, and two disk access, then there is a fraction (h) and access time (t) for each class of access.

Note : This calculation is the same type that computer architects use to calculate memory performance. *In that case, their access classes might be*

(1) Cached in L1

(2) Cached in L2

(3) RAM.

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PAGE SELECTION AND REPLACEMENT.

Once the hardware has provided basic capabilities for virtual memory, operating system must make two types of scheduling decisions:

1. **Page Selection :** When to bring pages into memory.
2. **Page replacement** : Which page(s) should be thrown out, and when.

Page Selection Algorithms

(*i*) **Demand paging :** It is start up process with no pages loaded, load a page when a page fault for it occurs, i.e. until it absolutely MUST be in memory. Almost all paging systems are like this.

(*ii*) **Request paging :** Let user say which pages are needed. The trouble is, users do not always know best, and are not always impartial. They will overestimate needs.

(*iii*) **Prepaging :** Bring a page into memory before it is referenced
e.g. when one page is referenced, bring in the next one, just in case. Hard to do effectively without a prophet, may spend a lot of time doing wasted work.

Page-Replacement Algorithms : Evaluate an algorithm by running it on a particular string of memory references and computing the number of page faults. The string of memory references is called *reference string.*

To illustrate page-replacement algorithms, following reference string are used for a memory with three frames:

7, 0, 1, 2, 0, 3, 0, 4, 2, 3, 0, 3, 2, 1, 2, 0, 1, 7, 0 1

1. FIFO (First in First Out) Algorithm.

It is the simplest page-replacement algorithm. A FIFO replacement algorithm associates with each page the time when that page is chosen. It is not necessary to record the time when a page is brought in. A FIFO queue can be created to hold all pages in memory. The page is replaced at the head of the queue. When a page is brought into memory, it is inserted at the tail of the queue.

Reference string

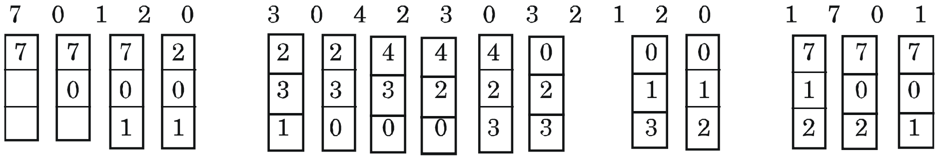

Fig. FIFO page-replacement algorithm.

There are 15 faults altogether. *Belady's anomaly* reflects the fact that, *for some page-replacement algorithms, the page-fault rate may increase as the number of allocated frames increases.*

2. Optimal Algorithm

Belady's anomaly shows the search for an *optimal* page-replacement algorithm. An optimal page-replacement algorithm has the lowest page-fault rate of all algorithms. An optimal algorithm will never suffer from Belady's anomaly. An optimal page-replacement algorithm exists, and has been called *OPT or MIN*. It simply replace the page that will not be used for the longest period of time.

Unfortunately, optimal page-replacement algorithm is difficult to implement, because it requires future knowledge of the reference string.

Reference string

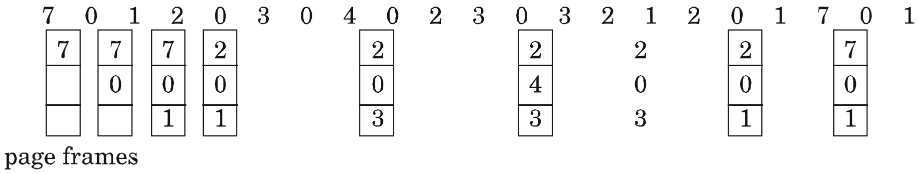

Fig. Optimal page-replacement algorithm

3. LRU (Least Recently Used) Algorithm

This approach replace the page that *has* not been used for the longest period of time.

LRU replacement associates with each page the time of that page's last use. When a page must be replaced, LRU chooses that page that has not been used for the longest period of time. This strategy is the *optimal page-replacement algorithm* looking backward in time, rather than forward. The result of applying

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LRU replacement to out example reference string is shown in the figure below.

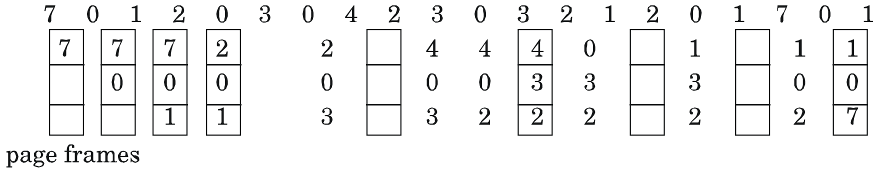

Fig. LRU page-replacement algorithm

LRU policy is often used as a page-replacement algorithm and is considered to be quite good. The major problem is *how* to implement LRU replacement. A LRU page-replacement algorithm may require substantial hardware assistance. The problem is to determine an order for the frames defined by the time of last use.

Approach to Implementing LRU Replacement :

Following two implementations are feasible :

(*i*) **Counters :** In the simplest case, associated with each page-table entry and add to the CPU a logical clock or counter. The clock is incremented for every memory reference. Whenever a reference to a page is made, contents of the clock register are copied to the time-of use field in the page table for that page. In this way, we always have "time" of the last reference to each page. We replace the page with the smallest time value. This scheme requires a search of the page table to find LRU page, and a write to memory (to the time-to-use field in the page table) for each memory access. The times must also be maintained when page tables are changed (due to CPU scheduling). Overflow of the clock must be considered.

(*ii*) **Stack :** Keep a *stack* of page numbers. Whenever a page is referenced, it is removed from the stack and put on the top. In this way, top of the stack is always the most recently used page and bottom is the LRU page. Because entries must be removed from the middle of the stack , it is best implemented by a doubly linked list, with a head and tail pointer.

Reference string

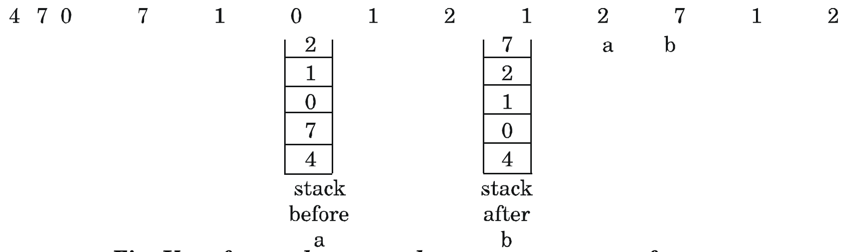

Fig. Use of a stack to record most recent page references

Implementing LRU : It need some form of hardware support, in order to keep track of which pages have been used recently.

Perfect LRU : Keep a register for each page, and store the system clock into that register on each memory reference. To replace a page, scan through all of them to find the one with the oldest clock. This is expensive if there are a lot of memory pages.

In practice, nobody implements perfect LRU. Instead, we settle for an approximation which is efficient. Just find an old page, not necessarily the oldest. LRU is just an approximation anyway.

4. Counting Algorithms.

There are many other algorithms that can be used for page replacement.

e.g., we could keep a counter of the number of references that have been made to each page, and develop following two schemes.

(*i*) ***Least frequently used* (*LFU*) page-replacement algorithm :** It requires that the page with the smallest count be replaced because an actively used page should have a large reference count. This algorithm suffers from the situation in which a page is used heavily during initial phase of a process, but then is never used again.

(*ii*) ***Most frequently used* (*MFU*) page-replacement algorithm :** It is based on the argument that the page with the smallest count was probably just brought in and has yet to be used.

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5. **Page buffering Algorithm.**
   Systems commonly keep a *pool* of free frames. When a page fault occurs, a victim frame is chosen as before. However, that desired page is read into a free frame from the pool before the victim is written out. This procedure allows the process to restart as soon as possible, without waiting for the victim page to be written out.

ALLOCATION OF FRAMES.

Under pure demand paging, all page faults would get free frames if available. If there are no free-frames, a page replacement algorithm would be used to swap out a process to get free frame.

Allocation Algorithms

1. **Equal allocation :** Every process is given an equal number of frames. In this, many frames may not be used by the process and hence wasted.
   e.g. If there are m frames to be divided amongst n frames, then an equal share of m/n frames are allocated. If there are 75 frames and 10 processes, then each process gets 7 frames each and leftover 5 frames could be sent to free-frame pool.
2. **Proportional allocation :** In this, a process is allocated frames according to its requirements.
   e.g. each process p_i is allocated memory s_i

$$S = \Sigma S_i = s_i + s_2 + \text{........} s_i$$

Number of Frames Allocated

Number of frames a_i allocated to a process p_i is $a_i \approx \frac{s_i}{S} \times m$

where, m = total number of frames

a_i should be an integer, which is greater than the minimum number of frames required by instruction set, with a sum not exceeding m.

e.g. If there are 75 frames to be divided amongst two processes. P_1 has 20 pages and P_2 has 50 pages, then each process gets

$$\frac{20}{70} \times 75 \approx 21 \text{ frames} ; \frac{20}{70} \times 75 \approx 53 \text{ frames}$$

- A process cannot be allocated more than the available frames (unless there is page sharing).
- As number of frames allocated to a process decreases, the rate of page fault increases, slowing the process execution.
- *Minimum number of frames* per process is defined by the architecture.
- *Maximum number of frames* is defined by the amount of available physical memory.

Example. Try the reference string A B C A B D A D B C B, assume there are three page frames of physical memory. Show memory allocation state after each memory reference.

FIFO Page Reference	FIFO Memory			LRU Page Reference	LRU Memory		
A				A			
B				B			
C				C			
A				A			
B				B			
D				D			
A				A			
D				D			
B				B			
C				C			
B				B			

MIN is optimal (cannot be beaten), but principle of locality states that past behavior predicts future behavior, thus LRU should do just about as well.

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CLOCK ALGORITHM, THRASHING.

This is an efficient way to approximate LRU.

Clock Algorithm

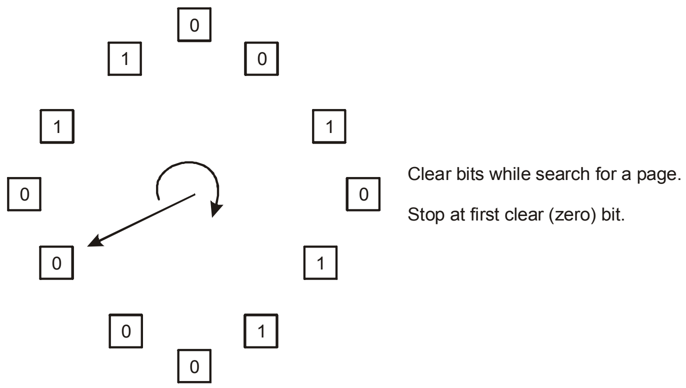
It keep '*use bit*' for each page frame; hardware sets the appropriate bit on every memory reference. Operating system clears the bits from time to time in order to figure out how often pages are being referenced. Introduce clock algorithm where to find a page to throw out the operating system circulates through the physical frames clearing use bits until one is found that is zero. Use that one. Show clock analogy.

Fancier Algorithm

It give pages a second (third? fourth?) chance. Store (in software) a counter for each page frame, and increment the counter if use bit is zero. Only throw the page out if counter passes a certain limit value. Limit = 0 corresponds to the previous case.

Some systems also use a '*dirty bit*' to give preference to dirty pages because it is more expensive to throw out dirty pages. Clean ones need not be written to disk.

Clock Algorithm

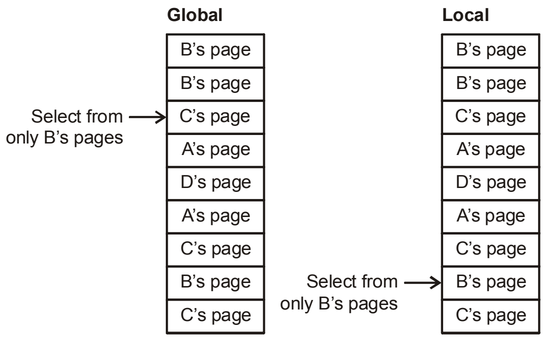


Global Replacement Algorithm

If all pages from all processes are lumped together by the replacement algorithm, then it is called *global replacement algorithm*. Under this scheme, each process competes with all of the other processes for page frames.

Per process replacement algorithm allocates page frames to individual processes: a page fault in one process can only replace one of that process' frames. This relieves interference from other processes. *Per job replacement algorithm* has a similar effect.

In *per-process* and *per-job allocation*, the allocations may change, but only slowly.



Thrashing

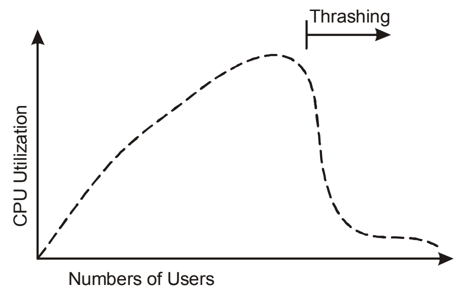
The high paging activity is called *thrashing*

Consider what happens when memory gets overcommitted.

Let there are many users, and between them their processes are making frequent references to 50 pages, but memory has 40 pages.

Each time one page is brought in, another page, whose contents will soon be referenced, is thrown out.

Compute average memory access time. The system will spend all of its time reading and writing pages. It will be working



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very hard but not getting anything done.

Thrashing was a sever problem in early demand paging systems. Thrashing occurs because the system does not know when it has taken on more work than it can handle.

LRU mechanisms order pages in terms of last access, but do not give absolute numbers indicating pages that *must not* be thrown out.

If a single process is too large for memory, then operating system cannot do anything and that process will simply thrash.

If problem arises because of the sum of several processes, then

(*i*) figure out how much memory each process needs.

(*ii*) change scheduling priorities to run processes in groups whose memory needs can be satisfied.

I/O AND FILE SYSTEMS

FILE SYSTEMS

File

It is a named collection of bits stored on disk. It consists of a bunch of blocks stored on the device. Programmer may actually see a different interface (bytes or records), but this does not matter to the file system (just pack bytes into blocks, unpack them again on reading).

Common Addressing Patterns

1. **Sequential.**
   Information is processed in order, one piece after the other. This is the most common mode.
   e.g. editor writes out new file, compiler compiles it, etc.
2. **Random Access.**
   It can address any record in the file directly without passing through its predecessors.
   e.g. data set for demand paging, also databases.
3. **Keyed.**
   Search for records with particular values
   e.g. hash table, associative database, dictionary.
   Usually not provided by operating system.

Modern File Systems

It must address following four general problems:

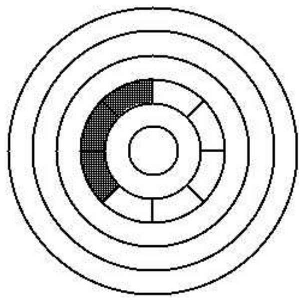
1. **Disk Management :** Efficient use of disk space, fast access to files, sharing of space between several users.
2. **Naming :** How do users select files?
3. **Protection :** All users are not equal.
4. **Reliability :** Information must last safely for long periods of time.

File Descriptor

How should the disk sectors be used to represent the blocks of a file? The structure used to describe which sectors represent a file is called *file descriptor*.

Continuous Disk allocation

Allocate files like segmented memory (give each disk sector a number from 0 up). Keep a free list of unused areas of the disk. When creating a file, make the user specify its length, allocate all the space at once. Descriptor contains location and size.



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Advantages :

(*i*) Easy access, both sequential and random.
(*ii*) Simple.
(*iii*) Few seeks.

Disadvantages

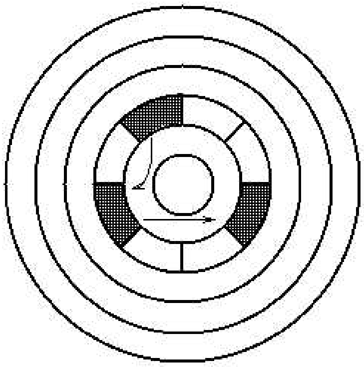
(*i*) Borrible fragmentation will preclude large files, hard to predict needs.
(*ii*) With interleaved user requests, still cannot eliminate all seeks.

Linked Files

In file descriptor, just keep pointer to first block. In each block of file, keep pointer to next block or keep a linked list of free blocks for the free list.

e.g. FAT (MSDOS) file system.

Linked Disk Allocation



Advantages :

(*i*) Files can be extended, no fragmentation problems.
(*ii*) Sequential access is easy: just chase links.

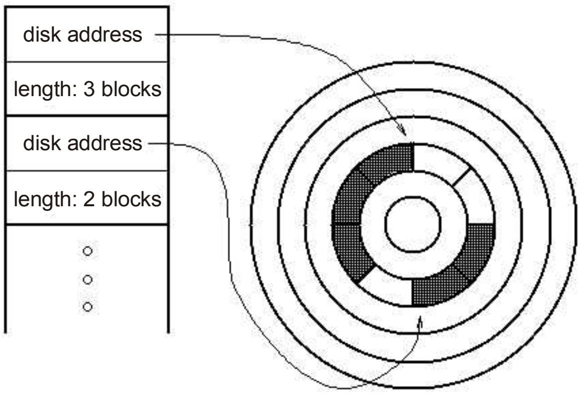
Disadvantages :

(*i*) Random access is virtually impossible.
(*ii*) Lots of seeking, even in sequential access.

Array of Block Pointers

File maximum length must be declared when it is created. Allocate an array to hold pointers to all the blocks, but do not allocate the blocks. Then fill in the pointers dynamically using a free list.

Block Pointer Allocation



Advantages : Not as much space wasted by over-predicting, both sequential and random access are easy.

Disadvantages : Still have to set maximum file size, and there will be lots of seeks.

DOS FAT (Allocation Table)

A single File Allocation Table (FAT) combines free list info and file allocation info. In file descriptor, keep pointer to first block.

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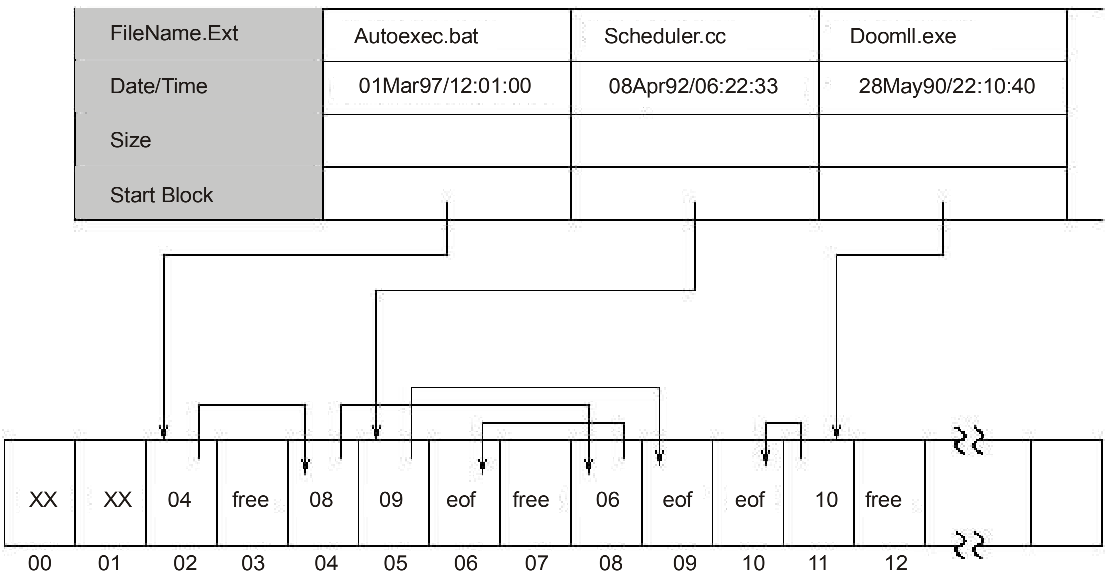
A FAT table entry contains either of the following :

(*i*) Block number of the next block in the file

(*ii*) A distinguished "*end of file*" (eof) value

(*iii*) A distinguished "*free*" value.

MS/DOS Directory Entires



File Access Table (FAT)

UNIX AND DEMOS DISK ALLOCATION

File descriptor information has to be stored on disk, so it will stay around even when operating system does not.

In Unix, all the descriptors are stored in a fixed size array on disk. Descriptors also contain protection and accounting information.

A special area of disk is used for this

Disk contains two parts:

(*i*) Fixed-size descriptor array

(*ii*) remainder, allocated for data and indirect blocks.

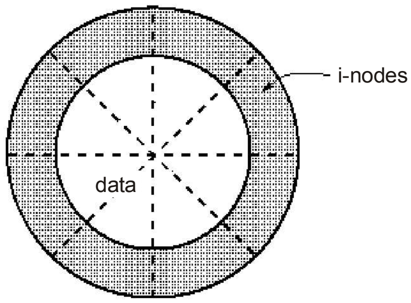
Size of the descriptor array is determined when disk is initialized, and cannot be changed.

In Unix, descriptor is called *i-node*, and its index in the array is called its *i-number*. Internally, operating system uses i-number to refer to the file.

When file is open, its descriptor is kept in main memory.

When file is closed, its descriptor is stored back to disk.

Unix Disk



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DIRECTORIES

Motivation

An approach for users need a way of finding the files that they created on disk is just to have users remember descriptor indexes.

Users want to use text names to refer to files. Special disk structures called *directories* are used to tell what descriptor indices correspond to what names.

Naming is one of the (if not *the*) most important issues in systems design.

Approach #1:

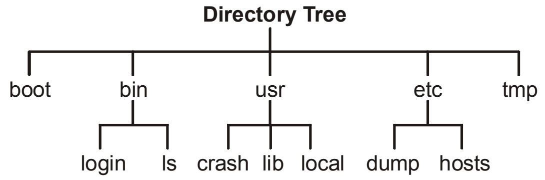
- have a single directory for the whole disk.
- Use a special area of disk to hold the directory.
- Directory contains pairs.
- If one user uses a name, no-one else can.

Approach #2:

- have a separate directory for each user (TOPS-10 approach). This is still clumsy: names from different projects get confused.

Unix Directories

Generalize the directory structure to a tree.



- Directories are stored on disk just like regular files (i.e. file descriptor with 13 pointers, etc.). User programs can read directories just like any other file. Only special system programs may write directories.
- Each directory contains pairs. The file pointed to by the index may be another directory. Hence, get hierarchical tree structure, name with /usr/local.
- There is one special directory, called *root*. This directory has no name, and is the file pointed to by descriptor 2 (descriptors 0 and 1 have other special purposes).
- Directories and file descriptors are separate, and the directories are implemented just like files. This simplifies implementation and management of the structure.

Working directory:

- It is cumbersome constantly to have to specify full path name for all files.
- In Unix, there is one directory per process, called *working directory*, that the system remembers.
- When it gets a file name, it assumes that file is in the working directory. "/" is an escape to allow full path names.
- Many systems allow more than one current directory.

  e.g., check first in A, then in B, then in C. This set of directories is called *search path* or *search list*.

  This is very convenient when working on large systems with many different programmers in different areas.
- e.g., in Unix, the shell will automatically check in several places for programs. However, this is built into the shell, not into Unix, so if any other program wants to do the same, it has to rebuild the facilities from scratch. Should be in the operating system.
- This is another example of locality.

WINDOWS (NT) FILE SYSTEM

Background

It was introduced with Windows NT 4.0 and is the standard file system on Windows 2000 and later systems, such as Windows XP. Its goal was to solve the size, performance, reliability, and flexibility limitations in the DOS (aka "FAT" file system).

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It has a general similarity to the FAT file system in that all files are described in a single table, called the *Master File Table* (MFT).

Characteristics

All components are files, including:

(*i*) Master File Table

(*ii*) Data files

(*iii*) Directories

(*iv*) Free list (bit map)

(*v*) Boot images

(*vi*) Recovery logs

File system also has features to support redundancy and transactions.

Disk Layout

Disks are divide in fixed size regions. Each region is called a *volume*.

Each volume can contain different types of file system, such as NTFS, FAT, or even Unix.

Since each volume is a separate file system, it has its own root directory :

(*i*) Multiple volumes allow for fixed limits on the growth of a particular file tree, such as limiting the size of temporary file space.

(*ii*) Multiple volumes allow a single disk to contain multiple, separating bootable operating system.

Master File Table (MFT)

Clusters.

These are key element to allocation. Logically, disk consists of allocation units called ***clusters***.

Cluster is a power-of-two multiple of the physical disk block size. Cluster size is set when disk is formatted. A small cluster provides a finer granularity of allocation, but may require more space to describe the file and more separate operations to transfer data to or from memory.

The free list is a **bitmap**, each of whose bits describe one cluster.

Clusters on the disk are numbered starting from zero to the maximum number of clusters (minus one). These numbers are called *logical cluster numbers* (LCN) and are used to name blocks (clusters) on disk.

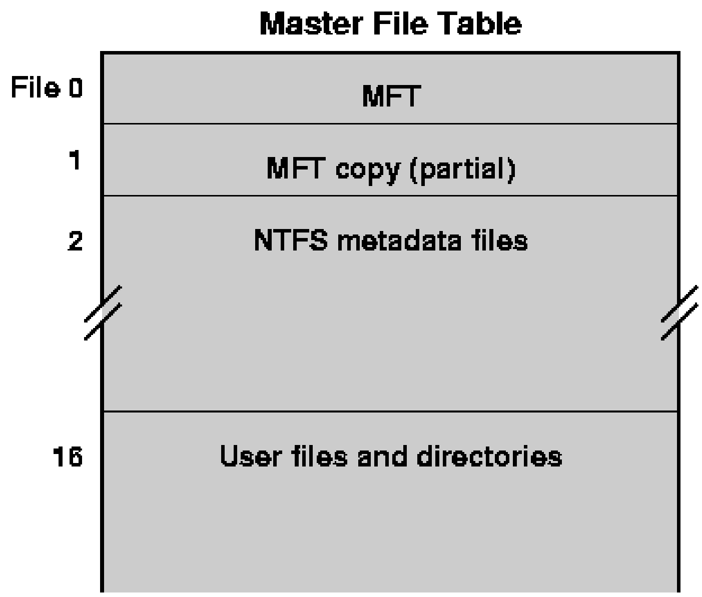
MFT is the major data structure on disk.

All files, and therefore all objects stored on disk are described by the MFT.

All files are logical stored *in* the MFT and, for small files are physically within the bounds of the MFT. In this sense, MFT *is* the file system.

MFT logically can be described as a table with one row per file.

First rows in the table described important configuration files, including for the MFT itself.



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MFT Entries

Each entry consists of (attribute, value) pairs, while conceptual design of NTFS is such that this set of pairs is extensible to include user-defined attributes, current version of NTFS have a fixed set.

Main attributes.

MFT Entry (Simplified)

Standard Information	File Name	Security Desriptor	Data

1. Standard information:

This attribute includes information that was standard in the MS-DOS world:

(*i*) Read/write permissions

(*ii*) Creation time

(*iii*) Last modification time

(*iv*) Count of how many directories point to this this file (hard link count).

2. File Name:

This attribute describes file's name in the Unicode character set.

Multiple file names are possible, when:

(*i*) the file has multiple links, or

(*ii*) the file has an MS-DOS short name.

3. Security Descriptor:

This attribute lists which user owns the file and which users can access it (and how they can access it).

4. Data:

This attribute either contains actual file data in the case of a small file or points to the data (or points to the objects that point to the data) in the case of larger files.

FILE SYSTEM CRASH RECOVERY

1. Unix File System Crash Recovery

Computers can crash at any time, and we want the file system to behave sensibly in the face of crashes. The key idea is called *consistency*:

File data and various control structures (descriptors, bitmaps) must be in agreement.

Since crashes can occur at any time, not all updates to the disk may be completed.

It must be insured that when the system reboots, it can return its file system to some sensible state.

Main constraint is that any file system write operation, in progress at the time of crash, either completely finishes or appears as if it never happened. This is called *atomicity* by the database folks.

Insuring consistency.

It requires following two things:

(*i*) Updates to the file system data structures must be done in the write order (and there is *only* one right order)

(*ii*) Proper steps must be taken at reboot time to bring the system back in to a consistent state.

Following three basic updates happen when data is written to a file :

(*i*) A block (or blocks) is allocated from the free list (bit map).

(*ii*) Data is written to the newly allocated block.

(*iii*) Inode is updated to include the new data.

Note : These operations must be done in the above order. If they are not, then it is possible to have a data block included in a file that might have garbage (uninitialized data) in the block.

After rebooting, recovery utility program on Unix, called "*fsck*", is going to traverse entire directory structure of the disk to insure that all free blocks are in the free list.

Recovery after a crash

It follows following steps:

(*i*) Allocate a temporary bit map, initialized to indicate that all disk blocks are free.

(*ii*) Start at the inode for the root directory.

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(*iii*) *Traverse the directory:*

(*a*) For each disk data block in the directory file, marks its blocks as "*allocated*" in the bit map.

(*b*) For each data file in this directory, marks its data blocks as "*allocated*" in the bit map.

(*c*) For each directory in this directory, perform the "*Traverse the directory*" steps above.

At completion of the algorithm, actual bit map can be compared to the temporary one to to find blocks that were allocated, but never made it into a file.

2. Windows File Sysem Crash Recovery

NTFS assures that the file system will remain consistent by use of a *write log*. This technique is similar to that used in a database system.

As in other file systems, consistency means that a write (or group of writes) to a file either complete or do not happen at all, it is not possible for a data block to be in an undefined state

e.g., allocated, but not written

Log file.

Log is one of those standard files stored at the beginning of the MFT. It is called *log file*.

Steps to write data to a file :

(*i*) A file update is written to the *in-memory* log buffer.

(*ii*) Updates to the *in-memory* file data and associated file system structures are made.

(*iii*) Log changes are flushed to disk.

(*iv*) File data and structure changes are flushed to disk.

Note : If system crashes during a file update, it is sufficient to go through the log an re-do each operation specified in the log.

Checkpoints.

The system occassioally creates checkpoints, so that it does not have to back to the beginning of the log for recovery.

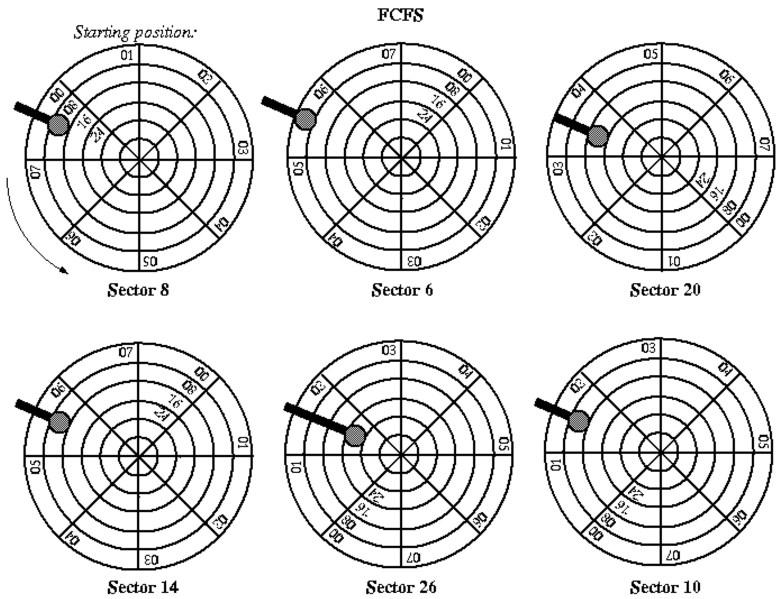
Advantages of Checkpoints :

It has following two main advantages:

(*i*) Log files can be truncated, reducing the space needed for the log.

(*ii*) Recovery time is faster if fewer log records need to be processed.

DISK SCHEDULING



In a system with many processes running, it can often be the case that there are several disk I/O's requested at the same time. The order in which requests are serviced may have a strong effect on the overall performance of disk.

First come first served (FIFO, FCFS) may result in a lot of unnecessary disk arm motion under heavy loads.

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PROTECTION AND SECURITY

The purpose of a *protection system* is to prevent accidental or intentional misuse of a system.

Type of Problems

1. **Accidents :** Problems of this type are easy to solve (can do things to make the likelihood small).
2. **Malicious abuse :** Problems of this type are very hard to completely eliminate (cannot leave any loopholes, cannot play on probabilities).

Problems Mechanism

There are three aspects to a protection mechanism:

1. **User identification (authentication) :** It make sure we know who is doing what.
2. **Authorization determination :** It must figure out what the user is and is not allowed to do. It need a simple database for this.

   Must indicate who is allowed to do what with what. Draw general form as an access matrix with one row per user, one column per file. Each entry indicates privileges of that user on that object.

   There are two general ways of storing this information:

   (*i*) Access Lists:

   With each file, indicate which users are allowed to perform which operations.
   - In the most general form, each file has a list of pairs.
   - It would be tedious to have a separate listing for every user, so they are usually grouped into classes. e.g., *in Unix there are three classes:*
     (*a*) Self
     (*b*) Group
     (*c*) Anybody else (nine bits per file).
   - Access lists are simple, and are used in almost all file systems.

   (*ii*) Capabilities:

   With each user, indicate which files may be accessed, and in what ways.
   - Store a list of pairs with each user. This is called a *capability list*.
   - Typically, capability systems use a different naming arrangement, where capabilities are the only names of objects. We cannot even name objects not referred to in capability list.
   - *In access-list systems*, default is usually for everyone to be able to access a file.
     In capability-based systems, default is for no-one to be able to access a file unless they have been given a capability. There is no way of even naming an object without a capability.
   - Capabilities are usually used in systems that need to be very secure. However, capabilities can make it difficult to share information, nobody can get access to stuff unless we explicitly give it to them.

MATRIX OF PROTECTION

Objects

Actors	1	2	3	A	B	C	
A	execute	read write		control			
B	execute		read write		control	control	capability
C	execute		read			control	

access list

   (*i*) Password :

   User identification is most often done with *passwords*. This is a relatively weak form of protection.
   - It is a secret piece of information used to establish the identity of a user.
   - It should not be stored in a readable form. One-way transformations should be used.
   - It should be relatively long and obscure.

   (*ii*) Badge or Key :
   - It does not have to be kept secret.
   - It should not be able to be forged or copied.
   - It can be stolen, but the owner should know if it is.

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Key paradox: Key must be cheap to make, hard to duplicate. This means there must be some trick (i.e. secret) that has to be protected.

Once identification is complete, the system must be sure to protect identity since other parts of the system will rely on it.

3. **Access Enforcement**

It must make sure there are no loopholes in the system.

Some part of the system must be responsible for enforcing access controls and protecting authorization and identification information.

Obviously, this portion of the system must run unprotected. Thus it should be as small and simple as possible.

e.g., portion of the system that sets up memory mapping tables.

Portion of the system that provides and enforces protection is called *security kernel*.

Most systems, like Unix, do not have a security kernel. As a consequence, the systems are not very secure.

A hierarchy of levels of protection is required with each level getting minimum privilege necessary to do its job. However, this is likely to be slow (crossing levels takes time).

Note : Even slightest flaw in any of these areas may ruin the whole protection mechanism.

SECURITY ABUSES

Generally, protecting a computer system is extremely difficult. There is no completely secure computer system in existence.

Some Common Problems:

(*i*) Abuse of valid privileges

(*ii*) Imposter

(*iii*) Trojan Horse

(*iv*) Listener

(*v*) Spoiler

Send weird escape sequences to terminals that cause commands to be echoed back from the terminal.

Once the system has been penetrated, it may be impossible to secure it again. Hooks could have been left around for the imposter to regain control. It is not always possible to tell when the system has been penetrated, since villain can clean up all traces behind himself. If we can never be sure that there are no bugs, then we can never be sure that the system is secure, since bugs could provide loopholes in the protection mechanisms.

Difference between Computers Humans

(*i*) Computer memory is volatile, humans do not forget.

(*ii*) Trust on computer are much more than of people. Privileges are given away freely in huge doses.

(*iii*) Computer programs are very poorly understood.

SECURITY IMPROVEMENTS, ENCRYPTION

Security Improvements

Some possibilities are given below :

1. **Logging.**

It record all important actions and uses of privilege in an indelible file. It can be used to catch imposters during their initial attempts and failures.

e.g. record all attempts to specify an incorrect password, all super-user logins. Even better is to get humans involved at key steps (this is one of the solutions for EFT).

2. **Principle of Minimum privilege ("need-to-know" Principle).**

Each piece of the system has access to the minimum amount of information, for minimum possible amount of time.

e.g., file system cannot touch memory map, memory manager cannot touch disk allocation tables. This reduces chances of accidental or intentional damage.

Note : Capabilities are an implementation of this idea. It is very hard to provide fool-proof information containment:

e.g., a trojan horse could write characters to a tty, or take page faults, in Morse code, as a signal to another process.

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3. **Correctness proofs.**

   These are very hard to do. This only proves that the system works according to spec. It does not mean that the spec is necessarily right, and it does not deal with Trojan Horses.

ENCRYPTION

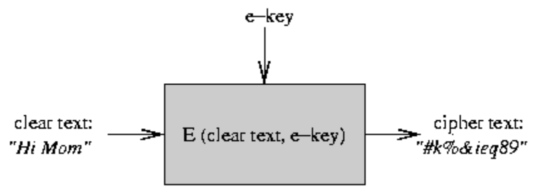
Store and transmit information in an encoded form that does not make any sense.

Basic mechanism

(1) Start with text to be protected. Initial readable text is called *clear text*.

(2) Encrypt the clear text, so that it does not make any sense at all. The nonsense text is called *cipher text*. The encryption is controlled by a secret password or number; this is called *encryption key*.

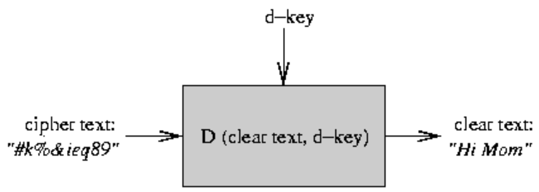
Encryption



(3) Encrypted text can be stored in a readable file, or transmitted over unprotected channels.

(4) To make sense of the cipher text, it must be *decrypted* back into clear text. This is done with some other algorithm that uses another secret password or number, called *decryption key*.

Decryption



All of this only works under following three conditions:

(1) Encryption function cannot easily be inverted.

(2) Encryption and decryption must be done in some safe place so the clear text cannot be stolen.

(3) Keys must be protected. In most systems, can compute one key from the other (sometimes the encryption and decryption keys are identical), so cannot afford to let either key leak out.

Public key encryption.

New mechanism for encryption where knowing the encryption key does not help to find decryption key, or vice versa.

User provides a single password, system uses it to generate two keys (use a one-way function, so cannot derive password from either key).

In these systems, keys are inverses of each other; could just as easily encrypt with decryption key and then use encryption key to recover clear text.

Each user keeps one key secret, publicizes the other. Cannot derive private key from public. Public keys are made available to everyone, in a phone book for example.

Safe Mail

- Use public key of destination user to encrypt mail.
- Anybody can encrypt mail for this user and be certain that only user will be able to decipher it.

In this scheme, only user has to remember one key, and all senders can use the same key.

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EXERCISE – I

MCQ TYPE QUESTIONS

1. In a time-sharing operating system, when the time slot given to a process is completed, the process goes from the RUNNING state to the
(*a*) BLOCKED state
(*b*) READY state
(*c*) SUSPENDED state
(*d*) TERMINATED state

2. Supervisor call
(*a*) is a call made by the supervisor of the system
(*b*) is a call with control functions
(*c*) are privileged calls that are used to perform resource management functions, which are controlled by the operating system
(*d*) is a call made by someone working in root directory

3. At a particular time, the value of a counting semaphore is 10. It will become 7 after
(*a*) 3 V operations
(*b*) 3 P operations
(*c*) 5 V operations and 2 P operations
(*d*) 13 P operations and 10 V operations

4. Mutual exclusion problem occurs between
(*a*) two disjoint processes that do not interact
(*b*) processes that share resources
(*c*) processes that do not use the same resource
(*d*) none of these

5. Some computer systems support dual mode operation—the user mode and the supervisor or monitor mode. These refer to the modes
(*a*) by which user programs handle their data
(*b*) by which the operating system executes user programs
(*c*) in which the processor and the associated hardware operate
(*d*) of memory access

6. Dijkstra's banking algorithm in an operating system, solves the problem of
(*a*) deadlock avoidance
(*b*) deadlock recovery
(*c*) mutual exclusion
(*d*) context switching

7. With a single resource, deadlock occurs
(*a*) if there are more than two processes competing for that resource
(*b*) if there are only two processes competing for that resource
(*c*) if there is a single process competing for that resource
(*d*) none of these

8. Pre-emptive scheduling is the strategy of temporarily suspending a running process
(*a*) before the CPU time slice expires
(*b*) to allow starving processes to run
(*c*) when it requests I/O
(*d*) none of these

9. In Round Robin CPU scheduling, as the time quantum is increased, the average turn around time
(*a*) increases (*b*) decreases
(*c*) remains constant (*d*) varies irregularly

10. Cascading termination refers to termination of all child processes before the parent terminates
(*a*) normally
(*b*) abnormally
(*c*) normally or abnormally
(*d*) none of these

11. A state is safe if the system can allocate resources to each process (up to its maximum) in some order and still avoid deadlock. Then
(*a*) deadlocked state is unsafe
(*b*) unsafe state may lead to a deadlock situation
(*c*) deadlocked state is a subset of unsafe state
(*d*) all of these

12. A system has 3 processes sharing 4 resources. If each process needs a maximum of 2 units, then
(*a*) deadlock can never occur
(*b*) deadlock may occur
(*c*) deadlock has to occur
(*d*) none of these

13. 'm' processes share 'n' resources of the same type. The maximum need of each process doesn't exceed 'n' and the sum all the their maximum needs is always less than m + n. In this set up
(*a*) deadlock can never occur
(*b*) deadlock may occur
(*c*) deadlock has to occur
(*d*) none of these

14. The methods for dealing with the deadlock problem is
(*a*) Use a protocol to make sure that the system never enters in to the deadlock state.
(*b*) Allow the system to enter a deadlock state and then recover.
(*c*) Ignore the problem, and pretend that deadlocks never occur in the system. The UNIX operating system uses this solution.
(*d*) all of these

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15. Working set (t, k) at an instant of time, t, is
   (*a*) the set of *k* future references that the operating system will make
   (*b*) the set of future references that the operating system will make in the next '*k*' time units
   (*c*) the set of *k* reference with high frequency
   (*d*) the set of pages that have been referenced in the last *k* time units

16. Dirty bit is used to show the
   (*a*) page with corrupted data
   (*b*) the wrong page in the memory
   (*c*) page that is modified after being loaded into cache memory
   (*d*) page that is less frequently accessed

17. Fence register is used for
   (*a*) CPU protection
   (*b*) memory protection
   (*c*) file protection
   (*d*) all of these

18. The first-fit, best-fit and the worst-fit algorithm can be used for
   (*a*) contiguous allocation of memory
   (*b*) linked allocation of memory
   (*c*) indexed allocation of memory
   (*d*) all of these

19. In a system that does not support swapping
   (*a*) the complier normally binds symbolic addresses (variables) to relocatable addresses
   (*b*) the compiler normally binds symbolic addresses to physical address
   (*c*) the loader binds relocatable addresses to physical addresses
   (*d*) binding of symbolic addresses to physical addresses normally takes place during execution

20. To obtain better memory utilization, dynamic loading is used. With dynamic loading, a routine is not loaded until it is called for. For implementing dynamic loading
   (*a*) special support from hardware is essential
   (*b*) special support from operating system is essential
   (*c*) special support from both hardware and operating system are essential
   (*d*) user programs can implement dynamic loading without any special support from the operating system or the hardware

21. Distributed systems should
   (*a*) meet prescribed time constraints
   (*b*) aim better resource sharing
   (*c*) aim better system utilization
   (*d*) aim low system overhead

22. In real-time opearting systems, which of the following is the most suitable scheduling scheme ?
   (*a*) round-robin   (*b*) first-come-first-served
   (*c*) preemptive   (*d*) random scheduling

23. A computer installation has 1000 K of main memory. The jobs arrive and finish in the following sequence :
   Job 1 requiring 200 K arrives
   Job 2 requiring 350 K arrives
   Job 3 requiring 300 K arrives
   Job 1 finishes
   Job 4 requiring 120 K arrives
   Job 5 requiring 150 K arrives
   Job 6 requiring 80 K arrives

   Among best fit and first fit, which performs better for this sequence ?
   (*a*) First fit
   (*b*) Best fit
   (*c*) Both perform the same
   (*d*) None of these

24. Consider the following heap :
   JOB1-50 FREE-150 JOB2-300 FREE-350 JOB3-600

   The sequence of requests for blocks of sizes 300, 25, 125, 50 can be satisfied if we use
   (*a*) either first fit or best fit policy
   (*b*) first fit, but not best fit
   (*c*) best fit, but not first fit
   (*d*) none of these

25. Of the following, the safe sequence is
   (*a*) P0, P1, P2, P3   (*b*) P1, P0, P3, P2
   (*c*) P2, P0, P1, P3   (*d*) None of these

26. A demand paging system, with page table held in registers, takes 5 ms to service a page fault if an empty page is available, or if the page to be replaced is not dirty. It takes 15 ms if the replaced page is dirty. Memory access time is 1m s. Assume we want an effective access time of 2m s and that the page to be replaced is dirty 60% of the time. What is the approximate maximum acceptable page fault rate to meet this access time requirement ?
   (*a*) 0.1%   (*b*) 1.0%
   (*c*) 2.5%   (*d*) 0.01%

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27. Fixed partitions

(*a*) are very common in current operating systems

(*b*) are very efficient in memory utilization

(*c*) are very inefficient in memory utilization

(*d*) are most used on large mainframe operating systems

28. Access to moving head disks requires three periods of delay before information is brought into memory. The response that correctly lists the three time delays for the physical access of data in the order of the relative speed from the slowest to the fastest is

(*a*) latency time, cache overhead time, seek time

(*b*) transmission time, latency time, seek time

(*c*) seek time, latency time, transmission time

(*d*) cache overhead time, latency time, seek time

29. An unpaged or read-ahead cache associates disk domains with the address of the read and continues for a specific length. The major disadvantage of unpaged cache is that

(*a*) it allows cache domain to contain redundant data

(*b*) it does not allow writes to be cached

(*c*) its access time is greater than that of paged caching

(*d*) read ahead cache domain blocks are necessarily fixed in size

30. Disk requests are received by a disk drive for cylinders 5, 25, 18, 3, 39, 8 and 35 in that order. 4 seek takes 5 m sec per cylinder moved. How much seek time is needed to serve these requests for a Shortest Seek First (SSF) algorithm ? Assume that the arm is at cylinder 20 when the last of these reefuests is made with none of the requests yet served

(*a*) 125 msec (*b*) 295 msec

(*c*) 575 msec (*d*) 750 msec

31. Which of the following statements is false ?

(*a*) Segmentation suffers from external fragmentation.

(*b*) Paging suffers from internal fragmentation.

(*c*) Segmented memory can be paged.

(*d*) Virtual memory is used only in multi-user systems.

32. The maximum amount of information that is available with one position of the disk access arm for a removal disk pack (without further movement of the arm with multiple heads) is

(*a*) a plate of data (*b*) a cylinder of data

(*c*) a track of data (*d*) a block of data

33. Cached and interleaved memories are ways of speeding up memory access between CPUs and slower RAM. Which memory models are best suited (i.e. improves performance the most) for which programs ?

(*i*) Cached memory is best suited for small loops.

(*ii*) Interleaved memory is best suited for small loops.

(*iii*) Interleaved memory is best suited for large sequential code.

(*iv*) Cached memory is best suited for large sequential code.

(*a*) (*i*) and (*ii*) are true

(*b*) (*i*) and (*iii*) are true

(*c*) (*iv*) and (*ii*) are true

(*d*) (*iv*) and (*iii*) are true

34. When used with I/O devices, the term intelligent implies

(*a*) a colour output capability

(*b*) speech processing capability

(*c*) high speed printing capability

(*d*) features to support off-line and online tesks

35. What is the storage capacity of a Hollerith card which is organized into nibbles ?

(*a*) 32 (*b*) 64

(*c*) 120 (*d*) 240

36. The digitizing technology that uses an electric field radiated from the tablet and picked by cursor is

(*a*) raster (*b*) electrostatic

(*c*) sonic (*d*) electromagnetic

37. Continuous line drawing are produced using

(*a*) chain printers

(*b*) daisy wheel printers

(*c*) plotters

(*d*) thermal devices

38. Bit-map terminal

(*a*) supports displays containing multiple windows

(*b*) requires considerable amount of video RAM

(*c*) requires tremendous amount of copying and hence low performance

(*d*) all of these

39. In a modem using dibit phase, encoding has the bit rate

(*a*) same as the baud rate

(*b*) higher than its baud rate

(*c*) lower than its baud rate

(*d*) independent of its baud rate

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40. Memory mapped displays
(*a*) are utilized for high resolution graphics such as maps
(*b*) uses ordinary memory to store the display data in character form
(*c*) stores the display data as individual bits
(*d*) are associated with electromechanical teleprinters

41. Plotter accuracy is measured in terms of repeatability and
(*a*) buffer size (*b*) resolution
(*c*) vertical dimension (*d*) intelligence

42. In electrostatic plotters, the term "resolution" refers to
(*a*) number of dots per inch
(*b*) speed per inch
(*c*) accuracy to which the plotter returns to a specific point
(*d*) dual axis tracking

43. In a serial communication, an extra clock is needed
(*a*) to make efficient use of RS - 232
(*b*) for programmed band rate control
(*c*) to synchronize the devices
(*d*) all of these

44. Which of the following statements is incorrect for a micro-processorsystem using I/O mapped I/O ?
(*a*) I/O address space is greater.
(*b*) I/O and memory spaces are distinct.
(*c*) Memory space available is greater.
(*d*) Not all data transfer instructions are available.

45. A DMA module is transferring characters to main memory using cycle-stealing (getting the system bus from CPU to do DMA transfer). The device is transmitting at 1 Mbps over a Local Area Network. The CPU fetches instructions at the rate of 1 MIPS (one million instructions per second). Assuming each memory location accessible in one bus cycle can hold 16 bits, the CPU instruction fetch speed will degrade down to
(*a*) .9500 MIPS (*b*) .9250 MIPS
(*c*) .9375 MIPS (*d*) .9345 MIPS

46. In the disk storage aspect of data bases and application programs,
(*a*) unblocked record files and locate mode I/O are faster
(*b*) blocked record files and locate mode I/O are faster
(*c*) unblocked record files and move mode I/O are faster
(*d*) blocked record files and move mode I/O are faster

47. On receiving an interrupt from an I/O device, the CPU
(*a*) halts for a predetermined time
(*b*) branches off to the interrupt service routine after completion of the current instruction
(*c*) branches off to the interrupt service routine immediately
(*d*) hands over control of address bus and data bus to the interrupting device

48. In a memory based I/O system, which of the following instruction will not be there ?
(*a*) IN, OUT (*b*) ADD
(*c*) LDA (*d*) All of these

49. Which of the following does not affect the resolution of a video display image ?
(*a*) Bandwidth
(*b*) Screen size
(*c*) Raster scan rate
(*d*) Vertical and horizontal lines of resolution

50. Advantages of COM are
(*a*) compact size, readability
(*b*) compact size, speed
(*c*) readability size, speed
(*d*) low cost, readability

51. EBCDIC can code upto how many different characters?
(*a*) 8 (*b*) 16
(*c*) 32 (*d*) 256

52. Two basic types of record-access methods are
(*a*) sequential and random
(*b*) direct and immediate
(*c*) sequential and indexed
(*d*) on line and real time

53. Which of the following types of terminals is entirely dependent for all its capabilities on the computer system to which it is connected?
(*a*) Smart terminal
(*b*) Dumb terminal
(*c*) Micro computer
(*d*) None of these

54. Which of the following is the principal difference between a monochrome monitor and an RGB monitor?
(*a*) Number of electron guns
(*b*) Resolution
(*c*) Size
(*d*) Cost

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55. Output hardware is often categorized according to whether it
(*a*) is expensive
(*b*) requires a large amount of electricity to work
(*c*) produces hardcopy or softcopy
(*d*) can fit on a desk top

56. Which of the following does not affect the resolution of a video display image?
(*a*) Band width
(*b*) Raster scan rate
(*c*) Vertical and horizontal lines of resolution
(*d*) Screen size

57. To produce high quality graphics (hard copy) in color, you would want to use
(*a*) RGB monitor (*b*) plotter
(*c*) ink jet printer (*d*) laser printer

58. The technique designed to support the effective access of micro filmed data is
(*a*) micro fiche retrieval
(*b*) COM
(*c*) micro graphics
(*d*) all of these

59. Which of the following printing devices provide an output composed of a series of dots ?
(*a*) wire-matrix printer
(*b*) band printer
(*c*) wang image printer
(*d*) either (*a*) or (*c*)

60. Which of the following statements is true?
(*a*) All hardcopy terminals use punched paper tapes
(*b*) Intelligent terminals provide hard copy outputs only
(*c*) Microfiche are always produced directly from printed outputs
(*d*) None of these

61. The Josephson tunnelling device illustrates principles associated with the advanced storage technique
(*a*) cryogenics (*b*) CCD
(*c*) EBAM (*d*) holographing

62. A mass storage system is a direct access device capable of storing within its array of cartridges
(*a*) over 500 megabytes of data
(*b*) almost 16 billion characters
(*c*) 2000 megabytes of data
(*d*) 472 billion characters

63. A serial card reader would be expected to be
(*a*) faster than a parallel reader
(*b*) more expensive than a parallel reader
(*c*) both (*a*) and (*b*)
(*d*) none of these

64. MICR has made possible to
(*a*) cashless society
(*b*) checkless society
(*c*) creditless society
(*d*) none of these

65. Disk sheduling involves deciding
(*a*) which disk should be accessed next
(*b*) the order in which disk access requests must be serviced
(*c*) the physical location where files should be accessed in the disk
(*d*) none of these

66. In a paged segmented scheme of memory management, the segment table itself must have a page table because
(*a*) the segment is spread over a number to hit in one page
(*b*) each segment is spread over a number of pages
(*c*) segment tables point to page tables and not to the physical location of the segment
(*d*) the processor's description base register points to a page table

67. The most efficient data set organization is a/an
(*a*) sequential file
(*b*) ISAM file
(*c*) variable depending upon the usage of the data set
(*d*) a partitioned data set

68. A partitioned data set is most used for
(*a*) storing program data
(*b*) storing ISAM files
(*c*) a program or source library
(*d*) storing backup information

69. An incremental backup
(*a*) uses more tapes
(*b*) should be done each month
(*c*) saves only files that have recently changed
(*d*) saves all files

70. Disaster recovery
(*a*) is needed by every installation
(*b*) is never needed
(*c*) varies in degree between installations
(*d*) requires off-site computer for immediate use

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71. Wild-card specifiers
(*a*) provide an easy way of finding groups of related files
(*b*) are only used when printing the contents of files
(*c*) can be used when writing a file
(*d*) allow several files to be read simultaneously

72. The allocation map
(*a*) is used to store program data
(*b*) specifies which blocks are used by which file
(*c*) is updated by applications programs
(*d*) allow programs to erase files

73. The activity of a file
(*a*) is a low percentage of number of records that are added to or deleted from a file
(*b*) is a measure of the percentage of existing records updated during a run
(*c*) refers to how closely the files fit into the allocated space
(*d*) is a measure of the number of records added or deleted from a file compassed with the original number of records

74. The volatility of a file refers to
(*a*) the number of records added or deleted from a file composed to the original number of records in that file
(*b*) efficiency with which non-sequential files are processed
(*c*) the extent where the records of the file are contiguous and in proximity to others
(*d*) percentage of records that has changed in a given time period.

75. Which types of file organization are supported by magnetic tape ?
(*a*) random files
(*b*) contiguous sequential file
(*c*) indexed sequential file
(*d*) all of these

76. Number of minimal set of required file operations are
(*a*) two (*b*) four
(*c*) five (*d*) six

77. How many common file type are there ?
(*a*) nine (*b*) six
(*c*) five (*d*) two

78. System supports two types of file, which are
(*a*) text files
(*b*) executable binary files
(*c*) both (*a*) and (*b*)
(*d*) none of these

79. The simplest directory structure is
(*a*) single level directory
(*b*) two level directory
(*c*) tree structure directory
(*d*) none of these

80. Solution of name collision problem is
(*a*) single level directory
(*b*) two level directory
(*c*) tree structure
(*d*) all of these

81. Path's name are
(*a*) absolute path (*b*) relative path
(*c*) both (*a*) and (*b*) (*d*) none of these

82. Which structure prohibits the sharing of files and directories ?
(*a*) tree structure
(*b*) one level structure
(*c*) two level structure
(*d*) none of these

83. Shared sub-directories and files are example of
(*a*) a cyclic graph directory
(*b*) tree structured direction
(*c*) one level directory
(*d*) none of these

84. Which type of design problem in file system ?
(*a*) How the file system should look to the user
(*b*) Algorithm and data structure must be created to map the logical file system onto the secondary storage device
(*c*) Both (*a*) and (*b*)
(*d*) None of these

85. In MS-DOS, relocatable object files and load modules have extensions
(*a*) .OBJ and .COM or .EXE respectively
(*b*) .COM and .OBJ respectively
(*c*) .EXE and .OBJ respectively
(*d*) .DAS and .EXE respectively

86. A file sometimes called a
(*a*) collection of input data
(*b*) data set
(*c*) temporary place to store data
(*d*) program

87. Access time is the highest in the case of
(*a*) floppy disk
(*b*) cache
(*c*) swapping devices
(*d*) magnetic disks

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88. A file organization component of a VSAM file is
(a) relative record data set
(b) keyed sequential data set
(c) entry sequential data set
(d) all of these

89. The file structure that redefines its first record at a base of zero uses the term
(a) relative organization
(b) key fielding
(c) dynamic reallocation
(d) hashing

90. A partitioned data set is mostly used for
(a) a program or source library
(b) storing program data
(c) storing back up information
(d) storing ISAM files

91. Consider six files : F_1, F_2, F_3, F_4, F_5, F_6 with corresponding sizes 100, 200, 70, 40, 250 and 50 respectively, the files are to be stored on a sequential device in such a way that as to optimize access time. In what order should the files be stored ?
(a) F_6, F_5, F_4, F_3, F_2, F_1
(b) F_1, F_2, F_3, F_4, F_5, F_6
(c) F_5, F_2, F_1, F_3, F_6, F_4
(d) F_4, F_6, F_3, F_1, F_2, F_5

92. Consider six files F_1, F_2, F_3, F_4, F_5 and F_6 of corresponding sizes 100, 200, 70, 40, 250 and 50 respectively.
If the files are stored in such a manner to optimize access time, what will be the approximate average access time of a record from one of the six files on the sequential device ?
(a) 131 (b) 198
(c) 286 (d) 433

93. Primitive disk operating system (POS) manages its disk files in contiguous blocks. A file is saved to the first available space that is large enough to hold the file. Assume that a disk has only 10 contiguous blocks of available free space. Which of the following set of file operations
$F_a = 1, F_b = 3, F_c = 5, F_d = 6$
(a) save (f_a), save (f_b), save (f_d), delete (f_d), save (f_c)
(b) save (f_c), save (f_a), delete (f_c), save (f_d)
(c) save (f_d), save (f_b), delete (f_d), save (f_d)
(d) save (f_b), save (f_a), delete (f_b), save (f_d)

94. Supervisor state is
(a) entered by programs when they enter the processor
(b) required to perform any I/O
(c) only allowed to the operating system
(d) never used

95. Trojan-Horse programs
(a) are legitimate programs that allow unauthorized access
(b) are hacker programs that do not show up on the system
(c) really do not usually work
(d) usually are immediately discovered

96. Link encryption
(a) is more secure than end-to-end encryption
(b) is less secure than end-to-end encryption
(c) cannot be used in a large network
(d) is used only to detect errors

97. Device busy
(a) is good, since that means the device is in use
(b) is bad, since that means the device is not available
(c) is bad only if the device busy rate is too high
(d) depends upon the system paging rate

98. Interrupts per second
(a) is a good measure of the system I/O activity
(b) is a good measure of the system paging activity
(c) should never be greater than 1000/ second
(d) is higher on smaller computers

99. Control-unit busy
(a) is the same as device busy
(b) shows the availability of the path of devices
(c) can be prevented with more devices
(d) can be prevented by reducing the paging rate

100. Real-time displays
(a) are used only for capacity planning
(b) are most useful for showing performance problems
(c) are useless for capacity planning
(d) cannot show the system paging rate

101. Seeks analysis
(a) is used for analyzing paging problems
(b) is used for analyzing device busy problems
(c) is used for analyzing control-unit busy problems
(d) is only shown on real-time displays

102. A long-term monitor
(a) should show any immediate performance problems
(b) should show I/O, paging, and processor activity
(c) need show only the I/O and processor activity
(d) usually reports only on terminal displays

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103. A long-term process display should
(*a*) show which processes used the most processor time
(*b*) show process activity by time interval
(*c*) show resources used by the most active processes
(*d*) all of these

104. A certain microprocessor requires 4.5 microseconds to respond to interrupt. Assume that the three interrupts I_1, I_2, I_3 require the following execution time after the interrupt is recognized.
(*i*) I_1 requires 25 microseconds
(*ii*) I_2 requires 35 microseconds
(*iii*) I_3 requires 20 microseconds
I_1 has the highest priority and I_3 has the lowest. What is the possible range of time for I_3 to be executed assuming that it may or may not occur simultaneously with other interrupts ?
(*a*) 24.5 microseconds to 93.5 microseconds
(*b*) 24.5 microseconds to 39.5 microseconds
(*c*) 4.5 microseconds to 24.5 microseconds
(*d*) 29.5 microseconds to 93.5 microseconds

105. The most difficult problem with co-processors is
(*a*) finding suitable applications
(*b*) keeping all processors busy
(*c*) managing shared resources
(*d*) cost

106. A file server uses a form of locking as a concurrency control technique. When a file is locked by a client, all attempts to use or lock the file by other clients are prevented by the server. A potential problem exists if a client requests a lock on a file and then crashes. This situation could result in the file being locked indefinitely. To prevent this from occurring, the file server starts a time whenever it sets a lock. If the timer runs out before the file is unlocked the server assumes that the client has crashed and releases the lock. Which of the following is (are) true of this strategy ?
(*a*) It provides a solution to the problem of preventing indefinite lockout.
(*b*) It may result in interleaved access to a file by two or more clients.
(*c*) It will guarantee mutual exclusion.
(*d*) It will fail to guarantee the prevention of indefinite lockout

107. Domain of protection is
(*a*) process (*b*) objects
(*c*) both (*a*) and (*b*) (*d*) none of these

108. In MULTICS
(*a*) has segmented address space
(*b*) each segment is file
(*c*) segment is associated with the rings
(*d*) all of these

109. Domains are represented by
(*a*) rows (*b*) columns
(*c*) tables (*d*) all of these

110. Objects are represented by
(*a*) rows (*b*) columns
(*c*) tables (*d*) none of these

111. Contents of the access matrix entries requires
(*a*) copy (*b*) columns
(*c*) control (*d*) all of these

112. Hydra
(*a*) is a capability based protection system
(*b*) provides considerable flexibility
(*c*) both (*a*) and (*b*)
(*d*) none of these

113. A hydra
(*a*) subsystem is built on top of its protection kernel and may require protection of its own components
(*b*) is memory management device
(*c*) is used in interfacing
(*d*) all of these

114. CAP
(*a*) is simpler and less powerful than that of hydra
(*b*) is complex and less powerful than that of hydra
(*c*) is simple and greater power than that of hydra
(*d*) none of these

115. Capability of CAP is
(*a*) data capability
(*b*) software capability
(*c*) both (*a*) and (*b*)
(*d*) none of these

116. Software capability
(*a*) is protected by CAP microcode
(*b*) it is not interpreted by microcode
(*c*) it is interpreted by protected procedure
(*d*) all of these

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117. Main motto of CAP system is
(*a*) use of software capability has allowed them to relative considerable economics in formulating and implementing protection policies
(*b*) system provides him with no library of procedures to be used
(*c*) both (*a*) and (*b*)
(*d*) none of these

118. Security violation due to
(*a*) malicious (*b*) accidental
(*c*) both (*a*) and (*b*) (*d*) none of these

119. Malicious access are
(*a*) unauthorized reading of data
(*b*) unauthorized modification of data
(*c*) unauthorized destruction of data
(*d*) all of these

120. Physical level
(*a*) computer systems must be physically secured against armed or surreptitious entry by intruders
(*b*) users must be screened carefully so that the chance of authorizing a user who then gives access to an intruder
(*c*) both (*a*) and (*b*)
(*d*) none of these

121. Human level
(*a*) computer system must be physically secured against armed or surreptitious entry by intruders
(*b*) users must be screened carefully so that the chance of authorizing a user, who then gives access to an intruder
(*c*) both (*a*) and (*b*)
(*d*) none of these

122. A major security problem for operating system is
(*a*) authentication problem
(*b*) physical problem
(*c*) human problem
(*d*) none of these

123. The most common approach to authenticating a user identity is
(*a*) user passwords (*b*) user log in
(*c*) hardware device (*d*) none of these

124. In one time password
(*a*) the password is different in each instance
(*b*) the password is same in each instance
(*c*) both (*a*) and (*b*)
(*d*) none of these

125. Program threats are
(*a*) Trojan Horse (*b*) trap doors
(*c*) both (*a*) and (*b*) (*d*) none of these

126. Trojan Horse
(*a*) A code segment that misuses its file is called Trojan horse
(*b*) good for file accessing
(*c*) both (*a*) and (*b*)
(*d*) none of these

127. Trap door
(*a*) could be included in a compiler
(*b*) pose a difficult problem because, to detect them
(*c*) both (*a*) and (*b*)
(*d*) none of these

128. Worm
(*a*) is a process that uses the spawn mechanism to clobber system performance
(*b*) spawns copies of itself, using up system resources and perhaps locking out system use by all other processes.
(*c*) both (*a*) and (*b*)
(*d*) none of these

129. Worm was made up
(*a*) one program (*b*) two programs
(*c*) three programs (*d*) all of these

130. Two programs of worm are
(*a*) grappling hook program
(*b*) main program
(*c*) both (*a*) and (*b*)
(*d*) none of these

131. Boot strap or vector is called
(*a*) grappling hook program
(*b*) main program
(*c*) secondary program
(*d*) primary storage

132. Viruses
(*a*) another form of computer attack is virus
(*b*) virus are designed to spread into other programs and can wreak havoc in a system
(*c*) it modifying or destroying files and causing system crashes and program malfunctions
(*d*) all of these

133. Viruses
(*a*) are usually spread by users downloading viral programs from public bulletin boards or exchanging floppy disks containing an infection
(*b*) are beneficial for program
(*c*) both (*a*) and (*b*)
(*d*) none of these

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134. Protection against computer viruses is
(*a*) preventation
(*b*) practice of safe computing
(*c*) purchasing unopened software from vendors
(*d*) avoiding free or pirated copies from public sources

135. Threat monitoring technique is
(*a*) the system can check for suspicious patterns of activity in an attempt to detect a security violation
(*b*) a time sharing a system that counts the number of incorrect passwords given
(*c*) both (*a*) and (*b*)
(*d*) none of these

136. The security of a system can be improved by
(*a*) threat monitoring (*b*) audit log
(*c*) both (*a*) and (*b*) (*d*) none of these

137. An audit log
(*a*) simply records the time
(*b*) user
(*c*) type of all access to an object
(*d*) all of these

138. Audit log can be used to
(*a*) determine how and when the problem occurred
(*b*) what amount of damage was done
(*c*) when security has been violated
(*d*) all of these

139. Window NT
(*a*) security model is based on the notion of user accounts
(*b*) creates a security access token that includes the security ID for users
(*c*) NT operates with client server model
(*d*) all of these

140. Poor response time are caused by
(*a*) Process or busy (*b*) High I/O rate
(*c*) High paging rates (*d*) All of these

141. Global locks
(*a*) synchronize access to local resources
(*b*) synchronize access to global resources
(*c*) are used to avoid local locks.
(*d*) prevent access to global resources

142. The file structure that redefines its first record at a base of zero uses the term
(*a*) relative organization
(*b*) key fielding
(*c*) dynamic reallocation
(*d*) all of these

143. A file organization component of a VSAM file is
(*a*) relative record data set.
(*b*) keyed sequential data set.
(*c*) entry sequential data set.
(*d*) all of these

144. Which of the following refers to the associative memory?
(*a*) Address of the data is generated by the CPU.
(*b*) Address of the data is supplied by the users.
(*c*) There is no need for an address ie the data is used as an address.
(*d*) Data are accessed sequentially.

NUMERICAL TYPE QUESTIONS

1. At a particular time of computation, the value of a counting semaphore is 7. Then 20 P operations and 'x' V operations were completed on this semaphore. If the final value of the semaphore is 5, x will be__________

2. A computer system has 6 tape drives, with 'n' processes competing for them. Each process may need 3 tape drives. The maximum value of 'n' for which the system is guaranteed to be deadlock free is _______

3. Consider a system having 'm' resources of the same type. These resources are shared by3 processes A, B, C, which have peak time demands of 3, 4, 6 respectively. The minimum value of 'm' that ensures that deadlock will never occur is______

4. The number of page faults is _______ when references to pages occur in the following order : 1, 2, 4, 5, 2, 1, 2, 4. Assume that the main memory can accommodate 3 pages and the main memory already has the pages 1 and 2, with page 1 having been brought earlier than page 2.
(LRU algorithm is used)

5. In a paged memory, the page hit ratio is 0.35. The time required to access a page in secondary memory is equal to 100 ns. The time required to access a page in primary memory is 10 ns. The average time required to access a page is ______ns

6. If there are 32 segments, each of size 1 K byte, then the logical address should have______ bits

7. Consider a computer with 8 Mbytes of main memory and a 128 K cache. The cache block size is 4 K. It uses a direct mapping scheme for cache management. The number of different main memory blocks can map onto a given physical cache block is _______.

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8. If a virtual memory system has 4 pages in real memory and the rest must be swapped to disk. The hit ratio for the following page address stream is _______ %

Assume that memory starts emply. Use the First In First Out (FIFO) algorithm.

9. If a disk has a seek time of 20 ms, rotates 20 revolutions per second, has 100 words per block, and each track has capacity of 300 words. Then the total time required to access one block is_____

10. The address sequence generated by tracing a particular program executing in a pure demand paging system with 100 records per page, with a free main memory frame is recorded as follows. The number of page faults is ______ ?

0100, 0200, 0430, 0499, 0510, 0530, 0560, 0120, 0220, 0240, 0260, 0320, 0370.

11. A computer system has 4 k word cache organised in a block-set-associative manner, with 4 blocks per set, 64 words per block. The number of bits in the SET and WORD fields of the main memory address format is _____

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. Consider a main memory with five page frames and the following sequence of page references : 3, 8, 2, 3, 9, 1, 6, 3, 8, 9, 3, 6, 2, 1, 3. Which one of the following is true with respect to page replacement policies First In First Out (FIFO) and Least Recently Used (LRU) ?

(*a*) Both incur the same number of page faults.

(*b*) FIFO incurs 2 more page faults than LRU

(*c*) LRU incurs 2 more page faults than FIFO

(*d*) FIFO incurs 1 more page faults than LRU

2. A system has 6 identical resources and N processes competing for them. Each process can request atmost 2 resources. Which one of the following values of N could lead to a deadlock?

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

3. Consider six memory partitions of sizes 200 KB, 400 KB, 600 KB, 500 KB, 300 KB and 250 KB, where KB refers to kilobyte. These partitions need to be allotted to four processes of sizes 357 KB, 210KB, 468 KB and 491 KB in that order. If the best fit algorithm is used, which partitions are NOT allotted to any process?

(*a*) 200KB and 300 KB

(*b*) 200KB and 250 KB

(*c*) 250KB and 300 KB

(*d*) 300KB and 400 KB

4. The maximum number of processes that can be in Ready state for a computer system with n CPUs is

(*a*) n (*b*) n^2

(*c*) 2^n (*d*) Independent of n

5. Consider the following policies for preventing deadlock in a system with mutually exclusive resources.

I. Processes should acquire all their resources at the beginning of execution. If any resources acquired so far are released.

II. The resources are numbered uniquely, and processes are allowed to request for resources only in increasing resource numbers.

III. The resources are numbered uniquely, and processes are allowed to request for resources only in decreasing resource numbers.

IV. The resources are numbered uniquely. A process is allowed to request only for a resource with resource number larger than its currently held resources.

When of the above policies can be used for preventing deadlock ?

(*a*) Any one of I and III but not II or IV

(*b*) Any one of I, III, and IV but not II

(*c*) Any one of II and III but not I or IV

(*d*) Any one of I, II, III, and IV

6. For the processes listed in the following table, which of the following scheduling schemes will the lowest average turnaround time ?

Process	Arrival Time	Processing Time
A	0	3
B	1	6
C	4	4
D	6	2

(*a*) First Come First Serve

(*b*) Non-preemptive Shortest Job First

(*c*) Shortest Remaining Time

(*d*) Round Robin with Quantum value two

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2014

7. Which one of the following is **FALSE**?

(*a*) User level threads are not scheduled by the kernel.

(*b*) When a user level thread is blocked, all other threads of its process are blocked.

(*c*) Context switching between user level threads is faster than context switching between kernel level threads.

(*d*) Kernel level threads cannot share the code segment.

8. A computer has twenty physical page frames which contain pages numbered 101 through 120. Now a program accesses the pages numbered 1, 2, .,.100 in that order, and repeats the access sequence THRICE. Which one of the following page replacement policies experiences the same number of page faults as the optimal page replacement policy for this program?

(*a*) Least-recently-used

(*b*) First-in-first-out

(*c*) Last-in-first-out

(*d*) Most-recently-used

9. Which of the following statements are CORRECT?

1. Static allocation of all data areas by a compiler makes it impossible to implement recursion.
2. Automatic garbage collection is essential to implement recursion.
3. Dynamic allocation of activation records is essential to implement recursion.
4. Both heap and stack are essential to implement recursion.

(*a*) 1 and 2 only (*b*) 2 and 3 only

(*c*) 3 and 4 only (*d*) 1 and 3 only

2013

10. Three concurrent processes X, Y, and Z execute three different code segments that access and update certain shared variables. Process X executes the P operation (i.e., wait) on semaphores a, b and c; process Y executes the P operation on semaphores b, c and d; process Z executes the P operation on semaphores c, d, and a before entering the respective code, segments. After completing the execution of its code segment, each process invokes the V operation (**i.e., signal**) on its three semaphores. All semaphores are binary semaphores initialized to one. Which one of the following represents a deadlock-free order of invoking the P operations by the processes?

(*a*) X: P (*a*) P (*b*) P (*c*)
Y: P (*b*) P (*c*) P (*d*)
Z: P (*c*) P (*d*) P (*a*)

(*b*) X: P (*b*) P (*a*) P (*c*)
Y: P (*b*) P (*c*) P (*d*)
Z: P (*a*) P (*c*) P (*d*)

(*c*) X: P (*b*) P (*a*) P (*c*)
Y:’P (*c*) P (*b*)P (*d*)
Z: P (*a*) P (*c*) P (*d*)

(*d*) X: P (*a*) P (*b*) P (*c*)
Y: P (*c*) P (*b*) P (*d*)
Z: P(*c*) P (*d*) P (*a*)

11. A shared variable x, initialized to zero, is operated on by four concurrent processes W, X, Y, Z as follows. Each of the processes W and X reads x from memory, increments by one, stores it to memory, and then terminates. Each of the processes Y and Z reads x from memory, decrements by two, stores it to memory, and then terminates. Each process before reading x invokes the P operation (i.e., wait) on a counting semaphore S and invokes the V operation (i.e., signal) on the semaphore S after storing x to memory. Semaphore S is initialized to two. What is the maximum possible value of x after all processes complete execution?

(*a*) –2 (*b*) –1

(*c*) 1 (*d*) 2

2012

12. A process executes the code

```
fork () ;
fork () ;
fork () ;
```

The total number of child processes created is

(*a*) 3 (*b*) 4

(*c*) 7 (*d*) 8

13. Consider the virtual page reference string

1, 2, 3, 2, 4, 1, 3, 2, 4, 1

on a demand paged virtual memory system running on a computer system that has main memory size of 3 page frames which are initially empty. Let LRU, FIFO and OPTIMAL denote the number of page faults under the corresponding page replacement policy. Then

(*a*) OPTIMAL < LRU < FIFO

(*b*) OPTIMAL < FIFO < LRU

(*c*) OPTIMAL = LRU

(*d*) OPTIMAL = FIFO

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14. Consider the 3 processes, P1, P2 and P3 shown in the table.

Process	Arrival time	Time Units Required
P1	0	5
P2	1	7
P3	3	7

The completion order of the 3 processes under the policies FCFS and RR2 (round robin scheduling with CPU quantum of 2 time units) are

(*a*) **FCFS:** P1, P2, P3 **RR2:** P1, P2, P3
(*b*) **FCFS:** P1, P3, P2 **RR2:** P1, P3, P2
(*c*) **FCFS:** P1, P2, P3 **RR2:** P1, P3, P2
(*d*) **FCFS:** P1, P3, P2 **RR2:** P1, P2, P3

15. Consider the following transactions with data items P and Q initialized to zero :

T_1 : read (P);
read (Q);
if P = 0 then Q : = Q + 1;
write (Q)
T_2 : read (Q);
read (P);
if Q = 0 then P : = P + 1;
write (P).

Any non-serial interleaving of T_1 and T_2 for concurrent execution leads to

(*a*) a serializable schedule
(*b*) a schedule that is not conflict serializable
(*c*) a conflict serializable schedule
(*d*) a schedule for which a precedence graph cannot be drawn

2011

16. A thread is usually defined as a "light weight process" because an operating system (OS) maintains smaller data structures for a thread than for a process. In relation to this, which of the following is **TRUE**?

(*a*) On per-thread basis, the OS maintains *only* CPU register state
(*b*) The OS does not maintain a separate stack for each thread
(*c*) On per-thread basis, the OS does maintain virtual memory state
(*d*) On per-thread basis, the OS maintains *only* scheduling and accounting information

17. A company needs to develop digital signal processing software for one of its newest inventions. The software is expected to have 40000 lines of code. The company needs to determine the effort in person-months needed to develop this software using the basic COCOMO model. The multiplicative factor for this model is given as 2.8 for the software development on embedded systems, while the exponentiation factor is given as 1.20. What is the estimated effort in person-months?

(*a*) 234.25 (*b*) 932.50
(*c*) 287.80 (*d*) 122.40

18. On a non-pipelined sequential processor, a program segment, which is a part of the interrupt service routine, is given to transfer 500 bytes from an I/O device to memory.

```
         Initialize the address register
         Initialize the count to 500
LOOP :   Load a byte from device
         Store in memory at address given by address register
         Increment the address register
         Decrement the count
         If count ! = 0 go to LOOP
```

Assume that each statement in this program is equivalent to a machine instruction which takes one clock cycle to execute if it is a non-load/store instruction. The load-store instructions take two clock cycles to execute.

The designer of the system also has an alternate approach of using the DMA controller to implement the same transfer. The DMA controller requires 20 clock cycles for initialization and other overheads. Each DMA transfer cycle takes two clock cycles to transfer one byte of data from the device to the memory.

What is the approximate speedup when the DMA controller based design is used in place of the interrupt driven program based input-output?

(*a*) 3.4 (*b*) 4.4
(*c*) 5.1 (*d*) 6.7

19. Let the page fault service time be 10 ms in a computer with average memory access time being 20 ns. If one page fault is generated for every 10^6 memory accesses, what is the effective access time for the memory?

(*a*) 21 ns (*b*) 30 ns
(*c*) 23 ns (*d*) 35 ns

2010

20. Consider the methods used by processes P1 and P2 for accessing their critical sections whenever needed, as given below. The initial values of shared boolean variables S1 and S2 are randomly assigned.

Method used by P1	Method used by P2
while (S1 = = S2) ; Critical Section S1 = S2 ;	while (S1 ! = S2) ; Critical Section S2 = not (S1) ;

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Which one of the following statements describes the properties achieved?

(*a*) Mutual exclusion but not progress

(*b*) Progress but not mutual exclusion

(*c*) Neither mutual exclusion nor progress

(*d*) Both mutual exclusion and progress

21. A system uses FIFO policy for page replacement. It has 4 page frames with no pages loaded to begin with. The system first accesses 100 distinct pages in some order and then accesses the same 100 pages but now in the reverse order. How many page faults will occur?

(*a*) 196 (*b*) 192

(*c*) 197 (*d*) 195

22. Which of the following statements are true?

I. Shortest remaining time first scheduling may cause starvation

II. Preemptive scheduling may cause starvation

III. Round robin is better than FCFS in terms of response time

(*a*) I only (*b*) I and III only

(*c*) II and III only (*d*) I, II and III

23. The following program consists of 3 concurrent processes and 3 binary semaphores. The semaphores are initialized as

S0 = 1, S1 = 0, S2 = 0.

Process P0	Process P1	Process P2

while (true) {
wait (S0);
print '0';
release (S1);
release (S2);
}
	wait (S1);
	release (S0);
		wait (S2)
		release (S0);

How many times will process P0 print '0'?

(*a*) at least twice (*b*) exactly twice

(*c*) exactly thrice (*d*) exactly once

24. A system has n resources $R_0...R_{n-1}$, and k processes $P_0..P_{k-1}$. The implementation of the resource request logic of each process P_i, is as follows:

```
if (i % 2 ==0) {
    if (i < n) request R_i ;
    if (i + 2 < n) request R_(i+2) ;
}
else {
    if (i < n) request R_(n–i) ;
    if (i + 2 < n) request R_(n–i–2);
}
```

In which one of the following situations is a deadlock possible?

(a) n = 40, k = 26 (b) n = 21, k = 12

(c) n = 20, k = 10 (d) n = 41, k = 19

2009

25. In which one of the following page replacement policies, Belady's anomaly may occur ?

(*a*) FIFO (*b*) Optimal

(*c*) LRU (*d*) MRU

26. The essential content (s) in each entry of a page table is/are

(*a*) virtual page number.

(*b*) page frame number.

(*c*) both virtual page number and page frame number

(*d*) access right information.

27. Consider a system with 4 types of resources R1 (3 units), R2 (2 units), R3 (3 units), R4 (2 units). A non-preemptive resource allocation policy is used. At any given instance, a request is not entertained if it cannot be completely satisfied. Three processes P1, P2, P3 request the resources as follows if executed independently.

Process P1 :	Process P2 :	Process P3 :
t = 0: requests 2 units of R2	t = 0: requests 2 units of R3	t = 0: requests 1 units of R4
t = 1: requests 1 units of R3	t = 2: requests 1 units of R4	t = 2: requests 2 units of R1
t = 3: requests 2 units of R1	t = 4: requests 1 units of R1	t = 5: requests 2 units of R1
t = 5: releases 1 units of R2 and 1 unit of R1	t = 6: releases 1 units of R3 t = 8: Finishes	t = 7: requests 1 units of R2 t = 8: requests 1 units of R3
t = 7: releases 1 units of R3		t = 9: Finishes
t = 8: requests 2 units of R4		
t = 10: Finishes		

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Which one of the following statements is TRUE if all three processes run concurrently starting at time t = 0 ?

(*a*) All processes will finish without any deadlock.

(*b*) Only P1 and P2 will be in deadlock.

(*c*) Only P1 and P3 will be in deadlock.

(*d*) All three processes will be in deadlock.

28. Consider a disk system with 100 cylinders. The requests to access the cylinders occur in following sequence:

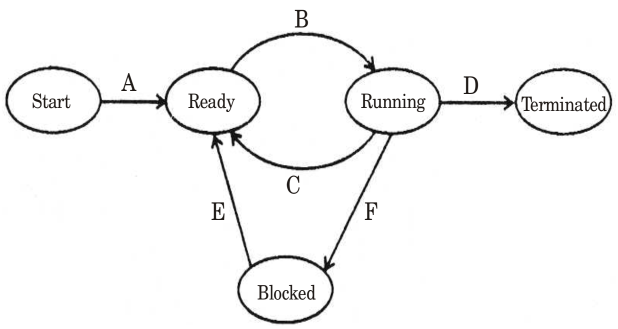
4, 34, 10, 7, 19, 73, 2, 15, 6, 20

Assuming that the head is currently at cylinder 50, what is the time taken to satisfy all requests if it takes 1 ms to move from one cylinder to adjacent one and shortest seek time first policy is used ?

(*a*) 95 ms (*b*) 119 ms

(*c*) 233 ms (*d*) 276 ms

29. In the following process state transition diagram for a uniprocessor system, assume that there are always some processes in the ready state:



Now consider the following statements:

I. If a process makes a transition D, it would result in another process making transition A immediately.

II. A process P2 in blocked state can make transition E while another process P1 is in running state.

III. The OS uses preemptive scheduling.

IV. The OS uses non-preemptive schedulig.

Which of the above statements are TRUE ?

(*a*) I and II (*b*) I and III

(*c*) II and III (*d*) II and IV

30. The enter_CS ( ) and leave_CS ( ) functions to implement critical section of a process are realized using test-and-set instruction as follows:

```
void enter_CS (X)
{
    while (test-and-set (X));
}
void leave_CS(X)
{
    X = 0;
}
```

In the above solution, X is a memory location associated with the CS and is initialized to 0. Now consider the following statements:

I. The above solution to CS problem is deadlock free.

II. The solution is starvation free.

III. The processes enter CS is FIFO order.

IV. More than one process can enter CS at the same time.

Which of the above statements are TRUE ?

(*a*) I only (*b*) I and II

(*c*) II and III (*d*) IV only

31. A multilevel page table is preferred in comparison to a single level page table for translating virtual address to physical address because

(*a*) it reduces the memory access time to read or write a memory location.

(*b*) it helps to reduce the size of page table needed to implement the virtual address space of a process.

(*c*) it is required by the translation lookaside buffer.

(*d*) it helps to reduce the number of page faults in page replacement algorithms.

2008

32. Which of the following system calls results in the sending of SYN packets?

(*a*) socket (*b*) bind

(*c*) listen (*d*) connect

33. The data blocks of a very large file in the Unix file system are allocated using

(*a*) contiguous allocation

(*b*) linked allocation

(*c*) indexed allocation

(*d*) an extension of indexed allocation

34. The P and V operations on counting semaphores, where is a counting semaphore, are defined as follows

P(s) : s =s – 1;

if s < 0 then wait;

V(s) : s = s + 1;

if s < = 0 then wakeup a process waiting on s;

Assume that P_b and V_b, the wait and signal operations on binary semaphores are provided. Two binary semaphores x_b and y_b are used to implement the semaphore operations P(s) and V(s) as follows

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```
P (s): P_b (x_b);
    s = s – 1;
    if (s < 0)
    {
    V_b (X_b);
    P_b (y_b);
    }
    else V_b (X_b);
V (s) : P_b (X_b);
    s = s + 1;
    if (s < = 0) V_b(y_b);
    V_b(X_b);
```

The initial values of x_b the y_b are respectively

(*a*) 0 and 0 (*b*) 0 and 1

(*c*) 1 and 0 (*d*) 1 and 1

35. Which of the following statements about synchronous and asynchronous I/O is NOT true?

(*a*) An ISR is invoked on completion of I/O in synchronous I/O but not in asynchronous I/O

(*b*) In both synchronous and asynchronous I/O, an ISR (Interrupt Service Routine) is invoked after completion of the I/O

(*c*) A process making a synchronous I/O call waits until I/O is complete, but a process making an asynchronous I/O call does not wait for completion of the I/O

(*d*) In the case of synchronous I/O, the process waiting for the completion of I/O is woken up by the ISR that is invoked after the completion of I/O

36. Which of the following is NOT true of deadlock prevention and deadlock avoidance schemes?

(*a*) In deadlock prevention, the request for resources is always granted if the resulting state is safe

(*b*) In deadlock avoidance, the request for resources is always granted if the resulting state is safe

(*c*) Deadlock avoidance is less restrictive than deadlock prevention

(*d*) Deadlock avoidance requires knowledge of resource requirements a *priori*

37. A process executes the following code

```
for (i = 0; i < n; i++) for k ();
```

The total number of child processes created is

(*a*) n (*b*) $2^n - 1$

(*c*) 2^n (*d*) $2^{n+1} - 1$

38. A processor uses 36 bit physical addresses and 32 bit virtual addresses, with a page frame size of 4 Kbytes. Each page table entry is of size 4 bytes. A three level page table is used for virtual-to-physical address translation, where the virtual address is used as follows

- bits 30-31 are used to index into the first level page table,
- bits 21-29 are used to index into the second level page table,
- bits 12-20 are used to index into the third level page table, and
- bits 0-11 are used as offset within the page.

The number of bits required for addressing the next level page table (or page frame) in the page table entry of the first, second and third level page tables are respectively

(*a*) 20, 20 and 20

(*b*) 24, 24 and 24

(*c*) 24, 24 and 20

(*d*) 25, 25 and 24

2007

39. Group 1 contains some CPU scheduling algorithms and Group 2 contains some applications. Match entries in Group 1 to entries in Group 2.

Group 1	Group 2
P. Gang Scheduling	1. Guaranteed Scheduling
Q. Rate Monotonic Scheduling	2. Real-time Scheduling
R. Fair Share Scheduling	3. Thread Scheduling

Codes :

	P	Q	R
(*a*)	3	2	1
(*b*)	1	2	3
(*c*)	2	3	1
(*d*)	1	3	2

40. Consider the following statements about user level threads and kernel level threads. Which one of the following statements is FALSE ?

(*a*) Context switch time is longer for kernel level threads than for user level threads.

(*b*) User level threads do not need any hardware support

(*c*) Related kernel level threads can be scheduled on different processors in a multiprocessor system

(*d*) Blocking one kernel level thread blocks all related threads

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41. An operating system uses Shortest Remaining Time first (SRT) process scheduling algorithm. Consider the arrival times and execution times for the following processes :

Process	Execution time	Arrival time
P1	20	0
P2	25	15
P3	10	30
P4	15	45

What is the total waiting time for process P2 ?

(*a*) 5 (*b*) 15
(*c*) 40 (*d*) 55

42. A virtual memory system uses first In First Out (FIFO) page replacement policy and allocates a fixed number of frames to a process. Consider the following statements :

P : Increasing the number of page frames allocated to a process sometimes increases the page fault rate.

Q : Some programs do not exhibit locality of reference.

Which one of the following is **TRUE** ?

(*a*) Both P and Q are true, and Q is the reason for P.
(*b*) Both P and Q are true, but Q is not the reason for P.
(*c*) P is false, but Q is true.
(*d*) Both P and Q are false.

43. A single processor system has three resource types X, Y, and Z, which are shared by three processes. There are 5 units of each resource type. Consider the following scenario, where the column ***alloc*** denotes the number of units of each resource type allocated to each process, and the column ***request*** denotes the number of units of each resource type requested by a process in order to complete execution. Which of these processes will finish **LAST**?

	alloc	***request***
	X Y Z	X Y Z
P0	1 2 1	1 0 3
P1	2 0 1	0 1 2
P2	2 2 1	1 2 0

(*a*) P0
(*b*) P1
(*c*) P2
(*d*) None of the above, since the system is in a deadlock.

44. Two processes, P1 and P2, need to access a critical section of code. Consider the following synchronization construct used by the processes :

/* P1 */	/* P2 */
while (true) {	While (true) {
wants1 = true ;	wants2 = true ;
while (wants2 ==	while (wants1 ==
true);	true);
/ *Critical	/ * Critical
Section * /	Section * /
wants 1 = false ;	wants 2 = false ;
}	}
/*Remainder section	*/ / *Remainder section*

Here, wants1 and want s2 are shared variables, Which are initialized to false. Which one of the following statements is **TRUE** about the above construct ?

(*a*) It does not ensure mutual exclusion.
(*b*) It does not ensure bounded waiting.
(*c*) It requires that processes enter the critical section in strict alternation.
(*d*) It does not prevent deadlocks, but ensures mutual exclusion.

2006

45. Consider three CPU-intensive processes, which require 10, 20, and 30 time units and arrive at times 0, 2, and 6, respectively. How many context switches are needed if the operating system implements a shortes remaining time first scheduling algorithm ? Do not count the context switches at time zero and at the end.

(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

46. The atomic fetch-and-set x, y instruction unconditionally sets the memory location x to 1 and fetches the old value of x in y without allowing any intervening access to the memory location x. Consider the following implementation of P and V functions on a binary semaphore S.

```
void p (binary_semaphore *S)
{
unsigned y;
unsigned *x = & (S->value);
}
    do
    {
    fetch – and – set x, y;
    } while (y);
}
void V (binary_semaphore *S) {
            {S–> value = 0;
}
```

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Which one of the following is true ?

(a) The implementation may not work if context switching is disabled in P

(b) Instead of using fetch-and-set, a pair of normal load/store can be used

(c) The implementation of V is wrong

(d) The code does not implement a binary semaphore

47. A CPU generates 32-bit virtual addresses. The page size is 4 kB. The processor has a translation look-aside buffer (TLB) which can hold a total of 128 page table entries and is 4-way set associative. The minimum size of the TLB tag is

(a) 11 bits (b) 13 bits

(c) 15 bits (d) 20 bits

48. Consider three processes (process id 0, 1, 2, respectively) with compute time bursts 2, 4, and 8 time units. All processes arrive at time zero. Consider the longest remaining time first (LRTF) scheduling algorithm. In LRTF ties are broken by giving priority to the process with the lowest process id. The average turn around time is

(a) 13 units (b) 14 units

(c) 15 units (d) 16 units

49. Consider three processes, all arriving at time zero, with total execution time of 10, 20, and 30 units, respectively. Each process spends the first 20% of execution time doing I/O, the next 70% of time doing computation, and the last 10% of time doing I/O again. The operating system uses a shortest remaining compute time first scheduling algorithm and schedules a new process either when the running process gets blocked on I/O or when the running process finishes its compute burst. Assume that all I/O operations can be overlapped as much as possible. For what percentage of time does the CPU remain idle ?

(a) 0% (b) 10.6%

(c) 30.0% (d) 89.4%

50. Consider the following snapshot of a system running n processes. Process i is holding x_i instances of a resource R, for $1 \le i \le n$. Currently, all instances of R are occupied. Further, for all i, process i has placed a request for an additional y_i instances while holding the x_i instances it already has. There are exactly two processes p and q such that $y_p = y_q = 0$. Which one of the following can serve as a necessary condition to guarantee that the system is not approaching a deadlock?

(a) $\min (x_p, x_q) < \max_{k \neq p,q} y_k$

(b) $x_p + x_q \ge \min_{k \neq p, q} y_k$

(c) $\max (x_p, x_q) > 1$

(d) $\min (x_p, x_q) > 1$

NUMERICAL TYPE QUESTIONS

2015

1. Consider a uniprocessor system executing three tasks T_1, T_2 and T_3, each o which is composed of an infinite sequence of jobs (or instances) which arrive periodically at intervals of 3, 7 and 20 milliseconds, respectively. The priority of each task is the inverse of its period, and the available tasks are scheduled in order of priority, with the highest priority task scheduled first. Each instance of T_1, T_2 and T_3 requires an execution time of 1, 2 and 4 milliseconds, respectively. Given that all tasks initially arrive at the beginning of the 1st millisecond and task preemptions are allowed, the first instance of T_3 completes its execution at the end of ________ milliseconds.

2. Suppose the following disk request sequence (track numbers) for a disk with 100 tracks is given: 45, 20, 90, 10, 50, 60, 80, 25, 70. Assume that the initial position of the R/W head is on track 50. The additional distance that will be traversed by the R/W head when the Shortest Seek Time First (SSTF) algorithm is used compared to the SCAN (Elevator) algorithm (assuming that SCAN algorithm moves towards 100 when it starts execution) is ______ tracks.

2014

3. Assume that there are 3 page frames which are initially empty. If the page reference string 1, 2, 3, 4, 2, 1, 5, 3, 2, 4, 6, the number of page faults using the *optimal replacement policy* is __________.

4. A system uses 3 page frames for storing process pages in main memory. It uses the Least Recently Used (LRU) page replacement policy. Assume that all the page frames are initially empty. What is the total number of page faults that will occur while processing the page reference string given below?

4, 7, 6, 1, 7, 6, 1, 2, 7, 2

5. A system contains three programs and each requires three tape units for its operation. The minimum number of tape units which the system must have such that deadlocks never arise is ________.

6. An operating system uses *shortest remaining time first* scheduling algorithm for pre-emptive scheduling of processes. Consider the following set of processes with their arrival times and CPU burst times (in milliseconds):

Process	Arrival Time	Burst Time
P1	0	12
P2	2	4
P3	3	6
P4	8	5

The average waiting time (in milliseconds) of the processes is _________.

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ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*b*)	**2.** (*c*)	**3.** (*b*)	**4.** (*b*)	**5.** (*c*)	**6.** (*a*)	**7.** (*d*)	**8.** (*a*)	**9.** (*d*)	**10.** (*c*)
11. (*d*)	**12.** (*a*)	**13.** (*a*)	**14.** (*d*)	**15.** (*d*)	**16.** (*c*)	**17.** (*b*)	**18.** (*a*)	**19.** (*a*)	**20.** (*d*)
21. (*b*)	**22.** (*c*)	**23.** (*a*)	**24.** (*b*)	**25.** (*c*)	**26.** (*d*)	**27.** (*c*)	**28.** (*c*)	**29.** (*a*)	**30.** (*b*)
31. (*d*)	**32.** (*b*)	**33.** (*b*)	**34.** (*d*)	**35.** (*d*)	**36.** (b)	**37.** (*c*)	**38.** (*d*)	**39.** (*b*)	**40.** (*b*)
41. (*b*)	**42.** (*a*)	**43.** (*b*)	**44.** (*a*)	**45.** (*c*)	**46.** (*b*)	**47.** (*b*)	**48.** (*a*)	**49.** (*b*)	**50.** (*b*)
51. (*d*)	**52.** (*a*)	**53.** (*b*)	**54.** (*a*)	**55.** (*c*)	**56.** (*d*)	**57.** (*b*)	**58.** (*c*)	**59.** (*d*)	**60.** (*d*)
61. (*d*)	**62.** (*d*)	**63.** (*a*)	**64.** (*a*)	**65.** (*b*)	**66.** (*b*)	**67.** (*c*)	**68.** (*c*)	**69.** (*c*)	**70.** (*c*)
71. (*a*)	**72.** (*b*)	**73.** (*a*)	**74.** (*a*)	**75.** (*b*)	**76.** (*d*)	**77.** (*a*)	**78.** (*c*)	**79.** (*a*)	**80.** (*b*)
81. (*c*)	**82.** (*a*)	**83.** (*a*)	**84.** (*c*)	**85.** (*a*)	**86.** (*b*)	**87.** (*d*)	**88.** (*d*)	**89.** (*a*)	**90.** (*a*)
91. (*d*)	**92.** (*c*)	**93.** (*c*)	**94.** (*c*)	**95.** (*a*)	**96.** (*b*)	**97.** (*c*)	**98.** (*a*)	**99.** (*b*)	**100.** (*b*)
101. (*b*)	**102.** (*b*)	**103.** (*d*)	**104.** (*a*)	**105.** (*b*)	**106.** (*a*)	**107.** (*c*)	**108.** (*d*)	**109.** (*a*)	**110.** (*b*)
111. (*d*)	**112.** (*c*)	**113.** (*a*)	**114.** (*a*)	**115.** (*c*)	**116.** (*d*)	**117.** (*c*)	**118.** (*c*)	**119.** (*d*)	**120.** (*c*)
121. (*b*)	**122.** (*a*)	**123.** (*a*)	**124.** (*a*)	**125.** (*c*)	**126.** (*a*)	**127.** (*c*)	**128.** (*c*)	**129.** (*b*)	**130.** (*c*)
131. (*a*)	**132.** (*d*)	**133.** (*a*)	**134.** (*d*)	**135.** (*c*)	**136.** (*a*)	**137.** (*d*)	**138.** (*d*)	**139.** (*d*)	**140.** (*d*)
141. (*b*)	**142.** (*a*)	**143.** (*d*)	**144.** (*c*)						

Numerical Type Questions

1. 22	**2.** 2	**3.** 11	**4.** 4	**5.** 68.5	**6.** 15	**7.** 64	**8.** 31%	**9.** 60	**10.** 7
11. 4,6									

EXERCISE – II

MCQ Type Questions

1. (*a*)	**2.** (*d*)	**3.** (*a*)	**4.** (*d*)	**5.** (*d*)	**6.** (*c*)	**7.** (*d*)	**8.** (*d*)	**9.** (*d*)	**10.** (*b*)
11. (*d*)	**12.** (*c*)	**13.** (*b*)	**14.** (*c*)	**15.** (*b*)	**16.** (*a*)	**17.** (*a*)	**18.** (*a*)	**19.** (*b*)	**20.** (*a*)
21. (*a*)	**22.** (*d*)	**23.** (*a*)	**24.** (*b*)	**25.** (*a*)	**26.** (*b*)	**27.** (*a*)	**28.** (*b*)	**29.** (*c*)	**30.** (*a*)
31. (*b*)	**32.** (*d*)	**33.** (*d*)	**34.** (*c*)	**35.** (*b*)	**36.** (*a*)	**37.** (*b*)	**38.** (*b*)	**39.** (*a*)	**40.** (*d*)
41. (*b*)	**42.** (*b*)	**43.** (*c*)	**44.** (*d*)	**45.** (*b*)	**46.** (*a*)	**47.** (*a*)	**48.** (*a*)	**49.** (*b*)	**50.** (*a*)

Numerical Type Questions

1. (12)	**2.** (10)	**3.** (7)	**4.** (6)	**5.** (7)	**6.** (5.5)

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EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. The process goes from running state to ready state.

2. Supervisor call are privileged calls that are used to perform resource management function, which are controlled by operating system.

3. V mean decrement and P mean increment

So $10 - 3 = 7$

i.e., after 3V operation.

4. Mutual exclusion occurs between processes that share resources.

5. To refer to nodes in which the processor and the associated hardware operate.

6. Rijkstra's algo solve the problem of deadlock avoidence.

7. With a single resource, deadlock occurs by none of the given options.

8. Suspending a running process before the CPU time slice expires.

9. The average turn around time varies irregularly.

10. Refer to termination of all child process before the parent terminal normally or abnormally.

11. All of these

12. Atleast one process will be holding 2 resources in case of a simultaneous demand from all the processes. That process will release the 2 resources, thereby avoiding any possible deadlock.

13. Using Banker's algorithm, one can show that one process has to acquire all its needed resources. This process, after completing its task, will release all its resources, thereby avoiding any possible deadlock.

15. t is the set of pages that have been referenced in the last k time units.

16. The page that is modified after being loaded into cache memory.

17. Fence register is used for memory protection.

18. Contiguoun allocation of memory.

19. The compiler normally binds synbatic address to reallocate addresses.

20. For implementing dynamic loading user program can the implement dynamic loading without any special support from the operating system or the handware.

21. Distributed system should a better resource sharing.

22. The most suitable scheduling scheme is preemptive.

23. The memory configuration after the arrival of the jobs 1, 2 and 3 and the termination of job 1 can be depicted as :

FREE-200 JOB2-350 JOB3-300 FREE-150

First fit algorithm will allocate the FREE-200 slot for job 4. But best fit algorithm will allocate the FREE-150 slot for job 4.

The memory configuration for the first fit and best fit will be

JOB4-120 FREE-80 JOB2-350
JOB3-300 FREE-150

and

FREE-200 JOB2-350 JOB3-300
JOB4-120 FREE-300

respectively.

When job 5 arrives, it will be allotted the FREE-150 slot by the first fit algorithm and FREE-200 slot by the best fit algorithm. Memory allocation table for the first fit and best fit will be

JOB4-120 FREE-80 JOB2-350
JOB3-300 JOB5-150

and

JOB5-150 FREE-50 JOB2-350
JOB3-300 FREE-30

When Job 6 arrives, it will be allotted the FREE-80 slot by the first fit algorithm. The best fit algorithm will find no room to store Job 5 as the needed 80 K, is not available contiguously. So, it has to wait till a job terminates. So, first-fit algorithm performs better in this case.

27. Fixed positions are very inefficient in memory utilization.

28. Seek time, latency time, transmission time.

29. Major disadvantage is it allows cache domain to contain redundant data.

30. Seek time = 295 ms

31. Virtual memory is used only in multiuser system.

34. Intelligent implies features to support offline and online task.

35. Storage capacity is 240.

36. The digitizing technology in electrostatic.

37. Continuous line drawing are produced using plotter.

39. Encoding has bit rate higher than band rate.

40. Memory mapped user ordinary memory to store the display data in character form.

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41. Plotter accuracy is measured in term of repeatability and resolution.

42. Resolution refers to number of dots per inch.

43. An extra clock is needed for programmed band rate control.

44. I/O address space is greater.

46. Blocked record filer and locate mode I/O are faster.

47. The CPU branches off to the interrupt service routine after completion of the current instruction.

48. In memory based I/O system. IN and OUT are not there.

49. Screen size doesnot effect the resolution of video display image.

50. Advantages of com are compact size and speed.

51. EBCDIC can code upto 256 different character.

52. Two basic types of record access method are sequential and random.

53. Dumb terminal is entirely dependent.

54. The principal difference is number of electron guns.

55. Output hardware is characterized whether it produces hardcopy or softy copy.

56. Screen size does not effect the resolution of video display image.

57. To produce high quality graphics, plotter is used.

58. Micro graphics support the effective access of micro filmed data.

59. (*a*) or (*c*) provides an output composed of series of dots.

61. The Josephson tunnelling device illustrates holographing.

62. It is capable of storing 472 billion character.

63. A serial card reader would be expected to be faster than a parallel reader.

64. MICR has made possible to cashless society.

65. Disk scheduling invalver deciding the order in which disk access requests must be serviced.

66. Each segment in spread over a number of pages.

67. The most efficient data set organisation in a variable depending upon the usage of the data set.

68. A partitioned data set is mostly used for a program or source library.

69. An incremental backup saves only filers that have recently changed.

70. Disaster recovery varies in degree between installations.

71. Wild card specifier provide an easy way of finding groups of related filer.

72. The allocation map specifier which blocks are used by which file.

73. The activity of a file in a low percentage of number of records that are added to or deleted from a file.

74. The volatility of a file refers to the number of records added or deleted from a file compared to the original number of records in that file.

75. Magnetic tape supports contiguous sequential file.

76. Number of minimal set of required file operations are six.

77. There are nine common file type.

78. Tent files and executable binary files are supported by syntex.

79. The simplest directory structure is single level directory.

80. Solution of name collision problem in two level directory.

81. Paths name are absolute and relative.

82. Tree structure prohibits the sharing of files and directiories.

83. It is example of a cyclic graph directory.

85. .OBJ and .COM or .EXE respectively.

86. A file sometimes called a data set.

87. Access time in the highest in case of magnetic disks.

89. The term used in relative organisation.

90. A partitioned data set is mostly used for a program or source library.

91. The order should be
$f_4, f_6, f_3, f_1, f_2, f_5$

92. Average access time is 286.

93. Save (f_d), save (f_b), delete (f_d), save (f_d)

94. Supervisor state is only allowed to the operating system.

95. Trojan horse program are legitimate programs that allows unauthorized access.

96. Link excryption in less secure than end-to-end encryption.

97. Device busy in bad only if the dervice busy rate is too high.

98. Interrupts per second is a good measure of the system I/O activity.

99. Control unit busy shows the availability of the path of devices.

100. Real time displays are most useful for showing performance problems.

101. Seeks analysis is used for analyzing device busy problems.

102. A long term monitor should show I/O, paging and processes activity.

103. A long term process display should show all of the given option.

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104. 24.5 ms to 93.5 ms.

105. The most difficult problem is keeping all processes busy.

106. Provides a solution to the problem of the preventing indefinite lockout.

109. Domains are represented by rows.

110. Objects are represented by columns.

113. A hydra subsystem is built on top of its protection kennal and may require protection of its own components.

114. CAP is simpler and ten powerful than that of hydra.

121. Human level users must be screened carefully so that the chance of authorizing a user, who then gives access to an intruder.

122. A major security problem for operation system is an whethercation problem.

123. The most common approach is user password.

124. In one time password, the password is different in each instance.

126. Trjan horse is a code segment that misusers its file.

129. Worns was made up two program.

131. Boot strap or vector is called grappling hook program.

133. Viruses are usually spread by users downloading viral program from public bulletin boards or exchanging floppy disks containing an infection.

134. Protection against computer virus is avoiding free or pirated copies from public source.

136. The security of a system can be improved by threat modeling.

141. Global locks synchronize access to global resources.

142. The term used is relative organization.

144. There is no need for an address i.e., the data is used as an address.

NUMERICAL TYPE QUESTIONS

1. Each P operation will decrease the semaphore value by 1 and V operation increases by 1.

If x is 18, then 7 P operations will make the semaphore value 0.

If this is followed by 7 V operations the value comes back to 7.

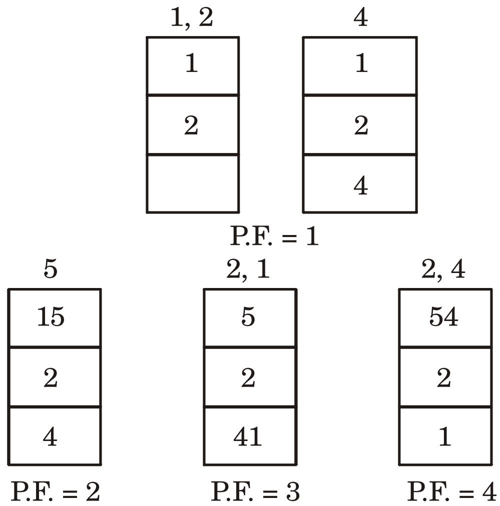
So, after 18 P and 18 V operations, the value of the semaphore will be 7.

The remaining 2 P operations result in the semaphore value 5.

2. Two processes can never lead to deadlock as the peak time demand of 6 (3 + 3) tape drives can be satisfied. But 3 processes can lead to a deadlock if each hold 2 drives and then demand one more.

3. Having 11 resources ensures that atleast 1 process will have no pending request. This process after using will release the resources and so deadlock can never occur.

4.



i.e., Number of page fault = 4 Ans.

5. Average time required $= 0.35 \times 10 + (1 - 0.35) \times 100$

$= 68.5$ ns

6. To specify a particular segment, 5 bits are required (since $2^5 = 32$).

Having selected a page, to select a particular byte one needs 10 bits (since $2^{10} = 1$ k byte).

So, totally 5 + 10 = 15 bits are needed.

7. Main memory bits $= 2^3 \cdot 2^{20} = 2^{23}$

i.e., 23

Cache block age $= 2^7 \cdot 2^{10} = 2^{17}$

i.e., 17 bits

8. Hit ratio = 31%

9. Total time = 60 ms

10. When it tries to access 0100, it results in a page fault as the memory is empty right now. So, it loads the second page (which has the addresses 100-199).

Trying to access 200 will result in a page fault, as it is not in memory right now. So the third page with the addresses from 200 to 299 will replace the second page in memory. Trying to access 430 will result in another page fault. Proceeding this way, we find trying to access the addresses 0510, 0120, 0220, 0320 will all result in page faults. So, altogether 7 page faults.

11. There are 64 words in a block.

So 4 K cache has $(4 \times 1024)/64 = 64$ blocks. Since 1 set has 4 blocks, there are 16 sets. 16 sets needs 4 bits for representation. In a set there are 4 blocks, which needs 2 bits. Each block has 64 words. So, the word field has 6 bits.

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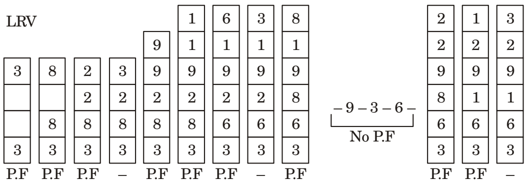
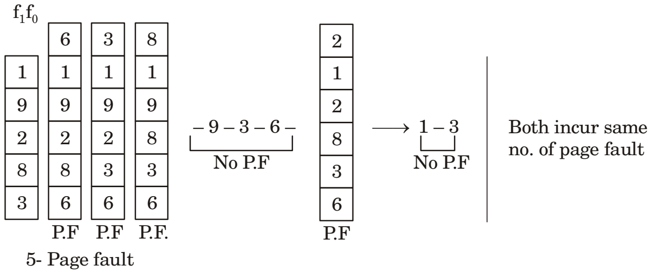
EXERCISE – II

MCQ TYPE QUESTIONS

1.



Total page fault = 9



Total page fault = 9

3. Consider the memory partition and using best fit algorithm

200	400	600	500	300	250
	357	491	468		201

Hence 200 and 300 are not alloted to any process.

4. The maximum number of processor is not dependent upon number of CPU's.

5. For deadlock prevention we need to dissatisfy any of the necessary condition for deadlock

1. For hold and wait if we are disatisfy it can be hold or wait for hold … before process start os assign all resources. While executing if process making new resource request it has to release all its assign resources that is waiting for release resources.
2. If we are dissatisfy circular wait condition suppose there are five resources uniquely numbered r1 to r5 and there five processes p1 to p5 suppose p2 hold r1 ,and requesting for r2,and r2 hold by p3 and wait for r3, r3 hold by p4 wait for r4 which is hold by p5 and wait for r5 which is held by p1 (this is happening in circular manner) (condition to come out from circular wait is a process can hold the resource and request for new resource only when holding resource id is lesser then requesting resource id) in above p1 is violating condition.
3. If we dissatisfy preemption if a process hold some resources requesting for new resources and that requesting resources are busy with some other process requesting process voluntarily release its held resources.
4. Answer, according to 1 and 3 above statements first option is correct.

According to above second statement second option is correct.

According to above second statement if we implement the logic in reverse order third option is also correct.

According to second statement condition fourth option is also correct.

6.

Process	AT	BT
A	0	3
B	1	6
C	4	4
D	6	2

Given Snapshot

(*i*) FcFs

Process	AT	BT	LT	TAT
A	0	3	3	3
B	1	6	9	8
C	4	4	13	9
D	6	2	15	9

$$\frac{29}{4} = 7.25$$

Gantt chart

A	B	C	D

0 3 9 13 15

(*ii*) SJF

Process	AT	BT	LT	TAT
A	0	3	3	3
B	1	6	9	8
C	4	4	15	11
D	6	2	11	5

$$\frac{27}{4} = 6.75$$

Gantt chart

A	B	D	C

0 3 9 11 15

B – 6 C – 9

D – 2

(*iii*) SRTF

Process	AT	BT	CT	TAT
A	0	3	3	3
B	1	6	15	14
C	4	4	8	4
D	6	2	10	4

$$\frac{25}{4} = 6.25$$

Gantt chart

A	A	B	C	C	D	B

0 1 3 4 6 8 10 15

A – 2 B – 5 C – 2 B – 5 B – 5
B – 6 C – 4 B – 5 D – 2
D – 2

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(*iv*) R.Q (Q = 2)

Process	AT	BT	CT	TAT
A	0	3	5	5
B	1	6	15	14
C	4	4	13	9
D	6	2	11	5

$$\frac{33}{4} = 8.25$$

Gantt Chart

R. Q

A	B	A	C	B	D	C	B

0 2 4 5 7 9 11 13 15

7. User threads are supported above the kernel and a managed without kernel support. The thread function library to implement user level threads usually runs on top of the system in user mode. Thus these threads with in a process are invisible to the operating system. Since the kernel is unaware of the existence of such threads; when one user level thread is blocked in the kernel all other threads of its process are blocked. So options (*a*) and (*b*) are true. (*c*) The OS is aware of kernel level threads. Kernel threads are scheduled by the OS's scheduling algorithms and require a "lightweight" context switch to switch between (that is, registers, PC and SP must be changed, but the memory context remains the same among kernel threads in the (same process). User level threads are much faster to switch between as there is not context switch.(*d*) False Kernel level threads within the same process share code section, data section and other operating system resources such as open files and signals.

8. Physical page frames are 20 contain pages numbered 101 to 120.

Program accessed page numbered 1, 2, - - - - 100 in that order and repeat thrice.

9. To implement recursion, activation record should be implemented by providing dynamic memory allocation. This dynamic allocation is done from runtime stack. Heap is essential to allocate memory for data structures at run-time, not for recursion. So statement 1 and 3 are correct.

10. (*b*) Correct

(*a*)

X	Y	Z
P(*a*)	P(*b*)	P(*c*)
P(*b*)	P(*c*)	P(*d*)
P(*c*)	P(*d*)	P(*a*)

wrong answer because:

suppose X execute first & executed P(*a*) & P(*b*) then process switches to Z, then Z executed P(*c*), P(*d*) & wait for P(*a*) then again process switches to X & X then wait for P(*c*).

so, X is waiting for C which is occupied by Z & Z is waiting for X so no one can execute & deadlock occur.

(*b*) Correct

X	Y	Z
P(*b*)	P(*b*)	P(*a*)
P(*a*)	P(*c*)	P(*c*)
P(*c*)	P(*d*)	P(*d*)

Correct because all process does not have reverse call for wait on any variable.

(*c*)

X	Y	Z
P(*b*)	P(*c*)	P(*a*)
P(*a*)	P(*b*)	P(*c*)
P(*c*)	P(*d*)	P(*d*)

wrong answer because sequence of P(*b*) & P(*c*) are reverse or opposite in X & Y respectively so deadlock may occur.

(*d*)

X	Y	Z
P(*a*)	P(*c*)	P(*c*)
P(*b*)	P(*b*)	P(*d*)
P(*c*)	P(*d*)	P(*a*)

wrong answer because sequence of P(*c*) & P(*a*) are opposite in Z & X. so deadlock may occur.

11. A shared variable x_1 initialized to zero, is opereated on by four concurrent processes W, X, Y, Z as follows. Each of the processes W and X reads X from memory, increments by one. Stores it to memory and the terminates. Each of processes Y and z reads x from memory, decrements by two, stores it to memory and then terminates. Each process before reading x invokes the P operation (i.e. wait) on a counting semaphore S and invokes the V operation (i.e. signal) on the semaphore s after storing x to memory. Semaphore S is initalized to two. What is the maximum possible value of x after all processes complete execution?

$\Rightarrow$ x , w reads x and increment x by 1.

y, z reads x and decrement by 2.

Start with x. will perform P(s) then $s = 1$. read $x = 0$ $x = x + 1 = 1$

then y will perform P(s) then $s = 0$. read $x = 0$

$x = x - 2 = -2$, then store $x \cdot v(s)$, $s = 1$

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then z will perform P(s) there s = 0, read $x = -2$

$x = x - 2 = -4$, then store x, $v(s)$, $s = 1$

then x will store $x \cdot v(s)$, $s = 2$, $x = 1$

then w will perform P(s), $s = 1$, read $x = 1$

$x = x + 1 = 2$, store x, $v(s)$, $s = 2$, $\boxed{x = 2}$

12. If fork is called n times, there will be total 2^n running processes including the parent process. So, there will be $2^n - 1$ child processes.

13. 1, 2, 3, 2, 4, 1, 3, 2, 4, 1

LRU	**1**	**1**	**1**	**1**	**3**	**2**	**4**	**1**	**3**	**2**
		2	**2**	**3**	**2**	**4**	**1**	**3**	**2**	**4**
			3	**2**	**4**	**1**	**3**	**2**	**4**	**1**
	F	**F**	**F**		**F**	**F**	**F**	**F**	**F**	**F = 9**
FIFO	**1**	**1**	**1**	**1**	**2**	**3**	**3**	**4**	**4**	**4**
		2	**2**	**2**	**3**	**4**	**4**	**1**	**1**	**1**
			3	**3**	**4**	**1**	**1**	**2**	**2**	**2**
	F	**F**	**F**		**F**	**F**		**F**		**= 6**
OPTIMAL	**1**	**1**	**1**	**1**	**4**	**4**	**4**	**2**	**2**	**2**
		2	**2**	**2**	**1**	**1**	**1**	**1**	**1**	**1**
			3	**3**	**3**	**3**	**3**	**4**	**4**	**4**
	F	**F**	**F**	**NF**	**F**			**F**		**= 5**

Optimal < FIFO < LRU
5 < 6 < 9

14. FCFS will complete in

P_1 P_2 P_3

and Round Robin

P_1 P_3 P_2

P_1	P_2	P_1	P_3	P_2	P_1	P_3

0 2 4 6 8 10 11 13

15. Let S be a non-serial schedule, without loss of generality assume that T1 has started earlier than T2. The first instruction of T1 is read (P) and the last instruction of T2 is write (P), so the precedence graph for S has an edge from T1 to T2, now since S is a non-serial schedule the first instruction of T2 (read(Q)) should be executed before last instruction of T1 (write(Q)) and since read and write are conflicting operations, the precedence graphs for S also contains an edge from T2 to T1, So we will have a cycle in the precedence graph which implies that any non serial schedule with T1 as the earliest transaction will not be conflict serializable.

In a similar way we can show that if T2 is the earliest transaction then also the schedule is not conflict serializable.

17. Effort person per month = a.(KDSI)^B

KDSI = kilo LOC

$= 2.8 \times (40)^{1.20}$

$= 2.8 \times 83.6511$

$= 234.22$ person per month

18. Number of clock cycles required by using load-store approach

$= 2 + 500 \times 7 = 3502$

and that of by using DMA

$= 20 + 500 \times 2 = 1020$

Required speed up $= \dfrac{3502}{1020} = 3.4$

19. P = page fault rate

EA = p × page fault service time

+ $(1 - p)$ × Memory access time

$$= \frac{1}{10^6} \times 10 \times 10^6 + \left(1 - \frac{1}{10^6}\right) \times 20$$

$\cong 29.9$ ns

20. Both process can't be in critical section simulteneously, so they should be in mutual exclusion.

21. Initially all pages are bring in physical memory and all are different, so that all time page fault occur.

So, 100 page fault after that in reverse order pages required.

So we see that 4 pages at last already in memory.

So there is no page fault for these.

So only 100 – 4 = 96 page faults occur.

∴ Total page faults = 100 + 96 = 196

22. SRTF (Shortest Remaining Time First)

Preemptive may cause of starvation and RR (Round Robin) is better than FCFS in response time because in RR if some process comes letter, then after some time it brings in process but in FCFS it is not upto finishing of First process.

Alternately

I. In SRTF case, if processes with small processing time keep on coming, then processes with higher processing time has to wait and may starve.

II. In preemptive scheduling, if processes of high priority keep on coming, then processes with low priority may starve.

III. In RR, every process is served for a quantam time period but it is not in FCFS.

Thus all I, II and III are true.

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23. Initial P1 and P2 does not go because they are not using semaphore initially. P0 uses semaphore whose value S0 = 1, and P1 and P2 uses S1 = S2 = 0, So initially P0 executes so it prints '0'.

Now, it release S1 and S2 means value of these now 1 = S1 = S2. Then process P1 & P2 executes after that they Release. So and it is S0 = 1 again. Then this prints '0' again means at least twice **0** print.

Alternately

- Initially P1 and P2 can't proceed because S1 and S2 are 0.
- P0 will proceed and print '0' then it release S1 and S2.
- Either of S1 and S2 will release so it prints '0' again.

24. If $n = 21$, $k = 12$

(if (i%2 = = 0) {if ($i < n$) Request Ri;

if ($i + 2 < n$) Request R_{i+2};

else (if ($i<n$) Request R_{n-i}, if (i+2<n) Request (R_{n-i-2i})

We seen that if value of k = 11, 12.

Now first $k = 11$ means $p_{11-1} = p_{10} = p_i \Rightarrow i = 10$

then according to logic

initially it goes if block because (i%2 = 0)

(10%2 = = 0)

then it Request R_{10}, R_{10+2} (R_{12})

$\Rightarrow R_{10}, R_{12}$ Resources

if now next $k = 12$ then $i = 11$

Now it goes in else block and

Request R_{n-i}, R_{n-i-2}

$\Rightarrow R_{21-11}, R_{21-11-2}$

$\Rightarrow R_{10}, R_8$

then it also Request R_{10} then

both $k = 11$, $k = 12$ request R_{10}.

It is problem of dead lock.

25. FIFO may have Belady's Anamoly

e.g. in FIFO for three frames the reference string

1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

There are 9 page faults and for four frames for the reference string there will be 10 faults.

26. Page frame number is essential content in each entry of the page table.

Alternately

The page table has the ability to mark on entry invalid through a valid invalid bit or special value of protection bit, which is often called page frame number.

27. *At resource use*

t = 0 R2 - 2units R3 - 2 units R4 - 1units

t = 2 R2 - 2units R3 - 3 units R4 - 2 units R1- 1units

t = 3 R2 - 2units R3 - 3 units R4 -2 units <u>R1- 4 units</u>

t = 4 R2 - 2units R3 - 3 units R4 - 2units <u>R1- 5 units</u>

t = 5 R2 - 1 units R3 - 3units R4 - 2units R1-2units

t = 6 R2 - 1units R3 - 2 units R4 - 2units R1-2units

t = 7 R2- 2 units R3 - 1 units R4 - 2units R1- 2units

t = 8 R2 - 2 units R3- 1 units <u>R4- 3 units</u> R1- 1units

t = 9

Underline show that at that time requirement of resource is less than avaiability but it does not cause dead lock. All process finsh without any dead lock.

28. Requests : 4, 34, 10, 7, 19, 73, 2, 15, 6, 20

Current Position of head is 50.

Current	50	34	20	19	15	10	7	6	4	2
Next Access	34	20	19	15	10	7	6	4	2	73
Difference	16	14	1	4	5	5	1	2	2	71

Total time = 16 + 14 + 1 + 4 + 5 + 3 + 1 + 2 + 2 + 71

= 119 ms

29. In premptive sheduling the process which allocated the CPU keeps it and release it only whne it switching to wating state. Also a process P_2 in blocked state can make transition E while other process P_1 in running.

Alternately

A process P_2 in blocked state can make transition E while another P_1 is in running state.

OS uses preemptive Scheduling.

30. In given program the process test -and-set in critical section and in function leaves _CS initialise memory location x to zero. So this process is dead lock free.

31. Most computer system support a large logical address space (2^{32} to 2^{64}). The page table become excessively large for example.

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Consider a system with a 32 bit logical address space. If page size 4K(2^{12}), then page table may consist 1 million page. For this we use multilevels page table.

32. Connect result in sending SYN packets (TCP handshaking).

33. Index allocation cannot allocate very large file.

34. P(I) : P_6 (x_1); S = S – 1; if (S < 0) {V_6 (K_6); p_6 (y_5);}
else V_6 (x_6) ; value of x_6 = 1 satisfy
V(S) : P_6 (x_6) ; P_6 (x_6) ; S = S + 2 ;
if (S < = 0) V_6 (y_6); V_6 (x_5) ;
Y_6 = 0 satisfy

35. ISR is invoked after completion of I/O irrespective of the type of I/O.

36. In deadlock avoidance, the request for resources is granted if only the state guarantees deadlock would never occur (safe state)

Deadlock prevention provides a set of methods that ensures that at least one of the necessary condition cannot hold – (mutual exclusion, hold & wait, no preemption & circular wait). These methods prevents deadlock by constraining how request can be made.

37. Number of child processes $2^n – 1$

38. Number of bits required for first lent = 24
Number of bits required for second lent = 24
Number of bits required for third lent = 24

39.

Group I	Group 2
P. Gang Scheduling	3. Thread Scheduling
Q. Rate Monotonic Scheduling	2. Real-time Scheduling
R. Fair Show Scheduling	1. Guaranteed Scheduling

40. Blocking one kernel level thread doesn't block all related threads.

41. Gant chart of scheduling can be given as

P_1	P_2	P_3	P_2	P_4

0 20 30 40 55 70

Waiting time of process, P_2 = (20 – 15) + 10 = 15

42. Both P and Q are true But Q is not the reason for P because page fault is concerned with pages present in frame not with pages nearly the page accessed recently.

43. In present situation free resources are 0.12, so P_1 can be processed, after this resources will be 213 so P_0 can be processed therefore P_2 will be last to be processed.

44. Since there is no way to synchronize two statementwants 1 = true (in P_1) and wants 2 = true (in P_2).

So both of them may get executed one after other. In this situation there will be a dead lock.

But if only of the two is true, other will not go beyond cuhile loop, hence ensure nutual exclusion.

45. Context-Switches will be needed only at the time of switching over to some other process. Following table shows **time remaining** for three processes at different instants of time

Time/Processes →	I	II	III	Process continued
0	10	-	-	I
2	8	20	-	I
6	4	20	30	I
10	0	20	30	II
30	0	0	30	III
60	0	0	0	-

Hence context-witches are needed only at times 10 and 30.

46. The P and V function is as follows :

1. void P (binary-semaphore * S) {
2. unsigned *y*;
3. unsigned * *x* = & (S → value);}
4. do {
5. fetch-and-set *x*, *y*;
6. } while (*y*);
7. }
8. void V (binary-semaphore *S) {
9. {S → value = 0;}
10. }

fetch-and-set instruction always sets the memory location *x* = 1 and fetches the old value of *x* into *y*. The binary-semphore * S takes only two values either 0 and 1. When we initiliaze S = 0 then in statement 3 this value will be start at location *x* and fetch-and-set instruction charge the value of *x* = 0 to *x* = 1 and *y* becomes 0. If there are more than two processes and context switching between processes is disabled in P then this implementation doesn't work properly and can't synchronize the processes.

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47. *Number of bits required for various purposes in the 32 bit address is as follows :*

To find memory cell out of 4-way associative memory – 2 bits are needed.

To find page out of 128 page table entries - 7 bits are needed.

To find required byte in the page of size 4 kB – 12 bits are needed.

Total bits consumed in this way = 21

TLB tag's minimum size = 32 – 21 = 11 bits

48.

Process	id	Arrival time	CPU Burst
P_0	0	0	2
P_1	1	0	4
P_2	2	0	8

Remaining Time

P_0	2
P_1	4
P_2	8

The Gannt chart for LRTF scheduling algorithm

P_0	P_1	P_2	P_2	P_2	P_2	P_2	P_1	P_2	P_1	P_2	P_0	P_1	P_2

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

Turn around time for P_0 = 12 – 0 = 12

Turn around time for P_1 = 13 – 1 = 12

Turn around time for P_2 = 14 – 2 = 12

Average Turn around Time = 36/3 = 12 ≈ 13

50. Total number of processes = n

Process i is holding x_i instances of a resource R for $1 \le i \le n$

Process i holding resource x_i and additional required y_i

There are two processes p and q such that $y_p = y_q = 0$

For no dead lock can occur

$\min(x_p, x_q) < (\max_{k \neq p,q} yk)$

NUMERICAL TYPE QUESTIONS

1. Gantt chart is

T_1^1	T_2^1	T_1^2	T_3^1	T_1^3	T_2^2	T_1^4	T_3^1	T_1^5

0 1 3 4 6 7 9 10 12

all tasks are arriving at beginning of 1 ms means at 0. T_1 has highest priority, then T_2 and the T_3. Hence first instance of T_1 by end of 12.

2. Using SSTF algorithm, total head moments are

5 + 15 + 10 + 10 + 10 + 65 + 5 + 10 = 130

using SCAN algorithm total head moments are

10 + 10 + 10 + 10 + 10 + 55 + 20 + 5 + 10 = 140

Hence additional distance covered = 140 – 130 = 10

3.

1	2	3	4	2	1	5	3	2	4	
		3	4	4	4	4	4	4	4	4
	2	2	2	2	2	2	2	2	2	6
1	1	1	1	1	1	5	3	3	3	3
F	F	F	F	H	H	F	F	H	H	F

7 page faults

4. Page reference string is

4, 7, 6, 1, 7, 6, 1, 2, 7, 2

Implementing LRU using 3 page frames

Total page faults = 6

~~4~~	1
~~2~~	2
~~6~~	7

5.

Maximum	Allocate	Need	Available
P_1 – 3	2	1	1
P_2 – 3	2	1	
P_3 – 3	2	1	

With the above given data, after allocating 2 units of tape to each process, with 1 available unit any of the 3 process can be satisfied in such a way, that No dead lock will be there. So answer is 7 tape units.

6. The Gantt chart for SRTF scheduling algorithm is as follows:

P_1	P_2	P_3	P_4	P_1

0 2 6 12 17 27

Average waiting time = $\dfrac{15+0+3+4}{4} = \dfrac{22}{4} = 5.5$

■■

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7

CHAPTER

Databases

ER-MODEL, RELATIONAL MODEL

DATABASE MANAGEMENT SYSTEMS

Database management system (DBMS), or simply a **database system** (DBS), consists of

(*i*) a collection of interrelated and persistent data (usually referred to as the **database** (DB)).

(*ii*) a set of application programs used to access, update and manage that data (which form the data management system (MS)).

The goal of a DBMS is to provide an environment that is both **convenient** and **efficient** to use in

(*i*) retrieving information from the database.

(*ii*) storing information into the database.

Databases are usually designed to manage **large** bodies of information. This involves

(*i*) definition of structures for information storage (data modeling).

(*ii*) provision of mechanisms for the manipulation of information (file and systems structure, query processing).

(*iii*) providing for the safety of information in the database (crash recovery and security).

(*iv*) concurrency control if the system is shared by users.

Components of Database systems.

(1) Data

(2) Hardware

(3) Software

(4) Users

Purpose of Database Systems

1. To see why database management systems are necessary, we see at a typical "file-processing system" supported by a conventional operating system.

Application is a savings bank:

(*i*) Savings account and customer records are kept in permanent system files.

(*ii*) *Application programs are written to manipulate files to perform the following tasks:*

(*a*) Debit or credit an account.

(*b*) Add a new account.

(*c*) Find an account balance.

(*d*) Generate monthly statements.

2. *Development of the system proceeds as follows:*

(*i*) New application programs must be written as the need arises.

(*ii*) New permanent files are created as required.

(*iii*) **but** over a long period of time files may be in different formats, and

(*iv*) Application programs may be in different languages.

3. *There are problems with the straight file-processing approach:*

(*i*) *Data redundancy and inconsistency*

(*a*) Same information may be duplicated in several places.

(*b*) All copies may not be updated properly.

(*ii*) *Difficulty in accessing data*

(*a*) May have to write a new application program to satisfy an unusual request.

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(*b*) *e.g.* find all customers with the same postal code.

(*c*) Could generate this data manually, but a long job.

(*iii*) *Data isolation*

(*a*) Data in different files.

(*b*) Data in different formats.

(*c*) Difficult to write new application programs.

(*iv*) *Multiple users*

(*a*) Want concurrency for faster response time.

(*b*) Need protection for concurrent updates.

(*v*) *Security problems*

(*a*) Every user of the system should be able to access only the data they are permitted to see. *e.g.* payroll people only handle employee records, and cannot see customer accounts; tellers only access account data and cannot see payroll data.

(*b*) Difficult to enforce this with application programs.

(*vi*) *Integrity problems*

(*a*) Data may be required to satisfy constraints. *e.g.* no account balance below ₹ 5000/-

(*b*) Difficult to enforce or to change constraints with the file-processing approach.

These problems and others led to the development of **database management systems**.

DATA ABSTRACTION

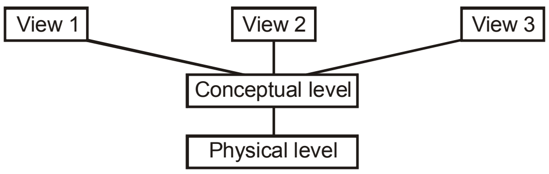
Main purpose of a database system is to provide users with an **abstract view** of the system.

The system hides certain details of how data is stored and created and maintained

Complexity should be hidden from database users.

Levels of Data base system

There are several levels of abstraction:



1. Physical Level:

- *How the data are stored :* *e.g.* index, B-tree, hashing.
- *Lowest level of abstraction :* *e.g.* Data Compression and encryption techniques
- *Complex low-level structures*

2. Conceptual Level:

- Next highest level of abstraction.
- Describes *what* data are stored.
- Describes the relationships among data.
- Database administrator level.

3. View Level:

- Highest level.
- Describes *part* of the database for a particular group of users.
- Can be many different views of a database. *e.g.* tellers in a bank get a view of customer accounts, but not of payroll data.

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DATA MODELS

These are a collection of conceptual tools for describing data, data relationships, data semantics and data constraints.

There are three different groups:

1. Object-based Logical models.

2. Record-based Logical models.

3. Physical Data models.

1. OBJECT-BASED LOGICAL MODELS.

(*i*) Describe data at the conceptual and view levels.

(*ii*) Provide fairly flexible structuring capabilities.

(*iii*) Allow one to specify data constraints explicitly.

(*iv*) *Include over 30 such models such as*

- Entity-relationship model (E-R model)
- Object-oriented model
- Binary model
- Semantic data model
- Infological model
- Functional data model

E-R Model

It is based on a perception of the world as consisting of a collection of basic **objects** (entities) and **relationships** among these objects.

- **Entity** is a distinguishable object that exists.
- Each entity has associated with it a set of **attributes** describing it.
  e.g. number and *balance* for an account entity.
- **Relationship** is an association among several entities.
  e.g. A *cust_acct* relationship associates a customer with each account he or she has.
- The set of all entities or relationships of the same type is called **entity set** or **relationship set**.
- Another essential element of the E-R diagram is the **mapping cardinalities**, which express the number of entities to which another entity can be associated via a relationship set.
- The overall logical structure of a database can be expressed graphically by an **E-R diagram**:
  (*a*) **rectangles** represent entity sets.
  (*b*) **ellipses** represent attributes.
  (*c*) **diamonds** represent relationships among entity sets.
  (*d*) **lines** attributes to entity sets and entity sets to relationships.

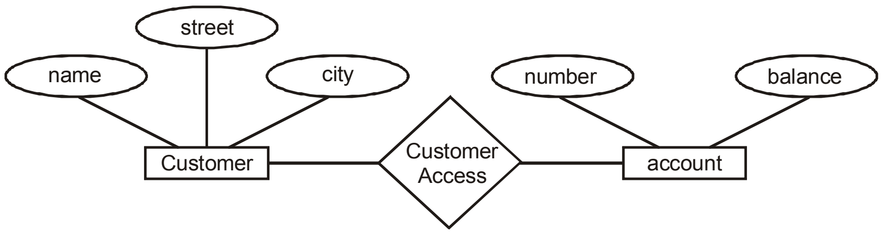

Fig. Sample E-R diagram.

Object-Oriented Model

It is based on a collection of objects, like the E-R model.

- An object contains values stored in **instance variables** within the object. Unlike the record-oriented models, these values are themselves objects. Thus objects contain objects to an arbitrarily deep level of nesting. An object also contains bodies of code that operate on the the object. These bodies of code are called **methods**.

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- Objects that contain the same types of values and the same methods are grouped into **classes**. A class may be viewed as a type definition for objects.
- *Analogy:* The programming language concept of an abstract data type.
- The only way in which one object can access the data of another object is by invoking the method of that other object. This is called **sending a message** to the object.
- Internal parts of the object, the instance variables and method code, are not visible externally.
- Result is two levels of data abstraction.

  e.g., consider an object representing a bank account.

  The object contains instance variables *number* and *balance*.

  The object contains a method *pay-interest* which adds interest to the balance.

  Under most data models, changing the interest rate entails changing code in application programs.

  In the object-oriented model, this only entails a change within the *pay-interest* method.
- Unlike entities in the E-R model, each object has its own unique identity, independent of the values it contains:

  (*a*) two objects containing the same values are distinct.

  (*b*) distinction is created and maintained in physical level by assigning distinct object identifiers.

2. RECORD BASED LOGICAL MODELS.

(*i*) Relational Model

Data and relationships are represented by a collection of **tables**. Each **table** has a number of columns with unique names, *e.g. customer, account*.

Name	Street	City	Number
Lowery	Maple	Queens	900
Shiver	North	Bronx	336
Shiver	North	Bronx	647
Hodgest	Sidehill	Brooklyn	801
Hodgest	Sidehill	Brooklyn	647

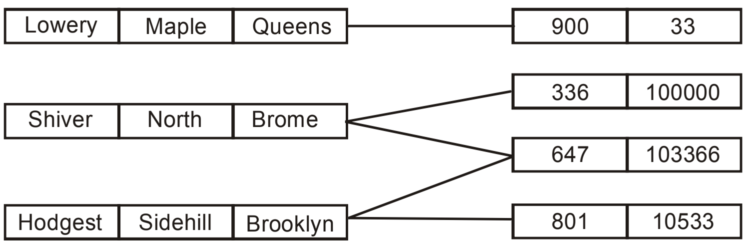
Name	Balance
900	33
336	100000
647	103366
801	105333

Sample relational database.

(*ii*) The Network Model

Data are represented by collections of records. Relationships among data are represented by links. Organization is that of an **arbitrary graph**.

Figure below shows a sample network database that is equivalent of the relational database shown above.



Sample network database

(*iii*) Hierarchical Model

It is similar to the network model. Organization of the records is as a collection of **trees**, rather than arbitrary graphs.

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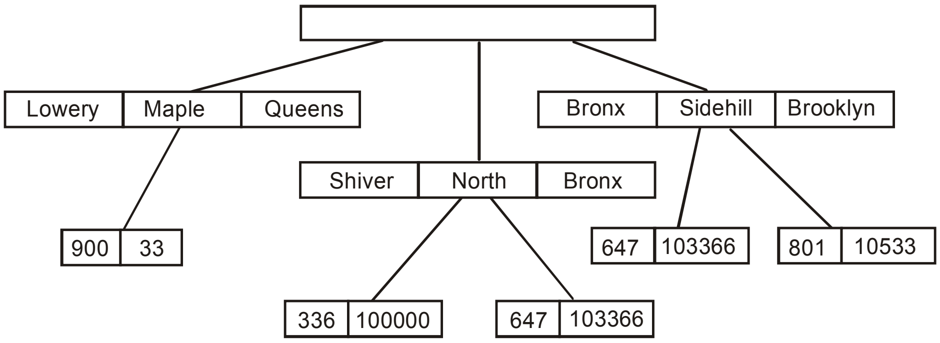
Figure below shows a sample hierarchical database that is equivalent of the relational database shown above.



Sample hierarchical database

The relational model does not use pointers or links, but relates records by the values they contain. This allows a formal mathematical foundation to be defined.

3. PHYSICAL DATA MODELS.

These are used to describe data at the lowest level.

These are very few models.

e.g. Unifying model, Frame memory.

DATA INDEPENDENCE

The ability to modify a scheme definition in one level without affecting a scheme definition in a higher level is called **data independence**.

Types of Data Independence

There are two types:

1. Physical data independence

The ability to modify the physical scheme without causing application programs to be rewritten Modifications at this level are usually to improve performance.

2. Logical data independence

The ability to modify the conceptual scheme without causing application programs to be rewritten.

Usually done when logical structure of database is altered.

Note: Logical data independence is harder to achieve as the application programs are usually heavily dependent on the logical structure of the data. An analogy is made to abstract data types in programming languages.

DATA LANGUAGES

Data Definition Language (DDL)

It is used to specify a database scheme as a set of definitions expressed in a DDL

Data Dictionary.

DDL statements are compiled, resulting in a set of tables stored in a special file called a **data dictionary** or **data directory**. The data directory contains **metadata** (data about data).

Data Storage.

The storage structure and access methods used by the database system are specified by a set of definitions in a special type of DDL called **data storage** and **definition language**.

Basic idea is to hide implementation details of the database schemes from the users.

Data Manipulation Language (DML)

A DML is a language which enables users to access and manipulate data. The goal is to provide efficient human interaction with the system.

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Data Manipulation : It is

(*i*) **retrieval** of information from the database

(*ii*) **insertion** of new information into the database

(*iii*) **deletion** of information in the database

(*iv*) **modification** of information in the database

Types of DML.

There are two types of DML:

1. **Procedural :** the user specifies *what* data is needed and *how* to get it.
2. **Nonprocedural :** The user only specifies *what* data is needed.

   It is easier for user. It may not generate code as efficient as that produced by procedural languages.

Query language

It is a portion of a DML involving information retrieval only. The terms DML and query language are often used synonymously.

RELATIONAL MODEL

RELATIONAL ALGEBRA.

- It is the formal description of how a relational database operates.
- It is an interface to the data stored in the database itself.
- It is the mathematics which underpin SQL operations.

Operators in relational algebra

These are not necessarily the same as SQL operators, even if they have the same name.

e.g. the SELECT statement exists in SQL, and also exists in relational algebra. These two uses of SELECT are not the same. The DBMS must take whatever SQL statements the user types in and translate them into relational algebra operations before applying them to the database.

Terminology

(1) Relation : Set of tuples.

(2) Tuple : Collection of attributes which describe some real world entity.

(3) Attribute : Real world role played by a named domain.

(4) Domain : Set of atomic values.

(5) Set : Mathematical definition for a collection of objects which contains no duplicates.

Operators - Write

1. **INSERT :** It provides a list of attribute values for a new tuple in a relation. This operator is the same as SQL.
2. **DELETE :** It provides a condition on the attributes of a relation to determine which tuple(s) to remove from the relation. This operator is the same as SQL.
3. **MODIFY :** It changes the values of one or more attributes in one or more tuples of a relation, as identified by a condition operating on the attributes of the relation. This is equivalent to SQL UPDATE.

Operators - Retrieval

There are two groups of operations :

1. **Mathematical set theory based relations.**

   UNION, INTERSECTION, DIFFERENCE, and CARTESIAN PRODUCT.
2. **Special database operations.**

   SELECT (not the same as SQL SELECT), PROJECT, and JOIN.

(*i*) Relational SELECT

SELECT is used to obtain a subset of the tuples of a relation that satisfy a *select condition*.

e.g., find all employees born after 1st Jan 1950:

$$\text{SELECT}_{\text{dob 01/JAN/1950'}}(\text{employee})$$

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(*ii*) Relational PROJECT

The PROJECT operation is used to select a subset of the attributes of a relation by specifying the names of the required attributes.

e.g., to get a list of all employees surnames and employee numbers:

$$\text{PROJECT}_{\text{surname, empno}}(\text{employee})$$

(*iii*) SELECT and PROJECT

SELECT and PROJECT can be combined together.

e.g., to get a list of employee numbers for employees in department number 1:

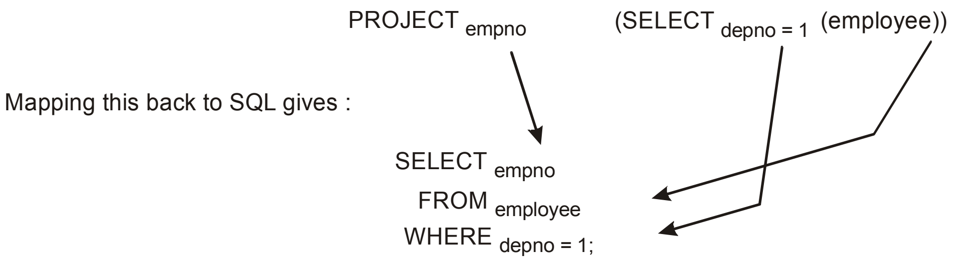


Fig. Mapping select and project

SET OPERATIONS.

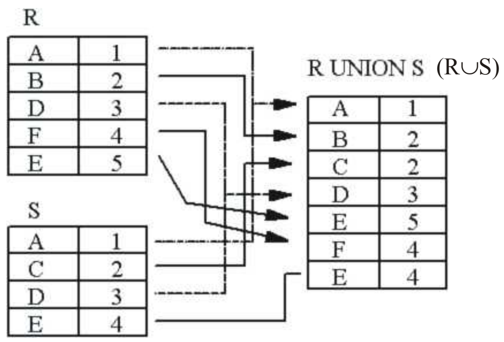
Semantics

Consider two relations R and S.

1. UNION of R and S.

The union of two relations is a relation that includes all the tuples that are either in R or in S or in both R and S. Duplicate tuples are eliminated.

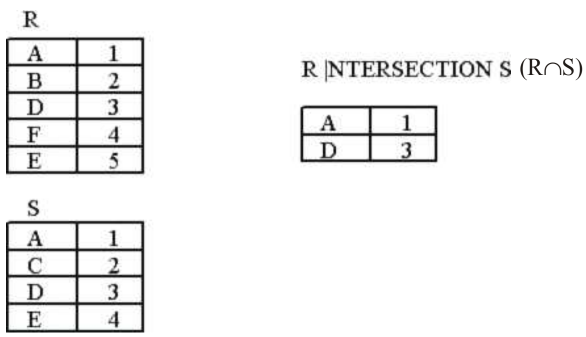
e.g.



2. INTERSECTION of R and S.

The intersection of R and S is a relation that includes all tuples that are both in R and S.

e.g.



3. DIFFERENCE of R and S.

The difference of R and S is the relation that contains all the tuples that are in R but that are not in S.

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e.g.

R

A	1
B	2
D	3
F	4
E	5

S

A	1
C	2
D	3
E	4

R DIFFERENCE S (R – S)

B	2
F	4
E	5

S DIFFERENCE R (S – R)

C	2
E	4

Requirements.

For set operations to function correctly the relations R and S must be union compatible. Two relations are union compatible if

(*i*) they have the same number of attributes

(*ii*) the domain of each attribute in column order is the same in both R and S.

CARTESIAN PRODUCT.

It is also an operator which works on two sets. It is sometimes called *Cross Product* or *Cross Join.*

It combines tuples of one relation with all the tuples of the other relation.

e.g.

R

A	1
B	2
D	3
F	4
E	5

S

A	1
C	2
D	3
E	4

R CROSS S (R × S)

A	1	A	1
A	1	C	2
A	1	D	3
A	1	E	4
B	2	A	1
B	2	C	2
B	2	D	3
B	2	E	4
D	3	A	1
D	3	C	2
D	3	D	3
D	3	E	4

F	4	A	1
F	4	C	2
F	4	D	3
F	4	E	4
E	5	A	1
E	5	C	2
E	5	D	3
E	5	E	4

JOIN Operator

JOIN is used to combine related tuples from two relations:

(1) In its simplest form, it is just the cross product of the two relations.

(2) As the join becomes more complex, tuples are removed within the cross product to make the result of the join more meaningful.

(3) JOIN allows to evaluate a join condition between attributes of the relations on which the join is undertaken.

Notation : R $\text{JOIN}_{\text{join condition}}$ S

e.g.

R

Col A	Col B
A	1
B	2
D	3
F	4
E	5

S

SColA	SColB
A	1
C	2
D	3
E	4

R JOIN S (R⋈S)
R Col A = S.SColA

A	1	A	1
D	3	D	3
E	5	E	4

R JOIN S (R⋈S)
R Col B = S.SColB

A	1	A	1
B	2	C	2
D	3	D	3
F	4	E	4

Symbolic Notation

Commonly used symbolic notation to represent the operators :

- SELECT → σ (sigma)
- PROJECT → π(pi)
- PRODUCT → × (times)

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- JOIN → |×| (bow-tie)
- UNION → = (cup)
- INTERSECTION→ ∩(cap)
- DIFFERENCE → – (minus)
- RENAME → ρ (rho)

Equivalences

The same relational algebraic expression can be written in many different ways. The order in which tuples appear in relations is never significant.

- A × B = B × A
- A ∩ B = B ∩ A
- A + B = B + A
- (A – B) is not the same as (B – A)
- $\sigma_{c1} (\sigma_{c2}(A)) = \sigma_{c2} (\sigma_{c1}(A)) = \sigma_{c1 \wedge c2}(A)$

TUPLE RELATIONAL CALCULUS.

It is a nonprocedural language. We must provide a formal description of the information desired.

A query in the tuple relational calculus is expressed as $\{t | P(t)\}$

where P is a **formula.**

Several tuple variables may appear in a formula.

The set of tuples t for which predicate P is true.

We also use the notation

- $t\ [\alpha]$ to indicate the value of tuple t on attribute α.
- $t \in r$ to show that tuple t is in relation r.

Tuple variables

A tuple variable is said to be a **free variable** unless it is quantified by a ∃ or a ∀. Then it is said to be a **bound variable**.

Atom

A formula is built of **atoms**. An atom is one of the following forms:

- $\delta \in r$, where s is a tuple variable, and r is a relation ($\notin$ is not allowed).
- $s[x]\ \theta$, where x and u are tuple variables, and x and y are attributes, and θ is a comparison operator (<, ≤, = ≠ , >, ≥).
- $s[x]\ \theta\ c$, where c is a constant in the domain of attribute c.

Formulae

These are built up from atoms using the following rules:

(*i*) An atom is a formula.

(*ii*) If P is a formula, then so are ¬P and (P).

(*iii*) If P_1 and P_2 are formulae, then so are $P_1 \vee P_2$, $P_1 \wedge P_2$ and $P_1 \Rightarrow P_2$.

(*iv*) If P(s) is a formula containing a free tuple variable s, then

$$\exists s \in r\left(P(s)\right) \text{ and } \forall s \in r\left(P(s)\right)$$

are formula also.

(*v*) *Some equivalences:*

- $P_1 \wedge P_2 = \neg\left(\neg P_1 \vee \neg P_2\right)$
- $\forall t \in r\left(P(t)\right) = \neg\exists t \in r\left(\neg P(t)\right)$
- $P_1 \Rightarrow P_2 = \neg P_1 \vee P_2$

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DATABASE DESIGN

DATABASE DESIGN

MODIFYING THE DATABASE.

Modifications are expressed using the assignment operator.

Deletion

It is expressed in much the same way as a query. Instead of displaying, the selected tuples are removed from the database. We can only delete whole tuples.

In relational algebra, a deletion is of the form

$$r \leftarrow r - E$$

where r is a relation and E is a relational algebra query.

Tuples in r for which E is true are deleted.

e.g. Delete all of Smith's account records.

$$\text{deposit} \leftarrow \text{deposit} - \sigma_{cn\ am\ s} = {}^{a}\text{Smith*(deposit)}$$

Insertions

To insert data into a relation, we either specify a tuple, or write a query whose result is the set of tuples to be inserted. Attribute values for inserted tuples must be members of the attribute's domain.

An insertion is expressed by $r \leftarrow r \cup \text{E}$

where r is a relation and E is a relational algebra expression.

e.g. To insert a tuple for Smith who has $1200 in account 9372 at the SFU branch.

$$\text{deposit} \leftarrow \text{deposit} \cup \{(\text{“SFU”}, 9372, \text{“Smith”}, 1200)\}$$

Updating

It allows us to change some values in a tuple without necessarily changing all.

We use the update operator, δ, with the form

$$\delta_{A \leftarrow E}(r)$$

where r is a relation with attribute Λ, which is assigned the value of expression E.

The expression E is any arithmetic expression involving constants and attributes in relation r.

e.g. To increase all balances by 5 percent.

$$\delta_{\text{balance} \neg \text{balance} + 1.05}\ (\text{deposit})$$

This statement is applied to every tuple in *deposit*.

Views

The term **view** refer to any relation, not part of the conceptual model, that is made visible to the user as a "virtual relation".

As relations may be modified by deletions, insertions and updates, it is generally not possible to store views

View Definition

- View is defined using the **create view** command:

  create view v as <query expression>

  where <query expression> is any legal query expression.

  The view created is given the name υ.

- To create a view *all-customer* of all branches and their customers:

  Create view all-customer as

$$\text{II}_{bn\ am\ s,\ cn\ ams}\ (\text{deposit}) \cup \text{II}_{bn\ am\ s,\ cn\ ams}\ (\text{borrow})$$

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- Having defined a view, we can now use it to refer to the virtual relation it creates. View names can appear anywhere a relation name can.
- We can now find all customers of the SFU branch by writing

$$\Pi_{cn\ am\ s}\ (\sigma_{bnams} = \text{"SFU"}\ (\text{all} - \text{customer}))$$

Integrity Constraints

These provide a way of ensuring that changes made to the database by authorized users do not result in a loss of data consistency.

Form of integrity constraint with E-R models:

(*i*) **Key declarations:** Stipulation that certain attributes form a candidate key for the entity set.

(*ii*) **Form of a relationship:** Mapping cardinalities 1-1, 1-many and many-many.

An integrity constraint can be any arbitrary predicate applied to the database.

They may be costly to evaluate, so we will only consider integrity constraints that can be tested with minimal overhead.

Domain Constraints

A domain of possible values should be associated with every attribute. These domain constraints are the most basic form of integrity constraint.

They are easy to test for when data is entered.

Domain types

(1) Attributes may have the same domain, e.g. *cname* and *employee-name*.

(2) It is not as clear whether *bname* and *cname* domains ought to be distinct.

(3) At the implementation level, they are both character strings.

(4) At the conceptual level, we do not expect customers to have the same names as branches, in general.

(5) Strong typing of domains allows us to test for values inserted, and whether queries make sense. Newer systems, particularly object-oriented database systems, offer a rich set of domain types that can be extended easily.

Referential Integrity

Often we wish to ensure that a value appearing in a relation for a given set of attributes also appears for another set of attributes in another relation. This is called *referential integrity.*

Dangling tuples.

Consider a pair of relations $r(R)$ and $s(S)$, and the natural join r 1 s.

There may be a tuple t_r in r that does not join with any tuple in s. *i.e.* there is no tuple t_s in s such that

$$t_r[\text{R} \cap \text{S}] = t_r[\text{R} \cap \text{S}].$$

This is called *dangling tuple.*

Dangling tuples may or may not be acceptable.

Distinction between these two situations:

bname is the primary key of *branch*, while it is not for *account*.

In account, *bname* is a **foreign key**, being the primary key of another relation.

- Let $r_1(\text{R}_1)$ and $r_2(\text{R}_2)$ be two relations with primary keys K_1 and K_2 respectively.
- We say that a subset α of R_2 is a *foreign key* referencing K_1 in relation r_1 if it is required that for every tuple t_2 in r_2 there must be a tuple t_1 in r_1 such that $t_1[\text{K}_1] = t_2(\alpha)$
- We call these requirements **referential integrity constraints**.
- Also known as **subset dependencies**, as we require

$$\Pi_{\alpha}(r_1) \subseteq \Pi\ \text{K}_1(r_1)$$

Referential Integrity in E-R model.

These constraints arise frequently. Every relation arising from a relationship set has referential integrity constraints.

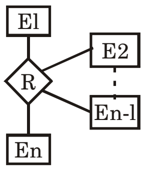

Fig. *n*-ary relationship set

Above figure shows an n-ary relationship set R relating entity sets $\text{E}_{1:}\ \text{E}_{2:} \ldots \text{E}_n$.

Let K_1 denote the primary key of E_1.

Attributes of the relation scheme for relationship set R include $\text{E}_1 \cup \text{K}_2 \cup \ldots \cup \text{K}_n$.

Each K_i in the scheme for R is a foreign key that leads to a referential integrity constraint.

Relation schemes for weak entity sets must include the primary key of the strong entity set on which they are existence dependent. This is a foreign key, which leads to another referential integrity constraint.

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Database Modification

These can cause violations of referential integrity.

To preserve referential integrity constraint

$$\Pi\alpha(r_2) \subseteq \Pi\, K_1(r_1)$$

in the following operations:

(*i*) **Insert:** if a tuple t_2 is inserted into r_2, the system must ensure that there is a tuple t_1 in t_1 such that $t_1[K] = t_2[\alpha]$, *i.e.*

$$t_2[\alpha] \in \Pi_K(r_1)$$

(*ii*) **Delete:** if a tuple t_1 is deleted from r_1, the system must compute the set of tuples in r_2 that reference t_1:

$$\sigma_\alpha = t_1[K](r_2)$$

If this set is not empty, either reject delete command, or delete also the tuples that reference t_1.

(*iii*) **Update :** Two cases

(*a*) *updates to referencing relation:* test similar to insert case must be made, ensuring if t'_2 is new value of tuple,

$$t'_2\,[\alpha] \in \Pi_K\,(r_1)$$

(*b*) *updates to referenced relation:* test similar to delete if update modifies values for primary key, must compute

$$\sigma_\alpha = t_1\,[K]\,(r_2)$$

to ensure that we are not removing a value referenced by tuples in r_2.

Referential Integrity in SQL.

An addition to the original standard allows specification of primary and candidate keys and foreign keys as part of the **create table** command:

- **Primary key** clause includes a list of attributes forming the primary key.
- **Unique key** clause includes a list of attributes forming a candidate key.
- **Foreign key** clause includes a list of attributes forming the foreign key, and name of the relation referenced by the foreign key.

Assertions

It is a predicate expressing a condition we wish the database to always satisfy.

Special forms of assertion are :

domain constraints, functional dependency and referential integrity

Where a constraint cannot be expressed in these forms, we use an assertion*, e.g.*

- ensuring sum of loan amounts for each branch is less than the sum of all account balances at the branch.
- ensuring every loan customer keeps a minimum of $1000 in an account.

An assertion in DQL-92 takes the form

create assertion assertion-name **check** predicate

Triggers

Another feature not present in the SQL standard is the **trigger**. Several existing systems have their own non-standard trigger features.

It is a statement that is automatically executed by the system as a side effect of a modification to the database.

We need to specify

(*i*) conditions under which the trigger is executed.

(*ii*) actions to be taken by the trigger.

e.g. Let an overdraft is intended to result in the account balance being set to zero, and a loan being created for the overdraft amount.

NORMALISATION

It is the process of taking data from a problem and reducing it to a set of relations while ensuring data integrity and eliminating data redundancy.

Data redundancy.

If data in the database can be found in two different locations (direct redundancy) or if data can be calculated from other data items (indirect redundancy), then the data is said to contain redundancy.

Normal Forms (NFs).

There are various types of NFs available. A relation is said to be in a particular normal form only if it satisfies the constraint (that it contains atomic values only to be able to call it as in 1NF).

A number of normal forms have been defined.

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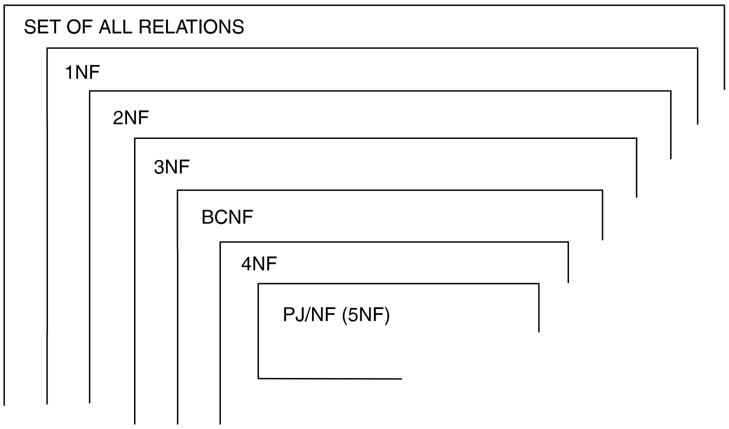
Set-diagram of NFs:



Insertion anomaly.

With the primary key including subject, we cannot enter a new student until they have at least one subject to study. We are not allowed NULLs in the primary key so we must have an entry in both matric_no and subject before we can create a new record. This is called *insertion anomaly.*

It is difficult to insert new records into the database. On a practical level, it also means that it is difficult to keep the data up to date.

Update anomaly.

If name of a student were changed, *e.g.* Smith, J. was changed to Green, J. this would require not one change but many one for every subject that Smith, J. studied.

Deletion anomaly.

If all of the records for the 'Databases' subject were deleted from the table,we would inadvertently lose all of the information on the student with matric_no 960145. This would be the same for any student who was studying only one subject and the subject was deleted. Again this problem arises from the need to have a compound primary key.

Full Functional Dependence.

Attribute B is fully functionally dependent on attribute A, if and only if, it is functionally dependent on A and NOT functionally dependent on any proper subset of A.

e.g. take a relation S = (SUPLR_NO, SUPL_NAME, SUPL_STATUS, CITY)

Transitive dependence.

If attribute A depends on B and B depends on C, due to which A depends on C, then A is said to be transitively dependent on C.

Transitive dependency causes problems in updation.

The data in the database can be considered to be in one of a number of 'normal forms'. Basically the normal form of the data indicates how much redundancy is in that data. The normal forms have a strict ordering:

1. First Normal Form **2.** Second Normal Form **3.** Third Normal Form **4.** BCNF

1. First Normal Form (1NF)

- A relation is in 1NF if it contains no repeating groups.
- To convert an unnormalised relation to 1NF either.
- Flatten the table and change the primary key, or decompose the relation into smaller relations, one for the repeating groups and one for the non-repeating groups.
- Put the primary key from the original relation into both new relations.
- This option is liable to give the best results.

2. Second Normal Form (2NF)

- A relation is in 2NF if it contains no repeating groups and no partial key functional dependencies
- **Rule:** A relation in 1NF with a single key field must be in 2NF
- To convert a relation with partial functional dependencies to 2NF create a set of new relations:
- One relation for the attributes that are fully dependent upon the key.
- One relation for each part of the key that has partially dependent attributes.

3. Third Normal Form (3NF)

- A relation is in 3NF if it contains no repeating groups, no partial functional dependencies, and no transitive functional dependencies.
- To convert a relation with transitive functional dependencies to 3NF, remove the attributes involved in the transitive dependency and put them in a new relation.

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- **Rule:** A relation in 2NF with only one non-key attribute must be in 3NF.
- In a normalised relation a non-key field must provide a fact about the key, the whole key and nothing but the key.
- Relations in 3NF are sufficient for most practical database design problems. However, 3NF does not guarantee that all anomalies have been removed.

4. Boyce-Codd Normal Form (BCNF).

- When a relation has more than one candidate key, anomalies may result even though the relation is in 3NF.
- 3NF does not deal satisfactorily with the case of a relation with overlapping candidate keys,
  i.e. composite candidate keys with at least one attribute in common.
- BCNF is based on the concept of a *determinant*.
- Determinant is any attribute (simple or composite) on which some other attribute is fully functionally dependent.
- A relation is in BCNF is, and only if, every determinant is a candidate key.

  Consider the following relation and determinants.

  R($\underline{a, b}$, c, d)

  a, c $\rightarrow$ b, d

  a, d $\rightarrow$ b

Here, first determinant suggests that the primary key of R could be changed from a,b to a,c. If this change was done all of the non-key attributes present in R could still be determined, and therefore this change is legal. However, the second determinant indicates that a,d determines b, but a,d could not be the key of R as a,d does not determine all of the non key attributes of R (it does not determine c). We would say that the first determinate is a candidate key, but the second determinant is not a candidate key, and thus this relation is not in BCNF (but is in 3rd normal form).

GRADE Relation.

Name	Student#	Course	Grade
James	23714539	353	A
Neelam	42717390	329	A
James	23714539	328	in prog
Mohan	388115183	456	C
Dilip	37116259	293	B
Deepak	82317293	491	C
Deepak	82317293	353	in prog
James	23714539	491	C
Raj	11011978	353	A +
Vikas	83910827	379	in prog

The problem in the relation GRADE is that it had two overlapping candidate keys.

In the Boyce Codd normal form (BCNF), which is stronger than the third normal form, the intention is to avoid to above anomalies. This is done by ensuring that for all non-trivial FDs implied by the relation, determine the FDs that involve a candidate keys.

The relation GRADE of above Table is not in BCNF because the dependencies

Student#$\rightarrow$ Name and Name $\rightarrow$ Student#

are non-trivial and their determinants are not super keys of GRADE.

Normal forms based on Primary keys and corresponding Normalization

Normal Form	*Test*	*Remedy (Normalization)*
First (1NF)	Relation should have no non-atomic attributes or nested relations.	Form new relations for each non-atomic attribute or nested relation
Second (2NF)	For relations where primary key contains multiple attributes, no non-key attribute should be functionally dependent on a part of the primary key.	Decompose and set up a new relation for each partial key with its dependent attribute(s). Make sure to keep a relation with the the original primary key and any attributes that are fully functionally dependent on it.
Third (3NF)	Relation should not have a non-key attribute functionally determined by non-key attributes. That is, there should be no transitive dependency of a non-key attributes on the primary key.	Decompose and set up a relation that includes the non-key attribute(s) that functionally determine(s) other non-key attribute(s).

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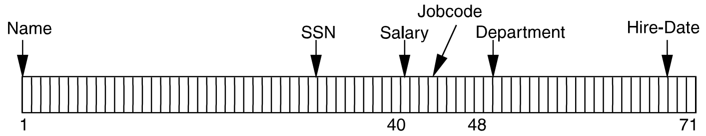
FILE STRUCTURES

A file is a *sequence* of records. In many cases, all records in a file are of the same record type.

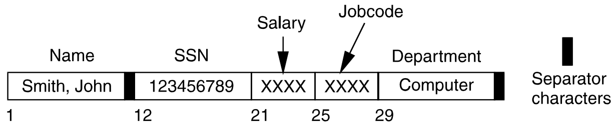
If every record in the file has exactly the same size (in bytes), the file is said to be made of *fixed-length records.*

If different records in the file have different sizes, the file is said to be made up of *variable-length records.*

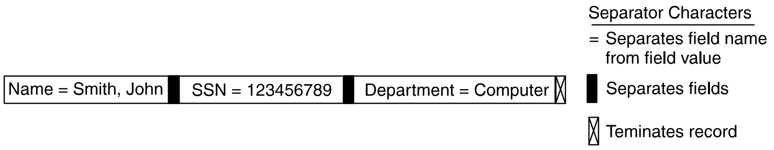
Fixed-length EMPLOYEE records in Fig. *(a)* have a record size of 71 bytes. Every record has the same fields, and field lengths are fixed, so the system can identify the starting byte position of each field relative to the starting position of the record.



(a) A fixed-length record with six fields and size of 71 bytes.



(b) A record with two variable-length fields and three fixed-length fields.



(c) A variable field record with three types of separator characters.

Fig. Three record storage formats.

For *variable-length fields*, each record has a value for each field, but we do not know the exact length of some field values. To determine the bytes within a particular record that represent each field, we can use special *separator* characters (such as ? or % $)—which do not appear in any field value—to terminate variable-length fields or we can store the length in bytes of the field in the record, preceding the field value.

SEQUENTIAL FILE ORGANIZATION

Sequential file. It is designed for efficient processing of records in **sorted order** on some **search key**.

- Records are chained together by pointers to permit fast retrieval in search key order.
- Pointer points to next record in order.
- Records are stored physically in search key order (or as close to this as possible).
- This minimizes number of block accesses.

Physical sequential order. It is difficult to maintain as records are inserted and deleted.

- Deletion can be managed with the pointer chains.
- Insertion poses problems if no space where new record should go.
- If space, use it, else put new record in an **overflow block**.
- Adjust pointers accordingly.
- If very few records in overflow blocks, this will work well.
- If order is lost, reorganize the file.
- Reorganizations are expensive and done when system load is low.

Note : If insertions rarely occur, we could keep the file in physically sorted order and reorganize when insertion occurs. In this case, the pointer fields are no longer required.

INDEXING TECHNIQUES.

Indexes are required to represent the sequence of stored record occurrences within their stored file.

Types of Indexing Techniques

1. **Nondense indexing :** File being indexed is divided into groups, with several record occurrences in each group, such that following conditions hold:

   (*i*) For any two groups, all the stored record occurrences in one precede all those in the other (with respect to the sequencing being imposed on the file).

   (*ii*) Within any one group, the file sequence is represented by physical contiguity.

   Index contains one entry per group, giving highest value of the indexed field occurring in the group and a pointer to the start of the group. The term "non-dense" refers to the fact that the index does not contain an entry for every stored record occurrence in the indexed file.

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2. Multilevel indexing : A multilevel (tree structure) index can contain any number of levels, each of which acts as a non-dense index to the level below. The top level contain a single entry.

B-trees : It is a particular form of multilevel or tree structure index. B-tree actually denotes the index set. The nodes of a B-tree do not normally contain the same number of value entries. They normally do contain a certain amount of free space. In general, a B-tree of order n has atleast n but not more than $2n$ value entries at any given node (and, if it has K value entries, then it also has $K + n$ pointers).

Advantage : B-tree insertion/deletion algorithm guarantees that the tree will remain balanced.

3. Indexing on field combinations : It is possible to construct an index on the basis of the values of two or more fields in combination. In general, an index on the combination of n fields F_1, F_2, ...F_n (in that order) will also serve. As an index on F_1, as an index on the combination F_1 F_2 (or F_2 F_1), as an index on the combination F_1 F_2 F_3 (in any order) and so on.

4. Selectivity in the index : It is not necessary to provide access via the index to every record occurrence in the indexed file. In some situations, it may be useful to have index entries for selected values only of the indexed field. *e.g.* in an employee file with 95% employees having status exempt and 5% non-exempt, it would be very useful to have an index pointing to all non-exempt employees.

5. Symbolic pointer representation : SRA-valued pointers can be replaced by the corresponding primary key values. This value can then be used to locate the corresponding record. An index using symbolic keys will clearly not need to be updated just because the indexed file has been reorganized. However, access via that index will be slower than access via an index that uses direct pointers.

CITY file

CITY	**S#**
Londons	S5
Paris	S1
	S4
Athens	S2
	S3

SUPPLIER file

S#	**SNAME**	**STATUS**
S1	Smith	20
S2	Jones	10
S3	Blake	30
S4	Clark	20
S5	Adams	30

Fig. Indexing on city (symbolic pointers)

B-TREES

A B-tree of order p, when used as an access structure on a *key field* to search for records in a data file, can be defined as follows :

(1) Each internal node in the B-tree Figure *(a)* is of the form

$$<P_1, <K_1, Pr_1>, P_2, <K_2, Pr_2>, ..., <K_{q-1}\ Pr_{q-1}>, P_q>$$

where $q \le p$.

Each P_i is a *tree pointer*–a pointer to another node in the B-tree.

Each Pr_i is a data pointer—a pointer to the record whose search key field value is equal to K_i (or to the data file block containing that record).

(2) With in each node, $K_1 < K_2 < ..... < K_{q-1}$.

(3) For all search key field values X in the subtree pointed at by P_i [i^{th} subtree, see Fig. *(a)*], we have

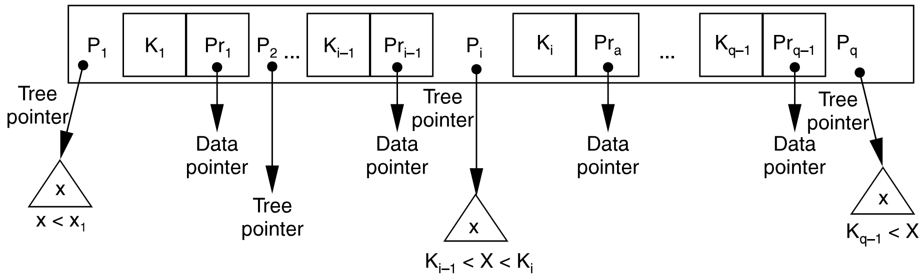
$$K_{i-1} < X < K_i \text{ for } 1 < i < q;$$
$$X < K_i \text{ for } i = 1;$$
and
$$K_{i-1} < X \text{ for } i = q.$$

(4) Each node has at most p tree pointers.

(5) Each node, except the root and leaf nodes, has at least $(p/2)$ tree pointers. The root node has at least two three pointers unless it is the only node in the tree.

(6) A node with q tree pointers, $q \le p$, has $q - 1$ search key field values (and hence has $q - 1$ data pointers).

(7) All leaf nodes are at the same level. Leaf nodes have the same structure as internal nodes except that all of their *tree pointers* P_i are null.



(a) A node in a B-tree with q – 1 search values

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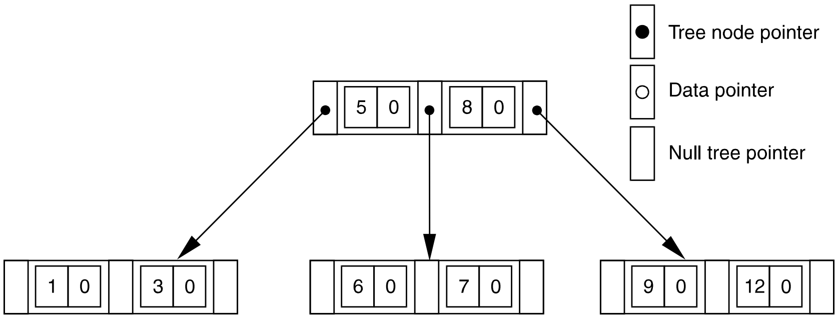

(b) A B-tree of order p = 3. The values were inserted in the order 8, 5, 1, 7, 3, 12, 9, 6

Fig. B-tree structures

Fig. (*b*) illustrates a B-tree of order $p = 3$. All search values K in the B-tree are unique because we assumed that the tree is used as an access structure on a key field. If we use a B-tree *on a nonkey field*, we must change the definition of the file pointers Pr_i to point to a block—or cluster of blocks—that contain the pointers to the file records.

B-tree starts with a single root node (which is also a leaf node) at level 0 (zero). Once the root node is full with $p - 1$ search key values and we attempt to insert another entry in tree, the root node splits into two nodes at level 1. Only the middle value is kept in the root node, and the rest of the values are split evenly between the other two nodes. When a nonroot node is full and a new entry is inserted into it, that node is split into two nodes at the same level, and the middle entry is moved to the parent node along with two pointers to the new split nodes. If the parent node is full, it is also split. Splitting can propagate all the way to the root node, creating a new level if the root is split.

If deletion of a value causes a node to be less than half full, it is combined with its neighbouring nodes, and this can also propagate all the way to the root. Hence, deletion can reduce the number of tree levels. It has been shown by analysis and simulation that, after numerous random insertions and deletions on a B-tree, the nodes are approximately 69 percent full when the number of values in the tree stabilizes. This is also true of B^+-trees. If this happens, node splitting and combining will occur only rarely, so insertion and deletion become quite efficient. If the number of values grow, the tree will expand without a problem—although splitting of nodes may occur, so some insertion will take more time.

B-trees are sometimes used as primary file organizations. In this case, whole records are stored within the B-tree nodes rather than just the <search key, record pointer> entries. This works well for files with a relatively *small number of records,* and a *small record size.* Otherwise, the fan-out and the number of levels become too great to permit efficient access.

B-trees provide a multilevel access structure that is a balanced tree structure in which each node is at least half full. Each node in a B-tree of order p can have at most $p - 1$ search values.

Advantages:

(1) Lack of redundant storage (but only marginally different).

(2) Some searches are faster (key may be in non-leaf node).

Disadvantages:

(1) Leaf and non-leaf nodes are of different size (complicates storage)

(2) Deletion may occur in a non-leaf node (more complicated)

Note: Generally, the structural simplicity of B^+ -tree is preferred.

B + -Tree Index Files

Most implementations of a dynamic multilevel index use a variation of the B-tree data structure called B^+-tree.In a B-tree, every value of the search field appears once at some level in the tree along with a data pointer.

In a B^+-tree, data pointers are stored *only at the leaf nodes* of the tree; hence, structure of leaf nodes differ from the structure of internal nodes. Leaf nodes have an entry for *every* value of the search field, along with a data pointer to the record (or to the block that contains this record) if search field is a key field. For a non-key search field, the pointer points to a block containing pointers to the data file records, creating an extra level of indirection.

Leaf nodes of the B^+-tree are usually linked together to provide ordered access on the search field to the record. These leaf nodes are similar to the first (base) level of an index. Internal nodes of the B^+-tree correspond to the other levels of a multilevel index. Some search field values from the leaf nodes are *repeated* in the internal nodes of the B^+-tree to guide the search.

- Primary disadvantage of index-sequential file organization is that performance degrades as the file grows. This can be remedied by costly re-organizations.
- B^+ -tree file structure maintains its efficiency despite frequent insertions and deletions. It imposes some acceptable update and space overheads.
- A B^+ -tree index is a *balanced tree* in which every path from the root to a leaf is of the same length.
- Each nonleaf node in the tree must have between $[n/2]$ and n children, where n is fixed for a particular tree.

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Structure of the internal nodes of a B⁺-tree of order p [Fig. (*a*)]

(1) Each internal node is of the form

$$<P_1, K_1\ P_2\ K_2, \ldots\ldots, P_{q-1}, K_{q-1}, P_q>$$

where $q \le p$ and each P_i is a tree pointer.

(2) Within each internal node, $K_1 < K_2 < \ldots\ldots < K_{q-1}$.

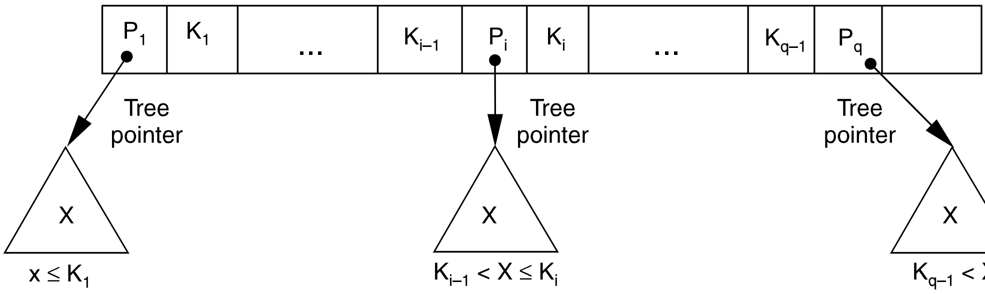
(3) For all search field values X in the subtree pointed at by P_i, we have

$K_{i-1} < X \le K_i$ for $1 < i < q$; $X \le K_i$ for $i = 1$; and $K_{i-1} < X$ for $i = q$ (see Figure).

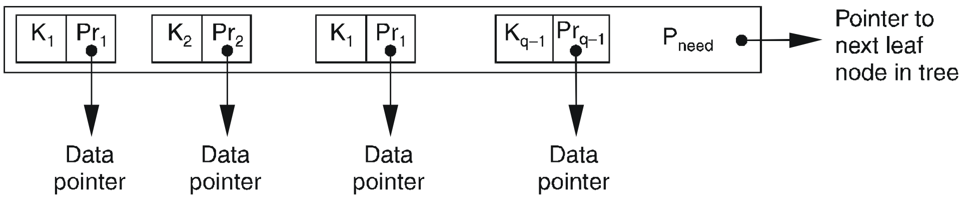
(4) Each internal node has at most p tree pointers.

(5) Each internal node, except the root, has at least $[(p/2)]$ tree pointers. The root node has at least two tree pointers if it is an internal node.

(6) An internal node with q pointers, $q \le p$, has $q - 1$ search field values.



(a) Internal node of a B⁺-tree with q – 1 search values.



(b) Leaf node of a B⁺-tree with q – 1 search values and q – 1 data pointers.

Fig. Nodes of a B⁺-tree

Structure of the leaf nodes of a B⁺-tree of order p

(1) Each leaf is of the form

$$<< K_1, Pr_1>, <K_2, Pr_2>, \ldots\ldots, <K_{q-1}, Pr_{q-1}>, P_{next}>$$

where $q \le p$, each Pr_i is a data pointer, and P_{next} points to the next *leaf node* of the B⁺-tree.

(2) Within each leaf node, $K_1 < K_2 < \ldots. < K_{q-1}$, $q \le p$.

(3) Each Pr_i is a data pointer that points to the record whose search field value is K_i or to a file block containing the record (or to a block of record pointers that point to records whose search field value is K_i if the search field is not a key).

(4) Each leaf node has at least $(p/2)$ values.

(5) All leaf nodes are at the same level.

The pointers in internal nodes are *tree pointers* to blocks that are tree nodes, whereas the pointers in leaf nodes are *data pointer* to the data file records or blocks—except for the P_{next} pointer, which is a tree pointer to the next leaf node. By starting at the leftmost leaf node, it is possible to traverse leaf nodes as a linked list using the P_{next} pointers. This provides ordered access to the data records on the indexing field. A $P_{previous}$ pointer can also be included. For a B⁺-tree on a nonkey field, so the Pr pointers are block pointers to blocks that contain a set of record pointers to the actual records in the data file.

Because entries in the *internal nodes* of a B⁺-tree include search values and tree pointers without any data pointers, more entries can be packed into an internal node of a B⁺-tree than for a similar B-tree. Thus for the same block (node) size, the order p will be larger for B⁺-tree than for the B-tree. This can lead to fewer B⁺-tree levels, improving search time. Because structures for internal and for leaf nodes of a B⁺-tree are different, the order p can be different. We will use p to denote the order for *internal nodes* and P_{leaf} to denote the order for *leaf nodes*, which we define as being the maximum number of data pointers in a leaf node.

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STRUCTURED QUERY LANGUAGE (SQL)

INTRODUCTION ABOUT SQL

(1) Structured Query Language (SQL) is the set of statements with which all programs and users access data in an Oracle database.

- Structured Query Language.
- A declarative query language.
- Its prototype, SEQUEL, was developed at IBM Research.
- Consists of DDL, DML and DCL.
- Basic elements -- statements/commands.
- Many other functions: defining views, indexes, embedded in a host language, etc.
- Available in almost all commercial DBMSs.
- SQL is case insensitive.

SQL Workings:

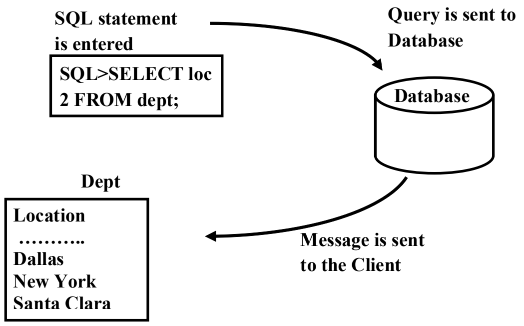

Fig. Working of SQL

- SQL is not Case Sensitive.
- Keywords cannot be split or abbreviated.
- SQL Statements can be split across lines.
- Clauses are placed in different lines, to promote readability.

Domain Types in SQL

- char(n): Fixed length character string, with user-specified length n.
- varchar(n): Variable length character strings, with user-specified maximum length n.
- int. Integer (a finite subset of the integers that is machine-dependent).
- smallint. Small integer (a machine-dependent subset of the integer domain type).
- numeric(p,d): Fixed point number, with user-specified precision of p digits, with n digits to the right of decimal point.
- real, double precision: Floating point and double-precision floating point numbers, with machine-dependent precision.
- float(n): Floating point number, with user-specified precision of at least n digits.

(2) DDL is Data Definition Language statements

Some examples of Keywords and their meaning:

- CREATE - to create new objects in the database
- ALTER - alters the structure of the database
- DROP - delete objects from the database
- TRUNCATE - remove all records from a table, including all spaces allocated for the records are removed
- COMMENT - add comments to the data dictionary
- GRANT - gives user's access privileges to database
- REVOKE - withdraw access privileges given with the GRANT command

Example : An SQL relation is defined using the create table command:

Create table r $(A_1\ D_1, A_2\ D_2, ..., A_n\ D_n)$
(integrity-constraint$_1$),
…,
(integrity-constraint$_k$))

* r is the name of the relation
* each A_i is an attribute name in the schema of relation r
* D_i is the data type of values in the domain of attribute A_i

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Example:

Create table instructor (
ID char(5),
name varchar(20) not null
dept_name varchar(20),
salary numeric(8,2))

- insert into instructor values ('10211', 'Smith', 'Biology', 66000);
- insert into instructor values ('10211', null, 'Biology', 66000);

Drop and Alter Table Constructs:

- **To Alter table:** The ALTER TABLE statement is used to add, delete, or modify column in an existing table.
- **To add column in a table –**
  ALTER TABLE table-name ADD column-name data type
- **To delete a column in a table –**
  ALTER TABLE table-name DROP COLUMN column-name
- **To change the data type of a column in a table –**
  ALTER TABLE table-name ALTER COLUMN column-name data type
- alter table r add A D
  * where A is the name of the attribute to be added to relation r and D is the domain of A.
  * All tuples in the relation are assigned null as the value for the new attribute.
- alter table r drop A
  * where A is the name of an attribute of relation r
  * Dropping of attributes not supported by many databases

DML is Data Manipulation Language statements. Some examples:

- SELECT - retrieve data from the a database
- INSERT - insert data into a table
- UPDATE - updates existing data within a table
- DELETE - deletes all records from a table, the space for the records remain
- CALL - call a PL/SQL or Java subprogram
- LOCK TABLE - control concurrency

INSERT statement allow user to insert a single record or multiple records into a table

INSERT INTO table
(Column-1, Column-2, . . . Column-n)
(Value-1, Value-2, ... Value-n);

DELETE statement is used to delete rows in a table.

DELETE FROM table-name
WHERE some-column = some-value

UPDATE statement is used to update existing records in a table.

UPDATE table-name
SET Column 1 = Value, column 2 = Value 2, - - -
WHERE some-column = some- value

DCL is Data Control Language statements. Some examples:

- COMMIT - save work done
- SAVEPOINT - identify a point in a transaction to which you can later roll back
- ROLLBACK - restore database to original since the last COMMIT
- SET TRANSACTION - Change transaction options like what rollback segment to use

The select Clause

- The **select** clause list the attributes desired in the result of a query
  * corresponds to the projection operation of the relational algebra
- Example: find the names of all instructors
  Select names
  from *instructor*
- **NOTE:** SQL names are case insensitive (i.e., you may use upper- or lower-case letters.)
  E.g. *Name* ≡ *NAME* ≡ *name*
  Some people use upper case wherever we use bold font.
- SQL allows duplicates in relations as well as in query results.

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- To force the elimination of duplicates, insert the keyword **distinct** after select.
- Find the names of all departments with instructor, and remove duplicates
  select distinct *dept_name*
  from *instructor*
- *The keyword **all** specifies that duplicates not be removed.*
  select all *dept_name*
  from *instructor*
- *An asterisk in the select clause denotes "all attributes"*

  select *
  from *instructor*
- *The **select** clause can contain arithmetic expressions involving the operation, +, –, *, and /, and operating on constants or attributes of tuples.*
- *The query:*

  select *ID, name, salary / 12*
  from *instructor*

would return a relation that is the same as the instructor relation, except that the value of the attribute salary is divided by 12.

The where Clause

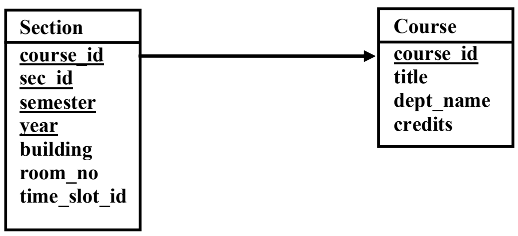
- *The **where** clause specifies conditions that the result must satisfy*
  * *Corresponds to the selection predicate of the relational algebra.*
- *To find all instructors in Comp. Sci. dept with salary > 80000* ***select*** *name*
  from *instructor*
  where *dept_name = 'Comp. Sci.'* ***and*** *salary > 80000*
- *Comparison results can be combined using the logical connectives **and**, **or**, and **not**.*
  Comparisons can be applied to results of arithmetic expressions

The from Clause

- The **from** clause lists the relations involved in the query
  * Corresponds to the Cartesian product operation of the relational algebra.
- Find the Cartesian product *instructor X teaches*
  Select *
  from *instructor, teaches*
  * generates every possible instructor – teaches pair, with all attributes from both relations
- Cartesian product not very useful directly, but useful combined with where-clause condition (selection operation in relational algebra)

(3) Joins

- For all instructors who have taught some course, find their names and the course ID of the courses they taught.
  Select *name, course.id*
  from *instructor, teaches*
  where *instructor.ID = teaches.ID*
- Find the course ID, semester, year and title of each course offered by the Comp. Sci. department
  Select *section. course.id semester, year, title*
  from *section, course*
  where *section.course.id = course.course.id* **and**
  dept_name = 'Comp. Sci.'



The Rename Operation

- The SQL allows renaming relations and attributes using the **as** clause:
  old-name ***as*** *new-name*

Example:

select *ID, name, salary / 12* **as** *monthly_salary*
from *instructor*

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- Find the names of all instructors who have a higher salary than some instructor in 'Comp. Sci'.

  select distinct *T. name*
  from *instructor* **as** *T, instructor* **as** *S*
  where *T.salary* > *S.salary* **and** *S.dept_name* = *'Comp. Sci.'*
- Keyword **as** is optional and may be omitted

  instructor **as** *T* = *instructor T*

SELECT [DISTINCT | ALL] {* | [column expression (AS newname)] [, ...]}
FROM table-name [alias] [,...]- Specifies the table or tables to be used
[WHERE condition] - Filters the rows subject to some condition
[GROUP BY column list] -Forms groups of rows with the same column value
[HAVING condition] - Filters the groups subject to some condition.
[ORDER BY column list]- Specifies the order of the output.

Some table Example:

Table 1 : Student table

Roll No	Stu_name	Age	Sex	Branch	Year	Marks	Phone
101	Sree	17	M	CSE	1	300	233607
102	Kavitha	19	F	ECE	2	345	
103	Gopi	22	M	IT	3	450	222468
104	Haritha	18	F	EEE	2	390	222230
105	Rajesh	22	M	CSE	4	430	
106	Keerthana	17	F	IT	1	440	454678
107	Raghav	17	M	CSE	1	350	
108	Abhilash	21	M	EEE	3	200	342567
109	Pramoda	19	F	EIE	2	460	
110	Chiranjeevi	21	M	IT	3	465	

Table 2 : Library table

Roll No	Book_No	Issue_Date	Return_Date	Fine
101	DB123	01-01-2005	10-01-2005	-
102	CD001	01-01-2005	08-01-2005	-
108	LP101	15-01-2005	05-02-2005	5
106	DB123	16-01-2005	17-01-2005	-
101	AI007	01-01-2005	13-01-2005	-
107	DS202	02-02-2005	10-02-2005	-
109	CO 128	05-01-2005	11-02-2005	-
108	CD101	06-01-2005	25-02-2005	5
103	CN202	16-02-2005	16-03-2005	15
103	DB201	16-02-2005	18-02-2005	-
109	OS245	01-01-2005	10-01-2005	-
101	TC501	20-02-2005	10-03-2005	35

Table 3 : CUSTOMERS

cid	cname	city	discnt
c001	Sree	Bombay	10.00
c002	Ram	Delhi	12.00
c003	Kumar	Delhi	8.00
c004	Murali	Bombay	8.00
c006	Murali	Chennai	0.00

Table 4 : AGENTS

aid	aname	city	percent
a01	Rai	Calcutta	6
a02	Gopal	Nagpur	6
a03	Balaji	Hyderabad	7
a04	Abdul	Calcutta	6
a05	Raju	Bombay	5
a06	Satheesh	Delhi	5

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Table 5 : PRODUCTS

Pid_	Pname	city	quantity	price
p0l	comb	Delhi	111400	0.50
p02	brush	Nagpur	203000	0.50
p03	razor	Bombay	150600	1.00
p04	pen	Bombay	125300	1.00
p05	pencil	Delhi	221400	1.00
p06	folder	Delhi	123100	2.00
p07	case	Nagpur	100500	1.00

Table 6 : ORDER

Ordno	month	cid	aid	pid	qty	dollars
1011	jan	c00l	a01	p01	1000	450.00
1012	jan	c00l	a01	p01	1000	450.00
1019	feb	c00l	a02	p02	400	180.00
1017	feb	c00l	a06	p03	600	540.00
1018	feb	c00l	a03	p04	600	540.00
1023	mar	c00l	a04	p05	500	450.00
1022	mar	c00l	a05	p06	400	720.00
1025	apr	c00l	a05	p07	800	720.00
1013	jan	c002	a03	p03	1000	880.00
1026	may	c002	a05	p03	800	704.00
1015	jan	c003	a03	p05	1200	1104.00
1014	jan	c003	a03	p05	1200	1104.00
1021	feb	c004	a06	p01	1000	460.00
1016	jan	c006	a01	p01	1000	500.00
1020	feb	c006	a03	p07	600	600.00
1024	mar	c006	a06	p01	800	400.00

Example

Employee *(**empID**, fName, IName, address, DOB, sex, position, deptNo)*
Department *(**deptNo**, deptName, mgrEmpID)*
Project *(**projNo**, projName, deptNo)*
WorksOn *(**empID**, **projNo**, hoursWorked)*

where Employee contains employee details and **empID** *is the key*

Department contains department details and deptNo **is** *the key.mgrEmpID identifies the employee who is the manager of the department. There is only one manager for each department.*

Project contains details of the projects in each department and the key is ***projNo*** *(no two departments can run the same project).*

And WorksOn contains details of the hours worked by employees on each project, and ***empID/projNo*** *form the key.*

(1) List all employees in alphabetical order of surname and within surname, first name.

```
SELECT *
FROM Employee
ORDER BY IName, fName:
```

(2) List all the details of employees who are female.

```
SELECT *
FROM Employee
WHERE sex = 'F';
```

(3) List the names and addresses of all employees who are Managers.

```
SELECT fName, IName, address
FROM Employee
WHERE position = 'Manager';
```

Or

```
SELECT fName, IName, address
FROM Employee e, Department d
WHERE e.empID = d.mgrEmpID;
```

(4) Produce a list of the names and addresses of all employees who work for the 'IT' department.

```
SELECT e.lName, e.address
FROM Employee e, Department d
WHERE e.deptNo = d.deptNo AND d.deptName = 'IT';
```

(5) Produce a complete list of all managers who are due to retire this year, in alphabetical order of surname.

```
SELECT IName
FROM Employee e, Department d
WHERE e.empID = d.mgrEmpID AND
    date_part ('year',DOB) <date_part('year', DATE('2001-10-01') - 65;
```

(student does not need to know exact date functions - just general idea)

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(6) Find out how many employees are managed by 'James Adams'.

```
SELECT COUNT(*)
FROM Employees e1,e2, Department d
WHERE e1 .IName = 'Adams' AND e1 .fName = 'James' AND
      e1 .empID = d.mgrEmpID AND d.deptNo = e2.deptNo;
```

(7) Produce a report of the total hours worked by each employee, arranged in order of department number and within department, alphabetically by employee surname.

```
SELECT e.empID, e.IName, e.fName, e.deptNo,
SUM(w.hoursWorked)
FROM Employee e, Project p, WorksOn w
WHERE e.deptNo = p.deptNo AND e.empID = w.empID
ORDER BY e.deptNo, e.lName;
```

(8) For each project on which more than two employees worked, list the project number, project name and the number of employees who work on that project.

```
SELECT e.projNo, e.projName. COUNT(*)
FROM Project p, WorksOn w
WHERE p.projNo = w. projNo
GROUP BY e.projNo, e.projName
HAVING COUNT(*) >2
```

(9) List the total number of employees in each department for those departments with more than 10 employees. Create an appropriate heading for the columns of the results table.

```
SELECT deptNo AS departmentNumber, COUNT(emplD) AS totalEmployees
FROM Employee
GROUP BY deptNo, empID
HAVING COUNT(emplD) > 10
```

Example:

Publisher	*(**pubID**, pubName, street, city, postcode, telNo, creditCode)*
Book Job	*(**jobID**, pubID, jobDate, description, jobType)*
PurchaseOrder	*(**jobID, polD,** poDate)*
POItem	*(**jobID, polD, itemID,** quantity)*
Item	*(**itemID**, description, onHand, price)*

*where Publisher contains publisher details and **pubID** is the key.*

*BookJob contains details of the printing jobs (books or part books) and **jobID** is the key.*

*PurchaseOrder A printing job requires the use of materials, such as paper and ink, which are assigned to a job via purchase orders. This table contains details of the purchase orders for each job and the key is **joblD/polD**. Each printing job may have several purchase orders assigned to it. Each purchase order (PO) may contain several PO items. This table contains details of the PO items and **joblD/polD/itemID** form the key and Item contains details of the materials which appear in POItem, and the key is **itemID***

(1) List all publishers in alphabetical order of name.

```
SELECT pubName
FROM Publisher
ORDER BY pubName;
```

(2) List all printing jobs for the publisher 'Gold Press'.

```
SELECT jobID
FROM BookJob b, Publisher p
WHERE b.pubID = p.pubID AND pubName = 'Gold Press';
```

(3) List the names and phone numbers of all publisher who have a rush job (jobType = 'R').

```
SELECT pubName, telNo
FROM BookJob b, Publisher p
WHERE b.pubID = p.pubID AND jobType = 'R-;
```

(4) List the dates of all the purchase orders for the publisher 'Gold Press'.

```
SELECT polD, poDate
FROM PurchaseOrder po, BookJob b, Publisher p
WHERE po.jobID = b.jobID AND b.pubID = p.pubID AND pubName = 'Gold Press';
```

(5) How many publisher fall into each credit code category?

```
SELECT creditCode, COUNT(*)
FROM Publisher
GROUP BY creditCode;
```

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(6) List all job type's with at least three printing jobs.

```
SELECT jobType, COUNT(*)
FROM BookJob
GROUP BY jobType
HAVING COUNT(*) >= 3;
```

(7) List the average price of all items.

```
SELECT AVG(price)
FROM Item;
```

(8) List all items with a price below the average price of an item.

```
SELECT *
FROM Item
WHERE price < (SELECT AVG(price) FROM item);
```

Example:

Patient (***patientNo***, *patName, patAddr, DOB)*

Ward (***wardNo***, *wardName, wardType, noOfBeds)*

Contains (***patientNo, wardNo, admissionDate***)

Drug (***drugNo***, *drugName, costPerUnit)*

Prescribed (***patientNo, drugNo***, *unitsPerDay,* ***startDate***, *finishDate)*

(1) List all the patients' details, alphabetically by name.

```
SELECT *
FROM Patient
ORDER BY patName
```

(2) List all the patients contained in the 'Surgical' ward.

```
SELECT p.patientNo, p.patName
FROM Patient p, Ward w, Contains c
WHERE w.wardNo = c.wardNo AND c.patientNo = p.patientNo AND wardName = 'Surgical'
```

(3) List all the patients admitted today.

```
SELECT p.patientNo, p.patName
FROM Patient p, Contains c
WHERE c.patientNo = p.patientNo AND admissionDate = 'today'
```

(4) Find the names of all the patients being prescribed 'Morphine'.

```
SELECT p.patName
FROM Patient p, Prescribed pr, Drug d
WHERE pr.patientNo = p.patientNo & pr.drugNo = d.drugNo & drugName = 'Morphine'.
```

(5) What is the total cost of Morphine supplied to a patient called 'John Smith'?

```
SELECT SUM(((finishDate - startDate) * unitsPerDay) * costPerUnit)
AS totalCost
FROM Patient p, Prescribed pr, Drug d
WHERE pr.patientNo = p.patientNo AND pr.drugNo = d.drugNo AND drugName = 'Morphine' AND
patName = 'John Smith'
```

Example: What is the maximum, minimum and average number of beds in a ward? Create appropriate column headings for the results table.

```
SELECT MAX(noOfBeds) AS Maximum, MIN(noOfBeds) AS Minimum, AVG(noOfBeds) AS Average FROM Ward
```

Example: For each ward that admitted more than 10 patients today, list the ward number, ward type and number of beds in each ward.

```
SELECT w.wardNo, wardType, noOfBeds
FROM Patient p, Ward w, Contains c
WHERE w.wardNo = c.wardNo AND c.patientNo = p.patientNo AND admissionDate = 'today' GROUP BY wardNo,
wardType, noOfBeds
HAVING COUNT(*) > 10
```

Example: List the numbers and names of all patients and the drugNo and number of units of their medication. The list should also include the details of patients that are not prescribed medication.

```
SELECT *
FROM Patient p Left Join Prescribed pr ON pr.patientNo = p.patientNo
```

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FILE STRUCTURES

SYSTEM STRUCTURE

Database systems are partitioned into modules for different functions. Some functions (e.g. file systems) may be provided by the operating system.

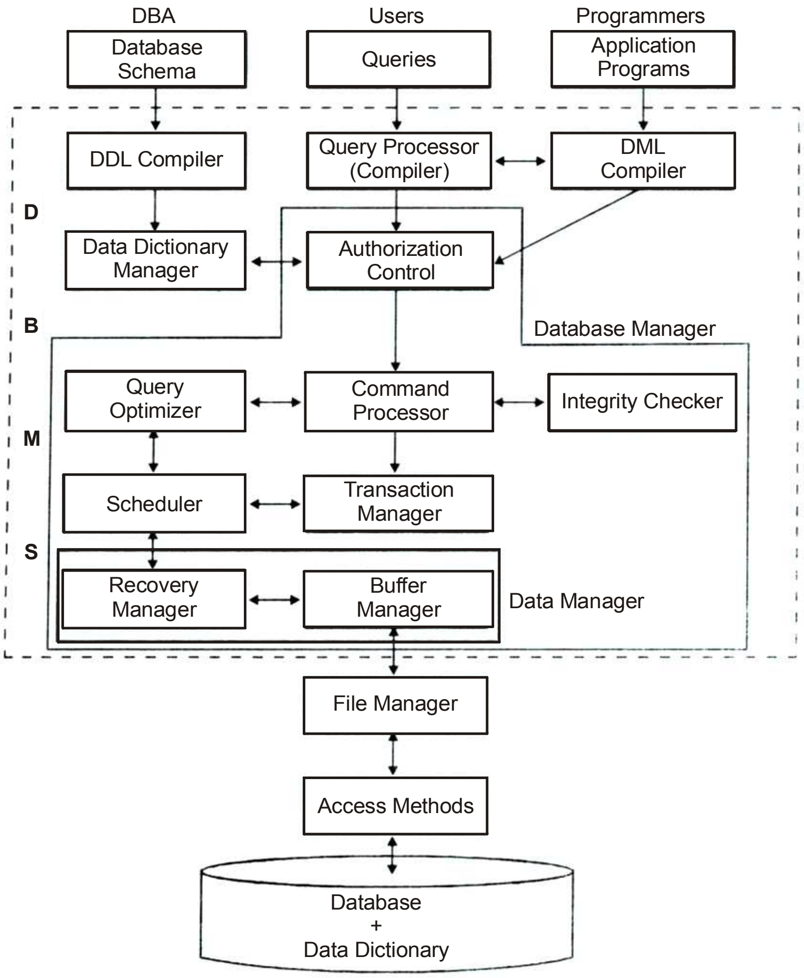

Fig. Database system structure showing components

Components of System Structure

(1) File manager : It manages allocation of disk space and data structures used to represent information on disk.

(2) Database manager : The interface between low-level data and application programs and queries.

(3) Query processor : It translates statements in a query language into low-level instructions the database manager understands. It may also attempt to find an equivalent but more efficient form.

(4) DML precompiler : It converts DML statements embedded in an application program to normal procedure calls in a host language. The precompiler interacts with the query processor.

(5) DDL compiler : It converts DDL statements to a set of tables containing metadata stored in a data dictionary.

Data structures required for physical system implementation:

(1) Data files : It store the database itself.

(2) Data dictionary : It stores information about the structure of the database. It is used **heavily**. Great emphasis should be placed on developing a good design and efficient implementation of the dictionary.

(3) Indices : It provide fast access to data items holding particular values.

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ENTITY RELATIONSHIP MODEL

- It is a design tool.
- It is a graphical representation of the database system.
- It provides a high-level conceptual data model.
- It supports user's perception of the data.
- It is DBMS and hardware independent.
- It had many variants.
- It is composed of entities, attributes, and relationships.

Entities

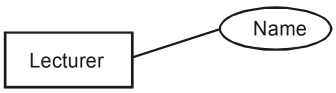
- It is any object in the system that we want to model and store information about.
- Individual objects are called *entities*.
  Groups of the same type of objects are called *entity types* or *entity sets*.
  Entities are represented by rectangles (either with round or square corners)

Chen's notation other notation

- *There are two types of entities :*
  (*i*) Weak entity types
  (*ii*) Strong entity types

Attribute

- All the data relating to an entity is held in its attributes. An attribute is a property of an entity. Each attribute can have any value from its domain.
- *Each entity within an entity type :*
  (*i*) may have any number of attributes.
  (*ii*) can have different attribute values than that in any other entity.
  (*iii*) have the same number of attributes.
- *Attributes can be*
  (*i*) simple or composite
  (*ii*) single-valued or multi-valued
- Attributes can be shown on ER models
- They appear inside ovals and are attached to their entity.



- Entity types can have a large number of attributes. If all are shown, then the diagrams would be confusing. Only show an attribute if it adds information to the ER diagram, or clarifies a point.

Keys

It is a data item that allows to uniquely identify individual occurrences or an entity type.

Types of keys

(1) Super key.

For an entity, it is a set of one or more attributes whose combined value uniquely identifies the entities in the entity set.

e.g. for an entity set Employees, the set of attributes (emp_name, address) can be considered to be a super key, if we assume that there are no two employees with the same name emp_name and same address.

(2) Primary key.

The primary key of a relation can be said to be a minimal super key.

The field or group of fields which forms the unique identifier for a table is called the table's primary key. The primary key uniquely identifies each record in the table and must never be the same for two records.

e.g. emp_code can be primary key for the entity set Employees.

The primary key should be chosen such that its attributes are never or very rarely changed. For instance, the address field of a person should not be part of the primary key, since it is likely to change. Emp_code, on the other thand, is guaranteed to never change, till he is in the organization.

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(3) Candidate key.

There is only one primay key in a table. But there can be multiple candidate keys. A *candidate key* is an attribute or set of attributes that uniquely identifies a record. These attributes or combinations of attributes are called *candidate key*. In such a case, one of the candidate key is chosen to be a primary key. The remaining candidate keys are called *alternate keys.*

(4) Composite key.

In many cases, as we design a database, we will have tables that will use more than one column as part of the primary key. These are called *composite keys* or *concatenated keys*. In other words, when a record cannot be uniquely identified by a single field, in such cases a composite key is used. A composite key is a group of fields that uniquely identify a record.

(5) Secondary key.

It is an attribute or combination of attributes that many not be a candidate key but classifies the entity set on a particular characteristic.

e.g. the entity set on a particular characteristic. For example, the entity set EMPLOYEE having the attribute *Department,* which identifies by its value which means all instances of EMPLOYEE who belong to a given department.

More than one employee may belong to a department, so the *Department* attribute is not a candidate key for the entity set EMPLOYEE, since it cannot uniquely identify an individual employee. However, the *Department* attribute does identify all employees belonging to a given *department.*

(6) Foreign key.

In a relation, column whose data values correspond to the values of a key column in another relation is called *foreign key. In a relational database, the foreign key of a relation would be the primary key of another relation.*

Relationships

Relationship type. It is a meaningful association between entity types.

It is an association of entities where the association includes one entity from each participating entity type.

Relationship types are represented on the ER diagram by a series of lines.

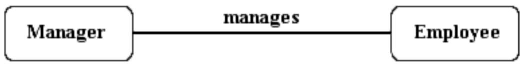
Notations.

Chen notation: In this, the relationship is placed inside a diamond,

e.g. managers manage employees:

Fig. Chens notation for relationships

For this module, alternative notation can be used, where relationship is a label on the line. Meaning is identical.



Degree of a Relationship.

The number of participating entities in a relationship is called *degree of relationship.*

It is also possible to have entities associated through two or more distinct relationships.

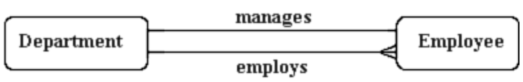
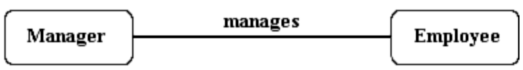


Fig. Multiple relationships

Binary relationship type.

If there are two entity types involved, it is a *binary relationship type*



Ternary relationship type

If there are three entity types involved, it is *ternary relationship type*

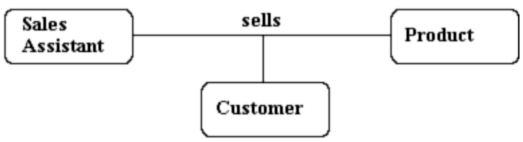


Fig. Ternary relationship

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Quaternary or Unary relationship type.

It is possible to have a n-ary relationship (e.g. quaternary or unary).

It is a relationship where the same entity participates more than once in different roles.

Unary relationships are also called a *recursive* relationship.

Examples.

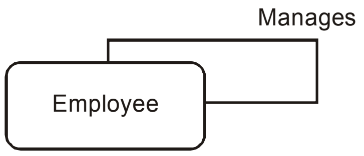


Fig. Recursive relationship

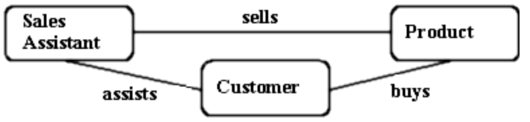
In this example, it is said that employees are managed by employees.

If more information about who manages whom, is needed a second entity type called *manager* is introduced.

Replacing ternary relationships

When ternary relationships occurs in an ER model, they should always be removed before finishing the model. Sometimes the relationships can be replaced by a series of binary relationships that link pairs of the original ternary relationship.

e.g.



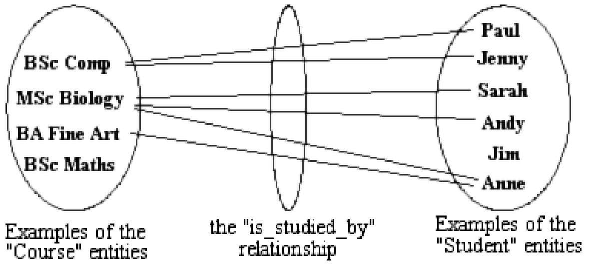
This can result in the loss of some information - It is no longer clear which sales assistant sold a customer a particular product.

Try replacing the ternary relationship with an entity type and a set of binary relationships.

Entity Sets

Sometimes it is useful to try out various examples of entities from an ER model to confirm the correct cardinality and optionality of a relationship. 'Entity set diagram' is used to show entity examples graphically.

Consider the example of 'course is_studied_by student'.



Confirming Correctness

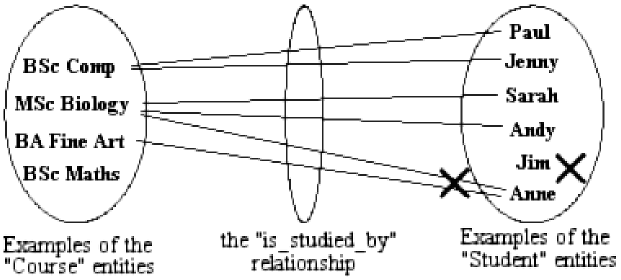


Fig. Entity set confirming errors

Use the diagram to show all possible relationship scenarios.

Go back to the requirements specification and check to see if they are allowed. If not, then put a cross through the forbidden relationships. This allows to show the cardinality and optionality of the relationship.

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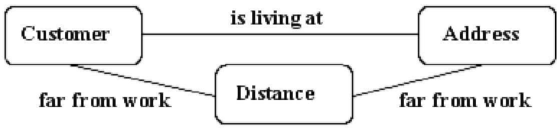
Redundant relationships

Some ER diagrams end up with a relationship loop.

Check to see if it is possible to break the loop without losing info.

Given three entities A, B, C, where there are relations A-B, B-C, and C-A, check if it is possible to navigate between A and C via B. If it is possible, then A-C was a *redundant relationship*.



Note : Always check carefully for ways to simplify your ER diagram. It makes it easier to read the remaining information.

Splitting n:m Relationships

A many to many relationship in an ER model is not necessarily incorrect. They can be replaced using an intermediate entity.

This should only be done where:

(*i*) m:n relationship hides an entity

(*ii*) resulting ER diagram is easier to understand.

e.g. Consider the case of a car hire company. Customers hire cars, one customer hires many card and a car is hired by many customers.

Fig. Many to Many example

The *many to many relationship* can be broken down to reveal a '*hire*' entity, which contains an attribute '*date of hire*'.

Fig. Splitting the Many to Many example

Construction of an ER model

Before start drawing ER model, read requirements specification carefully and document assumptions needed.

1. **Identify entities.** List all potential entity types. These are the object of interest in the system. It is better to put too many entities in at this stage and them discard them later if necessary.
2. **Remove duplicate entities.** Ensure that they really separate entity types or just two names for the same thing.

   Also do not include the system as an entity type
   e.g. if modelling a library, the entity types might be books, borrowers, etc.

   Note. Library is the system, thus should not be an entity type.
3. **List attributes of each entity** (all properties to describe entity which are relevant to the application).

   Ensure that the entity types are really needed. If any of them just attributes of another entity type, then keep them as attributes and cross them off the entity list.

   Do not have attributes of one entity as attributes of another entity.
4. **Mark the primary keys.**
   Which attributes uniquely identify instances of that entity type?

   This may not be possible for some weak entities.
5. **Define the relationships.**
   Examine each entity type to see its relationship to the others.
6. **Describe cardinality and optionality of the relationships.**
   Examine the constraints between participating entities.
7. **Remove redundant relationships.**
   Examine ER model for redundant relationships.

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TRANSACTION AND CONCURRENCY CONTROL

TRANSACTION AND CONCURRENCY CONTROL

TRANSACTIONS.

The goal in a 'concurrent' DBMS is to allow multiple users to access the database simultaneously without interfering with each other.

To control data access, first a concept is needed to allow us to encapsulate database accesses. Such encapsulation is called a '*Transaction*'.

It is a unit of logical work and recovery.

Transaction (ACID)

A - Atomicity (for integrity)

C - Consistency preservation

I - Isolation

D - durability

Available in SQL

Some applications require nested or long transactions.

Outcomes

After work is performed in a transaction, two outcomes are possible:

(1) Commit. Any changes made during the transaction by this transaction are committed to the database.

(2) Abort. All the changes made during the transaction by this transaction are not made to the database. The result of this is as if the transaction was never started.

Transaction Schedules

It is a tabular representation of how a set of transactions were executed over time. This is useful when examining problem scenarios.

Within the diagrams following nomenclatures are used:

(1) READ(*a*) : This is a read action on an attribute or data item called '*a*'.

(2) WRITE(*x*, *a*) : This is a write action on an attribute or data item called '*a*', where value '*x*' is written into '*a*'.

(3) *tn* (*e.g. t*1, *t*2, *t*10) : This indicates the time at which something occurred. The units are not important, but t*n* always occurs before *tn + 1*.

CONCURRENCY CONTROL.

In information technology and computer science, especially in the fields of computer programming, operating systems, multiprocessors, and databases, concurrency control ensures that correct results for concurrent operations are generated, while getting those results as quickly as possible.

Concurrency control in Database management systems ensures that *database transactions* are performed concurrently without violating data integrity of the respective databases. Thus concurrency control is an essential element for correctness in any system where two database transactions or more, executed with time overlap, can access the same data, e.g., virtually in any general-purpose database system.

Concurrency Control Mechanism

(1) Optimistic.

Delay the checking of whether a transaction meets the isolation and other integrity rules until its end, without blocking any of its (read, write) operations ("...and be optimistic about the rules being met..."), and then abort a transaction to prevent the violation, if desired rules are to be violated upon its commit. An aborted transaction is immediately restarted and re-executed, which incurs an obvious overhead (versus executing it to the end only once). If not too many transactions are aborted, then being optimistic is usually a good strategy.

(2) Pessimistic.

Block an operation of a transaction, if it may cause violation of the rules, until the possibility of violation disappears. Blocking operations is typically involved with performance reduction.

(3) Semi-optimistic.

Block operations in some situations, if they may cause violation of some rules, and do not block in other situations while delaying rules checking (if needed) to transaction's end, as done with optimistic.

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EXERCISE – I

MCQ TYPE QUESTIONS

1. Consider Join of a relation R with a relation S. If R has m tuples and S has n tuples, then maximum and minimum sizes of the Join respectively are

(*a*) $m + n$ and 0 (*b*) mn and 0

(*c*) $m + n$ and $|m - n|$ (*d*) mn and $m + n$

2. For two union compatible relations R_1 (A, B) and R_2 (C, D), what is the result of the operation $R_1 A = C\ AB = D\ R_2$?

(*a*) $R_1 \cup R_2$ (*b*) $R_1 \times R_2$

(*c*) $R_1 - R_2$ (*d*) $R_1 \cap R_2$

3. Relations produced from an E - R model will always be in

(*a*) 1 NF (*b*) 2 NF

(*c*) 3 NF (*d*) B CNF

4. A primary key, if combined with a foreign key creates

(*a*) parent child relationship between the tables that connect them

(*b*) many-to-many relationship between the tables that connect them

(*c*) network model between the tables connect them

(*d*) none of these

5. Network models are complicated by physical keys, but the relation model is

(*a*) faster becausse it uses logical keys

(*b*) faster because it uses physical keys

(*c*) slower because it uses physical keys

(*d*) slower because it uses logical keys

6. A relational model which allows non-atomic domains is

(*a*) nested relational data model

(*b*) non-atomic data model

(*c*) hierarchical data model

(*d*) none of these

7. Let R (a, b, c) and S (d, e, f) be two relations in which d is the foreign key of S that refers to the primary key of R.

Consider following four operations R and S

I. insert into R

II. insert into S

III. delete from R

IV. delete from S

Which of the following can cause violation of the referential integrity constraint above ?

(*a*) Both I and IV (*b*) Both II and III

(*c*) All of these (*d*) None of these

8. Which of the following is not true for the traditional approah to information processing ?

(*a*) Thre is common sharing of data among the various applications

(*b*) It is file oriented

(*c*) Programs are dependent on the files

(*d*) It is inflexible

9. The database environment has all of the following components except

(*a*) users

(*b*) separate files

(*c*) database

(*d*) database adminstration

10. The way a particular application views the data from the database that the application uses is a

(*a*) module (*b*) relational model

(*c*) schema (*d*) subschema

11. The data manipulation language (DML)

(*a*) refers to data using physical addresses

(*b*) cannot interface with high-level programming language

(*c*) is used to define the physical characteristics of each record

(*d*) none of these

12. The relational model uses some unfamiliar terminology. A tuple is a equivalent to

(*a*) record (*b*) field

(*c*) file (*d*) data base

13. Data base administrator is, in effect, the coordinator between ——— and —— .

(*a*) DBMS ; data base

(*b*) application program; data base

(*c*) data base; users

(*d*) application programs; users

14. A network structure

(*a*) is a physical representation of the data

(*b*) allows a many-to-many relationship

(*c*) is conceptually simple

(*d*) will be the dominant database of the future

15. An advantage of the database approach is

(*a*) elimination of the data redundancy

(*b*) ability to associate related data

(*c*) increases security

(*d*) all of these

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16. Which of the following hardware components is the most important to the operation of a database management system ?
(*a*) High resolution video display
(*b*) Printer
(*c*) High speed, large-capacity disk
(*d*) Plotter

17. Database management systems are intended to
(*a*) eliminate data redundancy
(*b*) establish relationship among records in different files
(*c*) manage file access
(*d*) all of these

18. Which of the following is not characteristics of a relational data base model ?
(*a*) Tables
(*b*) Treelike structure
(*c*) Complex logical relationships
(*d*) Records

19. Which of the following is not the responsibility of the utilities component of DBMS software ?
(*a*) Creating the physical and logical designs
(*b*) Removing flagged records for deletion
(*c*) Creating and maintaining the data dictionary
(*d*) Monitoring performance

20. Which of the following is a type of DBMS software?
(*a*) Data base manipulation language
(*b*) Query language
(*c*) Utilities
(*d*) All of these

21. A database administrator's function is
(*a*) Data base design
(*b*) Backing up the database
(*c*) Performance monitoring
(*d*) All of these

22. A data dictionary doesn't provide information about
(*a*) where data is located
(*b*) the size of the storage disk
(*c*) who owns or is responsible for the data
(*d*) how the data is used

23. Which of the following is a serious problem of file management systems ?
(*a*) Difficult to updata
(*b*) Lack of data independence
(*c*) Data redundancy
(*d*) All of these

24. A database management system
(*a*) allows simultaneous access to multiple file
(*b*) can do more than a record management system
(*c*) is a collection of programs for managing data in a single file
(*d*) both (a) and (b)

25. In a large DBMS
(*a*) each user can "see" only a small part of the entire data base.
(*b*) each subschema contains every field in the logical schema
(*c*) each user can access every subschema
(*d*) all of these

26. A transparent DBMS
(*a*) cannot hide sensitive information from users
(*b*) keeps its logical structure hidden from users
(*c*) keeps its physical structure hidden from users
(*d*) both (b) and (c)

27. Goals for the design of the logical schema include
(*a*) avoiding data inconsistency
(*b*) being able to construct queries easily
(*c*) being able to access data efficiently
(*d*) all of these

28. A top to botom relationship among the items in a database is established by a
(*a*) Hierarchical Schema
(*b*) Network Schema
(*c*) Relational Schema
(*d*) all of these

29. A Network Schema
(*a*) restricts the structure to a one to many relationships
(*b*) permits many to many relationships
(*c*) stores data in tables
(*d*) none of these

30. In a relational Schema, each tuple is divided in to fields called
(*a*) relations
(*b*) domains
(*c*) queries
(*d*) none of these

31. The on line soft copy displays a customer's charge account to respond to an inquiry is an example of
(*a*) forecasting report
(*b*) exception report
(*c*) regularly scheduled report
(*d*) on demand report

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32. If a field size is too small for the longest piece of data to be entered, then

(*a*) data base program will freeze

(*b*) field will automatically expand

(*c*) part of the data will be cut-off

(*d*) none of these

33. Which of the following is (are) logical data base structure?

(*a*) Network (*b*) Tree

(*c*) Chain (*d*) All of these

34. Which of the following is not a relational data of DBMS?

(*a*) Fox Pro

(*b*) dBASE IV

(*c*) 4th Dimension

(*d*) ReFlex

35. Consider a relation geq which represents "greater than or equal to", that is,

$(x, y) \in$ geq only if $y >= x$.

Create table geq (lb integer not null

ub integer not null

primary key lb

foreign key (ub) references geq on delete cascade)

Which of the following is possible if a tuple (x, y) is deleted ?

(*a*) a tuple (z, w) with $z > y$ is deleted

(*b*) a tuple (z, w) with $z > x$ is deleted

(*c*) a tuple (z, w) with $w < x$ is deleted

(*d*) the deletion of (x, y) is prohibited

36. Given relations $r(w, x)$ and $s(y, z)$, the result of

select distinct w, x

from r, s

is guaranteed to be the same as r, provided :

(*a*) r has no duplicates and s is non-empty

(*b*) r and s have no duplicates

(*c*) s has no duplicates and r is non-empty

(*d*) r and s have the same number of tuples

37. In SQL, relation can contain null values, and comparisons with null values are treated as unknown suppose all comparisons with a null value are treated as false.

Which of the following pairs is not equivalent ?

(*a*) $x = 5$ not (not $(x = 5)$)

(*b*) $x = 5$ $x > 4$ and $x < 6$, where x is an integer

(*c*) $x \neq 5$ not $(x = 5)$

(*d*) none of these

38. The SQL expression

Select distinct T. branchname from branch T, branch S where T. assets > S. assets and S. branchcity = "TENALI"

finds the names of

(*a*) all branches that have greater assets than some branch located in TENALI

(*b*) all branches that have greater assets than all branches located in TENALI

(*c*) the branch that has the greatest asset in TENALI

(*d*) any branch that has greater asset than any branch located in TENALI

39. The employee salary should not be greater than Rs. 2000. This is

(*a*) integrity constraint

(*b*) referential constraint

(*c*) over-defined constraint

(*d*) feasible constraint

40. Manager's salary details are hidden from the employee. This is

(*a*) conceptual level data hiding

(*b*) physical level data hiding

(*c*) external level data hiding

(*d*) none of these

41. Trigger is a

(*a*) statement that enables to start any DBMS

(*b*) statement that is executed by the user when debugging an application program

(*c*) condition the system tests for the validity of the database user

(*d*) statement that is executed automatically by the system as a side effect of a modification to the database

42. With regard to expressive power of the formal relational Query languages, relational algebra

(*a*) is more powerful than relational calculus

(*b*) has the same power as relational calculus

(*c*) has the same power as safe relational calculus

(*d*) none of these

43. If adjacency relation of vertices in a graph is represented in a table Adj (x, y), then which of the following Queries can not be expressed by a relational algebra expression of constant length ?

(*a*) List all vertices adjacent to a given vertex

(*b*) List all vertices which have self loops

(*c*) List all vertices which belong to cycle of less than three vertices

(*d*) List all vertices reachable from a given vertex

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44. Which of the following relational calculus expressions is not safe ?

(*a*) $\{t \mid \exists u \in R_1(t[A] = u[A]) \wedge \neg\exists s \in R_2(t[A] = s[A])\}$

(*b*) $\{t \mid \neg (t \in R_1)\}$

(*c*) $\{t \mid \exists u \in R_1(t[A] = u[A]) \wedge \exists s \in R_2(t[A] = s[A])\}$

(*d*) $\{t \mid \forall u \in R_1(u[A] = "x" \Rightarrow \exists s \in R_2(t[A] = s[A] \wedge s[A] = u[A]))\}$

45. The relational algebra expression equivalent to the tuple calculus expression

$\{t \setminus t \in r \wedge (t[A] = 10 \wedge t[B] = 20)\}$ is

(*a*) $\sigma_{(A = 10 \vee B = 20)}(r)$

(*b*) $\sigma_{(A=10)}(r) \cup \sigma_{(B=20)}(r)$

(*c*) $\sigma_{(A=10)}(r) \cap \sigma_{(B=20)}(r)$

(*d*) $\sigma_{(A=10)}(r) - \sigma_{(B=20)}(r)$

46. Which of the following Query transformations (i.e., replacing the L.H.S. expression by the R.H.S. expression) is incorrect ?

R_1 and R_2 are relations, C1 and C2 are selection conditions and A_1 and A_2 are attributes of R_1.

(*a*) $\sigma_{C1}(\sigma_{C1}(R_1)) \to \sigma_{C2}(\sigma_{C2}(R_1))$

(*b*) $\sigma_{C1}(\pi_{A1}(R_1)) \to \pi\sigma'_{A1}(\sigma_{C1}(R_1))$

(*c*) $\sigma_{C1}(R_1 \cup R_2) \to \sigma_{C1}(R_1) \cup \sigma_{C1}(R_2)$

(*d*) $\pi_{A2}(\sigma_{C1}(R_1) \to \sigma_{C1}(\pi_{A2}(R_1))$

47. Match the following and select the correct answer from the codes given below the lists :

List - I	**List - II**
A. secondary index	1. functional dependency
B. non-procedural Query language	2. B-Tree
C. closure of a set of attributes	3. domain calculus
D. natural - join	4. relational algebraic operations

Codes :

	A	**B**	**C**	**D**
(*a*)	2	3	1	4
(*b*)	3	2	4	1
(*c*)	3	1	2	4
(*d*)	2	1	4	3

48. If P and Q are predicates and P is the relational algebra expression, then which of the following equivalence are valid ?

(*a*) $\sigma_P(\sigma_Q(e)) = \sigma_Q(\sigma_P(e))$

(*b*) $\sigma_P(\sigma_Q(e)) = \sigma_{P \wedge Q}(e)$

(*c*) $\sigma_Q(\sigma_P(e)) = \sigma_{P \wedge Q}(e)$

(*d*) All of these

49. Which of the following contains a complete record of all activity that affected the contents of a database during a certain period of time ?

(*a*) Report writer
(*b*) Query language
(*c*) Data manipulation language
(*d*) Transaction log

50. Embedded pointer provide ?

(*a*) A secondary access path
(*b*) A physical record key
(*c*) An inverted index
(*d*) None of above

51. Updating a data base means

(*a*) revising the file structure
(*b*) reorganizing the database
(*c*) modifying or adding record occurrences
(*d*) all of these

52. One data dictionary software package is called

(*a*) DB/DC dictionary (*b*) TOTAL
(*c*) ACCESS (*d*) all of these

53. A logical schema

(*a*) is entire data base
(*b*) is a standard way of organizing information into a accessible part
(*c*) describes how data is actually stored on disk
(*d*) none of these

54. Subschema can be used to

(*a*) create very different personalized views of the same data
(*b*) present information in different formats
(*c*) hide sensitive information by omitting fields from the subschema description
(*d*) all of these

55. A data dictionary is a special file that contains

(*a*) names of all fields in all files
(*b*) data types of all fields in all files
(*c*) width of all fields in all files
(*d*) all of these

56. Queries to a data base

(*a*) are written in English
(*b*) can use aggregate functions like SUM and COUNT
(*c*) both (a) and (b)
(*d*) none of these

57. Data integrity control

(*a*) is used to set upper and lower limits on numeric data
(*b*) requires the use of passwords to prohibit unauthorized access to the file
(*c*) has the data dictionary to keep the data and time of last access, last back-up, and most recent modification for all files
(*d*) none of these

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58. Information can be transferred between DBMS and
(*a*) spread sheet program
(*b*) word processor program
(*c*) graphics program
(*d*) all of these

59. A race condition occurs when
(*a*) two concurrent activities interact to cause a processing error
(*b*) two users of the DBMS are interacting with different files at the same time
(*c*) both (a) and (b)
(*d*) none the these

60. Addresses are considered
(*a*) Numeric fields (*b*) Data fields
(*c*) Relational fields (*d*) Alpha fields

61. A data is often entered in a format such as 81/12/30 because
(*a*) military prefers this format
(*b*) reader can find the year faster
(*c*) it allow the user to sort by year
(*d*) none of these

62. Which of the following contains complete record of all activity that affected the contents of a database during a certain period of time?
(*a*) 4 GL (*b*) d-BASE
(*c*) Oracle (*d*) SQL

63. In a multiuser data base, if two users wish to update the same record at the same time, they are prevented from dong so by
(*a*) jamming
(*b*) password
(*c*) documentation
(*d*) record lock

64. A relational data base management (RDBM) package manages data in more than one file at once. It organize these files as
(*a*) tables
(*b*) relations
(*c*) tuple
(*d*) both (a) and (b)

65. Given relations γ (w, x) and S (y, z), the result of select distinct w, x From r, S is guaranteed to be the same as γ, empty
(*a*) γ has no duplicates and s is non empty
(*b*) γ and s have no duplicates
(*c*) S has no duplicates and γ is non empty
(*d*) γ and S have the same number of tuples.

66. The PROJECT command will create new table that has
(*a*) more fields than the original table
(*b*) more rows than original table
(*c*) both (a) and (b)
(*d*) none of these

67. A report generator is used to
(*a*) update files (*b*) print files on paper
(*c*) both (a) and (b) (*d*) none of these

68. Let domain set of an attribute consists of signed four digit numbers. What is the percentage of reduction in storage space of this attribute if it is stored as an integer rather than in character form?
(*a*) 80% (*b*) 20%
(*c*) 60% (*d*) 40%

69. Which of the following is/are correct?
(*a*) An SQL query automatically elimiates duplicates.
(*b*) An SQL query will not work if there are no indexes on the relations.
(*c*) SQL permits attributes name to be repeated in the same relation.
(*d*) None of these

70. Let R = (A, B, C, D, E, F) be a relation scheme with the following dependencies :
$$C \to F,\ E \to a,\ EC \to D,\ A \to B.$$
Which of the following is a key for R ?
(*a*) CD (*b*) EC
(*c*) AE (*d*) AC

71. Consider the schema R = (S T U V) and the dependencies
$S \to T$, $T \to U$, $U \to V$ and $V \to S$.
If R = (R1 and R2) be a decomposition such that $R1 \cap R2 = \phi$, then decomposition is
(*a*) not in 2 NF
(*b*) in 2 NF but not in 3 NF
(*c*) in 3 NF but not in 2 NF
(*d*) in both 2NF and 3 NF

72. Which normal form is considered adequate for normal relationship database design ?
(*a*) 2 NF (*b*) 3 NF
(*c*) 4 NF (*d*) 5 NF

73. A functional dependency of the form $x \to y$ is trivial if
(*a*) $y \subseteq x$ (*b*) $y \subset x$
(*c*) $x \subseteq y$ (*d*) $x \subset y$ and $y \subset x$

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74. If a relation scheme is in BCNF, then it is also in

(*a*) 1 NF (*b*) 2 NF

(*c*) 3 NF (*d*) none of these

75. Any Binary relation is in

(*a*) 1 NF (*b*) 2 NF

(*c*) 3 NF (*d*) B CNF

76. Normalization of database is used to

(*a*) eliminate redundancy

(*b*) improve security

(*c*) improve efficiency

(*d*) minimize errors

77. Functional dependencies are a generalization of

(*a*) key dependencies

(*b*) relation dependencies

(*c*) database dependencies

(*d*) none of these

78. Every Boyce-Codd Normal Form (BCNF) decomposition is

(*a*) dependency preserving

(*b*) not dependency preserving

(*c*) need be dependency preserving

(*d*) none of these

79. Report generators are used to

(*a*) Store data input by a user

(*b*) Retrieve information file

(*c*) Answer queries

(*d*) Both *b* and *c*

80. A phone no in a database is an example of a

(*a*) record (*b*) field

(*c*) sort (*d*) file

81. A library relational database system uses the following Schema

USERS (User #, User Name, Home Town)

BOOKS (Book # Book Title, Author Name)

ISSUED (Book #, User #, Data)

What each of the following relational alegebra queries is designed to determine is explained by one english sentence as

(*a*) σ user # = σC | | user #, Book Tittle (Users Issued) BOOKS)

(*b*) σ Author Name (Book (σ name town) = Delhi (Users ISSUED)))

(*c*) both (a) and (b)

(*d*) none of these

82. A relational database which is in 3NF may still have undesirable data redundancy because there may exist

(*a*) transitive functional dependencies.

(*b*) non-trivial functional dependencies involving prime attributes on the right side.

(*c*) non-trivial functional dependencies involving prime attributes only on the left side.

(*d*) non- trivial functional dependencies involving only prime attributes.

83. For a database relation R (*a*, *b*, *c, d*), where the domains of *a*, *b*, *c*, d include only atomic values, only the following functional dependencies and those that that can be inferred from them hold:

$$a \to c$$
$$b \to d$$

The relation is in

(*a*) first normal form but not in second normal form

(*b*) second normal form but not in third normal form

(*c*) third normal form

(*d*) none of these

84. Given following relation instance :

X	Y	Z
1	4	2
1	5	3
1	6	3
3	2	2

Which of the following functional dependencies are satisfied by the instance?

(*a*) XY → Z and Z → Y

(*b*) YZ → X and Y → Z

(*c*) YZ → X and X → Z

(*d*) XZ → Y and Y → X

85. An access path is provided by

(*a*) file key (*b*) physical record key

(*c*) both (*a*) and (*b*) (*d*) none of these

86. An indexing operation

(*a*) sorts a file using a single key

(*b*) sorts a file using two keys

(*c*) established an index for a file

(*d*) both (b) and (c)

87. The master list of an indexed file

(*a*) is sorted in ascending order

(*b*) contains only a list of keys and record numbers

(*c*) has a number assigned to each record

(*d*) both (a) and (b)

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88. Index consists of
(*a*) a list of keys
(*b*) pointers to the master list
(*c*) both (a) and (b)
(*d*) none of these

89. To have a file, hold a list, it is necessary to
(*a*) identify the records in the list
(*b*) identify the name, width and type of the fields of each record
(*c*) decide which fields will be used as sort or index keys
(*d*) all of these

90. If a numeric field has a width of 5.2, then value of the field could be
(*a*) 23.10 (*b*) 121.8
(*c*) 143.87 (*d*) both (a) and (b)

91. Varsatile report generators can provide
(*a*) columnar totals
(*b*) subtotals
(*c*) calculations
(*d*) all of these

92. Sort/Report generators
(*a*) are faster than index/report generators
(*b*) require more disk space than index/report generators.
(*c*) do not need to sort before generating report
(*d*) both (a) and (b)

93. A Report Form
(*a*) appears on the computer monitor during data entry
(*b*) is used during refort generation to format data
(*c*) both (*a*) and (*b*)
(*d*) none of these

94. A form defines
(*a*) where data is placed on the screen
(*b*) the width of each field
(*c*) both (*a*) and (*b*)
(*d*) none of these

95. The designer of a form includes
(*a*) field designators
(*b*) prompts
(*c*) data
(*d*) both (*a*) and (*b*)

96. If calculation is embedded in a form
(*a*) result of the calculations are strored with the form
(*b*) calculations are stored with the form
(*c*) both (*a*) and (*b*)
(*d*) none of these

97. A multiple form file management system allows
(*a*) defining different forms for different operations
(*b*) creating a look-up form with an associated read only password to present access by unauthorized users.
(*c*) both (*a*) and (*b*)
(*d*) none of these

98. The physical locations of a record is determined by a mathematical formual that transforms file key into a record location in a/an
(*a*) B-tree file (*b*) indexed file
(*c*) hashed file (*d*) all of these

99. Two files may be joined into a third file if they have
(*a*) row in common
(*b*) field in common
(*c*) no records with the same value in the common field
(*d*) both (*b*) and (*c*)

100. The main idea behind computer files is that it is convenient to
(*a*) arrange them
(*b*) store information together
(*c*) create them
(*d*) access them

101. A command that lets you change or more fields in a record is
(*a*) insert (*b*) modify
(*c*) look-up (*d*) none of these

102. Which command closes text file which has been created using
"SET ALTERNATIVE" <FILE NAME> "Command"?
(*a*) SET ALTERNATE OFF
(*b*) CLOSE DATABASE
(*c*) CLOSE ALTERNATE
(*d*) CLEAR ALL

103. Which command is fastest among the following?
(*a*) COPY TO < New File >
(*b*) COPY FILE < File 1 > < File 2 >
(*c*) COPY STRUCTURE TO < new file >
(*d*) COPY TO M FILE - DAT DELIMITED

104. GO BOTTOM and SKIP-3 Commands gives one after another in a database file of 30 records shift the control to
(*a*) 28th record (*b*) 27th record
(*c*) 3rd record (*d*) 4th record

105. Which of the following statements is true when structure of database file with 20 records is modified?
(*a*) ? EOF ( ) Prints. T (*b*) ? BOF ( ) Prints. F
(*c*) ? BOF ( ) Prints. T (*d*) ? EOF ( ) Prints. F

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106. In database software the key that acts as a toggle between Edit mode and Browse mode is

(*a*) F_5 (*b*) F_2

(*c*) F_1 (*d*) F_4

107. Which of the following commands permanently delete the record marked for deletion from the data base field?

(*a*) PACK (*b*) ZAP

(*c*) SEEK (*d*) SKIP

108. Which function is used to convert a data to a character string ?

(*a*) DTOC ( ) (*b*) DTOW ( )

(*c*) DOW ( ) (*d*) CDOW ( )

109. Display the name of author who have received highest royalty amount.

SQL > SELECT Author__Name

FROM Book

GROUP BY Author__Name

HAVING sum (Unit__Price * 0.15) > = all (select sum (Unit__Price * 0.15) from Book group by Author__Name); give data table

Title	Author Name	Publisher Name	Publishing Year	Royalty
Oracle	Arora	PHI	2008	5000
DBMS	Basu	Technical	2008	7000
ADBMS	Basu	Technical	2008	7000
Unix	Kapoor	Sci- Tech	2001	4500

(*a*) Basu (*b*) Arora

(*c*) Kapoor (*d*) None of these

110. Which of the following scenarios may lead to an irrecoverable error in a database system ?

(*a*) A transaction writes a data item after it is read by an uncommited transaction.

(*b*) A transaction reads a data item after it is read by an uncommited transaction.

(*c*) A transaction reads a data item after it is written by a committed transaction.

(*d*) A transaction reads a data item after it is written by an uncommitted transaction.

111. Consider the following SQL query select distinct $a_1, a_2, \ldots\ldots, a_n$

From $r_1, r_2, r_3 \ldots\ldots., r_m$

where P

For an arbitrary predicate P, this query is equivalent to which of the following relational algebra expressions.

(*a*) $a_1, a_2, \pi \ldots\ldots. a_n\ \sigma_p\ (r_1 \times r_2 \times \ldots\ldots. \times r_m)$

(*b*) $a_1, a_2, \pi \ldots\ldots. a_n\ \sigma_p\ (r_1 \bowtie r_2 \bowtie \ldots\ldots. \bowtie r_m)$

(*c*) $a_1, a_2, \pi \ldots\ldots. a_n\ \sigma_p\ (r_1 \cup r_2 \cup \ldots\ldots. \cup r_m)$

(*d*) $a_1, a_2, \pi \ldots\ldots. a_n\ \sigma_p\ (r_1 \cap r_2 \cap \ldots\ldots. \cap r_m)$

112. Consider the following functional dependencies in 'database'.

Date _ of _ Birth $\rightarrow$ Age, Age $\rightarrow$ Eligibility

Name $\rightarrow$ Roll _ Number, Roll_ number$\rightarrow$ Name

Course_ number $\rightarrow$ Course_name,

Course_number $\rightarrow$ Instructor

(Roll_Number; Course_Number) $\rightarrow$ Grade the relation

(Roll_Number; Name, Date_of_birth, Age) is

(*a*) In second normal form but not in third normal form

(*b*) In third normal form but not in BCNF

(*c*) In BCNF

(*d*) In none of these

113. Consider three data items A_1, A_2 and A_3 and the following execution Schedule of transaction T_1, T_2 and T_3. In the diagram R(A) and W(A) denoted the action and reading and writing the data item A respectively.

T_1	T_2	T_3
	R (A3)	
	R (A2)	
	W (A2)	
		R (A2)
		R (A2)
R (A1)		
W (A1)		
		W (A2)
		W (A3)
	R (A1)	
R (A2)		
W (A2)		
	W (A1)	

(*a*) The schedule is serializable as T2; T3; T1;

(*b*) The schedule is serializable as T2; T1; T3;

(*c*) The schedule is serializable as T3; T2; T1;

(*d*) The schedule is no serializable

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114. Consider the relation student (Name, sex, marks), where the primary key is shown underlined, pertaining to students in a class that has at least one boy and one girl. What does the following relational algebra expression produce ?

π_{name} (r_{sex} = Female$^{(\text{Student})}$) P_{name} (student $\bowtie r_n$, x, $m^{(\text{Student})}$)

(Sex = Female ^x = Male ^ Marks ≤ M)

(*a*) Names of girl students with the highest marks.

(*b*) Names of girl students with more marks than some boy student.

(*c*) Name of girl students with marks not less than some boy student.

(*d*) Name of girl students with more marks than all the boy students.

115. Consider a relation schema R = (A, B, C, D, E, H) on which the following functional Dependencies hold :

{A → B, BC → D, E → C, D → A}.

What are the candidate keys of R ?

(*a*) AE, BE

(*b*) AE, BE, DE

(*c*) AEH, BEH, BCH

(*d*) AEH, BEH, DEH

116. A clustring index is defined on the fields which are of type

(*a*) Non-key and ordering

(*b*) Non-key and non-ordering

(*c*) Key and ordering

(*d*) Key and non-ordering

117. The relation schema student performance (Name, Course No, Roll No, Grade) has the following functional dependencies :

Name, Course No. → Grade

Roll No., Course No. → Grade

Name → Roll No.

Roll No. → Name

The highest normal form of this relation schema is

(*a*) 2 NF (*b*) 3 NF

(*c*) BCNF (*d*) 4 NF

118. According to the levels of abstraction, the schema at the intermediate level is called

(*a*) Logical schema

(*b*) Physical schema

(*c*) Sub schema

(*d*) Conceptual schema

119. Consider a schema R (A, B, C, D) and functional dependencies A → B and C → D. Then the decomposition of R into R1 (A, B) and R2 (C, D) is

(*a*) dependency preserving and lossless join

(*b*) loss less join but not dependency preserving

(*c*) dependency preserving but not lossless join

(*d*) not dependency preserving and not lossless join

120. In SQL relationals can contain null values, and comparisons with null values are treated as unknown. Suppose all comparisons with a null value are treated as false.

Which of the following pairs is not equivalent?

(*a*) $x = 5$, (not ($x = 5$)

(*b*) $x = 5$, $x > 4$ and $x < 6$, where x is an integer

(*c*) $x < 5$, not ($x = 5$)

(*d*) None of the above

NUMERICAL TYPE QUESTIONS

1. Minimum number of record movements required to merge five files A (with 10 records), B (with 20 records), C (with 15 records), D (with 5 records) and E (with 25 records) is _______

2. If contents of a field cannot be zero or less than that, then in the range minimum value would be specified as_______

3. The first element in T [4, 3] is_____

4. The following schedule which perform the transaction as follows :

T_1 : lock - S (A);
read (A) ;
unlock (A);
lock-S (B)
read (B);
unlock (B);
display (A + B);

Suppose that the value of accounts A and B are Rs. 100 and Rs. 200, respectively. If these transactions are executed serially, then the result is ___________.

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5. Consider following Employee - salary relation

Employee – Salary			
Emp. Code	Name	Salary	Department
E0001	Hari	10000	Computer
E0002	Om	7000	IT
E0003	Smith	8000	Computer
E0004	Jay	5000	IT

and Gavg (salary) (employee-salary) ______ is the average salary of employees through aggregate function.

6. Let R (A, B, C) and F = (A → B).

Prove that the decomposition of R into R_1 (A, B) and R_2 (A, C) is lossless-join decomposition give the answer is Yes/No.

7. Consider following schedule, is the given schedule conflict serializable give the answer Yes/No. ?

T_1	T_2
Read (A)	
Write (A)	
	Read (A)
Read (B)	
	Write (A)
Write (B)	
	Read (B)
	Write (B)

8. The order of an internal node in a B * tree index is the maximum number of children it can have. Suppose that search field value takes 14 bytes, and the block size is 512 bytes. ______ is the order of the internal Node ?

9. Consider the following relation schema pertaining to a students database :

student (roll no, name, address)

enroll (roll no, course no, course name)

Where the primary keys are shown underlined. The number of tuples in the student and enroll tables are 120 and 8 respectively. ______are the maximum and minimum number of tuples that can be present in (student * enroll), where '*' denotes natural join ?

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2015

1. SELECT operation in SQL is equivalent to

(*a*) the selection operation in relational algebra

(*b*) the selection operation in relational algebra, except that SELECT in SQL retains duplicates

(*c*) the projection operation in relational algebra

(*d*) the projection operation in relational algebra, except that SELECT in SQL retains duplicates

2. Consider the following transaction involving two bank account x and y.

read (x); $x := x - 50$; write (x); read (y); $y := y + 50$; write (y)

The constraint that the sum of the accounts x and y should remain constant is that of

(*a*) Atomicity (*b*) Consistency

(*c*) Isolation (*d*) Durability

3. Consider two relations R_1(A, B) with the tuples (1.5), (3, 7) and R_2(A, C) = (1, 7), (4, 9). Assume that R(A, B, C) is the full natural outer join of R_1 and R_2. Consider the following tuples of the form (A, B, C): a = (1.5, null), b = (1, null, 7) c = (3, null, 9), d = (4, 7, null), e = (1, 5, 7), f = (3, 7, null), g = (4, null, 9).Which one of the following statements is correct?

(*a*) R contains a, b, e, f, g but not c, d.

(*b*) R contains all of a, b, c, d, e, f, g

(*c*) R contains e, f, g but not a, b

(*d*) R contains e but not f, g.

4. Consider a simple checkpointing protocol and the following set of operations in the log.

(Start, T4); (write, T4, y, 2, 3); (Start, T1); (commit, T4); (write, T1, z, 5, 7);

(checkpoint);

(Start,T_2); (write, T2, x,1, 9); (commit, T2); (start, T3), (write,T3, z, 7, 2);

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If a crash happens now and the system tries to recover using both undo and redo operations, what are the contents of the undo lists and the redo list?

(*a*) Undo T3, T1; Redo T2

(*b*) Undo T3, T1; Redo T2, T4

(*c*) Undo: none; redo :T2, T4, T3, T1

(*d*) Undo T3, T1; T4; Redo :T2

5. Consider the relation X(P, Q, R, S, T, U) with the following set of functional dependencies

F = {

$\{P, R\} \rightarrow \{S, T\}$

$\{P, S, U\} \rightarrow \{Q, R\}$

Which of the following is the trivial functional dependency in F^+, where F^+ is closure of F ?

(*a*) $\{P, R\} \rightarrow \{S, T\}$ (*b*) $\{P, R\} \rightarrow \{R, T\}$

(*c*) $\{P, S\} \rightarrow \{S\}$ (*d*) $\{P, S, U\} \rightarrow \{Q\}$

6. Two processes X and Y need to access a critical section. Consider the following synchronization construct used by both the processes

Process X	**Process Y**
/* other code for process X*/	/* other code for process Y*/
while (true)	while (true)
{	{
var P = true;	var Q = true;
while (var Q == true)	while (var P == true)
{	{
/* critical section */	/* critical section */
varP = false;	varQ = false;
}	}
}	}
/* other code for process X*/	/* other code for process X*/

Here, varP and varQ are shared variables and both are initialized to false. Which one of the following statements is true ?

(*a*) The proposed solution prevents deadlock but fails to guarantee mutual exclusion

(*b*) The proposed solution guarantees mutual exclusion but fails to prevent deadlock

(*c*) The proposed solution guarantees mutual exclusion and prevents deadlock

(*d*) The proposed solution fails to prevent deadlock and fails to guarantee mutual exclusion

7. Consider the following relation

Cinema (theater, address, capacity)

Which of the following options will be needed at the end of the SQL query

SELECT P1. address

FROM Cinema P1

Such that it always finds the addresses of theaters with maximum capacity ?

(*a*) WHERE P1. Capacity> = All (select P2. Capacity from Cinema P2)

(*b*) WHERE P1. Capacity> = Any (select P2. Capacity from Cinema P2)

(*c*) WHERE P1. Capacity> = All (select max(P2. Capacity) from Cinema P2)

(*d*) WHERE P1. Capacity> = Any (select max (P2. Capacity) from Cinema P2)

8. Consider the following partial Schedule S involving two transactions T1 and T2. Only the read and write operations have been shown. The read operation on data item P is denoted by read (P) and the write operation on data item P is denoted by write (P).

Time Instance	Transaction-id	
	T1	T2
1	read(A)	
2	write(A)	
3		read(C)
4		write(C)
5		read(B)
6		write(B)
7		read(A)
8		commit
9	read (B)	

Schedule S

Suppose that the transaction T1 fails immediately after time instance 9. Which one of the following statements is correct ?

(*a*) T2 must be aborted and then both T1 and T2 must be re-started to ensure transaction atomicity

(*b*) Schedule S is non-recoverable and cannot ensure transaction atomicity

(*c*) Only T2 must be aborted and then re-started to ensure transaction atomicity

(*d*) Schedule S is recoverable and can ensure atomicity and nothing else needs to be done

2014

9. Consider the relation scheme R = (E, F, G, H, I, J, K, L, M, N) and the set of functional dependencies $\{\{E,F\} \rightarrow \{G\}, \{F\} \rightarrow \{I, J\}, \{E,H\} \rightarrow \{K, L\}, \{K\} \rightarrow \{M\}, \{L\} \rightarrow \{N\}\}$ on R. What is the key for R?

(*a*) {E, F} (*b*) {E, F, H }

(*c*) {E, F, H, K, L} (*d*) {E}

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10. Given the following statements:

S1 : A foreign key declaration can always be replaced by an equivalent check assertion in SQL.

S2 : Given the table R(a,b,c) where a and b together form the primary key, the following is a valid table definition.

```
CREATE TABLE S (
a INTEGER,
d INTEGER,
e INTEGER,
PRIMARY KEY (d),
FOREIGN KEY (a) references R)
```

Which one of the following statements is CORRECT?

(*a*) S1 is TRUE and S2 is FALSE
(*b*) Both S1 and S2 are TRUE
(*c*) S1 is FALSE and S2 is TRUE
(*d*) Both S1 and S2 are FALSE

11. Consider the following four schedules due to three transactions (indicated by the subscript) using *read* and *write* on a data item x, denoted by r(x) and w(x) respectively. Which one of them is conflict serializable?

(*a*) $r_1(x); r_2(x); w_1(x); r_3(x); w_2(x)$
(*b*) $r_2(x); r_1(x); w_2(x); r_3(x); w_1(x)$
(*c*) $r_3(x); r_2(x); r_1(x); w_2(x); w_1(x)$
(*d*) $r_2(x); w_2(x); r_3(x); r_1(x); w_1(x)$

12. Given the following two statements:

S1: Every table with two single-valued attributes is in 1NF, 2NF, 3NF and BCNF

S2: $AB \to C, D \to E, E \to C$ is a minimal cover for the set of functional dependencies $AB \to C, D \to E, AB \to E, E \to C$.

Which one of the following is CORRECT?

(*a*) S1 is TRUE and S2 is FALSE.
(*b*) Both S1 and S2 are TRUE.
(*c*) S1 is FALSE and S2 is TRUE.
(*d*) Both S1 and S2 are FALSE.

13. Given the following schema:

employees(emp-id, first-name, last-name, hire-date, dept-id, salary)

departments(dept-id, dept-name, manager-id, location-id)

You want to display the last names and hire dates of all latest hires in their respective departments in the location ID 1700. You issue the following query:

```
SQL>SELECT last-name, hire-date
  FROM employees
  WHERE (dept-id, hire-date) IN
  (SELECT dept-id, MAX(hire-date)
  FROM employees JOIN departments USING(dept-id)
  WHERE location-id = 1700
  GROUP BY dept-id);
```

What is the outcome?

(*a*) It executes but does not give the correct result.
(*b*) It executes and gives the correct result.
(*c*) It generates an error because of pairwise comparison.
(*d*) It generates an error because the GROUP BY clause cannot be used with table joins in a sub-query.

14. Consider the following schedule S of transactions T1, T2, T3, T4:

T1	T2	T3	T4
	Reads(X)		
		Writes (X) Commit	
Writes (X) Commit			
	Writes(Y) Reads(Z) Commit		
			Reads(X) Reads(Y) Commit

Which one of the following statements is CORRECT?

(*a*) S is conflict-serializable but not recoverable
(*b*) S is not conflict-serializable but is recoverable
(*c*) S is both conflict-serializable and recoverable
(*d*) S is neither conflict-serializable nor is it recoverable

15. Consider a join (relation algebra) between relations r(R) and s(S) using the nested loop method. There are 3 buffers each of size equal to disk block size, out of which one buffer is reserved for intermediate results. Assuming size(r(R))<size(s(S)), the join will have fewer number of disk block accesses if

(*a*) relation r(R)is in the outer loop.
(*b*) relation s(S)is in the outer loop.
(*c*) join selection factor between r(R) and s(S) is more than 0.5
(*d*) join selection factor between r(R) and s(S) is less than 0.5

16. SQL allows duplicate tuples in relations, and correspondingly defines the multiplicity of tuples in the result of joins. Which one of the following queries always gives the same answer as the nested query shown below:

select * from R where a in (select S.a from S)

(*a*) select R.* from R, S where R.a = S . a
(*b*) select distinct R.* from R,S where R.a = S.a
(*c*) select R.* from R,(select distinct a from S) as S1 where R.a = S1.a
(*d*) select R.* from R, S where R.a = S.a and is unique R

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17. *A prime attribute* of a relation scheme R is an attribute that appears

(*a*) in all candidate keys of R.

(*b*) in some candidate key of R.

(*c*) in a foreign key of R.

(*d*) only in the primary key of R.

18. Consider the transactions T1, T2, and T3 and the schedules S1 and S2 given below.

T1 : r1 (X); r1 (Z) ; w1 (X) ; w1 (Z)

T2 : r2 (Y) ; r2 (Z) ; w2 (Z)

T3 : r3 (Y) ; r3 (X) ; w3 (Y)

S1: r1(X); r3(Y); r3(X); r2(Y); r2(Z); w3(Y); w2(Z); r1(Z); w1(X); w1(Z)

S2: r1(X); r3(Y); r2(Y); r3(X); r1(Z); r2(Z); w3(Y); w1(X); w2(Z); w1(Z)

Which one of the following statements about the schedules is TRUE?

(*a*) Only S1 is conflict-serializable.

(*b*) Only S2 is conflict-serializable.

(*c*) Both S1 and S2 are conflict-serializable.

(*d*) Neither S1 nor S2 is conflict-serializable.

19. Consider the relational schema given below, where eId of the relation dependent is a foreign key referring to empId of the relation employee. Assume that every employee has at least one associated dependent in the dependent relation.

employee (<u>empId</u>, empName, empAge)

dependent (<u>depId, eId</u>, depName, depAge)

Consider the following relational algebra query: Π_{empId} (employee) – Π_{empId} (employee $\bowtie_{\text{(empId = eID) Ù (emp Age £ depAge)}}$ dependent)

The above query evaluates to the set of empIds of employees whose age is greater than that of

(*a*) some dependent.

(*b*) all dependents.

(*c*) some of his/her dependents

(*d*) all of his/her dependents

20. Consider the following relational schema:

employee (<u>empId</u>, empName, empDept)

customer (<u>custId</u>, custName, sales RepId, rating)

salesRepId is a foreign key referring to empId of the employee relation. Assume that each employee makes a sale to at least one customer. What does the following query return?

```
SELECT empName
FROM employee E
WHERE NOT EXISTS (SELECT custId
                  FROM customer C
                  WHERE C.salesRepId = E. empId
                  AND C. rating < > 'GOOD');
```

(*a*) Names of all the employees with at least one of their customers having a 'GOOD' rating.

(*b*) Names of all the employees with at most one of their customers having a 'GOOD' rating.

(*c*) Names of all the employees with none of their customers having a 'GOOD' rating.

(*d*) Names of all the employees with all their customers having a 'GOOD' rating.

2013

21. An index is clustered, if

(*a*) it is on a set of fields that form a candidate key.

(*b*) it is on a set of fields that include the primary key.

(*c*) the data records of the file are organized in the same order as the data entries of the index.

(*d*) the data records of the file are organized not in the same order as the data entries of the index.

22. Consider the following relational schema.

Students(<u>rollno: integer,</u> sname: string)

Courses(<u>courseno: integer,</u> cname: string)

Registration(<u>rollno: integer, conrseno: integer,</u> percent: real)

Which of the following queries are equivalent to this query in English?

"Find the distinct names of all students who score more than 90% in the course numbered 107"

(I) SELECT DISTINCT S.sname
FROM Students as S, Registration as R
WHERE R.rollno = S.rollno AND R.courseno = 107 AND R.percent>90

(II) Π σ name (σ courseno=107 ^ percent > 90 (Registra-tion ∞ Students))

(III) {T | ∃ S ∈ Students, ∃ R ∈ Registration (S.rollno = R.rollno ^ R.courseno=107^ R. percent > 90 ^ T.sname = S.sname))

(IV) $\{\langle S_N\rangle \mid \exists S_R \exists R_P (\langle S_R, S_N\rangle \in \text{Students} \wedge \langle S_R, 107, R_P\rangle \in \text{Registration} \wedge R_P > 90)\}$

(*a*) I, II, III and IV (*b*) I, II and III only

(*c*) I, II and IV only (*d*) II, III and IV only

23. The preorder traversal sequence of a binary search tree is 30, 20, 10, 15, 25, 23, 39, 35, 42. Which one of the following is the postorder traversal sequence of the same tree?

(*a*) 10, 20, 15, 23, 25, 35, 42, 39, 30

(*b*) 15, 10, 25, 23, 20, 42, 35, 39, 30

(*c*) 15, 20, 10, 23, 25, 42, 35, 39, 30

(*d*) 15, 10, 23, 25, 20, 35, 42, 39, 30

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2012

24. Which of the following statements are TRUE about an SQL query?

P : An SQL query can contain a HAVING clause even if it does not have a GROUP BY clause

Q : An SQL query can contain a HAVING clause only if it has a GROUP BY clause

R : All attributes used in the GROUP BY clause must appear in the SELECT clause

S : Not all attributes used in the GROUP BY clause need to appear in the SELECT clause

(*a*) P and R (*b*) P and S

(*c*) Q and R (*d*) Q and S

25. Given the basic ER and relational models, which of the following is **INCORRECT**?

(*a*) An attribute of an entity can have more than one value

(*b*) An attribute of an entity can be composite

(*c*) In a row of a relational table, an attribute can have more than one value

(*d*) In a row of a relational table, an attribute can have exactly one value or a NULL value

26. Which of the following is TRUE?

(*a*) Every relation in 3NF is also in BCNF

(*b*) A relation R is in 3NF if every non-prime attribute of R is fully functionally dependent on every key of R

(*c*) Every relation in BCNF is also in 3NF

(*d*) No relation can be in both BCNF and 3NF

27. Suppose $R_1(A, B)$ and $R_2(C, D)$ are two relation schemas. Let r_1 and r_2 be the corresponding relation instances. B is a foreign key that refers to C in R_2. If data in r_1 and r_2 satisfy referential integrity constraints. which of the following is **ALWAYS TRUE**?

(*a*) $\Pi_B(r_1) - \Pi_C(r_2) = \varnothing$

(*b*) $\Pi_C(r_2) - \Pi_B(r_1) = \varnothing$

(*c*) $\Pi_B(r_1) = \Pi_C(r_2)$

(*d*) $\Pi_B(r_1) - \Pi_C(r_2) \neq \varnothing$

2011

28. Database table by name Loan_Records is given below.

Borrower	Bank_Manager	Loan_Amount
Ramesh	Sunderajan	10000.00
Suresh	Ramgopal	5000.00
Mahesh	Sunderajan	7000.00

What is the output of the following SQL query?

```
SELECT count (*)
FROM (
    (SELECT Borrower, Bank_Manager FROM
    Loan_Records) AS S
    NATURAL JOIN
    (SELECT Bank_Manager, Loan_Amount
    FROM Loan_Records) AS T );
```

(*a*) 3 (*b*) 9

(*c*) 5 (*d*) 6

29. Consider a relational table r with sufficient number of records, having attributes $A_1, A_2, \ldots A_n$ and let $1 \le p \le n$. Two queries Q_1 and Q_2 are given below.

Q1 : $\pi_{A_1 \cdots A_p}(\sigma_{A_p = c}(r))$ where c is a constant

Q2 : $\pi_{A_1 \cdots A_p}(\sigma_{c_1 \le A_p \le c_2}(r))$ where c_1 and c_2 are constants

The database can be configured to do ordered indexing on A_p or hashing on A_p. Which of the following statements is TRUE?

(*a*) Ordered indexing will always outperform hashing for both queries

(*b*) Hashing will always outperform ordered indexing for both queries

(*c*) Hashing will outperform ordered indexing on Q1, but not on Q2

(*d*) Hashing will outperform ordered indexing on Q2, but not on Q1

30. Consider a database table T containing two columns X and Y each of type integer. After the creation of the table, one record (X = 1, Y = 1) is inserted in the table.

Let MX and MY denote the respective maximum values of X and Y among all records in the table at any point in time. Using MX and MY, new records are inserted in the table 128 times with X and Y values of MX and MY change.

What will be the output of the following SQL query after the steps mentioned above are carried out?

```
SELECT Y FROM T WHERE X = 7;
```

(*a*) 127

(*b*) 255

(*c*) 129

(*d*) 257

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2010

31. One of the header fields in an IP datagram is the Time-to-Live (TTL) field. Which of the following statements best explains the need for this field?

(*a*) It can be used to prioritize packets

(*b*) It can be used to reduce delays

(*c*) It can be used to optimize throughput

(*d*) It can be used to prevent packet looping

32. Which one of the following is not a client-server application?

(*a*) Internet chat (*b*) Web browsing

(*c*) E-mail (*d*) Ping

33. Consider a B^+-tree in which the maximum number of keys in a node is 5. What is the minimum number of keys in any non-root node?

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

34. A relational schema for a train reservation database is given below.

Table: Passenger

pid	pname	Age
0	'Sachin'	65
1	'Rahul'	66
2	'Sourav'	67
3	'Anil'	69

Table: Reservation

pid	class	tid
0	'AC'	8200
1	'AC'	8201
2	'SC'	8201
5	'AC'	8203
1	'SC'	8204
3	'AC'	8202

Passenger (pid, pname, age)

Reservation (pid, class, tid)

What pids are returned by the following SQL query for the above instance of the tables?

```
SELECT   pid
FROM     Reservation
WHERE    class = □'AC' AND
         EXISTS(SELECT *
                  FROM Passenger
                  WHERE age > 65 and
                  Passenger. pid =
                  Reservation. pid)
```

(*a*) 1, 0 (*b*) 1, 2

(*c*) 1, 3 (*d*) 1, 5

35. Which of the following concurrency control protocols ensure both conflict serializability and freedom from deadlock?

I. 2-phase locking

II. Time-stamp ordering

(*a*) I only (*b*) II only

(*c*) Both I and II (*d*) Neither I nor II

36. Consider the following schedule for transactions T1, T2 and T3:

T1	T2	T3
Read (X)		
	Read (Y)	
		Read (Y)
	Write (Y)	
Write (X)		
		Write (X)
	Read (X)	
	Write (X)	

Which one of the schedules below is the correct serialization of the above?

(*a*) T1 → T3 → T2 (*b*) T2 → T1 → T3

(*c*) T2 → T3 → T1 (*d*) T3 → T1 → T2

37. The following functional dependencies hold for relations R (A, B, C) and S (B, D, E):

B → A,

A → C

The relation R contains 200 tuples and the relations S contains 100 tuples. What is the maximum number of tuples possible in the natural join R $\bowtie$ S?

(*a*) 100 (*b*) 200

(*c*) 300 (*d*) 2000

2009

38. Consider two transactions T_1 an T_2, and four schedules S_1, S_2, S_3, S_4 of T_1 and T_2 as given below :

$T_1 : R_1[x]\ W_1[x]\ W_1[y]$

$T_2 : R_2[x]\ R_2[y]\ W_2[y]$

$S_1 : R_1[x] R_2[x] R_2[y] W_1[x] W_1[y] W_2[y]$

$S_2 : R_1[x] R_2[x] R_2[y] W_1[x] W_2[y] W_1[y]$

$S_3 : R_1[x] W_1[x] R_2[x] W_1[y] R_2[y] W_2[y]$

$S_4 : R_4[x] R_2[y] R_1[x] W_1[x] W_1[y] W_2[y]$

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Which of the above schedules are conflict-serializable ?

(a) S_1 adn S_2 (b) S_2 and S_3

(c) S_3 only (d) S_4 only

39. The following key values are inserted into a B+ –tree in which order of the internal nodes is 3, and that of the leaf nodes is 2, is the sequence given below. The order of internal nodes in the maximum number of tree pointers in each node, and the order of leaf nodes is the maximum number of data items that can be stored in it. The B+ –tree is initially empty.

10, 3, 6, 8, 4, 2, 1

The maximum number of times leaf nodes would get split up as a result of these insertions is

(a) 2 (b) 3

(c) 4 (d) 5

40. Let R and S be relational schemes such that

$$R = \{a, b, c\} \text{ and } S = \{c\}.$$

Now consider the following queries on the database:

I. $\pi_{R-S}(r) - \pi_{R-S}(\pi_{R-S}(r) \times s - \pi_{R-S,S}(r))$

II. $\{t \mid t \in \pi_{R-S}(r) \wedge \forall u \in r$ $(\exists v \in s\,(u = v[s] \wedge t = v[R-S]))\}$

III. $\{t \mid t \in \pi_{R-S}(r) \wedge \forall v \in r$ $(\exists u \in s\,(u = v[s] \wedge t = v[R-S]))\}$

IV. Select R. a, R. b
from R, S
where R.c = S.c

Which of the above queries are equivalent ?

(a) I and II (b) I and III

(c) II and IV (d) III and IV

2008

41. A clustering index is defined on the fields which are of type

(a) non-key and ordering

(b) non-key and non-ordering

(c) key and ordering

(d) key and non ordering

42. Let R and S be two relations with the following schema

R(P, Q, R1, R2, R3)

S (P, Q, S1, S2)

where {P, Q} is the key for both schemas. Which of the following queries are equivalent?

I. $\Pi_P(R \bowtie S)$

II. $\Pi_P(R) \bowtie \Pi_P(S)$

III. $\Pi_P(\Pi_{P,Q}(R) \cap \Pi_{P,Q}(S))$

IV. $\Pi_P(\Pi_{P,Q}(R) - (\Pi_{P,Q}(R) - \Pi_{P,Q}(S)))$

(a) Only I and II

(b) Only I and III

(c) Only I, II and III

(d) Only I, III and IV

43. Consider the following relational schemes for a library database:

Book (Title, Author, Catalog_no, Publisher, Year, Price)

Collection (Title, Author, Catalog_no)

With the following functional dependencies:

I. Title Author → Catalong_no

II. Catalog_no → Title Author Publisher Year

III. Publisher Title Year → Price

Assume {Autho, Title} is the key for both schemes. Which of the following statements is true?

(a) Both Book and Collection are in BCNF

(b) Both Book and Collection are in 3 NF only

(c) Book is in 2NF and Collection is in 3NF

(d) Both Book and Collection are in 2NF only

44. Consider a file of 16384 records. Each record is 32 bytes long and its key field is of size 6 bytes. The file is ordered on a non-key field, and the file organization is unspanned. The file is stored in a file system with block size 1024 bytes, and the size of a block pointer is 10 bytes. If the secondary index is buit on the key field of the file, and a multi-level index scheme is used to store the secondary index, the number of first-level and second-level blocks in the multi-level index are respectively

(a) 8 and 0 (b) 128 and 6

(c) 256 and 4 (d) 512 and 5

2007

45. Information about a collection of students is given by the relation **studInfo** (studId, name, sex). The relation **enroll** (studId, courseId) gives which student has enrolled for (or taken) what course (s). Assume that every course is taken by at least one male and at least one female student. What does the following relational algebra expression represent ?

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$\Pi_{\text{courseId}}((\Pi_{\text{studId}}(\sigma_{\text{sex = "female"}}(\text{studInfo})) \;\; \Pi_{\text{courseId}}$ $(\text{enroll})) - \text{enroll})$

(*a*) Courses in which all the female students are enrolled.

(*b*) Courses in which a proper subset of female students are enrolled.

(*c*) Courses in which only male students are enrolled.

(*d*) None of these.

46. Consider the relation **employee** (name, sex, supervisorName(with *name* as the key. *supervisor Name* gives the name of the supervisor of the employee under consideration. What does the following Tuple Relational Calculus query produce ?

{e. name | employee (e) $\wedge$

($\forall$ x) [$\neg$ employee (x) $\vee$ x.supervisorName

$\neq$ e.name $\vee$ x. sex = "male"]}

(*a*) Names of employees with a male supervisor.

(*b*) Names of employees with no immediate male subordinates.

(*c*) Names of employees with no immediate female subordinates.

(*d*) Names of employees with a female supervisor.

47. Consider the table **employee** (empId, name, department, salary) and the two queries Q_1, Q_2 below. Assuming that department 5 has more than one employee, and we want to find the employees who get higher salary than anyone in the department 5, which one of the statements is **TRUE** for any arbitrary employee table ?

Q_1 : Select e.empId
From employee e
Where not exists
(Select * From employee s Where s. department = "5" and s. salary > = e.salary)

Q_2 : Select e. empId
From employee e
Where e. salary > Any
(Select distinct salary From employee s Where s. department = "5")

(*a*) Q_1 is the correct query.

(*b*) Q_2 is the correct query.

(*c*) Both Q_1 and Q_2 produce the same answer.

(*d*) Neither Q_1 nor Q_2 is the correct query.

48. Which one of the following statements is FALSE ?

(*a*) Any relation with two attributes is in BCNF.

(*b*) A relation in which every key has only one attribute is in 2NF.

(*c*) A prime attribute can be transitively dependent on a key in a 3NF relation.

(*d*) A prime attribute can be transitively dependent on a key in a BCNF relation.

49. The order of a leaf node in a B^+ –tree is the maximum number of (value, data record pointer) pairs it can hold. Given that the block size is 1K bytes, data record pointer is 7 bytes long, the value field is 9 bytes long and a block pointer is 6 bytes long, what is the order of the leaf node?

(*a*) 63 (*b*) 64

(*c*) 67 (*d*) 68

50. Consider the following schedules involving two transactions. Which one of the following statements is **TRUE** ?

$S_1 : r_1(X); r_1(Y); r_2(X); r_2(Y); w_2(Y); w_1(X)$

$S_2 : r_1(X); r_2(X); r_2(Y); w_2(Y); r_1(Y); w_1(X)$

(*a*) Both S_1 and S_2 are conflict serializable.

(*b*) S_1 is conflict serializable and S_2 is not conflict serializable.

(*c*) S_1 is not conflict serializable and S_2 is conflict serializable.

(*d*) Both S_1 and S_2 are not conflict serializable.

2006

51. Consider the following log sequence of two transactions on a bank account, with initial balance 12000, that transfer 2000 to a mortgage payment and, then apply a 5% interest.

1. T1 start
2. T1 B old = 12000 new = 10000
3. T1 M old = 0 new = 2000
4. T1 commit
5. T2 start
6. T2 B old = 10000 new = 10500
7. T2 commit

Suppose the database system crashed just before log record 7 is written. When the system is restarted, which one statement is true of the recovery procedure ?

(*a*) We must redo log record 6 to set B to 10500

(*b*) We must undo log record 6 to set B to 10000 and then redo log records 2 and 3

(*c*) We need not redo log records 2 and 3 because transaction T1 has committed

(*d*) We can apply redo and undo operations in arbitrary order because they are idempotent

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52. Consider the relation account (customer, balance) where customer is a primary key and there are no null values. We would like to rank customers according to decreasing balance. The customer with the largest balance gets rank 1. Ties are not broken but ranks are skipped: if exactly two customers have the largest balance they each get rank 1 and rank 2 is not assigned.

Query 1 : select A. customer, count (B. customer) from account A, account B where A. balance < = B. balance group by A. customer

Query 2 : select A. customer, 1 + count (B. customer) from account A, account B where A. balance < B. balance 7group by A. customer

Consider these statements about Query 1 and Query 2.

1. Query 1 will produce the same row set as Query 2 for some but not all databases
2. Both Query 1 and Query 2 are correct implementations of the specification
3. Query 1 is a correct implementation of the specification but Query 2 is not
4. Neither Query 1 nor Query 2 is a correct implementation of the specification
5. Assigning rank with a pure relational Query takes less time than scanning in decreasing balance order and assigning ranks using ODBC

Which two of the above statements are correct ?

(*a*) 2 and 5 (*b*) 1 and 3

(*c*) 1 and 4 (*d*) 3 and 5

53. Consider the relation enrolled (student, course) in which (student, course) is the primary key, and the relation paid (student, amount) where student is the primary key. Assume no null values and no foreign keys or integrity constraints. Given the following four queries :

Query 1 : select student form enrolled where student in (select student form paid)

Query 2 : select student from paid where student in (select student from enrolled)

Query 3 : select E. student from enrolled E, paid P where E. student = P. student

Query 4 : Select student from paid where exists (select * from enrolled where enrolled.student = paid. student)

Which one of the following statements is correct ?

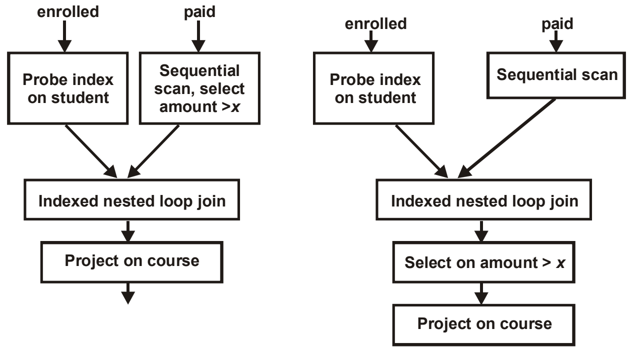
(*a*) All queires return identical row sets for any database

(*b*) Query 2 and Query 4 return identical row sets for all databases but there exist databases for which Query 1 and Query 2 return different row sets

(*c*) There exist databases for which Query 3 returns strictly fewer rows than Query 2

(*d*) There exist databases for which Query 4 will encounter an intergrity violation at runtime

54. Consider the relation enrolled (student, course) in which (student, course) is the primary key, and the relation paid (student, amount) where student is the primary key. Assume no null values and no foreign keys or integrity constraints. Assume that amounts 6000, 7000, 8000, 9000 and 10000 were each paid by 20% of the students. Consider these query plans (Plan 1 on left, Plan 2 on right) to "list all courses taken by students who have paid more than x."



A disk seek takes 4 ms, disk data transfer bandwidth is 300 MB/s and checking a tuple to see if amount is greater than x takes 10 µs. Which of the following statements is correct ?

(*a*) Plan 1 and Plan 2 will not output identical row sets for all databases

(*b*) A course may be listed more than once in the output of Plan 1 for some databases

(*c*) For $x = 5000$, Plan 1 executes faster than Plan 2 for all databases

(*d*) For $x = 9000$, Plan 1 executes slower than Plan 2 for all databases

55. The following functional dependencies are given :

$AB \to CD, AF \to D, DE \to F, C \to G, F \to E, G \to A.$

Which one of the following options is false ?

(*a*) $\{CF\}^+ = \{ACDEFG\}$

(*b*) $\{BG\}^+ = \{ABCDG\}$

(*c*) $\{AF\}^+ = \{ACDEFG\}$

(*d*) $\{AB\}^+ = \{ABCDFG\}$

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NUMERICAL TYPE QUESTIONS

2015

1. Consider the following relations :

Students

Roll Number	Student Name
1	Raj
2	Rohit
3	Raj

Performance

Roll Number	Course	Marks
1	Math	80
1	English	70
2	Math	75
3	English	80
2	Physics	65
3	Math	80

Consider the following SQL query.

SELECT S. Student_Name, sum (P.Marks)

FROM Student S, Performance P

WHERE S.Roll_No = P.Roll_No

GROUP BY S.Student_Name

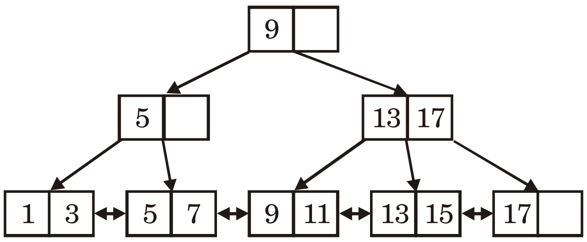
The number of rows that will be returned by the SQL query is _______.

2. Consider an Entity-Relationship (ER) model in which entity sets E_1 and E_2 are connected by an $m : n$ relationship R_{12}. E_1 and E_3 are connected by a $1 : n$ (1 on the side of E_1 and n on the side of E_3) relationship R_{13}.

E_1 has two single-valued attributes a_{11} and a_{12} of which a_{11} is the key attribute. E_2 has two single-valued attributes a_{21} and a_{22} of which a_{21} is the key attribute. E_3 has two single-valued attributes a_{31} and a_{32} of which a_{31} is the key attribute. The relationships do not have any attributes.

If a relational model is the derived from the above ER model, then the minimum number of relations that would be generated if all the relations are in 3NF is _______.

3. With reference to the B+ tree index of order 1 shown below, the minimum number of nodes (including the Root node) that must be fetched in order to satisfy the following query: "Get all records with a search key greater than or equal to 7 and less than 15" is _________



4. Consider a B+ tree in which the search Answer is 12 bytes long, block size is 1024 bytes, record pointer is 10 bytes long and block pointer is 8 bytes long. The maximum number of keys that can be accommodated in each non-leaf node of the tree is ____ .

2014

5. The maximum number of superkeys for the relation schema R(E,F,G,H) with E as the key is _____.

6. Given an instance of the STUDENTS relation as shown below:

StudentID	StudentName	StudentEmail	StudentAge	CPI
2345	Shankar	Shankar@math	X	9.4
1287	Swati	Swati@ee	19	9.5
7853	Shankar	Shankar@cse	19	9.4
9876	Swati	Swati@mech	18	9.3
8765	Ganesh	Ganesh@civil	19	8.7

For (StudentName, StudentAge) to be a key for this instance, the value X should NOT be equal to ______.

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ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*b*)	**2.** (*d*)	**3.** (*c*)	**4.** (*a*)	**5.** (*a*)	**6.** (*a*)	**7.** (*b*)	**8.** (*a*)	**9.** (*b*)	**10.** (*d*)
11. (*d*)	**12.** (*a*)	**13.** (*c*)	**14.** (*b*)	**15.** (*d*)	**16.** (*c*)	**17.** (*d*)	**18.** (*b*)	**19.** (*a*)	**20.** (*d*)
21. (*d*)	**22.** (*b*)	**23.** (*d*)	**24.** (*b*)	**25.** (*d*)	**26.** (*a*)	**27.** (*c*)	**28.** (*d*)	**29.** (*a*)	**30.** (*b*)
31. (*a*)	**32.** (*d*)	**33.** (*c*)	**34.** (*a*)	**35.** (*c*)	**36.** (*a*)	**37.** (*c*)	**38.** (*a*)	**39.** (*a*)	**40.** (*c*)
41. (*d*)	**42.** (*c*)	**43.** (*d*)	**44.** (*b*)	**45.** (*c*)	**46.** (*a*)	**47.** (*a*)	**48.** (*d*)	**49.** (*d*)	**50.** (*a*)
51. (*a*)	**52.** (*d*)	**53.** (*d*)	**54.** (*c*)	**55.** (*c*)	**56.** (*d*)	**57.** (*b*)	**58.** (*c*)	**59.** (*d*)	**60.** (*d*)
61. (*d*)	**62.** (*b*)	**63.** (*a*)	**64.** (*d*)	**65.** (*a*)	**66.** (*d*)	**67.** (*b*)	**68.** (*c*)	**69.** (*d*)	**70.** (*b*)
71. (*d*)	**72.** (*b*)	**73.** (*a*)	**74.** (*c*)	**75.** (*c*)	**76.** (*a*)	**77.** (*a*)	**78.** (*c*)	**79.** (*d*)	**80.** (*a*)
81. (*b*)	**82.** (*b,d*)	**83.** (*a*)	**84.** (*b*)	**85.** (*a*)	**86.** (*c*)	**87.** (*c*)	**88.** (*c*)	**89.** (*c*)	**90.** (*d*)
91. (*c*)	**92.** (*d*)	**93.** (*b*)	**94.** (*b*)	**95.** (*c*)	**96.** (*d*)	**97.** (*b*)	**98.** (*b*)	**99.** (*b*)	**100.** (*d*)
101. (*d*)	**102.** (*c*)	**103.** (*c*)	**104.** (*a*)	**105.** (*a*)	**106.** (*b*)	**107.** (*a*)	**108.** (*a*)	**109.** (*a*)	**110.** (*d*)
111. (*a*)	**112.** (*d*)	**113.** (*d*)	**114.** (*d*)	**115.** (*d*)	**116.** (*a*)	**117.** (*b*)	**118.** (*d*)	**119.** (*c*)	**120.** (*c*)

Numerical Type Questions

1. 65	**2.** 1	**3.** 12	**4.** 300	**5.** Rs.7500	**6.** Yes	**7.** yes	**8.** 26	**9.** 8

EXERCISE – II

MCQ Type Questions

1. (*d*)	**2.** (*b*)	**3.** (*c*)	**4.** (*a*)	**5.** (*c*)	**6.** (*a*)	**7.** (*a*)	**8.** (*b*)	**9.** (*b*)	**10.** (*d*)
11. (*d*)	**12.** (*a*)	**13.** (*b*)	**14.** (*c*)	**15.** (*a*)	**16.** (*c*)	**17.** (*b*)	**18.** (*a*)	**19.** (*d*)	**20.** (*d*)
21. (*c*)	**22.** (*a*)	**23.** (*d*)	**24.** (*c*)	**25.** (*c*)	**26.** (*c*)	**27.** (*a*)	**28.** (*c*)	**29.** (*c*)	**30.** (*a*)
31. (*d*)	**32.** (*b*)	**33.** (*b*)	**34.** (*c*)	**35.** (*b*)	**36.** (*a*)	**37.** (*a*)	**38.** (*b*)	**39.** (*b*)	**40.** (*c*)
41. (*a*)	**42.** (*c*)	**43.** (*c*)	**44.** (*c*)	**45.** (*b*)	**46.** (*c*)	**47.** (*b*)	**48.** (*d*)	**49.** (*b*)	**50.** (*c*)
51. (*c*)	**52.** (*c*)	**53.** (*a*)	**54.** (*c*)	**55.** (*c*)					

Numerical Type Questions

1. (2)	**2.** (4)	**3.** (5)	**4.** (50)	**5.** (8 to 8)	**6.** (19 to 19)

EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. The maximum size would be the case where every tuple of R is combined with every tuple of S to get mn tuples. The minimum case is when no tuple of R is able to combine with any tuple of S.
Inter section in set algebra corresponds to 'anding' in boolean algebra.

2. Give the two relation R_1 (A, B) and R_2 (C, D), and find the result

$$R_1A = C,$$
$$AB = DR_2$$

then,

$\therefore$ $R_1 (A, B) \cap R_2 (C, D)$

then finally find the result

$$= R_1A\,R_2C, R_1A\,R_2D, R_1B\,R_2C, R_1B\,R_2D$$
$$= R_1A\,(R_2C, R_2D)\,R_2B\,(R_2C, R_2D)$$

Both are common property

$$= R_1A = C$$
$$AB = R_2\,D$$

then find result $(R_1 \cap R_2)$

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3. Simply atomic value non multivalue attributes. Generally ERD is design level task and relation model is implementation. Whenever relational model is implemented. From ERD we get rid of multivalued attribute. Multivalued attributes are removed as M : N binary relation by repeating the tuples and hence relation becomes 1 NF.

4. Using the two relationships mother and father provides us a record of a child's mother, even if we are not aware of the father's identity ; a null value would be required if the ternary relationship parent is used. Using binary relationship sets is preferable in this case.

5. The network model are complicated by physical keys, but the relational model is faster because it uses logical keys. So we always used logical keys because this key is faster.

6. A relational model which allows non-atomic domains is hierarchical data model.

7. Let R (a, b, c) and S (d, e, f) be two relations in which d is the foreign key of S that refers to the primary key of R.

We know that insert in to S and delete from R can cause violation.

8. The traditional approach to information proceding have file oriented, programs are dependent on files and inflexible. Then there is common sharing of data among the various applications are not true.

9. The database environment has all the data files are separate save in database. Then all data record, are save separate file and SELECT and UPDATE are separate.

10. The particular application views the data from the database that the application uses is a sub schema are used.

11. Data manipulation language (DML) statement are used for managing data within schema objects. Some example. SELECT, INSERT, UPDATE, DELETE, MERGE, CALL, EXPLAINPLAN, LOCK TABLE all are DML command language.

12. The relational model uses some unfamiliar terminology. A tuple is a equivalent to record because the "Tuple, record, row, these all are same".

13. The database Administrator is in effect, the coordinator between database and users are always because the web application are used data back end and front end use client users.

14. The network structure is a newer type of organization structure vived as less hierarchical (multi "flat"), more decentralized, and more flexible than other structures. In a network structure, managers coordinate and control relationships that are both internal and external to the firm.

15. The main advantage of the database approach is

(*i*) Elimination of the data redundancy

(*ii*) Ability to associate related data

(*iii*) Increases security

(*iv*) Easily store data

16. The important to the operation of a database to the hardware consists of a set of physical electronic devices such as computers. It is impossible to implement the DBMs without the hardware devices. In a Network, a powerful computer with high data processing speed and a storage device with large storage capacity is required as database server.

17. The database management system are intended to

(*i*) eliminate data redundancy

(*ii*) establish relationship among records in different files.

(*iii*) Manage file access those all are properly used in database.

18. The main characteristics of the relational data base model are

(*i*) Tables

(*ii*) Complex logical relationships

(*iii*) Records

All are main characteristics but tree like structure not a part of relational data model. Then the tree like structure are not true.

19. The main responsibility of the utility component of DBMs software are

(*a*) Removing flagged records for deletion

(*b*) Creating and maintaining the data dictionary

(*c*) Monitoring performance

(*d*) Updating data all are responsibility of DBMs, creating the physical and logical designs are not the responsibility.

20. The main of a DBMs is the software. It is the set of programs used to handle the database and to control and manage the overall computerized data base.

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(*i*) Database Manipulation Language

(*ii*) Query Language

(*iii*) Utilities

(*iv*) Programing Language.

These all are DBMs software.

21. The database Administrator's manager are following tasks present a prioritized approach for designing, implementing and maintaining, database design. Backing up the database, performance monitoring all are true database function.

22. The data dictionary are provide in formula about

(*i*) Where data is located

(*ii*) Who owns or is responsible for the data

(*iii*) How the data is used and data dictionary doesn't provide information about the size of the storage disk.

23. The serious problem of file management system are

(*i*) Data redundancy

(*ii*) Data inconsistency

(*iii*) Difficulty in Accessing data

(*iv*) Data isolation

(*v*) Integrity problems

(*vi*) Atomicity problem

(*vii*) Lack of data independence.

They all are serious problem of management systems.

24. A database management system can do more than a record management system and this system software for creating and managing database. The DBMs provides users and programmers with a systematic way to create, retrieve, update and manage data.

25. In a large DBMs

(*i*) each user can "see" only a small part of the entire data base

(*ii*) each subschema contains every field in the logical schema

(*iii*) each user can access every subschema

All are large DBMs.

26. Transparent DBMs it is one, which keeps its physical structure hidden from user and can not hide sensitive information from users.

27. The main goals of the design of the logical schema include being able to access data efficiency, avoiding data inconsistency, data efficiently.

28. A top to bottom relationship among the items in a database is established by a

(*a*) Hierarchical schema

(*b*) Network schema

(*c*) Relational schemas are mainly of the data are relationship of the database.

29. A network schema restricts the structure of one to many relationships maintains.

One to many

30. In a relational schema, each tuple is divided into fields called domains. Domains are divided in row and column. The row and column combination are domains.

31. In a bank on line statement showing to the Customer's Charge Account to respond to an inquiry is an forecasting report. This type of report show online internet banking application provide.

32. If a field size is too small for the longest piece of data to be entered, then field will automatically expand in database, then they all options are false.

(*d*) none of these

33. The logical database structures include chain, table spaces, schema, objects, data block, extents and segments are all logical database structure.

34. There are many relational database of DBMS dBASE IV, 4^{th} Dimension, Reflex and Relational Model, Keys, Transaction all are relational data but foxpro are not. This is the platform.
We know that insert into S and delete from R can cause violation.

40. Manager's salary details are hidden from the employee. This is external level data hiding property in the database.

41. Trigger is a SQL procedure that initiates an action (fires an action) when an event (INSERT DELETE or UPDATE) occurs. Since triggers are event driven specialized procedures, they are stored in and managed by the DBMs. The condition the system tests for the validity of the database user.

42. Relational algebra has the same power as safe relational calculus.

43. Let adjacency relation of vertices in a graph is represented in a table Adj (x, y).

We know that "List of all vertices reachable from a given vertex" is a query can't be expressed by a relational algebra expression of constant length.

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44. Clearly $t \notin R_1$ so

$t \in R_2$ or R_3 or R_4 .... R_n or $t \in R_2$

and R_3, R_3 and R_5, R_3 and R_5 and R_7 .....

Hence it is completely not a safe.

45. Clearly $\{t / t \in r \wedge (t[A] = 10 \wedge t[b] = 20)\}$

is equal to $\sigma(A = 10)^{(r) \cap \sigma}(B = 20)^{(r)}$

46. Condition C_1 may be different from C_2.

47. B - trees follows secondary index.

Non - procedural Query language is also called the Domain calculus.

Closure set of attributes is also called *functional dependency*. One of the Relational Algebraic operations is also called *natural join*.

48. *a*, *b*, *c* are same type of equivalence expressions.

49. In the database contains a complete record of all activity that affected the contents of a database during transaction log affects certain period of time.

50. Embedded pointer is a pointer set in a data record instead of in a directory.

51. Updating a database means update the record and revising the file structure and update the needful data. Then option (*a*) is correct.

52. The data dictionary software package is called DB/DC dictionary, TOTAL, ACCESS, RETRIVE they all are software package. The answer are all of these.

53. A logical schema won't exist in your database. A logical schema is a design-centric database structure built to meet your business requirements.

Your model should be properly normalized.

54. A subschema lets the user, have access to different areas of application in which the user designed. The areas that are included in an application are set types, record types, data items and data aggregates. Following are the reasons used by subschema.

1. Subschema provide different views of the data to the user and programmer, who do not need to know all the data contained in the entire database.
2. Subschemas enhance security factors and prohibit data compromise.
3. Subschemas aid DBA while assuming data integrity. Then the high sensitive information by omitting fields from the subschema description.

55. A data dictionary is a special file that contains names of all fields in all files and data types of all fields in all files but the answer is width of all fields in all files.

56. A query is an inquiry into the database using the SELECT statement. A query is used to extract data from the database in a readable format according to the user's request. For instance, if you have an employee table, you might issue a SQL statement that returns the employee who is paid the most. This request to the database for usable employee information is a typical query that can be performed in a relational database.

57. Data integrity control requires the use of passwords to prohibit unauthorized access to the file.

58. Information can be transferred between DBMS and graphics program in the database.

59. A race condition is an undesirable situation that occurs when a device or system attempts to perform two or more operations at the same time, but because of the nature of the device or system, the operations must be done in the proper sequence in order to be done. Then they all options are none of these.

60. Addresses are considered Alpha fields only.

61. Data is often entered in a format such as 81/12/30 because this type of format is not used anywhere.

62. d-BASE contains complete record of all activity that affected the contents of a database during a certain period of time.

63. If two users wish to update the same record at the same time, they are prevented from doing so by jamming.

64. Relational database management (RDBM) Package manages data in more than one file at once. The file used tables, relations both are used in RDBM.

65. γ has no duplicates and *s* is non empty.

66. The PROJECT command will create new table that has to find row in a table but the given options are not matched.

67. Information system theory specific that information delivered to a target human reader must be Timely, Accurate and Relevant. Report generation software targets the final requirement by making sure that the information delivered is presented in the way most readily under stood by the target reader, the print files and paper.

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68. Maximum signed form digit number (± 9999) can be stored in two bytes as integer and will require 5 bytes storage space.

69. They all are the incorrect.

70. Find the closure set of all the options given, if any closure covers all the attributes of the relation R then that is the key.

Step 1 : Equate an attribute or attributes to X for which closure needs to be identified.

Step 2 : Take each FD (functional dependency) one by one and check whether the left side of FD is available in X, if yes then add the right side attributes to X, if it is not available.

Step 3 : Repeat step 2 as many times as possible to cover all FD's.

Step 4 : After no more attributes can be added to X declare it as the closure set.

FD : $C \to F$, $E \to A$, $EC \to D$, $A \to B$

Find closure set for CD.

$$\begin{aligned} X &= CD \\ &= CDF\ (C \to F) \end{aligned}$$

No more attributes can be added to X. Hence closure set of

$$CD = CDF$$

Find closure set for EC

$$\begin{aligned} X &= EC \\ &= ECF\ \{C \to F) \\ &= ECFA\ \{E \to A\} \\ &= ECFAD\ (EC \to D) \\ &= ECFADB\ \{A \to B\} \end{aligned}$$

Closure set of EC covers all the attributes of the relation R.

71. Given the schema R = (STUV) and the dependencies

$S \to T$, $T \to U$, $U \to V$, and $V \to S$

The fully functionaly dependent all area.

$S \to T$ {T is fully functional dependent on S}

$T \to U$ {T is fully functional dependent on S}

$U \to V$ {T is fully functional dependent on S}

$V \to S$ {T is fully functional dependent on S}

R = (R_1 and R_2) be a decomposition such that $R_1 \cap R_2 = \phi$ then decomposition the 2NF normal form and they have transative dependency then have 3NF.

72. The normal form is considered adequate for normal relationship database design through 3 NF.

73. A functional dependency of the form $x \to y$ is trivial because FDX $\to$ Y holds, where y is a subset of x, then it is called a trivial FD. Trivial FDs always hold.

74. BCNF is the strongest form of 3NF. BCNF is needed, in that case, when more than one candidate keys are there and an attribute is overlapped between them, we have to remove that overlapped attribute. BCNF, then it is also in 3NF

75. If relation schema is in BCNF then it is also in 3NF, 2NF, 1NF. Every binary relation is in BCNF, (because left hand side is always the superkey which satisfy the condition for BCNF).

76. Normalization is reduce the redundency of the data and increase the efficiency of the data, then eliminate redundancy.

77. Functional dependencies are a generalization of a key dependencies.

78. The relation is in BCNF if in every non-trivial functional dependency $x \to y$ is a super key. BCNF is free redundency and need be dependency preserving.

79. The report generators are used always retrive information from files and answer queries in database.

80. A phone number in a database is record.

81. According to relational algebra that is the data are the mainly of the data are USER, BOOKS ISSUED, the determined as

σ Author Name (Book (σ name town) = Delhi

(Users ISSUED)))

According to query process.

82. A relation database which is in 3NF may still have undesirable data redundancy because

1. Non-trivial functional dependencies involving prime attributes on the right side.
2. Non-trivial functional dependencies involving only prime attributes.

83. R(a, b, c, d) from here we can derive the key as [ab]

No partial dependencies = 2NF

No transitive dependencies = 3NF

as [ab] is the only minimal key available for the relation, so it is the super key. If [ab] is the superkey then R(a, b, c, d) is in BCNF also.

So answer is d.

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84. According to the FD the X $\rightarrow$ Y then X $\rightarrow$ Z the subset of the dependency then the given instance X and YZ the dependency YZ $\rightarrow$ X, and Y $\rightarrow$ Z that are the suitable fully functional dependency then

$$YZ \rightarrow X \text{ and } Y \rightarrow Z$$

85. The arrival sequence access path is based on the order in which the records arrive and are stored in the file. For reading or updating, records can be accessed.

Sequentially, where each record is taken from the next Sequential Physical Position in the file.

86. Indexing is a data structure technique to efficiently retrieve records from the database files based on some attributes on which the indexing has been done.

Indexing in database system is similar to what we see in books and established an index for a file.

87. The master list of an index file in database has front of the data file and has a number assigned to each record in database.

88. The index consists of the list of keys and pointers to the master list. So both are (*a*) and (*b*) option.

89. To the file, hold a list it is necessary to decide which fields will be used as sort or index keys, identify the records in the list, identify the name, width and type of the fields of each record, then answer is all of these.

91. Versatile report generators can provide calculations in database.

92. Sort/Report generators are faster than index/ report generator or require more disk space than index/report generators. Then answer (*a*) and (*b*).

94. The shape and structure of something as distinguished from its material. The shape the width of each field, then answer is (*b*).

95. The Designer of a form includes data in DBMs.

96. The calculation is embedded in a form the result of the calculation are printed in report.

97. Multiple form management system allows to perform different operation along with checks for the unauthorized access.

98. The physical location of a record is determined by hashed file that mathematical formula that transforms a file key into a record location.

99. If the two file is joined into a third file then they have field in common.

100. The main idea behind computer files is that it is convenient to store information together.

101. Sometimes we need to change the data type of a column. To do this, we use the ALTER TABLE Modify column command.

Syntax :

ALTER TABLE table_name

MODIFY column_name "New Data Type".

102. The "SET ALTERNATIVE" < FILE NAME > "Command" CLOSE ALTERNATE.

104. We have table named, it has 30 records if we give following command in order

GO BOTTOM then SKIP 3

What happen if this both command executes in the order.

$\therefore$ will it come to record number 3rd or 28th.

3rd because it will goes bottom then skip 3 will execute so went to 3rd record, or it will come upto 28th record.

106. Switch between Edit mode (with insertion point displayed) and navigation mode in a data sheet. When working in a form or report, press ESC to leave navigation mode.

107. Following commands permanently delete the record marked for deletion from the database, field records are marked for deletion, then the database is packed to permanently remove the records that use PACK.

108. The function is used to convert a data to a character string used DTOC( ), then option is (*a*) DTOC( ).

110. The transaction reads a data item after it is written then we can't recover the errors in this situation by an uncommitted transaction.

111. The given SQL is

Select distinct $a_1, a_2, \ldots\ldots a_n$

From $r_1, r_2 \ldots\ldots r_n$

where P

All possible combination of tuples from r_1, r_2, r_n is denoted by $r_1 \times r_2 \times \ldots\ldots \times r_n$.

If P is a predicate then select the all condition is denoted by $\sigma_p(r_1 \times r_2 \times \ldots\ldots \times r_n)$

If we want to select only some tuples in the relation then composite expression for above SQL is

$a_1, a_2, \ldots\ldots a_n\ \sigma_p\ (r_1 \times r_2 \times \ldots\ldots \times r_n)$

112. The functional dependency is as follows

Date_of_Birth $\rightarrow$ Age ... 1

Age $\rightarrow$ Eligibility ... 2

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Name → Roll_Number ... 3

Roll_number → Name ... 4

Course_Number → Course__Name ... 5

Course_Number → Instructor ... 6

(Roll_Number, Course_Number) → Grade Date _of _Birth → Age it violates the 3 NF and 2 NF).

So the relation (Roll_Number, Name, Date_of_Birth, Age is in 1 NF.

113. According to Serializable condition the data item A1, A2, A3 are following execution schedule of transaction T1, T2 and T3, then condition is serializable otherwise not serializable and this transaction is not serializable transaction.

114. The given query computes the names of girl students with more marks than all the boy students.

115. If R = {A, B, C, D, E, H}

Functional dependency

$= \{A \rightarrow B, BC \rightarrow D, E \rightarrow C, DD \rightarrow A\}$

The candidate keys of R is AEH, BEH, DEH because both of these generates the

{A, B, C, D, E, H}

116. The records of a file are physically ordered on a non key field which doesn't have a distinct value for each record that field is called the clustring field.

117. The functional dependency of the relation is

name, course no. → grade ..... (*a*)

roll no., course no. → grade ..... (*b*)

name, roll no → ..... (*c*)

roll no. name → ..... (*d*)

$\begin{cases} A \rightarrow B \\ B \rightarrow C \end{cases}$

Dependency c and d are transitive but they are prime attribute so we will break this relation up to 3 NF.

118. According to the levels of abstraction, the schema at the intermediate level is called conceptual schema.

119. In the above question R (A, B, C, D) is decomposed into R1 (A, B) and R2 (C, D) and there are only two FD's A → B and C → D.

So the decomposition is dependency preserving.

Lossless-join Decomposition : Decomposition of R into R1 and R2 is a lossless join decomposition if at least one of the following functional dependencies are in F_t (clouser of functional dependencies)

R1 ∩ R2 → R1

or

R1 ∩ R2 → R2

In the above question R (A, B, C, D) is decomposed into R1 (A, B) and R2 (C, D) and R1 ∩ R2 is empty.

So the decomposition is not lossless.

NUMERICAL TYPE QUESTIONS

1. Keep the records of A fixed and inset the records of the other files in A in sorted order. This would require

20 +15 + 5 + 25 = 65 record movement.

4. The value of account A = 100

B = 200

If these transactions are executed serially then insert the value in transaction T_1

T_1 : lock - S (100);

read (A);

unlock (100);

∴ when unlock then showing 100 value

lock -S (200);

read (B);

unlock (200);

∴ when unlock then showing 200 value

display (100 + 200);= Rs. 300

5. The given Aggregation Function then

Average salary = 10,000 + 7,000 + 8000 + 5000

$= \frac{30,000}{4} = \text{Rs.}7500$

Average salary = Rs.7500

6. The decomposition is lossless-join decomposition if either of following holds.

$R_1 \cap R_2 \rightarrow R_1$

$R_1 \cap R_2 \rightarrow R_2$

- $R_1 \cap R_2$ is A

  ∴ which is common attribute.

- A → B is the Functional Dependency in F. By Augmentation rule.

  Then A → AB which is relation R_1.

  Then $R_1 \cap R_2 \rightarrow R_1$ is satisfied. The decomposition is Lossless-Join decomposition.

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7. We can continue to swapping non conflict instructions :

- Swap the read (B) instruction of T_1 with read (A) instruction of T_2.
- Swap the write (B) instruction of T_1 with write (A) instruction of T_2.
- Swap the write (B) instruction of T_1 with the read (A) instuction of T_2.

The final result after swaps is shown :

T_1	T_2
read (A)	
write (A)	
read (B)	
write (B)	
	read (A)
	write (A)
	read (B)
	write (B)

This is serial schedule.

8. Size of child pointer = 6 byte

Size of search field value takes 14 bytes

Block size = 512

The order of internal node = P

$\Rightarrow$ $(P - 1)\ 14 + P * 6 \leq 512$

$= P = 260$

9. The minimum and maximum number of tuples in given natural join operation is minimum of these min (120, 8) = 8

EXERCISE – II

MCQ TYPE QUESTIONS

1. The select operation is SQL is same as

The projection operation is relational algebra except that SELECT is SQL retain duplicate.

2. Consistency : Consistancy properly ensure that any transaction will bring the database from one valid state to another

read (x); x : = x – 50; write (x) ;	
	read (y); y : = y + 50; write (y)

4. Undo T_3, T_{ij} Redo T_2

T_1 and T_3 will be undo, the transaction after checkpoint is T_2 which must be redo.

5. $\{P, S\} \rightarrow \{S\}$

On finding closure of F^+, we can define $\{P, S\} \rightarrow \{S\}$ is trival functional dependency.

6. Var P = Var Q = FALSE Initially.

Assume that, process X is willing to enter into critical section. So it makes $V_{ar}P$ = True, then if processor switches to process Y, then process Y can enter into critical section.

After entering into the critical section, then if processor switches to process X, then process X also can enter into the critical section.

It is clearly showing that both are in critical section at a time which leads to "failing to guarantee material exclusion"

To enter into the critical section process X is not waiting for process Y and vice versa. So we can "Prevent Deadlock" so, overall, option (*a*) is correct (or) TRUE.

7. Inner query collects capacities of all the theatres and in outer query we are filtering the tuples with the condition "capacity>=All". So the theatres which are having maximum capacity will be satisfy the conductivity and they will.

8. T_2 is reading the value written by T_1 and getting committed before T_1 commits. So it is non-recoverable schedule.

9. R (EFGHI, JKLMN)

F = {

$EF \rightarrow G$

$F \rightarrow IJ$

$EH \rightarrow KL$

$K \rightarrow M$

$L \rightarrow N$

}

$(EF)^+$ = EFGIJ, E & F

Together functionally derive GIJ and if we observe given FDs, H can't be determined by any other attributes. So H must be part of all the (candidate) keys. H along with E determines K and L, K & L functionally determine M and N respectively.

$\therefore$ $(EFH)^+$ = EFGIJHKLMN

$\therefore$ EFH is the only candidate for key.

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10. S_1 : Manger (<u>Name</u>, Dept ID)

Department (DeptName, <u>Deptid</u>)

In given relation Manager DeptID is a foreign key referencing Deptid (P.K) of relation Department.

Let's declare the foreign key by an equivalent check assertion as follows:-

```
CREATE TABLE Manager (
    Name Varchar (10)
    DeptID INT (6) check (DeptID IN (select
                         Deptid from Department)),
    PRIMARY KEY (Name)
    );
```

The above use of check assertion is good to declare the foreign key as far as insertion is considered for relation manager (will not insert any tuple in Manager containing such DeptID value which is not present in any tuple of Department).

But the above declaration will fail to implement changes done in Department relation in terms of deletion & updation. For an instance if a deptid present in Department gets deleted, then respective reference in Manager should also be deleted.

$\therefore$ S_1 is false

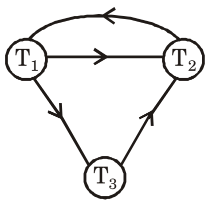
S_2 : The given table definition is not valid due to invalid foreign key declaration. Attribute a is declared as foreign key which is a single valued attribute and it is referencing the primary key (<u>ab</u>) of relation R (<u>a, b</u>, c), which is a composite key.

A single value attribute cannot refer a composite key.

4 S_2 is false.

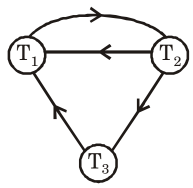
11. If there is a cycle in precedence graph, then the schedule is not conflict serializable.

(*a*)



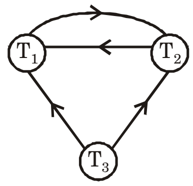
Not conflict serializable

(*b*)



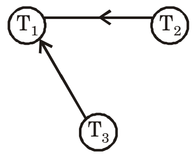
Not conflict serializable

(*c*)



Not conflict serializable

(*d*)



it is conflict equivalent to
$T_2 \rightarrow T_3 \rightarrow T_1$ &
$T_3 \rightarrow T_2 \rightarrow T_1$

12. S :True

Consider any table R with two attributes R (A.B)

The possible FD sets are

F_1 {
  $A \rightarrow B$
}

Key :A and is in BCNF

F_2 {
  $B \rightarrow A$
}

Key : B and is in BCNF

F_3 = {
  $A \rightarrow B$
  $B \rightarrow A$
}

Key : A & B it is in BCNF

F_4 = {

}

Key : AB and is in BCNF

If a table is in BCNF it is also in 1NF, 2NF and 3NF also

S_2 : False

First FD set cannot cover second FD set because in second FD set AB can functionally derive E but that is not happing in first FD set.

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13. In the inner sub query, "employees" and "departments" tables are joined by "using" clause (first Cartesian product of those two tables will be done and then and wherever these is a match on the dept-ids that tuple will be filtered). After this, the tuples of the resultant table will be filtered by using the condition "location-id=1700" and then will grouped on deptid(all the tuples having equal values under dept-id will come into one group). After grouping, the columns dept-id in location-id 1700 and maximum of hire dates in that respective dept-id will be selected. Format of the tuples in the resultant table will be dept-id in location-id 1700 along with the latest hire date in the respective dept (two columns). Outer query takes each tuple from "employees" table and it will check whether dept-id and hire-date pair for this tuple is contained in the table given by inner sub query. If this is the case it will display the last-name of respective employee

IN operator compares one or multiple expressions on the left side of the operator to a set of one or more values on the right side of the operator. When using multiple expressions (like 2 columns - pair wise comparison), the number and data types of expressions in the list must match on both sides of the operator.

14. Since, the graph contains no cycle, that's why it is called conflict serializable.

Since, in this schedule, transaction reads data item previously written by former transactions. The commit operation of former transactions occur before later transactions, hence it is recoverable also. That's why 'S' is both conflict serializable and recoverable.

15. A join between r(R) and s (S) using nested loop method will be as follows.

For each tuple r in R do

For each tuple s in S do

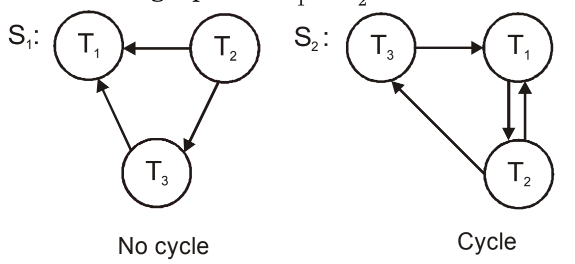
If r and s satisfy the join condition then output the tuple <r, s>

This algorithm will involve $n_r * b_s + b_r$ block transfers and $n_r + b_r$ seeks, where b_r and b_s are number of blocks in relations R and S respectively and n_r is number of tuple in relation R. Now to have less block accesses, n_r should be less and it is already given that $|R|<|S|$. Relation r(R) should be in the outer loop to have fewer number of disk block accesses.

16. When we execute query R * from (select distinct from S) as S_1 where $R \cdot a = S_1 \cdot a$, it will give the same results.

17. A prime attribute or key attribute of a relation scheme R is an attribute that appears in any of the candidate key of R, remaining attributes are known as non-prime or non-key tribute.

18. Precedence graph for S_1 & S_2 are as follows



$\therefore$ Only S_1 is conflict serializable.

19.

$$\underbrace{\Pi_{empId}(employee) - \overbrace{\Pi_{empId}(employee \bowtie_{empId=eID) \wedge (empAge \le depAge)} dependent)}^{A}}_{B}$$

Part A of the above given relational algebra query will give the set of empIds of those employees whose age is less than or equal to the age of some of his/her dependents. Now when set of empIds of all employees minus set of empIds obtained from part A is done, then we get the set of empIds of employees whose age is greater than that of all of his/her dependents.

20. The outer query will return the value (names of employees) for a tuple in relation E, only if inner query for that tuple will return no tuple (usage of NOT EXISTS).

The inner query will run for every tuple of outer query. It selects cust-id for an employee E, if rating of customer is NOT good. Such an employee should not be selected in the output of outer query.

So the query will return the names of all those employees whose all customers have GOOD rating.

21. **Clustered Index:** Sorts and store the data rows in the table or view based on their key values the column on which clustered index is created need not be primary key but it should always be not null.

22. Find the distinct names of all students who score more than 90% in the course no. 107.

1. SQL query

Condition would give all S. name having score > 90 and attending course no. 107 and DISTINCT S.sname will give distinct student names.

TRUE

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2. Relational Algebra

πsname gives projection of all students meeting the condition and 'π' gives DISTINCT value.

TRUE

3. Tuple Calculus

gives DISTINCT student name having score > 90 and course no. is 107. TRUE

4. Domain Calculus

domain calculus is equivalent to relational algebra and provide distinct value for the query.

TRUE

23. Given: Pre order traversal sequence of a binary search 30, 20, 10, 15, 25, 23, 39, 35, 42

To find: Post order traversal sequence

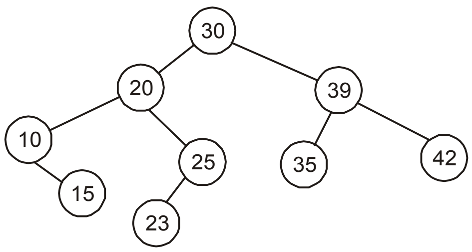
Analysis: Pre order traversal sequence is formed as follows:

First root, then left subtree, then right subtree and then recursively do this. This means 30 is the root. constructing tree for this:

(we also know, all elements <30 will be in left subtree and > 30 in right subtree, property of binary search tree) Left subtree now is 20, 10, 15, 25

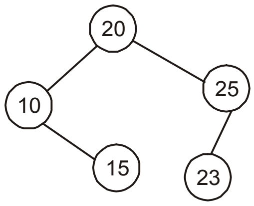
⇒ 20 is root of left subtree 10, 15 are its left subtree, 23, 25 its right subtree.

Going like this, use can construct the tree.

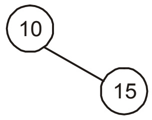


Getting the order:

Moving to left subtree of tree rooted at 30



Moving to left subtree



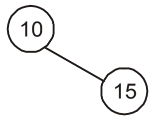
Left subtree is empty, so going to right subtree

15

Left is right subtree and are empty, so print root

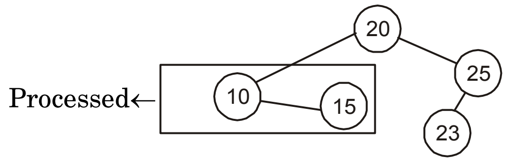
15

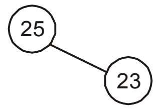
Going back from right subtree



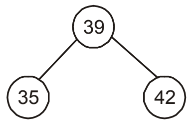
Print root 10

Backing from left subtree, going to right subtree





Similarly, here, we will print 23, 25 & go back to original root. Then move to its right subtree



Here, print order is 35, 42, 39 and at last root 30.

Hence, the order is:

15, 10, 23, 25, 20, 35, 42, 39, 30.

So, the answer is (*d*).

25. Row cannot have attribute more than one value.

26. Option A is false since BCNF is stricter than 3NF

Option B is false since the definition given here is of 2NF

Option C is true, since BCNF is stricter than 3NF, every relation in BCNF satisfies all the properties of 3NF.

27. Foreign key should refer to existing keys so:

$$\pi_B(r_1) - \pi_C(r_2) = \phi$$

28. S

Borrower	Bank_Manager
Ramesh	Sunderajan
Suresh	Ramgopal
Mahesh	Sunderjan

T

Bank_Manager	Loan_Amount
Sunderajan	10000.00
Ramgopal	5000.00
Sunderjan	7000.00

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After executing the given query, the output would be

Borrower	Bank_Manager	Loan_Amount
Ramesh	Sunderajan	10000.00
Ramesh	Sunderajan	7000.00
Suresh	Ramgopal	5000.00
Mahesh	Sunderajan	10000.00
Mahesh	Sunderajan	7000.00

30.

X	Y
1	1
2	3
3	7
4	15
5	31
6	63
7	127

31. It can be used to prevent packet looping because when Time-to-Live field reaches to '0' means when there is no any type of acknowledgement received, then TTL decrementet continuously and when it reaches '0', then it stop looping of packet.

33. Minimum number of keys = $\left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor \frac{5}{2} \right\rfloor = 2$

34. SELECT pid

FROM Reservation

WHERE class = 'AC'

EXISTS (SELECT * FROM Passanger WHERE age > 65 AND Passanger pid = Reservation. pid

Query A select where age > 65 and Passanger. pid matches with Reservation. pid

means Rahul, Sourav and Anil

Now we check in Query B from Reservation that class should 'AC' and Exit in Rahul, Sourav, Anil,

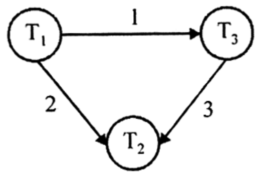
So that class of Rahul and Anil is AC.

So pid is 1 & 3.

35. 2-phase locking not ensure deadlock but it can ensure conflict serialization some how

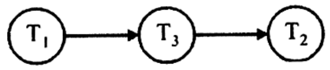
but Time-stamp locking can ensure both conflict searialization and deadlock.

36. By using the precedence graph we solve thus problem



1. Read (x) in T_1 is followed by write (x) in T_2
2. Read (x) in T_1 is followed by write (x) in T_3
3. Read (y) in T_3 is followed by write (y) in T_2

So it in clear from precedence graph



37. $B \rightarrow A$ where R (A, B, C) and S (B, D, E)

$A \rightarrow C$

R contains 200 tuples

S contains 100 tuples

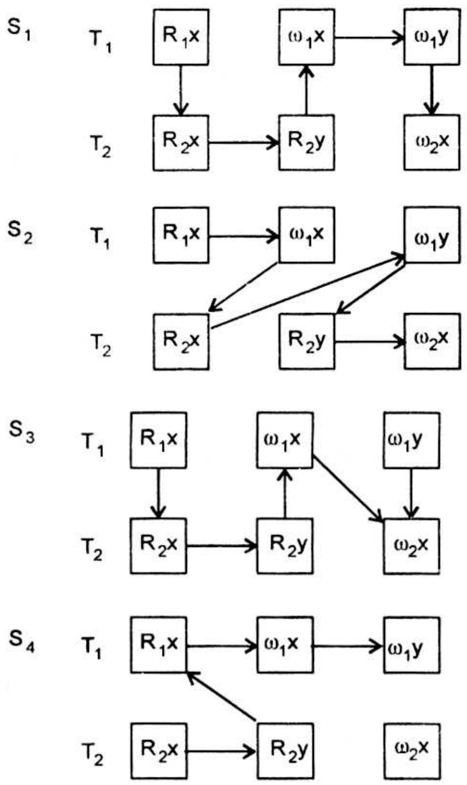
Natural join $\bowtie$ = R $\bowtie$ S = $\pi[\sigma (R \times S)]$

So R = 200 tuples and S = 100 tuples.

So R $\bowtie$ S = 100

[common in both or we can say Distinct equality b/w All common Attribute]

38.



Clearly S_2 and S_3 are conflict - serializable.

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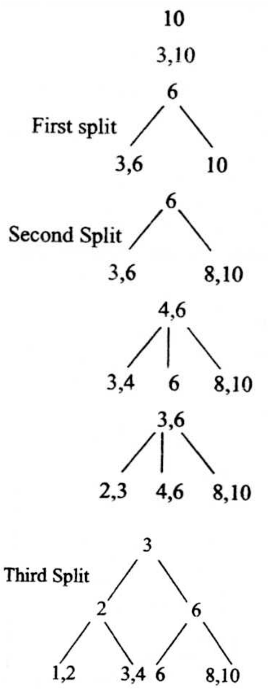
39.



40. Query (II) and (IV) given equal data so they are equivalent.

41. Clustering index in unique and is created automatically when primary key is declared it is ordered.

42. I, II and III express following quering select P from the join (natural) of R and S, Which can be expressed as select P, then joining (II) or selecting P from the union of R & S by first selecting R & Q (key values).

43. A table in the second normal from if all the non key fields fully dependent on whole key. A table is in third normal from if all the non key fields of the table are independent of all other non key field of table.

Alternately

If we have a function dependency of kind

$$\{A, B\} \rightarrow \{C, D\}$$
$$C \rightarrow B$$

then Tables are not in BCNF, so Book is not in BCNF.

If a non key value is depndent on other non key, then table is not in 3NF, of Book is also not in 3NF and collection is in 3NF.

44. Block size is given 1024 so first level and second level multiplication must be 2^{10}, so only (c) satisfy $2^8 \times 22 = 2^{10}$.

46. The query

$\{e \cdot \text{name} \mid \text{employee } (e) \wedge (\forall x) [\ulcorner \text{employee } (x)] \ vx \cdot$ supervisor Name $\neq e \cdot$ name v xssex = "male"$]\}$ computes the names of employees with no immediate female subordinates.

47. Query Q_1 doesn't give correct result because it contain not exists clause which lead to wrong answer. Q_2 is correct.

48. In BCNF, a prime attribute cannot be transitively dependent on a key.

49. Let the order be n

So, $\quad n \quad 7 + (n - 1)\, 9 = 1024$

$\Rightarrow \quad 16\, n = 1033$

$\Rightarrow \quad n = \left\lceil \dfrac{1033}{16} \right\rceil = 64$

50. Here S_1 is not conflict serializable whereas S_2 is conflict serializable.

51. Once a transaction commits, the changes made by it to the database get permanent and there is no effect of system failure over the data. Since T_2 is not allowed to commit, we must undo log record 6.

52. Lets take an example of the balances

1	20	1
2.	21	2
3.	21	2
4.	21	2
5.	22	5

Query 1 will assign rank 4 to account (4) which is wrong.

53. All queries give one and the same output.

54. In Plan 1, first the records from 'paid' is selected and then they are joined. On the other hand in Plan 2, all the records from 'paid' are joined and then the records are checked.

As join is an iterative and relatively expressive process. Plan 1 will execute faster though the output will be the same.

55. The algorithm used is (for α^+)

result $= \alpha$,

While (changes to result) do

for each functional dependency $\beta \rightarrow \gamma$ in F

do begin

if $\beta \subseteq$ result, then

result = result $v\gamma$,

end

For (C), $\quad \{AF\}^+ = \{AFDE\}$

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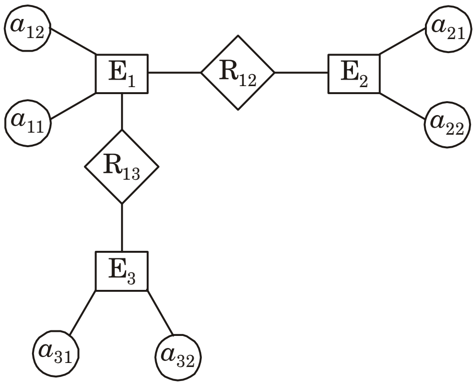
NUMERICAL TYPE QUESTIONS

1. The output is

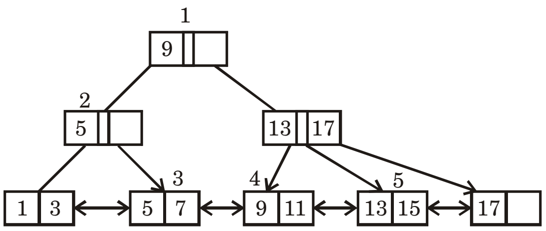
Raj	310
Rohit	140

Hence no of rows returned are 2

2. The E–R diagram is



3. In given B⁺ tree



So to find greater than or equal to 7 and less than 15. It will fatch 5 nodes.

As shown in above diagram it may be

1 – 2 – 3 – 4 – 5

or

1 – 6 – 5 – 4– 3

Ans = 5

4. Suppose that 'k' is order of the non-leaf node

$$k(8) + (k - 1)\,12 \le 1024$$

$$20\,k \le 1036$$

$$k \le \left\lceil \frac{1036}{20} \right\rceil \Rightarrow k \le 51$$

As the order is 51, maximum we can store 50 keys.

5. Since the number of super keys is given by 2^{n-1} where n is number of attributes.

Hence, total number of super keys with n attributes are= $2^{n-1} = 2^{4-1} = 2^8$

6. If we put value of student age to be 19, then student name, student age will not be key, that's why value of x should not be equal to 19 because it may be same for another student.

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8
CHAPTER

Computer Networks

ISO-OSI 7-LAYER NETWORK ARCHITECTURE

OSI REFERENCE MODEL

OSI (Open System Interconnection) deals with connecting upon system that are open for communication with other systems.

ISO-OSI 7-LAYER NETWORK ARCHITECTURE

According to the ISO standards, networks have been divided into 7 layers depending on the complexity of the fucntionality each of these layers provide.

Layers as defined by the standard in the increasing order of function complexity are given below :

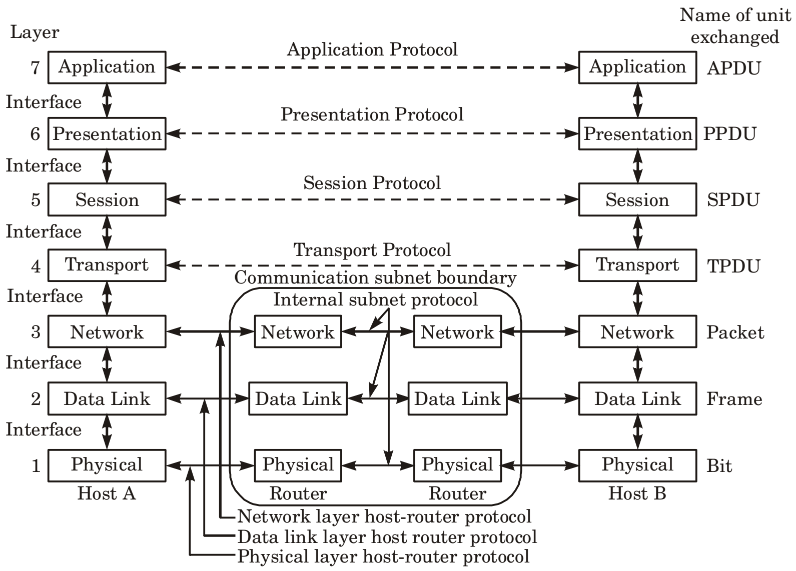

Fig. OSI reference model

1. **Physical Layer**

This layer is the lowest layer in the OSI model. It helps in the transmission of data between two machines that are communicating through a physical medium, which can be optical fibres,copper wire or wireless etc.

Main functions of the physical layer.

(***i***) **Hardware Specification:** Details of the physical cables, network interface cards, wireless radios, etc are a part of this layer.

(***ii***) **Encoding and Signalling:** How are the bits encoded in the medium is also decided by this layer. All the issues of modulation is dealt with in this layer. Binary phase shift keying can be used for representation of '1' and '0' rather than using different volatage levels if we have to transfer in RF waves.

(***iii***) **Data transmission and Reception:** This layer assures the transmission of each bit with a *high probability*. Transmission of the bits is not completely reliable as their is no error correction in this layer.

(***iv***) **Topology and Network design:** In network design, which part of the network is the router going to be placed, where the switches will be used, where we will put the hubs, how many machines is each switch going to handle, what server is going to be placed where, and many such concerns are taken care of by physical layer. It decides types of topologies that are to used like ring, bus, star or a hybrid of these topologies depending on the requirements.

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2. Data Link Layer

This layer provides reliable transmission of a packet by using services of the physical layer which transmits bits over the medium in an unreliable fashion.

This layer is concerned with :

(*i*) *Framing :* Breaking input data into frames (typically a few hundred bytes) and caring about the frame boundaries and the size of each frame.

(*ii*) *Acknowledgment :* Sent by the receiving end to inform the source that the frame was received without any error.

(*iii*) *Sequence numbering :* To acknowledge which frame was received.

(*iv*) *Error detection :* The frames may be damaged, lost or duplicated leading to errors.The error control is on **link to link** basis.

(*v*) *Retransmission :* The packet is retransmitted if the source fails to receive acknowledgment.

(*vi*) *Flow control :* Necessary for a fast transmitter to keep pace with a slow receiver.

Design Considerations.

(*i*) Reliable Delivery : Frames are delivered to the receiver reliably and in the same order as generated by the sender.

Connection state keeps track of sending order and which frames require retransmission.

e.g. receiver state includes which frames have been received, which ones have not, etc.

(*ii*) Best Effort : The receiver does not return acknowledgments to the sender, so the sender has no way of knowing if a frame has been successfully delivered.

Such a service will be appropriate

(*i*) When higher layers can recover from errors with little loss in performance, *i.e.* when errors are so infrequent that there is little to be gained by the data link layer performing the recovery. It is just as easy to have higher layers deal with occasional lost packet.

(*ii*) For real-time applications requiring "*better never than late*" semantics. Old data may be *worse* than no data.

(*iii*)Acknowledged Delivery : The receiver returns an acknowledgment frame to the sender indicating that a data frame was properly received. This sits somewhere between other two in that the sender keeps connection state, but may not necessarily retransmit unacknowledged frames. Likewise, the receiver may have received packets from higher layers in the order in which they arrive, regardless of the original sending order.

Typically, each frame is assigned a unique *sequence number*, which the receiver returns in an *acknowledgment frame* to indicate which frame the ACK refers to. The sender must *retransmit* unacknowledged (*e.g.,* lost or damaged) frames.

- In general, provide service to the network layer. The network layer wants to be able to send packets to its neighbors without worrying about the details of getting it there in one piece.

3. Network Layer

Basic functions of Network layer.

It is concerned with getting packets from the source all the way to the destination. The packets may require to make many hops at the intermediate routers while reaching the destination. This is the lowest layer that deals with end to end transmission. In order to achieve its goals, the network layer must know about the topology of the communication network. It must also take care to choose routes to avoid over loading of some of the communication lines while leaving others idle. The network layer-transport layer interface frequently is the interface between the carrier and the customer, that is the boundary of the subnet.

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Functions of Network layer.

(*i*) Routing : The process of transferring packets received from the Data Link Layer of the source network to the Data Link Layer of the correct destination network is called routing it. Involves decision making at each intermediate node on which to send the packet next so that it eventually reaches its destination. The node which makes this choice is called a *router*. For routing, some mode of addressing is required which is recognized by the Network Layer. This addressing is different from the MAC layer addressing.

It can be of three types :

(*a*) *Static :* Routes are based on static tables that are "*wired into*" the network and are rarely changed.

(*b*) *Dynamic :* All packets of one application can follow different routes depending upon the topology of the network, the shortest path and the current network load.

(*c*) *Semi-Dynamic :* A route is chosen at the start of each conversation and then all the packets of the application follow the same route.

Services provided by the network layer :

It can be of two types :

(*a*) **Connection less service:** Each packet of an application is treated as an independent entity. On each packet of the application, the destination address is provided and the packet is routed.

(*b*) **Connection oriented service:** Here, first a connection is established and then all packets of the application follow the same route.

(***ii***) **Congestion control:** A router can be connected to 4-5 networks. If all the networks send packet at the same time with maximum rate possible, then router may not be able to handle all the packets and may drop some/all packets. In this context, dropping of the packets should be minimized and the source whose packet was dropped should be informed. The control of such congestion is also a function of the network layer. Other issues related with this layer are transmitting time, delays, jittering.

If incoming rate of the packets arriving at any router is more than the outgoing rate, then congestion is said to occur. Congestion may be caused by many factors. If suddenly, packets begin arriving on many input lines and all need the same output line, then a queue will build up. If there is insufficient memory to hold all of them, packets will be lost. But even if routers have an infinite amount of memory, congestion gets worse, because by the time packets reach to the front of the queue, they have already timed out (repeatedly), and duplicates have been sent. All these packets are dutifully forwarded to the next router, increasing the load all the way to the destination. Another reason for congestion are slow processors. If router's CPUs are slow at performing the bookkeeping tasks required of them, queues can build up, even though there is excess line capacity. Similarly, low-bandwidth lines can also cause congestion.

(***iii***) **Internetworking:** Network layer is the same across all physical networks (such as Token-Ring and Ethernet). Thus, if two physically different networks have to communicate, the packets that arrive at the Data link layer of the node which connects these two physically different networks, would be stripped of their headers and passed to the Network Layer. Then network layer would pass this data to the Data link layer of the other physical network. These are multiple networks that are connected in such a way that they act as one large network, connecting multiple office or department networks. These are connected by networking hardware such as routers, switches, and bridges.

It is a solution born of following three networking problems:

(*a*) Isolated LANs

(*b*) Duplication of resources

(*c*) Lack of a centralized network management system.

With connected LANs, companies no longer have to duplicate programs or resources on each network. This in turn gives way to managing the network from one central location instead of trying to manage each separate LAN. We should be able to transmit any packet from one network to any other network even if they follow different protocols or use different addressing modes.

Note : Network layer **does not** guarantee that the packet will reach its intended destination. There are no reliability guarantees.

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4. Transport Layer

Main functions of the transport layer.

(*i*) **Multiplexing / Demultiplexing :** Normally this create distinct network connection for each transport connection required by the session layer. This may either create multiple network connections (to improve throughput) or it may multiplex several transport connections onto the same network connection (because creating and maintaining networks may be expensive). In the latter case, demultiplexing will be required at the receiving end. Communication is always carried out between two processes and not between two machines. This is also called *process-to-process communication.*

(*ii*) **Fragmentation and Re-assembly :** Data accepted by the transport layer from the session layer is split up into smaller units (fragmentation) if needed and then passed to the network layer. Correspondingly, the data provided by the network layer to the transport layer on the receiving side is re-assembled.

(*iii*) **Types of service :** It decides type of service that should be provided to the session layer. The service may be perfectly reliable, or may be reliable within certain tolerances or may not be reliable at all. The message may or may not be received in the order in which it was sent. The decision regarding the type of service to be provided is taken at the time when connection is established.

(*iv*) **Error control :** If reliable service is provided, then error detection and error recovery operations are also performed. It provides error control mechanism on **end to end** basis.

(*v*) **Flow control :** A fast host cannot keep pace with a slow one. Hence, this is a mechanism to regulate the flow of information.

(*vi*) **Connection Establishment / Release :** It establishes and releases the connection across the network. This requires some sort of naming mechanism so that a process on one machine can indicate with whom it wants to communicate.

5. Session Layer

It deals with the concept of *sessions,* i.e. when a user logins to a remote server he should be *authenticated* before getting access to the files and application programs. Another job of session layer is to establish and maintain sessions. If during the transfer of data between two machines, the session breaks down, it is the session layer which re-establishes the connection. It also ensures that the data transfer starts from where it breaks keeping it transparent to the end user e.g. in case of a session with a database server, this layer introduces **check points** at various places so that in case connectoin is broken and reestablished, the transition running on the database is not lost even if user has not committed. This activity is called *synchronization.*

Another function of this layer is *dialogue control* which determines whose turn is it to speak in a session. It is useful in video conferencing.

6. Presentation Layer

This layer is concerned with the *syntax* and *semantics* of the information transmitted. In order to make it possible for computers with different data representations to communicate data structures to be exchanged can be defined in abstract way alongwith standard encoding. It also manages these abstract data structres and allows higher level of data structres to be defined an exchange. It encodes the data in standard agreed way(network format).

Let there are two machines A and B one follows '*Big Endian*' and other '*Little Endian*' for data representation. This layer ensures that the data transmitted by one gets converted in the form compatibale to othe machine. This layer is concerned with the syntax and semantics of the information transmitted.In order to make it possible for computers with different data representations to communicate data structures to be exchanged canbe defined in abstract way alongwith standard encoding. It also manages these abstract data structres and allows higher level of data structures to be defined an exchange. Other functions include compression, encryption etc.

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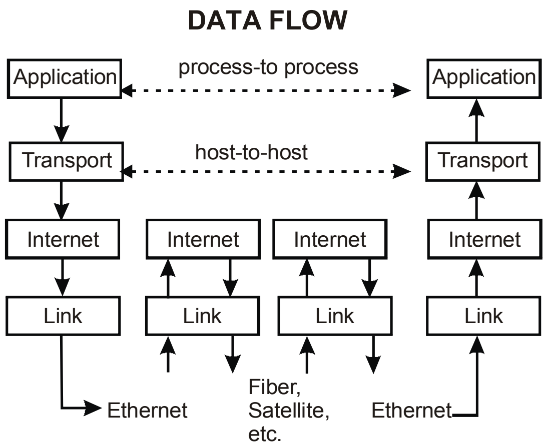
7. Application Layer

This layer contains application protocols with which the user gains access to the network. The choice of which specific protocols and their associated functions are to be used at the application level is up to the individual user. Thus boundary between presentation layer and the application layer represents a separation of the protocols imposed by the network designers from those being selected and implemented by the network users *e.g.* commonly used protocols are HTTP(for web browsing), FTP(for file transfer) etc.

TCP/IP REFRENCE MODEL.



There are four layers in the TCP/IP reference model.

1. **Application Layer :** This is the topmost layer of the TCP/IP suite. This is responsible for coding of the packet data.
2. **Transport Layer :** This layer monitors end to end path selections of the packets. It also provides service to the application layer.
3. **Internet Layer :** This layer is responsible for sending packets through different networks.
4. **Link Layer :** It is the closest layer to the network hardware. It provides service to Internet layer.

TCP/IP Model vs OSI Model

S. No.	TCP/IP Reference Model	OSI Reference Model
1	Defined after the advent of Internet.	Defined before advent of internet.
2	Service interface and protocols were not clearly distinguished before	Service interface and protocols are clearly distinguished
3	TCP/IP supports Internet working	Internet working not supported
4	Loosely layered	Strict layering
5	Protocol Dependant standard	Protocol independent standard
6	More Credible	Less Credible
7	TCP reliably delivers packets, IP does not reliably deliver packets	All packets are reliably delivered

LAN TECHNOLOGY

Point-to-point Communication

The first computer communication systems had each communication channel, e.g. a leased circuit, connecting exactly two computers. This is called *point-to-point communication.*

It has three useful properties.

(*i*) Each connection is independent of the others and can use appropriate hardware.

(*ii*) Two end points have exclusive access and can decide how to send data across the connection.

(*iii*) Since only two computers have access to the channel, it is easy to enforce security and privacy.

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Disadvantages.

The main disadvantage of point-to-point communications is the proliferation of connections, as shown below.

In Fig. (*a*), two computers require one connection. In Fig. (*b*), three computers require three connections. In Fig. (*c*), four computers requires six connections.

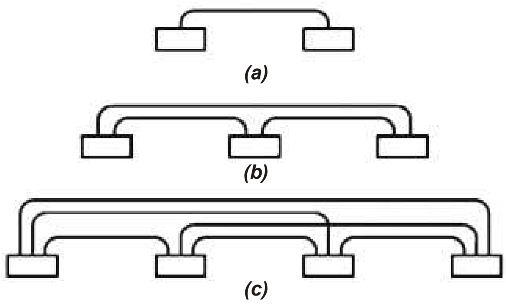


Fig. Point-to-point connections

Number of connections for N computers is proportional to the square of N.

In practice, the cost is high because many connections follow the same physical path. Figure below shows the situation with five computers located in two locations.

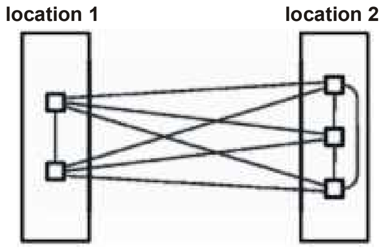


Fig. Different locations

There are six connections between the two locations, more than the total number of computers being connected. Another computer added to location 1 increases the number of connections between two locations to nine.

As point-to-point communication does not scale, as the physical distance between computers increases, the cost of installing the point-to-point connections becomes prohibitively expensive.

LAN Topologies

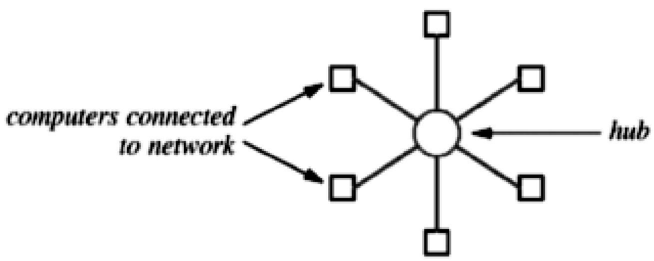
These are different from long-distance communications because they rely on sharing the network. Each LAN consists of a single shared medium, usually a cable, to which many computers are attached. The computers co-ordinate and take turns using the medium to send packets.

This mechanism does not scale. Co-ordination requires communication, and the time to communicate depends on distance - large geographic separation between computers introduces longer delays. Therefore, shared networks with long delays are inefficient. In addition, providing high bandwidth communication channels over long distances is very expensive.

Classification of LAN technology

There are a number of different LAN technologies. Each technology is classified into a category according to its *topology*, or general shape.

(1) Star topology.



The hub accepts data from a sender and delivers it to the receiver. In practice, a star network seldom has a symmetric shape; the hub often resides in a separate location from the computers attached to it.

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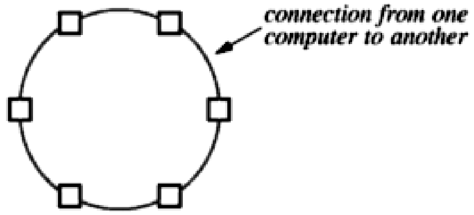
(2) Ring topology.

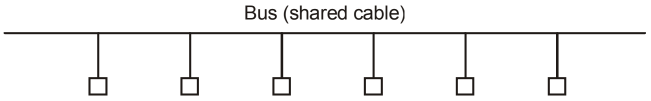
A network using a *ring topology* arranges the computers in a circle - the first computer is cabled to the second. Another cable connects the second computer to the third, and so on, until a cable connects the final computer back to the first.



Once again, the ring, like the star topology, refers to logical connections, not physical orientation.

(3) Bus topology.

A network that uses a *bus topology* consists of a number of computers all connected to a single, long cable. Any computer attached to the bus can send a signal down the cable, and all computers receive the signal.



The computers attached to a bus network must co-ordinate to ensure that only one computer sends a signal at any time. In addition, the ends of a bus network must be terminated to prevent electrical signals from reflecting back along the bus.

ETHERNET.

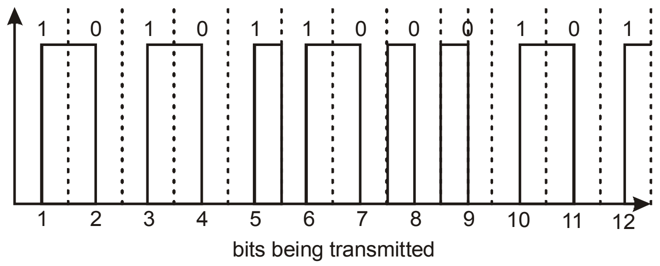
It is a widely used technology employing a bus topology. IEEE currently controls Ethernet standards, e.g. IEEE 802.3 was published in 1985.

In its original form, an Ethernet LAN consists of a single coaxial cable called *ether*, but often referred to as a *segment*. A segment is limited to 500 m in length, with a minimum separation of 3 m between each pair of connections. It operates at 10 Mbps; a later version, *Fast Ethernet*, operates at 100 Mbps; the latest version, *Gigabit Ethernet*, operates at 1,000 Mbps or 1 Gbps.

Manchester Encoding

The standard specifies that Ethernet frames are transmitted using *Manchester Encoding*, which uses the fact that hardware can detect a change in voltage more easily than a fixed value, e.g. RS-232.

Technically, hardware is *edge triggered*, with the changes called *rising* or *falling* edges. The sender transmits a falling edge to encode a 0 and a rising edge to encode a 1, as shown below :



The voltage change that encodes a bit occurs exactly half-way through the time slot. Exactly half-way through the first time slot, the voltage becomes positive (+0.85 V) to encode a 1. Similarly, exactly half-way through the second time slot, the voltage becomes negative (-0.85 V) to encode a 0. If two contiguous bits have the same value, an additional change in voltage occurs at the edge of the time slot.

Manchester encoding uses a *preamble* to allow for synchronisation. The preamble consists of 64 alternating 1s and 0s sent before the frame. These produce a square wave with transitions exactly in the middle of each slot. Receiving hardware uses the preamble to synchronise with the time slots. The last two bits of the preamble are both 1s to signal the end of the preamble.

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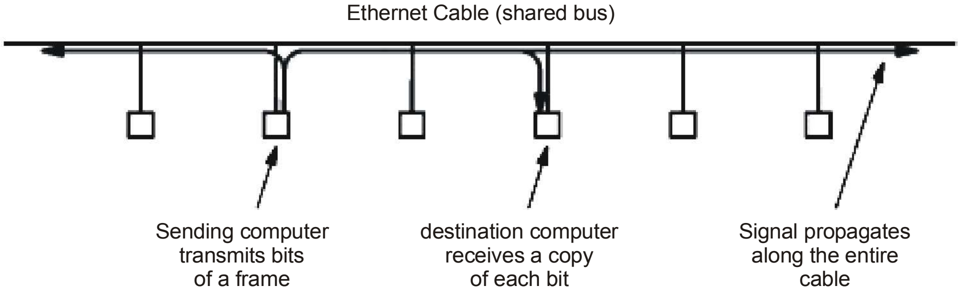
Sharing Ethernet

Ethernet requires multiple computers to share access to the other. A sender transmits a signal, which propagates towards both ends of the cable.



The sending computer has exclusive use of the cable during the transmission of the frame. Multiple frames cannot be sent at the same time, all other computers must wait.

Carrier Sense Multiple Access/Collision Detection (CSMA/CD)

All computers attached to the Ethernet use CSMA/CD to co-ordinate their activities. A computer wishing to transmit checks for electrical activity on the cable, informally called *carrier*. If there is no carrier, the computer can transmit. If a carrier is present, the computer waits for the sender to finish before proceeding.

However, it is possible for two or more computers to detect the lack of carrier and start transmission simultaneously. The signals travel at approximately 70% of the speed of light and interfere with one another. This interference is called *collision*. A sending computer monitors the signal on the cable and if it differs from the signal it is sending, then a collision has occurred and the computer stops transmitting.

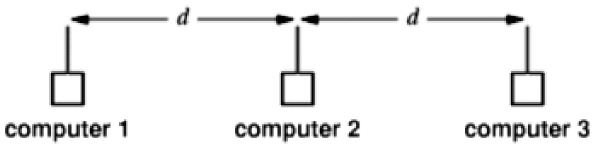
Following a collision, a computer waits for the cable to become idle before retransmitting. However, if the computers start transmitting as soon as the cable becomes free, another collision will occur. Ethernet requires each computer to delay after a collision. The standard specifies a maximum delay d, and requires each computer to choose a random delay less than d. In this case, the computer choosing the shortest delay will transmit first.

If subsequent collisions still occur, the computers double the maximum delay ($2d$, $4d$, ...) until the range is large enough for one computer to choose a short delay and transmit without a collision. This technique is called *binary exponential backoff*.

WIRELESS LANS.

Instead of transmitting signals across a cable, wireless LANs use antennas to broadcast RF signals through the air. All the computers using a wireless LAN share the same radio frequency, so they must take turns to transmit. Wireless LAN transmitters use low power, so the signal only propagates a short distance. In addition, metallic obstructions can block the signal.

This lack of full communication means that wireless LANs cannot use the CSMA/CD mechanism used by Ethernet. Consider the situation shown below :



Given a maximum tranmission distance d, computer 2 will receive all transmissions, but computers 1 and 3 will not receive transmissions from each other. Wireless LANs use *Carrier Sense Multiple Access / Collision Advoidance* (CSMA/CA) to share the air waves. Let computer 1 wants to send a frame to computer 2. It first sends a short *Request To Send* (RTS) control message. Computer 2 receives the RTS and responds with a short *Clear To Send* (CTS) control message. Now computer 1 can transmit its frame.

In the figure, computer 3 will not receive the RTS from computer 1, but it will receive CTS from computer 2 , so it knows that it must wait. In the event of computers 1 and 3, both sending a control message to computer 2 at the same time, they apply random backoff before resending. Since control messages are much shorter than data frames, the probability of a second collision is much lower than with conventional Ethernet.

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TOKEN RING.

Most LANs that employ ring technology use an access mechanism called *token passing*. A token ring operates as a single, shared medium. When a computer wants to send data, it must wait until it obtains the token, when it is in control of the ring as shown below.

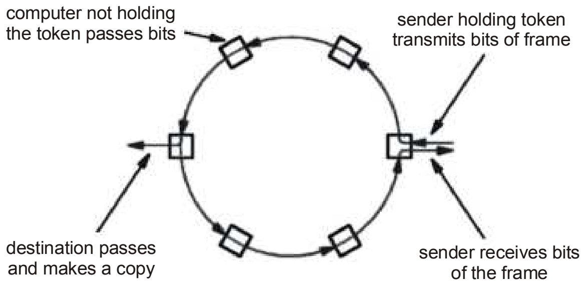

Fig. Token ring network

All stations except the sender forward bits around the ring. The sender compares the data being received with the data being sent to ensure that no transmission errors have occurred. The receiver makes a copy of the message as it passes the bits round the ring.

Co-ordination is achieved by using a special message called *token*. A computer wanting to transmit must wait for the token to arrive. It removes token from the ring and uses the ring to transmit data. After sending one frame it retransmits the token; this token passing guarantees that stations wanting to transmit will take turns. Ensuring there is one, and only one, token on the ring is handled by the ring hardware.

Fibre Distributed Data Interconnect (FDDI) Topology

A major disadvantage of token ring networks is that the failure of a single computer or connection disable the complete network. FDDI transmits data at 100 Mbps, uses optical fibres to interconnect computers, and uses redundancy to overcome failures.

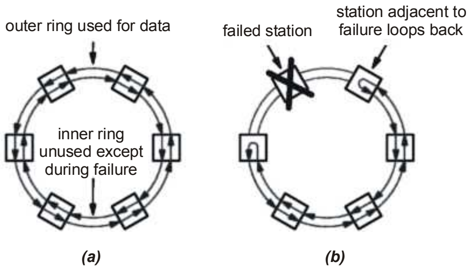

Fig. FDDI network

In Fig. (a), outer ring is used to transfer data. In Fig. (b), a station has failed and the adjacent stations detect the disconnection and reconfigure to use the reverse path to form a closed ring. This process of reconfiguring is called *self-healing* and FDDI is called *self-healing network*.

ERROR AND FLOW CONTROL

ERROR CONTROL.

It is concerned with insuring that all frames are eventually delivered (possibly in order) to a destination.

For this following three items are required :

1. **Acknowledgements :** Typically, reliable delivery is achieved using "*acknowledgments with retransmission*" paradigm, whereby receiver returns a special *acknowledgment* (ACK) frame to the sender indicating correct receipt of a frame.

   In some systems, receiver also returns a *negative acknowledgment* (NACK) for incorrectly-received frames. This is a hint to the sender so that it can retransmit a frame right away without waiting for a timer to expire.

2. **Timers :** One problem that simple ACK/NACK schemes fail to address is recovering from a frame that is lost, and as a result, fails to solicit an ACK or NACK.

   If an ACK or NACK becomes lost, then *Retransmission timers* are used to resend frames that don't produce an ACK. When sending a frame, schedule a timer to expire at some time after the ACK should have been returned. If the timer goes off, retransmit the frame.

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3. **Sequence Numbers :** Retransmissions introduce possibility of duplicate frames. To suppress duplicates, add sequence numbers to each frame, so that a receiver can distinguish between new frames and old copies.

FLOW CONTROL.

It deals with throttling speed of sender to match that of the receiver. Usually, this is a dynamic process, as receiving speed depends on changing factors such as load, and availability of buffer space.

One solution is to have the receiver extend *credits* to sender. For each credit, sender may send one frame. Thus, receiver controls transmission rate by handing out credits.

Link Management

In some cases, the data link layer service must be "*opened*" before use.

The data link layer uses open operations for allocating buffer space, control blocks, agreeing on the maximum message size, etc.

Synchronize and initialize send and receive sequence numbers with its peer at the other end of the communications channel.

ERROR DETECTION AND CORRECTION.

In data communication, line noise is a fact of life

e.g., signal attenuation, natural phenomenon such as lightning, and telephone repairman.

Moreover, noise usually occurs as bursts rather than independent, single bit errors.

e.g., a burst of lightning will affect a set of bits for a short time after the lightning strike.

Detecting and correcting errors requires *redundancy* — sending additional information along with the data.

There are two types of attacks against errors:

1. **Error detecting codes :** These include enough redundancy bits to *detect* errors and use ACKs and retransmissions to recover from the errors.
2. **Error correcting codes :** These include enough redundancy to detect *and* correct errors.

   To understand errors, consider the following:

   (*i*) Messages (frames) consist of m data (message) bits and r redundancy bits, yielding an $n = (m+r)$-bit *codeword*.

   (*ii*) *Hamming distance*. Given any two codewords, we can determine how many of the bits differ. Simply exclusive or (XOR) the two words, and count the number of 1 bits in the result.

   (*iii*) If two codewords are d bits apart, d errors are required to convert one to the other.

   (*iv*) A code's *Hamming distance* is defined as the minimum Hamming distance between any two of its legal codewords (from all possible codewords).

   (*v*) In general, all 2^m possible data words are legal. However, by choosing check bits carefully, the resulting codewords will have a large Hamming distance. The larger the Hamming distance, the better able the code can detect errors.

   To detect d 1-bit errors requires having a Hamming distance of at least d+1 bits, because to correct d errors requires $2d+1$ bits. Intuitively, after d errors, garbled messages is still closer to the original message than any other legal codeword.

Parity Bits

Consider parity. A single *parity bit* is appended to each data block (e.g. each character in ASCII systems) so that number of 1 bits always adds up to an even (odd) number.

1000000(1) 1111101(0)

Hamming distance for parity is 2, and it cannot correct even single-bit errors (but can detect single-bit errors).

Now, consider a 10-bit code used to represent 4 possible values:

"00000 00000", "00000 11111", "11111 00000", and "11111 11111".

Its Hamming distance is 5, and we can correct 2 single-bit errors:

e.g., "10111 00010" becomes "11111 00000" by changing only two bits.

However, if sender transmits "11111 00000" and receiver sees "00011 00000", the receiver will not correct the error properly.

Finally, in this example we are guaranteed to catch all 2-bit errors, but we might do better, if "00111 00111" contains 4 single-bit errors, we will reconstruct the block correctly.

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Single-Bit Error Correction

Design a code containing $n=m+r$ bits that corrects all single-bit errors

Here, m is number of message (data) bits and r is number of redundant (check) bits.

1. There are 2^m legal messages

   e.g., legal bit patterns.

2. Each of the m messages has n illegal codewords a distance of 1 from it, *i.e.,* systematically inverting each bit in the corresponding n-bit codeword, we get n illegal codewords a distance of 1 from the original.

   Thus, each message requires n+1 bits dedicated to it (n that are one bit away and 1 that is the message).

3. Total number of bit patterns = $(n+1)2^m \leq 2^n$

   i.e., all $(n+1)2^m$ encoded messages should be unique, and there can't be fewer messages than the 2^n possible codewords.

4. Since $n = m + r$, we get $$(m+r+1)2^m \leq 2^{m+r}$$

   or $$(m+r+1) \leq 2^r$$

   This formula gives absolute lower limit on the number of bits required to detect (and correct!) 1-bit errors.

   Hamming developed a code that meets this lower limit.

   Note :

- Bits are numbered left-to-right starting at 1.
- Bit numbers that are powers of two (e.g., 1, 2, 4, 8, etc.) are check bits; remaining bits are actual data bits.
- Each check bit acts as a parity bit for a set of bits (both data and check).
- To determine which parity bits in the codeword cover bit k of the codeword, rewrite bit position k as the sum of powers of two (e.g., 19 = 1+2+16). A bit is checked by only those check bits in the expansion (e.g., check bits 1, 2, and 16).
- When a codeword arrives, examine each check bit k to verify that it has the correct parity. If not, add k to a counter. At the end of the process, a zero counter means no errors have occurred; otherwise, the counter gives the bit position of the incorrect bit.

e.g., consider ascii character "a" = "1100001".

Therefore

- check bit 1 covers all odd numbered bits (e.g, 1, 3, 5, ...)
- check bit 2 covers bits 2, 3, 6, 7, 10, 11, ...
- check bit 3 covers bits 4, 5, 6, 7, 12, 13, 14, 15, ...
- check bit 4 covers bits 8, 9, 10, 11, 12, etc.

Thus:

- check bit 1 equals: ?+1+1+0+0+1 = 1
- check bit 2 equals: ?+1+0+0+0+1 = 0
- check bit 3 equals: ?+1+0+0 = 1
- check bit 4 equals: ?+0+0+1 = 1

giving k

Note: Hamming codes correct only single bit errors. To correct burst errors, b blocks can be sent, distributing the burst over each of the b blocks.

For instance, build a b-row matrix, where each row is one block. When actually sending the data, send it one column at a time. If a burst error occurs, each block (row) see a fraction of the errors, and may be able to correct its block.

Use of Error correction.

Error correction is most useful in three contexts:

(*i*) *Simplex links*

e.g., those that provide only one-way communication

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(*ii*) *Long delay paths,* where retransmitting data leads to long delays
e.g., satellites

(*iii*) Links with very high error rates, where there is often one or two errors in each frame. Without forward error correction, most frames would be damaged, and retransmitting them would result in the frames becoming garbled again.

ERROR DETECTION.

Error correction is relatively expensive (computationally and in bandwidth).

e.g., 10 redundancy bits are required to correct 1 single-bit error in a 1000-bit message but in, detecting a single bit error requires only a single-bit, no matter how large the message.

CRC Checksums.

The most popular error detection codes are based on *polynomial codes* or *cyclic redundancy codes.*

- Represent a *k*-bit frame as coefficients of a polynomial expansion ranging from τ^{k-1} to τ^0, *with high-order bit corresponding to the coefficient of* τ^{k-1}.
  e.g., represent string "11011" as the polynomial : $\tau^4 + \tau^3 + \tau + 1$
- Perform modulo 2 arithmetic
  e.g. XOR of the bits
- Sender and receiver agree on a generator polynomial : G(*x*).
  G(*x*) must be smaller than the number of bits in the message.
- Append a checksum to message; let's call the message M(*x*), and combination T(*x*).
  Checksum is computed as follows:
  (*i*) Let *r* be the degree of G(*x*), append *r* zeros to M(*x*). Then new polynomial becomes $\tau^r M(\tau)$
  (*ii*) Divide $\tau^r M(\tau)$ by G(x) using modulo 2 arithmetic.
  (*iii*) Subtract remainder from $\tau^r M(\tau)$, we get T(*x*).
- When receiver gets T(*x*), it divides T(*x*) by G(*x*); if T(*x*) divides cleanly (e.g., no remainder), no error has occurred. The presence of a remainder indicates an error.

Assume:

- *Receiver gets* $T(x) + E(x)$, where each bit in $E(x)$ corresponds to an error bit.
- *k* 1 bits indicate *k* single-bit errors.
- Receiver computes $[T(x) + E(x)]/G(x) = E(x)/G(x)$.

Detect:

- *Single bit errors*.
  If a single-bit error occurs, $G(x)$ will detect it if it contains more than one term.
  If it contains only one term, it may or may not detect the error, depending on $E(x)$ and $G(x)$.
- *Two isolated single-bit errors*.
  Consider two single-bit errors,
  $E(t) = \tau^i + \tau^j = \tau^i (1 + \tau^{j-i})$
  Note : τ^i is not divisible by G(x) if it contains two or more terms. Thus, double-bit errors can be detected if G(x) does not divide $(\tau^k + 1)$ for any k up to the message size.
  Satisfactory generator polynomials can be found.
  e.g., $G(\tau) = \tau^{15} + \tau^{14} + 1$ does not divide $\tau^k + 1$ for $k \le 32768$.
- Odd number of bits.
- Burst errors less than or equal to degree.
  Note : A polynomial with r check bits will detect all burst errors of length .

CRC Standards.

There are three international standards :

(1) CRC-12 : $\tau^{12} + \tau^{11} + \tau^3 + \tau^2 + \tau + 1$

(2) CRC-16 : $\tau^{16} + \tau^{15} + \tau^2 + 1$

(3) CRC-CCITT : $\tau^{16} + \tau^{12} + \tau^5 + 1$

Note : 16-bit CRCs detect all single and double errors, all errors with odd number of bits, all burst errors of length ≤ 16 bits, and 99.997% of 17-bit errors.

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ROUTING ALGORITHMS

ROUTING ALGORITHMS

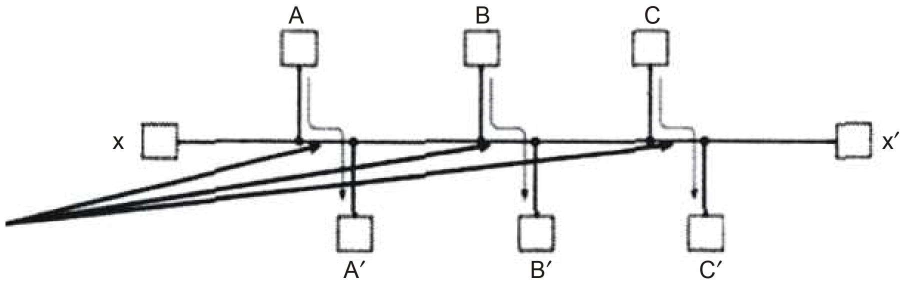
ROUTING.

It is the process of forwarding of a packet in a network so that it reaches its intended destination.

A router used to decide, for each incoming Packet, which output link the Packet should be transmitted on

(*i*) ***Datagram packet-switching:*** This decision must be made for every packet

(*ii*) ***Virtual circuit packet-switching:*** This decisions made only at v.c. set-up



Main goals of Routing

1. **Correctness :** Routing should be done properly and correctly so that the packets may reach their proper destination.
2. **Simplicity :** Routing should be done in a simple manner so that overhead is as low as possible. With increasing complexity of the routing algorithms, the overhead also increases.
3. **Robustness :** Once a major network becomes operative, it may be expected to run continuously for years without any failures. The algorithms designed for routing should be robust enough to handle hardware and software failures and should be able to cope with changes in the topology and traffic without requiring all jobs in all hosts to be aborted and the network rebooted every time some router goes down.
4. **Stability :** Routing algorithms should be stable under all possible circumstances.
5. **Fairness :** Every node connected to the network should get a fair chance of transmitting their packets. This is generally done on a first come first serve basis.
6. **Optimality:** Routing algorithms should be optimal in terms of throughput and minimizing mean packet delays. Here there is a trade-off and one has to choose depending on his suitability.

Routing Algorithms

Least-cost routing.

- *A value is assigned to each link in the network :* This is *cost* of using this link
- Cost of a route is combination of the values of its links
- Best route is the one with the lowest cost

Cost assigned to a Link

- **One for each link :** Finds route with the *fewest hops*
- **Cost (financial) of using link :** Finds *cheapest* route
- **Packet delay** on **the link :** Finds minimum-delay route
- **Packet transmission time on the link :** Finds *maximum-bandwidth* route or some combination of these, or other factors.
- Once *cost of each link* is known, the routers can run a routing algorithm to determine best routes for each possible sender-receiver transmission
- In practice, routing algorithm should be *adaptive* and *de-centralised*

Two most common Routing algorithms

1. **Distance-vector :** *Each router exchanges information about the entire network with neighbouring routers at regular intervals*
   - *Neighbouring routers :* connected by a direct link (e.g. a LAN)
   - *Regular intervals :* e.g. every 30 seconds

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2. **Link-state :** *Each router exchanges information about its neighbourhood with all routers in the network when there is a change*
   - **Neighbourhood of a router :** Set of neighbour routers for this router
   - **Each router's neighbourhood information** *is flooded* through the network
   - **Change :** e.g. if a neighbouring router does not reply to a status message link-state *converges* faster in practice, so more widely used

CLASSIFICATION OF ROUTING ALGORITHMS.

1. **Adaptive Routing Algorithm :** These algorithms change their routing decisions to reflect changes in the topology and traffic. These get their routing information from adjacent routers or from all routers.

   Optimization parameters

   (*i*) Distance

   (*ii*) Number of hops

   (*iii*) Estimated transit time.

   This can be further classified as follows:

   (***i***) **Centralized.**

   In this type, some central node in the network gets entire information about

   (*a*) network topology

   (*b*) traffic

   (*c*) other nodes

   Then this transmits this information to the respective routers.

   Advantage :

   Only one node is required to keep the information.

   Disadvantage :

   If central node goes down, the entire network is down, i.e. single point of failure.

   (***ii***) **Isolated :** In this method, node decides the routing without seeking information from other nodes. The sending node does not know about the status of a particular link.

   Disadvantage :

   Packet may be send through a congested route resulting in a delay.

   Some examples of isolated type of algorithm for routing :

   (***a***) **Hot Potato:** When a packet comes to a node, it tries to get rid of it as fast as it can, by putting it on the shortest output queue without regard to where that link leads. A variation of this algorithm is to combine static routing with the hot potato algorithm. When a packet arrives, the routing algorithm takes into account both static weights of the links and the queue lengths.

   (***b***) **Backward Learning:** In this method, routing tables at each node gets modified by information from the incoming packets. One way to implement backward learning is to include identity of the source node in each packet, together with a hop counter that is incremented on each hop. When a node receives a packet in a particular line, it notes down the number of hops it has taken to reach it from the source node. If previous value of hop count stored in the node is better than the current value, then nothing is done but if current value is better, then the value is updated for future use. The problem with this is that, when best route goes down, then it cannot recall the second best route to a particular node. Hence all the nodes have to forget stored informations periodically and start all over again.

   (***iii***) **Distributed :** In this, the node receives information from its neighbouring nodes and then takes the decision about which way to send the packet.

   Disadvantage : If in between the interval it receives information and sends the paket something changes then the packet may be delayed.

2. **Non-Adaptive Routing Algorithm :** These algorithms do not base their routing decisions on measurements and estimates of the current traffic and topology. Instead the route to be taken in going from one node to the other is computed in advance, off-line, and downloaded to the routers when the network is booted. This is also called *static routing*.

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This can be further classified as follows :

(*i*) **Flooding.**

It adapts the technique in which every incoming packet is sent on every outgoing line except the one on which it arrived.

One problem with this method is that, packets may go in a loop. As a result of this, a node may receive several copies of a particular packet which is undesirable.

Following Techniques adapted to overcome these problems :

(***a***) **Sequence numbers :** Every packet is given a sequence number. When a node receives the packet, it sees its source address and sequence number. If node finds that it has sent the same packet earlier, then it will not transmit the packet and will just discard it.

(***b***) **Hop count :** Every packet has a hop count associated with it. This is decremented(or incremented) by one by each node which sees it. When hop count becomes zero(or a maximum possible value) the packet is dropped.

(***c***) **Spanning tree:** The packet is sent only on those links that lead to the destination by constructing a spanning tree routed at the source. This avoids loops in transmission but is possible only when all the intermediate nodes have knowledge of the network topology.

Flooding is not practical for general types of applications. But in cases where high degree of robustness is desired such as in military applications, flooding is of great help.

(***ii***) **Random Walk.**

In this method a packet is sent by the node to one of its neighbours randomly. This algorithm is highly robust. When the network is highly interconnected, this algorithm has the property of making excellent use of alternative routes. It is usually implemented by sending the packet onto the least queued link.

DELTA ROUTING.

It is a hybrid of the centralized and isolated routing algorithms. Here each node computes the cost of each line (i.e., some functions of the delay, queue length, utilization, bandwidth etc.) and periodically sends a packet to the central node giving it these values which then computes the **k** best paths from node **i** to node **j**.

Let **Cij1** be the cost of the best **i-j** path, **Cij2** be the cost of the next best path and so on.

If **Cijn - Cij1 < delta**, (**Cijn** - cost of **n'th** best **i-j** path, **delta** is some constant), then path **n** is regarded equivalent to the best **i-j** path since their cost differ by so little.

When **delta -> 0,** this algorithm becomes centralized routing and when **delta -> infinity** all the paths become equivalent.

MULTIPATH ROUTING.

In algorithms it can been assumed that there is a single best path between any pair of nodes and that all traffic between them should use it. In many networks however there can be several paths between pairs of nodes that are almost equally good. Sometimes in order to improve the performance, multiple paths between single pair of nodes are used. This technique is called *multipath routing* or *bifurcated routing*. In this, each node maintains a table with one row for each possible destination node. A row gives the best, second best, third best, etc outgoing line for that destination, together with a relative weight. Before forwarding a packet, the node generates a random number and then chooses among the alternatives, using the weights as probabilities. The tables are worked out manually and loaded into the nodes before the network is brought up and not changed thereafter.

HIERARCHICAL ROUTING.

In this method of routing, nodes are divided into regions based on hierarchy. A particular node can communicate with nodes at the same hierarchial level or the nodes at a lower level and directly under it. Here, path from any source to a destination is fixed and is exactly one if heirarchy is a tree.

DIJKSTRA'S ALGORITHM.

- This algorithm builds a *shortest'path spanning tree* for the router. Such a tree has a route to all possible destinations, and no loops.

  Router running algorithm is the root of its shortest-path spanning tree.

  Even if all routers' link-state databases are identical, trees determined by the routers are different (since root of each tree is different)

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- **Node is either a network or a router.**

  Arc. Nodes are connected by *arcs*

  Algorithm keeps track of 2 sets of nodes and arcs

  (*i*) *Temporary*

  (*ii*) *Permanent*

  Initially, temporary set contains all neighbour nodes of the router itself, and arcs connecting them to the router; only the router is initially permanent.

  When all nodes and arcs are in the permanent set, the algorithm has terminated.

Routing Table

Once a router has found its shortest-path spanning tree, it can build its routing table

e.g., router A's link-state routing table given below :

Net	Cost	Next router
08	4	E
14	1	---
23	2	---
55	2	B
66	5	B
78	3	---
92	6	F

- In large networks, the memory required to store link-state database and the computation time to calculate link-state, routing table can be significant
- In practice, since link-state packet receptions are not synchronised, routers may be using different link-state databases to build their routing tables.

Link-State routing

Link-state routing algorithms have several desirable properties,

e.g. rapid convergence; small amount ol'trafiic generated; rapid response to lopology changes

Open Shortest Path First (OSPE).

Examples from the Internet are, Open Shortest Path First (OSPF) and Intermediate System to Intermediate System (IS-IS) routing protocols

- Link costs can be configured in OSPF.

  Possible link costs include:

  (*a*) I for each link

  (*b*) reliability: assigned by adminisiralor. indicates how often the link fails

  (*c*) packet delay

  (*d*) link bandwidth

  (*e*) financial cost of the link
- OSPF requires a lot of memory: Each router holds its routing table & link-state database
- OSPF can consume a lot of bandwidth if network topology changes often

TRANSPORT LAYER PROTOCOL

TCP was specifically designed to provide a reliable end to end byte stream over an unreliable internetwork. Each machine supporting TCP has a TCP transport entity either a user process or part of the kernel that manages TCP streams and interface to IP layer.

The IP layer gives no guarantee that datagram will be delivered properly, so it is up to TCP to timeout and retransmit, if needed. Duplicate, lost and out of sequence packets are handled using the sequence number, acknowledgements, retransmission, timers, etc to provide a reliable service. Connection is a must for this service. Bit errors are taken care of by the CRC checksum. One difference from usual sequence numbering is that each byte is given a number instead of each packet. This is done so that at the time of transmission in case of loss, data of many small packets can be combined together to get a larger packet, and hence smaller overhead.

TCP connection is a duplex connection, i.e. there is no difference between two sides once the connection is established.

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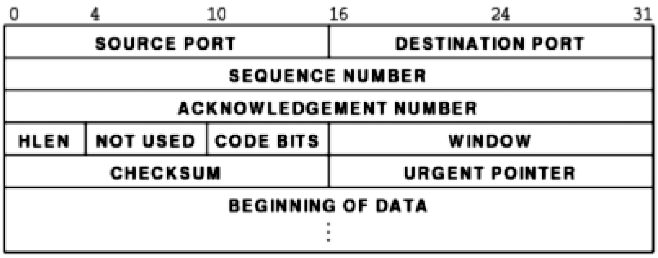
Salient Features of TCP

1. **Piggybacking of Acknowledgments :** ACK for the last received packet need not be sent as a new packet, but gets a free ride on the next outgoing data frame(using ACK field in the frame header). The technique is temporarily delaying outgoing ACKs, so that they can be hooked on the next outgoing data frame is called *piggybacking*. But ACK can't be delayed for a long time if receiver(of the packet to be acknowledged) does not have any data to send.
2. **Flow and congestion control :** TCP takes care of flow control by ensuring that both ends have enough resources and both can handle the speed of data transfer of each other, so that none of them gets overloaded with data. The term congestion control is used in almost the same context except that resources and speed of each router is also taken care of. The main concern is network resources in the latter case.
3. **Multiplexing / Demultiplexing :** Many application can be sending/receiving data at the same time. Data from all of them has to be multiplexed together. On receiving some data from lower layer, TCP has to decide which application is the recipient. This is called *demultiplexing*. TCP uses the concept of port number to do this.

TCP SEGMENT HEADER.



Header Fields

1. **Source and destination port :** These fields identify local endpoint of the connection. Each host may decide for itself, how to allocate its own ports starting at 1024. The source and destination socket numbers together identify the connection.
2. **Sequence and ACK number :** This field is used to give a sequence number to each and every byte transferred. This has an advantage over giving the sequence numbers to every packet because data of many small packets can be combined into one at the time of retransmission, if needed. ACK signifies next byte expected from the source and not the last byte received. ACKs are cumulative instead of selective. Sequence number space is as large as 32-bit although 17 bits would have been enough if packets were delivered in order.

   If packets reach in order, then according to following formula :

   (sender's window size) + (receiver's window size) < (sequence number space)

   the sequence number space should be 17-bits. But packets may take different routes and reach out of order. So, we need a larger sequence number space. And for optimisation, this is 32-bits.
3. **Header length :** This field tells how many 32-bit words are contained in the TCP header. This is needed because options field is of variable length.
4. **Flags.**

   There are six one-bit flags.

   (*i*) **URG :** This bit indicates whether urgent pointer field in this packet is being used.

   (*ii*) **ACK :** This bit is set to indicate ACK number field in this packet is valid.

   (*iii*) **PSH :** This bit indicates PUSHed data. The receiver is requested to deliver the data to the application upon arrival and not buffer it until a full buffer has been received.

   (*iv*) **RST :** This flag is used to reset a connection that has become confused due to a host crash or some other reason.It is also used to reject an invalid segment or refuse an attempt to open a connection. This causes an abrupt end to the connection, if it existed.

   (*v*) **SYN :** This bit is used to establish connections. The connection request(1st packet in 3-way handshake) has SYN=1 and ACK=0. The connection reply (2nd packet in 3-way handshake) has SYN=1 and ACK=1.

   (*vi*) **FIN :** This bit is used to release a connection. It specifies that the sender has no more fresh data to transmit. However, it will retransmit any lost or delayed packet. Also, it will continue to receive data from other side. Since SYN and FIN packets have to be acknowledged, they must have a sequence number even if they do not contain any data.

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5. **Window Size :** Flow control in TCP is handled using a variable-size sliding window. The Window Size field tells how many bytes may be sent starting at the byte acknowledged. Sender can send the bytes with sequence number between (ACK#) to (ACK# + window size - 1) A window size of zero is legal and says that the bytes upto and including ACK# -1 have been received, but receiver would like no more data for the moment. Permission to send can be granted later by sending a segment with the same ACK number and a nonzero Window Size field.

6. **Checksum :** This is provided for extreme reliability. It checksums the header, the data, and the conceptual pseudoheader.

   Pseudoheader contains

   (*i*) 32-bit IP address of the source and destination machines

   (*ii*) protocol number for TCP(6)

   (*iii*) byte count for the TCP segment (including header).

   Including the pseudoheader in TCP checksum computation helps detect misdelivered packets, but doing so, violates protocol hierarchy since IP addresses in it belong to the IP layer, not the TCP layer.

7. **Urgent pointer :** It indicates a byte offset from the current sequence number at which urgent data are to be found. Urgent data continues till the end of the segment. This is not used in practice. The same effect can be had by using two TCP connections, one for transferring urgent data.

8. **Options :** It provides a way to add extra facilities not covered by the regular header

   e.g.,

   (***i***) Maximum TCP payload that sender is willing to handle. The maximum size of segment is called *MSS (Maximum Segment Size)*. At the time of handshake, both parties inform each other about their capacity. Minimum of the two is honoured. This information is sent in the options of the SYN packets of the three way handshake.

   (***ii***) Window scale option can be used to increase window size. It can be specified by telling the receiver that the window size should be interpreted by shifting it left by specified number of bits. This header option allows window size up to 230.

9. **Data.**

   This can be of variable size. TCP knows its size by looking at the IP size header.

UDP (User Datagrom PROTOCOL).

- It is simpler, faster and cheaper than TCP.

  UDP is connectionless and unreliable which means that it does not establish a virtual circuit like TCP, nor does it demand an acknowledge. It merely sends out the message.

  UDP headers are all 8 bytes, while TCP headers can be 20-60 bytes long TCP, UDP provides delivery of segments using IP.

  Diagram of UDP header :

1 byte 1 byte	1 byte 1 byte
Souce Port Address	Destination Port Address
Total Length	Checksum

  Description of header components and their size in bits :

Source Port Address (16)	:	Address of the application that is generating the user datagram.
Destination Port Address (16)	:	Address of the application that will receive the user datagram.
Total Length (16)	:	Total length of the user datagram is bytes
Checksum (16)	:	Error Detection

- UDP is used for things like FINGER and other programs which prioritize speed over reliability, and whose data is not worth the overhead that establishing a TCP connecton demands.

  UDP is neither intelligent enough to compensate for congestion and receiver's sped, nor is it complex enough to care. There is no guarantee that the segments will arrive in order, or at all. It's also possible, if a long enough delay causes a re-send, to receive duplicate segments.

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UDP Datagrams

UDP network traffic is organized in the form of datagrams. A datagram comprises one message unit. The first eight (8) bytes of a datagram contain header information and the remaining bytes contain message data.

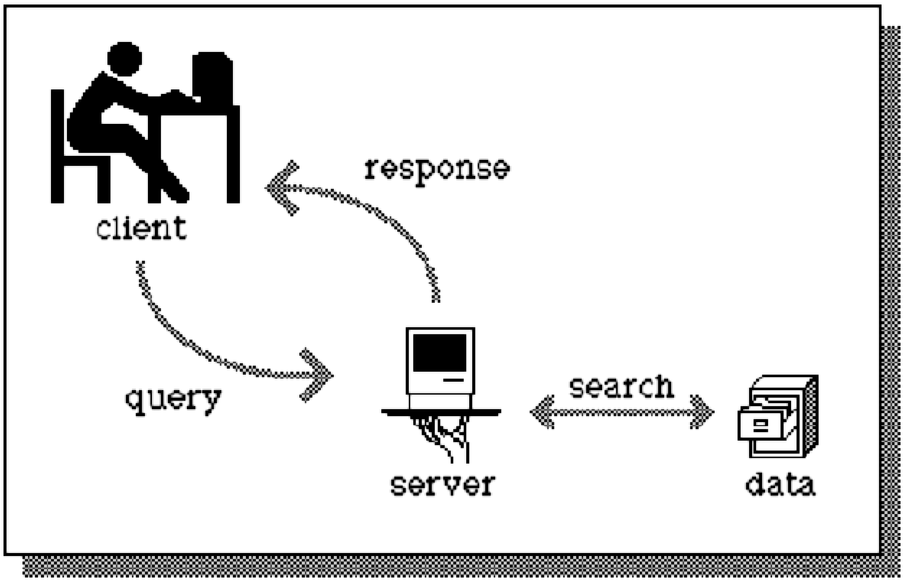
UDP datagram header consists of four (4) fields of two bytes each :

1. **Source port number :** It allow different applications to maintain their own channels for data similar to TCP.
2. **Destination port number :** UDP port headers are two bytes long; therefore, valid UDP port numbers range from 0 to 65535.
3. **Datagram size :** It is a count of total number of bytes contained in header and data sections. As header length is a fixed size, this field effectively tracks length of the variable-sized data portion (sometimes called *payload*). The size of datagrams varies depending on the operating environment but has a maximum of 65535 bytes.
4. **Checksums :** It protect message data from tampering. The checksum value represents an encoding of the datagram data calculated first by the sender and later by the receiver. Should an individual datagram be tampered with or get corrupted during transmission, the UDP protocol detects a checksum calculation mismatch. In UDP, checksumming is optional as opposed to TCP where checksums are mandatory.

SOCKET

CLIENT SERVER ARCHITECTURE

In this a machine(called *client*) makes a request to connect to another machine (called *server*) for providing some service. The services running on the server run on known ports(application identifiers) and client needs to know the address of the server machine and this port in order to connect to the server. On the other hand, the server does not need to know about the address or the port of the client at the time of connection initiation. The first packet which the client sends as a request to the server contains these informations about the client which are further used by the server to send any information. Client acts as the active device which makes the first move to establish the connection whereas the server passively waits for such requests from some client.



Client Server Model

UNIX SOCKET PROGRAMMING

In unix, whenever there is a need for inter process communication within the same machine, mechanism like signals or pipes(named or unnamed) is used. Similarly, when we desire a communication between two applications possibly running on different machines is desired, **sockets** is needed. Sockets are treated as another entry in the unix open file table. So all the system calls which can be used for any IO in unix can be used on socket. The server and client applications use various system calls to conenct which use the basic construct called **socket**. A socket is one end of the communication channel between two applications running on different machines.

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Steps followed by Client to establish the connection:

1. Create a socket
2. Connect the socket to the address of the server
3. Send/Receive data
4. Close the socket

Steps followed by Server to establish the connection:

1. Create a socket
2. Bind the socket to the port number known to all clients
3. Listen for the connection request
4. Accept connection request
5. Send/Receive data

Basic Data structures used in Socket programming

Socket Descriptor: A simple file descriptor in Unix.

int

Socket Address: This construct holds the information for socket address

```
struct sockaddrs {
   unsigned short    sa_family;    // address family, AF_xxx or PF_xxx
   char              sa_data[14];  // 14 bytes of protocol address
};
```

AF stands for Address Family and PF stands for Protocol Family. In most modern implementations only the AF is being used.

Types of AF.

Name	Purpose
AF_UNIX, AF_LOCAL	Local communication
AF_INET	IPv4 Internet protocols
AF_INET6	IPv6 Internet protocols
AF_IPX	IPX - Novell protocols
AF_NETLINK	Kernel user interface device
AF_X25	ITU-T X.25 / ISO-8208 protocol
AF_AX25	Amateur radio AX.25 protocol
AF_ATMPVC	Access to raw ATM PVCs
AF_APPLETALK	Appletalk
AF_PACKET	Low level packet interface

Note : In all the sample programs given below, we will be using AF_INET.

struct sockaddr_in: This construct holds information about the address family, port number, Internet address, and size of the struct sockaddr.

```
struct sockaddr_in {
   short int          sin_family;  // Address family
   unsigned short int sin_port;    // Port number
   struct in_addr     sin_addr;    // Internet address
   unsigned char      sin_zero[8]; // Same size as struct sockaddr
};
```

Some systems are

(*i*) Little Endian systems : Least signficant byte is stored in the higher address,

(*ii*) Big endian systems : Most significant byte is stored in the higher address.

Consider a situation where a Little Endian system wants to communicate with a Big Endian one. If there is no

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standard for data representation, then data sent by one machine is misinterpreted by the other. So standard has been defined for the data representation in the network (called *Network Byte Order*) which is Big Endian.

System calls that help to convert a short/long from Host Byte order to Network Byte Order and viceversa :

- htons() — "Host to Network Short"
- htonl() — "Host to Network Long"
- ntohs() — "Network to Host Short"
- ntohl() — "Network to Host Long"

INTERNET PROTOCOL (IP)

ADDRESSING SCHEME.

Address Classes

IP specifications divide addresses into following classes :

- Class A - For large networks

0	7 bits of the network address	24 bits of host address

- *Class B - For medium networks*

1	0	14 bits of the network address	16 bits of host address

- *Class C - For small networks*

1	1	0	21 bits of the network address	8 bits of host address

- *Class D - For multi-cast messages (multi-cast to a "group" of networks)*

1	1	1	0	28 bits for some sort of group address

- *Class E - Currently unused, reserved for potential uses in the future*

1	1	1	1	28 bits

Special Addresses

There are some special IP addresses :

1. **Broadcast Addresses**

   These are of two types :

   (*i*) ***Limited Broadcast :*** It consists of all 1's, i.e. address is 255.255.255.255 . It is used only on the LAN, and not for any external network.

   (*ii*) ***Directed Broadcast :*** It consists of the network number and all other bits as1's. It reaches the router corresponding to the network number, and from there it broadcasts to all the nodes in the network. This method is a major security problem, and is not used anymore. So now if we find that all the bits are 1 in the host no. field, then the packet is simply dropped. Therefore, only broadcast in own network using Limited Broadcast can be done.

2. **Network ID = 0**

   It means referring to this network and for local broadcast the host ID is made zero.

3. **Host ID = 0**

   This is used to refer to the entire network in the routing table.

4. **Loop-back Address.**

   Here addresses are of the type 127.x.y.z. It goes down way upto the IP layer and comes back to the application layer on the same host. This is used to test network applications before they are used commercially.

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APPLICATION LAYER PROTOCOL

APPLICATION LAYER PROTOCOL

Application layer provides interface to the communications environment which is used by the application process.

Classification of Application Layer Protocols

These are classified into

1. **Common Application Specific Elements (CASE)**
2. **Specific Application Specific Elements (SASE).**

These are commonly required by user elements and SASEs.

These are only included in the application process when their particular service is specifically required.

Defined CASEs.

There are four CASEs currently defined :

1. **Association Control ACSE :** It which enables users to establish or terminate an association between application processes.
2. **Reliable Transfer RTSE :** This enables reliable transfer of information between peers.
3. **Remote Operation ROSE :** This enables users to initiate operations at a remote site.
4. **Recovery ,CCR :** This enable users to recover from failure during execution of a task

Figure below shows the classified application protocols, interface with the application program.

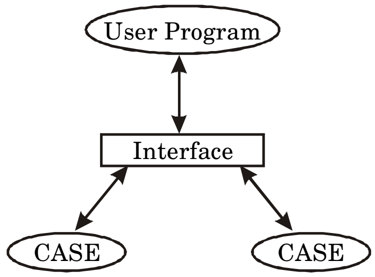

Fig. Datagram Application Process

APPLICATION SERVICES.

This is the actual internet service or access that we follow to get work or services done through the internet. With the advent of the world wide web, business has seized the opportunity to use the internet for communication, marketing, advertising and selling of different products.

Services Available on the Internet.

1. **World Wide Web :** It is the fastest growing service on the internet. It describes vast collection of hypertext-based files available on the internet and provides quick and easy way to retrieve these files, so they can be viewed using a browser. It's popularity comes from its ability to handle files containing many different elements, such as graphics, sound, video and animation.
2. **Search engines :** These help to find information on the internet. We can enter words and phrases relating to the information we are looking for and the search engine will find the closest match in its database. A list of alternatives (or results) is then displayed.
3. **Electronic mail :** It is an electronic postal system through which all kinds of data can be sent to other users of the internet using a Simple Mail Transfer Protocol (SMTP) which gives simple electronic mail facilities. The data to be transferred include- documents, pictures, sounds, spreadsheets and programs.
4. **Chat forms and Discussion groups :** These enable to take part in real-time discussion on any one of a wide variety of topics. We can take part in discussion by typing messages that can be read by other people in the group and replied to immediately. You can also view all the others conversations in the group.
5. **Newsgroups :** These are effectively global notice boards where anyone can post messages on a particular subject.
6. **HTTP :** It is the protocol used in the world wide web (www) and can be used for client-server applications involving hypertext.

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7. **FTP (File Transfer Protocol) :** It is the standard way to transfer files between different machines. In order to transfer files from the FTP server on the Unix/Telnet, we need an FTP client program on computer.

   FTP (File Transfer Protocol) : Site contains millions of files, many of which we can download for own benefits.

Note : Before transferring files, make sure there is enough disk space on the machine. FTP program uses TCP/IP protocol to transfer files to and from remote nodes. Sometimes it will read the host file to determine the IP address. The TCP/IP and the OSI protocol have coexisted for sometime and will continue to coexist in the internet community because as more and more OSI hosts are fielded on the internet, the requirement for gateways between the protocol suites become more pressing.

This report will describe an application layer gateway providing interoperability between TCP/IP file Transfer Protocol (FTP) and the OSI File Transfer, Access and Management (FTAM) protocol. The proposed application layer is based on a bi-directional set of mappings between FTP and FTAM protocols. Since the protocols have quite different command structures, the mappings between them are not one-to-one. This report assumes the knowledge of File Transfer Protocol (FTP) and the File Transfer, Access and Management Protocol (FTAM)

Goals of Mappings.

1. To provide FTP users with as much emulated FTP capability on an FTAM Responder as possible.
2. To provide FTAM users with as much emulated FTAM capability on an FTP server as possible.

Though it is anticipated that application layer gateway will be implemented on full protocol suites of both TCP/IP and OSI, atleast one implementation of such a gateway (included in the ISO Development Environment) can be configured to operate FTAM over, either OSI or TCP/IP lower layer services.

FTP Commands

- **ftp- starts FTP.** Here if a machine is specified, a connection to that machine is made immediately. e.g. telgate.uel.ac.uk, otherwise the FTP command is displayed.
- **open** - opens a connection to the specified machine.
- **cd**- change directory on the remote machine.
- **lcd** - change directory on the local machine.
- **pwd** - print working directory on the remote machine.
- **ls** - short directory listing on the remote machine.
- **lls** - short directory listing on the remote machine.
- **dir**- long directory listing on the local machine.
- **ascii** - set ascii transfer type (for text files)

POP3 and IMAP

POP3 (Post Office Protocol 3)

It is the most recent version of a standard protocol for receiving e-mail. It is a client/server protocol in which e-mail. It is received and held for you by your Internet server. Periodically, you (or your client e-mail receiver) check your mail-box on the server and download any mail, probably using POP3. This standard protocol is built into most popular e-mail products, such as Eudora and Outlook Express. It's

POP3 is designed to delete mail on the server as soon as the user has downloaded it. However, some implementations allow users or an administrator to specify that mail be saved for some period of time. POP can be thought of as a *store-and-forward* service.

Internet Message Access Protocol (IMAP). It is an alternative protocol. It provides the user more capabilities for retaining e-mail on the server and for organizing it in folders on the server. IMAP can be thought of as a remote file server.

Note : POP and IMAP deal with the receiving of e-mail and are not to be confused with the Simple Mail Transfer Protocol (SMTP), a protocol for transferring e-mail across the Internet. You send e-mail with SMTP and a mail handler receives it on your recipient's behalf, then the mail is read using POP or IMAP.

The conventional port number for POP3 is 110.

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BASIC CONCEPTS OF HUBS, SWITCHES, GATEWAYS AND ROUTERS

NETWORK WORKS

HUBS.

In computer networking, hub is a small, simple, inexpensive device that joins multiple computers together. Many network hubs available today support the Ethernet standard. Other types including USB hubs also exist, but Ethernet is the type traditionally used in home networking.

Working With Ethernet Hubs

To network a group of computers using an Ethernet hub, first connect an Ethernet cable into the unit, then connect other end of the cable to each computer's network interface card (.NIC). All Ethernet hubs accept the RJ-45 connectors of standard Ethernet cables.

To expand a network to accommodate more devices, Ethernet hubs can also be connected to each other, to switches, or to routers.

Characteristics of Ethernet Hubs

Ethernet hubs vary in the speed (network data rate or bandwidth) they support. Earlier, Ethernet hubs offered only 10 Mbps rated speeds but now hubs offer 100 Mbps Ethernet. Some support both 10 Mbps and 100 Mbps (so-called *dual-speed* or *10/100* hubs).

The number of ports an Ethernet hub supports also varies. Four- and five-port Ethernet hubs are most common in home networks, but eight- and 16-port hubs can be found in some home and small office environments.

Older Ethernet hubs were relatively large in size and sometimes noisy as they contained built in fans for cooling the unit. Newer devices are much smaller, designed for mobility, and noiseless.

Use of an Ethernet Hub.

Ethernet hubs operate as Layer 2 devices in the QSI_.m_Q.d_eJ, the same as network switches. Although offering comparable functionality, nearly all mainstream home network equipment today utilizes network switch technology instead of hubs due to the performance benefits of switches. Hub is useful for temporarily replacing a broken network switch or when performance is not a critical factor on the network.

SWITCH.

It is a small hardware device that joins multiple computers together within one local area network (LAN). Technically, network switches operate at layer two (Data link layer) of the QSI model.

Network switches appear nearly identical to network hups, but a switch generally contains more intelligence (and a slightly higher price tag) than a hub. Unlike hubs, network switches are capable of inspecting data packets as they are received, determining source and destination device of each packet, and forwarding them appropriately. By delivering messages only to the connected device intended, a network switch conserves network bandwidth and offers generally better performance than a hub.

As with hubs, Ethernet implementations of network switches are the most common. Mainstream Ethernet network switches support either 10/100 Mbps Fast Ethernet or Gigabit Ethernet (10/100/1000) standards.

Different models of network switches support differing numbers of connected devices. Most consumer-grade network switches provide either four or eight connections for Ethernet devices. Switches can be connected to each other, a so-called *daisy chaining* method to add progressively larger number of devices to a LAN.

BRIDGES.

It is a box with ports (usually two) to LAN segments. It operates in promiscuous mode at the data link layer (i.e. at the level of frames, not signals), it examines all frames and it recognizes where they came from, and where they are going to. It selectively *(frame filtering)* transfers frames from any port to other ports. It does not propagate noise signals and defective frames as it was the case for repeaters (at the physical layer). It adaptively recognizes which machines are reacheable from a port. It reduces traffic on each port and it improves security since each port will only transmit frames directed to nodes reacheable from that port (thus one does not overhear irrelevant traffic).

ROUTERS.

It is a box (usually a regular computer) with at least two ports, used to connect also dissimilar networks. It differs from bridges since it operates at the network level. It will also use different addresses, e.g. a bridge may use Ethernet addresses while a router uses IP addresses. It does all the transformations that may be required by the transfer of packets across the networks it connects.

Routing involves two basic activities:

(*i*) Running routing algorithms to determine routes, as expresed by *routing tables*

(*ii*) Using the routing tables to move packets across the network

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GATEWAYS.

It used to mean the same as packet switch, now it usually means a device that works above the network layer and can perform complete translations between different protocol stacks.

ARP, RARP, ICMP PROTOCOLS

Address Resolution Protocol (ARP)

If a machine talks to another machine in the same network, it requires its physical or MAC address. But, since application has given the destination's IP address, it requires some mechanism to bind the IP address with its MAC address. This is done through Address Resolution protocol (ARP). IP address of the destination node is broadcast and the destination node informs the source of its MAC address.

Reverse Address Resolution Protocol (RARP)

It is a protocol by which a physical machine in a local area network can request to learn its IP address from a gateway server's Address Resolution Protocol table or cache. This is needed since the machine may not have permanently attached disk where it can store its IP address permanently

Internet Control Message Protocol (ICMP)

This protocol discusses a mechanism that gateways and hosts use to communicate control or error information. The internet protocol provides unreliable, connectionless datagram service, and a datagram travels from gateway to gateway until it reaches one that can deliver it directly to its final destination. If a gateway cannot route or deliver a datagram, or if gateway detects an unusual condition, like network congestion, that affects its ability to forward the datagram, it needs to instruct the original source to take action to avoid or correct the problem. The ICMP allows gateways to send error or control messages to other gateways or hosts; ICMP provides communication between Internet protocol software on one machine and the Internet protocol software on another. This is a special purpose message mechanism added by the designers to the TCP/IP protocols. This is to allow gateways in an internet to report errors or provide information about unexpected circumstances. The IP protocol itself contains nothing to help the sender test connectivity or learn about failures.

NETWORK SECURITY

NETWORK SECURITY

It's very important to understand that in security, one simply cannot say "*what's the best firewall*?"

There are two extremes :

1. **Absolute security.**

   The closest we can get to an absolutely secure machine is one unplugged from the network, power supply, locked in a safe, and thrown at the bottom of the ocean. It isn't much useful in this state.

2. **Absolute access.**

   A machine with absolute access is extremely convenient to use: it's simply there, and will do whatever we tell it, without questions, authorization, passwords, or any other mechanism. This isn't much practical, either: the Internet is a bad neighborhood now, and it isn't long before some bonehead will tell the computer to do something like self-destruct, after which, it isn't much useful to us.

   Every organization needs to decide for itself where between the two extremes of total security and total access they need to be. A policy needs to articulate this, and then define *how* that will be enforced with practices and such. Everything that is done for security, then, must enforce that policy uniformly.

TYPES AND SOURCES OF NETWORK THREATS.

1. **Denial-of-Service (DoS)**

   It's attacks are probably the nastiest, and most difficult to address, because they're very easy to launch, difficult (sometimes impossible) to track, and it isn't easy to refuse the requests of the attacker, without also refusing legitimate requests for service.

   The premise of a DoS attack is simple: send more requests to the machine than it can handle. There are toolkits available in the underground community that make this a simple matter of running a program and telling it which host to blast with requests. The attacker's program simply makes a connection on some service port, perhaps forging the packet's header information that says where the packet came from, and then dropping the connection. If host is able to answer 20 requests per second, and the attacker is sending 50 per second, obviously the host will be unable to service all of the attacker's requests, much less any legitimate requests.

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Such attacks are now less popular.

The risk of being stung by a denial of service attack can be reduced by

(*i*) not running your visible-to-the-world servers at a level too close to capacity

(*ii*) using packet filtering to prevent obviously forged packets from entering into your network address space.

Obviously forged packets would include those that claim to come from your own hosts, addresses reserved for private networks as defined in RFC 1918 [4], and the *loopback* network (127.0.0.0).

(*iii*) keeping up-to-date on security-related patches for your hosts' operating systems.

2. **Unauthorized Access :** It is a very high-level term that can refer to a number of different sorts of attacks. The goal of these attacks is to access some resource that your machine should not provide the attacker.

   e.g. a host might be a web server, and should provide anyone with requested web pages. However, that host should not provide command shell access without being sure that the person making such a request is someone who should get it, such as a local administrator.

3. **Executing Commands Illicitly :** It is undesirable for an unknown and untrusted person to be able to execute commands on your server machines.

   There are two main classifications of the severity of this problem :

   (*i*) Normal user access: A normal user can do a number of things on a system (such as read files, mail them to other people, etc.) that an attacker should not be able to do. This might, then, be all the access that an attacker needs.

   (*ii*) Administrator access: An attacker might wish to make configuration changes to a host (perhaps changing its IP address, putting a start-up script in place to cause the machine to shut down every time it's started, or something similar). In this case, the attacker will need to gain administrator privileges on the host.

4. **Confidentiality Breaches :** We need to examine the threat model: what is it that you're trying to protect yourself against? There is certain information that could be quite damaging if it fell into the hands of a competitor, an enemy, or the public. In these cases, it's possible that compromise of a normal user's account on the machine can be enough to cause damage (perhaps in the form of PR, or obtaining information that can be used against the company, etc.)

   While many of the perpetrators of these sorts of break-ins are merely thrill-seekers interested in nothing more than to see a shell prompt for your computer on their screen, there are those who are more malicious.

5. **Destructive Behavior**

   Among the destructive sorts of break-ins and attacks, there are two major categories.

   (*i*) Data Diddling : The data diddler is likely the worst sort, since the fact of a break-in might not be immediately obvious. Perhaps he's toying with the numbers in your spreadsheets, or changing the dates in your projections and plans. Maybe he's changing the account numbers for the auto-deposit of certain paychecks. In any case, rare is the case when you'll come in to work one day, and simply know that something is wrong. An accounting procedure might turn up a discrepancy in the books three or four months after the fact. Trying to track the problem down will certainly be difficult, and once *that* problem is discovered, how can any of your numbers from that time period be trusted? How far back do you have to go before you think that your data is safe?

   (*ii*) Data Destruction : Some of those perpetrate attacks are simply twisted jerks who like to delete things. In these cases, the impact on your computing capability and consequently your business can be nothing less than if a fire or other disaster caused your computing equipment to be completely destroyed.

PROTECTION AGAINST NETWORK THREATS

If confidentiality or accuracy of information is of any value at all, it should be protected to an appropriate level. If the unauthorised disclosure or alteration of the information could result in any negative impact, it should be secured.

However, the means to achieve the requisite protection are usually far from obvious.

Commonly employed Mechanisms

1. Controlling access to the computer system or media.

   e.g., through '*logon*' authentication via passwords.

2. Employing an access control mechanism (such as profiling)

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3. Restricting physical access
   e.g., keeping media locked away or preventing access to the computer itself.

Cryptography

All the above approaches can be valuable and effective, but equally all can have serious shortcomings. A more fundamental approach to data security is *cryptography*.

(1) Conventional access control mechanisms can often be bypassed
    e.g., via hacking

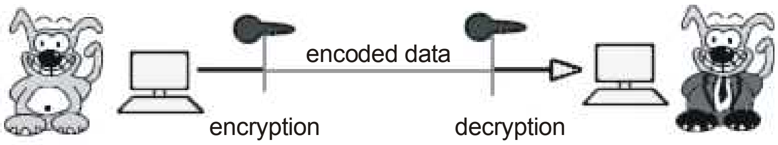
(2) What if data has to be transmitted, or if the data media (*e.g.,* floppy disk) has to be moved outside the secure environment?

(3) What if a number of people are sharing the computer environment? Is a technique designed to protect information in following situations.

Types of Cryptography

1. Private-key (or secret-key) cryptography

It is also called *Symmetric encryption* or *secret-key encryption*. It involves using the same key for encryption and decryption.



Encryption involves applying an operation (an algorithm) to the data to be encrypted using the private key to make them unintelligible. The slightest algorithm (such as an exclusive OR) can make the system nearly tamper proof (there being so such thing as absolute security).

To be completely secure, private-key systems need to use keys that are atleast as long as the message to be encrypted. Moreover, symmetric encryption requires that a secure channel be used to exchange the key, which seriously diminishes the usefulness of this type of encryption system.

Disadvantage : Symmetric encryption is based on the exchange of a secret (keys), thus problem of key distribution arises.

2. Public-key cryptography :

A cryptographic system uses two keys

(*i*) *Public key* known to everyone

(*ii*) *Private* or *Secret key* known only to the recipient of the message.

In public key system, public and private keys are related in such a way that only the public key can be used to encrypt messages and only the corresponding private key can be used to decrypt them. Moreover, it is virtually impossible to deduce the private key if you know the public key.

Public-key systems, such as Pretty Good Privacy (PGP), are becoming popular for transmitting information via the Internet. They are extremely secure and relatively simple to use.

The only difficulty with public-key systems is that you need to know the recipient's public key to encrypt a message for him or her. What's needed, therefore, is a global registry of public keys, which is one of the promises of the new LDAP technology.

Working of Public Key Cryptography (PKC) :

An encryption method has the property that publicly revealing an encryption key does not thereby reveal the corresponding decryption key. This has two important consequences:

(1) Couriers or other secure means are not needed to transmit keys.

(2) A message can be 'signed' using a privately held decryption key. Anyone can verify this signature using the corresponding publicly revealed encryption key.

The security of the standard Public key cryptography algorithm RSA is founded on the mathematical difficulty of finding two prime factors of a very large number.

Main idea of PKC is the use of two unique keys for each participant, with a bi-directional encryption mechanism that can use either key to decrypt information encrypted with the other key :

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- One of the keys allocated to each person is called *public key*, and is published in an open directory somewhere where anyone can easily look it up, *e.g.,* by *email address*.
- Each person keeps their other key secret, which is then called their *private key*.

This powerful architecture has three profound consequences :

(***i***) **Geography :** The sender and the recipient no longer need to meet or use some other potentially insecure method to exchange a common secret key. Since everyone has their own set of keys, then anyone can securely communicate with anyone else by first looking up their public key and using that to encrypt the message, enabling secure communication even across great distances over a network (like Internet).

(***ii***) **Security :** The disclosure of a key doesn't compromise all of the communications on a network, since disclosure of public keys is intended, and only messages sent to one person are affected by the disclosure of a private key.

(***iii***) **Digital signatures :** A sender can digitally sign their message by encrypting their name (or some other meaningful document) with their secret key and then attaching it to a message. The recipient can verify that the message came from the sender by decrypting their signature with their public key. If the decryption works and produces a readable signature, then the message came from the sender because only they could have encrypted the signature with their private key in the first place.

> *A message can be 'signed' using a privately held encryption key. Anyone can verify this signature using the corresponding publicly revealed encryption key. Signatures cannot be forged, and a signer cannot later deny the validity of his signature. This has obvious applications in* ***electronic mail*** *and* ***electronic funds transfer*** *systems.*

Public Key Cryptography (*PKC*) enables to send a digital signature across the *Internet* to verify a document actually came from you.

Digital signature is like a normal signature, in that it identifies you uniquely and is difficult to fake. Digital signature can be added to an *email*, document, or other file that you send to someone else using PKC, enabling the addressee to verify that the communication actually came from you. In practice, real implementation of digital signature systems was a prerequisite to widespread adoption of financial and legal communications on the Internet.

Note : Everyone using PKC has two keys, one private and one public, either of which can decrypt messages encrypted with the other.

Digital Signature process :

(***a***) **Encrypt message.** First the sender encrypts the message with the recipient's public key.

(***b***) **Add signature.** Then sender adds their signature to the encrypted message, perhaps some text and then encrypts the whole thing with their own *private* key.

(***c***) **Decrypt signature.** The recipient receives the message and decrypts it with the sender's public key, which produces the digital signature and the encrypted message.

(***d***) **Decrypt message.** Then recipient decrypts the remaining message with their own private key.

If this unwrapping procedure works, revealing a legible digital signature and a legible message, then recipient can be sure that the message was sent by the sender, since only they are in possession of the private key used to encrypt the entire message.

In practice, *PGP* system implements digital signatures by encrypting only a compact cryptographic checksum of the sender's message with the private key, thereby providing a smaller and more efficient attachment that still ensures that the signature is for the message it was sent with.

FIREWALLS.

In Internet and similar networks, connecting an organization to the Internet provides a two-way flow of traffic. This is undesirable in many organizations, as proprietary information is often displayed freely within a corporate *intranet* (*i.e.*, a TCP/IP network, modeled after the Internet that only works within the organization).

In order to provide some level of separation between an organization's intranet and the Internet, *firewalls* have been employed. *Firewall* is simply a group of components that collectively form a barrier between two networks.

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Bastion host

It is a general-purpose computer used to control access between internal network private intranet and the Internet (or any other untrusted network). Typically, these are hosts running a flavor of the Unix operating system that has been customized in order to reduce its functionality to only what is necessary in order to support its functions. Many of the general-purpose features have been turned off, and in many cases, completely removed, in order to improve security of the machine.

Router

A special purpose computer for connecting networks together. Routers also handle certain functions, such as *routing* , or managing the traffic on the networks they connect.

Access Control List (ACL)

Many routers now have the ability to selectively perform their duties, based on a number of facts about a packet that comes to it. This includes things like origination address, destination address, destination service port, and so on. These can be employed to limit the sorts of packets that are allowed to come in and go out of a given network.

Demilitarized Zone (DMZ)

It is a critical part of a firewall. It is a network that is neither part of the untrusted network, nor part of the trusted network, but this is a network that connects untrusted to the trusted. The importance of a DMZ is that someone who breaks into your network from the Internet should have to get through several layers in order to successfully do so. Those layers are provided by various components within the DMZ.

Proxy

This is the process of having one host act in behalf of another. A host that has the ability to fetch documents from the Internet might be configured as a *proxy server* , and host on the intranet might be configured to be *proxy clients* . In this situation, when a host on the intranet wishes to fetch the `<http://www.interhack.net/` > web page, for example, the browser will make a connection to the proxy server, and request the given URL. The proxy server will fetch the document, and return the result to the client. In this way, all hosts on the intranet are able to access resources on the Internet without having the ability to direct talk to the Internet.

Types of Firewalls

There are three basic types of firewalls :

1. **Application Gateways.**

   These are also called *proxy gateways*. These are made up of bastion hosts that run special software to act as a proxy server. This software runs at the *Application Layer* of ISO/OSI Reference Model. Clients behind the firewall must be *proxitized* (*i.e.*, must know how to use the proxy, and be configured to do so) in order to use Internet services. These are most secure, because these don't allow anything to pass by default, but need to have the programs written and turned on in order to begin passing traffic.

   These are also typically the slowest, because more processes need to be started in order to have a request serviced.

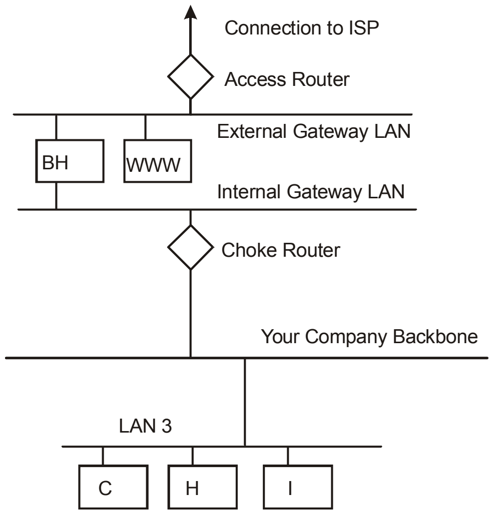

Fig. sample application gateway

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2. Packet Filtering.

It is a technique whereby routers have *ACLs* (Access Control Lists) turned on. By default, a router will pass all traffic sent it, and will do so without any sort of restrictions. Employing ACLs is a method for enforcing security policy with regard to what sorts of access is allowed to the outside world to have to internal network, and vice versa.

There is less overhead in packet filtering than with an application gateway, because feature of access control is performed at a lower ISO/OSI layer (typically, transport or session layer). Due to lower overhead and the fact that packet filtering is done with routers, which are specialized computers optimized for tasks related to networking, a packet filtering gateway is often much faster than its application layer cousins.

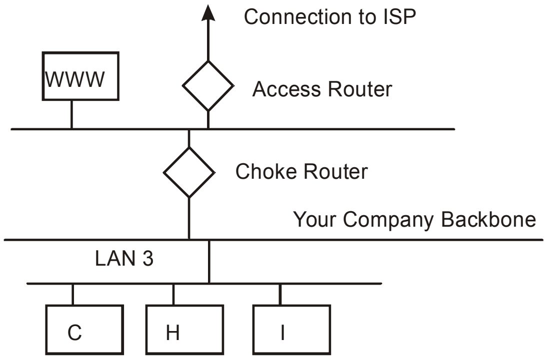

Fig. sample packet filtering gateway

Because we are working at a lower level, supporting new applications either comes automatically, or is a simple matter of allowing a specific packet type to pass through the gateway.

TCP/IP has absolutely no means of guaranteeing that the source address is really what it claims to be. As a result, we have to use layers of packet filters in order to localize the traffic. We can't get all the way down to the actual host, but with two layers of packet filters, we can differentiate between a packet that came from the Internet and one that came from our internal network. We can identify which network the packet came from with certainty, but we can't get more specific than that.

3. Hybrid Systems

This system has principle of both security of the application layer gateways and flexibility speed of packet filtering.

In some of these systems, new connections must be authenticated and approved at the application layer. Once this has been done, remainder of the connection is passed down to the session layer, where packet filters watch the connection to ensure that only packets that are part of an ongoing (already authenticated and approved) conversation are being passed.

Other possibilities include using both packet filtering and application layer proxies. The benefits here include providing a measure of protection against your machines that provide services to the Internet (such as a public web server), as well as provide the security of an application layer gateway to the internal network. Additionally, using this method, an attacker, in order to get to services on the internal network, will have to break through the access router, the bastion host, and the choke router.

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EXERCISE – I

MCQ TYPE QUESTIONS

1. Which of the following is possible in a token passing bus network ?
(*a*) in-service expansion
(*b*) unlimited number of stations
(*c*) both (*a*) and (*b*)
(*d*) unlimited distance

2. Which of the following is not relevant to networking ?
(*a*) Low-end stand alone hubs
(*b*) Stackable hubs
(*c*) Mesh network
(*d*) Bus hubs

3. In networking terminology UTP means
(*a*) Unshielded Twisted pair
(*b*) Ubiquitious Teflon port
(*c*) Uniformly Terminating port
(*d*) Unshielded T-connector port

4. Simple network management protocol (SNMP) is implemented with a daughter board in
(*a*) the nodes
(*b*) the server
(*c*) the hubs
(*d*) a separate PC that manages the network

5. The amount of uncertainty in a system of symbol is called
(*a*) Bandwidth (*b*) Entropy
(*c*) Loss (*d*) Quantum

6. What is the name of network architecture that was developed by IBM ?
(*a*) SNA (*b*) DECNET
(*c*) XNS (*d*) Novell

7. Which organisation draws up standards for modems ?
(*a*) CCITT (*b*) BELL
(*c*) AT and T (*d*) Hayes

8. Which one of the following is not a class of LAN ?
(*a*) Broad band (*b*) CSMA/CD
(*c*) Token bus (*d*) Token ring

9. Working of the WAH generally involves
(*a*) telephone lines (*b*) micro waves
(*c*) satellites (*d*) all of these

10. Modem is used in data trasmission. When was it involved and in which country ?
(*a*) 1963, USA (*b*) 1965, Germany
(*c*) 1950, USA (*d*) 1950, Japan

11. A local area network
(*a*) that connects thirty personnel computers can provide more computing power than a minicomputer
(*b*) controls error detection and correction
(*c*) constructs packets of data and sends them across the network
(*d*) none of these

12. Network operating system that does not support symmetric multiprocessing (SMP) is
(*a*) Banyan (VINES)
(*b*) Microsoft NT Advanced server
(*c*) SCO unix
(*d*) Novell network 3.X

13. Different computers are connected to a LAN by a cable and
(*a*) modem (*b*) interface card
(*c*) special wires (*d*) telephone lines

14. Method of communication in which transmission takes place in both directions, but only in one direction at a time, is called
(*a*) simplex (*b*) four wire circuit
(*c*) full duplex (*d*) half duplex

15. Error detection at the data link level is achieved by
(*a*) Bit stuffing
(*b*) Cyclic redundancy codes
(*c*) Hamming codes
(*d*) Both b and c

16. The topology with highest reliability is
(*a*) bus topology (*b*) star topology
(*c*) ring topology (*d*) mesh topology

17. Baud means the
(*a*) number of bits transmitted per unit time
(*b*) number of bytes transmitted per unit time
(*c*) rate at which the signal changes
(*d*) none of these

18. Unmodulated signal coming from a transmitter is known as
(*a*) carrier signal (*b*) baseband signal
(*c*) primary singal (*d*) none of these

19. Bit stuffing refers to
(*a*) inserting a '0' in user stream to differentiate it with a flag
(*b*) inserting a '0' in flag stream to avoid ambiguity
(*c*) appending a nibble to the flag sequence
(*d*) appending a nibble to the use data stream

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20. Which of the following ISO level is more closely related to the physical communications facilities?
(*a*) Application (*b*) Session
(*c*) Network (*d*) Data link

21. In a broad sense, a railway track is an example of
(*a*) simplex (*b*) half-duplex
(*c*) full-duplex (*d*) all of these

22. Frequency range at which the land coaxial cables are used, is
(*a*) 10^6 to 10^8 Hz (*b*) 10^{10} to 10^{11} Hz
(*c*) 10^3 to 10^4 Hz (*d*) 10^{14} to 10^{15} Hz

23. Which topology requires a central controller or hub ?
(*a*) Mesh (*b*) Star
(*c*) Bus (*d*) Ring

24. Which topology requires a multipoint connection?
(*a*) Mesh (*b*) Star
(*c*) Bus (*d*) Ring

25. In which topology, if there are n devices in a network, each device has n-1 ports for cables ?
(*a*) Mesh (*b*) Star
(*c*) Bus (*d*) Ring

26. A network that contains multiple hubs is most likely configured in which topology ?
(*a*) Mesh (*b*) Tree
(*c*) Bus (*d*) Star

27. Which model shows how the network functions of a computer ought to be organized ?
(*a*) ITU-T (*b*) OSI
(*c*) ISO (*d*) ANSI

28. In context of OSI or TCP/IP computer network models, which of the following is FALSE ?
(*a*) Besides span of geographical area, the other major difference between LAN and WAN is that the later uses switching element
(*b*) A repeater is used just to forward bits from one network to another one
(*c*) IP layer is connected oriented layer in TCP/IP
(*d*) A gateway is used to connect incompatible networks

29. The physical layer in reference to the OSI model defines
(*a*) data link procedures that provide for the exchange of data via frames that can be sent and received
(*b*) interface between X.25 network and packet mode device
(*c*) virtual circuit interface to packet-switched service
(*d*) all of these

30. What uses a physical star topology ?
(*a*) 10 base 5 (*b*) 10 base 2
(*c*) 10 base T (*d*) None of these

31. What can happen at a Token Ring station ?
(*a*) Examination of the destination address
(*b*) Regeneration of the frame
(*c*) Passing of the frame to the next station
(*d*) All of these

32. In Token Ring, when a frame reaches its destination station, then
(*a*) message is copied
(*b*) four bits in the packet are changed
(*c*) message is taken off the ring and replaced by the token
(*d*) both (*a*) and (*b*)

33. Which of the following is not a transceiver function?
(*a*) Transmission and receipt of data
(*b*) Checking of line voltages
(*c*) Addition and subtraction of headers
(*d*) Collision detection

34. The station-to-hub distance in which of the following is 2000 meters ?
(*a*) 100 Base-TX (*b*) 100 Base-FX
(*c*) 100 Base - T4 (*d*) 100 Base - T1

35. Ether LAN uses
(*a*) polar encoding
(*b*) differential manchester encoding
(*c*) manchester encoding
(*d*) NRZ

36. Which houses the switches in Token Ring ?
(*a*) NIC (*b*) MAU
(*c*) Nine-pin connector (*d*) Transceiver

37. Which of the following uses an 8B/6T encoding scheme?
(*a*) 100 Base-TX (*b*) 100 Base-FX
(*c*) 100 Base-T4 (*d*) 100 Base-T1

38. IBM's Token-Ring network uses
(*a*) adaptive routing (*b*) source routing
(*c*) alternative routing (*d*) spanned tree routing

39. In which circuit switching, delivery of data is delayed because data must be stored and retrieved from RAM ?
(*a*) space-division (*b*) time-division
(*c*) virtual (*d*) packet

40. To interface a computer terminal with a modem, required physical layer standard is
(*a*) RS 424 - A (*b*) RS 232 - C
(*c*) RS 449 (*d*) Either (*b*) or (*c*)

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41. How much bandwith is there in 0.1 micron of spectrum at a wavelength of 1 micron ?

(*a*) 20,000 GHz (*b*) 25,000 GHz

(*c*) 30,000 GHz (*d*) None of these

42. It is desired to send a sequence of computer screen images over an optical fiber. The screen is 480×640 pixels, each pixel being 24 bits. There are 60 screen images per second. How much bandwidth is needed, and how many microns of wavelenth are needed for this band at 1.30 microns?

(*a*) 1.0×10^5 microns (*b*) 2×10^5 microns

(*c*) 2.5×10^5 microns (*d*) 5×10^5 microns

43. A packet-switching network

(*a*) is free

(*b*) can reduce the cost of using an information utility

(*c*) allows communications channel to be shared among more than one user

(*d*) both (*b*) and (*c*)

44. The main purpose of a data link content monitor is to

(*a*) detect problems in protocols

(*b*) determine the type of transmission used in data link

(*c*) determine the type of switching used in data link

(*d*) determine the flow of data

45. Establishing a virtual connection is virtually equivalent to

(*a*) connecting as virtual memory

(*b*) physically connecting a DTE and DCE

(*c*) placing a telephone call prior to a conversion

(*d*) placing a modem prior to a conversion

46. Which company develops the TCP/IP protocol for networking ?

(*a*) IBM (*b*) DEC

(*c*) NOVELL (*d*) DARPA

47. How many digits of the network user Address are known as the DNIC (Data Network Identification Code) ?

(*a*) First three (*b*) First four

(*c*) First five (*d*) First seven

48. How many digits of the DNIC (Data Network Identification Code) identify the country ?

(*a*) First three (*b*) First four

(*c*) First five (*d*) First six

49. Which of the following digits are known as the area code of the network user address (NUA) ?

(*a*) 5 - 7 (*b*) 1 - 4

(*c*) 8 - 12 (*d*) 13 - 14

50. Which of the following digits are known as the terminal number of the network user address (NUA) ?

(*a*) 5 - 7 (*b*) 1 - 4

(*c*) 8 - 12 (*d*) 13 - 14

51. A station in a network forward incoming packets by placing them on its shortest output queue. What routing algorithm is being used ?

(*a*) Hot potato routing (*b*) Flodding

(*c*) Static routing (*d*) Delta routing

52. Which of the following program is used to copy files to or from another using timesharing system over a single link ?

(*a*) VMTP (*b*) TFTP

(*c*) UUCP (*d*) UART

53. Working of the WAH generally involves

(*a*) telephone lines (*b*) micro waves

(*c*) satellites (*d*) all of these

54. Which of the following is a wrong example of network layer ?

(*a*) Internet protocol (I/P) ARPA NET

(*b*) X.25 packet land protocols (PLP-ISO)

(*c*) Source routing and Domains Naming Usenet

(*d*) X.25 level 2-ISO

55. PURE ALOHA

(*a*) Does not require global time synchronization

(*b*) does require global time synchronization

(*c*) both (*a*) and (*b*)

(*d*) none of these

56. Slotted ALOHA

(*a*) divide time into discrete intervals

(*b*) require global time synchronization

(*c*) both (*a*) and (*b*)

(*d*) none of these

57. CSMA/CD

(*a*) is an important protocol

(*b*) is IEEE 802.3 (ethernet)

(*c*) both (*a*) and (*b*)

(*d*) none of these

58. Start and stop bits are used in serial communication for

(*a*) error detection

(*b*) error correction

(*c*) synchronization

(*d*) slowing down the communication

59. Manchester code is a

(*a*) non-return to zero code

(*b*) polar code

(*c*) both (*a*) and (*b*)

(*d*) none of these

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60. In the carrier sense network, if prevailing condition is a 'channel busy', then technique used is
(*a*) non-persistent then it results in randomised wait and sense
(*b*) 1-persistent then the channel is continually sensed
(*c*) p-persistent then randomised retransmission is done
(*d*) both (*a*) and (*b*)

61. Non-polling system is
(*a*) TDMA (*b*) stop and wait
(*c*) continuous ARQ (*d*) none of these

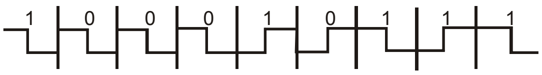
62. The systems that can be used in both priority and non-priority modes is
(*a*) TDM
(*b*) Register insertion
(*c*) Carrier sense systems
(*d*) All of these

63. BSC is a
(*a*) character-oriented protocol
(*b*) bit-oriented protocol
(*c*) full-duplex protocol
(*d*) none of these

64. In Ethernet CSMA/CD, the special bit sequence transmitted by media access management collision handling is called
(*a*) preamble (*b*) postamble
(*c*) jam (*d*) none of these

65. Adaptive or dynamic directory used in packet routing changes
(*a*) within each user session
(*b*) with each user session
(*c*) at system generation time only
(*d*) none of these

66. Differential Manchester code for the given figure is



(*a*) 111100101 (*b*) 100010111
(*c*) 101001111 (*d*) 101001101

67. Which of the following measures the number of lost or garbled messages as a fraction of the total sent in the sampling period ?
(*a*) Residual error rate
(*b*) Transfer failure probability
(*c*) Connection release failure probability
(*d*) Connection establishment failure probability

68. A field in the Ethernet message packet is
(*a*) Type (*b*) Data
(*c*) Address (*d*) All of these

69. In a broad sense, a railway track is an example of
(*a*) simplex (*b*) half-duplex
(*c*) full-duplex (*d*) all of these

70. Which of the following access control methods is probabilstic ?
(*a*) Polling (*b*) Contention
(*c*) Token passing (*d*) Sliding window

71. In stop-and-wait ARQ, if data 1 has an error, then receiver sends which frame ?
(*a*) NAK 0 (*b*) NAK 1
(*c*) NAK 2 (*d*) NAK

72. In which ARQ, when a NAK is received, all frames sent since the last frame acknowledged are retransmitted
(*a*) stop-and-wait (*b*) go-back-n
(*c*) selective-reject (*d*) both (*a*) and (*b*)

73. In which communication, poll/select method is used to determine control of the line ?
(*a*) peer-to peer
(*b*) peer-to-primary
(*c*) primary-to-peer
(*d*) primary-to-secondary

74. Poll/select line discipline requires what to identify the packet recipient ?
(*a*) Timer (*b*) Buffer
(*c*) Address (*d*) Dedicated line

75. The secondary device in a multipoint configuration sends data in response to which of the following event ?
(*a*) ACK (*b*) ENQ
(*c*) Poll (*d*) Sel

76. In sliding window flow control, if window size is 63, what is the range of sequence numbers ?
(*a*) 0 to 63 (*b*) 0 to 64
(*c*) 1 to 63 (*d*) 1 to 64

77. Which of the following decides the role (sender or receiver) of a device on a network ?
(*a*) Line connection (*b*) Link connection
(*c*) Line discipline (*d*) Link decision

78. For a sliding window of size n-1 (n sequence numbers), there can be maximum of how many frames sent but unacknowledged ?
(*a*) 0 (*b*) $n - 1$
(*c*) n (*d*) $n + 1$

79. For stop-and-wait flow control, for n data packets sent, how many acknowledgments are needed ?
(*a*) n (*b*) $2n$
(*c*) $n - 1$ (*d*) $n + 1$

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80. Select incorrect statement in error retransmission used in continuous ARQ method.

(*a*) Go-back-N method requires more storage at the receiving site

(*b*) Selective repeat involves complex login than GO-back-N

(*c*) Go-back-N has better line utilization

(*d*) Selective repeat has better line utilization

81. IEEE Project 802 divides the data link layer into which upper sublayer and which lower sublayer ?

(*a*) LLC, MAC (*b*) MAC, LLC

(*c*) PDU, HDLC (*d*) HDLC, PDU

82. Collision domain can be compared to which distance data travels between two stations ?

(*a*) Minimum (*b*) Maximum

(*c*) Virtual (*d*) Both (*a*) and (*b*)

83. The collision domain of tranditional Ethernet and Fast Ethernet respectively are

(*a*) 250m, 250m (*b*) 250m, 2500m

(*c*) 2500m, 250m (*d*) 2500m, 2500m

84. The systems that can be used in both priority and nonpriority modes, is

(*a*) TDM

(*b*) register insertion

(*c*) carrier sense systems

(*d*) token passing

85. Which of the following is a time - division switch?

(*a*) TSI (*b*) TDM bus

(*c*) Crosspoint (*d*) Both (*a*) and (*b*)

86. In a time - division switch, which governs the destination of a packet stored in RAM ?

(*a*) TDM bus (*b*) Crosspoint

(*c*) Crossbar (*d*) Control unit

87. In which routing method do all the routers have a common database ?

(*a*) Distance vector

(*b*) Link state

(*c*) Link vector

(*d*) None of these

88. A group of $2^m - 1$ routers are interconnected in a centralized binary tree. With router at each tree node. Router I communicates with router J by sending a message to the root of the tree. If root then sends the message back down J, the mean path length will be

(*a*) 2 (m – 2) (*b*) 2 (2^m – 1)

(*c*) m – 1 (*d*) (2m – 1)/mJ

89. Data link layer retransmits the damaged frames in most networks. If probability of a frame's being damaged is p, then what is the mean number of transmissions required to send a frame if acknowledgements are never lost ?

(*a*) $\dfrac{K}{K-p}$ (*b*) $\dfrac{1}{K-p}$

(*c*) $\dfrac{K}{K(1+p)}$ (*d*) $\dfrac{p}{K+1}$

90. A circuit switching network has the following characteristics :

Set up time = 5 sec

Propagation = 5 sec

Message size = 1 M bits

Data transmission rate = 1 M bits

Data transmission rate = 1 Kbps

Number of hops = 10

When will the last bit of the message reach the receiver ?

(*a*) t = 520 sec (*b*) t = 1000 ms

(*c*) t = 250 sec (*d*) t = 1025 sec

91. Two networks each provide reliable connection-oriented service. One of them offers a reliable byte stream and the other offers a reliable message stream. A process writes 1024 bytes, then

(*a*) both networks will receive 2048 bytes as a single unit

(*b*) message stream network will receive 1024 bytes only at one time. But byte stream network will receive 2048 bytes as a single unit.

(*c*) message stream network receives 2048 bytes as a whole. But bytes stream 1024 bytes only at a time.

(*d*) both will receive 1024 bytes at one time since there is a gap between writes.

92. Frames are generated by

(*a*) DLL in OSI mode, SATNET in TCP/IP, TC in ATM

(*b*) DLL in OSI mode, TC in TCP/IP, SATNET in ATM

(*c*) DLL in OSI mode, ARPANET in TCP/IP, PMD in ATM

(*d*) DLL in OSI mode, PMD in TCP/IP, ARPANET in ATM

93. A packet switching network (highly loaded) has the following characteristics:

Propagation delay = 2 sec/hp.

Data transmission rate = 1.

Packet size = r bit = 512 bits.

Number of of hops = 10.

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At what time the last bit of the last packet will reach the receiver ?

(*a*) 1024.5 sec (*b*) 1038 sec
(*c*) 2048 sec (*d*) 512 sec.

94. Television channels are 6 MHz. How many bits can be sent if four level digital signals are used (assume a noiseless channel)?

(*a*) 24 Mbps (*b*) 12 Mbps
(*c*) 6 Mbps (*d*) 36 Mbps

95. What is the maximum burst length on an 155.52 Mbps ATM ABR connection whose PCR value 200,000 and whose L value is 25 msec ?

(*a*) 9 cells (*b*) 12 cells
(*c*) 15 cells (*d*) 18 cells

96. Which of the following network access standard disassembler is used for connection station to a packet switched network ?

(*a*) X.3 (*b*) X.21
(*c*) X.25 (*d*) X.75

97. Which of the following network access standard is used for connecting station to a circuit switched network ?

(*a*) X.3 (*b*) X.21
(*c*) X.25 (*d*) X.75

98. IP address can be used to specify a broadcast and map to hardware broadcast if available. By conversion broadcast address has hosted with all bits

(*a*) 0 (*b*) 1
(*c*) both (*a*) and (*b*) (*d*) none of these

99. ARP (address resolution protocol) is

(*a*) A TCP/IP protocol used to dynamically bind a high level IP address to a low level physical hardware address
(*b*) ATCP/IP high level protocols for transferring files from one machine to another
(*c*) a protcol used to monitor computers
(*d*) a protocol that handles error and control messages.

100. ICMP (internet control message protocol) is

(*a*) a protocol that handles error and control messages
(*b*) a protocol used to monitor computers
(*c*) both (*a*) and (*b*)
(*d*) none of these

101. Which of the following TCP/IP protocol is used for file transfer with minimal capability and minimal overhead ?

(*a*) RARP (*b*) FTP
(*c*) TFTP (*d*) TELNET

102. Which of the following TCP/IP protocol is used to monitor IP gateway and the networks to which they attach ?

(*a*) SGMP (*b*) SUMP
(*c*) FTP (*d*) both (*a*) and (*b*)

103. How many class A, B and C networks Ids can exist?

(*a*) 2,113,658 (*b*) 16,382
(*c*) 126 (*d*) 128

104. Which of the following TCP/IP internet protocol a diskless machine uses to obtain its IP address from a server ?

(*a*) RDP (*b*) ARP
(*c*) RIP (*d*) X.25

105. An example of a network layer is

(*a*) Internet Protocol (IP)-ARPANET
(*b*) x.25 Packet Level Protocol (PLP)-ISO
(*c*) Source routing and domain naming-USENET
(*d*) all of these

106. A modern constellation diagram has data points at (0, 1) and (0, 2). The modern uses

(*a*) phase modulation
(*b*) amplitude modulation
(*c*) both (*a*) and (*b*)
(*d*) none of these

107. ICI (Interface Control Information) is

(*a*) used to transfer user data from layer to layer
(*b*) used to exchange information by peer entities at different sites on the network to instruct an entity to perform a service function
(*c*) a combination of service data unit (SDU) and protocol control information (PCI)
(*d*) a temporary parameter passed between N and N – 1 layers to involve service functions between two layers

108. In an Ethernet network, what is the relationship between round-trip time and the collision domain?

(*a*) Round-trip time increases; collision domain decreases
(*b*) Round-trip time decreases; collision domain decreases
(*c*) Round-trip time decreases; collision domain increases
(*d*) None of these

109. Repeaters function in

(*a*) physical layer (*b*) data link layer
(*c*) network layer (*d*) both (*a*) and (*b*)

110. What are the primary characteristics that distinguish a cell from a packet ?

(*a*) Cells are generally smaller that packets
(*b*) Cells do not incorporate physical address
(*c*) All cells have the same fixed length
(*d*) Packet cannot be switched

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111. Adaptive or dynamic directory used in packet routing changes

(*a*) within each user session

(*b*) with each user session

(*c*) at system generation time only

(*d*) both (*a*) and (*b*)

112. Three routers have to be inter-connected in a point-to-point network. Each pair of routers may be connected by a high speed line, a medium speed line, a low speed line or no line. If time taken to generate and inspect each topology, is 100 ms, how long will it take to inspect all of them to find one that best matches the expected load ?

(*a*) 12 sec (*b*) 14.4 sec

(*c*) 6.4 sec (*d*) 7.0 sec

113. A networking system has an n layers. Application generated message of length M bytes. At each of the layer, a h-byte header is added, what fraction of the network bandwidth is filled with headers ?

(*a*) M/(n × M) bytes (*b*) h × n × M bytes

(*c*) (n × h)/M bytes (*d*) M/(n × m)

114. For a packet switching network where

packet size = p bits

message size = x bits

propagation delay = d sec/hop

number of hops = k hops

Time required for the last bit of the last packet to reach the receiver is

(*a*) $t = \frac{x}{p} + k\,(d + p/d)$ sec

(*b*) $t = \frac{x}{p} + k\,(d + b)$ sec

(*c*) $t = \frac{x}{p} + k\,(d + b)$ sec

(*d*) $t = x + b + (k - 1)\,xp/b + kd$ sec

115. For a 1–Gbps network operating over 4000 km, the delay is the limiting factor, not the bandwidth. Consider a MAN with the average source and destination 20 km apart. At what data rate does the round-trip delay due to the speed of light equal the transmision delay for a1-KB packet ?

(*a*) 24 Mbps (*b*) 36 Mbps

(*c*) 40 Mbps (*d*) 48 Mbps

116. What is the bit rate for transmitting compressed VGA color with 8 bits/pixel at 40 frames/sec?

(*a*) 63.7 Mbps (*b*) 73.7 Mbps

(*c*) 83.7 Mbps (*d*) None of these

117. A group of $2^n - 1$ routers are interconnected in a centralised binary tree, with a router at each tree node. Router i communicates with router j by sending a message to the root of the tree. The root then sends the message back down to j. The mean number of hops per message for large n, assuming that all router pairs are equally likely will be

(*a*) $n - 4$ (*b*) $2n - 1$

(*c*) $n - 2$ (*d*) $2n - 4$

118. Imagine that you have trained your St. Bernard, Bernie, to carry a box of three 8 mm Exabyte instead of a flask of brandy. (When your disk fills up, you consider that an emergency). These tapes each contain 7 gigabytes. The dog can travel to your side, wherever you may be, at 18 khm/hour. For what range of distances does Bernie have a x higher data rate than a 155 Mbps ATM line ?

(*a*) $x < 2.4$ (*b*) $x < 4.2$

(*c*) $x < 5.4$ (*d*) $x < 7.2$

119. One important characteristics of the hub architecture of ARC - net is

(*a*) directionalized trasmission

(*b*) access control and addressing

(*c*) multiple virtual networks

(*d*) alternate routing

120. Select the chip that an active hub contains

(*a*) multiport repeater chip

(*b*) UART

(*c*) 386 SX

(*d*) none of these

121. Which of the following device is used with an X-25 network to provide service to asynchronous terminals ?

(*a*) Repeater (*b*) Bridges

(*c*) Gateway (*d*) Packet assembled

122. Satelite Switched Time-Division Multiple Access (SS/TDMA) is

(*a*) the method of determining which device has access to the trasmission medium at any time

(*b*) a medium access control technique for multiple access transmission media.

(*c*) a form of TDMA in which circuit switching is used to dynamically change the channel assignments

(*d*) all of these

123. End-to-end connectivity is provided from host-to-host in the

(*a*) network layer

(*b*) transport layer

(*c*) session layer

(*d*) none of these

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124. Frequency of failure and network recovery time after a failure are measures of what concept of a network ?
(*a*) Performance (*b*) Reliability
(*c*) Security (*d*) Feasibility

125. When a protocol specifies that address of the sender must occupy the first four bytes of a message, what is this issue ?
(*a*) Syntax (*b*) Semantics
(*c*) Timing (*d*) None of these

126. When a protocol specifies that address of the sender means the most recent sender and not the original source, what does this mean ?
(*a*) Syntax (*b*) Semantics
(*c*) Timing (*d*) None of these

127. When data is transmitted from device A to device B, the header from A's layer 5 is read by B's
(*a*) physical layer (*b*) transport layer
(*c*) session layer (*d*) presentation layer

128. Which layer functions as a liaison between user support layers and network support layers ?
(*a*) Network layer (*b*) Physical layer
(*c*) Transport layer (*d*) Session layer

129. Which of the following are session layer checkpoints ?
(*a*) Allow just a portion of a file to be resent
(*b*) Detect and recover errors
(*c*) Control the addition of headers
(*d*) Are involved in dialog control

130. Which of the following two networks can use possible hubs ?
(*a*) Ethernet (*b*) Arcnet
(*c*) Token ring (*d*) All of these

131. Gateways can function all the way upto
(*a*) transport layer (*b*) session layer
(*c*) presentation layer (*d*) application layer

132. A bridge has access to which address of a station on the same network ?
(*a*) Physical (*b*) Network
(*c*) Service access point (*d*) All of these

133. In session layer, during data transfer, the data stream responsible for the control purpose (i.e. control of the session layer itself) is
(*a*) regular data (*b*) typed data
(*c*) capability data (*d*) expedited data

134. A high speed communication equipment typically would not be needed for
(*a*) E-mail
(*b*) transferring large volume of data
(*c*) supporting communication between nodes in a LAN
(*d*) all of these

135. Protection of data from a natural disaster such as a tornado belongs to which of the following network issue ?
(*a*) Performance (*b*) Reliability
(*c*) Security (*d*) Management

136. If a protocol specifies that data should be sent at 100 Mbps, what is this issue ?
(*a*) Syntax (*b*) Semantics
(*c*) Timing (*d*) None of these

137. Decryption and encryption of data are responsibility of
(*a*) physical layer (*b*) data link layer
(*c*) presentation layer (*d*) session layer

138. Match the following

List-I

A. Bridge
B. Routers
C. Gateways

List-II

P. Operate at the top 3 layer of OSI.
Q. Operate at the bottom two layer of OSI.
R. Operate at the network layer of OSI.

Codes :

	A	**B**	**C**
(*a*)	P	R	Q
(*b*)	Q	R	P
(*c*)	R	P	Q
(*d*)	R	Q	P

139. Segmentation is done in
(*a*) Transport layer
(*b*) Network layer
(*c*) Data link layer
(*d*) Physical layer

140. Which of the following OSI Layer performs error checking of data?
(*a*) Network (*b*) Transport
(*c*) Data link (*d*) Physical

141. Match the following

P. SMTP	1. Application layer
Q. BGP	2. Transport layer
R. TCP	3. Data link layer
S. PPP	4. Network layer
	5. Physical layer

Codes :

	P	**Q**	**R**	**S**
(*a*)	2	1	3	5
(*b*)	1	4	2	3
(*c*)	1	4	2	5
(*d*)	2	4	1	5

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142. Choose the best matching between Group 1 and Group 2

Group – 1		Group – 2	
P.	Data link layer	1.	Ensures reliable transport of data over a physical point-to-point link.
Q.	Network Layer	2.	Encodes/decodes data for physical transmission.
		3.	Allow end-to-end communication between two processes.
R.	Transport layer	4.	Routes data from one network node to the next.

	P	**Q**	**R**
(*a*)	1	4	3
(*b*)	2	4	1
(*c*)	2	3	1
(*d*)	1	3	2

143. Which of the following statement is false ?

(*a*) Packets switching leads or better utilizations of bandwidth resources than circuit switching.

(*b*) Packet switching results in less variations in delay than circuit switching.

(*c*) Packet switching requires more per packet processing than circuit switching.

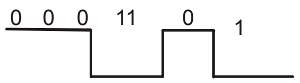
(*d*) Packet switching can lead to recordering unlike in circuit switching.

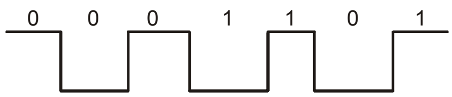
144. In Ethernet, MAC sub-layer uses access method

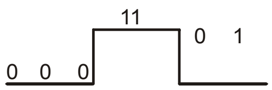
(*a*) ALOHA (*b*) CSMA/CD

(*c*) Slotted CSMA (*d*) None

145. Sketch the NRZL encoding for the bit 0001101

(*a*) 

(*b*) 

(*c*) 

(*d*) None

146. Consider a parity check code with three data bit and four parity check bit. Three of the code words are 0101011, 10001101, and 1110001, which of the following are also code words?

I. 0010111 II. 0110110

III. 1011010 IV. 0111010

(*a*) I and III (*b*) I, II, III

(*c*) II and IV (*d*) I, II, III and IV

147. Network Layer deals with

(*a*) End to End transmission

(*b*) Node to Node transmission

(*c*) Host to Host transmission

(*d*) None

148. Count-to-infinity problem is present in

(*a*) Distance-vector routing

(*b*) Link-state routing

(*c*) Hierarchical routing

(*d*) None of the above

149. Which of the following is an interior routing protocol ?

(*i*) RIP

(*ii*) OSPF

(*iii*) BGP

(*a*) (*i*) and (*ii*) (*b*) (*ii*) and (*iii*)

(*c*) (*i*) and (*iii*) (*d*) (*i*), (*ii*) and (*iii*)

150. Match with the suitable one :

List-I	**List-II**
A. Interior routing	1. Distance vector routing
B. Exterior routing	2. OSPF
C. RIP	3. IGMP
D. Multicast routing	4. BGP

Codes :

	A	**B**	**C**	**D**
(*a*)	2	4	2	1
(*b*)	2	4	1	3
(*c*)	3	1	4	2
(*d*)	4	3	1	2

151. Range of host address in class D is

(*a*) 128.0.0.0 to 191.255.255.255

(*b*) 192.0.0.0 to 223.255.255.255

(*c*) 224.0.0.0 to 239.255.255.255

(*d*) None

152. Convert the IP address whose hexadecimal representation is C22F1582 to dotted decimal notation.

(*a*) 194.47.21.130 (*b*) 194.47.21.125

(*c*) 194.47.25.130 (*d*) 194.47.21.210

153. Suppose computers A and B have IP address 10.105.1.113 and 10.105.1.91 respectively and they both use the same net mask N. Which of the values of N given below should not be used if, A and B should belong to the same network?

(*a*) 255.255.255.0 (*b*) 255.255.255.128

(*c*) 255.255.255.192 (*d*) 255.255.255.224

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154. A router uses the following routing table :

Destination	Mask	Interface
144.16.0.0	255.255.0.0	Eth 0
144.16.64.0	255.255.224.0	Eth 1
144.16.68.0	255.255.255.0	Eth 2
144.16.68.64	255.255.255.224	Eth 3

A packet bearing a destination address 144.16.68.117 arrives at the router. On which interfaces will be forwarded?

(*a*) Eth 0 (*b*) Eth 1
(*c*) Eth 2 (*d*) Eth 3

155. What is the IP address used for when a DHCP client sends out a discover message to locate a DHCP server?

(*a*) 255.255.255.255 (*b*) 255.255.255.0
(*c*) 0.0.0.0 (*d*) 255.0.0.0

156. A client message usually needs ------ to send a mail.

(*a*) SMTP (*b*) POP
(*c*) Both (*a*) and (*b*) (*d*) None of the above

157. Which one of the following uses UDP as the transport protocol?

(*a*) HTTP (*b*) Telnet
(*c*) DNS (*d*) SMTP

158. HELO and PORT, respectively are commands from the protocols.

(*a*) FTP and HTTP
(*b*) TELNET and POP 3
(*c*) HTTP and TELNET
(*d*) SMTP and FTP

NUMERICAL TYPE QUESTIONS

1. Fibre optics have maximum segment of_______m

2. Twisted pair have maximum segment is _____ m

3. Thick coax have maximum segment is____m

4. The monitor station in _______ standard ensures that one and only one token is circulating

5. If a network designer wants to connect 5 routers as point-to-point simplex line, then total number of lines required would be_______

6. A 100 km long cable runs at the T1 data rate. The propagation speed in the cable is half the speed of light. The ______ number of bits fit in the cable ?

7. Four bits are used for packed sequence numbering in a sliding window protocol used in a computer network. The maximum window size is _____

8. Number of cross point needed for 10 lines in a cross point switch which is full duplex in nature and there are no self-connection is_____

9. Maximum data rate of a channel for a noiseless 3-kHz binary channel is _______ bps

10. Maximum data rate of a channel of 3000 Hz bandwidth and SNR of 30 dB is _____bps

11. Minimum number of bits required to represent B is______

12. Minimum number of required to represent all the symbols together is_______

13. A 3000 Hz bandwidth noisy channel transmits bits with a signal to thermal noise ratio of 30 dB. The maximum data rate of the channel is _____kbps

14. If data rate of ring is 20 Mbps, signal propagation speed is 200 b/ms, then number of bits that can be placed on the channel of 200 km is ______bits

15. The signal-to-noise ratio is needed to put a T1 carrier on a T0-kHz line is ________db

16. In a crossbar with 1000 crosspoints, The ________ number of statistically are in use at any time

17. The ______ numbers of crosspoints are needed in a single-stage switch with 40 inputs and 50 outputs

18. The number of cross-points needed for 10 lines in a cross-point switch which is full duplex in nature and there are no self connection, is_____

19. A terminal multiplexer has six 1200 bps terminals and 'n' 300 bps terminals connected to it. The outgoing line is 9600 bps. The maximum value of n is_____

20. If a designer wants to design a point-to-point subnetwork with 10 routers of full duplex line, then total number of lines among them would be ______

21. Maximum data rate of a noiseless 4 kHz channel using T1 PCM system is ______kbps

22. The percentage of over head on a T1 carrier (1.544 Mbps) is _______

23. Phase shift keying (PSK.) method is used to modulate digital signals at 9600 bps using 16 levels. The line signals speed (i.e. modulation rate) will be_______ bands

24. A noiseless 3 kHz channel transmits bits with binary level signals. What is the maximum data rate is _______kbps

25. Given an infinite amount of buffer space. In the text it was stated that a satellite with two uplink and one downlink slotted ALOHA channels can achieve a downlink utilization of _______

26. The ______ number of hosts are attached to each of the local area network at your site

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27. The ______ number of bits internet address is assigned to each host on a TCP/IP internet which is used in all communication with the host

28. The ______ number of characters per sec (7 bits + 1 parity) can be transmitted over a 2400 bps line if the transfer is synchronous (1 start and 1 stop bit) ?

29. A terminal multiplexer has six 1200 bps terminals and 'n' 300 bps terminals connected to it. If outgoing line is 9600 bps, then maximum value of n is ________

30. In time division switches, if each memory access takes 100 ns and one frame period is 125ms, then maximum number of lines that can be supported is _______

31. The number of _______cross points needed for 12 lines in a cross point switch which is full duplex in nature and there are no self connection is

32. A transmission network is designed. ______the delay for utilization values of 25%. (Assume that the network delay is 20 ms).

33. ________connections are possible in a point to point network with 10 devices .

34. We want to send a 1000 KB file (K = 1000, and B byte) in 1 KB packets. The distance is 10 km and the signal propagation speed is 2×10^5 km/sec. The bandwidth is 1.5 mb/sec. (note M = 1000,000). How long will it take to send the file?

35. In a token ring network the transaction speed is 10 bps and the propagation speed is 200 meters/ ms. The 1-bit delay in this network is equivalent to _______

36. The maximum utilization of the token ring is ___________

When $t_i = 3, f_p = 5$ ms, N = 10

37. In an odd parity checking schema, _______is the value of the parity bit for the following sequence 0101101 ?

38. _________character per second (7 bits and 1 parity) can be transmitted over a 2400 line in case of synchronous transfer (1 stop and 1 start bit)?

39. A client uses TCP to send data to server. The data are 32 bytes. Calculated efficiency of this transmission at the TCP level. Assume the padding field is 10 bytes.

EXERCISE – II

(QUESTIONS FROM PREVIOUS GATE EXAMS)

MCQ TYPE QUESTIONS

2014

1. Consider the following three statements about link state and distance vector routing protocols, for a large network with 500 network nodes and 4000 links.

S1 : The computational overhead in link state protocols is higher than in distance vector protocols.

S2 : A distance vector protocol (with split horizon) avoids persistent routing loops, but not a link state protocol.

S3 : After a topology change, a link state protocol will converge faster than a distance vector protocol.

Which one of the following is correct about S1, S2, and S3?

(*a*) S1, S2, and S3 are all true
(*b*) S1, S2, and S3 are all false.
(*c*) S1 and S2 are true, but S3 is false
(*d*) S1 and S3 are true, but S2 is false.

2. Which one of the following are used to generate a message digest by the network security protocols?

P. RSA   Q. SHA-1
R. DES   S. MD5

(*a*) P and R only   (*b*) Q and R only
(*c*) Q and S only   (*d*) R and S only

3. Identify the correct order in which the following actions take place in an interaction between a web browser and a web server.

1. The web browser requests a webpage using HTTP.
2. The web browser establishes a TCP connection with the web server.
3. The web server sends the requested webpage using HTTP.
4. The web browser resolves the domain name using DNS.

(*a*) 4, 2, 1, 3
(*b*) 1, 2, 3, 4
(*c*) 4, 1, 2, 3
(*d*) 2, 4, 1, 3

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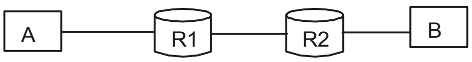
4. Which one of the following is TRUE about the interior gateway routing protocols – Routing Information Protocol (RIP) and Open Shortest Path First (OSPF)?
   (*a*) RIP uses distance vector routing and OSPF uses link state routing
   (*b*) OSPF uses distance vector routing and RIP uses link state routing
   (*c*) Both RIP and OSPF use link state routing
   (*d*) Both RIP and OSPF use distance vector routing

5. Which one of the following socket API functions converts an unconnected active TCP socket into a passive socket?
   (*a*) connect (*b*) bind
   (*c*) listen (*d*) accept

6. Consider the store and forward packet switched network given below. Assume that the bandwidth of each link is 10^6 bytes / sec. A user on host A sends a file of size 10^3 bytes to host B through routers R1 and R2 in three different ways. In the first case a single packet containing the complete file is transmitted from A to B. In the second case, the file is split into 10 equal parts, and these packets are transmitted from A to B. In the third case, the file is split into 20 equal parts and these packets are sent from A to B. Each packet contains 100 bytes of header information along with the user data. Consider only transmission time and ignore processing, queuing and propagation delays. Also assume that there are no errors during transmission. Let T1, T2 and T3 be the times taken to transmit the file in the first, second and third case respectively. Which one of the following is CORRECT?



   (*a*) T1 < T2 < T3
   (*b*) T1 > T2 > T3
   (*c*) T2 = T3, T3 < T1
   (*d*) T1 = T3, T3 > T2

7. An IP machine Q has a path to another IP machine H via three IP routers R1, R2, and R3.
   Q–R1–R2–R3–H
   H acts as an HTTP server, and Q connects to H via HTTP and downloads a file. Session layer encryption is used, with DES as the shared key encryption protocol. Consider the following four pieces of information:
   [11] The URL of the file downloaded by Q
   [12] The TCP port numbers at Q and H
   [13] The IP addresses of Q and H
   [14] The link layer addresses of Q and H
   Which of I1, I2, I3, and I4 can an intruder learn through sniffing at R2 alone?
   (*a*) Only I1 and I2
   (*b*) Only I1
   (*c*) Only I2 and I3
   (*d*) Only I3 and I4

8. In the following pairs of OSI protocol layer/sub-layer and its functionality, the INCORRECT pair is
   (*a*) Netwrok layer and Routing
   (*b*) Data Link Layer and Bit synchronization
   (*c*) Transport layer and End-to-end process communication
   (*d*) Medium Access Control sub-layer and Channel sharing

9. Host A (on TCP/IP v4 network A) sends an IP datagram D to host B (also on TCP/IP v4 network B). Assume that no error occurred during the transmission of D. When D reaches B, which of the following IP header field(s) may be different from that of the original datagram D?
   (*i*) TTL (*ii*) Checksum
   (*iii*) Fragment Offset
   (*a*) (*i*) only
   (*b*) (*i*) and (*ii*) only
   (*c*) (*ii*) and (*iii*) only
   (*d*) (*i*), (*ii*) and (*iii*)

10. An IP router with a Maximum Transmission Unit (MTU) of 1500 bytes has received an IP packet of size 4404 bytes with an IP header of length 20 bytes. The values of the relevant fields in the header of the third IP fragment generated by the router for this packet are
   (*a*) MF bit: 0, Datagram Length: 1444; Offset: 370
   (*b*) MF bit: 1, Datagram Length: 1424; Offset: 185
   (*c*) MF bit: 1, Datagram Length: 1500; Offset: 370
   (*d*) MF bit: 0, Datagram Length: 1424; Offset: 2960

11. Suppose that everyone in a group of N people wants to communicate secretly with the N-1 others using symmetric key cryptographic system. The communication between any two persons should not be decodable by the others in the group. The number of keys required in the system as a whole to satisfy the confidentiality requirement is
   (*a*) 2N (*b*) N(N-1)
   (*c*) N(N-1)/2 (*d*) $(N-1)^2$

12. Which one of the following fields of an IP header is NOT modified by a typical IP router?
   (*a*) Checksum (*b*) Source address
   (*c*) Time to Live (TTL) (*d*) Length

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13. Suppose two hosts use a TCP connection to transfer a large file. Which of the following statements is/are FALSE with respect to the TCP connection ?

I. If the sequence number of a segment is m, then the sequence number of the subsequent segment is always m + 1.

II. If the estimated round trip time at any given point of time is *t* sec the value of the retransmission timeout is always set to greater than or equal to t sec.

III. The size of the advertised window never changes during the curse of the TCP connection.

IV. The number of unacknowledged bytes at the sender is always less than or equal to the advertised window.

(*a*) III only (*b*) I and III only

(*c*) I and IV only (*d*) II and IV only

14. In one of the pairs of protocols given below, both the protocols can use multiple TCP connections between the same client and the server. Which one is that ?

(*a*) HTTP, FTP (*b*) HTTP, TELNET

(*c*) FTP, SMTP (*d*) HTTP, SMTP

15. Identify the correct order in which a server process must invoke the function calls accept, bind, listen, and recv according to UNIX socket APL

(*a*) listen, accept, bind recv

(*b*) bind, listen, accept, recv

(*c*) bind, accept, listen, recv

(*d*) accept, listen, bind recv

16. Which one of the following statements is NOT correct about HTTP cookies?

(*a*) A cookie is a piece of code that has the potential to compromise the security of an internet user

(*b*) A cookie gains entry to the user's work area through an HTTP header

(*c*) A cookie has an expiry date and time

(*d*) Cookies can be used to track the browsing pattern of a user at a particular site

17. Host A sends a UDP datagram containing 8880 bytes of user data to host B over an Ethernet LAN. Ethernet frames may carry data up to 1500 bytes (i.e. MTU = 1500 bytes). Size of UDP header is 8 bytes and size of IP heard is 20 bytes.There is no option field in IP header. How many total number of IP fragments will be transmitted and what will be the contents of offset field in the last fragment?

(*a*) 6 and 925 (*b*) 6 and 7400

(*c*) 7 and 1110 (*d*) 7 and 8880

18. Consider the following routing table at an IP router:

Network No.	Net Mask	Next Hop
128.96.170.0	255.255.254.0	Interface 0
128.96.168.0	255.255.254.0	Interface 1
128.96.166.0	255.255.254.0	R2
128.96.164.0	255.255.252.0	R3
0.0.0.0	Default	R4

For each IP address in Group I identify the correct choice of the next hop from Group II using the entries from the routing table above.

Group I	Group II
i. 128.96.171.92	a. Interface 0
ii. 128.96.167.151	b. Interface 1
iii. 128.96.163.151	c. R2
iv. 128.96.165.121	d. R3
	e. R4

(*a*) i – a, ii – c, iii – e, iv – d

(*b*) i – a, ii – d, iii – b, iv – e

(*c*) i – b, ii – c, iii – d, iv – e

(*d*) i – b, ii – c, iii – e, iv – d

19. Assume that the bandwidth for a TCP connection is 1048560 bits/sec. Let α be the value of RTT in milliseconds. (rounded off to the nearest integer) after which the TCP window scale option is needed. Let β be the maximum possible window size the window scale option. Then the values of α and β are

(*a*) 63 milli seconds, 65535×2^{14}

(*b*) 63 milli seconds, 65535×2^{16}

(*c*) 500 milliseconds, 65535×2^{14}

(*d*) 500 milliseconds, 65535×2^{16}

20. Consider the following statements.

I. TCP connections are full duplex

II. TCP has no option for selective acknowledgment

III. TCP connections are message streams

(*a*) Only I is correct

(*b*) Only I and III are correct

(*c*) Only II and III are correct

(*d*) All of I, II and III are correct

21. Consider a CSMA/CD network that transmits data at a rate of 100 Mbps (10^8 bits second) over a 1 km(kilometer) cable with no repeaters. If the minimum frame size required for this network is 1250 bytes, what is the signal speed (km/sec) in the cable ?

(*a*) 8000 (*b*) 10000

(*c*) 16000 (*d*) 20000

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2013

22. Match the problem domains in GROUP I with the solution technologies in GROUP II.

GROUP I

(P) Service oriented computing

(Q) Heterogeneous communicating systems

(R) Information representation

(S) Process description

GROUP II

(1) Interoperability

(2) BPMN

(3) Publish-find-bind

(4) XML

(*a*) P-1, Q-2, R-3, S-4

(*b*) P-3, Q-4, R-2, S-1

(*c*) P-3, Q-1, R-4, S-2

(*d*) P-4, Q-3, R-2, S-1

23. The transport layer protocols used for real time multimedia, file transfer, DNS and email, respectively are:

(*a*) TCP, UDP, UDP and TCP

(*b*) UDP, TCP, TCP and UDP

(*c*) UDP, TCP, UDP and TCP

(*d*) TCP, UDP, TCP and UDP

24. Using public key cryptography, X adds a digital signature σ to message M, encrypts <M, σ>, and sends it to Y, where it is decrypted. Which one of the following sequences of keys is used for the operations?

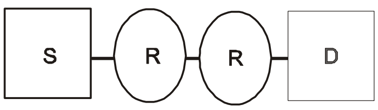
(*a*) Encryption: X's private key followed by Y's private key; Decryption: X's public key followed by Y's public key.

(*b*) Encryption: X's private key followed by Y's public key; Decryption: X's public key followed by Y's private key.

(*c*) Encryption: X's public key followed by Y's private key; Decryption: Y's public key followed by X's private key.

(*d*) Encryption: X's private key followed by Y's public key; Decryption: Y's private key followed by X's public key.

25. Assume that source S and destination D are connected through two intermediate routers labeled R. Determine how many times each packet has to visit the network layer and the data link layer during a transmission from S to D.



(*a*) Network layer - 4 times and Data link layer - 4 times

(*b*) Network layer - 4 times and Data link layer - 3 times

(*c*) Network layer - 4 times and Data link layer - 6 times

(*d*) Network layer - 2 times and Data link layer - 6 times.

26. Determine the maximum length of the cable (in km) for transmitting data at a rate of 500 Mbps in an Ethernet LAN with frames of size 10,000 bits. Assume the signal speed in the cable to be 2,00,000 km/s.

(*a*) 1 (*b*) 2

(*c*) 2.5 (*d*) 5

27. In an IPv4 datagram, the M bit is 0, the value of HLEN is 10, the value of total length is 400 and the fragment offset value is 300. The position of the datagram, the sequence numbers of the first and the last bytes of the payload, respectively are

(*a*) Last fragment, 2400 and 2789

(*b*) First fragment, 2400 and 2759

(*c*) Last fragment, 2400 and 2759

(*d*) Middle fragment, 300 and 689

2012

28. In the IPv4 addressing format, the number of networks allowed under Class C addresses is

(*a*) 2^{14} (*b*) 2^{7}

(*c*) 2^{21} (*d*) 2^{24}

29. Which of the following transport layer protocols is used to support electronic mail?

(*a*) SMTP (*b*) IP

(*c*) TCP (*d*) UDP

30. The protocol data unit (PDU) for the application layer in the Internet stack is

(*a*) Segment

(*b*) Datagram

(*c*) Message

(*d*) Frame

31. Consider an instance of TCP's Additive Increase Multiplicative Decrease (AIMD) algorithm where the window size at the start of the slow start phase is 2 MSS and the threshold at the start of the first transmission is 8 MSS. Assume that a timeout occurs during the fifth transmission. Find the congestion window size at the end of the tenth transmission.

(*a*) 8 MSS

(*b*) 14 MSS

(*c*) 7 MSS

(*d*) 12 MSS

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32. Consider a source computer (S) transmitting a file of size 10^6 bits to destination computer (D) over a network of two routers (R_1 and R_2) and three links (L_1, L_2, and L_3), L1 connects S to R_1 : L_2 connects R_1 to R_2; and L_3 connects R_2 to D. Let each link be of length 100 km. Assume signals travel over each link at a speed of 10^8 meters per second. Assume that the link bandwidth on each link is 1 Mbps. Let the file be broken down into 1000 packets each of size 1000 bits. Find the total sum of transmission and propagation delays in transmitting file from S to D?

(*a*) 1005 ms (*b*) 1010 ms

(*c*) 3000 ms (*d*) 3003 ms

33. An Internet Service Provider (ISP) has the following chunk of CIDR-based IP addresses available with it: 245.248.128.0/20. The ISP wants to give half of this chunk of addresses to Organization A, and a quarter to Organization B, while retaining the remaining with itself. Which of the following is a valid allocation of addresses to A and B?

(*a*) 245.248.136.0/21 and 245.248.128.0/22

(*b*) 245.248.128.0/21 and 245.248.128.0/22

(*c*) 245.248.132.0/22 and 245.248.132.0/21

(*d*) 245.248.136.0/24 and 245.248.132.0/21

2011

34. Which one of the following is NOT desired in a good Software Requirement Specifications (SRS) document?

(*a*) Functional Requirements

(*b*) Non-Functional Requirements

(*c*) Goals of Implementation

(*d*) Algorithms for Software Implementation

35. HTML (Hyper Text Markup Language) has language elements which permit certain actions other than describing the structure of the web document. Which one of the following actions is NOT supported by pure HTML (without any server or client side scripting) pages?

(*a*) Embed web objects from different sites into the same page

(*b*) Refresh the page automatically after a specified interval

(*c*) Automatically redirect to another page upon download

(*d*) Display the client time as part of the page

36. Let the time taken to switch between user and kernel modes of execution be t_1 while the time taken to switch between two processes be t_2. Which of the following is **TRUE**?

(*a*) $t_1 > t_2$

(*b*) $t_1 = t_2$

(*c*) $t_1 < t_2$

(*d*) nothing can be said about the relation between t_1 and t_2

37. Consider different activities related to email.

m1 : Send an emial from a mail client to a mail server

m2 : Download an email from mailbox to a mail client

m3 : Checking email in a web browser

Which is the application level protocol used in each activity?

(*a*) m1 : HTTP m2 :SMTP m3 : POP

(*b*) m1 : SMTP m2 :FTP m3 : HTTP

(*c*) m1 : SMTP m2 :POP m3 : HTTP

(*d*) m1 : POP m2 :SMTP m3 : IMAP

38. A layer-4 firewall (a device that can look at all protocol headers up to the transport layer) **CANNOT**

(*a*) block entire HTTP traffic during 9:00 PM and 5:00 AM

(*b*) block all ICMP traffic

(*c*) stop incoming traffic from a specific IP address but allow outgoing traffic to the same IP address

(*d*) block TCP traffic from a specific user on a multi-user system during 9:00 PM and 5:00 AM

2010

39. One of the header fields in an IP datagram is the Time-to-Live (TTL) field. Which of the following statements best explains the need for this field?

(*a*) It can be used to prioritize packets

(*b*) It can be used to reduce delays

(*c*) It can be used to optimize throughput

(*d*) It can be used to prevent packet looping

40. Which one of the following is not a client-server application?

(*a*) Internet chat

(*b*) Web browsing

(*c*) E-mail

(*d*) Ping

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41. Suppose computers A and B have IP addresses 10.105.1.113 and 10.0105.1.91 respectively and they both use the same netmask N. Which of the values of N given below should not be used if A and B should belong to the same network?

(*a*) 255.255.255.0 (*b*) 255.255.255.128
(*c*) 255.255.255.192 (*d*) 255.255.255.224

2009

42. In the RSA public key cryptosystem, the private and public keys are (e, n) and (d, n) respectively, where $n = p * q$ and p and q are large primes. Besides, n is public and p and q are private. Let M be a integer such that $0 < M < n$ ad $\phi(n) = (p-1)(q-1)$.

Now cosider the following equatios.

I. $M' = M^e \bmod n$

$M = (M')^d \bmod n$

II. ed $\equiv 1 \bmod n$

III. ed $\equiv 1 \bmod \phi n$

IV. $M' = M^e \bmod \phi(n)$

$M = (M')^d \bmod \phi(n)$

While of the above equations correctly represent RSA cryptosystem ?

(*a*) I and II (*b*) I and III
(*c*) II and IV (*d*) III and IV

43. Which opening a TCP connection, the initial sequence number is to be derived using a time-of-day (ToD) clock that keeps running even when the host is down. The low order 32 bits of the counter of the ToD clock is to be used for the initial sequence numbers. The clock counter increments once per millisecond. The maximum packet lifetime is given to be 64s.

Which one of the choices given below is closest to the minimum permissible rate at which sequence numbers used for packets of a connection can increase ?

(*a*) 0.015/s (*b*) 0.064/s
(*c*) 0.135/s (*d*) 0.327/s

44. Let $G(x)$ be the generator polynomial used for CRC checking. What is the condition that should be satisfied by $G(x)$ to detect odd number of bits in error?

(*a*) $G(x)$ contains more than two terms.

(*b*) $G(x)$ does not divide $1 + x^k$, for any k not ex ceeding the frame length.

(*c*) $1 + x$ is a factor of $G(x)$.

(*d*) $G(x)$ has an odd number of terms.

2008

45. What is the maximum size of data that the application layer can pass on to the TCP layer below?

(*a*) Any size
(*b*) 2^{16} bytes – size of TCP header
(*c*) 2^{16} bytes
(*d*) 1500 bytes

46. In the slow start phase of the TCP congestion control algorithm, the size of the congestion window

(*a*) does not increase
(*b*) increases linearly
(*c*) increases quadratically
(*d*) increases exponentially

47. If a class B network on the Internet has a subnet mask of 255.255.248.0, what is the maximum number of hosts per subnet?

(*a*) 1022 (*b*) 1023
(*c*) 2046 (*d*) 2047

48. A computer on a 10 Mbps network is regulated by a token bucket. The token bucket is filled at a rate of 2Mbps. It is initially filled to capacity with 16 megabits. What is the maximum duration for which the computer can transmit at the full 10 Mbps?

(*a*) 1.6 seconds (*b*) 2 seconds
(*c*) 5 seconds (*d*) 8 seconds

49. A client process P needs to make a TCP connection to a server process S. Consider the following situation: the server process S executes a socket (), a bind () and a listen () system call in that order, following which it is preempted. Subsequently, the client process P executes a socket () system call followed by connect () system call to connect to the server proccess S. The server process has not executed any accept () system call. Which one of the following events could take place?

(*a*) connect () system call returns successfully
(*b*) connect () system call blocks
(*c*) connect () system call returns an error
(*d*) connect () system call results in a core dump

2007

50. In Ethernet when Manchester encoding is used, the bit rate is

(*a*) Half the baud rate
(*b*) Twice the baud rate
(*c*) Same as the baud rate
(*d*) None of these

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51. Which one of the following uses UDP as the transport protocol ?

(*a*) HTTP (*b*) Telnet
(*c*) DNS (*d*) SMTP

52. There are n stations in a slotted LAN. Each station attempts to transmit with a probability p in each time slot. What is the probability that **ONLY** one station transmits in a given time slot ?

(*a*) $np(1-p)^{n-1}$ (*b*) $(1-p)^{n-1}$
(*c*) $p(1-p)^{n-1}$ (*d*) $1-(1-p)^{n-1}$

53. In a token ring network the transmission speed is 10^7 bps and the propagation speed is 200 metres / μs. The 1 –bit delay in this network is equivalent to :

(*a*) 500 metres of cable.
(*b*) 200 metres of cable.
(*c*) 20 metres of cable.
(*d*) 50 metres of cable.

54. The address of a class B host is to be split into subnets with a 6 – bit subnet number. What is the maximum number of subnets and the maximum number of hosts in each subnet ?

(*a*) 62 subnets and 262142 hosts.
(*b*) 64 subnets and 262142 hosts.
(*c*) 62 subnets and 1022 hosts.
(*d*) 64 subnets and 1024 hosts.

55. The message 11001001 is to be transmitted using the CRC polynomial $x^3 + 1$ to protect it from errors. The message that should be transmitted is :

(*a*) 11001001000
(*b*) 11001001011
(*c*) 11001010
(*d*) 110010010011

56. The distance between two stations M and N is L kilometres. All frames are K bits long. The propagation delay per kilometre is t seconds. Let R bits/second be the channel capacity. Assuming that processing delay is negligible, the *minimum* number of bits for the sequence number field in a frame for maximum utilization, when the *sliding window protocol* is used, is:

(*a*) $\left\lceil \log_2 \frac{2LtR + 2K}{K} \right\rceil$ (*b*) $\left\lceil \log_2 \frac{2LtR}{K} \right\rceil$

(*c*) $\left\lceil \log_2 \frac{2LtR + K}{K} \right\rceil$ (*d*) $\left\lceil \log_2 \frac{2LtR + K}{2K} \right\rceil$

57. Match the following :

P. SMTP	1. Application layer
Q. BGP	2. Transport layer
R. TCP	3. Data link layer
S. PPP	4. Network layer
	5. Physical layer

Codes :

	P	Q	R	S
(*a*)	2	1	3	5
(*b*)	1	4	2	3
(*c*)	1	4	2	5
(*d*)	2	4	1	3

2006

58. Station A uses 32 byte packets to transmit messages to Station B using a sliding window protocol. The round trip delay between A and B is 80 milliseconds and the bottleneck bandwidth on the path between A and B is 128 kbps. What is the optimal window size that A should use ?

(*a*) 20 (*b*) 40
(*c*) 160 (*d*) 320

59. Two computers C1 and C2 are configured as follows. C1 has IP address 203. 197.2.53 and netmask 255. 255. 128.0. C2 has IP address 203.197.75.201 and netmask 255. 255.192.0. Which one of the following statements is true ?

(*a*) C1 and C2 both assume they are on the same network
(*b*) C2 assumes C1 is on same network, but C1 assumes C2 is on a different network
(*c*) C1 assumes C2 is on same network, but C2 assumes C1 is on a different network
(*d*) C1 and C2 both assume they are on different networks

60. Station A needs to send a message consisting of 9 packets to Station B using a sliding window (window size 3) and go-back-n error control strategy. All packets are ready and immediately available for transmission. If every 5^{th} packet that A transmits gets lost (but no acks from B ever get lost), then what is the number of packets that A will transmit for sending the message to B ?

(*a*) 12
(*b*) 14
(*c*) 16
(*d*) 18

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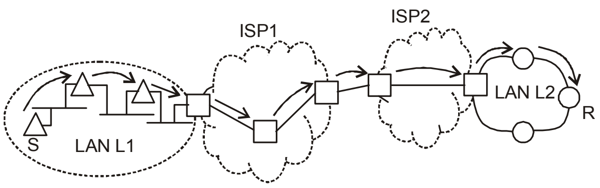
NUMERICAL TYPE QUESTIONS

2014

1. Consider a token ring network with a length of 2km having 10 stations including a monitoring station. The propagation speed of the signal is 2×10^8 m/s and the token transmission time is ignored. If each station is allowed to hold the token for 2 μsec, the minimum time for which the monitoring station should wait (in μsec) before assuming that the token is lost is _______.
2. Consider a selective repeat sliding window protocol that uses a frame size of 1 KB to send data on a 1.5 Mbps link with a one-way latency of 50 msec. To achieve a link utilization of 60%, the minimum number of bits required to represent the sequence number field is ________.
3. In the diagram shown below, L1 is an Ethernet LAN and L2 is a Token-Ring LAN. An IP packet originates from sender S and traverses to R, as shown. The links within each ISP and across the two ISPs, are all point-to-point optical links. The initial value of the TTL field is 32. The maximum possible value of the TTL field when R receives the datagram is ____________ .



4. An IP router implementing Classless Inter-domain routing (CIDR) receives a packet with address 131.23.151.76. The router's routing table has the following entries:

Prefix	Output Interface Identifier
131.16.0.0/12	3
131.28.0.0/14	5
131.19.0.0/16	2
131.22.0.0/15	1

The identifier of the output interface on which this packet will be forwarded is ______.

5. Every host in an IPv4 network has a 1-second resolution real-time clock with battery backup. Each host needs to generate up to 1000 unique identifiers per second. Assume that each host has a globally unique IPv4 address. Design a 50-bit globally unique ID for this purpose. After what period (in seconds) will the identifiers generated by a host wrap around?
6. Suppose that the stop-and-wait protocol is used on a link with a bit rate of 64 kilobits per second and 20 milliseconds propagation delay. Assume that the transmission time for the acknowledgement and the processing time at nodes are negligible. Then the minimum frame size in bytes to achieve a link utilization of at least 50% is _________.
7. Consider a LAN with four nodes S_1,S_2, S_3 and S_4. Time is divided into fixed-size slots, and a node can begin its transmission only at the beginning of a slot. A collision is said to have occurred if more than one node transmit in the same slot. The probability of generation of a frame in a time slot by S_1, S_2, S_3 and S_4 are 0.1, 0.2, 0.3 and 0.4, respectively. The probability of sending a frame in the first slot without any collision by any of these four stations is ________.
8. A link has a transmission speed of 10^6 bits/sec. It uses data packets of size 1000 bytes each. Assume that the acknowledgement has negligible transmission delay, and that its propagation delay is the same as the data propagation delay. Also assume that the processing delays at the nodes are negligible. The efficiency of the stop-and-wait protocol in this setup is exactly 25%. The value of the one-way propagation delay (in milliseconds) is ___________.
9. Consider a network connected two systems located 8000 kilometers apart. The bandwidth of the network is 500×10^6 bits per second. The propagation speed of the media is 4×10^6 meters per second. It is needed to design a Go-Back-N sliding window protocol for this network. The average packet size is 10^7 bits. The network is to be used to its full capacity.

   Assume that processing delays at nodes are negligible. Then the minimum size in bits of the sequence number field has to be __________.
10. In the network 200.20.11.144/27, the fourth octet (in decimal) of the last IP address of the network which can be assigned to a host is ________.
11. Two hosts are connected via a packet switch with 10^7 bits per second links. Each link has a propagation. Delay of 20 microseconds. The switch begins forwarding a packet 35 microseconds after it receives the same. If 1000 bits of data are to be transmitted between the two hosts using a packet size of 5000 bits, the time elapsed between the transmission of the first bit of data and the reception of the last of the data in microsecond is _________.

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ANSWERS

EXERCISE – I

MCQ Type Questions

1. (*a*)	**2.** (*d*)	**3.** (*a*)	**4.** (*c*)	**5.** (*b*)	**6.** (*a*)	**7.** (*a*)	**8.** (*a*)	**9.** (*d*)	**10.** (*c*)
11. (*a*)	**12.** (*d*)	**13.** (*b*)	**14.** (*d*)	**15.** (*b*)	**16.** (*d*)	**17.** (*c*)	**18.** (*b*)	**19.** (*a*)	**20.** (*d*)
21. (*b*)	**22.** (*a*)	**23.** (*b*)	**24.** (*c*)	**25.** (*a*)	**26.** (*b*)	**27.** (*b*)	**28.** (*c*)	**29.** (*d*)	**30.** (*c*)
31. (*d*)	**32.** (*d*)	**33.** (*c*)	**34.** (*b*)	**35.** (*c*)	**36.** (*b*)	**37.** (*c*)	**38.** (*a*)	**39.** (*b*)	**40.** (*d*)
41. (*c*)	**42.** (*c*)	**43.** (*d*)	**44.** (*a*)	**45.** (*c*)	**46.** (*d*)	**47.** (*b*)	**48.** (*a*)	**49.** (*a*)	**50.** (*c*)
51. (*a*)	**52.** (*c*)	**53.** (*d*)	**54.** (*d*)	**55.** (*a*)	**56.** (*c*)	**57.** (*c*)	**58.** (*c*)	**59.** (*c*)	**60.** (*b*)
61. (*a*)	**62.** (*c*)	**63.** (*a*)	**64.** (*c*)	**65.** (*a*)	**66.** (*b*)	**67.** (*a*)	**68.** (*d*)	**69.** (*b*)	**70.** (*d*)
71. (*d*)	**72.** (*b*)	**73.** (*d*)	**74.** (*c*)	**75.** (*c*)	**76.** (*a*)	**77.** (*c*)	**78.** (*b*)	**79.** (*a*)	**80.** (*c*)
81. (*a*)	**82.** (*b*)	**83.** (*c*)	**84.** (*c,d*)	**85.** (*d*)	**86.** (*d*)	**87.** (*b*)	**88.** (*a*)	**89.** (*b*)	**90.** (*d*)
91. (*b*)	**92.** (*a*)	**93.** (*a*)	**94.** (*a*)	**95.** (*b*)	**96.** (*c*)	**97.** (*b*)	**98.** (*b*)	**99.** (*a*)	**100.** (*a*)
101. (*c*)	**102.** (*d*)	**103.** (*a*)	**104.** (*b*)	**105.** (*d*)	**106.** (*b*)	**107.** (*d*)	**108.** (*b*)	**109.** (*a*)	**110.** (*c*)
111. (*a*)	**112.** (*c*)	**113.** (*c*)	**114.** (*d*)	**115.** (*c*)	**116.** (*b*)	**117.** (*d*)	**118.** (*c*)	**119.** (*a*)	**120.** (*a*)
121. (*d*)	**122.** (*c*)	**123.** (*b*)	**124.** (*b*)	**125.** (*a*)	**126.** (*b*)	**127.** (*c*)	**128.** (*c*)	**129.** (*a*)	**130.** (*a, b*)
131. (*d*)	**132.** (*a*)	**133.** (*c*)	**134.** (*a*)	**135.** (*b*)	**136.** (*c*)	**137.** (*c*)	**138.** (*b*)	**139.** (*a*)	**140.** (*c*)
141. (*b*)	**142.** (*a*)	**143.** (*b*)	**144.** (*b*)	**145.** (*a*)	**146.** (*a*)	**147.** (*a*)	**148.** (*a*)	**149.** (*a*)	**150.** (*b*)
151. (*c*)	**152.** (*a*)	**153.** (*d*)	**154.** (*c*)	**155.** (*a*)	**156.** (*c*)	**157.** (*c*)	**158.** (*d*)		

Numerical Type Questions

1. 200	**2.** 100	**3.** 100	**4.** 802.5	**5.** 10	**6.** 772	**7.** 15	**8.** 45	**9.** 6000
10. 30,000	**11.** 3	**12.** 12	**13.** 30	**14.** 20,000	**15.** 93	**16.** 250	**17.** 2000	**18.** 100
19. 8	**20.** 90	**21.** 56	**22.** 13%	**23.** 2400	**24.** 6	**25.** 0.736	**26.** 254	**27.** 32
28. 300	**29.** 8	**30.** 625	**31.** 66	**32.** 26.66	**33.** 45	**34.** 5.33	**35.** 200	**36.** 0.857
37. 1	**38.** 300	**39.** 0.46						

EXERCISE – II

MCQ Type Questions

1. (*d*)	**2.** (*c*)	**3.** (*a*)	**4.** (*a*)	**5.** (*c*)	**6.** (*d*)	**7.** (*c*)	**8.** (*b*)	**9.** (*d*)	**10.** (*a*)
11. (*c*)	**12.** (*b*)	**13.** (*b*)	**14.** (*a*)	**15.** (*b*)	**16.** (*a*)	**17.** (*c*)	**18.** (*a*)	**19.** (*c*)	**20.** (*a*)
21. (*d*)	**22.** (*b*)	**23.** (*c*)	**24.** (*d*)	**25.** (*c*)	**26.** (*b*)	**27.** (*c*)	**28.** (*c*)	**29.** (*c*)	**30.** (*c*)
31. (*c*)	**32.** (*a*)	**33.** (*a*)	**34.** (*d*)	**35.** (*d*)	**36.** (*c*)	**37.** (*c*)	**38.** (*a*)	**39.** (*d*)	**40.** (*d*)
41. (*d*)	**42.** (*b*)	**43.** (*b*)	**44.** (*c*)	**45.** (*a*)	**46.** (*d*)	**47.** (*c*)	**48.** (*b*)	**49.** (*c*)	**50.** (*b*)
51. (*c*)	**52.** (*a*)	**53.** (*c*)	**54.** (*c*)	**55.** (*b*)	**56.** (*b*)	**57.** (*b*)	**58.** (*b*)	**59.** (*c*)	**60.** (*c*)

Numerical Type Questions

1. (28μs to 30μs)	**2.** (5)	**3.** (26 to 26)	**4.** (1)	**5.** (256)	**6.** (160)
7. (0.40–0.46)	**8.** (12)	**9.** (8)	**10.** (158)	**11.** (1575)	

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EXPLANATIONS

EXERCISE – I

MCQ TYPE QUESTIONS

1. Token passing bus network in-service expansion.

2. Bus hubs are not relevant to networking.

3. UTP (Unshielded Twisted Pair).

4. SNMP is implemented with the hubs.

5. Entropy is the amount of uncertainty in a system.

6. SNA network developed by IBM.

7. CCITT organisation draws up standards for modems.

8. Broad band

9. WAH generally used telephone lines, microwaves, satellites, embedded systems.

10. Modem involved in 1963, USA.

11. Local area network that connects thirty personnel computers can provide more computing power than a minicomputer.

12. SMP is Novell network 3.X.

13. The different computer connected to a LAN by a interface card.

14. The communication direction at a time only one side that called is half duplex.

15. Data link level is achieved by cyclic redundancy codes.

16. Mesh topology is highest reliability.

17. Baud means the rate at which the signal changes.

18. Unmodulated signal coming from a baseband signal.

19. Bit stuffing is required when there is a flag of bits to represent one of the incidents like start of frame, end of frame, etc, If same flag of bits appear in the data stream, a zero can be inserted. The receiver deletes this zero from the data stream.

20. Data link layer.

21. Railway track is an example of half duplex.

22. Land coaxial cables 10^6 to 10^8 Hz.

23. Star topology and tree topology requires central controller or hub. Mesh, ring, bus topologies doesn't need hubs.

24. Bus topology requires a multipoint connection. Because, in this there is a single communication channel, that is shared by all the systems in a net work. So bus topology require multipoint connection.

25. Mesh Topology.

26. *Tree topology* is a variation of star topology. Tree topology consists of multiple hub, one is *primary hub* and remaining are *secondary hubs*.

27. *Open system interconnection model* shows the network function of a computer developed by international standards organization.

28. *Internet protocol layer* (IP layer) provides totally connectionless services, but in OSI Model it provides both connection oriented and connection less services.

29. The physical layer data send in frame and interface provide packet mode and device and virtual circuit interface to PSS service.

30. 10 base T cable is also called as *Twisted-pair-Ethernet*. A star topology LAN using unshared twisted pair instead of coaxial cable.

31. In a token ring each station with capture the token and checks the destination address if it is addressed to any other station then the frame will be regenerated and passed to the next station.

32. In Token ring, when a frame reaches its destination station, the entire message is copied and four bits in the packet are changed.

33. Transceiver is an acronym for transmission receiver, which receives data after checking the line voltage, hence it can't add headers.

34. The distance between station-to-hub in 100 Base - TX is 100 meters. But in 100-Base-FX design. The distance between station-to-hub is 2000 meters.

35. Base band Ethernet LAN uses Manchester encoding.

36. *MAU (Multistation Access Unit) :*

Individual automatic switches are combined into a hub called a multistation access unit (MAU) and MAU can support up to eight stations. Token ring uses the MAU.

37. 100 base-T4 requires four pairs of unshielded twisted pair cable to reduce baud rate of transmission (number of signal changes) 8B/6T (eight binary / six ternary) is used in which each block of eight bits is transformed into six bauds of three voltages levels.

38. IBM's Token Ring Network uses adaptive routing.

39. *TSI (Time Slot Interchanger)* in time division switches consists of random access memory (RAM) with several memory locations. The RAM fills up with incoming data from the time slots in the order received. Hence in time division switches deliver

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of data is delayed because data must be stored and retrieved from RAM.

40. Physical layer standard is RS 232-C and RS 449 used.

41. Given : $\Delta\lambda = 10^{-7}$, and $\lambda = 10^{-7}$

$$\therefore \quad \Delta f = \frac{c\Delta\lambda}{\lambda^2} = 30{,}000 \text{ GHz}$$

42. Data rate is 480 × 640 × 24 × 60 bps, which is 442 Mbps.

Assume 1 bps per Hz.

$$\Delta\lambda = 4.42 \times 10^5$$

From the ralation $\lambda = \Delta\lambda c$

$$\Rightarrow \quad \Delta\lambda = \frac{\lambda^2}{c}$$

$$\Delta\lambda = 4.42 \times 10^5$$

$$\therefore \quad \lambda = 2.5 \times 10^5 \text{ microns.}$$

43. Packet-switching network can reduce the cost of using an information utility and allows communications channel to be shared among more than one user, then both *b* and *c*.

44. Data link content monitor is to detect problems in protocols.

45. Establishing a virtual connection is virtually equivalent to connecting as virtual memory.

46. TCP/IP protocol developed by DARPA.

47. DNIC uses first four address area.

48. First three digits of DNIC identify the country.

49. NUA (Network User Address) are used 5-7 number.

50. Terminal number used to NUA 8-12.

51. Routing algorithm is being used in hot potao routing.

52. UUCP single link.

53. WAH generally involves microwaves, telephone lines, satellites.

54. The X. 25 level 2-ISO is the wrong example of the network layer.

55. PURE ALOHA does not require global time synchronization.

56. Slotted ALOHA divide time into discrete interval require global time synchronization.

57. CSMA/CD is an important protocol, is IEEE 802.3 (ethernet).

58. Start and stop bits are used in serial communication for synchronization.

59. In bipolar code, the singal varies among three levels.

In non-return to zero code, the signal remains the same through-out the bit cell.

In unipolar code, there will be no signal either below zero or above zero.

In Manchester code, the signal level will not vary in the middle and is unipolar.

60. 1-persistent then the channel is continually sensed.

61. Non-polling system is TDMA.

62. Carrier sense systems.

63. BSC is a character-oriented protocol.

64. Ethernet CSMA/CD, the special bit sequence transmitted by media access management is jam.

65. Adaptive or dynamic directory used in packet routing changes within each user session.

66. 100010111

67. The sampling period of residual error rate.

68. Ethernet message packet is type, data and address.

69. The railway track is an example of half-duplex.

70. Access control methods is sliding window.

71. Stop-and-wait ARQ (Automatic Repeat Request) is similar to stop - and - wait flow control protocol, but it includes retransmission of data in case of lost or damaged frames.

Both data and ACK frames are numbered alternately 0 and 1. But NAK frames are not numbered. If any error occurs the Receiver sends a NAK frame, it tells the sender to retransmit the frame last sent.

Sender maintains a copy of frame until the Acknowledgement is back.

72. In the case of go-back-n protocol, when a NAK is received, all the frames sent since the last frame acknowledged are retransmitted.

Go ------ > Back --------- > n.

73. Poll/select method is used in primary - to - secondary communication.

ENQ/ACK is used in peer-to-peer communication method.

74. In Pol/select method, every device on a link has an address that can be used for identification.

75. Poll/select is a line discipline method.

The primary device always initiates communication with either a poll or select (SEL) frame. In this point the secondary device sends data in response to a poll frame event.

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76. In sliding window flow control the window size is 63, the range of sequence numbers are 0 to 63 {0 .... 63}.

77. Line discipline co-ordinate the link system. It determines which device can send and when it can send. Line discipline decides the role of a device on a network.

78. In sliding window protocol, the window size is n – 1, then its maximum capability is n – 1.

79. For stop-and-wait flow control for n data packets sent, in acknowledgements are needed.

80. Go-back-N has better line utilization is incorrect statement. Because if there is an error at the receiver the frames must be retransmitted where the error occurred in the transmission.

81. IEEE project 802 divides the data link layer into two sub layer logical link control (LLC) and Medium access control (MAC).

84. System are both priority and non-priority modes are carrier sense systems and token passing.

85. In *time division switch*, slots are divided by time switching accomplished using time division multiplexing (TDM) and another device called time slot interchange (TSI).

86. In *time division switch*, based on the control unit decisions order of slots and destinations of a packet stored in RAM are governed.

87. In link state routing each router creates its own link state packet (LSP). Every other router receives this LSP through the flooding process. All routers therefore have the same information. This is compiled into the link state database.

88. The mean of the data are mainly of 2 (m-2).

89. $\dfrac{1}{\text{K} - \text{p}}$

90. $t = 1025$ sec

91. A process writes 1024 bytes, then message stream network will receive 1024 bytes only at one time. But byte stream network will receive 2048 bytes as a single unit.

92. Frames are generated by DLL in OSI mode, SATNET in TCP/IP, TC in ATM.

93. Maximum data rate of a noise less 4kHz channel using T1 Pcm system is 56kbps.

94. The television channels are 6 MHz the bits are true the 4 bits ring in 6 × 4 = 24 Mbps noiseless.

95. Given : T = 5 μses

L = 25 μsec

d = 2.73 μsec.

$$\text{N} = \frac{1+L}{(T-\delta)} = 12.01 \text{ i.e. } 12 \text{ cells.}$$

96. For windows above 9 frames, the full 64 kbps is used.

98. The broadcast address has hosted with all bits 1.

99. ARP is a TCP/IP protocol used to dynamically bind a high level IP address to a low level physical hardware address.

100. ICMP is a protocol that handles error and control messages.

101. TCP/IP protocol is used for file transfer with minimal capability and minimal overhead TFTP.

102. The network to attached SGMP and SUMP.

103. Class A, B and C network IDs can exist 2,113, 658.

104. The diskless machine uses to obtain IP address from a server is ARP.

105. Network layer is IP-ARPANET, X.25 Packet Level Protocol-ISO, source routing and domain naming_ USENET.

106. The modern uses Amplitude modulation.

107. ICI a temporary parameter passed between N and N-1 layers to involve service functions between two layers.

108. Ethernet network is relation between Round trip time decreases, collision domain decreases.

109. Repeaters function in Physical Layer.

110. The primary characteristics in a packet is all cells have the same fixed length.

111. Adaptive and dynamic directorary used in packet routing changes within each user session.

112. 6.4 sec.

113. Bandwidth is $(n \times h)$/M bytes.

114. Time requried for the last bit of the last packet to reach the receiver

$$t = x + b + (k - 1)\, xp/\text{b} + \text{kd sec.}$$

115. Speed of light in fiber and copper ≈ 200 km/msec.

For a 20-km line, the delay is 100 μsec one way and 200 μsec round trip.

A 1 KB packet has 8192 bits.

If the time to send 8192 bits and get the acknowledgement is 200 μsec, the transmission and propagation delays are equal.

If B is the bit time, then

$8192 = 2 \times 10^{-4}$ sec.

$\therefore$ Data rate, $\dfrac{1}{\text{B}} = 40$ Mbps. (approx)

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116. Number of bits / sec $\approx 640 \times 480 \times 30 \times 8$ or 73.728 Mbps.

117. Mean router-router path is twice the mean router-root path.

Number levels of the tree with the root as 1 and the deepest level as n.

Path from the root to level n requires $n - 1$ hops, and 0.50 of the routers are at this level.

Path from root to level $n - 1$ has 0.25 of the routers, and a length of $n - 2$ hops.

$\therefore$ Mean path length

$$l = 0.5 \times (n-1) + 0.25 \times (n-2) + 0.125 \times (n-3) + ..$$

$$l = \sum_{i=1}^{\infty} n(0.5)i - \sum_{i=1}^{\infty} i(0.5)i$$

The expression reduces to $l = n - 2$

$\therefore$ Mean router-router path = $2n - 4$.

118. Dog can carry 21 gigabytes, or 168 gigabits.

Speed of 20 km/hour equals = 0.005 km/sec.

$$\text{Time to travel distance } x \text{ km} = \frac{x}{0.005} = 200x \text{ sec.}$$

$$\therefore \text{Data rate} = \frac{168}{200x} \text{ Gbps or } \frac{840}{x} \text{ Mbps.}$$

For $x < 5.4$ km, the dog has a higher rate than an ATM line.

124. Reliability can be measured by a frequency of failure and network recovery time after a failure. *Catastrophic events*. In this case frequency of failure and network recovery time after a failure are the reliability issues.

125. If a protocol specifies that format of a data (or) message, so it is *syntax issue* (General Format).

126. *Protocol* is a set of rules and regulations which manages the data communication system.

Syntax refers to the structure or format of the data.

Semantics means meaning of data. Here this issue is semantic issue.

127. Layer 5 is session layer. The header from A's layer 5 is read by B's session layer.

128. *7 layer of OSI model are divided into the following groups :*

Physical, datalink, network transport, - Network support layers,

Application, Presentation, Session - User support layers.

129. The purpose of check points or synchronization is allow just a portion of file to be resent. To avoid the retransmission of frames, session layer adds check points to the data unit.

131. *Gateways* can function all the way upto the application layer. Because Gateways operate in all the layers of the OSI model (1 - 7).

132. A bridge operates at the data link layer, giving it access to the physical address of all stations connected to it. Answer is physical addresses.

133. Any layer can transfer data basing on its bandwidth (data handling capacity) i.e. it is limited to certain extent, i.e. capability data.

135. Protection of data from a natural disaster such as tornado, earthquakes, (or) fire etc. are the network reliability issues. These are also called *catastrophic events*.

136. If a protocol specifies that data should by sent at 100 mbps, this issue is *timing issue*.

137. To send any important information (or) any sensitive information that must be able to assure privacy.

Encryption means encoding of original information, *decryption* means decoding.

Encryption (decryption (original message)

= original message.

138. Bridge $\rightarrow$ Operate at the bottom two layer of OSI.

Router $\rightarrow$ Operate at the network layer of the OSI model.

Gateway $\rightarrow$ Operate at the top 3 layer of OSI.

139. Transport layer is responsible for segmentation.

140. Data link layer perform error checking.

141. SMTP ------------- Application layer

BGP ------------- Network layer

TCP ------------- Transport layer

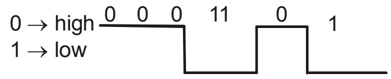
PPP ------------- Data link layer

142. **1. Data link layer :** Ensure reliable transport of data over a physical point to point link.

2. Network layer : Routes data from one network node to the next.

3. Transport layer : Allow end to end communication.

144. MAC sub-layer uses CSMS/CD access method.

145. 

146. Maximum data rate possible = speed of satellite transmission rate = 20 kbps

147. Network layer $\rightarrow$ End to End transmission

148. Distance-vector routing $\rightarrow$ count-to-infinity

149. RIP and OSPF is an interior routing protocol.

151. Class D : 224.0.0.0 to 239.255.255.255

152. C22F1582 equals 194.47.21.130

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153.

10.105.1.113	10.105.1.91
255.255.255.224	255.255.255.224
10.105.1.106	10.105.1.64

Both are different, for all the others it will show the same Id.

154. Eth 0 and Eth 2 has a destination address which can prefer 144.16.68.117. But eth 0 will be general and Eth 2 will be more specific.

156. SMTP and POP needs to send a mail.

157. DNS uses UDP as the transport protocol.

158. HELO and PORT are commands SMTP and FTP.

NUMERICAL TYPE QUESTIONS

1. Fiber optics have maximum segment 2000 m.

2. Twisted pair have maximum segment is 100 m.

3. Thick coax have maximum segment is 500 m.

4. Here 802.3 is Ethernet, it uses CSMA/CD. But 802.5 is Token ring. It will have only one token in circulation. These are IEEE standards but FDDI is ANSI standard.

6. Propagation speed in the cable

= 200,000 km/sec = 200 km/μsec.

Hence 100 km cable will be filled in 500 μsec.

This corresponds to four 193-bit frames, or 772 bits on the cable.

7. The maximum window size is 15.

8. As all lines are full-duplex and there are no self-connections, only the cross points above the diagonal are needed. Hence formula for the number of cross points needed is $\frac{n(n-1)}{2}$

9. Maximum data rate = $2H \log_2 V$ bps,

where, H = bandwidth

V= discrete levels

Here H is 3 kHz and V is 2.

10. Maximum number of bps = $H\log_2(1 + SNR)$.

11. Minimum number of bits required to represent B is 3.

12. Minimum number of required to represent all the symbols together is 15.

13. rate of ring 20 Mbps

speed = 200 b/ms

channel = 200 km

$$= \frac{\text{speed} \times \text{channel}}{\text{ring}}$$

$$= \frac{200 \times 200}{20} = 2000 \text{ bits}$$

15. To send a T1 signal we need

$$H\log_2\left(1+\frac{S}{N}\right) = 1.544 \times 10^6 \text{ with H } 50{,}000.$$

$$\frac{S}{N} = 230 - 1{,}54 \approx 93 \text{ db}.$$

16. A crossbar switch connects n inputs to m outputs in a grid, using electronic micro switches at each cross point, totally n × m cross points are required for crossbar switch. In general 25 percent of the cross points are used at a given time. The rest are idle so, totally 1000 cross points, so 25% = 250 crosspoint points are in use.

17. In a single stage switch n inputs and m outputs. Totally n × m crosspoints are needed.

Here, n = 40 and m = 50

n × m= 2000

18. The full duplex of needed for 10 lines and 10 port is 10 × 10 = 100

19. The size is 1200 and outgoing line 9600

$$n = \frac{9600}{1200} = 8$$

22. T1 carrier (1.544 Mbps) = 13%

25. The probability of success (e.e. exactly one frame in a slot) = $\frac{1}{e} \approx 0.368$.

Then probability of a failure (no frames or multiple frames) ≈ 0.632.

Thus joint probabilities of the two uplink channels are :

success on 1 and success on 2 : 0.135 (2 frames sent)

success on 1 and failure on 2 : 0.233 (1 frames sent)

failure on 1 and success on 2 : 0.233 (1 frames sent)

failure on 1 and success on 2 : 0.399 (0 frames sent)

Then expected number of success per slot.

$$E = 0.35 \times x + 0.233 \times 1 + 0.233 \times 1 + 0.399 \times 0$$
$$= 0.270 + 0.233 + 0.233 = 0.736$$

26. The number of hosts that are attached to each of the local area network at a site is 254.

27. All communication with the host is 32 bits.

28. Start and stop bits are not needed in synchronous transfer of data.

Hence, it is $\frac{2400}{8} = 300$.

29. Since there are six 1200 bps terminals, therefore

$$6 \times 1200 + n \times 300 = 9600$$
$$\Rightarrow \quad n = 8.$$

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30. In time division switches
$2nT = 1$ frame period, where T = memory access time.

31. Number of cross point = $\frac{n(n-1)}{2}$

$= \frac{12 \times 11}{2} = 66$

32. We know that the effective delay of a network can be determind using the formula :

$$D = \frac{1}{(1-U)}$$

Given, I = 20 ms

When utilization is 25%

$$D = \frac{20}{(1-0.25)} = \frac{20}{(0.75)}$$

$= 26.66$ ms

33. We know that the number of direct connection

$$= \frac{(N^2 - N)}{2}$$

Substituting the value 6 in the above equation, we get 15 connections.

34. $\because$ 1 propagation delay + transmission time for 1000 kB

Propogation delay is :

$$\frac{10 \text{ km}}{2 \times 10^5 \text{ km/sec}} = 5 \times 10^{-5} \text{ sec}$$

Transmission time is (kB = 8000b)

$$\frac{8 \times 10^6 \text{kb}}{1.5 \times 10^6 \text{b}} = 5.3333 \text{ sec}$$

Total time is 5.3333 + .00005 = 5.33338

In effect, propogation delay is negligible here.

35. Min frame size $= 2 \times \frac{d}{v} \times \text{data rate}$

$$= 2 \times \frac{200}{2 \times 10^8} \times 10^9$$

$= 2 \times 10^2 \times 10^1$

$= 2 \times 10^3 = 2000$ bits

$= 250$ bytes

36.

$t_i = 3$ ms

$t_p = 5$ ms

N = 10

$\because$ Maximum utilization = 0.857

37. Odd parity means number of 1 should be odd, here number of 1 is 4. The parity bit would be 1 to bring the number of 1's in the data and the parity to 5 an odd number. Parity bit = 1

38. For synchronous transfer, start bit and stop bit are ignored.

So for synchronous transfer $= \frac{2400}{8} = 300$

39. Data field $\rightarrow$ 32 bytes

TCP Header Size $\rightarrow (20 + 8 + 10) = 38$ bytes

$$\because \quad \text{Efficiency} = \frac{32}{32+38}$$

$$= \frac{32}{70} = 0.46$$

EXERCISE – II

MCQ TYPE QUESTIONS

1. Statement S1

The Distance Vector routing protocols rely on the information from their directly connected neighbours in order to calculate and accumulate route information. Distance Vector routing protocols require very little overhead as compared to Link State routing protocols as measured by memory and processor power while the Link State routing protocols do not rely solely on the information from the neighbours or adjacent router in order to calculate route information. Instead, Link State routing protocols have a system of databases that they use in order to calculate the best route to destinations in the network. This is TRUE

Statement S3

Distance Vector exchanges the routing updates periodically whether the topology is change or not, this will maximize the convergence time which increases the chance of routing loops while the Link State routing protocols send triggered change based updates when there is a topology change. After initial flood, pass small event based triggered link state updates to all other routers. This will minimize the convergence time that's why there is no chance of routing loops. This is

TRUE.

2. RSA and DES are for Encryption where MD5 and SHA – 1 are used to generate Message Digest.

3. First of all the browser must now know what IP to connect to. For this purpose browser takes help of Domain name system (DNS) servers which are used for resolving hostnames to IP addresses. As browser is an HTTP client and as HTTP is based

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on the TCP/IP protocols, first it establishes a TCP connection with the web server and requests a webpage using HTTP, and then the web server sends the requested webpage using HTTP. Hence the order is 4, 2, 1, 3 .

4. We know that RIP uses distance vector routing and QSPF uses link state routing.

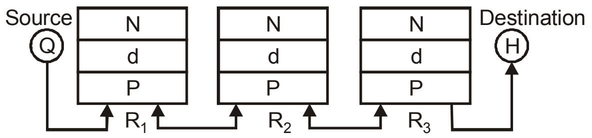
5. Listen connects an unconnected active TCP socket into a passive socket.

6. Bandwidth of link is 10^6 bytes/sec

File size = 10^3 bytes

T_1 & T_3 will take same time and $T_3 > T_2$ means T_3 will take more time then T_2.

7. $Q - R_1 - R_2 - R_3 - H$



P-Physical Layer, d-data link layer, N-Network layer

Since, the router R_1 contains TCP part numbers of Q and R_2 contains the TCP part numbers of H at Data link layer, and with both routers contains IP addresses of Q and H at Network layer. So, only statement I_2 and I_3 will be true.

8. (*a*) One of the main functionality of Network Layer is Routing. So Option (a) is CORRECT.

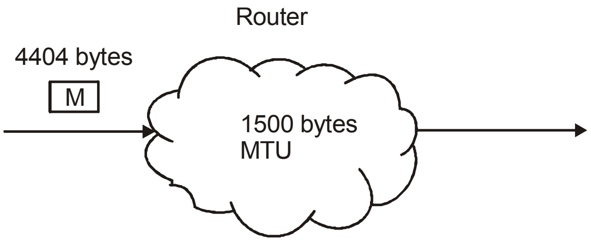
(*b*) Bit Synchronization is always handled by Physical Layer of OSI model but not Data Link Layer. So Option (*b*) is INCORRECT.

(*c*) End – to – End Process Communication is handled by Transport Layer. So Option (c) is CORRECT.

(*d*) MAC sub layer have 3 types of protocols (Random, Controlled and Channelized Access).

9. While an IP Datagram is transferring from one host to another host, TTL, Checksum and Fragmentation Offset will be changed.

10.



Data length	1480 +	1480 +	1444
Header length	20	20	
MF	1	1	0

11. If there are N-people, then total no of key required are $N(N-1)/2$.

12. In a IP header, the source address is never modified checksum, TTL and length may vary for each router.

13. In a TCP connection

- If the sequence number of a segments is m. Then it is not necessary the subsequent segment is $m + 1$, because it depend upon no of packet sent is current segment.
- Advertise window can not be fixed it may change on changing the capacity of receiver window.

14. HTTP and FTP both can use multiple TCP connection between same client and server.

15. bind, listen, accept, recv

17. 7 and 1110

18. Network ranges are

0 · 0 · 0 · 0 – default

128 · 96 · 164 · 0 to 128 · 96 · 165 · 0 — R_3

128 · 96 · 166 · 0 to 128 · 96 · 167 · 255 — R_2

128 · 96 · 168 · 0 to 128 · 96 · 169 · 255 — interface 1

128 · 96 · 170 · 0 to 128 · 96 · 171 · 255 — interface 0

Ans = A

A – 1

B – 3

C – 5

D – 4

20. TCP connections are full duplex. It allow to send and receive packets at same time.

TCP allow for selective acknowledgement depend upon cengestion window and there are not message stream.

21. Given L = 1250 Bytes

B = 100 mbps

d = 1 km

V = ?

In CSMA/CD, $L = 2 \times \frac{d}{v} \times B$

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$$\Rightarrow \qquad V = \frac{2dB}{L} = \frac{2\times10^3\times10^8}{10^4}$$

$$\Rightarrow \qquad V = 20{,}000 \text{ km/sec}$$

22. **Service Oriented Computing (SOC) :** Publish-Find-BIND

Heterogeneous Comm. System: Interpretability

Information Representation: XML

Process description: BPMN

Reference: Wikipedia .org/wiki/XML/Bussines_ Process_ Model_and_Notation

Trick: Info representation mapped to XML in only one option.

23. **1.** **Real Time Multimedia:** Can be unreliable but have to be fast so UDP

2. **File Transfer:** Has to be secure & reliable so uses TCP

3. **DNS:** Work or part 953, can be both TCP and UDP

4. **Email:** uses TCP, for reliability

∴ UDP, TCP, TCP/UDP, TCP

24. **Given:** X encrypts a message and send it to Y.

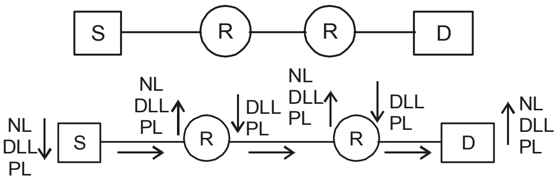
To find: Correction encryption - decryption order

Analysis: To make sure that Y receives the message & is able to decrypt it and no one else is able to X. The message over the network should be encrypted by Y's public key.

So, order of encryption is X's private key, Y's public key. On receiving the encrypted message, Y will have to decrypt it using its private key & then use X's public key for signature. So, order of decryption is Y's private key followed by X's public key.

So, the answer is (*d*)

25. Assume the source S & destination D are connected through two intermediate routers labled R. Determine how many times each packet **HAS TO VISIT THE NETWORK LAYER AND DATA LINK LAYER DURING A TRANSMISSION FROM S TO D.**



⇒ Network Layer 4 times

⇒ Data link layer 6 times

26. Propagation time = Transmission time + collision signal time (Acknowledge)

$$\frac{\text{Frame size}}{\text{Propagation Time}} = \frac{\text{Length}}{\text{Signal Speed}} + \frac{\text{Length}}{\text{Signal Speed}}$$

$$\frac{10000 \text{ bit}}{500\times10^6 \text{bits/sec}} = \frac{2\times\text{Length}}{2\times10^5 \text{ km/sec}}$$

$$\boxed{\text{Length} = 2\text{km}}$$

* An ethernet node must be transmitting a frame for the slot time for a collision with that frame to detected.

27. Since M bit is 0, so there is no fragments after this fragment. Hence this <u>fragment is the "last fragment</u>" Tow, H LEN defines the length of Header in datagram. Since H LEN is 10 so size of header is 10 * 4 = 40B.

Length of data = Total length – Leader length
= 400 – 40 = 360 B.

Few, fragment offset of data in original datagram is measured in units of 8B. So to find first Byte of this fragment,

$$\frac{\text{first Byte}}{B} = \text{fragment offset}$$

⇒ <u>first Byte</u> = 300 ∗ 8 = <u>"2400"</u> B

since length of data is 360B

so last Byte on this datagram will be <u>"2759"</u> B

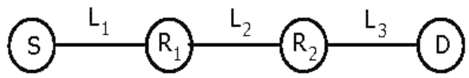
$$\boxed{i.e. \begin{matrix} \text{``first} & \text{last''} \\ \multicolumn{2}{c}{\underbrace{2400 - 2759}_{\text{360B length}}} \end{matrix}}$$

28. For class C address, size of network field is 24 bits. But first 3 bits are fixed as 110; hence total number of networks possible is 2^{21}

29. E-mail uses SMTP, application layer protocol which intern uses TCP transport layer protocol.

30. The PDU for Datalink layer, Network layer, Transport layer and Application layer are frame, datagram, segment and message respectively.

32.

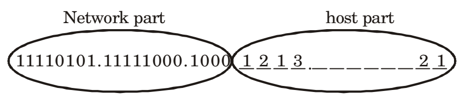


Transmission delay for 1 packet from each of S, R_1 and R_2 will take 1ms

Propagation delay on each link L_1, L_2 and L_3 for one packet is 1ms

Therefore the sum of transmission delay and propagation delay on each link for one packet is 2ms.

33.



Since half of 4096 host addresses must be given to organization A, we can set 12th bit to 1 and include that bit into network part of organization A, so the valid allocation of addresses to A is 245.248.136.0/21

Now for organization B, 12th bit is set to '0' but since we need only half of 2048 addresses, 13th bit can be set to '0' and include that bit into

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network part of organization B so the valid allocation of addresses to B is 245.248.128.0/2.

36. Process switching also involves mode changing.

37. Sending an email will be done through user agent and message transfer agent by SMTP, downloading an email from mail box is done through POP, checking email in a web browser is done through HTTP

38. Since it is a layer 4 firewall it cannot block application layer protocol like HTTP.

39. It can be used to prevent packet looping because when Time-to-Live field reaches to '0' means when there is no any type of acknowledgement received, then TTL decrementet continuously and when it reaches '0', then it stop looping of packet.

40. Internet chat, web browsing and E-mail all are client-server application. Ping is a utility. It is mainly used to check the connection between two computer, there is a chance both are client or one is client and another server. In chat system first user authentication required and it requires server.

41. Given : 10.105.1.113 and 10.105.1.91

When these are in same network, then AND with netmask give the address of same ID otherwise these are in different class in these only AND with

255.255.255.224 are in different class

This is 96 and 64. Then these are different ID means 10.105.1.96 and 10.105.1.64 are different ID. So this is not in same network.

Alternately

10.105.1.113	→ 00001010	01101001	00000001	01110001
	→ 11111111	11111111	11111111	11100000
	00001010	01101001	00000001	01100000
	10 .	105 .	1 .	96

When we do the similar operation with 2nd IP, it gives 10.105.1.64

Here, it gives different IP

but for the rest it gives same IP.

42. In RSA public key cryptography, a public key provide for every one but they have their seprate private key .

According to formula of public key cryptography

I. $M' = M^e$ mod, $M = (M')^d$ mod n

IV. $M' = M^e \bmod \phi(n)$, $M = (M')^d \bmod \phi(n)$

represent RSA cryptosystem.

43. *Sequence number* : Frame from a sending station are numbered sequentially we need to set a limit.

44. When polynomial code method is employed, the sender and receiver must agree upon a generator polynomial G(x). Both high and low order bit of the generator must be 1 .

So $1 + x$ is a factor of G(x).

Alternately

For odd number of bits error, error function will be having a factor of $(1 + x^2 + x^4 + ...)$ which can not be divided by the generator having $(1 + x)$ as a factor.

45. Application layer just takes the data from user, pass it on the TCP layers below. It is the work of other TCP layer to actually disintegrate the data into packets accroding to network parameters to transmit in the network and assemble different packets on receiving them and handling it to the application layer. So, application layer doesnot bother about size.

46. Congestion window increases exponentially.

47. Starting address : 255. 255. 248. 0

End address : 255. 255. 255. 254.

48. Let duration is t sec.

$\therefore \quad 16 + 2t = 10 \times t$

$\Rightarrow \quad t = 2$ sec

49. As there is no accept() at server side, the connect() at client could not get anything that can respond to its requests, so error occurs.

50. In Manchester encoding, bit rate is twice the baud rate.

51. DNS uses UDP as the transport protocal.

52. Let 1 : event when station 1 transmits

2 : event when station 2 transmits

N : event when station N transmits

Given : P (i) = P $\quad 1 \le i \le N$

$P(\bar{i}) = (1 - p) \quad (\le i \le N)$

Probability that only one station transmits is given as

$$P(1\,\bar{2}\,\bar{3} \ldots \bar{N} \cup \bar{1}\,2\,\bar{3}\,\bar{4} \ldots \bar{N} \cup \bar{1}\,\bar{2}\,3\,\bar{4}\,5 \ldots \bar{N} \cup \ldots)$$

$$= P(1\,\bar{2}\,\bar{3} \ldots \bar{N}) + P(\bar{1}\,2\,\bar{3}\,\bar{4} \ldots \bar{N}) + P(\bar{1}\,\bar{2}\,3\,\bar{4}\,\bar{5} \ldots \bar{N}) + \ldots$$

Since all events are mutually exclusive, hence probability

$$= P(1)\,P(\bar{2})\,P(\bar{3}) \ldots P(\bar{N}) + P(\bar{1})\,P(2)\,P(\bar{3})\,P(\bar{4}) \ldots P(\bar{N}) + P(\bar{1})\,P(\bar{2})\,P(3)\,P(\bar{4})\,P(\bar{5}) \ldots P(\bar{N}) + \ldots$$

Since all events are independent, hence probability

$$= p\,(1-p)\,(1-p) \ldots (1-p) + (1-p)\,p\,(1-p) \ldots (1-p) + (1-p)\,(1-p)\,p\,(1-p) \ldots (1-p) + \ldots$$
$$= n\,p\,(1-p)^{n-1}$$

53. 10^7 bits needs 1 second

$\therefore$ 1 bits needs $\dfrac{1}{10^7}$ second $= \dfrac{1}{10}$ μs.

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Since 1 μs corresponds to 200 metre cable.

$\therefore \quad \frac{1}{10}$ μs corresponds to 20 metre cable.

54. Maximum number of subnets

$= 2^6 - 2 = 62$

Maximum number of host in each subnet

$= 2^{10} - 2 = 1022$

55. $P(x) = 11001001$

divisor $D(x) = 1001$ and CRC remainder is 011 so the transmitted message is 11001001011.

56. Total propagation delay = Lt sec.

Roundtrip time = 2 Lt = 2 Lt sec

Number of bits transmitted in roundtrip

= 2 LtR bits

$$\text{Number of frames} = \frac{2LtR}{K}$$

Let number of bits in sequent numbers be b.

$$\therefore \quad 2^b = \frac{2LtR}{K}$$

$$\Rightarrow \quad b = \left[\log_2 \frac{2LtR}{K}\right]$$

57. TCP protocol works on Transport layer.

SMTP protocol works on Application layer.

PPP Protol works on Data link layer.

BGP protol works on Physical layer.

58. Band width delay product

= Round trip delay × bottle neck bind width

$= 80 \times 128 \times 10^{-3} \times 1024$ bits

$$= \frac{80 \times 128 \times 10^{-3} \times 1024}{8} \text{ bytes}$$

$\therefore$ Optimal size window

$$= \frac{80 \times 128 \times 10^{-3} \times 1024}{8 \times 32} = 40$$

59. The two computer C1 and C2 are configured as follows :

	C1		C2
IP Address	203.197.2.53		203.197.75.201
AND		AND	
Netmask	255.255.128.0		255.255.192.0
	203.197.0.0		203.197.64.0
	Network Address		Network Address

203.197.2.53 → 203.197.00000010.00111001

255.255.192.0 →255.255.11000000.00000000

203.197.0.0 203.197.0.0

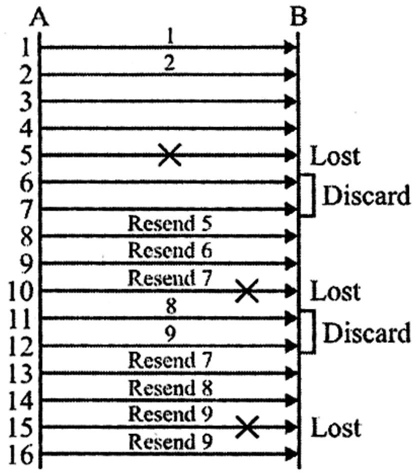
203.197.75.201

255.255.128.0

203.192.0.0

$\therefore$ C1 assumes C2 is on same network but C2 assumes C1 is on different network.

60.



NUMERICAL TYPE QUESTIONS

1. Given Length (d) = 2 Km

No. of Stations (m) = 10

Propagation Speed (v) = 2×10^8 m/s

THT = 2μs

So, Max. TRT = T_P in the Ring + No. of Active Stations * THT

$= 10 \times 10^{-6} + 10 \times 2 \times 10^{-6} = 30$ μs

2. Given L = 1KB

B = 1.5Mbps

T = 50ms

$\eta = 60\%$

Efficiency formula for SR protocol is

$$\eta = \frac{W}{1+2a} \Rightarrow \frac{60}{100} = \frac{W}{1+2a} \qquad \left(\because a = \frac{T_p}{T_x}\right)$$

$$T_x = \frac{L}{B} = \frac{8 \times 10^3}{1.5 \times 10^6} = 5.3 \text{ ms}$$

$$a = \frac{T_p}{T_x} = \frac{50}{5.3} = \frac{500}{53} = 9.43$$

$$\Rightarrow \frac{60}{100} = \frac{W}{19.86} \Rightarrow W = 11.9 \approx 12$$

$\Rightarrow$ $W = 2^{n-1} = 12 \quad \Rightarrow 2^n = 24$

$\Rightarrow$ $2^n = 24 \approx 2^5 \Rightarrow n = 5$

3. From S to R_1, there are 10 nodes but only 6 of them will decrement the TTL value. That's why, the maximum value of TTL field when R receives datagram is 32 – 6 = 26.

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4. Given address 131.23. 151.76. coming to the first field of given routing table

$\Rightarrow$ 131.16. 0.0/12

131.0001 0111.151.76

131.0001 0000.0.0 ($\because$ given mask bits = 12)

$\Rightarrow$ 131.16.0.0 Matched

Coming to the 2nd field of given Routing table

$\Rightarrow$ 131.28.0.0/14

131.0001 0111. 151. 76

131.0001 0100.0.0 ($\because$ given mask bits = 14)

$\Rightarrow$ 131.20.0.0 not matched

Coming to the 3rd field of given Routing table

Error ! Not a valid link 131.19.0.0/16

131.0001 0111.151.76

131.0001 0111.0.0 ($\because$ given mask bits = 16)

$\Rightarrow$ 131.23.0.0 Not matched

Coming to the 4^{th} field of given Routing table

$\Rightarrow$ 131.22.0.0/15

131.0001 0111.151.76

131.0001 0110.0.0 ($\because$ given mask bits = 15)

$\Rightarrow$ 131.22.0.0 Matched

We are getting 1st and 4th entries are matched so among them we have to picked up the longest mask bit, so output interface identifier is 1.

5. Given that each host has a globally unique IPv4 Address and we have to design 50 – bit unique Id. So, 50 – bit in the sense (32 + 18). So, It is clearly showing that IP Address (32 – bit) followed by 18 bits.

1000 unique Ids => 1Sec

2^{18} unique Ids => $2^{18} / 1000 = 2^8 = 256$

6. Given Bit rate = 64 kbps, T_p = 20 ns

L $\geq$ BR then

$$L = 64 \times 10^3 \times 20 \times 10^{-3}$$
$$= 1280 \text{ bits}$$
$$= 160 \text{ byte}$$

8. The efficiency of stop-and-wait protocol is 25%, then

$$\frac{1}{4} = \frac{1}{1+2a} \text{ i.e } 1 + 2a = 4$$

$$a = \frac{3}{2}$$

Now, $$T_X = \frac{8 \times 10^3}{10^6} = 8\text{ms}$$

as $$a = \frac{3}{2} \Rightarrow \frac{T_P}{T_X}$$

$$2T_P = 3T_X$$
$$2T_P = 3 \times 8$$
$$T_P = 12 \text{ ms}$$

9. Given, $$\left.\begin{aligned} L &= 10^7 \text{ bits} \\ B &= 500 \times 10^6 \text{ bps} \end{aligned}\right\} T_x$$

$$\Rightarrow \quad \frac{L}{B} = \frac{10 \times 10^6}{500 \times 10^6} = 0.02 \text{ sec}$$

$$\left.\begin{aligned} d &= 8000 \text{ km} \\ v &= 4 \times 10^6 \text{ m/s} \end{aligned}\right\} \Rightarrow T_p$$

$$\frac{d}{v} = \frac{8 \times 10^6}{4 \times 10^6} = 2 \text{ sec}$$

$$\eta = 100\%$$
$$n = ?$$

$$\therefore \quad a = \frac{T_p}{T_n} = \frac{2}{0.02} = 100$$

Given Protocol, Go back N protocol.

So $$\eta = \frac{w}{1+2a}$$

where $$w = 2^n - 1$$

$$\frac{100}{100} = \frac{w}{1+2a}$$

$$\Rightarrow \quad w = 1 + 2a$$
$$\Rightarrow \quad 2^{n-1} = 1 + 2(100)$$
$$\Rightarrow \quad 2^n - 1 = 201$$
$$\Rightarrow \quad 2^n = 202$$
$$\Rightarrow \quad 2^n = 2^8$$
$$\Rightarrow \quad n = 8$$

10. Given IP address 200.20.11.144/27,

To find out the loss address in a block, we have to set $(32 - n)$ no. of right most bits to 1.

$$n = 27$$
$$32 - n = 32 - 27 = 5$$

200.20.11.10010000

200.20.11.10011111

200.20.11.159

$\therefore$ CIDR Address range is 200.20.11.128/27-200.20.11.159/27

But w.r.t. the question, the value of the last octet of last host in this network is 200.10.11.158.

■■

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Mock Tests
and
Solved Paper

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MOCK TEST - 1

INSTRUCTIONS

1. Total of 65 questions carrying 100 marks, out of which 10 questions carrying a total of 15 marks are in General Aptitude (GA)
2. The Engineering Mathematics will carry around **15% of the total marks,** the General Aptitude section will carry **15% of the total marks** and the **remaining 70% of the total marks.**
3. **Types of Questions**
   (*a*) Multiple Choice Questions (MCQ) carrying 1 or 2 marks each in all papers and sections. These questions are objective in nature, and each will have a choice of four options, out of which the candidate has to mark the correct answer(s).
   (*b*) Numerical Answer Questions of 1 or 2 marks each in all papers and sections. For these questions the answer is a real number, to be entered by the candidate using the virtual keypad. No choices will be shown for these type of questions.
4. For **1-mark** multiple-choice questions, **1/3 marks** will be deducted for a wrong answer. Likewise, for **2-marks** multiple-choice questions, **2/3** marks will be deducted for a wrong answer. There is no negative marking for numerical answer type questions.

GENERAL APTITUDE (GA)

(Q.1 – 5) : Carry One Mark Each

1. Complete the sentence: Universalism is to particularism as diffuseness is to ____

(*a*) specific (*b*) neutrality
(*c*) generality (*d*) adapt

2. Which of the following option is closest in meaning of word below GRAVID ?

(*a*) Convert (*b*) Covet
(*c*) Pregnant (*d*) Cornivorous

3. Select the choice pair that has the same relation as the pair given below

dilate : expand

(*a*) can : discharge (*b*) idyllic : carefree
(*c*) exit : missionary (*d*) fear : profile

4. Find the odd man out from the group of four words

(*a*) Progression (*b*) Headway
(*c*) Forge ahead (*d*) Deviant

5. What is the sum of all odd numbers from 1 to 100.?

(Q.6 –10) : Carry Two Marks Each

6. A person starts his journey from his house towards north-east and travel for 4 km, then the turn to his right and travel for 5 km. Then he turn to his left to travel another 4 km and turn his right and travel 1 km. Which direction is he facing when he complete his journey and how far is the place from his house ?

(*a*) North-west, 8 km (*b*) North-east, 10 km
(*c*) South-west, 8 km (*d*) South-east, 10 km

7. An article costing ₹ 9000 is sold at a discount which is equal to the simple interest on ₹ 3000 for N months. Find N if the rate of discount is same as the rate of interest.

8. If 137 + 276 = 435, how much is 371 + 726 ?

9. If GO = 32 and SHE = 49, then what is SOME?

10. Though one eye is kept firmly on the ______, the company now also promotes ___ contemporary art.

(*a*) Present, Experimental
(*b*) Present, Popular
(*c*) Future, Popular
(*d*) Market, Popular

TECHNICAL SECTION

(Q.1 – 25) : Carry One Mark Each

1. Consider the language $L_1 = \{a, b\}^*$ and $L_2 = \{ab\}^*$ Which of the following is true?

(*a*) $L_1 \in L_2$ (*b*) $L_2 = L$
(*c*) $L_1 \cap L_2 = d$ (*d*) $L_1 \cup L_2 = L_2$

2. How many minimum number of states are required for designing a DFA of

$$L = \{\{a, b\}^* \mid n_a \bmod 3 = 0\}$$

3. Which of the following regular expression does not contain bab as a substring

(*a*) $a^*\, b^*\, (ba)^*\, a^*$
(*b*) $a^*\, b^*\, a^*\, (ba)b^*$
(*c*) $a^*\, b^*\, (ba)^*\, b^*$
(*d*) $(ab)^*\, b^*\, (ab)^*b^*$

4. What is the value of x to satisfy the following equation

$$(123)_5 = (38)_x$$

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5. Which of the following is not correct?
(*a*) ARP is applied on transport layer
(*b*) ARP is applied on data link layer
(*c*) ARP stands for address resolution protocol
(*d*) ARP is used for finding physical address of destination machine.

6. Which sorting algorithm is best when the list is already sorted?
(*a*) Head Sort (*b*) Merge Sort
(*c*) Insertion Sort (*d*) Quick Sort

7. Consider the C function
int foo [A, 1, 9]
{
$x \leftarrow$ A[9]
$i \leftarrow 0$
for ($i \leftarrow$ 1 to 8)
do if A[j] $\leq x$
$i \leftarrow i + 1$
exchange A[i] $\leftrightarrow$ A[j]
exchange A[i + 1] $\leftrightarrow$ A[a]
return i + 1
}
What is the value returned by above function if the array A is

1	2	3	4	5	6	7	8	9
4	8	9	7	6	3	8	7	5

8. Which sorting algorithm perform best in case of sorting ?
(*a*) Heap Sort (*b*) Quicks Sort
(*c*) Merge Sort (*d*) Insertion Sort

9. Inheritance is not possible in
(*a*) Linear programming languages
(*b*) Object oriented programming language
(*c*) Java
(*d*) C++

10. Which of the following is TRUE?
(*a*) EDF is used for scheduling real time
(*b*) RMA is used only for scheduling non-preempting
(*c*) Task
(*d*) EDF is used for energy minimization. FIFO is real time scheduling algorithm.

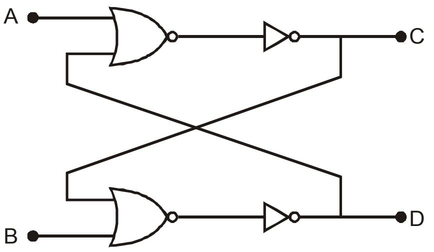
11. Which of the following is not public IP address ?
(*a*) 12.0.0.1 (*b*) 222.13.15
(*c*) 192.168.0.1 (*d*) 171.12.5.18

12. A process has been allocated 3 page frames. Assume that none of the pages of processes are available in the memory. If the LRU page replacement policy is used then how many page fault will occurs for a given string
4, 3, 4, 2, 6, 3, 1, 2, 3

13. How many page replacements will occur for above question?

14. Which of the following is not the function of data link layer?
(*a*) Error control (*b*) Transmission control
(*c*) Flow control (*d*) Access control

15. In the circuit shown below with input A = 0, B = 0, the possible logic states C and D are.



(*a*) C = 0, D = 1 (*b*) C = 1, D = 0
(*c*) C = 0, D = 0 (*d*) C = 1, D = 1

16. If a matrix is given as $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$, then determinant of A^{-1} is _______?

17. If one of the eigen value of matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is zero and $a, b, c, d \neq 0$, then
(*a*) $ad + bc = 0$ (*b*) $ad - bc = 1$
(*c*) $ad - bc = 0$ (*d*) None

18. A fair coin is flipped untill the first head occurs. If x is random variable which takes number of flip required, then the entropy in bits is
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

19. The derivation of $|x|$ with respect to x when $x \neq 0$ is
(*a*) –1 (*b*) Undefined
(*c*) $\frac{|x|}{x}$ (*d*) 1

20. The value of $f(f(x))$ is equal to ________ if
$f(a) = 2x - a$
(*a*) x (*b*) $2a$
(*c*) $2x$ (*d*) a

21. In a binary tree n node, every node has an odd number of descendants. Every node is considered to be its own descendants. What is the number of node in the tree that have exactly one child ?
(*a*) 0 (*b*) 1
(*c*) $\frac{(n-1)}{2}$ (*d*) $n - 1$

22. The best data structure to check whether an arithmetic expression has balanced paranthesis is a
(*a*) Queue (*b*) Stack
(*c*) tree (*d*) List

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23. int func (int num)

```
{
    int count = 0;
    while (num)
     {
     count ++;
     num >> = 1;
     }
    return (count);
```

The value of func (435) is ______

24. Which one of the following array represents a binary man-heap?

(*a*) 25, 12, 16, 13, 10, 8, 14

(*b*) 25, 14, 13, 16, 10, 8, 12

(*c*) 25, 14, 16, 13, 10, 8, 12

(*d*) 25, 14, 12, 13, 10, 8, 16

25. The following post fell expression with single digit operands is evaluated using a stack ;

8 2 3 ^ ? 2 3 * + 5 1 * –

the top two element of the stack after the first * is evaluated are

(Q.26 – 55) : Carry Two Marks Each

26. (function

int f (int n)

{ static int i = 1;

if (n > = 5) return n ;

$n + n + 1$;

i++ ;

return (f(n));

}

Find f(1)

27. Two alternative packages A and B are available for processing database having 10^n records. Package A required 0.0001 n^2 time unit and package B require 10 n $\log_{10}$n time unit to process n records. What is smallest value of K for which package B will preferred over A ?

28. The max number of binary tree that can be formed with three unlabelled node is

29. Which of the following sorting algorithm has the lowest worst-ease complexity?

(*a*) Merge sort (*b*) Heap sort

(*c*) Insertion sort (*d*) Selection sort

30. What is the minimum possible weight of a spanning tree T in this graph such that vertex O is a leaf node in the tree T?

31. How many distinct binary search trees can be created out of 4 distinct key?

32. If L and I are recursively enumerable, the L is

(*a*) regular (*b*) content free

(*c*) content sensitive (*d*) recursive

33. The regular expression 0*(10*)* denotes the solve set as

(*a*) (1 + 0)* 1* (*b*) 0 + (0 + 10)*

(*c*) (0 + 1)* 10(0 + 1)* (*d*) None of these

34. Consider the regular language L = (111 + 11111)* The minimum number of state in any DFA accepting this language is

35. Which of the following strings is generated by the grammar?

S – aAa/^, A – bAb/^

(*a*) aaadbb (*b*) aabbbb

(*c*) aabbab (*d*) abbbba

36. In which one of the following page replacement policies Bleady's anomaly may occur?

(*a*) FIFO (*b*) Optimal

(*c*) LRO (*d*) MRU

37. Where does the swap space reside?

(*a*) Round Robin

(*b*) FIFO

(*c*) Multilevel Queue scheduling

(*d*) Multilevel Queue scheduling with feedback

38.

Process	Arrival time	Burst time
P_0	0 ms	9 ms
P_1	1 ms	4 ms
P_2	2 ms	9 ms

The pro-emptive shortest job first scheduling is carried out only at arrival or completion of processes. What is arrival waiting time for the three processes.

39. What is the value of P, if $P = \lim_{x\to\infty}\left(\frac{x}{1+x}\right)^x$?

(*a*) e (*b*) $\frac{1}{e}$

(*c*) $-e$ (*d*) none

40. What is the density of undirected graph G with |V| vertices and |E| edges?

(*a*) $D = \frac{2|E|}{|V|(|V|-1)}$ (*b*) $D = \frac{2|E|}{|V|(|V|+1)}$

(*c*) $D = \frac{|E|}{|V|(|V|-1)}$ (*d*) $D = \frac{|E|}{|V|(|V|+1)}$

41. The value of the integral given below is

$$\int_0^{\pi} x^2 \cos x \, dx$$

(*a*) -2π (*b*) π

(*c*) $-\pi$ (*d*) 2π

42. $f(x) = (x-1)(x-2)(x-3)$ has a maximum value when x is equal to

(*a*) $2-\frac{1}{\sqrt{3}}$ (*b*) $2+\frac{1}{\sqrt{3}}$

(*c*) $1+\frac{2}{\sqrt{3}}$ (*d*) $1-\frac{2}{\sqrt{3}}$

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43. Consider the basic block given below.

$a = b + c,\quad c = a + d,\quad d = b + c,$

$e = d - b,\quad a = e + b$

The minimum number of nodes edges in the DAG representation of the above basic block respectively are

(*a*) 6 and 6 (*b*) 8 and 10

(*c*) 9 and 12 (*d*) 4 and 4

44. R(A, B, C, D) is a relation which of the following does not have a loss less join, dependency preserving BCNF decomposition?

(*a*) $A \to B, B \to CD$ (*b*) $A \to B, B \to C, C \to D$

(*c*) $AB \to C, C \to AD$ (*d*) $A \to BCD$

45. $(1217)_8$ is equivalent to $(\ )_{16}$.

46.

cd \ ab	00	01	11	10
00	1	1		1
01	×			
11	×			
10	1	1		×

(*a*) $\bar{b}\bar{d} + \bar{a}\bar{d}$ (*b*) $\bar{a}\bar{d} + \bar{b}\bar{d} + \bar{a}b\bar{d}$

(*c*) $\bar{b}\bar{d} + \bar{a}b\bar{d}$ (*d*) $\bar{a}\bar{b} + \bar{b}\bar{d} + \bar{a}\bar{d}$

47. What is the maximum number of different Boolean function involving *n* Boolean variable?

(*a*) n^2 (*b*) 2^n

(*c*) 2^{2n} (*d*) 2^{n^2}

48. X′Y′ + XY + XY is equivalent to

(*a*) Y′ + Y′ (*b*) X + Y

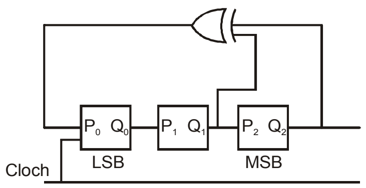
(*c*) X + Y′ (*d*) X′ + Y

49. The number 43 in 2's complement representation is

(*a*) 01010101 (*b*) 11010101

(*c*) 00101011 (*d*) 10101011

50. Consider the circuit given below with initial $Q_0 = 1, Q_1 = Q_2 = 0$ the state of the circuit is given by the value $4Q_2 + 2Q_1 + Q_0$.



Which one of the following is the correct of sequence of the circuit?

(*a*) 1, 3, 4, 6, 7, 5, 2

(*b*) 1, 2, 5, 3, 7, 6, 4

(*c*) 1, 2, 7, 3, 5, 6, 4

(*d*) 1, 6, 5, 7, 2, 3, 4

51. Which of the following is not desired in a good software requirement specification (SRS) document?

(*a*) Functional Requirement

(*b*) Non functional Requirement

(*c*) Goods of Implementation

(*d*) Algorithm for s/w implementation

52. Which of the following system calls result in sending of SYN packets ?

(*a*) Socket (*b*) Bind

(*c*) Listen (*d*) Connect

53. Which out of the following uses UDP at the transfer protocol?

(*a*) HTTP (*b*) Telnet

(*c*) DNS (*d*) SMTP

54. Pockets of the same section may be routed through different paths in

(*a*) TCP but not UDP (*b*) TCP and UDP

(*c*) UDP but not TCP (*d*) Neither TCP not UDP

55. The maximum window size for data transmission using the selection reject protocol with a n-bit frame sequence number is

(*a*) 2^n (*b*) $2n$–1

(*c*) 2^{n-1} (*d*) 2^{n-2}

ANSWERS

General Aptitude (GA)

1. (*a*) **2.** (*c*) **3.** (*a*) **4.** (*d*) **5.** (2500) **6.** (*d*) **7.** (36) **8.** (11119) **9.** (76) **10.** (*d*)

Technical Section

1. (*a*) **2.** (3) **3.** (*a*) **4.** (10) **5.** (*a*) **6.** (*c*) **7.** (3) **8.** (*b*) **9.** (*a*) **10.** (*a*)
11. (*c*) **12.** (7) **13.** (4) **14.** (*b*) **15.** (*c*) **16.** (0.5) **17.** (*c*) **18.** (*b*) **19.** (*c*) **20.** (*a*)
21. (*a*) **22.** (*b*) **23.** (9) **24.** (*c*) **25.** (6, 1) **26.** (7) **27.** (10^6) **28.** (5) **29.** (*a*) **30.** (10)
31. (14) **32.** (*d*) **33.** (*d*) **34.** (9) **35.** (*c*) **36.** (*a*) **37.** (*b*) **38.** (5) **39.** (*b*) **40.** (*a*)
41. (*a*) **42.** (*c*) **43.** (*a*) **44.** (*c*) **45.** (028F) **46.** (*d*) **47.** (*c*) **48.** (*d*) **49.** (*c*) **50.** (*b*)
51. (*d*) **52.** (*d*) **53.** (*c*) **54.** (*b*) **55.** (*c*)

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MOCK TEST - 2

INSTRUCTIONS

1. Total of 65 questions carrying 100 marks, out of which 10 questions carrying a total of 15 marks are in General Aptitude (GA)
2. The Engineering Mathematics will carry around **15% of the total marks,** the General Aptitude section will carry **15% of the total marks** and the **remaining 70% of the total marks.**
3. **Types of Questions**
   (*a*) Multiple Choice Questions (MCQ) carrying 1 or 2 marks each in all papers and sections. These questions are objective in nature, and each will have a choice of four options, out of which the candidate has to mark the correct answer(s).
   (*b*) Numerical Answer Questions of 1 or 2 marks each in all papers and sections. For these questions the answer is a real number, to be entered by the candidate using the virtual keypad. No choices will be shown for these type of questions.
4. For **1-mark** multiple-choice questions, **1/3 marks** will be deducted for a wrong answer. Likewise, for **2-marks** multiple-choice questions, **2/3** marks will be deducted for a wrong answer. There is no negative marking for numerical answer type questions.

GENERAL APTITUDE (GA)

(Q.1–5) : Carry One Mark Each

1. The sum of three prime number is 100. If one of them exceeds another by 36, then one of the number is _______.

2. This is not the first time that the management has done some ________.

(*a*) tough talk (*b*) firm talk
(*c*) tough talking (*d*) firm talking

3. Choose the word which is closest in meaning of the word given below

Twirl

(*a*) Twinkle (*b*) Twist
(*c*) Entangle (*d*) Tug

4. Find the least number which on being divided by 12, 21 and 35 will have in each case the same remainder 6.

5. What is single word for "Close connection relationship"

(*a*) affinity (*b*) affable
(*c*) aleck (*d*) agile

(Q.6–10) : Carry Two Marks Each

6. Most political leaders _____ their position by causing a large number of people to believe that these leaders are _____ by altrusitic desire.

(*a*) acquire, actuated
(*b*) desire, actuated
(*c*) acquire, convinced
(*d*) desire, convinced

7. Find three consecutive integer such that the square of their sum exceeds the sum of their square by 214

(*a*) 5, 6 7
(*b*) 9, 10, 11
(*c*) 10, 11, 12
(*d*) 7, 8, 9

8. A and B start from same point to run in opposite direction round a circle path 550 yards in length. A giving a start of 100 yards. They pass each other when A runs 250 yards. Who will come first to the starting point and what distance will they be apart.

(*a*) A, 20 yards
(*b*) A, 10 yards
(*c*) B, 20 yards
(*d*) B, 10 yards

9. What is the value of $5+\cfrac{1}{5+\cfrac{1}{5+\cfrac{1}{5+\cfrac{1}{5}\cdots}}}$?

10. Given the sequence of term AZ, BY, CX, DW, the next term is

(*a*) EX
(*b*) FV
(*c*) EU
(*d*) EV

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TECHNICAL SECTION

(Q.1–25) : Carry One Mark Each

1. The number of generators of group {0, 1, 2, .., 14} under the group operation modulo 15 is _____.

2. Given a system with n processes, how many possible ways can these processes be scheduled?

(*a*) 2^n (*b*) $n - 1$

(*c*) $n!$ (*d*) n

3. What is the value of $\int_{-\infty}^{\infty} \frac{1}{1+x^2}.dx$

(*a*) π (*b*) $\frac{\pi}{2}$

(*c*) $\frac{\pi}{4}$ (*d*) none of these

4. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then the value of x is

(*a*) 1 (*b*) 2

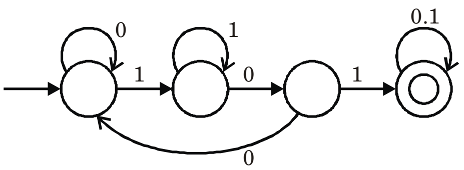
(*c*) $\frac{1}{2}$ (*d*) none of these

5. The probability function of a continuous function $f(x) = ae^{-b|x|}$ where $-\infty < x < \infty$. The relation between 'a' and 'b' is

(*a*) $\frac{a}{b} = \frac{1}{2}$ (*b*) $a = b$

(*c*) $\frac{b}{a} = \frac{1}{2}$ (*d*) $\frac{a}{b} = \frac{1}{3}$

6. Consider the given DFSM given below. The language accept all string over {0, 1}



(*a*) ends with 0 or 1.

(*b*) starts with 0 or 1 and ends with 0 or 1.

(*c*) contain substring 101.

(*d*) contain substring 01.

7. Which of the following is TRUE?

(*a*) For all NFA, there is an equivalent DFA.

(*b*) For all NPDA, there is an equivalent PDA.

(*c*) For all CFG, there is an equivalent DFA.

(*d*) For all CFG, there is an equivalent regular grammer.

8. Which of the following is not a context free language?

(*a*) $L = \{a^n\, b^{m+k}\ n, m, k \geq 0\}$

(*b*) $L = \{a^n\, b^{m+k}\ n, m \geq 0\}$

(*c*) $L = \{a^n\, b^{n^m}\ n, m \geq 0.$ m is prime$\}$

(*d*) $L = [a^n\, b^m]n, m \geq 0$

9. Which of the following statement is TRUE?

(*a*) SLR parser is more powerful than LALR.

(*b*) LALR is more powerful than CLR.

(*c*) CLR and SLR have same power.

(*d*) CLR is more powerful than any other parser.

10. The range of n-bit unsigned binary number is-

(*a*) $-(2^{n-1} - 1)$ to $2^{n-1} - 1$

(*b*) 0 to $2^n - 1$

(*c*) $-(2^n - 1)$ to $(2^n - 1)$

(*d*) 0 to $2^{n-1} - 1$

11. What is the value of the memory system that transfer 64 bits of data per request has a latency of 25 ns per operation and has a precharge time of 5 ns between operations?

(*a*) 2.7×10^8 bits/sec (*b*) 2.7×10^7 bits/sec

(*c*) 2.3×10^7 bits/sec (*d*) 2.8×10^8 bits/sec

12. Which of the following is not internet topology?

(*a*) Ring (*b*) Star

(*c*) Diamond (*d*) Bus

13. The worst case time required to search a given element in a sorted list of length n is

(*a*) O(1) (*b*) O(n)

(*c*) O(log n) (*d*) O(n log n)

14. Consider the given C program

```
main ( )
  {
  unsigned int a ;
  into c = 0 ;
  & scanf ("%u", &a);
  for (; a ; a > = 1)
  }
    if (a & 1)
    c++ ;
  }
  print f("%d", c) ;
  }
```

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(*a*) Count the number of bits which are on number a

(*b*) Sets all bits in number a to 0.

(*c*) Sets bits in the number a to 0.

(*d*) Garbage value.

15. Consider an undirected connected graph contain w be the minimum weight among all edge. Let e be the specific edge of weight. Which of the following is FALSE?

(*a*) There is a minimum spanning tree containing e of weight w.

(*b*) There may be more than one spanning tree and every spanning tree contains e.

(*c*) Every minimum spanning tree has an edge of weight w.

(*d*) If e is not in minimum spanning, there is another edge of same weight in minimum spanning tree.

16. Union operation in database is used to perform union of

(*a*) Two sets who combine all the types from both sets.

(*b*) Two similar structured table

(*c*) All types and attributes will be combined.

(*d*) All types and attributes will be combined.

17. Suppose a sender A wants to send data to receiver B. A apply digital signature on the data and send. The sender energy the data on

(*a*) Application layer

(*b*) Presentation layer

(*c*) Network layer

(*d*) Data link layer

18. The following expression is evaluated using a stack. What is the value on the top of stack after * operation

$\rightarrow$ 251 + 3 / * 2 *

19. Cascading termination refers to termination of all child processes before the parent terminates

(*a*) normally

(*b*) abnormally

(*c*) both *a* and *b*

(*d*) none of these

20. Which of the following does not affect the resolution of a video display image?

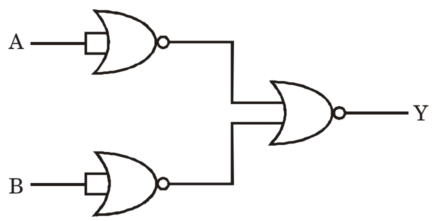
(*a*) Bandwidth

(*b*) Screen size

(*c*) Raster screen rate

(*d*) Vertical and horizontal lines of resolution.

21. Consider the circuit given below :



What is output Y when A = 0 and B = 1

(*a*) 0

(*b*) 1 to 0

(*c*) 1

(*d*) none of these

22. The simplest form of Boolean expression

$A\overline{BC} + A\overline{B}C + AB\overline{C} + ABC$ is

(*a*) $A\overline{B}C$ (*b*) A

(*c*) $\overline{A}$ (*d*) BC

23. What is the minimal form of K-map shown below? Assume that X denotes a don't care term

AB \ CD	00	01	11	10
00	1			×
01				
11		×		
10	1	1	×	1

(*a*) $\overline{BD} + A\overline{B}$ (*b*) $\overline{BD} + AD$

(*c*) $\overline{BD} + \bar{A}B$ (*d*) $\overline{BD} + \overline{AD}$

24. When a cache is 10 times faster than main memory, and a cache can be used 90% of the time, how much speed we gain by using cache?

25. Let = $\{a^n\, b^n\, c^n \mid n \geq 0\}$ the given language is

(*a*) Context free

(*b*) Regular

(*c*) Regular as well as context free

(*d*) Context sensitive

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(Q.26–55) : Carry Two Marks Each

26. The number of ways in which the numbers 1, 2, 3, 4, 5, 6, 7 can be inserted in an empty binary search tree, such that the resulting tree has height 6, is _____

Note : The height of a tree with a single node is 0.

27. Consider tye grammer

$S \to asa \mid bsb \mid a \mid b$

The language generated by the given grammer is

(*a*) Even length palindrome

(*b*) Odd length palindrome

(*c*) Equal number of *a* and *b*

(*d*) Odd number of *a* and *b*

28. Which of the following is not TRUE

(*a*) All context free language is recursive language.

(*b*) All context free language are regular.

(*c*) All singular language are context free.

(*d*) All singular language are recursive language.

29. Match the List I with List II

List I	List II
A. Finite Automatar	1. Context free languages
B. Push Down Automata	2. Recursive languages
C. Linear bounded Automata	3. Regular language
D. Turing Machine	4. Context sensitive languages

	A	B	C	D
(*a*)	3	1	4	2
(*b*)	3	1	2	4
(*c*)	3	4	2	1
(*d*)	4	1	2	3

30. Consider the inorder traversal of a binary tree is

c b a e f d g

Which of the following is pre-order traversal?

(*a*) c b f e g d a

(*b*) c b e f g d a

(*c*) a b d e c g f

(*d*) a b c d e f g

31. If $z = e^x \sin y, x = \log_e t$ and $y = t^2$, then $\frac{dz}{dt}$ is equal to

(*a*) $\frac{e^x}{t}(\sin y - 2t^2 \cos y)$

(*b*) $\frac{e^x}{t}(\cos y + 2t^2 \sin y)$

(*c*) $\frac{e^x}{t}(\sin y + 2t^2 \cos y)$

(*d*) $\frac{e^x}{t}(\cos y - 2t^2 \sin y)$

32. What is the value of

$^5C_5 + {}^6C_5 + {}^7C_5 + {}^8C_5 + {}^9C_5 + {}^{10}C_5$?

33. Which of the following argument is valid?

(*a*) $p \wedge (p \to r) \wedge ((p \wedge r) \to (q \vee r)) \to q$

(*b*) $((p \wedge q) \to r) \wedge (7q \vee 7r) \to p$

(*c*) $((p \wedge q) \to r) \wedge 7q \wedge (p \to 7r) \to (q \to 7q)$

(*d*) $(p \wedge 7q) \wedge (p \to (q \to 7r)) \to 7r$

34. Which of the following statement is true

S_1 : The union of two equivalent relations on a set A is an equivalent relation.

S_2 : The union of two reffexive relations on a set A is reflexive relation.

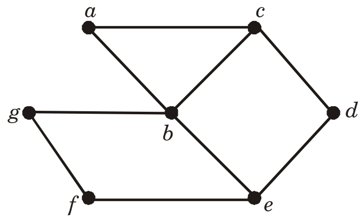
(*a*) S_1 and S_2 both true

(*b*) S_1 is true and S_2 is false

(*c*) S_1 and S_2 both false

(*d*) S_1 is false and S_2 is true

35. Consider the following graph



Which of the following statement is true about the graph?

(*a*) It has a Eulerian path, but not Eulerian cycle

(*b*) It has a Eulerian cycle, but not Eulerian path

(*c*) It has both Eulerian cycle and path

(*d*) It does not have any Eulerian cycle or path.

36. In a circular linked list organization, insertion of a record involve the modification of

(*a*) 1 pointer (*b*) 2 pointer

(*c*) 3 pointer (*d*) 4 pointer

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37. Consider the given C program

```
Main ( )
{
int foo ( );
int (* fune.ptr) ();
fune-ptz = foo ;
(* fune-ptr) ( ) ;
}
int foo ( )
{
puts ("Gate 2015") ;
}
```

What is the output of the code?

(*a*) Garbage value

(*b*) Gate 2015

(*c*) Run time error

(*d*) Compile time error

38. For merging two sorted list of size m and n into a sorted list of size $m + n$, we repair comparison of

(*a*) $O(n)$ (*b*) $O(m)$

(*c*) $O(n + m)$ (*d*) $O(\log m.n)$

39. Which of the following is not TRUE?

(*a*) $O(\log n) \leq O(n) \leq O(n^2) \leq O(a^n)$

(*b*) $O(\log n) \leq O(n \log n) \leq O(n^2 \log n) \leq O(a^n)$

(*c*) $O(\log n) \leq O(n) \leq O(n \log n) \leq O(a^n) \leq O(n^2 \log n)$

(*d*) $O(\log n) \leq O(n) \leq O(n \log n) \leq O(n^2 \log n) \leq O(a^n)$

40. If $f(n)$ and $g(n)$ are two functions defined on the positive integers. Then $f(n) = og(n)$ if and only if there exist < > O and $n > n_0$ such that

(*a*) $O \leq f(n) \leq Cg(n)$

(*b*) $O \leq C_1 g(n) \leq f(n) < C_2 g(n)$

(*c*) $O \leq Cg(n) \leq f(n)$

(*d*) $O \leq C_1 g(n) \leq C_2 g(n) \leq f(n)$

41. Consider the graph in which Dijkstra Algorithm is used for finding single source shortest path and floyed-warshall's algo is used for finding all pair shortest path. Which of the above use greedy and which one use dynamic programming approach.

(*a*) Dijkstra-greedy, Warshall-dynamic programming

(*b*) Dijkstra-dynamic programming, warshall's greedy

(*c*) Both uses-dynamic programming

(*d*) Both uses greedy

42. Consider the following sequence of page refrences.

2 3 4 2 6 4 3 6 2 4

The process has allocated 3 page frames and assume that none of the pages are available in the memory initially. How many page fault will occur if LRU page replacement policy is used.

43. How many page fault will occur if optional page replacement policy is used in the above problem.

44. An operating system uses shortest remaining line first (SRT) process scheduling algorithm.

Consider the arrival time and execution time for the following proesses

Process	**Execution time**	**Arrival time**
P_0	18	0
P_1	20	10
P_2	15	20
P_3	10	30

What is the total waiting time of the process P_1.

45. What is the average waiting time of the above problem?

46. A computer uses a memory unit iwth 256 k words of 32 bit each. A binary instruction code in one word of memory. The instruction has 4 parts ; an indirect bit, an operand code, a register code to specify one of 64 registers and an address part.

How many bits are there in operation code ?

47. How many bits are there in data part ?

48. Consider a full binary tree which n leaves node. How many total number of nodes are in the tree?

(*a*) n nodes (*b*) $\log_2 n$ nodes

(*c*) 2^n–1 nodes (*d*) $2n$ nodes

49. A divide by 78 counter can be realized using

(*a*) 6 numbers of mod-13 counters

(*b*) One mode- 13 counter followed by 1 mode-6 counter

(*c*) 78 mod 6 counter

(*d*) None of these

50. A relative mode brand type of instruction is stered in memory at the address equivalent to decimal 790.

The branch is mode to an address equivalent to decimal 600.

What should be the value of relative address field of decimal instruction?

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51. What is the most appropriate match of the item

A.	Indirect Addressing	1.	Array implementation
B.	Indexed Addressing	2.	Implementation writing
C.	Base Register Addressing	3.	Passing array as parameter

	A	B	C
(*a*)	3	1	2
(*b*)	1	2	3
(*c*)	2	3	1
(*d*)	1	3	2

52. The schema R(ABCDE) has the functional dependency F = A → C, B → D, find the normal form :

(*a*) 7NF (*b*) 2NF
(*c*) 7NF (*d*) CNF

53. The schema R(ABCDE) has the functional find the key of the schema

(*a*) AB, θ (*b*) AB, E
(*c*) AB, C (*d*) AB

54. DDL and DML statement are database language which is used by

(*a*) Data model (*b*) transaction manager
(*c*) Storage Manager (*d*) Query processor

55. Find the number of roots in the output of the following sub-query

select *
From Table A
Where acists (select count (*)
From Table B
Where TBZ = 'a');

(*a*) 0 rows (*b*) > 0 rows
(*c*) error (*d*) 3 rows

ANSWERS

General Aptitude (GA)

1. (67) **2.** (*b*) **3.** (*b*) **4.** (426) **5.** (*b*) **6.** (*c*) **7.** (*a*) **8.** (*b*) **9.** $\frac{3640}{701}$ **10.** (*d*)

Technical Section

1. (8) **2.** (*c*) **3.** (*a*) **4.** (*c*) **5.** (*a*) **6.** (*c*) **7.** (*a*) **8.** (*c*) **9.** (*d*) **10.** (*b*)
11. (*a*) **12.** (*c*) **13.** (*b*) **14.** (*a*) **15.** (*b*) **16.** (*b*) **17.** (*b*) **18.** (8) **19.** (*c*) **20.** (*b*)
21. (*a*) **22.** (*b*) **23.** (*a*) **24.** (5.3) **25.** (*d*) **26.** (64) **27.** (*b*) **28.** (*b*) **29.** (*a*) **30.** (*d*)
31. (*c*) **32.** (462) **33.** (*c*) **34.** (*d*) **35.** (*b*) **36.** (*b*) **37.** (*b*) **38.** (*c*) **39.** (*c*) **40.** (*b*)
41. (*a*) **42.** (7) **43.** (5) **44.** (23) **45.** (7.0) **46.** (7) **47.** (32) **48.** (*c*) **49.** (*b*) **50.** (190)
51. (*a*) **52.** (*a*) **53.** (*b*) **54.** (*d*) **55.** (*d*)

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MOCK TEST - 3

INSTRUCTIONS

1. Total of 65 questions carrying 100 marks, out of which 10 questions carrying a total of 15 marks are in General Aptitude (GA)
2. The Engineering Mathematics will carry around **15% of the total marks,** the General Aptitude section will carry **15% of the total marks** and the **remaining 70% of the total marks.**
3. **Types of Questions**
   (*a*) Multiple Choice Questions (MCQ) carrying 1 or 2 marks each in all papers and sections. These questions are objective in nature, and each will have a choice of four options, out of which the candidate has to mark the correct answer(s).
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4. For **1-mark** multiple-choice questions, **1/3 marks** will be deducted for a wrong answer. Likewise, for **2-marks** multiple-choice questions, **2/3** marks will be deducted for a wrong answer. There is no negative marking for numerical answer type questions.

GENERAL APTITUDE (GA)

(Q1–5) : Carry One Mark Each

1. Which one of the following is the closest in the meaning to the word given below

Beatific

(*a*) Beautiful (*b*) Bawdy
(*c*) Celestial (*d*) Beau

2. Choose the most appropriate word to complete the sentence

Learning is more efficient when it is ______, less efficient when it is ______

(*a*) fast, slow
(*b*) rapid, turtle-slow
(*c*) tedious, like a joy ride
(*d*) fun, drudgery

3. If $\log(P) = \frac{1}{2}\log(Q) = \left(\frac{1}{3}\right)\log(R)$, then which of the following is true ?

(*a*) $P^2 = Q^3R^3$ (*b*) $Q^2 = PR$
(*c*) $Q^2 = R^3P^2$ (*d*) $R = P^2Q^2$

4. Find the harmonic mean of two numbers, whose geometric mean and arithmetic mean is 8 and 5 respectively.

(*a*) 12.8
(*b*) 12
(*c*) 13.5
(*d*) 14.6

5. Find the word odd in given below

(*a*) detest (*b*) abhor
(*c*) ardour (*d*) loathe

(Q.6-10) : Carry Two Marks Each

6. A person invest Rs. 1000 at 10% annual compound interest for 2 years. At the end of two years the whole amount is invested at an annual simple interest of 12% for 5 years. The total value of the investment finally is _____

7. Identify the incorrect sentence or sentences

A. Harish told Raj to plead guilty
B. Raj pleaded guilty of stealing money from the shop
C. The court found Raj guilty of all the crimes he was charged with
D. He was sentenced for three years in jail

(*a*) A and B
(*b*) A, C and D
(*c*) B and D
(*d*) B, C and D

8. Given the sequence of term AD, CG, _____, JP, OV find the missing term

(*a*) FK (*b*) OW
(*c*) ED (*d*) FJ

9. What is the value of 'a' will make $x^2 - ax + 25$ as perfect square?

(*a*) 10 (*b*) 20
(*c*) 50 (*d*) – 25

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10. A clock loses 5 seconds in an hour and is set right on sunday at noon. What time will it indicate on the following monday at noon

(*a*) 11.56 am. (*b*) 12.02 am.

(*c*) 11.58 am. (*d*) 12.04 am.

TECHNICAL SECTION

(Q.1–25) : Carry One Mark Each

1. Which of the following is TRUE for a simple connected undirected graph with more than two vertices?

(*a*) No two vertices have same degree

(*b*) All vertices have equal degree

(*c*) At least two vertices have same degree

(*d*) No one vertex have same degree.

2. Which of the following proposition is a Tautology

(*a*) $P \to (p \to q)$ (*b*) $(P \vee q) \to P$

(*c*) $P \vee (q \to p)$ (*d*) $P \vee (p \to q)$

3. The value of integral $\int_{-1}^{1} \frac{1}{x^2} dx$ is

(*a*) 2

(*b*) – 2

(*c*) ∞

(*d*) 0

4. Eight coins are tossed simultaneously. The probability of getting at least 6 head is

(*a*) $\frac{7}{64}$ (*b*) $\frac{57}{64}$

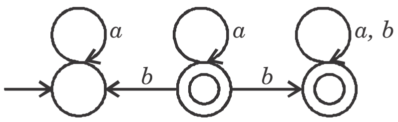
(*c*) $\frac{37}{256}$ (*d*) $\frac{249}{256}$

5. The Eigen value of matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

(*a*) 6, 1 (*b*) 5, 2

(*c*) 6, 2 (*d*) 5, 1

6. Consider the NFSM shown below



The language accepted by given NFSM is

(*a*) L = {(abb)* (ab)*}

(*b*) L = {a*bb(a*b*)*}

(*c*) L = ϕ

(*d*) L = {^}

7. For a language L = {set of all string contains at least two 0's} over $\Sigma\{0, 1\}$, which of the following is false.

(*a*) (0 + 1)* 01* (0 + 1)* 0(0 + 1)*

(*b*) (0 + 1)* 0 (0 + 1)* 1*0 (0 + 1)*

(*c*) (0 + 1)* 0* (0 + 1)* 0* (0 + 1)*

(*d*) (0 + 1)* 0 (0 + 1)* 0 (0 + 1)*

8. Let L_1 be a recursive language. Let L_2 be the language that is recursively enumerable but not recursive. Which of the following is true.

(*a*) $L_2 - L_1$ is recursively enumerable

(*b*) $L_1 - L_2$ is recursively enumerable

(*c*) $L_2 - L_1$ is recursive but not RE.

(*d*) $L_1 - L_2$ is recursive but not RE.

9. If SR catch made by cross-coupling two NAND gates, if S and R both input are set to 0, then results in

(*a*) Q = 0, Q′ = 1 (*b*) Q = 1, Q′ = 0

(*c*) Q = 1, Q′ = 1 (*d*) Indeterminant state

10. Convert $(028\text{ F})_{16}$ to $(\ )_8$

11. Which of the following operation is performed at run time using relative addressing mode.

(*a*) Instruction fatch (*b*) Program relocation

(*c*) Load instruction (*d*) None of above

12. Consider a disk pack with 16 surfaces 128 tracks per surface and 256 sectors per track, 1024 bytes of data is stored in bit serial manner in a sector. What is capacity of disk pack.

13. The tightest lower bound on the number of comparisions in worst case for comparision based sorting is the order of

(*a*) n (*b*) n^2

(*c*) $n \log n$ (*d*) $n^2 \log n$

14. Which of the following sorting algorithm has lowest worst case complexity ?

(*a*) Merge sort

(*b*) Bubble sort

(*c*) Selection sort

(*d*) Quick sort

15. Which of the following is the function of data link layer?

(*a*) Error control

(*b*) Flow control

(*c*) Access control

(*d*) All of above

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16. Maximum through put of a linear pipeline is equal to

(*a*) Frequency (*b*) Efficiency

(*c*) Through put (*d*) Some constant k

17. The recurrence $T(n) = T(\sqrt{n}) + 1$, then $T(n)$ can be defined as

(*a*) $0(\log_2 n)$ (*b*) $0(\log(\log_2 n))$

(*c*) $0(\sqrt{n}\log_2 n)$ (*d*) $0(n\log_2 n)$

18. What is the maximum possible height of an AVL tree with 10 nodes ?

19. The minimum number of page frame that must be allocated to a running process in a virtual memory environment is determined by

(*a*) page size

(*b*) instruction set architecture

(*c*) physical memory size

(*d*) number of process

20. The address resolution protocol (ARP) is used for

(*a*) Finding MAC address of source machine.

(*b*) Finding MAC address that correspond to an IP address on WAN.

(*c*) Finding MAC address that correspond to an IP address on LAN.

(*d*) Finding MAC address from DNS.

21. Which one of the following is not desired in a good software requirement specification (SRS) document?

(*a*) Functional requirement

(*b*) Non-functional requirement

(*c*) Goals implementation

(*d*) Algorithm for software implementation

22. What is the channel capacity of a printer with 400 Hz bandwidth and a signal to noice ratio is 7 dB?

23. Rate monotonic scheduling is used for scheduling

(*a*) Real time task scheduling

(*b*) Threads scheduling

(*c*) Frames scheduling

(*d*) Process scheduling

24. Let a given expression

$a = (x > y)\ ?\ ((x > z)\ ?\ x : z) : ((y > z)\ ?\ y : z)$

If the value of $x = 3$, $y = 4$ and $z = 2$, what is the value of a.?

25. Which of the three is not transport layer protocol

(*a*) ARP

(*b*) TCP

(*c*) UDP

(*d*) RSVP

(Q.26–55) : Carry Two Marks Each

26. If a class B network on the internet has a subnet mask of 255.255.248.0. What is the maximum number of host per subnet.

27. A minimum state DFA accepting the language $L = \{w \mid w \in \{0, 1\}^*, |w| \text{ is divisible by } 5\}$ has _____ number of states.

28. The language $L = \{0^n\ 12^{n+1} \mid n \geq 0\}$ over $\Sigma = \{0, 1, 2\}$ is,

(*a*) regular language

(*b*) recursive language

(*c*) not recursive

(*d*) CFL but not deterministic

29. The rank of a given matrix

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & 1 & 4 \end{bmatrix} \text{is}$$

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 0

30. Find $\int \frac{dx}{a^2 - x^2} = ?$

(*a*) $\frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right)$ (*b*) $\frac{1}{2a}\ln\left(\frac{ax}{a-x}\right)$

(*c*) $\frac{1}{a}\ln\left(\frac{a+x}{a-x}\right)$ (*d*) $\frac{1}{a}\ln\left(\frac{a-x}{a+x}\right)$

31. For a binomial distribution with n-observations, the difference between mean (μ) and variance (σ^2) is

(*a*) $\frac{\mu}{n}$ (*b*) $\frac{\sigma^2}{n}$

(*c*) $\frac{\mu^2}{n}$ (*d*) $\frac{\sigma}{n}$

32. What is the time complexity of Bellman - Ford Single source shortest path algorithm on a complete graph of n vertices ?

(*a*) $\Theta(n^2)$ (*b*) $\Theta(n^2 \log n)$

(*c*) $\Theta(n^3)$ (*d*) $\Theta(n^3 \log n)$

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33. Let G be connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes ____

(*a*) 1000
(*b*) 995
(*c*) 2000
(*d*) 1995

34. Consider the following grammar

$S \rightarrow AB \mid CA$
$A \rightarrow a$
$B \rightarrow Bc \mid AB$
$C \rightarrow aB \mid b$

What is the reduced grammar after eliminating useless symbols

(*a*) $S \rightarrow CA$
$A \rightarrow a$
$C \rightarrow b$

(*b*) $S \rightarrow CA$
$A \rightarrow a$
$C \rightarrow aB \mid b$

(*c*) $S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow a \mid b$

(*d*) $S \rightarrow AB \mid CA$
$A \rightarrow a$
$B \rightarrow a \mid b$
$C \rightarrow b$

35. The following prefix expression with single digit operand is evaluated using a stack, what is the top element of stack after first * is evaluated.

732 + – 3 ↑ 82 ------ 3*

note that ↑ is exponentation operator.

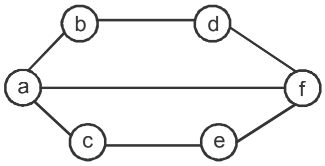
36. Consider the following c code

```
int i, n ;
static int i = 1 ;
while (i < = n)
i = i * 2 ;
```

The number of comparisions made in the execution of loop for n > 0 is

(*a*) $[\log_2 n] + 1$ (*b*) $(\log_2 n)$
(*c*) $[\log_2 n] - 1$ (*d*) $[n \log_2 n]$

37. Consider the following graph



Which is depth first search traversal of above graph

(*a*) abcdef
(*b*) acdefb
(*c*) abdcef
(*d*) acbdef

38. Assume that the operators +, – and * are left associative and ^ is right associative, the order of precedence from lowest to highest is –, +, * and ^. The infix expression corrospond to given prefix expression abc* + def^^ – is

(*a*) a + b * c – d ^ e ^ f
(*b*) a * b + c ^ d – e ^ f
(*c*) a + b * c ^ d – e ^ f
(*d*) a * b + c – d ^ e ^ f

39. Consider the code

```
x ← A[r]
i ← P – 1
for i ← P to r – 1
    do if A[j] ≤ n
    then i ← i + 1
    exchange A[i] ↔ A[j]
    exchange A[i + 1] ↔ A[r]
    return i + 1
```

if we have an array A[ ] = {2, 8, 7, 1, 3, 5, 6, 4} P and r are first and last index of array then what will be return by above code for given array?

40. A multilevel page table is preferred in comparision to a single level page table for translating virtual address to physical address because

(*a*) If help to reduced the number of page fault in page replacement algorithm.
(*b*) A look a side buffer is used for translation.
(*c*) It reduce memory access time to read and write.
(*d*) It help to reduce the size of page table need to implement the virtual address space.

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41. The employee information of an organization is stored as

Emp (name, sex, salary, dept, DOB)

Consider the following query

Select dept from emp →employee

avg → $\alpha\varpi\varepsilon\rho\alpha\gamma\varepsilon$

where sex = " f "

group by dept

having avg (salary) > (select avg salary from emp)

The above query returns

(*a*) The average salary of female employee is more than the avg salary in company.

(*b*) The avg salary of female employee is more than avg salary in dept.

(*c*) The avg salary of female employee is more than male employee in dept.

(*d*) The avg salary is more than female employee in dept.

42. Consider the following k-map

AB \ CD	00	01	10	11
00	X	1	1	X
01			1	
10			1	
11	1		1	

What is the nominal for m of above k-map

1. $\overline{AB} + C\,D$
2. $C\,D + \bar{A}\,D$
3. $C\,D + \bar{A}\,\bar{C}\,D$
4. $\bar{A}\,D + A\,C\,\bar{D}$

(*a*) All 1, 2, 3 and 4 (*b*) 1 and 2 only

(*c*) 1, 2, 3 only (*d*) 2 only

43. Suppose the round trip propogation delay for a 10 Mbps ethernet having 48 bit jaming signal in 48.6 μs. The minimum frame size is ?

44. Consider the following C program

```
char c [10]
char * s = "string" ;
int str-length = strlen (s) ;
for (i = 0 ; i < length ; i ++)
c [i] = s [length – i] ;
Print f ("% s", c) ;
```

The output of program is

(*a*) string (*b*) gstrin

(*c*) ngstri (*d*) no output is printed

45. Which are the essential prime implecant of following Boolean function ?

f(a, b, c) = a′c + ac′ + b′c

(*a*) a′c and b′c (*b*) a′c and ac′

(*c*) ac′ and b′c (*d*) a′c and bc′

46. Consider the following code

if key < A [i]

then error "new key is smaller than A [i]"

A [i] ← key

while i > 1 and A [Parent (i)] < A [i]

do exchange A[i] ↔ A [Parent (i)]

i ← parent (i)

What is the running time of above code

(*a*) 0 (log n) (*b*) 0 (n)

(*c*) 0 (n log n) (*d*) 0 (n^2)

47. Consider the following grammar

S → A C | C B

C → a C b | ∈

A → a A | a

B → b B | b

What language is generated by given grammar?

(*a*) L = $\{a^n\, b^m \mid n, m \geq 1\}$

(*b*) L = $\{a^n\, b^m \mid n, m \geq 1, n \neq m\}$

(*c*) L = $\{a^n\, b^n \mid n, m \geq 1, n \geq m\}$

(*d*) L = $\{a^n\, b^m \mid n, m \geq 1, n \leq m\}$

48. Consider a system using paging and segmentation. The virtual address space consists of upto 8 segment and each segment is 2^{29} bytes long. The system pages each segment into 2^8 bytes per page. How many bits in to virtual address specify the segment number?

49. Consider a system using paging and segmentation. The virtual address space consists of upto 8 segment and each segment is 2^{29} bytes long. The system pages each segment into 2^8 bytes per page. How many bits in virtual address specify page number?

50. Consider the following relational database

employee (employee-name, street, city)

works (employee-name, company-name, salary)

company (company-name, city)

manage (employee-name, manager-name)

What is returned by given expression

$\pi_{\text{emp-name}}$ (employee ⋈ works ⋈ company)

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(*a*) name of all employee who live in same city as the company of which they work.

(*b*) name of all employee who live in same city

(*c*) name of all employee who works for same company

(*d*) name of all employee who live in same street.

51. Consider the following relational database

employee (employee-name, street, city)

works (employee-name, company-name, salary)

company (company-name, city)

manage (employee-name, manager-name)

What does the following expression returns :

$\pi_{\text{emp-name, street, city}}$ (σ (company-name = "ABC" ^ salary) tank 1000 works ⋈ employee)

(*a*) The name and city of residence of all employee who works for "ABC".

(*b*) Name, street and city of residence of all employee who works for ABC and earn more than 1000.

(*c*) Name, street and city of all employee of ABC.

(*d*) Name and street of employee who works for ABC.

52. Consider the following grammar production

$S \to B\ B$

$B \to a\ B \mid b$

LR (i) item [$B \to a\ .\ B$, a] is valid for which of the following viable prefix

(*a*) a a (*b*) a a a

(*c*) a a a a (*d*) B a a

53. Consider the following grammar production

$S \to B\ B$

$B \to a\ B \mid b$

LR (i) item [$B \to a\ .\ B$, \$] is valid for which of the following viable prefix.

(*a*) B a a

(*b*) a B a

(*c*) B a B

(*d*) a B

54. The process has allocated 3 page frames. Assume that none of the page of the process are available in the memory initially. The process make the following sequence of page reference :

2 3 1 2 4 6 7 3 6 5 4 3

If the FIFO page replacement policy is used how many page fault occurs for the above string?

55. The process has allocated 3 page frames. Assume that none of the page of the process are available in the memory initially. The process make the following sequence of page reference :

2 3 1 2 4 6 7 3 6 5 4 3

Least recently used (LRU) page replacement policy is a practical approximation to optional page replacement. For above sequence, how many more page fault occurs with LRU, then?

ANSWERS

General Aptitude (GA)

1. (*c*) **2.** (*d*) **3.** (*b*) **4.** (*a*) **5.** (*c*) **6.** (1936) **7.** (*a*) **8.** (*a*) **9.** (*a*) **10.** (*c*)

Technical Section

1. (*c*) **2.** (*b*) **3.** (*b*) **4.** (*c*) **5.** (*a*) **6.** (*c*) **7.** (*c*) **8.** (*a*) **9.** (*c*) **10.** $(1217)_8$

11. (*b*) **12.** (512 MB) **13.** (*c*) **14.** (*a*) **15.** (*d*) **16.** (*a*) **17.** (*b*) **18.** (3) **19.** (*b*)

20. (*c*) **21.** (*d*) **22.** (1200) **23.** (*a*) **24.** (4) **25.** (*a*) **26.** (2046) **27.** (5) **28.** (*d*) **29.** (*c*)

30. (*d*) **31.** (*a*) **32.** (*c*) **33.** (*b*) **34.** (*a*) **35.** (6) **36.** (*a*) **37.** (*b*) **38.** (*a*) **39.** (4)

40. (*d*) **41.** (*a*) **42.** (*b*) **43.** (512) **44.** (*d*) **45.** (*b*) **46.** (*a*) **47.** (*b*) **48.** (3) **49.** (21)

50. (*a*) **51.** (*b*) **52.** (*b*) **53.** (*a*) **54.** (10) **55.** (10)

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SOLVED PAPER - 2016

INSTRUCTIONS

1. Total of 65 questions carrying 100 marks, out of which 10 questions carrying a total of 15 marks are in General Aptitude (GA)
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GENERAL APTITUDE (GA)

(Q.1 – 5) : Carry One Mark Each

1. Out of the following four sentences, select the most suitable sentence with respect to grammar and usage.

(*a*) I will not leave the place until the minister does not meet me.

(*b*) I will not leave the place until the minister doesn't meet me.

(*c*) I will not leave the place until the minister meet me.

(*d*) I will not leave the place until the minister meets me.

2. A rewording of something written or spoken is a__________.

(*a*) paraphrase (*b*) paradox

(*c*) paradigm (*d*) paraffin

3. Archimedes said, "Give me a lever long enough and a fulcrum on which to place it, and I will move the world."

The sentence above is an example of a ________ statement.

(*a*) figurative (*b*) collateral

(*c*) literal (*d*) figurine

4. If 'relftaga' means carefree, 'otaga' means careful and 'fertaga' means careless, which of the following could mean 'aftercare'?

(*a*) zentaga (*b*) tagafer

(*c*) tagazen (*d*) relffer

5. A cube is built using 64 cubic blocks of side one unit. After it is built, one cubic block is removed from every corner of the cube. The resulting surface area of the body (in square units) after the removal is _______.

(*a*) 56

(*b*) 64

(*c*) 72

(*d*) 96

(Q.6 –10) : Carry Two Marks Each

6. A shaving set company sells 4 different types of razors, Elegance, Smooth, Soft and Executive. Elegance sells at Rs. 48, Smooth at Rs. 63, Soft at Rs. 78 and Executive at Rs. 173 per piece. The table below shows the numbers of each razor sold in each quarter of a year.

Quarter \product	Elegance	Smooth	Soft	Executive
Q1	27300	20009	17602	9999
Q2	25222	19392	18445	8942
Q3	28976	22429	19544	10234
Q4	21012	18229	16595	10109

Which product contributes the greatest fraction to the revenue of the company in that year?

(*a*) Elegance

(*b*) Executive

(*c*) Smooth

(*d*) Soft

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7. Indian currency notes show the denomination indicated in at least seventeen languages. If this is not an indication of the nation's diversity, nothing else is.

Which of the following can be logically inferred from the above sentences?

(*a*) India is a country of exactly seventeen languages.

(*b*) Linguistic pluralism is the only indicator of a nation's diversity.

(*c*) Indian currency notes have sufficient space for all the Indian languages.

(*d*) Linguistic pluralism is strong evidence of India's diversity.

8. Consider the following statements relating to the level of poker play of four players P, Q, R and S.

I. P always beats Q

II. R always beats S

III. S loses to P only sometimes

IV. R always loses to Q

Which of the following can be logically inferred from the above statements?

(*i*) P is likely to beat all the three other players

(*ii*) S is the absolute worst player in the set

(*a*) (*i*) only

(*b*) (*ii*) only

(*c*) (*i*) and (*ii*)

(*d*) neither (*i*) nor (*ii*)

9. If $f(x) = 2x^7 + 3x - 5$, which of the following is a factor of $f(x)$?

(*a*) $(x^3 + 8)$

(*b*) $(x - 1)$

(*c*) $(2x - 5)$

(*d*) $(x + 1)$

10. In a process, the number of cycles to failure decreases exponentially with an increase in load. At a load of 80 units, it takes 100 cycles for failure. When the load is halved, it takes 10000 cycles for failure. The load for which the failure will happen in 5000 cycles is _____.

(*a*) 40.00

(*b*) 46.02

(*c*) 60.01

(*d*) 92.02

TECHNICAL SECTION

(Q.1 – 25) : Carry One Mark Each

1. Let p, q, r, s represent the following propositions.

$p : x \in \{8, 9, 10, 11, 12\}$

$q : x$ is a composite number

$r : x$ is a perfect square

$s : x$ is a prime number

The integer $x \geq 2$ which satisfies $\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ is ______.

2. Let a_n be the number of n-bit strings that do NOT contain two consecutive 1s. Which one of the following is the recurrence relation for a_n?

(*a*) $a_n = a_{n-1} + 2a_{n-2}$

(*b*) $a_n = a_{n-1} + a_{n-2}$

(*c*) $a_n = 2a_{n-1} + a_{n-2}$

(*d*) $a_n = 2a_{n-1} + 2a_{n-2}$

3. $\lim\limits_{x \to 4} \dfrac{\sin(x-4)}{x-4}$ = ______.

4. A probability density function on the interval [a, 1] is given by $1/x^2$ and outside this interval the value of the function is zero. The value of a is ______ .

5. Two eigenvalues of a 3 × 3 real matrix P are $\left(2+\sqrt{-1}\right)$ and 3. The determinant of P is______.

6. Consider the Boolean operator # with the following properties:

$x\#0 = x$, $x\#1 = \bar{x}$, $x\#x = 0$ and $x\#\bar{x} = 1$, Then $x\#y$ is equivalent to

(*a*) $x\bar{y} + \bar{x}y$ (*b*) $x\bar{y} + \bar{x}\,\bar{y}$

(*c*) $\bar{x}y + xy$ (*d*) $xy + \bar{x}\,\bar{y}$

7. The 16-bit 2's complement representation of an integer is 1111 1111 1111 0101; its decimal representation is ______.

8. We want to design a synchronous counter that counts the sequence 0-1-0-2-0-3 and then repeats. The minimum number of J-K flip-flops required to implement this counter is ________.

9. A processor can support a maximum memory of 4 GB, where the memory is word-addressable (a word consists of two bytes) . The size of the address bus of the processor is at least ________ bits.

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10. A queue is implemented using an array such that ENQUEUE and DEQUEUE operations are performed efficiently. Which one of the following statements is **CORRECT** (n refers to the number of items in the queue) ?

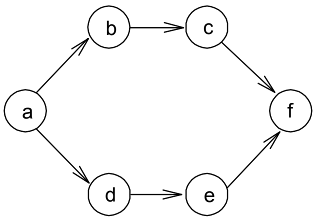
(a) Both operations can be performed in O(1) time

(b) At most one operation can be performed in O(1) time but the worst case time for the other operation will be $\Omega(n)$

(c) The worst case time complexity for both operations will be $\Omega(n)$

(d) Worst case time complexity for both operations will be $\Omega(\log n)$

11. Consider the following directed graph:



The number of different topological orderings of the vertices of the graph is _________.

12. Consider the following C program.

```
void f(int,short);
void main()
{
    int i = 100;
    short s = 12;
    short *p = &s;
    __________ ;   // call to f ()
}
```

Which one of the following expressions, when placed in the blank above, will NOT result in a type checking error?

(a) f (s,*s)

(b) i = f (i, s)

(c) f (i, *s)

(d) f (i, *p)

13. The worst case running times of *Insertion sort, Merge sort and Quick sort*, respectively, are:

(a) $\Theta(n \log n)$, $\Theta(n \log n)$, and $\Theta(n^2)$

(b) $\Theta(n^2)$, $\Theta(n^2)$, and $\Theta(n \log n)$

(c) $\Theta(n^2)$, $\Theta(n \log n)$, and $\Theta(n \log n)$

(d) $\Theta(n^2)$, $\Theta(n \log n)$, and $\Theta(n^2)$

14. Let G be a weighted connected undirected graph with distinct positive edge weights. If every edge weight is increased by the same value, then which of the following statements is/are **TRUE**?

P : Minimum spanning tree of G does not change

Q : Shortest path between any pair of vertices does not change

(a) P only (b) Q only

(c) Neither P nor Q (d) Both P and Q

15. Consider the following C program.

```
#include<stdio.h>
void mystery (int *ptra, int *ptrb) {
    int *temp;
    temp = ptrb;
    ptrb = ptra;
    ptra = temp;
}
int main () {
    int a=2016, b=0, c=4, d=42;
    mystery (&a, &b) ;
    if (a < c)
        mystery (&c, &a) ;
    mystery (&a, &d) ;
    printf("%d\n", a);
}
```

The output of the program is ______ .

16. Which of the following languages is generated by the given grammar?

$$S \to aS \mid bS \mid \varepsilon$$

(a) $\{a^n b^m \mid n, m \geq 0\}$

(b) $\{w \in \{a, b\}^* \mid w$ has equal number of a's and b's $\}$

(c) $\{a^n \mid n \geq 0\} \cup \{b^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$

(d) $\{a, b\}^*$

17. Which of the following decision problems are undecidable?

I. GivenNFAs N_1 and N_2, is $L(N_1) \cap L(N_2) = \Phi$?

II. Given a CFG $G = (N, \Sigma, P, S)$ and a string $x \in \Sigma^*$, does $x \in L(G)$?

III. Given CFGs G_1 and G_2, is $L(G_1) = L(G_2)$?

IV. Given a TM M, is $L(M) = \Phi$?

(a) I and IV only (b) II and III only

(c) III and IV only (d) II and IV only

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18. Which one of the following regular expressions represents the language: *the set of all binary strings having two consecutive 0s and two consecutive* 1*s*?

(*a*) $(0 + 1)^*0011\,(0 + 1)^* + (0 + 1)^*1100\,(0 + 1)^*$

(*b*) $(0 + 1)^*(00\,(0 + 1)^*\,11 + 11(0 + 1)^*00)\,(0 + 1)^*$

(*c*) $(0 + 1)^*00\,(0 + 1)^* + (0 + 1)^*\,11\,(0 + 1)^*$

(*d*) $00(0 + 1)^*\,11 + 11(0 + 1)^*00$

19. Consider the following code segment.

```
x = u – t;
y = x * v;
x = y + w;
y = t – z;
y = x * y;
```

The minimum number of *total* variables required to convert the above code segment to *static single assignment* form is _______ .

20. Consider an arbitrary set of CPU-bound processes with unequal CPU burst lengths submitted at the same time to a computer system. Which one of the following process scheduling algorithms would minimize the average waiting time in the ready queue?

(*a*) Shortest remaining time first

(*b*) Round-robin with time quantum less than the shortest CPU burst

(*c*) Uniform random

(*d*) Highest priority first with priority proportional to CPU burst length

21. Which of the following is **NOT** a superkey in a relational schema with attributes V, W, X, Y, Z and primary key V Y?

(*a*) VXYZ

(*b*) VWXZ

(*c*) VWXY

(*d*) VWXYZ

22. Which one of the following is **NOT** a part of the ACID properties of database transactions?

(*a*) Atomicity

(*b*) Consistency

(*c*) Isolation

(*d*) Deadlock-freedom

23. A database of research articles in a journal uses the following schema.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, YEAR, PRICE)

The primary key is (VOLUME, NUMBER, STARTPAGE, ENDPAGE) and the following functional dependencies exist in the schema.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE) → TITLE

(VOLUME, NUMBER) → YEAR

(VOLUME, NUMBER, STARTPAGE, ENDPAGE) → PRICE

The database is redesigned to use the following schemas.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, PRICE)

(VOLUME, NUMBER, YEAR)

Which is the weakest normal form that the new database satisfies, but the old one does not?

(*a*) 1NF (*b*) 2NF

(*c*) 3NF (*d*) BCNF

24. Which one of the following protocols is **NOT** used to resolve one form of address to another one?

(*a*) DNS (*b*) ARP

(*c*) DHCP (*d*) RARP

25. Which of the following is/are example (s) of stateful application layer protocols?

(*i*) HTTP (*ii*) FTP

(*iii*) TCP (*iv*) POP3

(*a*) (*i*) and (*ii*) only (*b*) (*ii*) and (*iii*) only

(*c*) (*ii*) and (*iv*) only (*d*) (*iv*) only

(Q.26 – 55) : Carry Two Marks Each

26. The coefficient of x^{12} in $(x^3 + x^4 + x^5 + x^6 + \ldots)^3$ is ______.

27. Consider the recurrence relation $a_1 = 8$, $a_n = 6n^2 + 2n + a_{n-1}$. Let $a_{99} = K \times 10^4$. The value of K is ______.

28. A function $f : \mathbb{N}^+ \to \mathbb{N}^+$, defined on the set of positive integers $\mathbb{N}^+$, satisfies the following properties:

$f(n) = f(n / 2)$ **if** n is even

$f(n) = f(n + 5)$ **if** n is odd

Let $R = \{i \mid \exists j : f(j) = i\}$ be the set of distinct values that f takes. The maximum possible size of R is ______.

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29. Consider the following experiment.

Step 1. Flip a fair coin twice.

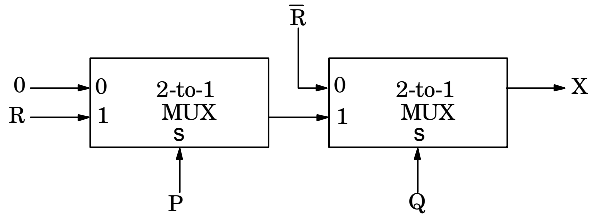
Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.

Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS) , then output N and stop.

Step 4. If the outcomes are (TAILS, TAILS) , then go to Step 1.

The probability that the output of the experiment is Y is (up to two decimal places) ________.

30. Consider the two cascaded 2-to-l multiplexers as shown in the figure.



The minimal sum of products form of the output X is

(*a*) $\bar{P}\bar{Q} + PQR$ (*b*) $\bar{P}Q + QR$

(*c*) $PQ + \bar{P}\bar{Q}R$ (*d*) $\bar{Q}R + PQR$

31. The size of the data count register of a DMA controller is 16 bits. The processor needs to transfer a file of 29,154 kilobytes from disk to main memory. The memory is byte addressable. The minimum number of times the DMA controller needs to get the control of the system bus from the processor to transfer the file from the disk to main memory is ______.

32. The stage delays in a 4-stage pipeline are 800, 500, 400 and 300 picoseconds. The first stage (with delay 800 picoseconds) is replaced with a functionally equivalent design involving two stages with respective delays 600 and 350 picoseconds. The throughput increase of the pipeline is ______ percent.

33. Consider a carry lookahead adder for adding two n-bit integers, built using gates of fan-in at most two. The time to perform addition using this adder is

(*a*) $\Theta(1)$

(*b*) $\Theta(\log(n))$

(*c*) $\Theta(\sqrt{n})$

(*d*) $\Theta(n)$

34. The following function computes the maximum value contained in an integer array p [ ] of size n (n >= 1) .

```
int max (int *p, int n) {
int a=0, b=n-1;
while (_______) {
      if (p[a] <= p[b])    { a = a+1; }
      else                 {b = b –1; }
}
return p[a];
}
```

The missing loop condition is

(*a*) a ! = n (*b*) b ! = 0

(*c*) b > (a + 1) (*d*) b ! = a

35. What will be the output of the following C program?

```
void count (int n) {
   static int d=1;
   printf (“%d” n) ;
   printf (“%d” d) ;
   d++;
   if (n>l) count (n-1) ;
   printf (“%d” d) ;
}
void main () {
   count (3) ;
}
```

(*a*) 3 1 2 2 1 3 4 4 4 (*b*) 3 1 2 1 1 1 2 2 2

(*c*) 3 1 2 2 1 3 4 (*d*) 3 1 2 1 1 1 2

36. What will be the output of the following pseudo-code when parameters are passed by reference and dynamic scoping is assumed?

```
a=3;
void n (x) {x = x * a; print (x) ;}
void m (y) {a = 1; a = y - a; n (a) ; print (a) ;}
void main () {m (a) ; }
```

(*a*) 6, 2

(*b*) 6, 6

(*c*) 4, 2

(*d*) 4, 4

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37. An operator delete (i) for a binary heap data structure is to be designed to delete the item in the i-th node. Assume that the heap is implemented in an array and i refers to the i-th index of the array. If the heap tree has depth d (number of edges on the path from the root to the farthest leaf) , then what is the time complexity to re-fix the heap efficiently after the removal of the element?

(a) O(1)

(b) O(d) but not O(1)

(c) O(2^d) but not O(d)

(d) O($d2^d$) but not O(2^d)

38. Consider the weighted undirected graph with 4 vertices, where the weight of edge $\{i, j\}$ is given by the entry W_{ij} in the matrix W.

$$W = \begin{bmatrix} 0 & 2 & 8 & 5 \\ 2 & 0 & 5 & 8 \\ 8 & 5 & 0 & x \\ 5 & 8 & x & 0 \end{bmatrix}$$

The largest possible integer value of x, for which at least one shortest path between some pair of vertices will contain the edge with weight x is ______.

39. Let G be a complete undirected graph on 4 vertices, having 6 edges with weights being 1, 2, 3, 4, 5, and 6. The maximum possible weight that a minimum weight spanning tree of G can have is ______.

40. G = (V, E) is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTs) of G is/are **TRUE**?

I. If e is the lightest edge of some cycle in G, then every MST of G includes e

II. If e is the heaviest edge of some cycle in G, then every MST of G excludes e

(a) I only

(b) II only

(c) both I and II

(d) neither I nor II

41. Let Q denote a queue containing sixteen numbers and S be an empty stack. Head (Q) returns the element at the head of the queue Q **without** removing it from Q, Similarly Top (S) returns the element at the top of S **without** removing it from S, Consider the algorithm given below.

```
while Q is not Empty do
    if S is Empty OR Top (S) ≤ Head (Q) then
        x:=Dequeue (Q);
        Push(S; x);
    else
        x:= Pop(S);
        Enqueue (Q,x);
    end
end
```

The maximum possible number of iterations of the **while** loop in the algorithm is ________.

42. Consider the following context-free grammars:

$G_1 : S \to aS \mid , B \to b \mid bB$

$G_2 : S \to aA \mid bB, A \to aA \mid B \mid \varepsilon, B \to bB \mid \varepsilon$

Which one of the following pairs of languages is generated by G_1 and G_2, respectively?

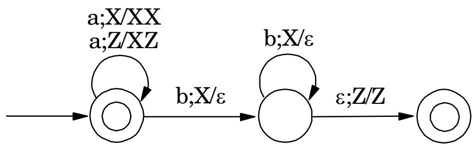
(a) $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$

(b) $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n \geq 0\}$

(c) $\{a^m b^n \mid m \geq 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$

(d) $\{a^m b^n \mid m \geq 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$

43. Consider the transition diagram of a PDA given below with input alphabet $\Sigma = \{a,b\}$ and stack alphabet $\Gamma = \{X,Z\}$. Z is the initial stack symbol. Let L denote the language accepted by the PDA.



Which one of the following is TRUE?

(a) $L = \{a^n b^n \mid n \geq 0\}$ and is not accepted by any finite automata

(b) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is not accepted by any deterministic PDA

(c) L is not accepted by any Turing machine that halts on every input

(d) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is deterministic context-free

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44. Let X be a recursive language and Y be a recursively enumerable but not recursive language. Let W and Z be two languages such that $\overline{Y}$ reduces to W, and Z reduces to $\overline{X}$ (reduction means the standard many-one reduction). Which one of the following statements is **TRUE**?

(*a*) W can be recursively enumerable and Z is recursive.

(*b*) W can be recursive and Z is recursively enumerable.

(*c*) W is not recursively enumerable and Z is recursive.

(*d*) W is not recursively enumerable and Z is not recursive.

45. The attributes of three arithmetic operators in some programming language are given below.

Operator	Precedence	Associativity	Arity
+	High	Left	Binary
—	Medium	Right	Binary
*	Low	Left	Binary

The value of the expression **2 – 5 + 1 – 7*3** in this language is ______ .

46. Consider the following Syntax Directed Translation Scheme (SDTS), with non-terminals {S, A} and terminals **{a, b}**.

S → **a**A { print 1 }

S → **a** { print 2 }

A → S**b** { print 3 }

Using the above SDTS, the output printed by a bottom-up parser, for the input **aab** is:

(*a*) 1 3 2 (*b*) 2 2 3

(*c*) 2 3 1 (*d*) syntax error

47. Consider a computer system with 40-bit virtual addressing and page size of sixteen kilobytes. If the computer system has a one-level page table per process and each page table entry requires 48 bits, then the size of the per-process page table is ______ megabytes.

48. Consider a disk queue with requests for I/O to blocks on cylinders 47, 38, 121, 191, 87, 11, 92, 10. The C-LOOK scheduling algorithm is used. The head is initially at cylinder number 63, moving towards larger cylinder numbers on its servicing pass. The cylinders are numbered from 0 to 199. The total head movement (in number of cylinders) incurred while servicing these requests is ______ .

49. Consider a computer system with ten physical page frames. The system is provided with an access sequence $(a_1, a_2, ..., a_{20}, a_1, a_2, ..., a_{20})$, where each a_i is a distinct virtual page number. The difference in the number of page faults between the last-in-first-out page replacement policy and the optimal page replacement policy is ______ .

50. Consider the following proposed solution for the critical section problem. There are n processes: P_0 ... P_{n-1}, In the code, function pmax returns an integer not smaller than any of its arguments. For all i, t [i] is initialized to zero,

Code for P_i:

```
do {
   c[i]=1; t[i] = pmax (t [0] ,...,t [n-1] ) + 1; c[i]=0;
   for every j ≠ i in {0,..., n – 1}  {
      while (c[j]) ;
      while (t [ j] != 0 && t[j] <= t[i]) ;
   }
   Critical Section ;
   t[i] = 0;
   Remainder Section;
} while (true) ;
```

Which one of the following is **TRUE** about the above solution?

(*a*) At most one process can be in the critical section at any time

(*b*) The bounded wait condition is satisfied

(*c*) The progress condition is satisfied

(*d*) It cannot cause a deadlock

51. Consider the following two phase locking protocol. Suppose a transaction T accesses (for read or write operations), a certain set of objects $\{O_1,..., O_k\}$. This is done in the following manner:

Step 1. T acquires exclusive locks to O_1, ..., O_k in increasing order of their addresses.

Step 2. The required operations are performed.

Step 3. All locks are released.

This protocol will

(*a*) guarantee serializability and deadlock-freedom

(*b*) guarantee neither serializability nor deadlock-freedom

(*c*) guarantee serializability but not deadlock-freedom

(*d*) guarantee deadlock-freedom but not serializability

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52. Consider that B wants to send a message m that is digitally signed to A. Let the pair of private and public keys for A and B be denoted by K_x^- and K_x^+ for x = A, B, respectively. Let $K_x(m)$ represent the operation of encrypting m with a key K_x and H(m) represent the message digest. Which one of the following indicates the **CORRECT** way of sending the message m along with the digital signature to A?

(a) $\{m, K_B^+(H(m))\}$ (b) $\{m, K_B^-(H(m))\}$

(c) $\{m, K_A^-(H(m))\}$ (d) $\{m, K_A^+(H(m))\}$

53. An IP datagram of size 1000 bytes arrives at a router. The router has to forward this packet on a link whose MTU (maximum transmission unit) is 100 bytes. Assume that the size of the IP header is 20 bytes.

The number of fragments that the IP datagram will be divided into for transmission is _______.

54. For a host machine that uses the token bucket algorithm for congestion control, the token bucket has a capacity of 1 megabyte and the maximum output rate is 20 megabytes per second. Tokens arrive at a rate to sustain output at a rate of 10 megabytes per second. The token bucket is currently full and the machine needs to send 12 megabytes of data. The minimum time required to transmit the data is ______ seconds.

55. A sender uses the Stop-and-Wait ARQ protocol for reliable transmission of frames. Frames are of size 1000 bytes and the transmission rate at the sender is 80 Kbps (1Kbps = 1000 bits/second). Size of an acknowledgement is 100 bytes and the transmission rate at the receiver is 8 Kbps. The one-way propagation delay is 100 milliseconds.

Assuming no frame is lost, the sender throughput is ______ bytes/second.

ANSWERS

General Aptitude (GA)

1. (d) **2.** (a) **3.** (a) **4.** (c) **5.** (d) **6.** (b) **7.** (d) **8.** (a) **9.** (b) **10.** (b)

Technical Section

1. (11) **2.** (b) **3.** (1) **4.** (0.5) **5.** (15) **6.** (a) **7.** (–11) **8.** (4) **9.** (31) **10.** (a)
11. (6) **12.** (d) **13.** (d) **14.** (d) **15.** (2016) **16.** (d) **17.** (c) **18.** (b) **19.** (7) **20.** (a)
21. (b) **22.** (d) **23.** (2NF) **24.** (a) **25.** (c) **26.** (10) **27.** (198) **28.** (3) **29.** (0.33) **30.** (d)
31. (456) **32.** (33.28) **33.** (b) **34.** (d) **35.** (a) **36.** (c) **37.** (b) **38.** (11) **39.** (7) **40.** (c)
41. (256) **42.** (d) **43.** (d) **44.** (c) **45.** (9) **46.** (c) **47.** (384 MB) **48.** (346) **49.** (1)
50. (a) **51.** (a) **52.** (b) **53.** (13) **54.** (1.1) **55.** (2500)

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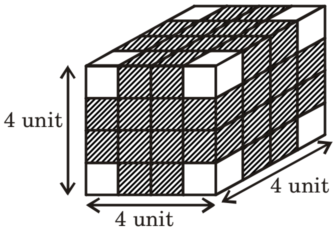
EXPLANATIONS

General Aptitude (GA)

1. I will not leave the place until the minister meets me.
2. Rewording of something written or spoken is PARAPHRASE.
3. Figurative statement
4. relftaga $\longrightarrow$ carefree
   otaga $\longrightarrow$ careful
   fertaga $\longrightarrow$ careless
   taga $\longrightarrow$ care
   aftercare $\longrightarrow$ taga____.
   Option (*c*) matches i.e. tagazen.
5. 

   Required Surface Area = $(6 \times 4) + (8 \times 6) + (3 \times 8)$
   $= 24 + 48 + 24$
   $= 96$ units.
6. Elegance $\Rightarrow (27300 + 25222 + 28976 + 21012) \times 48$
   Executive $\Rightarrow (9999 + 8942 + 10234 + 10109) \times 173$
   Smooth $\Rightarrow (20009 + 19392 + 22429 + 18229) \times 63$
   Soft $\Rightarrow (17602 + 18445 + 19544 + 16595) \times 78$
   Executive has the highest revenue.
8. (*i*) is followed.
   (*ii*) is not followed.
9. $f(x) = 2x^7 + 3x - 5$
   Putting $x = 1$
   $f(1) = 2 + 3 - 5 = 0$
   So, $(x - 1)$ is the factor of $f(x)$.

Technical Section

1. $p : x \in \{8, 9, 10, 11, 12\}$
   q : x is a composite number.
   r : x is a perfect square.
   s : x is a prime number.
   $\{x \geq 2\}$
   $\neg((p \Rightarrow q) \wedge (\neg r \text{ v } \neg s))$
   $\Rightarrow \neg((\neg p \vee q) \wedge (\neg r \vee \neg s))$
   $\Rightarrow ((p' + q) \,.\, (r' + s'))'$
   $\Rightarrow (p' + q)' + (r' + s')'$
   $\Rightarrow pq' + rs$
   $\Rightarrow ps + rs$ $\quad q \rightarrow$ Composite Number
   $\Rightarrow \underbrace{(p + r)s}_{\downarrow\; 11}$ $\quad q' \rightarrow$ Prime Number
   So $q' = s$
   The prime number from set x is 11
2. Let n length of string
   $$T(n) = a_n$$
   $$\underset{T(n-2)}{10\ldots\ldots\ldots\ldots} \quad \text{or} \quad \underset{T(n-1)}{0\ldots\ldots\ldots\ldots}$$
   $$T(n) = T(n-2) + T(n-1)$$
   $$= a_{n-2} + a_{n-1}$$
3. $$\lim_{x \to 4} \frac{\sin(x-4)}{(x-4)} \qquad \left(\frac{0}{0}\text{form}\right)$$
   Apply L' Hospital Rule :
   $$\lim_{x \to 4} \left[\frac{\cos(x-4)}{1}\right]$$
   $$\Rightarrow \frac{\cos 0}{1} \Rightarrow 1.$$
4. Total probability = 1
   So $$\int_a^1 \frac{1}{x^2} = 1$$
   $$\left[-\frac{1}{x}\right]_a^1 = 1$$
   $$\frac{1}{a} = 2$$
   $$a = \frac{1}{2} = 0.5$$
5. $(2 + i)$ is an eigenvalue
   So $(2 - i)$ will be other.
   So Determinant will be product of eigenvalues.
   So $\quad |A| = (2 + i)(2 - i) \times 3$
   $(2^2 - i^2) \times 3 = 15$
6. $x \# 0 = x, \; x \# 1 = \bar{x}, \; x \# x = 0, \; x \# \bar{x} = 1.$
   $x \# y = ?$

x	y	x'	y'	xy	xy'	$x'y$	$x'y'$	$xy + \bar{x}\,\bar{y}$
0	0	1	1	0	0	0	1	1
0	1	1	0	0	0	1	0	0
1	0	0	1	0	1	0	0	1
1	1	0	0	1	0	0	0	0

Case $\rightarrow$

$x\bar{y} + \bar{x}y$
0
1
1
0

This satisfy the above 4 conditions.

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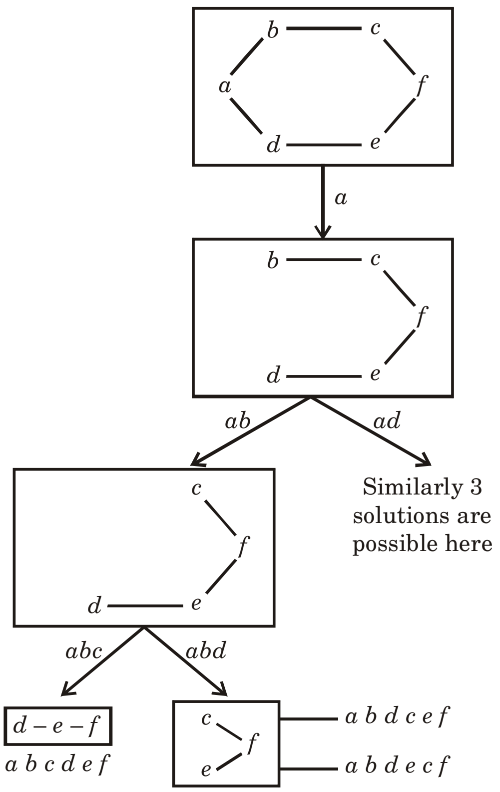
7. 1111 1111 1111 0101
-1
1111 1111 1111 0100
$\Downarrow$
1111 1111 1111 1011
-11

9. $\dfrac{2^{30+2}}{2} = 2^{31}$

So 31 bits required.

10. Both ENQUEUE and DEQUEUE operations can be performed in O(1) time.

11.



So total solution 6.

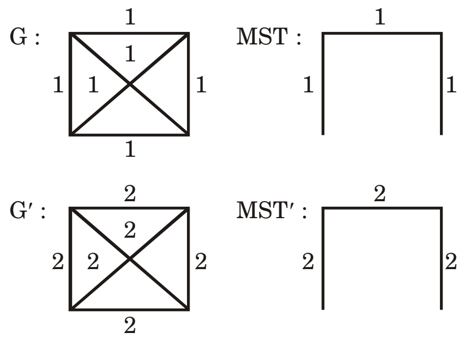
12. $f(i, * p)$

```
void f(int, short);
    void main( )
{
    int i = 100;
    short s = 12;
    short * p = &s;
    f(i, * p);
}
```

13.

		Worst Case
Insertion sort	$\rightarrow$	$O(n^2)$
Merge sort	$\rightarrow$	$O(n \log n)$
Quick sort	$\rightarrow$	$O(n^2)$

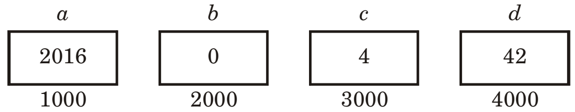
14. Example :



- Minimum spanning tree of G does not change.
- Shortest path between any pair of vertices does not change.

15.

```
int main ( )
{
    int a = 2016, b = 0, c = 4, d = 42;
```




```
    mystery (&a, &b);
    if (a < c)
    mystery (&c, &a);
    mystery (&a, &d);
    printf("%d \ n", a);
}
void mystery (int * ptra, int * ptrb)
{
    int * temp;
    temp = ptrb;
    ptrb = ptra;
    ptra = temp;
}
```

Output $\rightarrow$ 2016.

16. $S \rightarrow aS \mid bS \mid \varepsilon$

$S \rightarrow \varepsilon$, $S \rightarrow a$, $S \rightarrow b$, $S \rightarrow aa$

$S \rightarrow ab$, $S \rightarrow ba$, $S \rightarrow bb$ ......

Hence $\{a, b\}^*$.

17. $L[N_1] \cap L[N_2] \neq \phi$ – undecidable

$x \in L(G)$ – decidable

$L(G_1) = L(G_2)$ – decidable

$L[M] = \phi$ is also undecidable.

18. $(0 + 1)^* \, (00(0 + 1)^* \, 11 + 11(0 + 1)^*00) \, (0 + 1)^*$

20. Shortest remaining time first minimizes the average waiting time in the ready queue.

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21. Super key

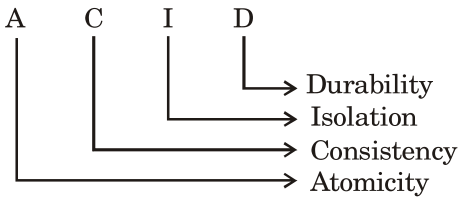
= Candidate key + Other Attribute of set

= Primary key + Other Attribute of set.

Given, Primary key = VY

So, VXYZ, VWXY, VWXYZ are super keys except VWXZ.

22.



23. It is 2NF because initially it contains partial dependency.

24. In DNS (mapping of IP address & URL)

IP address	URL
197.12.0.7	www.xyz.com

25. FTP and POP3 are non-HTTP protocols

These are stateful application layer protocols.

26. $(x^3 + x^4 + x^5 + x^6 \ldots\ldots)^3$

$$\Rightarrow \left(\frac{x}{1-x} - x - x^2\right)^3$$

$$\Rightarrow \left(\frac{x - x(1-x) - x^2(1-x)}{(1-x)}\right)^3$$

$$\Rightarrow \left(\frac{x - x - x^2 + x^2 + x^3}{(1-x)}\right)^3$$

$$\Rightarrow \left(\frac{x^3}{1-x}\right)^3$$

$\Rightarrow$ Coefficient of x^{12} in $\left(\frac{x^3}{1-x}\right)^3$

$\Rightarrow$ Coefficient of x^3 in $\left(\frac{1}{1-x}\right)^3$

$\Rightarrow$ Coefficient of x^3 in $(1-x)^{-3}$

$$f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0)$$

$f(x) = (1-x)^{-3}$

$f'(x) = +\,3(1-x)^{-4}$

$f''(x) = +\,12(1-x)^{-5}$

$f'''(x) = +\,60(1-x)^{-6}$

Coefficient of x^3 in $(1-x)^{-3} = \frac{60}{3!} = 10$

27. $a_n = 6n^2 + 2n + a_{n-1}$, $a_1 = 8$.

$a_2 = 6 \times 2^2 + 2 \times 2 + a_1$

$a_3 = 6 \times 3^2 + 2 \times 3 + a_2$

$= 6 \times 3^2 + 2 \times 3 + 6 \times 2^2 + 2 \times 2 + a_1$

$\vdots$

$a_{99} = 6 \times 99^2 + 2 \times 99 + 6 \times 98^2 + 2 \times 98 + \ldots. + 6 \times 2^2 + 2 \times 2 + 8$

$= 6[1^2 + 2^2 + 3^2 + \ldots. + 99^2] + 2[1 + 2 + \ldots 99]$

$$= 6.\frac{(99(99+1)(2\times 99+1))}{6} + 2\frac{(99(99+1))}{2}$$

$= 99 \times 100 \times 199 + 99 \times 100$

$= 100 \times 99\ (199 + 1)$

$= 2 \times 99 \times 10^4 = \underbrace{\boxed{198}}_{\text{value of K}} \times 10^4$

28. Maximum number of distinct values f takes is only 3.

$1^{st} \Rightarrow f(1) = f(2) = f(3) = f(4) = f(8)$

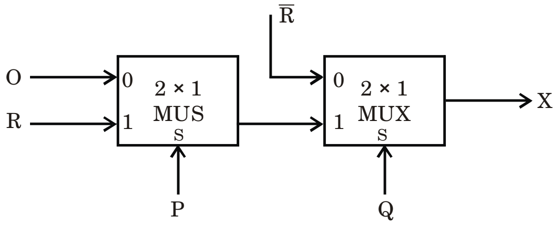
$2^{nd} \Rightarrow f(5) = f(10)$

$3^{rd} \Rightarrow f(6) = f(7) = f(9)$

Remaining all other n values will provide only one of these three values.

29. Probability $= \frac{1}{3} = 0.33$.

30.



P′ then 0 $\Rightarrow$ P′ 0 = 0

P then R $\Rightarrow$ PR

Q′ then R $\Rightarrow$ Q′R

Q then (0 + PR) $\Rightarrow$ Q(0 + PR) = QPR.

X = Q′R + PQR

31. Data Count Register of DMA controller = 16-bits

Count Value = 2^{16} = 64 KB

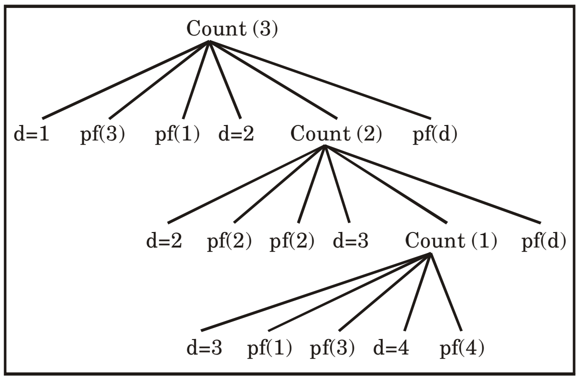
Number of controls to transfer = 29154 K

456 times System Bus control is needed.

33. Time to perform addition using this adder is $O(\log n)$.

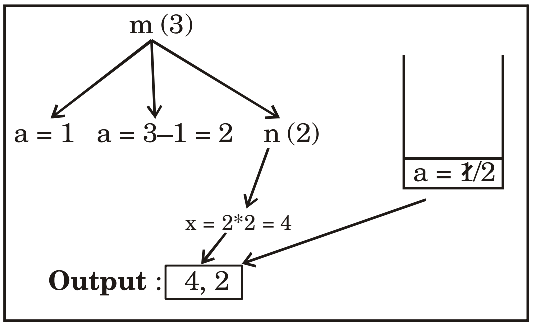
34. b ! = a

35.



Output : 3 1 2 2 1 3 4 4 4

36.



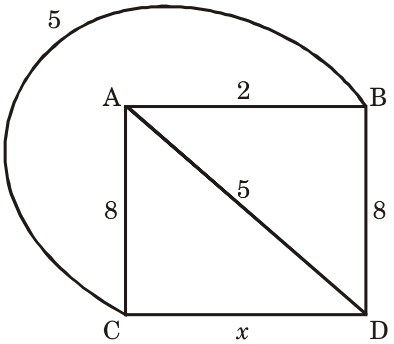
37. In a Binary heap

The re-fix takes log$n$ time

Here d is depth

Hence O(d).

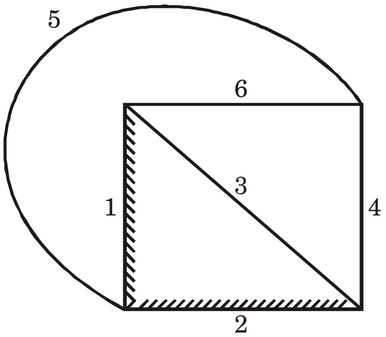
38.



Smallest path between C & D is 12.

So maximum possible value of x is 11.

39.



Here 3 can't be taken because it will produce a cycle so take 4.

This solution is 1 + 2 + 4 = 7

40. Both I and II

41. for n = 16.

Algorithm runs while loop 16 × 16 = 256.

42. a can be 0.

b can't be 0.

43. String will be of form $a^n b^n$

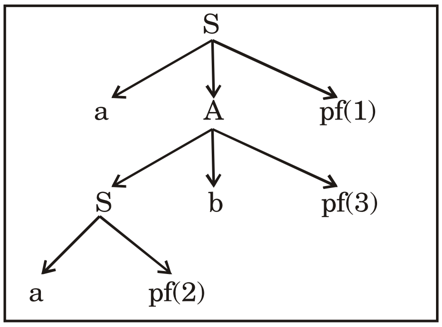
45. 2 – (5 + 1) – 7 * 3

2 – (6 – 7) * 3

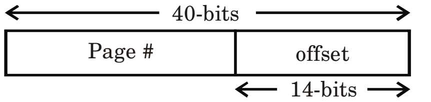
2 – (–1) * 3

3 * 3 = 9

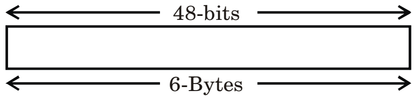
46.



47. V.A. :



P.T.E. :



P.T. size = #entries in P.T. * P.T.E. size

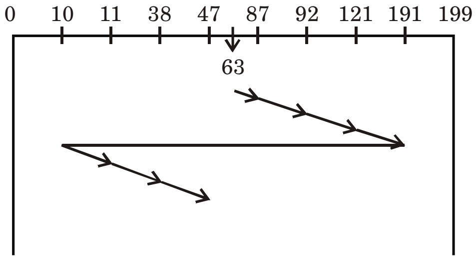
$= \frac{2^{40}}{2^{14}} * 6$

$= 2^{26} * 6$

= 64 MB × 6

= 384 MB

48.



Total head movement

= (191 – 63) + (191 – 10) + (47 – 10)

= 128 + 181 + 37

= 346

49. Frame size = 10

Sequence $\{a_1, a_2, ..., a_{20}, a_1, a_2, ..., a_{20}\}$

⇒ Find difference of page fault of LIFO & optimal page replacement policy.

Consider a small case :

Frame size = 2

Sequence = $a_1 a_2 a_3 a_4 a_1 a_2 a_3 a_4$

LIFO :

a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
	$\cancel{a_2}$	$\cancel{a_3}$	a_4	$\cancel{a_4}$	$\cancel{a_2}$	$\cancel{a_3}$	a_4
a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1
F	F	F	F	H	F	F	F

⇒ 7 faults

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Optional :

a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
	$\cancel{a_2}$	$\cancel{a_3}$	a_4	a_4	a_4	a_4	a_4
a_1	a_1	a_1	a_1	$\cancel{a_1}$	$\cancel{a_2}$	a_3	a_3
F	F	F	F	H	F	F	H

$\Rightarrow$ 6 faults

Difference = LIFO page faults – Optimal page faults

= 7 – 6

= 1

Similarly for :

Frame size = 10

Sequence = $a_1\ a_2\ a_3\ a_4\ ......\ a_{20}\ a_1\ a_2\ a_3\ ...\ a_{20}$

Difference = LIFO page faults – Optimal page faults

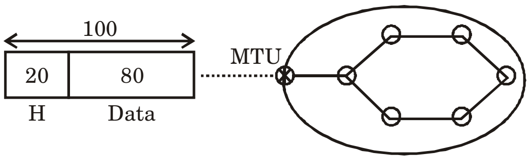
= 31 – 30

= 1

53. IP datagram = 1000 bytes.

MTU = 100 bytes

IP header = 20 bytes.



Number of fragments $\Rightarrow \left\lceil \frac{1000}{80} \right\rceil \Rightarrow [12.4] \Rightarrow 13$

54. C + ps = mS

$$S = \frac{1MB}{(20-10)MBPs} = 0.1 \text{ sec.}$$

Data transmission in 0.1 sec.

= 0.1 × output data rate

= 2 MB.

Data Remaining = (12 MB – 2 MB) = 10 MB.

To transmit 1 MB $\xrightarrow{\text{takes}}$ 0.1 sec.

10 MB $\longrightarrow 10 \times 0.1$ sec.

Total time = 1 + 0.1 = 1.1 sec.

55.
$$\text{Throughput} = \frac{\text{output}}{\text{time}} = \frac{L}{T_{data} + T_{ack} + 2P_d}$$

$$T_d = \frac{1000 \times 8}{80 \times 1000} = \frac{1}{10} \text{ sec} = 100 \text{ ms}$$

$$T_{ack} = \frac{100 \times 8}{8 \times 1000}$$

$$= \frac{1}{10} \text{ sec} = 100 \text{ ms}$$

$$= \frac{1000}{100 + 100 + 2 \times 100} = 2500$$

■■

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SOLVED PAPER - 2017

INSTRUCTIONS

1. Total of 65 questions carrying 100 marks, out of which 10 questions carrying a total of 15 marks are in General Aptitude (GA)
2. The Engineering Mathematics will carry around **15% of the total marks,** the General Aptitude section will carry **15% of the total marks** and the **remaining 70% of the total marks.**
3. **Types of Questions**
   (*a*) Multiple Choice Questions (MCQ) carrying 1 or 2 marks each in all papers and sections. These questions are objective in nature, and each will have a choice of four options, out of which the candidate has to mark the correct answer(s).
   (*b*) Numerical Answer Questions of 1 or 2 marks each in all papers and sections. For these questions the answer is a real number, to be entered by the candidate using the virtual keypad. No choices will be shown for these type of questions.
4. For **1-mark** multiple-choice questions, **1/3 marks** will be deducted for a wrong answer. Likewise, for **2-marks** multiple-choice questions, **2/3** marks will be deducted for a wrong answer. There is no negative marking for numerical answer type questions.

Chapter-Wise Analysis

#	Chapters	Marks		
		1	2	Total
1	General Aptitude	5	5	15
2	Engineering Mathematics	5	6	17
3	Digital Logic	3	2	7
4	Computer Organization and Architecture	1	3	7
5	Data Structures and Algorithms	6	6	18

#	Chapters	Marks		
		1	2	Total
6	Theory of Computation	3	3	9
7	Compiler Design	2	2	6
8	Operating System	2	3	8
9	Databases	2	3	8
10	Computer Networks	1	2	5

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TECHNICAL SECTION

(Q.1 – 25) : Carry One Mark Each

1. The representation of the value of a 16-bit unsigned integer X in hexadecimal number system is BCA9. The representation of the value of X in octal number system is

(*a*) 571244 (*b*) 736251
(*c*) 571247 (*d*) 136251

2. In a file allocation system, which of the following allocation scheme(*s*) can be used if no external fragmentation is allowed?

I. Contiguous
II. Linked
III. Indexed

(*a*) I and III only
(*b*) II only
(*c*) III only
(*d*) II and III only

3. A circular queue has been implemented using a singly linked list where each node consists of a value and a single pointer pointing to the next node. We maintain exactly two external pointers FRONT and REAR pointing to the front node and the rear node of the queue, respectively. Which of the following statements is/are CORRECT for such a circular queue, so that insertion and deletion operation can be performed in O(1) time?

I. Next pointer of front node points to the rear node.
II. Next pointer of rear node points to the front node.

(*a*) I only (*b*) II only
(*c*) Both I and II (*d*) Neither I nor II

4. Identify the language generated by the following grammar, where S is the start variable.

$S \rightarrow XY$
$X \rightarrow aX \mid a$
$Y \rightarrow aYb / \in$

(*a*) $\{a^m b^n \mid m \geq n, n > 0\}$
(*b*) $\{a^m b^n \mid m \geq n, n \geq 0\}$
(*c*) $\{a^m b^n \mid m > n, n \geq 0\}$
(*d*) $\{a^m b^n \mid m > n, n > 0\}$

5. The minimum possible number of states of a determinstic finite automaton that accepts the regular language

$L = \{w_1 a\, w_2 \mid w_2, w_2 \in \{a, b\}^*, \mid w_1 \mid = 2, \mid w_2 \geq 3\}$ is ______ .

6. Match the following according to input (from the left column) to the complier phase (in the right column) that processes it.

List I	List II
(P) Syntax tree	(*i*) Code generator
(Q) Character stream	(*ii*) Syntax analyzer
(R) Intermediate representation	(*iii*) Semantic analyzer
(S) Token stream	(*iv*) Lexical analyzer

(*a*) $P \rightarrow (ii), Q \rightarrow (iii), R \rightarrow (iv), S \rightarrow (i)$
(*b*) $P \rightarrow (ii), Q \rightarrow (i), R \rightarrow (iii), S \rightarrow (iv)$
(*c*) $P \rightarrow (iii), Q \rightarrow (iv), R \rightarrow (i), S \rightarrow (ii)$
(*d*) $P \rightarrow (i), Q \rightarrow (iv), R \rightarrow (ii), S \rightarrow (iii)$

7. If $f(x) = R \sin\left(\frac{\pi x}{2}\right) + S$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)\, dx = \frac{2R}{\pi}$, then the constants R and S are. respectively.

(*a*) $\frac{2}{\pi}$ and $\frac{16}{\pi}$ (*b*) $\frac{2}{\pi}$ and 0
(*c*) $\frac{4}{\pi}$ and 0 (*d*) $\frac{4}{\pi}$ and $\frac{16}{\pi}$

8. An ER model of a database consists of entity types A and B. These are connected by a relationship R which does not have its own attribute. Under which one of the following conditions, can the relational table for R be merged with that of A?

(*a*) Relationship R is one-to-many and the participation of A in R is total.
(*b*) Relationship R is one-to-many and the participation of A in R is partial.
(*c*) Relationship R is many-to-one and the participation of A in R is partial.
(*d*) Relationship R is many-to-one and the participation of A in R is partial.

9. Given the following binary number in 32-bit (single precision) IEEE-754 format:

00111110011011010000000000000000

The decimal value closest to this floating-point number is

(*a*) 1.45×10^{1} (*b*) 1.45×10^{-1}
(*c*) 2.27×10^{-1} (*d*) 2.27×10^{1}

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10. Let L_1, L_2 be any two context-free language and R be any regular language. Then which of the following is are CORRECT?

I. $L_1 \cup L_2$ is context-free

II. L_1 is context-free

III. $L_1 - R$ is context-free

IV. $L_1 \cap L_2$ is context-free

(*a*) I, II and IV only (*b*) I and III only

(*c*) II and IV only (*d*) I only

11. Consider a quadratic equation $x^2 - 13x + 36 = 0$ with coefficients in a base b. The solution of this equation in the same base b are $x = 5$ and $x = 6$. Then b = _____ .

12. Consider the following table T_1 and T_2.

T_1

P	Q
2	2
3	8
7	3
5	8
6	9
8	5
9	8

T_2

R	S
2	2
8	3
3	2
9	7
5	7
7	2

In table T_1, P is the primary key and Q is the foregein key referencing R in table T_2 with on-delete cascade and on-update cascade. In table T_2. R is the primary key and S is the foreign key referencing P in table T_1 with on-delet set NULL and on-update cascade. In order to delete record (3, 8) from table T_1, the number of additional records that need to be deleted from table T_1 is _______ .

13. Consider socket API on a Linux machine that supports connected UDP sockets. A connected UDP socket is a UDP socket on which connect function has already been called. Which of the following statement is/are CORRECT?

I. A connected UDP socket can be used to communicate with multiple peers simultaneously.

II. A process can successfully call connect function again for an already connected UDP socket.

(*a*) I only (*b*) II only

(*c*) Both I and II (*d*) Neither I nor II

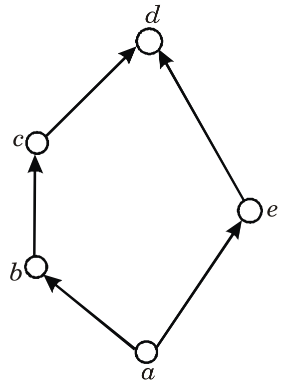
14. G is an undirected graph with n vertices and 25 edges such that each vertex of G has degree at least 3. Then the maximum possible value of n is _____ .

15. Consider the set X = {*a, b, c, d, e*} under the partial ordering

$R = \{(a, a), (a, b), (a, c), (a, d), (a, e), (b, b), (b, c), (b, e), (c, c), (c, e), (d, d), (d, e), (e, e)\}$.

The Hase diagram of the partial order (X, R) is shown below :

List II



The minimum number of ordered pairs that need to be added to R to make (X, R) a lattice is ______.

16. Which of the following statements about parser is/are CORRECT?

I. Canonical LR is more powerful than SLR.

II. SLR is more powerful than LALR.

III. SLR is more powerful than Canonical LR.

(*a*) I only (*b*) II only

(*c*) III only (*d*) II and III only

17. Let $P = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and

$Q = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$ be two matrices.

Then the rank of P + Q is ______ .

18. Match the following

List–I

(P) Static char var;

(Q) m = malloc (10); m = NULL;

(R) char *ptr [10];

(S) register int var 1;

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List–II

(*i*) Sequence of memory locations to store addresses

(*ii*) A variable located in data section of memory

(*iii*) Request to allocate a CPU register to store data

(*iv*) A lost memory which cannot be freed

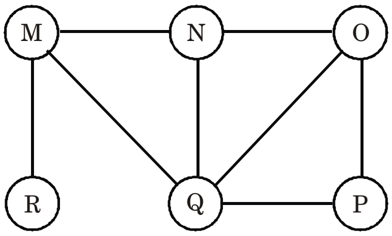
(*a*) P → (*ii*), Q → (*iv*), R → (*i*), S → (*iii*)

(*b*) P → (*ii*), Q → (*i*), R → (*iv*), S → (*iii*)

(*c*) P → (*ii*), Q → (*iv*), R → (*iii*), S → (*i*)

(*d*) P → (*iii*), Q → (*iv*), R → (*i*), S → (*ii*)

19. The Breadth First Search (BFS) algorithm has been implemented using the queue data structure. Which one of the following is a possible order of visiting the nodes in the graph below?



(*a*) MNOPQR (*b*) NQMPOR
(*c*) QMNROP (*d*) POQNMR

20. Let *p, q, r* denote the statements "*It is raining*". "*It is cold*", and "*It is pleasant*", respectively. Then the statement "*It is nor raining and it is pleasant, and it is not pleasant only if it is raining and it is cold*" is represented by

(*a*) $(\neg p \wedge r) \wedge (\neg r \to (p \wedge q))$

(*b*) $(\neg p \wedge r) \wedge ((p \wedge q) \to \neg r)$

(*c*) $(\neg p \wedge r) \vee ((p \wedge q) \to \neg r)$

(*d*) $(\neg p \wedge \mathrm{r}) \vee (r \to (p \wedge q))$

21. Consider the following function implemented in C:

```
void printxy (int x, int y)
{
   int * ptr ;
       x = 0 ;
         ptr= &x ;
          y=*ptr ;
   *ptr = 1;
   printf ("'% d, % d", x, y);
}
```

The output of invoking printexy (1, 1) is

(*a*) 0, 0 (*b*) 0, 1
(*c*) 1, 0 (*d*) 1, 1

22. Match the algorithms with their time complexities:

List I : Algorithm

(P) Towers of Hanoi with *n* disks

(Q) Binary search given *n* sorted numbers

(R) Heap sort given *n* numbers at the worst case

(S) Addition of two $n \times n$ matrices

List II : Time Complexity

(*i*) $\Theta(n^2)$

(*ii*) $\Theta(n \log n)$

(*iii*) $\Theta(2^n)$

(*iv*) $\Theta(\log n)$

(*a*) P → (*iii*), Q → (*iv*), R → (*i*), S → (*ii*)

(*b*) P → (*iv*), Q → (*iii*), R → (*i*), S → (*ii*)

(*c*) P → (*iii*), Q → (*iv*), R → (*ii*), S → (*i*)

(*d*) P → (*iv*), Q → (*iii*), R → (*ii*), S → (*i*)

23. Consider the following about the routing protocols. Routing Information Protocol (RIP) and Open Shortest Path First (OSPF) in an IPv4 network.

I. RIP uses distance vector routing

II. RIP packets are sent using UDP

III. OSPF packets are sent using TCP?

IV. OSPF operation is based on link-state routing

Which of the statement above are CORRECT?

(*a*) I and IV only

(*b*) I, II and III only

(*c*) I, II and IV only

(*d*) II, III and IV only

24. The maximum number of IPv4 router addreses that can be listed in the record route (RR) oprion field of an IPv4 header is ______.

25. Which of the following is/are shared by all the threads in a process?

I. Program contact

II. Stack

III. Address space

IV. Registers

(*a*) I and II only (*b*) III only
(*c*) IV only (*d*) III and IV only

(Q.26 – 55) : Carry Two Marks Each

26. P and Q are considering to apply for a job. The probability that P applies for the job is $\frac{1}{4}$, the probability that P applies for the job given that Q applies for the job is $\frac{1}{2}$, and the probability

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that Q applies for the job given that P applies for the job is $\frac{1}{3}$. Then the probability that P does not apply for the job given that Q does not apply for the job is

(*a*) $\frac{4}{5}$ (*b*) $\frac{5}{6}$

(*c*) $\frac{7}{8}$ (*d*) $\frac{11}{12}$

27. Consider the following expression grammar G :

$E \to E - T \mid T$
$T \to T + F \mid F$
$F \to (E) \mid id$

Which of the following grammars is not left recursive, but is equivalent to G?

(*a*) $E \to E - T \mid T$
$T \to T + F \mid F$
$F \to (E) \mid id$

(*b*) $E \to TE'$
$E' \to - TE' \mid \in$
$T \to T + F \mid F$
$F \to (E) \mid id$

(*c*) $E \to TX$
$X \to - TX \mid \in$
$T \to FY$
$Y \to + FY \mid \in$
$F \to (E) \mid id$

(*d*) $E \to TX \mid (TX)$
$X \to - TX \mid + TX \mid \in$
$T \to id$

28. Consider a binary code that consists of only four valid codewords as given below:

00000, 01011, 10101, 11110

Let the minimum Hamming distance of the code be p and the maximum number of erroneous bits that can be corrected by the code be q. Then the values of p and q are

(*a*) $p = 3$ and $q = 1$
(*b*) $p = 3$ and $q = 2$
(*c*) $p = 4$ and $q = 1$
(*d*) $p = 4$ and $q = 2$

29. Consider the following C Program.

```
#include<stdio.h>
#include<string.h>
{
   int main()
   char*c = "GATECSIT2017";
   char* p=c;
   printf("%d",int)strlen(c+2[p]-6[p]-1));
   return 0;
}
```

The ouput of the program is _____.

30. In a two-level cache system, the access times of L_1 cache and L_2 are 1 and 8 clock cycles, respectively. The miss penalty from the L_2 cache to main memory is 18 clock cycles. The miss rate of L_1 cache is twice that of L_2. The average memory access time (AMAT) of this cache system is 2 cycles. The miss rates of L_1 and L_2 respectively are :

(*a*) 0.111 and 0.056 (*b*) 0.056 and 0.111
(*c*) 0.0892 and 0.1784 (*d*) 0.1784 and 0.0892

31. If the ordinary generating function of a sequence $\{a_n\}_{n=0}^{\infty}$ is $\frac{1+z}{(1-z)^3}$. then $a_3 - a_0$ is equal to ______.

32. Let δ denote the transition function and $\hat{\delta}$ denote the extended transition function of the ε-NFA whose transition table is given below:

δ	ε	a	b
$\to q_0$	$\{q_z\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	(q_3)
q_2	$\{q_0\}$	Φ	Φ
q_3	Φ	Φ	$\{q_2\}$

Then $\hat{\delta}(q_2, aba)$ is

(*a*) ϕ (*b*) (q_0, q_1, q_3)
(*c*) (q_0, q_1, q_2) (*d*) (q_0, q_2, q_3)

33. Consider the following database table named top_scorer

top_scorer

Player	Country	Goals
Klose	Germany	16
Ronaldo	Brazil	15
G Muller	Germany	14
Fontaine	France	13
Pele	Brazil	12
Klinsmann	Germany	11
Kocsis	Hungary	11
Batistuta	Argentina	10
Cubillas	Peru	10
Lato	Poland	10
Lineker	England	10
T Muller	Germany	10
Rahn	Germany	10

Consider the following SQL query
we get

```
SELECT ta. player FROM top_score as ta
WHERE ta.goals > ALL (SELECT tb.goals
          FROM top_scorer as tb
          WHERE tb.country = 'Spain')
AND ta.goals> ANY (SELECT tc.goals
          FROM top_scorer as tc
          WHERE tc.country = 'Germany')
```

The number of tuples returned by the above SQL query is _______

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34. In a B^+ tree, if the search value is 8 bytes long, the block size is 512 bytes and the block pointer size is 2 bytes, then the maximum order of the B^+ tree is _____ .

35. Given $f(w, x, y, z) = \Sigma_m(0, 1, 2, 3, 7, 8, 10) + \Sigma_d$ (5, 6, 11, 15), where d represents the *don't-care* condition in Karnaugh maps. Which of the following is a minimum product-of-sums (POS) form of $f(w, x, y, z)$?

(*a*) $f = (\overline{w} + \overline{z})(\overline{x} + z)$

(*b*) $f = (\overline{w} + z)(x + z)$

(*c*) $f = (w + z)(\overline{x} + z)$

(*d*) $f = (w + \overline{z})(\overline{x} + z)$

36. The next state table of a 2-bit saturating up-counter is given below.

Q_1	Q_0	Q_1^+	Q_0^+
0	0	0	1
0	1	1	0
1	0	1	1
1	1	1	1

The counter is built as a synchronous sequential circuit using T flip-flops. The expression for T_1 and T_0 are

(*a*) $T_1 = Q_1Q_0$ $\quad T_0 = \bar{Q}_1\bar{Q}_0$

(*b*) $T_1 = \bar{Q}_1\bar{Q}_0$ $\quad T_0 = \bar{Q}_1 + \bar{Q}_0$

(*c*) $T_1 = Q_1 + Q_0$ $\quad T_0 = \bar{Q}_1 + \bar{Q}_0$

(*d*) $T_1 = \bar{Q}_1Q_0$ $\quad T_0 = Q_1 + Q_0$

37. Consider the recurrence function

$$T(n) = \begin{cases} 2T(\sqrt{n}) + 1, & n > 2 \\ 2, & 0 < n \le 2 \end{cases}$$

Then $T(n)$ in terms of Θ notation is

(*a*) $\Theta(\log \log n)$ (*b*) $\Theta(\log n)$

(*c*) $\Theta(\sqrt{n})$ (*d*) $\Theta(n)$

38. A message is made up entirely of characters from the set X = {P, Q, R, S, T}. The table of probabilities for each of the characters is shown below :

Character	Probability
P	0.22
Q	0.34
R	0.17
S	0.19
T	0.08
Total	1.00

If a message of 100 characters over X is encode using Huffman coding, then the expected length of the encoded message in bits is _______ .

39. Consider the following C Program.

```
# include<stdio.h>
int main ()
{
  int m = 10 ;
   int n,n1 ;
      n = ++m ;
    n1 = m++ ;
    n - -  ;
    - -n1 ;
    n -= n1 ;
   printf ("%d", n) ;
   return 0 ;
}
```

The output of the program is ________ .

40. If w, x, y, z are Boolean variables, then which one of the following is INCORRECT?

(*a*) $wx + w(x + y) + x(x + y) = x + wy$

(*b*) $\overline{w\overline{x}(y + \overline{z})} + \overline{w}x = \overline{w} + x + \overline{y}z$

(*c*) $(w\overline{x}(y + x\overline{z}) + \overline{w}\overline{x})y = x\overline{y}$

(*d*) $(w + y)(wxy + wyz) = wxy + wyz$

41. For any discrete random variable X with probability mass function

$P(X = j) = p_j$, $p_j \ge 0$. $j \in \{0......N\}$, and $\sum_{j=0}^{N} p_j = 1$,

define the polynomial function $g_x(z) = \sum_{j=0}^{N} p_j z^j$.

For a certain discrete random variable Y, there exists s scalar $\beta \in [0, 1]$ such that $g_y(z) = (1 - \beta + \beta z)^N$. The expectation of Y is

(*a*) $N\beta(1 - \beta)$

(*b*) $N\beta$

(*c*) $N(1 - \beta)$

(*d*) Not expressible in terms of N and β alone

42. Consider the set of processes with arrival time (in milliseconds), CPU burst time (in milliseconds). and priority (0 is the highest priority) shown below. None of the processess have I/O burst time.

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Process	Arrival Time	Burst Time	Priority
P_1	0	11	2
P_2	5	28	0
P_3	12	2	3
P_4	2	10	1
P_5	9	16	4

The average waiting time (in milliseconds) of all the processes using preemptive priority scheduling algorithm is __________ .

43. Consider a machine with a byte addressable main memory of 2^{32} bytes divided into blocks of size 32 bytes. Assume that a direct mapped cache having 512 cache lines is used with this machine. The size of the tag field in bits is ________ .

44. Consider two hosts X and Y, connected by a single direct link of rate 10^6 bits/sec. The distance between the two hosts is 10,000 km and the propagation speed along the link is 2×10^8 m/sec. Host X sends a file of 50,000 bytes as one large message to host Y continuously. Let the transmission and propagation delays be p milliseconds and q milliseconds, respectively.

Then the values of p and q are

(*a*) $p = 50$ and $q = 100$
(*b*) $p = 50$ and $q = 400$
(*c*) $p = 100$ and $q = 50$
(*d*) $p = 400$ and $q = 50$

45. If a random variable X has a Poisson distribution with mean 5, then the expectation $E[(X + 2)^2]$ equals _______ .

46. Consider the following snippet of a C program. Assume that swap (&x, &y) exchanges the contents of x and y.

```
int main()
{
   int array[] = {3, 5, 1, 4, 6, 2};
   int done = 0;
   int i;
   while {done == 0)
   {
    done = 1;
    for (i = 0; j<=4; i++)
    {
      if (array[i] < array[i+1])
      {
        swap (&array[i], &array[i+1]);
      }
    }
    for (i=5, i>=1; i--)
      {
        if (array[i] > array[i-1])
        {
        swap (&array[i], &array[i-1]);
        done = 0;
      }
    }
  printf{"%d",array[3]);
 }
```

The output of the program is _____.

47. A system shares 9 tape drives. The current allocation and maximum requirement of tape drives for three processes are shown below :

Process	Current Allocation	Maximum Requirement
P1	3	7
P2	1	6
P3	3	5

Which of the following best describes current state of the system?

(*a*) Safe, Deadlocked
(*b*) Safe, Not Deadlocked
(*c*) Not Safe, Deadlocked
(*d*) Not Safe, Not Deadlocked

48. Consider the following C function.

```
int fun(int n)
{
   int i, j;
   for(i = 1; i <= n; i++)
   {
     for(j = 1; j < n; j += i)
     {
       printf (" %d %d", i,j);
     }
   }
 }
```

Time complexity of fun in terms of Θ notation is

(*a*) $\Theta(n\sqrt{n})$ (*b*) $\Theta(n^2)$
(*c*) $\Theta(n \log n)$ (*d*) $\Theta(n^2 \log n)$

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49. Let L(R) be the language represented by regular expression R. Let L(G) be the language generated by a context free grammer G. Let L(M) be the language accepted by a Turing machine M. Which of the following decision problems are undecidable?

I. Given a regular expression R and a string w, is $w \in L(R)$?

II. Given a context-free grammer G, is $L(G) = \Phi$?

III. Given a context-free grammer G, is $L(G) = \Sigma^*$ for some alphabet Σ?

IV. Given a Turing machine M and a string w, is $w \in L(M)$?

(*a*) I and IV only

(*b*) II and III only

(*c*) II, III and IV only

(*d*) III and IV only

50. If the characteristic polynomial of a 3 × 3 matrix M over $\mathbb{R}$ (the set of real numbers) is $\lambda^3 - 4\lambda^2 + a\lambda + 30$, $a \in \mathbb{R}$, and one eigenvalue of M is 2, then the largest among the absolute values of the eigenvalues of M is ________.

51. Consider the following languages.

$$L_1 = \{a^p \mid p \text{ is a prime number}\}$$

$$L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\}$$

$$L_3 = \{a^n b^n c^{2n} \mid n \geq 0\}$$

$$L_4 = \{a^n b^n \mid n \geq 1\}$$

Which of the following are CORRECT?

I. L_1 is context-free but not regular.

II. L_2 is not context-free.

III. L_3 is not context-free but recursive.

IV. L_4 is deterministic context-free.

(*a*) I, II and IV only

(*b*) II and III only

(*c*) I and IV only

(*d*) III and IV only

52. Two transactions T_1 and T_2 are given as

$$T_1 : r_1(X)w_1(X)r_1(Y)w_1(Y)$$

$$T_2 : r_2(Y)w_2(Y)r_2(Z)w_2(Z)$$

where $r_i(V)$ denotes a *read* operation by transaction T_i on a variable V and $w_1(V)$ denotes a *write* operation by transaction T_i on a variable V. The total number of conflict serializable schedules that can be formed by T_1 and T_2 is ______ .

53. The read access times and the hit ratios for different caches in a memory hierarchy are as given below.

Code	Read access time (in nanoseconds)	Hit ratio
I-cache	2	0.8
D-cache	2	0.9
L2-cache	8	0.9

The read access time of main memory is 90 nanoseconds. Assume that the caches use the referred word-first read policy and the write back policy. Assume that all the caches are direct mapped caches. Assume that the dirty bit is always 0 for all the blocks in the caches. In execution of a program. 60% of memory reads are for instruction fetch and 40% are for memory operand fetch. The average read access time in nanoseconds (up to 2 decimal places) is _________.

54. Consider the C program fragment below which is meant to divide `x` by `y` using repeated subtractions. The variables `x, y, q` and `r` are all `unsigned int`.

```
while (r >= y)
{
        r = r - y;
        q = q + 1;
}
```

Which of the following conditions on the variables `x, y, q` and `r` before the execution of the fragment will ensure that the loop terminates in a state satisfying the condition `x==(y*q + r)`?

(*a*) `(q == r) && (r == 0)`

(*b*) `(x > 0) && (r == x) && (y > 0)`

(*c*) `(q == 0) && (r == x) && (y > 0)`

(*d*) `(q == 0) && (y > 0)`

55. The pre-order traversal of a binary search tree is given by 12, 8, 6, 2, 7, 9, 10, 16, 15, 19, 17, 20. Then the post-order traversal of this tree is :

(*a*) 2, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 20

(*b*) 2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 12

(*c*) 7, 2, 6, 8, 9, 10, 20, 17, 19, 15, 16, 12

(*d*) 7, 6, 2, 10, 9, 8, 15, 16, 17, 20, 19, 12

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GENERAL APTITUDE

(Q.1 – 5) : Carry One Mark Each

1. A test has twenty questions worth 100 marks in total. There are two types of questions. Multiple choice questions are worth 3 marks each and essay questions are worth 11 marks each. How many multiple choice questions does the exam have ?

(*a*) 12 (*b*) 15
(*c*) 18 (*d*) 19

2. Choose the option with words that are not synonyms.

(*a*) aversion, dislike
(*b*) luminous, radiant
(*c*) plunder, loot
(*d*) yielding, resistant

3. There are five buildings called V, W, X, Y and Z in a row (not necessarily in that order). V is to the West of W. Z is to the East of X and the West of V. W is to the West of Y. Which is the building in the middle ?

(*a*) V (*b*) W
(*c*) X (*d*) Y

4. There are 3 red socks, 4 green socks and 3 blue socks. You choose 2 socks. The probability that they are of the same colour is

(*a*) 1/5 (*b*) 7/30
(*c*) 1/4 (*d*) 4/15

5. Saturn is _____ to be seen on a clear night with the naked eye.

(*a*) enough bright
(*b*) bright enough
(*c*) as enough bright
(*d*) bright as enough

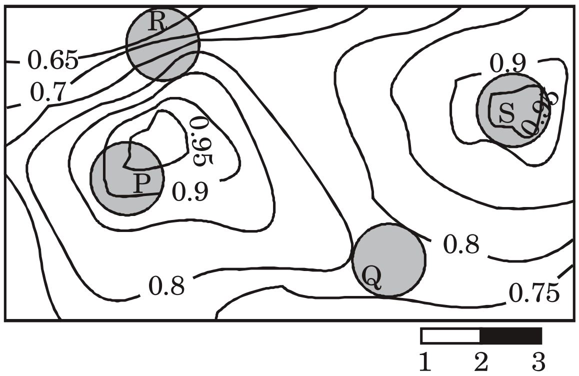
(Q.6 – 10) : Carry Two Marks Each

6. "We lived in a culture that denied any merit to literary works, considering them important only when they were handmaidens to somethings seemingly more urgent – namely ideology. This was a country where all gestures, even the most private, were interprated in political terms."

The author's belief that ideology is not as important as literature is revealed by the word :

(*a*) 'culture' (*b*) 'seemingly'
(*c*) 'urgent' (*d*) 'political'

7. An air pressure contour line joins locations in a region having the same atmospheric pressure. The following is an air pressure contour plot is a geographical region. Contour lines are shown at 0.05 bar intervals in this plot.



If the possibility of a thunderstorm is given by how fast air pressure rises or drops over a region, which of the following regions is most likely to have a thunderstorm ?

(*a*) P (*b*) Q
(*c*) R (*d*) S

8. The number of roots of $e^x + 0.5x^2 - 2 = 0$ in the range [–5, 5] is

(*a*) 0
(*b*) 1
(*c*) 2
(*d*) 3

9. There are three boxes. One contains apples another contains oranges and the last one contains both apples and oranges. All three are known to be incorrectly labelled. If you are permitted to open just one box and then pull out and inspect only one fruit, which box would you open to determine the contents of all three boxes?

(*a*) The box labelled 'Apples'
(*b*) The box labelled 'Apples and Oranges'
(*c*) The box labelled 'Oranges'
(*d*) Cannot be determined

10. X is a 30 digit number starting with the digit 4 followed by the digit 7. Then the number X^3 will have

(*a*) 90 digits
(*b*) 91 digits
(*c*) 92 digits
(*d*) 93 digits

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ANSWERS

Techanical Section

1.(*d*)	**2.**(*d*)	**3.**(*b*)	**4.**(*c*)	**5.**(8)	**6.**(*c*)	**7.**(*c*)	**8.**(*c* & *d*)	**9.**(*c*)	**10.**(*b*)
11.(8)	**12.**(0)	**13.**(*b*)	**14.**(16)	**15.**(0)	**16.**(*a*)	**17.**(2)	**18.**(*a*)	**19.**(*d*)	**20.**(*a*)
21.(*c*)	**22.**(*c*)	**23.**(*c*)	**24.**(9)	**25.**(*b*)	**26.**(*a*)	**27.**(*c*)	**28.**(*a*)	**29.**(2)	**30.**(*a*)
31.(15)	**32.**(*c*)	**33.**(7)	**34.**(52)	**35.**(*a*)	**36.**(*b*)	**37.**(*b*)	**38.**(225)	**39.**(0)	**40.**(*c*)
41.(*b*)	**42.**(29)	**43.**(18)	**44.**(*d*)	**45.**(54)	**46.**(3)	**47.**(*b*)	**48.**(*c*)	**49.**(*d*)	**50.**(5)
51.(*d*)	**52.**(54)	**53.**(2.74)	**54.**(*c*)	**55.**(*b*)					

General Aptitude

1.(*b*)	**2.**(*d*)	**3.**(*a*)	**4.**(*d*)	**5.**(*b*)	**6.**(*c*)	**7.**(*c*)	**8.**(*c*)	**9.**(*a*)	**10.**(*a*)

EXPLANATIONS

TECHNICAL SECTION

1. The 16 bit is further divided into:

$$X = (B\ \ C\ \ A\ \ 9)_{16}$$

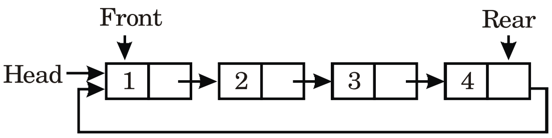
for octal number divide from the right in to 3 bits and derive the below number

$$= 1\ \underbrace{011}\ \underbrace{110}\ \underbrace{010}\ \underbrace{101}\ \underbrace{001}$$

$$= (1\quad 3\quad 6\quad 2\quad 5\quad 1)_8$$

2. Only as linked and indexed allocations are non cantagious, they will not suffer from external fragmentation.

3.



As in a queue, insertion is done from REAR and deletion is done from FRONT. As the next pointer of REAR node is pointing to the FRONT node, it leads both insertion and deletion operations in O(1) time.

4. As given:

$$S \to XY$$
$$X \to aX|a \Rightarrow X \to \{a^m | m \geq 1\}$$
$$Y \to aYb|\in \Rightarrow Y \to \{a^n b^n | n \geq 0\}$$
$$S \to XY \Rightarrow S \to \{a^m b^n | m > n,\ n \geq 0\}$$

$m > n$ because at least 1 will be attached on left of $a^n b^n$.

Hence the language generated here is

$$\{a^m b^n \mid m > n, n \geq 0\}$$

5. Given the regular language is :

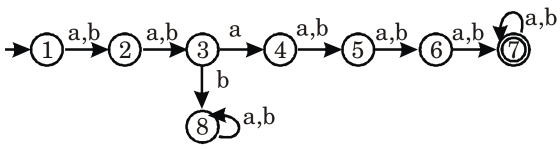
Min DFA for L

$$= \left\{w_1 a w_2 \middle| w_1, w_2 \in \{a,b\}^*, |w_1| = 2, |w_2| \geq 3\right\}$$

Hence the regular expression for L is

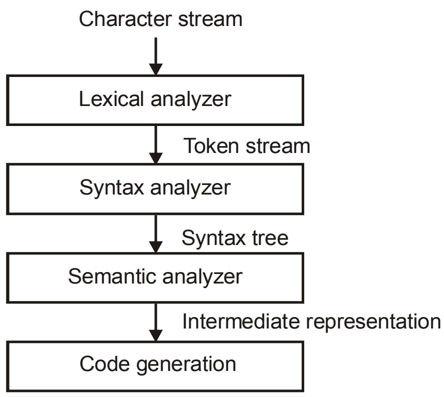
$$(a + b)\ (a + b)\ a\ (a + b)\ (a + b)(a + b)\ (a + b)^*$$

The minimal DFA is



Hence the minimum possible number of states of DFA is 8.

6. According to the compiler phase process, we get the below structure:



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7. Given,

$$f(x) = R\sin\left(\frac{\pi x}{2}\right) + S, \qquad ...(1)$$

Given $$f'\left(\frac{1}{2}\right) = \sqrt{2} \qquad ...(2)$$

$$\int_0^1 f(x)dx = \frac{2R}{\pi} \qquad ...(3)$$

Now we need to find R and S.

$$f'(x) = R\cos\left(\frac{\pi x}{2}\right)\frac{\pi}{2}$$

$$f'\left(\frac{1}{2}\right) = R\cos\left(\frac{\pi}{4}\right)\times\frac{\pi}{2} = \sqrt{2}$$

$$\Rightarrow \quad \frac{R}{\sqrt{2}}\times\frac{\pi}{2} = \sqrt{2}$$

$$\Rightarrow \quad R = \frac{4}{\pi}$$

Now, $$\int f(x)dx = \int R\sin\frac{\pi x}{2} + S$$

Putting $R = \frac{4}{\pi}$ we get

$$\int f(x)dx = \int \frac{4}{\pi}\sin\left(\frac{\pi x}{2}\right)dx + \int S\,dx$$

$$= \frac{4}{\pi}\times -\frac{\cos\left(\frac{\pi x}{2}\right)}{\frac{\pi}{2}} + Sx$$

$$= \frac{-8}{\pi^2}\cos\left(\frac{\pi x}{2}\right) + Sx$$

Putting limit 0 and 1

$$\int_0^1 f(x)dx = \frac{-8}{\pi^2}\left(\cos\frac{\pi}{2} - \cos(0)\right) + S(1-0)$$

$$= \frac{2R}{\pi}$$

$$\Rightarrow \quad \frac{-8}{\pi^2}(0-1) + S = \frac{2R}{\pi}$$

Put $R = \frac{4}{\pi}$ and solve for S

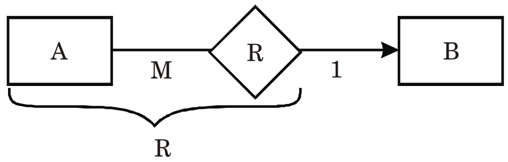
$$\Rightarrow \quad S = 0$$

So, $R = \frac{4}{\pi}$ and $S = 0$ is answer.

8. There are 2 entities sets A, B

1 relationship set R

As relation R merges with that of A



As many to one relationship set can merge towards entity set 'A'

The partipation towards A side can be total/partial

9. The 32- bit number is further divided as below:

1 bit	8 bit	23 bit
0	01111100	1101101000...
S	BE	M

Step 1. Sing = 0

= +ve

Step 2. AE = BE – Bias

$$\begin{array}{r} \text{BE} = 0\,1\,1\,1\,1\,1\,0\,0 \\ \text{Bias} = 0\,1\,1\,1\,1\,1\,1\,1 \\ \hline \text{AE} = \underline{1}\,1\,1\,1\,1\,1\,0\,1 \\ \hline \end{array}$$

AS sign of AE is negative so take two complement of AE.

i.e., 00000010

1

00000011

$\Rightarrow$ – 3

Step 3.Mantissa

Hence normal Mantissa = 1.M = 1.1101101

Data + 1.1101101 $\times 2^{-3}$ $\{\pm M \times B^{\pm e}\}$

Now mantissa is aligned to right upto 3 times

↓

+0.0011101101

↓

0.228

↓

Hence 2.28×10^{-1}

10. As given :

I. $L_1 \cup L_2$ is context- free = CFL $\cup$ CFL = CFL.

So True.

II. $\overline{L}_1$ is context-free = $\overline{\text{CFL}}$ = CSL but not CFL.

So False.

III. L_1 –R is context-free = CFL $\cap$ $\overline{\text{Regular}}$ = CFL.

So,True

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IV. $L_1 \cap L_2$ is context-free = $CFL \cap CFL$ = CSL.

So, False

11. $x^2 - 13x + 36 = 0$

In base b $\quad 13 = 1 \times b^1 + 3 \times b^0 = b + 3$

In base b $\quad 36 = 3 \times b^1 + 6 \times b^0$

$= 3b + 6$

So the equation becomes

$x^2 - (b + 3)x + (3b + 6) = 0$

Now since it is given that x = 5 is a solution , so

$5^2 - (b + 3)\,5 + (3b + 6) = 0$

$\Rightarrow \quad -2b + 16 = 0$

$\Rightarrow \quad b = 8$

Same can be obtained by putting $x = 6$ also.

12. As we have the primary key of one table as foreign key of another table, from these tables :

on delete set null
on update cascade

on delete set null
on update cascade

P	Q
2	2
~~3~~	~~8~~
7	3
5	8
6	9
8	5
9	8

R	S
2	2
8	~~3~~ null
3	2
9	7
5	7
7	2

No other record need to delete because of deletion of (3, 8) record from T_1

13. Bind () function is used to create local address

Connect () function specifies remote address. An unconnected UDP socket is just a Bind () function.

But a connected UDP socket is connect () function which behaves like a TCP.

14. In the undirected graph G, we have n vertices and 25 edges

Hence

$n \le ?$

$e = 25$

As each vertex has at least 3 degree

and $\quad 2e = \Sigma$degree

i.e., $\quad 2e \ge 3n$

$n \le 2e/3$

$\Rightarrow \quad n \le \dfrac{2 \times 25}{3} \le 16.66$

$\therefore \quad n$ is at the maximum possible value of 16.

15. There is no need to add any ordered pair as the given hasse diagram is already a lattice.

16. Among all the LR(k) parsers, cononical LR is the most powerful parsers.

17. By adding both P and Q

$$P + Q = \begin{bmatrix} 0 & -1 & -2 \\ 8 & 9 & 10 \\ 8 & 8 & 8 \end{bmatrix}$$

Magnitude of P & Q

$|P + Q| = -16 + 16 = 0$

So rank $\ne 3$

rank < 3

Take the 2×2 minor $\begin{bmatrix} 0 & -1 \\ 8 & 9 \end{bmatrix} = 8 \ne 0$

So, rank of P + Q is 2

18. This refers to a variable located in data section of memory.

static chr var; : Initialization of a variable located in data section of memory.

***m* = malloc (10); *m* = NULL; :** A lost memory which can't be freed because free (m) is missed in code.

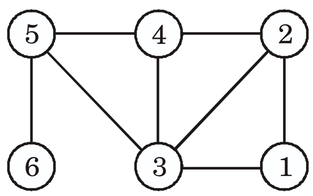
char * ptr[10]; : Sequence of memory locations to store addresses.

register int var 1; : Request to allocate a CPU register to store data.

19. While considering each option, we can see that:

- After vertices 'M' and 'N', vertex 'O' can't be traversed.
- After visting vertex 'M', vertex 'O' should be traversed.
- After 'N' either of the vertices 'O' or 'P' should be traversed

Hence we derive the below order of visiting the nodes



20. As given :

p : "It is raining"

q : "It is cold", and

r : "It is pleasant",

So the correct representation of "*It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold"* is

$\neg p \wedge r \wedge \neg$ r only if $p \wedge q$

$\equiv (\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$

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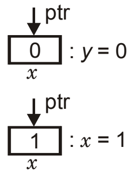
21. print $xy(1, 1)$



As the last line is printf (" % d, % d", x, y)

the output will be (1, 0)

22. As per the given algorithms and time complexities we derive:

Towers of Hanoi with n disks

$$= 2T(n - 1) + 1 = \Theta(2^n)$$

Binary search given n sorted numbers

$$= T(n/2) + 1 = \Theta(\log n)$$

Heap sort given n numbers at the worst case

$$= 2T(n/2) + n = \Theta(n \log n)$$

Addition of two $n \times n$ matrices

$$= 4T(n/2) + 1 = \Theta(n^2)$$

23. As we know:

- RIP uses distance vector routing.
- OSPF uses link-state routing protocols.
- RIP uses UDP as transport protocol.
- OSPF neither uses TCP nor UDP.

[Link state packet should be given to all routers in subnet so it's not possible with TCP. These link state packets should be reliable at that same time which is not possible with UDP.]

24. In IPv4, 40 bytes are alloted for options and padding.

and maximum nine routers addresses are allowed.

Each IPv4 address is 32 bits or 4 bytes

Hence $4 \times 9 = 36$ bytes

The remaining byte are used for the option.

25. Only address space is shared by all the threads in a process, the other entities such as stack , PC, resisters are not shared as each thread has its own.

26. Given that

$$p(P) = \frac{1}{4} \quad ...(i)$$

$$p(P \mid Q) = \frac{1}{2} \quad ...(ii)$$

$$p(Q \mid P) = \frac{1}{3} \quad ...(iii)$$

$$p(\overline{P} \mid \overline{Q}) = ?$$

First solve for $p(Q)$ and $p(P \cap Q)$ from equation (ii) and (iii) as follows :

From equation (ii)

$$p(P \mid Q) = \frac{p(P \cap Q)}{p(Q)} = \frac{1}{2} \quad ...(iv)$$

From equation (iii) $p(Q \mid P) = \dfrac{p(P \cap Q)}{p(P)} = \dfrac{1}{3}$

$$\Rightarrow \quad p(P \cap Q) = \frac{1}{3} \times p(P)$$

$$= \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

Substitute in equation (iv) get

$$p(Q) = \frac{p(P \cap Q)}{1/2} = \frac{1/12}{1/2}$$

$$= \frac{2}{12} = \frac{1}{6}$$

$$p(P) = \frac{1}{4}$$

$$p(Q) = \frac{1}{6}$$

and $\quad p(P \cap Q) = \dfrac{1}{12}$

we need to find

$$p(\overline{P} \mid \overline{Q}) = \frac{p(\overline{P} \cap \overline{Q})}{p(\overline{Q})}$$

$$= \frac{1 - (P \cup Q)}{1 - p(Q)}$$

$$= 1 - \frac{[p(P) + p(Q) - p(P \cap Q)]}{1 - p(Q)}$$

$$= \frac{1 - \left[\frac{1}{4} + \frac{1}{6} - \frac{1}{12}\right]}{1 - \frac{1}{6}} = \frac{\frac{2}{3}}{\frac{5}{6}} = \frac{4}{5}$$

$$p(\overline{P} \mid \overline{Q}) = \frac{4}{5}$$

27.

$$E \to E - T \mid T$$
$$T \to T + F \mid F$$
$$F \to (E) \mid id$$

There are 2 left recurssion and both have to be removed.

$$E' \to - TE' \mid \in$$
$$E \to TE'$$
$$T' \to + FT' \mid \in$$
$$Y \to FT'$$
$$F \to (E) \mid id$$

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Now by putting E′ as X and T′ as Y,

$$E \rightarrow TX$$
$$X \rightarrow -TX \mid \in$$
$$T \rightarrow FY$$
$$Y \rightarrow +FY \mid \in$$
$$F \rightarrow (E) \mid id$$

So option (*c*) is correct

28. By question, codewords are

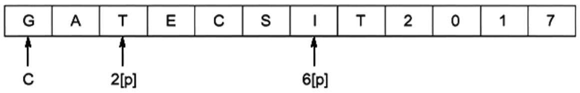
00000
01011
10101
11110

Minimum hamming distance = min (hamming distance between every 2 pair) = 3

for correcting errors, we know that

$$2d + 1 = 3$$
$$\Rightarrow \quad d = 1$$

29. (C + 2[p] – 6[p] – 1)



C + 'T' – 'I' – 1
= C + 11 – 1
= C + 10

Finally printf will print '2'.

30. Given data

Hit time L_1 = 1 cycle
Hit time L_2 = 8 cycles

Miss penality L_2 = 18 cycles

Tavg = 2 ns

Miss rate $L_1 = 2x$
Miss rate $L_2 = x$

- Tavg = Hit time L_1 + (Miss rate L_1 × Miss penality L_1)
  T_{avg} = 2ns
- Miss penality L_1 = Hit time L_2 + (Miss rate L_2 × Miss penality L_2)
- Substitute the above data and verifying with respect to the given options.
- $L_1 = 0.111 \qquad L_2 = 0.056$

31. Then generating function $\{a_n\}_{n=0}^{\infty}$ is $\dfrac{1+z}{(1-z)^3}$

$$A(z) = \sum a_r z^r$$
$$= \frac{1+z}{(1-z)^3}$$

We can replace z by x, we get

$$A(x) = \sum a_r x^r$$
$$= \frac{1+x}{(1-x)^3}$$
$$= \frac{1}{(1-x)^3} + \frac{x}{(1-x)^3}$$
$$= \sum_{r=0}^{\infty} {}^{3-1+r}C_r x^r + x\sum_{r=0}^{\infty} {}^{3-1+r}C_r x^r$$
$$= \sum_{r=0}^{\infty} {}^{r+2}C_2 x^r + \sum_{r=0}^{\infty} {}^{r+2}C_2 x^{r+1}$$

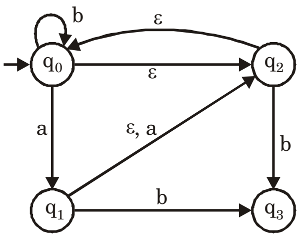
$${}^2C_2x^0 + {}^3C_2x^1 + {}^4C_2r^2 + {}^5C_2x^3 ....$$
$$+ {}^2C_2x^1 + {}^3C_2x^2 + {}^4C_2x^3 + ...$$

Now we read to find a_0 and a_3 which are coefficient of x^0 and x^3 respectively.

$$a_0 = \text{Coefficient } x^0$$
$$= {}^2C_2 = 1$$
$$a_3 = \text{Coefficient } x^3$$
$$= {}^5C_2 + {}^4C_2 = 16$$

So, $\quad a_3 - a_0 = 16 - 1 = 15$

32. Draw the table from state diagram, we get,



$\hat{\delta}(q_2, aba)$ = All states reachable from q_2 by "aba"

If aba is broken as ε.a.ε.ε.b.a. Then from q_2 we can reach q_1 and from there by null transition we can reach state q_2 as well as q_0.

$$\hat{\delta}(q_2, aba) = (q_0, q_1, q_2\}$$

33. Select ta.player
FROM top_scorer as ta
WHERE ta.goals > ALL (SELECT tb.goals)
FROM top_scorer as tb
WHERE tb.country = 'Spain')
AND ta.goals > ANY (SELECT tc. goals
FROM top_scorer as tc
WHERE tc.country = 'Germany')

So number of tuples in result 7.

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34. B^+ tree

Search key : 8 bytes

Block size : 512 bytes

Block pointer : 2 bytes

Maximum order of B^+ tree node??

$P \times B_P + (P - 1) \times \text{Key} \leq \text{Block size}$

$$P \times 2 + (P - 1)8 \leq 512$$

$$10P \leq 520$$

$$P = \left[\frac{520}{10}\right] = 52$$

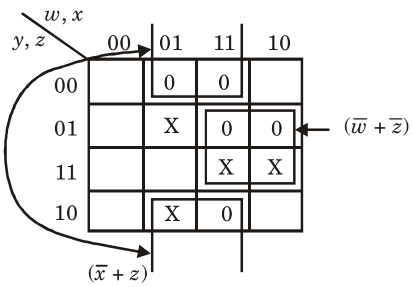
Maximum order of B^+ tree is 52

35. Given

$f(w, x, y, z) = \Sigma_m\{0, 1, 2, 3, 7, 8, 10) + d(5, 6, 11, 15)$

$= \pi_M \{4, 9, 12, 13, 14). d(5, 6, 11, 15)$

draw K-map



$f(w, x, y, z) = (\overline{w}+\overline{z})(\overline{x}+z)$]

This is minimum POS form of $f(w, x, y, z)$

36. State table showing input of flip-flope

FF inputs

Q_1	Q_0	Q_1^+	Q_0^+	T_1	T_0
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	0	0

From the table

$$T_1(Q_1, Q_0) = \overline{Q}_1 Q_0$$

$$T_2(Q_1, Q_0) = \overline{Q}_1\overline{Q}_0 + \overline{Q}_1 Q_0 + Q_1\overline{Q}_0$$

minimize T_2

$$= \overline{Q}_1 + \overline{Q}_0$$

37. $$T(n) = 2T\left(\sqrt{n}\right)+1 \quad n > 2 \ ...(i)$$

$$T(n) = 2T\left(\sqrt[2]{n}\right)+1 \quad ...(ii)$$

Put (ii) in (i)

$$T(n) = 2 * 2T\left(\sqrt[2]{n}\right)+2$$

$$T(n) = 2^2T\left(\sqrt[2]{n}\right)+2 \quad ...(iii)$$

$$T\left(\sqrt[2]{n}\right) = 2T\left(\sqrt[3]{n}\right)+1 \quad ...(iv)$$

Substituting (iv) in (iii)

$$T(n) = 2^3T\left(\sqrt[3]{n}\right)+3$$

Running the same till K times,

$$T(n) = 2^K T\left(\sqrt[K]{n}\right)+K$$

$$\sqrt[K]{n} = 2$$

$$K = \log_2 n$$

Solving this will give

$$T(n) = \Theta(\log n)$$

39. By given C program, we have :

1. int m = 10; // m = 10
2. int n, n1 ;
3. n = ++m; // n = 11
4. n1 = m++; // n1 = 11, m = 12
5. n – –; // n =10
6. – – n1 ; // n1 = 10
7. n – = n1 ; // n = 0
8. printf ("%d", n);

Now, by above steps, we get output = 0

40. $$\left(w\overline{x}\left(y+x\overline{z}\right)+\overline{w}\,\overline{x}\right)y = \overline{x}y$$

$$\left(w\overline{x}y+\overline{w}\,\overline{x}\right)y$$

$$w\overline{x}y+\overline{w}\,\overline{x}y$$

$\overline{x}y\left(w+\overline{w}\right)$, since $\left(w+\overline{w}\right)$ evaluate to 1

$$\overline{x}y \neq x\overline{y} \qquad \text{False}$$

41. $g_Y(z) = ((1-\beta)+\beta z)^N$ Scalar $\beta \in [0,1]$

If $g_Y(z)$ is expanded, we would get a bionomial distribution with $n = N$ and $p = \beta$

So the $E[Y] = np = N\beta$

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42. As given in the table

Process	Arrival Time	Burst Time	Priority	C.T.	T.A.T.	W.T.
P_1	0	11	2	49	49	38
P_2	5	28	0 (high)	33	28	0
P_3	12	2	3	51	39	37
P_4	2	10	1	40	38	28
P_5	9	16	4	67	58	42
					Total	145

P_1	P_4	P_2	P_4	P_1	P_3	P_5

0 2 5 33 40 49 51 67

Hence

$$\text{Average waiting time} = \frac{145}{5} = 29 \text{ Ms}$$

43. From the given question:

MM size = 2^{32} B, Block size = 32 B

Direct CM

\# lines = 512

Address format is (32 bit)

Tag	LO	WO
18 bit	$\log_2 512$ = 9 bit	$\log_2 32$ = 5 bit

Hence Tag field = 18

44. Transmission time is ratio of Data size to Bandwidth

$$\text{Transmission time} = \frac{\text{Data size}}{\text{Bandwidth}} = \frac{50000 \times 8 \text{ bits}}{10^6 \text{ bits/sec}} = 400 \text{ m sec}$$

Also

$$\text{Propagation time} = \frac{\text{Distance}}{\text{Velocity}} = \frac{10000 \times 10^3 \text{ m}}{2 \times 10^8 \text{ m/sec}} = \frac{10^3 \times 10^{-4}}{2} = 50 \text{ msec}$$

45. We know that in Poisson distribution representation

$$E(X) = V(X) = \lambda$$

Given $\lambda = 5$

$$E(X) = V(X) = 5$$

We need $E[(X+2)^2]$

$$= E(X^2 + 4x + 4)$$
$$= E(X^2) + 4E(X + 4)$$

To find $E(X^2)$ we write,

$$V(X) = E(X^2) - (E(X))^2$$
$$5 = E(X^2) - 5^2$$

So, $E(X^2) = 5^2 + 5 = 30$

required value $= 30 + 4 \times 5 + 4 = 54$

46. Here, by question we have

3	5	1	4	6	2
0	1	2	3	4	5

For first for loop

($i = 0$) 5 3 1 4 6 2
($i = 1$) 5 3 1 4 6 2
($i = 2$) 5 3 4 1 6 2
($i = 3$) 5 3 4 6 1 2
($i = 4$) 5 3 4 6 2 1

For second for loop

($i = 5$) 5 3 4 6 2 1
($i = 4$) 5 3 4 6 2 1
($i = 3$) 5 3 6 4 2 1
($i = 2$) 5 6 3 4 2 1
($i = 1$) 6 5 3 4 2 1

Now since done is '0', So for loop will executed again and so.

for first for loop :

($i = 0$) 6 5 3 4 2 1
($i = 1$) 6 5 3 4 2 1
($i = 2$) 6 5 4 3 2 1
($i = 3$) 6 5 4 3 2 1
($i = 4$) 6 5 4 3 2 1

for second for loop :

($i = 5$) 6 5 4 3 2 1
($i = 4$) 6 5 4 3 2 1
($i = 3$) 6 5 4 3 2 1
($i = 2$) 6 5 4 3 2 1
($i = 1$) 6 5 4 3 2 1

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Value of done is still '0', hence the for loop will execute again.

First for loop :

This time there will be no change by the for loop.

The value of done is '1'. Hence the loop terminates as

6	5	4	3	2	1
0	1	2	3	4	5

The output of the program will be '3'

47. Draw the table

Process	Current Allocation	Maximum Requirement	Remaining Need	Current Available
P1	3	7	4	9 – 7 = 2
P2	1	6	5	5
P3	3	5	2	8
				9

From table

Safe sequence ⇒ P3 → P1 → P2

Safe and Not deadlocked

48. By the given C function

First loop will execute 'n' times and the inner loop will execute $\Theta(n\log n)$ times.

Hence the complexity will be $\Theta(n\log n)$

49. I. Membership of regular language (Decidable)

II. Emptiness of CFL (Decidable)

III. L = E* problem of CFL (Undecidable)

IV. Membership of RE language (Undecidable)

So, only III and IV are undecidable. So correct answer is (*d*).

50. $f(\lambda) = \lambda^3 - 4\lambda^2 + a\lambda + 30 = 0$

Given $m = 2$ will be one root of polynomial

So, $2^3 - 4 \times 2^2 + a \times 2 + 30 = 0$

$\Rightarrow \quad 8 - 16 + 2a + 30 = 0$

$\Rightarrow \quad a = -11$ (satisfy the equation)

So, the equation is $\lambda^3 - 4\lambda^2 - 11\lambda + 30 = 0$

Divided polynomials by $(\lambda - 2)$

$$\frac{\lambda^3 - 4\lambda^2 - 11\lambda + 30}{\lambda - 2} = \lambda^2 - 2\lambda - 15$$

roots of $\lambda^2 - 2\lambda - 15 = 0$ are

$$\lambda = \frac{2 \pm \sqrt{4+60}}{2}$$

$$= \frac{2+8}{2} = 5 \text{ and } -3$$

$\lambda_1 = 2$ $\lambda_2 = 5$ $\lambda_3 = 3$ the maximum absolute eigen value is 5.

51. $L_1 = \{a^p \mid p \text{ prime}\}$ is a CSL but not CFL (Prime number checking involve divsion)

$L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\}$ is CFL (one comparision)

$L_3 = \{a^n b^n c^{2n} \mid n \geq 0\}$ is CSL (two comparision)

$L_4 = \{a^n b^n \mid n \geq 1\}$ is a DCFL

I. L_1 is CFL but not regular is false.

II. L_2 is not CFL is false.

III. L_3 is not CFL but recursive is true since every CSL is recursive.

IV. L_4 is DCFL is true.

Only III and IV are true and correct.

52. $T_1 : r_1(X)\ w_1(X)\ r_1(Y)\ w_1(Y)$

$T_2 : r_2(Y)\ w_2(Y)\ r_2(Z)\ w_2(Z)$

(*i*) Number of conflict serializable on $T_1 \to T_2$: 1

$r_1(X)\ w_1(X)\ r_1(Y)\ w_1(Y) r_2(Y)\ w_2(Y)\ r_2(Z) w_2(Z)$

(*ii*) Number of conflict seralizable on $T_2 \to T_1$:53

S : $r_2(Y)\ w_2(Y)\ r_1(Y)\ w_1(Y)$

$r_1(X)\ w_1(X)$ must be before $r_1(Y)$

so that $(r_2(Y)\ w_2(Y))\ (r_1(X)\ w_1(X))$ can place.

$${}^4C_2 = \frac{4!}{2! \times 2!} = 6 \text{ ways}$$

1. $r_2(Y)\ w_2(Y)\ r_1(X)\ w_1(X)\ r_1(Y)\ w_1(Y)$

   $r_2(Z)\ w_2(Z)$ can place in ${}^6C_2 = \frac{6!}{4! \times 2!}$ =15 ways

   Like wise

2. $r_2(Y)\ r_1(X)\ w_1(X)\ w_2(Y)\ r_1(Y)\ w_1(Y)$

   $r_2(Z)\ w_2(Z)$ can place in ${}^4C_2$ = 6 ways

3. $r_2(Y)\ r_1(X)\ w_2(Y)\ w_1(X)\ r_1(Y)\ w_1(Y)$

   $r_2(Z)\ w_2(Z)$ can place in ${}^5C_2$ = 10 ways

4. $r_1(X)\ w_1(X)\ r_2(Y)\ w_2(Y)\ r_1(Y)\ w_1(Y)$

   $r_2(Z)\ w_2(Z)$ can place in ${}^4C_2$ = 6 ways

5. $r_1(X)\ r_2(Y)\ w_2(Y)\ w_1(X)\ r_1(Y)\ w_1(Y)$

   $r_2(Z)\ w_2(Z)$ can place in ${}^5C_2$ = 10 ways

6. $r_1(X)\ r_2(Y)\ w_1(X)\ w_2(Y)\ r_1(Y)\ w_1(Y)$

   $r_2(Z)\ w_2(Z)$ can place in ${}^4C_2$ = 6 ways

So total no of conflict serializable schedules of T_1 and T_2 = 53 +1 = 54 ways

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53. $T_{avg} = H_1T_1 + (1 - H_1)H_2(T_2 + T_1) + (1 - H_1)(1 - H_2) H_3(T_3 + T_2 + T_1) + (1 - H_1)(1 - H_2)(1 - H_3) H_4(T_4 + T_3 + T_2 + T_1)$

Put data

$$T_{avg} = (0.8 \times 2) + (1 - 0.8)\, 0.9 \times (2 + 2) + (1 - 0.8)(1 - 0.9)\, 0.9 \times (8 + 2 + 2) + (1 - 0.8)(1 - 0.9)(1 - 0.9) \times (90 + 8 + 2 + 2)$$

$$= 1.6 + 0.72 + 0.216 + 0.204 = 2.32 + 0.420$$

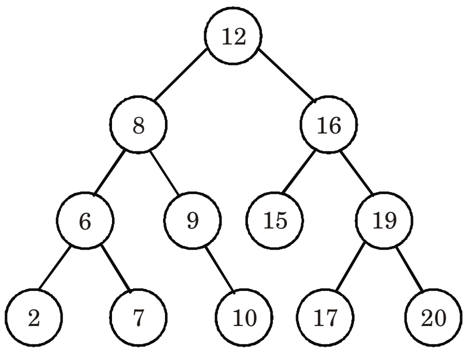
$$= 0.740 = 2.74 \text{ ns}$$

55. Given

Preorder : 12, 8, 6, 2, 7, 9, 10, 16, 15, 19, 17, 20

Inorder: 2, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 20

Tree will be, formed as



Postorder will be,

2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 12

GENERAL APTITUDE

1. Let number of multiple choice question be 'x'

$\therefore$ Total number of essay type question = $20 - x$

Now according to question

$$x \times 3 + (20 - x)11 = 100$$

$\therefore$ $3x + 220 - 11x = 100$

$\therefore$ $220 - 100 = 8x$

$\therefore$ $x = \dfrac{120}{8}$

$\therefore$ $x = 15$

$\therefore$ Opation (b) is correct

3. From the given condition of sequence we get following order of building in

X, Z, V, W, Y

Hence 'V' building is in the middle

$\therefore$ Option (a) is correct

5. Saturn is bright enough to be seen on a clear night with the naked eye.

$\therefore$ Option (b) is correct

2. Aversion means a strong dislike or dinclination. Hence aversion is synonyms of dislike.

Aversion means giving of light which is synonym of radiant plunder means loot while yielding and resistant are not synonyms of each other.

$\therefore$ Option (d) is correct.

4.
$$n(R) = 3$$
$$n(G) = 4$$
$$n(B) = 3$$

Probability that 2 socks choosen are of both Red or Both Green or Both blue

$$\text{Same colour} = \frac{3}{10} \times \frac{2}{9} + \frac{4}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{2}{9}$$

$$= \frac{6}{90} + \frac{12}{90} + \frac{6}{10} = \frac{24}{90} = \frac{4}{15}$$

$\therefore$ Option (d) is correct.

6. The author's belief that ideology is not as important as literature is revealed by the word 'urgent' in above paragraph.

$\therefore$ Option (c) is correct.

8.
$$e^x + 0.5x^2 - 2 = 0$$

Differention w.r.t to 'x' we get

$$e^x + x = 0$$

$\therefore$ Hence 'x' has total number of root which lies in the range [– 5,5] is '2'.

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SOLVED PAPER - 2018

INSTRUCTIONS

1. Total of 65 questions carrying 100 marks, out of which 10 questions carrying a total of 15 marks are in General Aptitude (GA)
2. The Engineering Mathematics will carry around **15% of the total marks,** the General Aptitude section will carry **15% of the total marks** and the **remaining 70% of the total marks.**
3. **Types of Questions**
   (*a*) Multiple Choice Questions (MCQ) carrying 1 or 2 marks each in all papers and sections. These questions are objective in nature, and each will have a choice of four options, out of which the candidate has to mark the correct answer(s).
   (*b*) Numerical Answer Questions of 1 or 2 marks each in all papers and sections. For these questions the answer is a real number, to be entered by the candidate using the virtual keypad. No choices will be shown for these type of questions.
4. For **1-mark** multiple-choice questions, **1/3 marks** will be deducted for a wrong answer. Likewise, for **2-marks** multiple-choice questions, **2/3** marks will be deducted for a wrong answer. There is no negative marking for numerical answer type questions.

Chapter-Wise Analysis

#	Chapters	Marks		
		1	2	Total
1	General Aptitude	5	5	15
2	Engineering Mathematics	5	7	19
3	Theory of Computation	1	2	5
4	Digital Logic	2	1	4
5	Computer Organization & Architecture	3	4	11
6	Programming & Data Structures	4	4	12
7	Algorithm	1	3	7

#	Chapters	Marks		
		1	2	Total
8	Compiler Design	2	2	6
9	Operating System	4	3	10
10	Data Bases	0	2	4
11	Computer Networks	3	2	7
12	*Software Engineering	0	0	0
13	*Web Technology	0	0	0

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GENERAL APTITUDE

1. "From where are they bringing their books? ______ bringing ______ books from ______."

The words that best fill the blanks in the above sentence are

(*a*) Their, they're, there
(*b*) They're, their, there
(*c*) There, their, they're
(*d*) They're, there, there

2. "A ______investigation can sometimes yield new facts, but typically organized ones are more successful."

The word that best fills the blank in the above sentence is

(*a*) meandering (*b*) timely
(*c*) consistent (*d*) systematic

3. The area of a square is d. What is the area of the circle which has the diagonal of the square as its diameter?

(*a*) πd (*b*) πd^2
(*c*) $\frac{1}{4}\pi d^2$ (*d*) $\frac{1}{2}\pi d$

4. What would be the smallest natural number which when divided either by 20 or by 42 or by 76 leaves a remainder of 7 in each case?

(*a*) 3047 (*b*) 6047
(*c*) 7987 (*d*) 63847

5. What is the missing number in the following sequence?

2, 12, 60, 240, 720, 1440, _______, 0

(*a*) 2880 (*b*) 1440
(*c*) 720 (*d*) 0

6. In appreciation of the social improvements completed in a town, a wealthy philanthropist decided to gift Rs. 750 to each male senior citizen in the town and Rs. 1000 to each female senior citizen. Altogether, there were 300 senior citizens eligible for this gift. However, only 8/9th of the eligible men and 2/3rd of the eligible women claimed the gift. How much money (in Rupees) did the philanthropist give away in total?

(*a*) 1,50,000 (*b*) 2,00,000
(*c*) 1,75,000 (*d*) 1,51,000

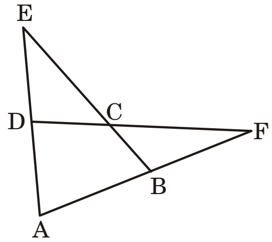
7. In $pqr \neq 0$ and $p^{-x} = \frac{1}{q}, q^{-y} = \frac{1}{r}, r^{-z} = \frac{1}{p}$, what is the value of the product xyz?

(*a*) –1 (*b*) $\frac{1}{pqr}$
(*c*) 1 (*d*) pqr

8. In a party, 60% of the invited guests are male and 40% are female. If 80% of the invited guests attended the party and if all the invited female guests attended, what would be the ratio of males of females among the attendees in the party?

(*a*) 2 : 3 (*b*) 1 : 1
(*c*) 3 : 2 (*d*) 2 : 1

9. In the figure below, $\angle DEC + \angle BFC$ is equal to ___.



(*a*) $\angle BCD - \angle BAD$
(*b*) $\angle BAD + \angle BCF$
(*c*) $\angle BAD + \angle BCD$
(*d*) $\angle CBA + \angle ADC$

10. A six sided unbiased die with four green faces and two red faces is rolled seven times. Which of the following combinations is the most likely outcome of the experiment?

(*a*) Three green faces and four red faces.
(*b*) Four green faces and three red faces.
(*c*) Five green faces and two red faces.
(*d*) Six green faces and one red face.

COMPUTER SCIENCE

1. Which one of the following is a closed form expression for the generating function of the sequence $\{a_n\}$, where $a_n = 2n + 3$ for all $n = 0, 1, 2, ...$?

(*a*) $\frac{3}{(1-x)^2}$ (*b*) $\frac{3x}{(1-x)^2}$
(*c*) $\frac{2-x}{(1-x)^2}$ (*d*) $\frac{3-x}{(1-x)^2}$

2. Consider the following C program:

```
#include <stdio.h>
struct ournode {
    char x, y, z;
} ;
int main () {
    struct ournode p = {'1', '0', 'a' + 2};
    struct ournode *p = &q;
    printf("%c, %c", *((char*) q + 1), *((char*) q + 2));
    return 0;
}
```

The output of this program is:

(*a*) 0, c (*b*) 0, a + 2
(*c*) '0', 'a + 2' (*d*) '0', 'c'

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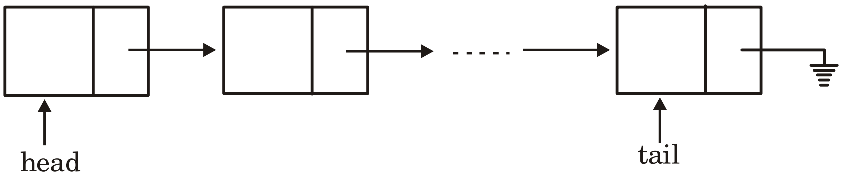
3. A queue is implemented using a non-circular singly linked list. The queue has a head pointer and tail pointer, as shown in the figure. Let n denote of number of nodes in the queue. Let enqueue be implemented by inserting a new node at the head and dequeue be implemented by deletion of a node from the tail.



Which one of the following is the time complexity of the most time-efficient implementation of enqueue and dequeue, respectively, for this data structure?

(*a*) $\theta(1), \theta(1)$ (*b*) $\theta(1), \theta(n)$

(*c*) $\theta(n), \theta(1)$ (*d*) $\theta(n), \theta(n)$

4. Let $\oplus$ and $\odot$ denote the Exclusive OR and Exclusive NOR operations, respectively. Which one of the following is NOT CORRECT?

(*a*) $\overline{P \oplus Q} = P \odot Q$

(*b*) $\overline{P} \oplus Q = P \odot Q$

(*c*) $\overline{P} \oplus \overline{Q} = PQ\oplus$

(*d*) $(P \oplus \overline{P}) \oplus Q = (P \odot \overline{P}) \odot \overline{Q}$

5. Consider the following processor design characteristics:

I. Register-to-register arithmetic operations only.

II. Fixed-length instruction format.

III. Hardwired control unit.

Which of the characteristics above are used in the design of a RISC processor?

(*a*) I and II only (*b*) II and III only

(*c*) I and III only (*d*) I, II and III

6. Let N be an NFA with n states, Let k be the number of states of a minimal DFA which is equivalent to N. Which one of the following is necessarily true?

(*a*) $k \geq 2^n$ (*b*) $k \geq n$

(*c*) $k \leq n^2$ (*d*) $k \leq 2^n$

7. The set of all recursively enumerable language is

(*a*) closed under complementation

(*b*) closed under intersection

(*c*) a subset of the set of all recursive languages

(*d*) an uncountable set

8. Which one of the following statements is FALSE?

(*a*) Context-free grammar can be used to specify both lexical and syntax rules.

(*b*) Type checking is done before parsing.

(*c*) High-level language programs can be translated to different intermediate Representations.

(*d*) Arguments to a function can be passed using the program stack.

9. The following are some events that occur after a device controller issues an interrupt while process L is under execution.

(P) The processor pushes the process status of L onto the control stack.

(Q) The processor finishes the execution of the current instruction.

(R) The processor executes the interrupt service routine.

(S) The processor pops the process status of L from the control stack.

(T) The processor loads the new PC value based on the interrupt.

Which one of the following is the correct order in which the events above occur?

(*a*) QPTRS (*b*) PTRSQ

(*c*) TRPQS (*d*) QTPRS

10. Consider a process executing on an operating system that uses demand paging. The average time for a memory access in the system is M units if the corresponding memory page is available in memory and D units if the memory access causes a page fault. It has been experimentally measured that the average time taken for a memory access in the process is X units.

Which one of the following is the correct expression for the page fault rate experienced by the process?

(*a*) (D – M)/(X – M)

(*b*) (X – M)/(D – M)

(*c*) (D – X)/(D – M)

(*d*) (X – M)/(D – X)

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11. In an Entity-Relationship (ER) model, suppose R is a many-to-one relationship from entity set E1 to entity set E2. Assume that E1 and E2 participate totally in R and that the cardinality of E1 is greater than the cardinality of E2.

Which one of the following is true about R?

(*a*) Every entity in E1 is associated with exactly one entity in E2.

(*b*) Some entity in E1 is associated with more than one entity in E2.

(*c*) Every entity in E2 is associated with exactly one entity in E1.

(*d*) Every entity in E2 is associated with at most one entity in E1.

12. Consider the following two tables and four queries in SQL.

Book (isbn, bname), Stock (isbn, copies)

Query 1:

SELECT B.isbn, S copies
FROM Book B INNER JOIN Stock S
ON B. isbn = S.isbn;

Query 2:

SELECT B. isbn, S.copies
FROM Book B LEFT OUTER JOIN Stock S
ON B. isbn = S.isbn;

Query 3:

SELECT B. isbn, S.copies
FROM Book B RIGHT OUTER JOIN Stock S
ON B. isbn = S.isbn;

Query 4:

SELECT B. isbn, S.copies
FROM Book B FULL OUTER JOIN Stock S
ON B. isbn = S.isbn;

Which one of the queries above is certain to have an output that is superset of the outputs of the other three queries?

(*a*) Query 1 (*b*) Query 2

(*c*) Query 3 (*d*) Query 4

13. Match the following:

Field	Length in bits
P. UDP Header's Port Number	I. 48
Q. Ethernet MAC Address	II. 8
R. IPv6 Next Header	III. 32
S. TCP Header's Sequence Number	IV. 16

(*a*) P-III, Q-IV, R-II, S-I

(*b*) P-II, Q-I, R-IV, S-III

(*c*) P-IV, Q-I, R-II, S-III

(*d*) P-IV, Q-I, R-III, S-II

14. Consider the following statements regarding the slow start phase of the TCP congestion control algorithm. Note the *cwnd* stands for the TCP congestion window and MSS denotes the Maximum Segment Size.

(*i*) The *cwnd* increases by 2 MSS on every successful acknowledgment.

(*ii*) The *cwnd* approximately doubles on every successful acknowledgment.

(*iii*) The *cwnd* increases by 1 MSS every round trip time.

(*iv*) The *cwnd* approximately doubles every round trip time.

Which one of the following is correct?

(*a*) Only (*ii*) and (*iii*) are true

(*b*) Only (*i*) and (*iii*) are true

(*c*) Only (*iv*) is true

(*d*) Only (*i*) and (*iv*) are true

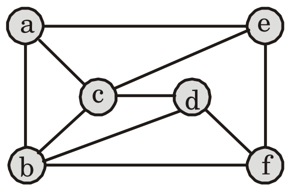
15. Two people. P and Q, decide to independently roll two identical dice, each with 6 faces, Numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial a throw of the dice by P and Q, Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is _____.

16. The value of $\int_0^{\pi/4} x\cos\left(x^2\right)dx$ correct to three decimal places (assuming that $\pi = 3.14$) is ______.

17. Consider a matrix $A = uv^T$ where $u = \begin{pmatrix}1\\2\end{pmatrix}, v = \begin{pmatrix}1\\1\end{pmatrix}$.

Note that v^T denotes the transpose of v. The largest eigenvalue of A is _____.

18. The chromatic number of the following graph is ______.



19. Let G be a finite group on 84 elements. The size of a largest possible proper subgroup of G is _____.

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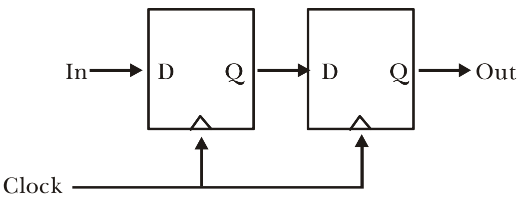
20. The post order traversal of a binary tree is 8, 9, 6, 7, 4, 5, 2, 3, 1. The inorder traversal of the same tree is 8, 6, 9, 4, 7, 2, 5, 1, 3. The height of a tree is the length of the longest path from the root to any leaf. The height of the binary tree above is ______.

21. Consider the following C program:

```
#include <stdio.h>
int counter = 0;
int calc (int a, int b) {
    int c;
    counter ++;
    if (b = = 3) return (a*a*a);
    else {
        c = calc (a b/3);
        return (c*c*c);
    }
}
int main ( ) {
    calc(4, 81);
    print("%%d", counter);
}
```

The output of this program is _____.

22. Consider the sequential circuit shown in the figure, where both flip-flops used are positive edge-triggered D flip-flops.



The number of states in the state transition diagram of the circuit that have a transition back to the same state on some value of "in" is ______.

23. A 32-bit wide main memory unit with a capacity of 1 GB is built using 256 M × 4 bit- DRAM chips. The number of rows of memory cells in the DRAM chip is 2^{14}. The time taken to perform one refresh operation is 50 nanoseconds. The refresh period is 2 milliseconds. The percentage (rounded to the closest integer) of the time available for performing the memory read/write operations in the main memory unit is ______.

24. Consider a system with 3 processes that share 4 instance of the same resource type. Each process can request a maximum of K instances. Resource instances can be requested and released only one at a time. The largest value of K that will always avoid deadlock is ______.

25. Consider a long-lived TCP session with an end-to-end bandwidth of 1 Gbps (= 10^9 bits-per-second). The session starts with a sequence number of 1234. The minimum time (in seconds, rounded to the closest integer) before this sequence number can be used again is _______.

26. Consider a matrix P whose only eigenvectors are the multiples of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Consider the following statements:

I. P does not have an inverse.

II. P has a repeated eigenvalue.

III. P cannot be diagonalized.

Which one of the following options is correct?

(*a*) Only I and III are necessarily true

(*b*) Only II is necessarily true

(*c*) Only I and II are necessarily true

(*d*) Only II and III are necessarily true

27. Let N be the set of natural numbers. Consider the following sets:

P. Set of rational numbers (positive and negative).

Q. Set of functions from {0, 1} to N.

R. Set of functions from N to {0, 1}.

S. Set of finite subsets of N.

Which of the sets above are countable?

(*a*) Q and S only

(*b*) P and S only

(*c*) P and R only

(*d*) P, Q and S only

28. Consider the first-order logic sentence

$$\varphi \equiv \exists s \exists t \exists u \forall v \forall w \forall x \forall y \psi(s,t,u,v,w,x,y)$$

where $\psi(s, t, u, v, w, x, y)$ is a quantifier-free first-order logic formula using only predicate symbols and possibly equality, but no function symbols. Suppose φ has a model with a universe containing 7 elements.

Which one of the following statements is necessarily true?

(*a*) There exists at least one model of φ with universe of size less than or equal to 3.

(*b*) There exists no model of φ with universe of size less than or equal to 3.

(*c*) There exists no model of φ with universe of size greater than 7.

(*d*) Every model of φ has a universe of size equal to 7.

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29. Consider the following C program:

```
#include <stdio.h>
void fun1 (char *s1, char *s2) {
    char *tmp;
    tmp = s1;
    s1 = s2;
    s2 = tmp;
}
void fun2 (char **s1, char **s2) {
    char *tmp;
    tmp = *s1;
    *s1 = *s2;
    *s2 = tmp;
}
int main () {
    char *str 1 = "Hi", *str2 = "Bye";
    fun1 (str1, str2); print f("%s %s, str1, str2);
    fun2 (&str1, &str2); print f("%s %s, str1, str2);
    return 0;
}
```

The output of the program above is

(*a*) Hi Bye Bye Hi (*b*) Hi Bye Hi Bye

(*c*) Bye Hi Hi Bye (*d*) Bye Hi Bye Hi

30. Let G be a simple undirected graph. Let T_D be a depth first search tree of G. Let T_B be a breadth first search tree of G. Consider the following statements:

I. No edge of G is a cross edge with respect to T_D. (A cross edge in G is between two nodes neither of which is an ancestor of the other in T_D).

II. For every edge (u, v) of G, if u is at depth i and v is at depth j in T_B, then $|i - j| = 1$.

Which of the statements above must necessarily be true?

(*a*) I only (*b*) II only

(*c*) Both I and II (*d*) Neither I nor II

31. Assume that multiplying a matrix G_1 of dimension p × q with another matrix G_2 of of dimension q × r requires *pqr* scalar multiplications. Computing the product of n matrices $G_1G_2G_3...G_n$ can be done by parenthesizing in different ways. Define $G_i\,G_{i+1}$ as an explicitly computed pair for a given paranthesization if they are directly multiplied. For example, in the matrix multiplication chain $G_1G_2G_3G_4G_5G_6$ using parenthesization $(G_1(G_2G_3))(G_4(G_5G_6))$. G_2G_3 and G_5G_6 are the only explicitly computed pairs.

Consider a matrix multiplication chain $F_1F_2F_3F_4F_5$, where matrices F_1, F_2, F_3, F_4 and F_5 are of dimensions 2 × 25, 25 × 3, 3 × 16, 16 × 1 and 1 × 1000, respectively. In the parenthesization of $F_1F_2F_3F_4F_5$ that minimizes the total number of scalar multiplications, the explicitly computed pairs is/are

(*a*) F_1F_2 and F_3F_4 only (*b*) F_2F_3 only

(*c*) F_3F_4 only (*d*) F_1F_2 and F_4F_5 only

32. Consider the following C code. Assume that unsigned long int type length is 64 bits.

```
unsigned long int fun (unsigned long int n) {
    unsigned long int i, j = 0, sum = 0;
    for (i = n; i > 1; i = i/2) j++;
    for (; j > 1; j = j/2) sum ++;
    return (sum);
}
```

The value returned when call fun with the input 2^{40} is

(*a*) 4 (*b*) 5

(*c*) 6 (*d*) 40

33. Consider the unsigned 8-bit fixed point binary number representation below:

$b_7\,b_6\,b_5\,b_4\,b_3\,.\,b_2\,b_1\,b_0$

where the position of the binary points is between b_3 and b_2. Assume b_7 is the most significant bit. Some of the decimal numbers listed below cannot be represented exactly in the above representation:

(i) 31.500 (ii) 0.875

(iii) 12.100 (iv) 3.001

Which one of the following statements is true?

(*a*) None of (i), (ii), (iii), (iv) can be exactly represented.

(*b*) Only (ii) cannot be exactly represented.

(*c*) Only (iii) and (iv) cannot be exactly represented.

(*d*) Only (i) and (ii) cannot be exactly represented.

34. The size of the physical address space of a processor is 2^P bytes. The word length is 2^W bytes. The capacity of cache memory is 2^N bytes, The size of each cache block is 2^M words. For a K-way set-associative cache memory, the length (in number of bits of the tag field is

(*a*) $P - N - \log_2 K$

(*b*) $P - N + \log_2 K$

(*c*) $P - N - M - W - \log_2 K$

(*d*) $P - N - M - W + \log_2 K$

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35. Consider the following languages:

I. $\{a^m b^n c^p d^q \mid m + p = n + q, \text{ where } m, n, p, q \geq 0\}$

II. $\{a^m b^n c^p d^q \mid m = n \text{ and } p = q, \text{ where } m, n, p, q \geq 0\}$

III. $\{a^m b^n c^p d^q \mid m = n = p \text{ and } p \neq q, \text{ where } m, n, p, q \geq 0\}$

IV. $\{a^m b^n c^p d^q \mid mn = p + q, \text{ where } m, n, p, q \geq 0\}$

Which of the language above are context-free?

(*a*) I and IV only (*b*) I and II only

(*c*) II and III only (*d*) II and IV only

36. Consider the following L(G) denotes the language generated by a grammar G. L(M) denotes the language accepted by a machine M.

I. For an unrestricted grammar G and a string w, whether $w \in$ L(G).

II. Given a Turing Machine M, whether L(M) is regular.

III. Given two grammars G_1 and G_2, whether $L(G_1) = L(G_2)$.

IV. Given an NFAN, whether there is a deterministic PDA P such that N and P accept the same language.

Which one of the following statements is correct?

(*a*) Only I and II are undecidable

(*b*) Only III is undecidable

(*c*) Only II and IV are undecidable

(*d*) Only I, II and III are undecidable

37. A lexical analyzer uses the following patterns to recognize three tokens T_1, T_2 and T_3 over the alphabet {a, b, c}.

T_1 : a? (b | c)*a

T_2 : b? (a | c)*b

T_3 : c? (b | a)*c

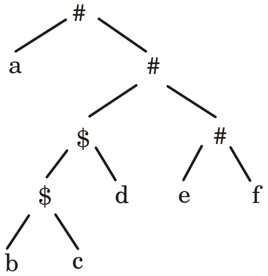
Note that 'x?' means 0 or 1 occurrence of the symbol x. Note also that the analyzer outputs the token that matches the longest possible prefix.

If the string bbaacabc is processed by the analyzer, which one of the following is the sequence of tokens it outputs?

(*a*) $T_1T_2T_3$ (*b*) $T_1T_1T_3$

(*c*) $T_2T_1T_3$ (*d*) T_3T_3

38. Consider the following parse tree for the expression a#bcd#e#f, involving two binary operators $ and #.



Which one of the following is correct for the given parse tree?

(*a*) $ has higher precedence and is left associative; # is right associative

(*b*) # has higher precedence and is left associative; $ is right associative

(*c*) $ has higher precedence and is left associative; # is left associative

(*d*) # has higher precedence and is right associative; $ is left associative

39. In a system, there are three types of resources: E, F and G. Four processes P_0, P_1, P_2 and P_3 execute concurrently. At the outset, the processes have declared their maximum resource requirements using a matrix named Max as given below. For example Max[P_2,F] is the maximum number of instances of F that P_2 would require. The number of instances of the resources allocated to the various processes at any given state is given by a matrix named Allocation.

Consider a state of the system with the Allocation matrix as shown below and in which 3 instances of E and 3 instances of F are the only resources available.

Allocation			
	E	F	G
P_0	1	0	1
P_1	1	1	2
P_2	1	0	3
P_3	2	0	0

Max			
	E	F	G
P_0	4	3	1
P_1	2	1	4
P_2	1	3	3
P_3	5	4	1

From the perspective of deadlock avoidance, which one of the following is true?

(*a*) The system is in safe state.

(*b*) The system is not in safe state, but would be safe if one more instance of E were available.

(*c*) The system is not in safe state, but would be safe if one more instance of F were available.

(*d*) The system is not in safe state, but would safe if one more instance of G were available.

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40. Consider the following solution to the producer-consumer synchronization problem. The shared buffer size is N. Three semaphores empty, full and mutex are defined with respective initial values of 0, N and 1. Semaphore empty denotes the number of available slots in the buffer, for the consumer to read from. Semaphore full denotes the number of available slots in the buffer, for the producer to write to. The placeholder variables, denoted by P, Q, R and S, in the code below can be assigned either empty or full. The valid semaphore operations are: wait ( ) and signal ( ).

Producer	Consumer

do {
    wait (P);
    wait (mutex);
    //Add item to buffer
    signal (mutex);
    signal (Q);
} while (1);
	do {
	    wait (R);
	    wait (mutex);
	    //Consume item from buffer
	    signal (mutex);
	    signal (S);
	} while (1);

Which one of the following assignment to P, Q, R and S will yield the correct solution?

(*a*) P : full, Q : full, R: empty, S: empty (*b*) P : empty, Q : empty, R: full, S: full

(*c*) P : full, Q : empty, R: empty, S: full (*d*) P : empty, Q : full, R: full, S: empty

41. Consider the relations $r(A, B)$ and $s(B, C)$, where $s.B$ is a primary key and $r.B$ is a foreign key referencing $s.B$. Consider the query

$Q : r \bowtie (\sigma_{B<5}(s))$

Let LOJ denote the natural left outer-join operation. Assume that r and s contain no null values.

Which one of the following queries is not equivalent to Q?

(*a*) $\sigma_{B<5}(r \bowtie s)$

(*b*) $\sigma_{B<5}(r \text{LOJ } s)$

(*c*) $r\text{LOJ }(\sigma_{B<5}(S))$

(*d*) $\sigma_{B<5}(r)$ LOJ s

42. Consider the following relational schemas. For each schema, all non-trivial functional dependencies are listed. The underlined attributes are the respective primary keys.

Schema I: Registration (<u>rollno</u>, courses)

Field 'courses' is a set-valued attribute containing the set of courses a student has registered for.

Non-trivial functions dependency:

rollno $\rightarrow$ courses

Schema II: Registration (<u>rollno</u>, courseid, email)

Non-trivial functional dependencies.

rollno, courseid $\rightarrow$ email

email $\rightarrow$ rollno

Schema III: Registration (<u>rollno</u>, <u>courseid</u>, marks, grade)

Non-trivial functional dependencies.

rollno, courseid $\rightarrow$ marks, grade

marks $\rightarrow$ grade

Schema IV: Registration (<u>rollno, *courseid*</u>, credit)

Non-trivial functional dependencies.

rollno, courseid $\rightarrow$ credit

courseid $\rightarrow$ credit

Which one of the relational schemas above is in 3NF but not in BCNF?

(*a*) Schema I (*b*) Schema II

(*c*) Schema III (*d*) Schema IV

43. Let G be a graph with 100! vertices, with each vertex labelled by a distinct permutation of the numbers 1, 2, ... 100. There is an edge between vertices u and v if and only if the label of u can be obtained by swapping two adjacent numbers in the label of v. Let y denote the degree of a vertex in G and z denote the number of connected components is G.

Then, $y + 10z$ = ______.

44. Consider Guwahati (G) and Delhi (D) whose temperatures can be classified as high (H), medium (M) and low (L). Let $P(H_G)$ denote the probability that Guwahati has high temperature. Similarly, $P(M_G)$ and $P(L_G)$ denotes the probability of Guwahati having medium and low temperatures respectively. Similarly, we use $P(H_D)$, $P(M_D)$ and $P(L_D)$ for Delhi.

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The following table gives the conditional probabilities for Delhi's temperature given Guwahati's temperature.

	H_D	M_D	L_D
M_G	0.40	0.48	0.12
M_G	0.10	0.65	0.25
L_G	0.01	0.50	0.49

Consider the first row in the table above. The first entry denotes that if Guwahati has high temperature (H_G) then the probability of Delhi also having a high temperature (H_D) is 0.40; i.e, $P(H_D | H_G) = 0.40$. Similarly, the next two entries are $P(M_D | H_G) = 0.48$ and $P(L_D | H_G) = 0.12$. Similarly for the other rows.

If it is known that $P(H_G) = 0.2$, $P(M_G) = 0.5$ and $P(L_G) = 0.3$, then the probability (correct to two decimal places) that Guwahati has high temperature given that Delhi has high temperature is ______.

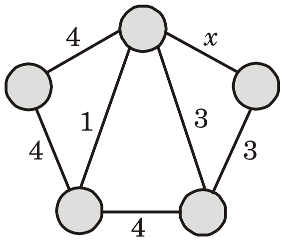
45. Consider the following program written in pseudo-code. Assume that x and y are integers:

```
Count (x, y) {
    if (y ! = 1) {
        if (x ! = 1) {
            print ("*");
            Count (x/2, y);
        }
        else {
            y = y – 1;
            Count (1024, y);
        }
    }
}
```

The number of times that the print statement is executed by the call Count (1024, 1024) is _____.

46. The number of possible min-heaps containing each value from {1, 2, 3, 4, 5, 6, 7} exactly once is _____.

47. Consider the following undirected graph G:



Choose a value of x that will maximize the number of minimum weight spanning trees (MWSTs) of G. The number of MWSTs of G for this value of x is ______.

48. Consider the weights and values of items listed below. Note that there is only one unit of each item.

Item Number	Weight in (Kgs)	Value in(Rupees)
1	10	60
2	7	28
3	4	20
4	2	24

The task is to pick subset of these items such that their total weight is not more than 11 Kgs and their total value is maximized. Moreover, no item may be split. The total value of items picked by an optimal algorithm is denoted by V_{opt}. A greedy algorithm sorts the items by their value-to-weight ratios in descending order and packs them greedily, starting from the first item in the ordered list. The total value of items picked by the greedy algorithm is denoted by V_{greedy}.

The value of $V_{opt} - V_{greedy}$ is ______.

49. Consider the minterm list form of a Boolean function F given below:

$F(P, Q, R, S) = \Sigma m(0, 2, 5, 7, 9, 11) + d(3, 8, 10, 12, 14)$

Here, m denotes a minterm and d denotes a don't care term. The number of essential prime implicants of the function F is ______.

50. The instruction pipeline of a RISC processor has the following stages. Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO) and Writeback (WB). The IF, ID, OF and WB stages take 1 clock cycle each for every instruction. Consider a sequence of 100 instructions. In the PO stage, 40 instructions take 3 clock cycles each, 35 instructions take 2 clock cycles each, and the remaining 25 instructions take 1 clock cycle each. Assume that there are no data hazards and no control hazards. The number of clock cycles required for completion of execution of the sequence of instructions is ______.

51. A processor has 16 integer registers (R0, R1, ..., R15) and 64 floating point registers (F0, F1, ..., F63). It uses a 2-byte instruction format. There are four categories of instructions: Type-1, Type-2, Type-3 and Type-4. Type-1 category consists of four instructions, each with 3 integer register operands (3Rs). Type-2 category consists of eight

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instructions, each with 2 floating point register operands (2F2). Type-3 category consists of fourteen instructions, each with one integer register operand and one floating point register operand (1R + 1F). Type-4 category consists of N instructions, each with a floating point register operand (IF).

The maximum value of N is ______.

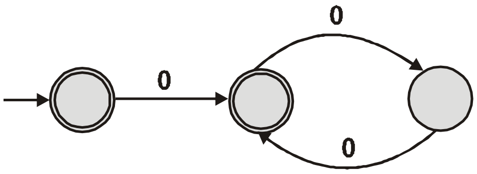
52. Given a language L, define L^i as follows:

$L^0 = \{\in\}$

$L^i = L^{i-1}.\ L$ for all $i > 0$

The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$.

Consider the language L_1 (over alphabet 0) accepted by the following automation.



The order of L_1 is ______.

53. Consider a storage disk with 4 platters (numbered as 0, 1, 2 and 3), 200 cylinders (numbered as 0, 1, ..., 199) and 256 sectors per track (numbered as 0, 1, ..., 255). The following 6 disk requests of the form [sector number, cylinder number, platter number] are received by the disk controller at the same time:

[120, 72, 2], [180, 134, 1], [60, 20, 0], [212, 86, 3,], 56, 116, 2], [118, 16, 1]

Currently the head is positioned at sector number 100 of cylinder 80 and is moving towards higher cylinder numbers. The average power dissipation in moving the head over 100 cylinders is 20 milliwatts and for reversing the direction of the head movement once is 15 milliwatts. Power dissipation associated with rotational latency and switching of head between different platters is negligible.

The total power consumption in milliwatts to satisfy all of the above disk requests using the Shortest Seek Time First scheduling algorithm is ______.

54. Consider an IP packet with a length of 4500 bytes that includes a 20-byte IPv4 header and a 40-byte TCP header. The packet is forwarded to an IPv4 router that supports a Maximum Transmission Unit (MTU) of 600 bytes. Assume that the length of the IP header in all the outgoing fragments of this packet is 20 bytes. Assume that the fragmentation offset value stored in the first fragment is 0.

The fragmentation offset value stored in the third fragment is ______.

55. Consider a simple communication system where multiple nodes are connected by a shared broadcast medium (like Ethernet or wireless). The nodes in the system use the following carrier-sense based medium access protocol. A node that receives a packet to transmit will carrier-sense the medium for 5 units of time. If the node does not detect any other transmission in this duration, it starts transmitting its packet in the next time unit. If the node detects another transmission, it waits until this other transmission finishes, and then begins to carrier-sense for 5 time units again. Once they start to transmit, nodes do not perform any collision detection and continue transmission even if a collision occurs. All transmissions last for 20 units of time. Assume that the transmission signal travels at the speed of 10 meters per unit time in the medium. Assume that the system has two nodes P and Q, located at a distance d meters from each other. P starts transmitting a packet at time t = 0 after successfully completing its carrier-sense phase. Node Q has a packet to transmit at time t = 0 and begins to carrier-sense the medium.

The maximum distance d(in meters, rounded to the closest integer) that allows Q to successfully avoid a collision between its proposed transmission and P's ongoing transmission is ______.

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ANSWERS

General Aptitude (GA)

1. (*b*)	**2.** (*a*)	**3.** (*d*)	**4.** (*c*)	**5.** (*b*)	**6.** (*b*)	**7.** (*c*)	**8.** (*b*)	**9.** (*a*)	**10.** (*c*)

Technical Section

1. (*d*)	**2.** (*a*)	**3.** (*b*)	**4.** (*d*)	**5.** (*d*)	**6.** (*d*)	**7.** (*b*)
8. (*b*)	**9.** (*a*)	**10.** (*b*)	**11.** (*a*)	**12.** (*d*)	**13.** (*c*)	**14.** (*c*)
15. 0.023	**16.** 0.289	**17.** 3	**18.** (3)	**19.** 42	**20.** 4	**21.** 4
22. 2	**23.** (60)	**24.** 2	**25.** 34	**26.** (*d*)	**27.** (*d*)	**28.** (*a*)
29. (*a*)	**30.** (*a*)	**31.** (*c*)	**32.** (*b*)	**33.** (*c*)	**34.** (*b*)	**35.** (*b*)
36. (*d*)	**37.** (*d*)	**38.** (*a*)	**39.** (*a*)	**40.** (*c*)	**41.** (*c*)	**42.** (*b*)
43. 109	**44.** 0.60	**45.** 10230	**46.** 80	**47.** 4	**48.** 16	**49.** 3
50. 219	**51.** 32	**52.** 2	**53.** 85	**54.** 144	**55.** 50	

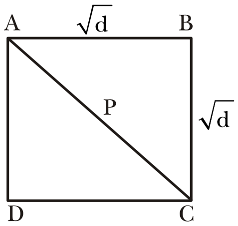
EXPLANATIONS

GENERAL APTITUDE

1. They're used for pointing group.
Their is pointing people.
There is used for place.

2. Meandering : wandering aimlessly/indirect.

3.



Area of square = d

Side one side of square = $\sqrt{d}$

Now Diagonal of square (AC) = $\sqrt{d+d} = \sqrt{2d}$

Now, Area of circle = πr^2

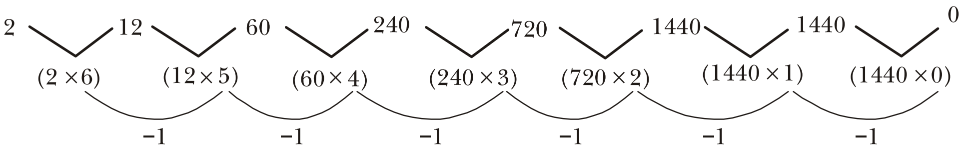
$$= \pi\left(\frac{\sqrt{2d}}{2}\right)^2 \quad \left(\because r = \frac{\text{diameter}}{2}\right)$$

$$= \pi\frac{d}{2}$$

$$= \frac{1}{2}\pi d$$

4. Number is divided by either by 20 or 42 or by 76
K × LCM (20, 42, 76) + constant difference
= 7890 K + 7 (K is natural number)
Least number will be 7890 + 7 = 7897.

5.



So, 1440 must be answer.

6. Male (M) + Female (F) = 300

$$\text{Total money} = \frac{8}{9}\text{M}\times 750 + \frac{2}{3}\text{F}\times 1000 \quad ...(i)$$

$$= \frac{6000}{9}\text{M} + \frac{6000}{9}\text{F} = \frac{6000}{9}(\text{M}+\text{F})$$

From equation (i) we get,

$$\text{Total money} = \frac{6000\times 300}{9} = 2{,}00{,}000$$

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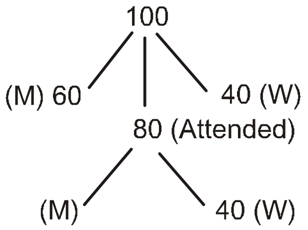
7. $p^x = q \Rightarrow x \log p = \log q$

and $q^y = r \Rightarrow y \log q = \log r$

and $z^2 = p \Rightarrow z \log r = \log p$

$\therefore \quad xyz = \dfrac{\log q}{\log p} \times \dfrac{\log r}{\log q} \times \dfrac{\log p}{\log r} = 1$

8. Let total number of people are 100.



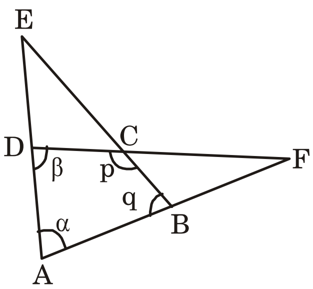
So, male attended the party 80 – 40 = 40

Ratio of male to female who attended the party

M : W = 40 : 40

= 1 : 1

9.



$\angle E + \angle F = ?$

$\alpha + q + E = 180$ ....(i)

and $\alpha + \beta + F = 180$ ....(ii)

and $\alpha + \beta + p + q = 360$ ....(iii)

Adding equations (i) and (ii) and equating it to (iii) we get

$\alpha + q + E + \alpha + \beta + F = \alpha + \beta + p + q$

$\therefore \quad E + F = p - \alpha$

10. Four green, two red face

$$P(G) = \frac{4}{6} = \frac{2}{3}$$

$$q(R) = 1 - P(G)$$

$$= \frac{1}{3}$$

$$n = 7$$

Now from options

From option (a), $P(G = 3) = {}^7C_3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{4}\right)^4$

$$\frac{35 \times 2^3}{(3)^7} = \frac{35 \times 2^3}{(3)^7}$$

From option (b), $P(G = 4) = {}^7C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^3$

$$\frac{35 \times 2^4}{(3)^7} = \frac{35 \times 2^4}{(3)^7}$$

From option (c), $P(G = 5) = {}^7C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^2$

$$\frac{21 \times 2^5}{(3)^7} = \frac{42 \times 2^4}{(3)^7}$$

From option (d), $P(G = 6) = {}^7C_6 \left(\frac{2}{3}\right)^6 \times \left(\frac{1}{3}\right)$

$$\frac{7 \times 2^6}{(3)^7} = \frac{28 \times 2^4}{(3)^7}$$

Option c is maximum value.

COMPUTER SCIENCE

1. Given, $a_n = 2n + 3$

Since generating function for 1 is $\dfrac{1}{1-x}$ and n is $\dfrac{x}{(1-x)^2}$, the generating function for a_n is

$$A(x) = \frac{2x}{(1-x)^2} + \frac{3}{1-x}$$

$$\frac{2x + 3(1-x)}{(1-x)^2} = \frac{3-x}{(1-x)^2}$$

which is option (d).

4. (A) $\overline{P \oplus Q} = P \odot Q$ true

(B) $\bar{P} \oplus Q = P \odot Q$ true

(C) $\bar{P} \oplus \bar{Q} = P \oplus Q$ true

(D) $(P \oplus \bar{P}) \oplus Q = (P \odot \bar{P}) \odot \bar{Q}$ false

$(P \oplus \bar{P}) \oplus Q = 1 \oplus Q = \bar{Q}$

$(P \odot \bar{P}) \odot \bar{Q} = 0 \oplus \bar{Q} = Q$

$(P \oplus \bar{P}) \oplus Q \neq (P \odot \bar{P}) \odot \bar{Q}$

5. RISC processor characteristics:

1. It supports mode registers, so ALU operations are performed only on a register data.
2. It support fixed length instructions.
3. It uses hard-wired control unit.

6. n is number of states of given nfa (may not be minimal) k is number of states of equivalent min dfa. First we convert nfa to dfa using subset construction algorithm and we get an equivalent dfa which will have almost 2^n states. Then we can convert this dfa to a minimal dfa and get a minimal dfa with k states where $k \le 2^n$.

7. The set of RE languages is closed under intersection, not closed under complementation, is not subset of set of REC language and is a countable set.

8. Type checking is done before parsing is clearly false because in compiler type checking is done after parsing phase.

9. Processor required to handle the interrupt:

1. Processor finishes the execution of the current instruction.
2. Processor pushes the process status of L onto the control stack.
3. Processor loads the new PC value based on the interrupt.
4. Processor executes the interrupt service routine.
5. Processor pops the process status of "L" from the control stack.

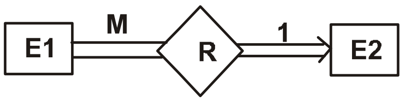
10. EMAT = P * S + (1 – P) * M

X = P * D + (1 – P) * M

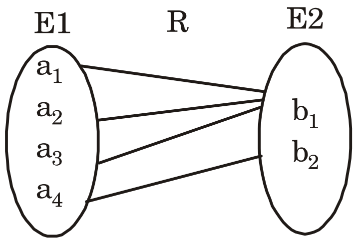
X = P * D + M – P * M

X – M = P (D – M)

$$P = \frac{X - M}{D - M}$$

11. 

E1 entries > E2 entities



Every entity in E1 associated with exactly one entity in E2.

12.

Book (isbn, bname)

isbn	bname
2	A
4	B
6	C
8	D
10	E

Stock (isbn, copies)

isbn	copies
4	100
6	200
10	200
12	400

Query 1:

isbn	copies
4	100
6	200
10	200

Query 2:

isbn	copies
4	100
6	200
10	200
2	Null
8	Null

Query 3:

isbn	copies
4	100
6	200
10	200
12	400

Query 4:

isbn	copies
4	100
6	200
10	200
2	Null
8	Null
12	400

Query 4 is full outer join so that full order join record set superset of records compare to inner join, left outer join & right outer join.

13. UDP Header's Port Number ⇒ 16 bit

Ethernet MAC Address ⇒ 48 bit

IPv6 Next Header ⇒ 8 bit

TCP Header's Sequence Number ⇒ 32 bit

14. *cwnd* approximately doubles every round trip time.

15. P(one of them wins in 3rd trial)

= P(1st trial is Tie) × P (IInd trial is Tie) × P(one of them wins 3rd trial)

P(Tie in any trial)

= P(P = 1 and Q = 1) + P(P = 2 and Q = 2) + ... + P(P = 6 and Q = 6)

$$= \frac{6}{36} = \frac{1}{6}$$

∴ P(one of them wins) = 1 – P(Tie)

$$= 1 - \frac{1}{6} = \frac{5}{6}$$

So, required probability

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{216} = 0.023$$

(rounded to 3 decimal places)

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16. $\int_0^{\pi/4} x\cos(x^2)\,dx$

Let, $t = x^2$

$\therefore \quad \dfrac{dt}{dx} = 2x$

$dt = 2x\,dx$

$\Rightarrow \quad xdx = xdx = \dfrac{dt}{2}$

when $x = 0 \Rightarrow t = 0$ and when $x = \dfrac{\pi}{4} \Rightarrow t = \left(\dfrac{\pi}{4}\right)^2$

So required integral reduce to

$$\int_0^{(\pi/4)^2} \cos t\,dt = [\sin t]_0^{(\pi/4)^2}$$

$$= \sin\left(\frac{\pi}{4}\right)^2 - \sin(0)$$

$$= \sin\left(\frac{\pi}{4}\right)^2 = 0.28898$$

$\simeq 0.289$

(rounded to 3 decimal places)

17. $\begin{pmatrix}1\\2\end{pmatrix}, v = \begin{pmatrix}1\\1\end{pmatrix}$

$A = uv^T$

$= \begin{pmatrix}1\\2\end{pmatrix}\begin{bmatrix}1 & 1\end{bmatrix}$

$= \begin{bmatrix}1 & 1\\2 & 2\end{bmatrix}$

Now $|A - \lambda I| = 0$

$$\left|\begin{bmatrix}1 & 1\\2 & 2\end{bmatrix} - \begin{bmatrix}\lambda & 0\\0 & \lambda\end{bmatrix}\right| = 0$$

$$\begin{vmatrix}1-\lambda & 1\\2 & 2-\lambda\end{vmatrix} = 0$$

$\therefore \quad (1-\lambda)(2-\lambda) - 2 = 0$

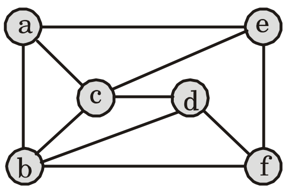
$\lambda^2 - 3\lambda = 0$

$\lambda(\lambda - 3) = 0$

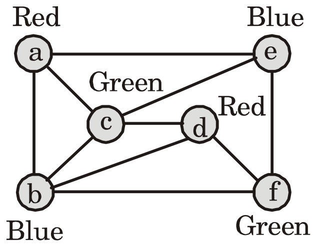
$\lambda = 0$ or $\lambda = 3$

$\therefore$ The largest eigen value is 3.

18.



Since the largest complete sub graph is K_3, chromatic number is atleast 3. We can try for a chromatic number of 3 by using 3 colors, as follows:



Since we have successfully, properly coloured all vertices with only 3 colors, the chromatic number of this graph is 3.

19. Given $|G| = 84$

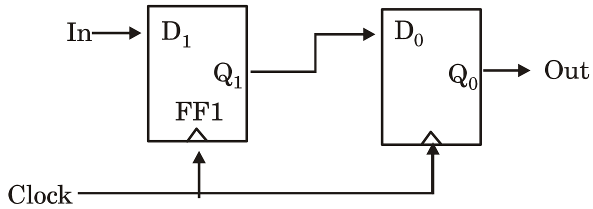
By Lagrang's theorem any subgroup size is a divisor of 84.

But a proper subgroup cannot have same size as group.

So largest divisor of 84, other than 84 is 42.

So, largest proper subgroup can have in size of 42.

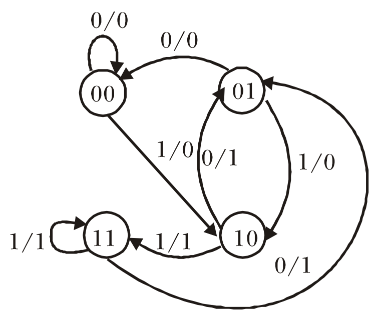
22.



State Table:

P.S		Input	FF Inputs		N.S		
Q_1	Q_0	x	D_1=x	D_0=Q_1	Q_1	Q_0	Out = Q_0
0	0	0	0	0	0	0	0
0	0	1	1	0	1	0	0
0	1	0	0	0	0	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	1	1
1	0	1	1	1	1	1	1
1	1	0	0	1	0	1	1
1	1	1	1	1	1	1	1

State Transition Diagram:



The question is the number of self loop states.

The number of self loop states are 00 and 11.

Hence answer is 2.

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24. P_1 P_2 P_3

| | | No deadlock

|

Maximum each process can request for 2 resources so, that there will not be any deadlock, because we, have only 4 resource available.

So, K value is '2'.

25. 1 sec = 10^9 bits

$$\frac{2^{32} \times 8}{10^9} = 2^{32} \text{ bytes}$$

$\Rightarrow$ 34.35 sec

26. Only Eigen vector is $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ multiples means that eigen value is repeated since if eigen values were distinct we will get one more independent eigen vector. So, II P has repeated eigen values is true. I need not be true since $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ has repeated eigen values and yet it is invertible. III is true since if matrix has repeated eigen values then it cannot be diagonalizable.

27. P : Set of rational number $\rightarrow$ countable

Q : Set of functions from {0, 1} to N $\rightarrow$ N

0	1

$N \times N$

0 can be assigned in N ways

1 can be assigned in N ways

There are N × N functions, cross product of countable set in countable.

R : Set of functions from N to {0, 1}

1	2	3	4	5	6				

Each of thus boxes can be assigned to 0 or 1 so each such function is a binary number with infinite number of bits.

Example: 0000 ..... is the binary number corresponding to 0 is assigned to all boxes and so on.

Since each such binary number represents a subset of N (the set of natural numbers) by characteristic function method, therefore, the set of such function is same as power set of N which is uncountable due to Cantor's theorem, which says that power set of a countably infinite set is always uncountably infinite.

S : Set of finite subsets of N $\rightarrow$ countably infinite since we are counting only finite subsets.

So, P, Q and S are countable.

33. Binary code: $(b_7\, b_6\, b_5\, b_5\, b_3 \,.\, b_2\, b_1\, b_0)_2$

$(31.5)_{10} = (11111{,}1)_2$

$(0.875)_{10} = (0.111)_2$

$(12.100)_{10} = (01100.000110....)_2$

$\downarrow$

only 3 bits of fraction space available

So can't be stored.

$(3.001)_{10} = (00011.000000.....)_2$

It is also not accurate storage.

34. MM space = 2^P bytes

Physical Address (PA) size = P bits

CM size = 2^N bytes

Block size = 2^M words

= 2^M words * 2^W bytes/word

= $2^{(M+W)}$ bytes

$$\text{Number of lines} = \frac{\text{CMsize}}{\text{Block size}} \Rightarrow \frac{2^N}{2^{M+W}}$$

$\Rightarrow 2^{N-M-W}$

$$\text{Number of sets} = \frac{\text{Number in cm}}{\text{P - way}}$$

$$= \frac{2^{N-M-W}}{K}$$

Address format

$\longrightarrow \underset{P\ bits}{PA} \longrightarrow$

tag	so	wo

$$\log_2 \frac{2^{N-M-W}}{K} \quad \log_2 2^{M+W}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\log_2 \frac{2^{N-M-W}}{\log_2 K} \quad (M+W)\,bits$$

$$\downarrow$$

$$\log_2 2^{N-M-W} - \log_2 K$$

$\Rightarrow$ (N – M – W – $\log_2$ K)

$\therefore$ Tag size

$\Rightarrow$ P – (M + W – $\log_2$ K + M + W)

$\Rightarrow$ P – N + $\log_2$ K

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35. I. $\{a^m b^n c^p d^q \mid m + p = n + q\}$ is clearly CFL since, we can rearrange the equation as m – n + p – q = 0 which can be done by push, pop, push and pop and check if stack is empty at end.

II. $\{a^m b^n c^p d^q \mid m = n \text{ and } p = q\}$ is clearly CFL since, one comparison at a time can be done by pda.

III. $\{a^m b^n c^p d^q \mid m = n = p \text{ and } p \neq q\}$ is not CFL since m = n = p is a double comparison which cannot be done by PDA.

IV. $\{a^m b^n c^p d^q \mid mn = p + q\}$ is not a CFL, since mn involves multiplying number of a's and number b's which cannot be done by a PDA.

So, only I and II are CFL's.

36. I. Membership problem RE $\rightarrow$ undecidable

II. Regularity problem for RE $\rightarrow$ undecidable

III. Equivalence problem for RE $\rightarrow$ undecidable

IV. Since DPDA P exists for every *nfa* N and equivalent to it, this problem is trivially decidable.

37. Ans is T_3T_3 because from first T_3 $\boxed{\text{bbaac}}$ is taken from second T_3 $\boxed{\text{abc}}$ is taken longest possible prefix.

Hence T_3T_3 token output.

38. If any given parse tree or syntax free low level operations having higher precedence than upper level operators.

Hence here $ is higher precedence than # $ is left associative because in the sub expression b $ c $ d, b $ c will be evaluated first as per given tree.

As per the given tree structure right # if higher precedence than left #.

Hence it is right associative.

39.

Max need			
	E	F	G
P_0	4	3	1
P_1	2	1	4
P_2	1	3	3
P_3	5	4	1

Current allocation			
	E	F	G
P_0	1	0	1
P_1	1	1	2
P_2	1	0	3
P_3	2	0	0

Current available			
	E(3)	F(3)	G(0)
P_0	4	3	1
P_1	5	3	4
P_2	6	4	6
P_3	8	4	6

Remaining need			
	E	F	G
P_0	3	3	0
P_1	1	0	2
P_2	0	3	0
P_3	3	4	1

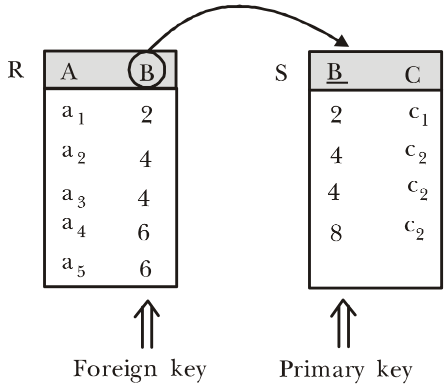
Safe sequence: P_0, P_2, P_1, P_3

Safe state

40. Full = N, empty = 0, mutex = 1

- Initially buffer will be empty, so consumer should not start first, so option b, D are eliminated.
- With option A consumer will never consume the item, so it is wrong.
- Option 'c' is correct answer which proper functionality of produce and consumer.

41.



Given query: r $\bowtie(\sigma_{B<5}(s))$

A	B	C
a_1	2	c_1
a_2	4	c_1
a_3	4	c_1

A : $\sigma_{B<5}(r \bowtie s)$

A	B	C
a_1	2	c_1
a_2	4	c_1
a_3	4	c_1

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B : $\sigma_{B<5}(r \bowtie s)$

A	B	C
a_1	2	c_1
a_2	4	c_1
a_3	4	c_1

C : $r \bowtie (\sigma_{B<5}(s))$

A	B	C
a_1	2	c_1
a_2	4	c_1
a_3	4	c_1
a_4	6	Null
a_5	6	Null

D : $\sigma_{B<5}(r) \bowtie s$

A	B	C
a_1	2	c_1
a_2	4	c_1
a_3	4	c_1

Option "C" query result not equal to given query.

42. Schema II : Registration (<u>rollno, courseid</u>, email)

Primary key [<u>rollno, courseid</u>]

Non-trivial functional dependencies:

{rollno, courseid → email

email → rollno}

candidate keys {<u>rollno, courseid,</u>}

<u>email courseid</u>}

Given relation is in 3NF but not in BCNF.

43. The graph has 100! vertices which each vertex labelled by one of the 100! permutation.

Let us find degree of each vertex.

Let us take a vertex whose labelling is say 1, 2, 3, 4,....100.

Now it will be connected to all vertices where exactly 2 of the adjacent numbers all swapped.

The two swapped numbers could be (1, 2), (2, 3), (3, 4) .... etc. upto (99, 100) which makes for 99 edges for each such vertex.

So the graph is a regular graph with each vertex connected to 99 other vertices.

So $\quad y = 99$

The number of connected components = z = 1 since we can go from any vertex to any other vertex by only swapping 2 adjacent number at a time, many times i.e. there is a path from any vertex to any other vertex.

Graph is connected.

So $\quad z = 1$

So $y + 10z = 99 + 10 \times 1 = 109$

44. The condition probability table given is

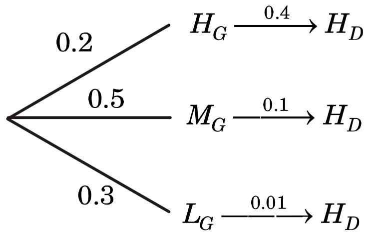
	H_D	M_D	L_D
M_G	0.40	0.48	0.12
M_G	0.10	0.65	0.25
L_G	0.01	0.50	0.49

$P(H_G) = 0.2$

$P(M_G) = 0.5$

$P(L_G) = 0.3$

Drawing the tree diagram for HD we get.



$$P(H_G \mid H_D) = \frac{P(H_G \cap H_D)}{P(H_D)}$$

From diagram,

$$P(H_G \cap H_D) = 0.2 \times 0.4$$

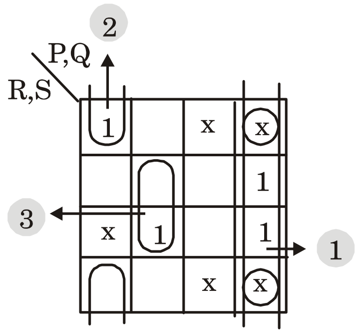
$$P(H_D) = 0.2 \times 0.4 + 0.5 \times 0.1 + 0.3 \times 0.01$$
$$= 0.133$$

Required probability, $P(H_G \mid H_D)$

$$= \frac{0.2 \times 0.4}{0.133} = 0.60$$

(rounding upto 2 decimal place)

49. $F(P, Q, R, S) = \Sigma m(0, 2, 5, 7, 9, 11) + d(3, 8, 10, 12, 14)$



Number of EPI = 3

50.

	IF	ID	OF	PO	WB
Type(I_1)	1	1	1	3	1
Type(I_2)	1	1	1	2	1
Type(I_3)	1	1	1	1	1

$I_1 : 1 + 1 + 1 + 3 + 1 = 7$

$\left.\begin{array}{l} I_2 : \\ I_3 : \end{array}\right\}$ usually takes cycles each due to overlapping but I_1(PO) takes 2 cycles to add "1" cycle extra (2 + 1)

$\Rightarrow$ 7 + 8 + 1 = 10

	1	2	3	4	5	6	7	8	9	10
I_1	IF	ID	OF	PO	PO	PO	WB			
I_2		IF	ID	OF	OF	OF	PO	PO	WB	
I_3			IF	ID	ID	ID	OF	OF	PO	PO

	IF	*ID*	*OF*	*PO*	*WB*
I_1	1	1	1	3	1
I_2	1	1	1	3	1
I_3	1	1	1	2	1
I_4	1	1	1	2	1
I_5	1	1	1	1	1
I_6	1	1	1	1	1

If Type 1 – 2 instruction

Type 2 – 2 instruction

Type 3 – 2 instruction

I_1 : 1 + 1 + 1 + 3 + 1 = 7

I_2 : 1 + 2 extra cycles = 3

I_3 : 1 + 1 extra cycles = 2

I_4 : 1 + 1 extra cycles = 2

I_5 : 1 extra cycles = 1

I_6 : 1 extra cycles = 1

= 16

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
I_1	IF	ID	OF	PO	PO	PO	WB									
I_2		IF	ID	OF	OF	OF	PO	PO	PO	WB						
I_3			IF	ID	ID	ID	OF	OF	OF	PO	PO	WB				
I4				IF	IF	IF	ID	ID	ID	OF	OF	PO	PO	WB		
I_5							IF	IF	IF	ID	ID	OF	OF	PO	WB	
I_6										IF	IF	ID	ID	OF	PO	WB

In this way, if we compute 100 instruction number of cycles required is

If ID OF PO WB

$\Rightarrow$ $1 + 1 + 1 + [(40 \times 3) + (35 \times 2)\ (25 \times 1)] + 1$

$\Rightarrow$ 219

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51. Instruction size = 16 bit

Type I instruction design:

16 bit

opcode	IR	IR	IR
4 bit	4 bit	4 bit	4 bit

Number of operations = 2^4

= 16

↓

Face opcodes = 16 – 4 = 12

Type II instruction design:

16 bit

opcode	FR	FR
4 bit	6 bit	6 bit

Face opcodes after type 1 instruction = 12

Face opcodes after type 2 instruction = 12 – 8 = 4

Type III instruction design:

16 bit

opcode	IR	FR
6 bit	4 bit	4 bit

Expand opcode size = 6 bit

↓

Number of opcodes = $4 \times 2^2 = 16$

∴ Number of free opcodes after type 3 instruction

= 16 – 14 = 2

Type IV instruction design:

16 bit

opcode	*FR*
10 bit	6 bit

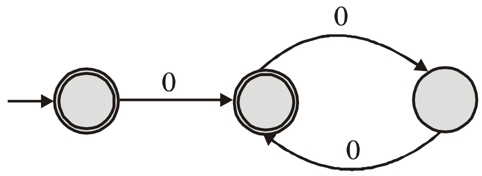
Expand opcodes size = 10 bit

↓

Number of opcodes = $2 \times 2^4 = 2^5 = 32$

52. We need to find smallest value of k which satisfies

$L_1^k = L_1^{k+1}$



$L_1 = \in(00)^*$

Try $k = 0$ $\quad L_1^0 = L_1^1$

$\Rightarrow \quad \in = L_1$ which is false.

So order is not 0.

Try k = 1: $\quad L_1^1 = L_1^2$

$\Rightarrow \quad L_1^2 = L_1$

Now, $\quad L_1^2 = (\in +0(00)^*)(\in +0(00)^*)$

$= \in +0(00)^* + 00(00)^* = 0^*$

Clearly $\quad L_1^2 \neq L_1$

So order is not 1.

Try k = 2: $\quad L_1^2 = L_1^3$

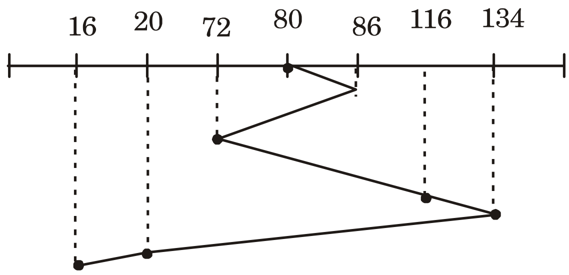
Now, $\quad L_1^3 = L_1^2 . L_1$

$= 0^*(\in +0(00)^*) = 0^*$

Clearly $\quad L_1^3 = L_1^2 = 0^*$

(So order of L_1 is 2)

53.



(86 – 80) + (86 – 72) + (134 – 72)

+ (134 – 16) 62 + 118 + 14

= 200

$100 \rightarrow 20$

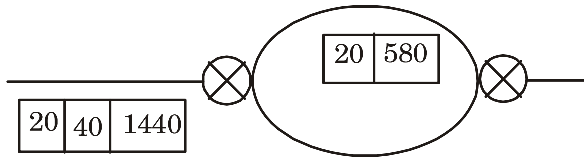
$200 \rightarrow ?$

$$\frac{200}{100} \times 20 = 40$$

3 direction changes $3 \times 15 = 45$

40 + 45 = 85

54.



Fragment Bytes

1st		2nd		3rd		4th	
20	576	20	576	20	576	20	576

Fragment Offset

(0 – 71) (72 – 143) (144 – 215) (216 – 287)

■■

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